some hints on Q1 (e)-(f), Assignment 2

The fundamental gradient inequality plays an important role under the case that f is convex.
It's easy to check the following key inequality:

$$\frac{\alpha L_f}{2k}\left\|x^*-x^p\right\|^2 \geq f(x^p)-f^* \geq f(x^{2p})-f^*+\frac{\beta}{2\alpha^2 L_f}\sum_{n=p}^{2p-1}\left\|\nabla f(x^n)\right\|^2 \geq \frac{\beta p}{2\alpha^2 L_f}\min_{n=0,1,\cdots,2p}\left\|\nabla f\left(x^n\right)\right\|^2.$$

On the other hand we check the sequence $\left\{\left\|\nabla f\left(x^k\right)\right\|\right\}_{k\geq 0}$ is nonincreasing by two claims:

1. recall the Baillon-Haddad theorem;
2. a simple relation:

$$\langle a-b, a\rangle \geq \|a-b\|^2 \Rightarrow \|a\|^2 \geq \|b\|^2.$$