University of Technology
Department of Electrical Engineering
Sem.(2) Final Term Examination 2019-2020

Subject: Mathematics 2 | ***Year: 1st***
Division: Electrical Engineering | ***Time: 10 days***
Examiner: Dr. Jasim F. H. | ***End Date: 11/7/2020***

Write A Report

***Note*: write the report with no more than 5 pages: [21 Marks]**

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Q1) a) $I = \int \sec 2t \, dt + \int \frac{\ln(\sec x)}{\cot x} dx$

$\int \sec 2t \, dt = \frac{1}{2} \ln|\sec 2t + \tan 2t|$

$\int \frac{\ln(\sec x)}{\cot x} dx$

نفرض أن :

$u = \ln(\sec x) \Rightarrow du = \frac{\sec x . \tan x}{\sec x} . dx$

$\Rightarrow du = \frac{1}{\cot x} . dx$

إذاً :

$\int \frac{\ln(\sec x)}{\cot x} dx = \int u . du = \frac{u^2}{2}$

$= \frac{1}{2}(\ln|\sec x|)^2$

$I = \frac{1}{2} \ln|\sec 2t + \tan 2t| + \frac{1}{2}(\ln|\sec x|)^2 + C$

Q1) b) $J = \int \frac{\tan y}{\ln(\sec y)}\,dy + \int \ln(6)^{\sec\theta}\,d\theta$

$$\int \frac{\tan y}{\ln(\sec y)}\,dy$$

$u = \ln(\sec y)$ بفرض آن

$\Rightarrow du = \tan y \cdot dy$

$$\Rightarrow \int \frac{\tan y}{\ln(\sec y)}\,dy = \int \frac{du}{u} = \ln|u| = \ln|\ln(\sec y)|$$

$$\int \ln(6)^{\sec\theta}\,d\theta = \ln(6)\int \sec\theta \cdot d\theta = \ln(6)\left(\ln|\sec\theta + \tan\theta|\right)$$

$$J = \ln|\ln(\sec y)| + \ln 6 \cdot \ln|\sec\theta + \tan\theta| + C$$

Note: for this question, I think there is something wrong with it, I couldn't solve it until I replaced the y^2 by y^3.

Q1) $\boxed{C}$ $M = \int \frac{x\, y^3\, dy}{1+y^8} + \int \frac{y t x\, dt}{t+t^3} + \int \frac{x y z^2}{(1-z^3)^2} dz$

$$\int \frac{xy^3}{1+y^8} dy = x \int \frac{y^3}{1+(y^4)^2} dy$$

$$= \frac{x}{4} \int \frac{4y^3}{1+(y^4)^2} dy = \frac{x}{4} \tan^{-1}(y^4) + C$$

$$\int \frac{y t x}{t+t^3} dt = yx \int \frac{1}{1+t^2} dt = yx \tan^{-1}(t^2) + C$$

$$\int \frac{x y z^2}{(1-z^3)^2} dz = \frac{-xy}{3} \int -3z^2 (1-z^3)^{-2} dz$$

$$= \frac{xy}{3(1-z^3)} + C$$

وإذاً يصبح التكامل:

$$M = \frac{x}{4} \tan^{-1}(y^4) + yx \tan^{-1}(t^2) + \frac{xy}{3(1-z^3)} + C$$

Q_1 (d) $K=\int \frac{x^2 . p . \tan^{-1} p^2}{1+p^4} dp + \int \frac{y \sin^{-1} \alpha}{\sqrt{4-\alpha^3}} dy + \int \frac{\sqrt{xy}}{|z| \sqrt{z^2-1}} dz$

الحل:

$\int \frac{x^2 . p . \tan^{-1} p^2}{1+p^4} dp$

نفرض أن:

$u = \tan^{-1} p^2$

$\Rightarrow du = \frac{2p}{1+p^4} dp \Rightarrow \frac{du}{2} = \frac{p}{1+p^4} dp$

بالتعويض يصبح التكامل:

$\int \frac{x^2}{2} u . du = \frac{x^2}{4} u^2 + C = \frac{x^2}{4} (\tan^{-1} p^2)^2 + C$

