Q1

a) $$\lim_{(x,y)\to(1,2)} \frac{\ln(1-x+xy)}{x-1}$$

$$= \lim_{(x,y)\to(1,2)} \frac{\ln(2)}{x-1}$$

as $x \to 1^+$

$$\lim_{(x,y)\to(1,2)} \frac{\ln(2)}{x-1} = \infty$$

as $x \to 1^-$

$$\lim_{(x,y)\to(1,2)} \frac{\ln(2)}{x-1} = -\infty$$

So $\lim_{(x,y)\to(1,2)} \frac{\ln(1-x+xy)}{x-1}$ does not exist

b) $$\lim_{(x,y)\to(0,0)} (x+y)^{x+y+xy} = 0^0 = 0$$

limit exists

Q2

$$f(x,y) = \begin{cases} \dfrac{2x+1}{x^2+y^2} \sin(x^2+y^2) & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{2x+1}{x^2+y^2} \sin(x^2+y^2)$$

$$= \lim_{(x,y)\to(0,0)} \frac{2x+1}{x^2+y^2} \sin(x^2+y^2)$$

$$= \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}(2x+1)$$

$$= \lim_{(x,y)\to(0,0)} 1\,(2x+1)$$

$$= \lim_{(x,y)\to(0,0)} 2x+1 = 1$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0) = 1$$

So, $f(x,y)$ is continuous at origin $(0,0)$

Q3 $\quad 3x^2+y^2+z^3=3$

$$f(x,y,z)=3x^2+y^2+z^3=3$$

$$\vec{\nabla}F = \langle 6x, 2y, 2z\rangle$$

$$\vec{\nabla}F = \sqrt{36x^2+4y^2+4z^2}$$

$$\hat{n} = \frac{\vec{\nabla}F}{|\vec{\nabla}F|} = \frac{1}{\sqrt{36x^2+4y^2+4z^2}}\langle 6x, 2y, 2z\rangle$$

at $(1,0,0)$

$$\hat{n} = \frac{1}{\sqrt{36}}\langle 6,0,0\rangle = \frac{1}{6}\langle 6,0,0\rangle$$

$$= \langle 1,0,0\rangle$$

$$f(x,y,z) = \frac{\sqrt{x^2+y^2+\cancel{z^2}}}{(y+z+1)^2}$$

we need only f_x since $\hat{n}$ is in direction of x

$$f_x = \frac{x}{(y+z+1)^2\sqrt{x^2+y^2+z^2}}$$

$$f_x(1,0,0) = \frac{1}{(1)^2\sqrt{1}} = 1$$

$$\vec{\nabla}f\cdot\hat{n} = \langle 1, f_y, f_z\rangle\cdot\langle 1,0,0\rangle$$

$$= \underline{\underline{1}}$$

Q4

$$xy + z + 2^{xy} = 4 \qquad P(1,1,1)$$

$$F(x,y,z) = xy + z + 2^{xy} = 4$$

$$\vec{\nabla} F = \langle y + 2^{xy} y \ln(2),\ x + 2^{xy} x \ln(2),\ 1 \rangle$$

$$\vec{\nabla} F(1,1,1) = \langle 1 + 2\ln 2,\ 1 + 2\ln 2,\ 1 \rangle$$

Normal vector is in the direction of

$$\vec{\nabla} F(1,1,1) = \langle 1 + 2\ln 2,\ 1 + 2\ln 2,\ 1 \rangle$$

Equation of tangent plane is

$$(1+2\ln 2)(x-1) + (1+2\ln 2)(y-1) + 1(z-1) = 0$$

$$(1+2\ln 2)x + (1+2\ln 2)y = 3 + 4\ln 2$$

Q5

$$f(x,y,z) = x^{5/2} + y^{5/2} + z^{5/2}$$

$$x^2+y^2+z^2=1$$

Lagrangian $L(x,y,z,\lambda) = x^{5/2}+y^{5/2}+z^{5/2}+\lambda(x^2+y^2+z^2-1)$

$$\frac{\partial L}{\partial x} = 2\lambda x + \frac{5x^{3/2}}{2} = 0$$

$$\frac{\partial L}{\partial y} = 2\lambda y + \frac{5y^{3/2}}{2} = 0$$

$$\frac{\partial L}{\partial z} = 2\lambda z + \frac{5z^{3/2}}{2} = 0$$

$$\frac{\partial L}{\partial \lambda} = x^2+y^2+z^2-1 = 0$$

solving yields $(x,y,z) = (0,0,1)$, [illegible]

