HW4.

P218-233. 1, 2, 3, 4, 6, 7, 8, 9, 10*, 13

14*, 15, 17, 19, 20, 23.

1. 证明: $\bar{P}(\bar{A}_x) = \dfrac{\bar{A}_x}{\bar{a}_x} \overset{\delta=0}{=\!=} \dfrac{1}{\bar{a}_x} = 1\Big/\int_0^\infty v^t\,{}_tp_x\,dt = 1\Big/\int_0^\infty {}_tp_x\,dt = \dfrac{1}{\mathring{e}_x}$

2. 证明: 利用 $\dfrac{d\bar{a}_x}{dx} = (\mu(x)+\delta)\bar{a}_x - 1$

$$\frac{d\bar{A}_x}{dx} = (\mu(x)+\delta)\bar{A}_x - \mu(x), \quad \bar{P}(\bar{A}_x) = \bar{A}_x/\bar{a}_x$$

有. $\left(1+\dfrac{d\bar{a}_x}{dx}\right)\bar{P}(\bar{A}_x) = (\mu(x)+\delta)\,\bar{a}_x \times \dfrac{\bar{A}_x}{\bar{a}_x} = (\mu(x)+\delta)\,\bar{A}_x = \dfrac{d\bar{A}_x}{dx} + \mu(x)$

所以. $\left(1+\dfrac{d\bar{a}_x}{dx}\right)\bar{P}(\bar{A}_x) - \dfrac{d}{dx}\bar{A}_x = \mu(x)$

3. 解: 记 $T_i(x)$ 为第 i 个个体的未来生存时间.

$$L_i = 1000\,v^{T_i(x)} - 3.5\,\bar{a}_{\overline{T_i(x)}|} = 1070\,v^{T_i(x)} - 70$$

则有 $L_{Agg} = \sum_{i=1}^{100} E(L_i) = 100 \times 1070 \times \bar{A}_x - 100 \times 70 = -580$

$$Var(L_{Agg}) = 100\,Var(L_i) = 100 \times 1070^2\left({}^2\bar{A}_x - \bar{A}_x^2\right) = 45796$$

6. 解: $A_{35} = 1 - d\ddot{a}_{35} = 0.12872$

$$P(\bar{A}_{35}) = \frac{\bar{A}_{35}}{\ddot{a}_{35}} = \frac{i}{\delta}\frac{A_{35}}{\ddot{a}_{35}} = 0.00861$$

7. 解：由 $A^{1}_{x:\overline{2}|} = vq_x + v^2 p_x q_{x+1} = 0.2358$

$\ddot{a}_{x:\overline{2}|} = 1 + vp_x = 1.81$

$\Rightarrow$ 年净保费 $P = A^{1}_{x:\overline{2}|} / \ddot{a}_{x:\overline{2}|} \approx 0.1303$

$E(L) = 0 \Rightarrow E(L^2) = (v-P)^2 q_x + (v^2 - P - vP)^2 p_x q_{x+1} + (1+v)^2 P^2 p_x p_{x+1} \approx 0.1603$

8. 解：设每年保费为 P，则 $E(L) = A_x - P\ddot{a}_x = A_x - P \cdot \frac{1-A_x}{d}$

由 ~~var~~ $Var(L) = \left(1+\frac{P}{d}\right)^2 \left({}^2A_{49} - (A_{49})^2\right) = 0.1$

$\Rightarrow P = 0.03679$ & $E(L) = -0.2545$

9. 解：由 ${}_{k|}q_x = \frac{0.9^{k+1}}{9}$ $(k=0,1,2,\cdots)$

得 ${}_k p_x = \sum_{j=k}^{\infty} {}_{j|}q_x = 0.9^k$ $(k=0,1,2,\cdots)$

所以 $p_{x+k} = e^{-\mu(x+k)} = \frac{{}_{k+1}p_x}{{}_k p_x} = 0.9 \Rightarrow \mu(x+k) = -\ln 0.9$

故对 $\forall t$, $\mu(x+t) = -\ln 0.9 \Rightarrow A_x = \sum_{k=0}^{\infty} v^{k+1} {}_{k|}q_x = \frac{5}{9}$

$\ddot{a}_x = \frac{1-A_x}{d} = 6 \Rightarrow P_x = \frac{A_x}{\ddot{a}_x} = \frac{5}{54}$

又在常数死亡力假设下，$\bar{P}(\bar{A}_x) = \mu = -\ln 0.9$

$\Rightarrow 1000\left(\bar{P}(\bar{A}_x) - P_x\right) = 1000\left(-\ln 0.9 - \frac{5}{54}\right) \approx 12.7674$

10*. 解：(1) $E(L) = A_x - P\ddot{a}_x = A_x - 0.05 \times \frac{1-A_x}{d} = -0.23$

$var(L) = 2.05^2({}^2A_x - (A_x)^2) = 0.1681$

(2) $E(L_{Agg}) = 135E(L) + 10 \times 10E(L) = -54.05$

$var(L_{Agg}) = 135\,var(L) + 10 \times 10^2\,var(L) = 190.7935$

(3) $P(L_{Agg} > 45) = P\left(\frac{L_{Agg} - E(L_{Agg})}{\sqrt{var(L_{Agg})}} > \frac{45 + 54.05}{\sqrt{190.7935}}\right)$

