南方科技大学
SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY

**Course Name:** MA-127, Calculus II A **Dept.:** Mathematics
**Exam Duration:** 120 minutes **Exam Paper Setters:** V. Didenko & Xuli Han

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
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| Score | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 | 10 |

This exam paper contains 10 problems and the score is 100 Marks in total.

1. Determine whether the following limits exist:

$$(a) \qquad \lim_{(x,y)\to(1,2)} \frac{\ln(1-x+xy)}{x-1}.$$

$$(b) \qquad \lim_{(x,y)\to(0,0)} (x+y)^{x+y+xy}.$$

2. Let $f$ be the function defined by

$$f(x,y)=\begin{cases} \dfrac{2x+1}{x^2+y^2}\sin(x^2+y^2) & \text{if} \quad (x,y)\neq(0,0), \\ 1 & \text{if} \quad (x,y)=(0,0). \end{cases}$$

Determine if the function $f$ is continuous at the origin.

3. Let $\mathbf{n}$ be the normal unit vector pointing inside the surface $3x^2+y^2+z^2=3$. Compute the directional derivative of the function

$$f(x,y,z)=\frac{\sqrt{x^2+y^2+z^2}}{(y+z+1)^2}$$

at the point $(1,0,0)$ in the direction $\mathbf{n}$.

4. Find the equations of the tangent plane and the normal line for the surface

$$xy+z+2^{xy}=4$$

at the point $(1,1,1)$.

5. Use the Lagrange multipliers to find the minimal and maximal values of the function

$$f(x,y,z)=x^{\frac{5}{2}}+y^{\frac{5}{2}}+z^{\frac{5}{2}}$$

on the sphere $x^2+y^2+z^2=1$.

6. Compute the integral

$$I=\iint_D x^2y^2\,dxdy,$$

where $D$ is the plane domain bounded by the curves $x=1/2$ and $y^2=2x$.

7. Determine the area of the region bounded by the curves $r=\sin\theta$ and $r=\cos\theta$.

8. Find the volume of the solid bounded by the surfaces $S_1$ and $S_2$,

$$S_1 := \{(x,y,z) : x^2 + y^2 + 4z^2 = 9,\ z \geq 0\},$$
$$S_2 := \left\{(x,y,z) : z = \sqrt{x^2+y^2}\right\}.$$

9. Determine the work done by the vector field

$$\mathbf{F} = e^{y+2z}(\mathbf{i} + x\mathbf{j} + 2x\mathbf{k})$$

along the curve of the intersection of the surfaces $x^2 + 2y^2 + 3z^2 = 3$ and $x + y + z = 0$ joining the points $A(1,-1,0)$ and $B(-1,1,0)$.

10. Calculate the circulation of the vector field

$$\mathbf{F} = yz^2\mathbf{i} + 2xz^2\mathbf{j} + xyz\mathbf{k}$$

along the curve of intersection of the sphere $x^2 + y^2 + z^2 = 1$ and the cone $z = \sqrt{x^2+y^2}$ traversed in the counterclockwise direction around the $z$-axis when viewed from above.