$f(0,0,1)=1$

take the point $(x,y,z) = \left(\frac{1}{100}, \frac{1}{100}, \frac{\sqrt{9998}}{100}\right)$

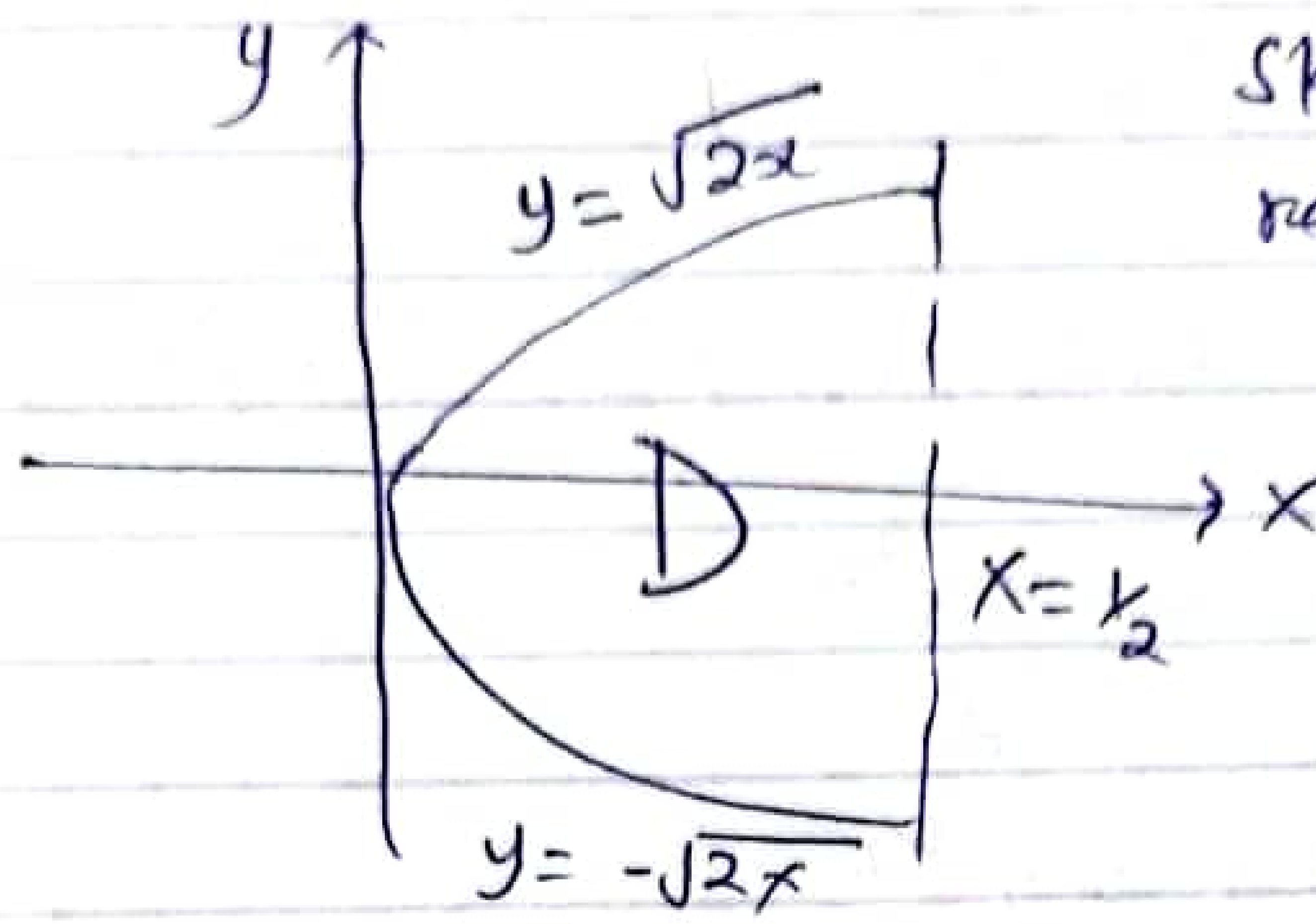
Since $f\left(\frac{1}{100}, \frac{1}{100}, \frac{\sqrt{9998}}{100}\right) = \frac{1}{50000} + \frac{4999\sqrt[4]{9998}}{50000} < 1$

