HW 6.

习题10. P264-265 1, 3, 6, 7, 8.

1. 证明.

因为 $C_j = v b_{j+1} I_{\{K(x)=j\}} - \pi_j I_{\{K(x) \geq j\}}$,

$C_n = v b_{n+1} I_{\{K(x)=n\}} - \pi_n I_{\{K(x)\geq n\}}$,

所以 $E(C_j) = v b_{j+1}\, {}_jp_x\, q_{x+j} - \pi_j\, {}_jp_x$

$E(C_n) = v b_{n+1}\, {}_np_x\, q_{x+n} - \pi_n\, {}_np_x$

对 $j<n$

$$E(C_jC_n) = E\big(v^2 b_{j+1} b_{n+1} I_{\{K(x)=j\}} I_{\{K(x)=n\}} - \pi_j I_{\{K(x)\geq j\}} v b_{n+1} I_{\{K(x)=n\}}$$
$$- v b_{j+1} I_{\{K(x)=j\}} \pi_n I_{\{K(x)\geq n\}} + \pi_j \pi_n I_{\{K(x)\geq j\}} I_{\{K(x)\geq n\}}\big)$$
$$= 0 - \pi_j v b_{n+1}\, {}_np_x\, q_{x+n} - 0 + \pi_j \pi_n\, {}_np_x$$

所以 $\mathrm{cov}(C_j, C_n) = E(C_jC_n) - E(C_j)E(C_n)$

$$= \pi_j \pi_n\, {}_np_x - \pi_j v b_{n+1}\, {}_np_x\, q_{x+n} - (v b_{j+1}\, {}_jp_x\, q_{x+j} - \pi_j\, {}_jp_x) \times (v b_{n+1}\, {}_np_x\, q_{x+n} - \pi_n\, {}_np_x)$$

$$= {}_np_x \pi_j (\pi_n - v b_{n+1} q_{x+n}) + {}_jp_x (v b_{j+1} q_{x+j} - \pi_j)\, {}_np_x (\pi_n - v b_{n+1} q_{x+n})$$

$$= {}_np_x (\pi_n - v b_{n+1} q_{x+n}) (\pi_j + v b_{j+1} q_{x+j}\, {}_jp_x - \pi_j\, {}_jp_x)$$

$$= {}_np_x (\pi_n - v b_{n+1} q_{x+n}) (\pi_j\, {}_jp_x + v b_{j+1}\, {}_jp_x\, q_{x+j}).$$

5. 解：由 ${}_2V = {}_3V - \pi_n = 1.56$ 得 $i = 0.2$

由定义有 $b_n = 3$，$\pi_n = 0.94$，${}_0V = 0$，${}_1V = 0.66$，${}_2V = 1.56$，${}_3V = 3$.

由 ${}_1V + \pi_n = v b_2 q_{x+1} + v\, {}_2V p_{x+1}$ 得 $q_{x+1} = 0.25$

由 ${}_0V + \pi_n = v b_1 q_x + v\, {}_1V p_x$ 得 $q_x = 0.2$

故 $var({}_0L) = var({}_0L \mid K(x) \geq 0) = \sum_{j=0}^{2} v^{2j} [v(b_{j+1} - {}_{j+1}V)]^2 \, {}_jp_x \, p_{x+j} \, q_{x+j}$

$= 0.7584$

$var({}_1L \mid K(x) \geq 1) = \sum_{j=1}^{2} v^{2(j-1)} [v(b_{j+1} - {}_{j+1}V)]^2 \, {}_{j-1}p_{x+1} \, p_{x+j} \, q_{x+j}$

$= v^2 (b_2 - {}_2V)^2 p_{x+1} q_{x+1} = 0.27$

6. E

$${}_{10}V = 1000 A_{x+10} = \frac{({}_9V + P) \times 1.06 - q_{x+9} \times 1000}{p_{x+9}} = 369.13$$

$$\ddot{a}_{x+10} = \frac{1 - 0.36913}{\frac{0.06}{1.06}} = 11.14537 \Rightarrow 1000 P_{x+10} = \frac{1000 A_{x+10}}{\ddot{a}_{x+10}} \approx 33.12$$

7. E. $var(\pi_1 + {}_1V) = [v(b_1 - {}_1V)]^2 p_{50} q_{50} = 358664.09$

$var(\pi_2 + {}_2V) = [v(b_2 - {}_2V)]^2 p_{51} q_{51} = 101015.09$

$var(\pi_3 + {}_3V) = 0$

$\Rightarrow var({}_0L) = var(\pi_1 + V_1) + v^2 var(\pi_2 + {}_2V) \approx 454000$

8 C. ${}_{20}V = 0$，${}_1V = \dfrac{({}_0V + P) \times 1.07 - (1000 + {}_1V) \times q}{1 - 0.03}$

$\Rightarrow {}_1V = 1.07P - 1000 \times 0.03$ ∵ ${}_{20}V = 0$

${}_2V = 1.07^2 P - 1000 \times 0.03 \times (1.07 + 1)$ ∴ $P = 317.82 \approx 318$

${}_3V = 1.07^3 P - 1000 \times 0.03 (1.07^{19} + 1.07^{18} + \cdots + 1)$