HW7

习题11. P279-280 1 (1) (2) (3*), 2, 3, 7, 8, 9.

1. 解: (1) 有 ${}_{17}V = 1000A_{85:\overline{3}|} - 55.62\ddot{a}_{85:\overline{3}|} = 739.65$

$${}_{2}V = 1000P_{68}\ddot{s}_{68:\overline{3}|} - {}_{2}k_{68}1000$$

$$= 1000\left(1 - \frac{\ddot{a}_{70}}{\ddot{a}_{68}}\right) = 63.01$$

(2) 由 ${}_5V + \pi_5 = vb_6 \cdot q_{73} + v_{\#6}V p_{73}$, 得

$$\pi_5 = \frac{1}{1.05}(1000 - 292) \times 0.0433 + \frac{1}{1.05} \times 292 - 157 = 150.2918$$

(3) 由均衡保费:

$$1000P_{68}\ddot{a}_{68:\overline{3}|} + v^5{}_5p_{68}(\pi_5 - 1000P_{68})$$

$$= 1000A_{68:\overline{3}|} + (b_{16} - 1000)v^{16}{}_{15}p_{68}$$

得 $b_{16} = 6303.26$.

因为 $\ddot{a}_{68} = \ddot{a}_{68:\overline{9}|} + v^9{}_9p_{68}\ddot{a}_{77}$, ${}_9p_{68} = \frac{l_{77}}{l_{68}}$

得 $b_{\#} = \ddot{a}_{68:\overline{9}|} = 6.59$

故第9个保单年度末,保费累计的精算终值为:

$$[55.62 \cdot \ddot{a}_{68:\overline{9}|} + (1010 - 55.62){}_5p_{68}v^5]/{}_9E_{68} = 1146.579$$

因为 $d\ddot{a}_{68:\overline{9}|} + A_{68:\overline{9}|} = d\ddot{a}_{68:\overline{9}|} + A'_{68:\overline{9}|} + {}_9E_{68} = 1$

所以 $1000\,A'_{68:\overline{9}|}/{}_9E_{68} = 546.967$

因此 $_9V = 1146.579 - 546.967 = 599.612$

2. 解 由 $\begin{cases} {}_2V_x = 1 - \dfrac{\ddot{a}_{x+2}}{\ddot{a}_x} = 0.5 \\ \ddot{a}_{x+2} = 1.1 \end{cases}$ $\Rightarrow \ddot{a}_x = 2.2$

又由 $\ddot{a}_x = \dfrac{1}{P_x + d}$，$P_x = \dfrac{4}{11} \Rightarrow d = \dfrac{1}{11}$ 即 $i = 0.1$

3. 解 因为 $_{10}V_{25} = 1 - \dfrac{\ddot{a}_{25}}{\ddot{a}_{25}} = 0.1$，所以 $\dfrac{\ddot{a}_{25}}{\ddot{a}_{25}} = 0.9$.

因为 $_{10}V_{25} = 1 - \dfrac{\ddot{a}_{45}}{\ddot{a}_{35}} = 0.2$ 所以 $\dfrac{\ddot{a}_{45}}{\ddot{a}_{35}} = 0.8$

有 $\dfrac{\ddot{a}_{45}}{\ddot{a}_{25}} = 0.8 \times 0.9 = 0.72$

所以 $_{20}V_{25} = 1 - \dfrac{\ddot{a}_{45}}{\ddot{a}_{25}} = 0.28$

7. B. 有 $_1V + P = 3vq_{x+1} + {}_2V \cdot vp_{x+1}$

$\Rightarrow (0.898 + 0.834) = \dfrac{3}{1.1} q_{x+1} + \dfrac{1.893}{1.1}(1 - q_{x+1})$

$\Rightarrow q_{x+1} = 0.011$

8. E, $_0L = 3V - 3P_{x:\overline{3}|} = 3\times\frac{1}{1.1} - 0.839 = 1.89$

9. A. 由于知 $q_{x+1} = 0.011$ 及 $p_{x+1} = 0.989$

$$\begin{aligned}\mathrm{Var}[{}_0L \mid K(x)\geq 1] &= \mathrm{Var}[\Lambda_1 \mid K(x)\geq 1] + v^2 p_{x+1}\,\mathrm{Var}[{}_2L \mid K(x)\geq 2] \\ &= v^2(b_2 - {}_2V)^2 p_{x+1} q_{x+1} + 0 = 0.011\end{aligned}$$