$\int \frac{y \sin^{-1} \alpha}{\sqrt{4-\alpha^3}} dy = \frac{y^2 \sin^{-1} \alpha}{2\sqrt{4-\alpha^3}} + C$

$\int \frac{\sqrt{xy}}{|z| \sqrt{z^2-1}} dz = \sqrt{xy} . \sec^{-1} |z| + C$

إذاً يصبح التكامل:

$K = \frac{x^2}{4} . (\tan^{-1} p^2)^2 + \frac{y^2 \sin^{-1} \alpha}{2\sqrt{4-\alpha^3}} + \sqrt{xy} . \sec^{-1} |z| + C$

Q2) A ① $\int \sin(\ln e^{\sqrt{\theta}})\, d\theta$

$$= \int \sin\left(\tfrac{1}{2}\ln e^{\theta}\right) d\theta = \int \sin\left(\tfrac{1}{2}\theta\right) d\theta$$

$$= -2\cos\left(\tfrac{1}{2}\theta\right) + C$$

Q2) $\boxed{B}$ ⑥ $\int \frac{t^3\, dt}{\sqrt{t^2+a^2}} = \int \frac{t(t^2+a^2-a^2)}{\sqrt{t^2+a^2}}\, dt$

$$= \int \frac{t(t^2+a^2) - a^2 t}{\sqrt{t^2+a^2}}\, dt = \cancel{\int t\sqrt{t^2+a^2}\, dt - \int \frac{a^2 t}{\sqrt{t^2+a^2}}\, dt}$$

$$= \int \left(t\sqrt{t^2+a^2} - \frac{a^2 t}{\sqrt{t^2+a^2}}\right) dt = \frac{1}{3}(t^2+a^2)^{\frac{3}{2}} - a^2\sqrt{t^2+a^2} + C$$

Q2) $\boxed{C}$ 8

$$\int \frac{\omega^3}{\omega^3-4\omega}\,d\omega = \int \frac{\omega^2}{\omega^2-4}\,d\omega = \int \frac{\omega^2-4+4}{\omega^2-4}\,d\omega$$

$$= \int \left(1+\frac{4}{\omega^2-4}\right) d\omega = \int \left(1+\frac{1}{\omega-2}-\frac{1}{\omega+2}\right) d\omega$$

$$= \omega + \ln|\omega-2| - \ln|\omega+2| + C$$

Q3) a) $x = y^3-5y$ and $x = 3y-y^3$

$L(y) = y^3-5y-(3y-y^3) = 0 \Rightarrow 2y^3-8y=0 \Rightarrow 2y(y^2-4)=0$

$\Rightarrow 2y(y-2)(y+2)=0$

$\Rightarrow y=0$ or $y=2$ or $y=-2$

y	$-\infty$		-2		0		2		$+\infty$
$L(y)$		$-$	0	$+$	0	$-$	0	$+$	

$$\text{Area} = \int_{-2}^{0} L(y)\,dy + \int_{2}^{0} L(y)\,dy = \int_{-2}^{0} (2y^3-8y)\,dy + \int_{2}^{0} (2y^3-8y)\,dy$$

$$= \left[\frac{y^4}{2}-4y^2\right]_{-2}^{0} + \left[\frac{y^4}{2}-4y^2\right]_{2}^{0}$$

$$= 0-\left(\frac{16}{2}-16\right) + 0 - \left(\frac{16}{2}-16\right) = 16$$

Q4) e) $y = 2^x$, $y = x^2 + 4x$

$g(x) = 2^x - (x^2 + 4x) = e^{x\ln(2)} - x^2 - 4x$

$$\text{Area} = \int_{-4}^{0} g(x)\,dx = \int_{-4}^{0} \left(e^{x\ln(2)} - x^2 - 4x\right) dx$$

$$= \left[\frac{1}{\ln(2)} e^{x\ln(2)} - \frac{x^3}{3} - 2x^2\right]_{-4}^{0}$$

$$= \frac{1}{\ln(2)} - \left(\frac{2^{-4}}{\ln(2)} + \frac{64}{3} - 32\right) =$$

$$= \frac{1}{\ln(2)} - \frac{1}{16\ln(2)} - \frac{64}{3} + 32 = \frac{15}{16}\ln(2) + \frac{32}{3}$$