$\approx 1 - \Phi(7.17)$

13. 解 $\begin{cases} A_{45} = A^{1}_{45:\overline{15}|} + A_{45:\overline{15}|}^{\ \ \ 1} \times A_{60} \\ A^{1}_{45:\overline{15}|} + A_{45:\overline{15}|}^{\ \ \ 1} = A_{45:\overline{15}|} \end{cases}$

同除 $\ddot{a}_{45:\overline{15}|}$

$\Rightarrow \begin{cases} {}_{15}P_{45} = P^{1}_{45:15} + P_{45:\overline{15}|}^{\ \ \ 1} \times A_{60} \\ P^{1}_{45:15} + P_{45:\overline{15}|}^{\ \ \ 1} = P_{45:\overline{15}|} \end{cases}$

$\Rightarrow P^{1}_{45:\overline{15}|} = 0.008, \quad P_{45:\overline{15}|}^{\ \ \ 1} = 0.048.$

14*. B.

$L = v^{k+1} - \sum_{j=0} 1.04^j P v^j I_{\{k>j\}} = v - P\sum_{j=0} I_{\{k>j\}}$

$E(L) = v - P\sum_{j=0} {}_jP_x = v - P(1 + \sum_{j=1} {}_jP_x) = v - P(1+e_x) = 0$

$\Rightarrow P = \frac{v}{1+e_x}$

15, A.

$$E(L) = A_x - \pi_x \frac{1-A_x}{d} = -0.08$$

$$var(L) = \left(1+\frac{\pi_x}{d}\right)^2 ({}^2A_x - A_x^2) = 0.0496$$

$$P(L_{agg} > 0) = P\left(\frac{nL+0.08n}{\sqrt{0.0496n}} > \frac{0.08n}{\sqrt{0.0496n}}\right) = 1-\Phi\left(\frac{0.08n}{\sqrt{0.0496n}}\right) < 0.05$$

$\Rightarrow n > 20.97$ 取 21.

17. B.

$$\delta = \ln(i+1) = \ln(0.04+1) = 0.039, \quad d = \frac{i}{1+i} = \frac{0.04}{1+0.04} = \frac{1}{26}$$

$$\bar{A}^1_{x:\overline{n}|} = \bar{A}_{x:\overline{n}|} - {}_nE_x = 0.804 - 0.6 = 0.204$$

$$A^1_{x:\overline{n}|} = \frac{\delta}{i}\bar{A}^1_{x:\overline{n}|} = \frac{0.039}{0.04} \times 0.204 = 2\times10^{-3}$$

$$A_{x:\overline{n}|} = \bar{A}_{x:\overline{n}|} - \frac{i-\delta}{\delta} A^1_{x:\overline{n}|} = 0.804 - \frac{0.04-0.039}{0.039} \times 2\times10^{-3} = 0.8$$

$$\ddot{a}_{x:\overline{n}|} = \frac{1-A_{x:\overline{n}|}}{d} = \frac{1-0.8}{1/26} = 5.2$$

$$1000P(\bar{A}_{x:\overline{n}|}) = 1000\,\frac{\bar{A}_{x:\overline{n}|}}{\ddot{a}_{x:n}} = 1000\,\frac{0.804}{5.2} = 154.6$$

19. C.

$$d = \frac{i}{i+1} = \frac{0.05}{0.05+1} = \frac{1}{21}, \quad \ddot{a}_{49} = \frac{1-A_{49}}{d} = \frac{1-0.29224}{1/21} = 14.86$$

$$E(L) = A_{49} - \pi\ddot{a}_{49} = 0.29224 - 14.86 \times 0.036 = -0.25$$

20. C.

$$ {}_{47}p_{30} = \frac{l_{77}}{l_{30}} = 0.508 $$

$$ {}_{48}p_{30} = \frac{l_{78}}{l_{30}} = 0.477 $$

$$ d = \frac{i}{i+1} = \frac{0.06}{0.06+1} = \frac{3}{53} $$

$$ \ddot{a}_{30} = \frac{1-A_{30}}{d} = 36.8 $$

23. B.

$$ \pi = \frac{\bar{A}_x}{\bar{a}_x} = \frac{1-\bar{a}_x\delta}{\bar{a}_x} = \frac{1-5\times 0.08}{5} = 0.12 $$

$$ \pi^* = 0.12 \times 1.25 = 0.15 $$

$$ E(L^*) = \bar{A}_x - 1.25\pi\bar{a}_x - 0.25\pi\bar{a}_x = -0.15 $$

$$ Var(L^*) = Var(L)\frac{(1+\frac{\pi^*}{\delta})^2}{(1+\frac{\pi}{\delta})^2} = 0.5625 \times \frac{(1+\frac{0.15}{0.08})^2}{(1+\frac{0.12}{0.08})^2} = 0.7439 $$

$$ E(L^*) + \sqrt{Var(L^*)} = -0.15 + \sqrt{0.7439} = 0.71 $$