Real Analysis Final 2023 Fall

1. $E_t = \{x \mid |f(x)| \geq t\}$, proof $\int_{\mathbb{R}} |f| = \int_0^{+\infty} m(E_t)\, dt$

2. Exercise 3.19

3. Proof $\sqrt{x}$ absolutely continuous on $[0,1]$

4. Proof 3.13 & 3.14

5. $f(x,y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$, μ defined as

   (i) The first is the discrete example with X a countable set, $X = \{x_n\}_{n=1}^{\infty}$, $\mathcal{M}$ the collection of all subsets of X, and the measure μ determined by $\mu(x_n) = \mu_n$, with $\{\mu_n\}_{n=1}^{\infty}$ a given sequence of (extended) non-negative numbers. Note that $\mu(E) = \sum_{x_n \in E} \mu_n$. When $\mu_n = 1$ for all n, we call μ the **counting measure**, and also denote it by $\#$. In this case integration will amount to nothing but the summation of (absolutely) convergent series.

   m defined as Lesbegue measure

   Compute $\int_X \int_Y f\, d\mu\, dm$ and $\int_Y \int_X f\, dm\, d\mu$ with $X \times Y = [0,1] \times [0,1]$