Question 1 (10 points)

- Partial Credit:
  - This question is only based on the **answer**, not the process.
  - **No partial credit** is available for the process.
- Grading Criteria:

  (1) Yes (or other positive answers, **5 points**)

  (2) No (or other negative answers, **5 points**)
- Explanation:
  - Illegible or intentionally confusing handwriting will be considered incorrect and receive **0 point**.

Question 2 (10 points)

- Partial Credit:
  - Proving $|A| = |B|$ part earns **2 points**.
  - Proving $|A| < |B|$ part earns **4 points**.
  - Proving $|A| > |B|$ part earns **4 points**.
- Grading Criteria:
  - **Exampling is not the proof!** The example will be ignored if it is a proof question.
  - Deductions may be made for the use of **imprecise or flawed methods of proof**, such as "obvious," "easily seen," "definitely," or simple drawings without explanation.
  - Deductions may be made for **errors** in conditions, reasoning, or conclusions during the proof.

Question 3 (10 points)

- Partial Credit:
  - Correctly writing $(B - A) = B \cap \overline{A}$ (**2 points**) and $(C - A) = C \cap \overline{A}$ (**2 points**).
  - Expanding and calculating or directly applying set identity theorems to get $(B \cup C) \cap \overline{A}$ (**4 points**).
  - Using definitions to derive the conclusion $(B \cup C) - A$ (**2 points**).
- Grading Criteria:
  - If it is not proven **using the set identities based method**, the **maximum score** should not exceed **5 points**.

Question 4 (10 points)

- 1 point for **each** correct answer.

Question 5 (10 points)

- Partial Scores for Completing the Process:

1. By definition

   $|A| \leq |B|$ means that there is (**1 point**) a injective function f: $A \to B$ (**1 point**)

   $|B| = |C|$ means that there is (**1 point**) a bijective function g: $B \to C$ (**1 point**)

2. Then, by definition we need to show there is a injective function from A to C(**1 point**). It suffices to show the composition $g \circ f$ (**1 point**) is injective: i.e., for any $x, y \in A$ such that $x \neq y$, we have $g \circ f(x) \neq g \circ f(y)$.

3. The above holds because f and g are both injective:

   f injective: for any $x, y \in A$ such that $x \neq y$, we have $f(x) \neq f(y)$ (**1 point**)

   g injective: for any $x, y \in B$ such that $x \neq y$, we have $g(x) \neq g(y)$ (**1 point**)

4. So, for $x, y \in A$ such that $x \neq y$, we have $f(x) \neq f(y)$ with $f(x), f(y) \in B$, and hence $g(f(x)) \neq g(f(y))$, i.e., $g \circ f(x) \neq g \circ f(y)$. (**2 point**)

- General Scoring Criteria:
  - Process 1:

    Mention there is a injective (one to one) function from $A \to B$, give **2 point**. (incomplete/inaccurate statement give **1 point**, otherwise **0 point**)

    Mention there is a bijective (injective) function from $B \to C$, give **2 point**. (incomplete/inaccurate statement give **1 point**, otherwise **0 point**)

  - Process 2:

    If mentioned there is a injective function from A to C, give **1 point**. If mentioned composition $g \circ f$ is injective and correctly write the term of composition, give **1 point** (otherwise, **0 point**)

  - Process 3 and 4:

    Contrapositive statement derive by definition, give **4 point**. Incomplete/inaccurate statement give **1-3 point**, otherwise **0 point**.

  - Other way to prove (**discretionary grading**).

Contact

If you have any questions, contact corresponding TA:

Q1-3: 12112910 罗嘉诚

Q4-5 and Grade computation: 12332414 李昊洋