Ordinary differential equation A

15th April, 14:00-15:50

1. (20 marks)

   (a) Find the general solution of $y' = 4y + x^2 e^{4x}$.

   (b) Solve the initial value problem

   $$y' = \frac{1 + 3x^2}{3y^2 - 6y}, \qquad y(0) = 1,$$

   and determine the interval in which the solution is valid.

2. (30 marks) Solve the following problems.

   (a) $y^{(6)} + y = 0$.

   (b) $y' = \frac{x-y-1}{x+y-2}$.

   (e) $y'' - 3y' - 4y = 2e^{-t}$.

3. (20 marks) Provide $y = t$ is one of the solution for

   $$t^2 y'' - 2ty' + 2y = 0, \; t > 0.$$

   Then find the general solution for

   $$t^2 y'' - 2ty' + 2y = 4t^2, \; t > 0.$$

4. (20 marks) Consider the differential equation $2y\,dx + (x + y)\,dy = 0$.

   (a) Without finding it, prove that the equation has an integrating factor that is a function of y.

   (b) Find the integrating factor and use it to solve (implicitly) the differential equation.

5. (10 marks) Solve the initial value problem

   $$y' = y^{1/4}, \qquad y(0) = 0.$$