

CS201: Discrete Mathematics (Fall 2023)
Written Assignment #1
(100 points maximum but 105 points in total)
Deadline: 11:59pm on Oct 12 (please submit via Blackboard)
PLAGIARISM WILL BE PUNISHED SEVERELY

Q.1 (5p) Consider the following propositions:

p : You get an A on the final.

q : You do all the assignments.

r : You get an A in this course.

Translate the following statements to formulas using p , q , r and logical connectives.

- (a) (1p) You get an A either in this course or on the final.
- (b) (1p) To get an A in this course, it is necessary for you to do all the assignments.
- (c) (1p) You do all the assignments, but you don't get an A on the final; nevertheless, you get an A in this course.
- (d) (1p) If you don't get an A in this course, then you don't get an A on the final or don't do all the assignments.
- (e) (1p) You get an A in this course if and only if you do all the assignments and get an A on the final.

Q.2 (10p) Construct a truth table for each of the following compound propositions:

- (a) (1p) $p \oplus \neg p$
- (b) (2p) $(p \rightarrow q) \wedge (\neg p \leftrightarrow q)$
- (c) (2p) $(p \oplus q) \rightarrow (p \vee \neg q)$
- (d) (5p) $(p \rightarrow \neg q) \leftrightarrow (r \rightarrow (p \vee \neg q))$

Q.3 (15p) Use logical equivalences to prove the following statements.

- (a) (4p) $\neg(p \rightarrow q) \rightarrow p$ is a tautology.
- (b) (3p) $(p \wedge \neg q) \rightarrow r$ and $p \rightarrow (q \vee r)$ are equivalent.
- (c) (8p) $(p \rightarrow q) \rightarrow ((r \rightarrow p) \rightarrow (r \rightarrow q))$ is a tautology.

Q.4 (10p) Determine whether or not the following pairs of statements are logically equivalent, and explain your answer. (Truth tables are not necessary if your explanation is clear.)

- (a) (2p) $p \oplus q$ and $\neg p \vee \neg q$
- (b) (2p) $\neg q \wedge (p \leftrightarrow q)$ and $\neg p$
- (c) (3p) $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$

(d) **(3p)** $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$

Q.5 **(5p)** Determine for which values of p, q, r the statement $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true and for which values it is false, and explain your reasoning. Do not use a truth table.

Q.6 **(5p)** Prove that if $p \rightarrow q, \neg p \rightarrow \neg r, s \vee r$, then $q \vee s$.

Q.7 **(5p)** Prove that if $p \wedge q, q \rightarrow \neg(p \wedge r), s \rightarrow r$, then $\neg s$.

Q.8 **(5p)** Let $P(x)$ be the statement “ x can speak Russian” and let $Q(x)$ be the statement “ x knows the computer language C++”. Express each of these sentences in terms of $P(x), Q(x)$, quantifiers, and logical connectives. The domain for quantifiers consists of all students at SUSTech.

(a) **(1p)** There is a student who either can speak Russian or knows C++.

(b) **(1p)** There is a student who can speak Russian but who doesn’t know C++.

(c) **(1p)** Every student can speak Russian and knows C++.

(d) **(1p)** No student can speak Russian or knows C++.

(e) **(1p)** If a student can speak Russian then he/she does not know C++.

Q.9 **(8p)** Let $L(x, y)$ be the statement “ x loves y ”, where the domain for both x and y consists of all people in the world. Translate the following statements to quantified formulas. (Hint: you can use $=, \neq$ to connect variables.)

(a) **(1p)** Everybody loves somebody.

(b) **(2p)** There is someone who loves only himself or herself but no other person.

(c) **(5p)** There are exactly two people whom Lynn loves.

Q.10 **(5p)** Express the negations of each of the following statements such that all negation symbols *immediately precede* predicates.

(a) **(1p)** $\exists z \forall y \forall x T(x, y, z)$

(b) **(2p)** $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$

(c) **(2p)** $\forall x \exists y (P(x, y) \rightarrow Q(x, y))$

Q.11 **(10p)** Consider this argument: “All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners.” Answer the following questions.

(a) **(3p)** Define the predicates and translate each sentence to a quantified formula.

(b) **(7p)** Show the formal proof steps and explain which rule of inference is used for each step.

Q.12 **(5p)** Prove that $\sqrt[3]{2}$ is irrational.

Q.13 **(5p)** Prove that there is an irrational number between every two distinct rational numbers. (Hint: you can use the in-class learned fact that $\sqrt{2}$ is irrational.)

Q.14 (12p) Prove that all integral solutions to the equation

$$\frac{1}{m} + \frac{1}{n} = \frac{1}{e} + \frac{1}{2}$$

such that $m, n \geq 3$ and $e > 0$ are in this table:

m	n	e
3	3	6
3	4	12
3	5	30
4	3	12
5	3	30

(Hint: Use proof by cases.)