it can be stated that 1 is the maximum

$f(0,0,1) = 1$ is maximum

There is No minimum

Q6 $\iint_D x^2y^2\,dx\,dy \qquad x=\frac{1}{2} \quad y^2=2x$



Sketch region of integration

$$\int_0^{1/2}\int_{-\sqrt{2x}}^{\sqrt{2x}} x^2y^2\,dy\,dx$$

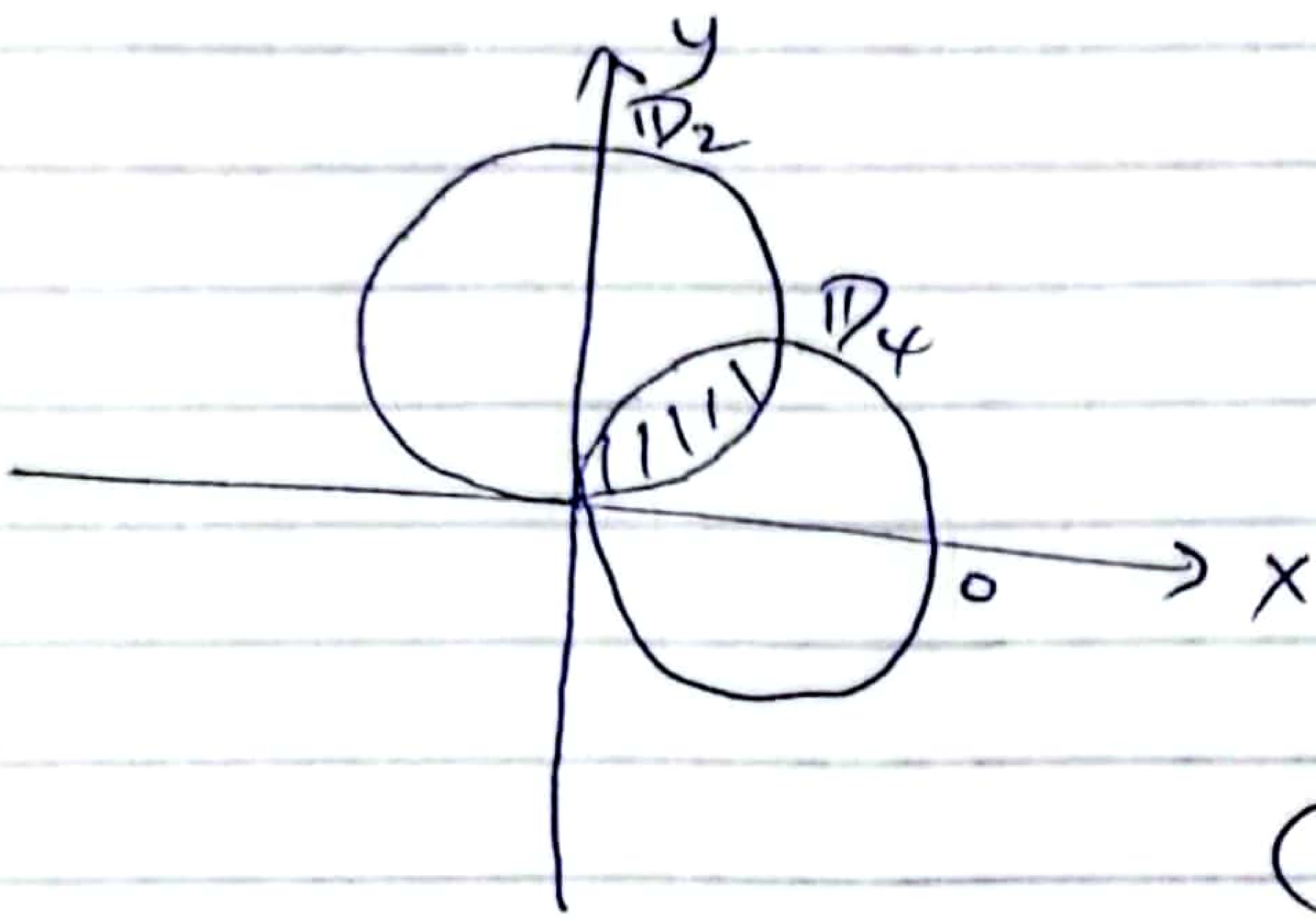
or $$\int_{-1}^{1}\int_{y^2/2}^{1/2} x^2y^2\,dx\,dy = \int_{-1}^{1} \frac{x^3y^2}{3}\Bigg|_{y^2/2}^{1/2} dy$$

