

Find the first three $n \geq 1$ such that $6n+1$, $12n+1$ and $18n+1$ are all prime

(a) if $6n+1$, $12n+1$ and $18n+1$ are prime for a $n \geq 1$

prove $N = (6n+1)(12n+1)(18n+1)$ is a carmichael number ($a^{N-1} \equiv 1 \pmod{N}$) for all a that are relatively prime to N

(b) let $n \geq 1$ be an odd integer. let x, y, z be integers such that $\gcd(x, y) = 1$ $xy = z^n$
 prove there exist integers a, b such that $x = a^n, y = b^n$

(c) prove the only integral solution to $y^2 - y = x^3$ are $(0, 0), (0, 1)$

let $a > 1, b > 1$ be two relatively prime integers

(a) prove no exist no negative integers x, y $xa + yb = ab - a - b$

(b) proof for any $N > ab - a - b$ there are not negative integer x, y $xa + yb = N$

let p be an odd prime

$$\sum_{a=0}^{p-1} \left(\frac{aa+1}{p} \right), \quad \sum_{a=0}^{p-1} \left(\frac{a^2+1}{p} \right)$$

$\left(\frac{\cdot}{p} \right)$ is the legendre symbol