$$= \int_{-1}^{1} \frac{y^2-y^8}{24}\,dy$$

$$= \left[\frac{y^3}{72} - \frac{y^9}{216}\right]_{-1}^{1}$$

$$= \frac{1}{54}$$

Q7 $\quad r = \sin\theta \quad r = \cos\theta$



$$r^2 = r\sin\theta$$
$$x^2 + y^2 - y = 0$$
$$x^2 + (y - \tfrac{1}{2})^2 = (\tfrac{1}{2})^2$$

$$r^2 = r\cos\theta$$
$$x^2 + y^2 - x = 0$$
$$(x - \tfrac{1}{2})^2 + y^2 = (\tfrac{1}{2})^2$$

$$\text{Area} = \frac{1}{2}\int_{\pi/4}^{\pi/2} \cos^2\theta \, d\theta + \frac{1}{2}\int_0^{\pi/4} \sin^2 d\theta$$

$$= \frac{1}{4}\int_{\pi/4}^{\pi/2} (1 + \cos(2\theta))\, d\theta + \frac{1}{4}\int_0^{\pi/4} (1 - \cos(2\theta))\, d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{1}{2}\sin(2\theta)\right]_{\pi/4}^{\pi/2} + \frac{1}{4}\left[\theta - \frac{1}{2}\sin(2\theta)\right]_0^{\pi/4}$$

$$= \frac{1}{4}\left[\frac{\pi}{2} - \left(\frac{\pi}{4} + \frac{1}{2}\right)\right] + \frac{1}{4}\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{4} - \frac{1}{2}\right] + \frac{1}{4}\left[\frac{\pi}{4} - \frac{1}{2}\right]$$

$$= \frac{\pi}{8} - \frac{1}{4}$$

Q8 $S_1 := \{(x,y,z) : x^2+y^2+4z^2 = 9,\ z \geq 0\}$

$S_2 := \{(x,y,z) : z = \sqrt{x^2+y^2}$

$$x^2+y^2+4(x^2+y^2) = 9$$

$$x^2+y^2 = 9/5 \qquad \text{radius} = \frac{3}{\sqrt5}$$

limits $\quad r \leq z \leq \dfrac{\sqrt{9-r^2}}{2}$

$$0 \leq r \leq \frac{3}{\sqrt5} \qquad 0 \leq \theta \leq 2\pi$$

$$V = \int_0^{2\pi}\int_0^{\frac{3}{\sqrt5}}\int_r^{\frac{\sqrt{9-r^2}}{2}} r\,dz\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{3}{\sqrt5}} rz\Big]_r^{\frac{\sqrt{9-r^2}}{2}} dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{3}{\sqrt5}} \left(\frac{r}{2}\sqrt{9-r^2} - r^2\right) dr\,d\theta$$

$$= \int_0^{2\pi}\left[\frac{-1}{6}(9-r^2)^{3/2} - r^3/3\right]_0^{\frac{3}{\sqrt5}} d\theta$$

$$= \int_0^{2\pi} \frac{9(\sqrt5-2)}{2\sqrt5}\,d\theta = \frac{9\pi(\sqrt5-2)}{\sqrt5}$$

Q9 $\vec{F} = e^{y+2z}(i + xj + 2xk)$

$$\text{Curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{y+2z} & xe^{y+2z} & 2xe^{y+2z} \end{vmatrix}$$

$$= (2xe^{y+2z} - 2xe^{y+2z})i - (2e^{y+2z} - 2e^{y+2z})j$$

$$+ (\cancel{x}e^{y+2z} - e^{y+2z})k$$

$= \langle 0, 0, 0 \rangle$ so $\vec{F}$ is Conservative

$$\int_C \vec{F} \cdot d\vec{r} = f(B) - f(A)$$

$$\frac{\partial f}{\partial x} = e^{y+2z}, \quad \frac{\partial f}{\partial y} = xe^{y+2z}, \quad \frac{\partial f}{\partial z} = 2xe^{y+2z}$$

$\Rightarrow f(x,y,z) = xe^{y+2z}$ is the potential function

$$\int_C \vec{F} \cdot d\vec{r} = f(-1,1,0) - f(1,-1,0)$$

$$= (-e^{1+0}) - (e^{-1+0})$$

$$= -e - e^{-1} = -e - \frac{1}{e}$$

Work done $= \int_C \vec{F} \cdot d\vec{r} = \underline{\underline{-e - \frac{1}{e}}}$

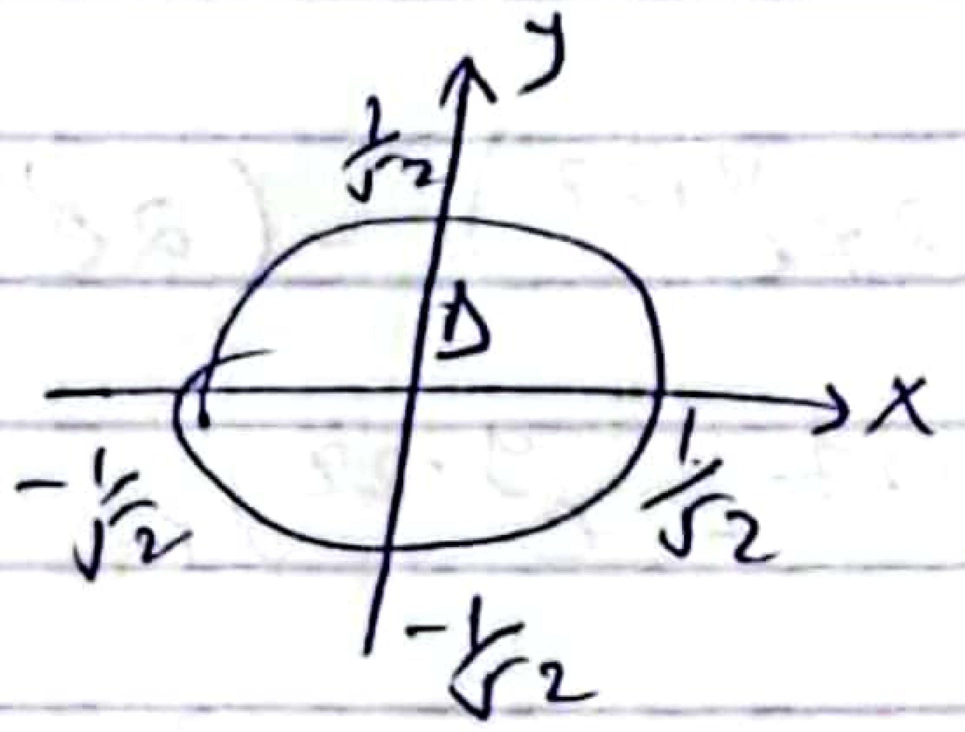
Q10

$$F = yz^2 i + 2xz^2 j + xyz \hat{k}$$

$$x^2+y^2+z^2 = 1, \quad \sqrt{x^2+y^2} = z$$

at point of intersection $x^2+y^2+x^2+y^2 = 1$

$$x^2+y^2 = \frac{1}{2}$$



$$\text{Curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz^2 & 2xz^2 & xyz \end{vmatrix}$$

$$= \langle -3xz, yz, z^2 \rangle$$

normal vector is $\hat{n} = \langle 0, 0, 1 \rangle$

parametric

$$\langle -3xz, yz, z^2 \rangle \cdot \langle 0,0,1 \rangle = z^2$$

at point of intersection $z = \frac{1}{\sqrt{2}}$, $z^2 = \frac{1}{2}$

$$\text{Circulation} = \int_0^{2\pi} \int_0^{\frac{1}{\sqrt{2}}} \frac{1}{2}\, dr\, d\theta = \int_0^{2\pi} \frac{r}{2} \Big|_0^{\frac{1}{\sqrt{2}}} d\theta$$

$$= \int_0^{2\pi} \frac{1}{2\sqrt{2}} \theta \Big]_0^{2\pi} = \frac{2\pi}{2\sqrt{2}} = \frac{\pi}{\sqrt{2}}$$