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Homework 1

~~1. If $c' < x_1$, f~~

1. Suppose $f_j(c) = \sum_{i=j}^{2n-1-j} |x_i - c|$, we need to ~~ad~~ minimize $f_1(c)$.

If $c' < x_1$, $f_1(c') = \sum_{i=1}^{2n-1} (x_i - c') > f_1(x_1)$

Similarly, $c' > x_{2n-1}$, $f_1(c') > f_1(x_{2n-1})$

So $x_1 \le c \le x_{2n-1}$, then $|x_1 - c| + |x_{2n-1} - c| = x_{2n-1} - x_1$ is a constant.

$f_1(c) = x_{2n-1} - x_1 + f_2(c)$, now we need to minimize $f_2(c)$.

Iterate this process for n-1 times, we get we need to minimize $|x_n - c|$, then we take $c = x_n$.

Hence, $\arg\min_c \sum_{i=1}^{2n-1} |x_i - c| = x_n$

2. (1) By definition, (E) is correct.

(2) $P(X=1 | W=2) = P_{W=2}(X=1) = \lim_{\varepsilon \to 0} \int_1^{1+\varepsilon} (1 - \frac{1}{2}x)\,dx = \lim_{\varepsilon \to 0} (x - \frac{1}{4}x^2)\Big|_1^{1+\varepsilon} = 0$

(3) When $w=2$, $p(x) = \begin{cases} 0 & , x<0 \\ 1-\frac{1}{2}x & , 0 \le x \le 2 \\ 0 & , x>2 \end{cases}$, so $p(1) = \frac{1}{2}$

3. (1) $E_X(E(Y|X)) = \int_X p_X(x) E(Y|X)\,dx$

$= \int_X p_X(x) \int_Y y P(Y|X=x)\,dy$

$= \int_X \int_Y p_X(x)\, y\, P(y|X=x)\,dy\,dx$

$= \int_X \int_Y y\, p(x,y)\,dy\,dx$

$= \int_Y y \int_X p(x,y)\,dx\,dy$

$= \int_Y y\, p_Y(y)\,dy$

$= E_Y(Y)$

(2) $E(Y|X=x) = \frac{\int_Y y\, p(x,y)\,dy}{p_X(x)} = \frac{\int_Y y\, p_X(x) p_Y(y)\,dy}{p_X(x)} = \int_Y y\, p_Y(y)\,dy = E(Y)$

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(3) Define $f(c) = E[(Y-c)^2|X=x] = \int_Y (y-c)^2 p(y|X=x)dy$

$f'(c) = 2\int_Y (c-y)\, p(y|X=x)\, dy$

For a given x, $f'(c)$ is increasing.

Hence c should satisfy $f'(c)=0$

$\int_Y (c-y)\, p(y|X=x)dy = 0 \Leftrightarrow E[(c-y)|X=x]=0$

$c = E(y|X=x)$

4. (1) (a) Positivity. $|A \triangle B| \geq 0$, $|\Omega| \geq 0 \Rightarrow R_s(A,B) \geq 0$

If $R_s(A,B)=0$, $|A\triangle B|=0$, which means $A=B$

(b) $R_s(A,B) = \frac{|A\triangle B|}{|\Omega|} = \frac{|B\triangle A|}{|\Omega|} = R_s(B,A)$

(c) Consider $\forall x \in A\triangle B$, ~~we~~ $x\in A$ or $x \in B$

We may assume $x\in A$, $x \notin B$

case 1, $x\in C$, since $x\notin B$, $x \in B\triangle C$, $x\in (A\triangle C)\cup(B\triangle C)$

case 2, $x\notin C$, since $x\in A$, $x\in A\triangle C$, $x\in (A\triangle C)\cup(B\triangle C)$

From arbitrariness of x, $A\triangle B \subset (A\triangle C)\cup(B\triangle C)$

Hence $|A\triangle B| \leq |(A\triangle C)\cup(B\triangle C)| \leq |A\triangle C| + |B\triangle C|$

Then $R_s(A,B) \leq R_s(A,C) + R_s(B,C)$

(2)(a) $|A\triangle B| \geq 0$, $|A\cup B| \geq 0 \Rightarrow J_s(A,B) \geq 0$

If $J_s(A,B)=0$, $|A\triangle B|=0$, which means $A=B$

(b) $J_s(A,B) = \frac{|A\triangle B|}{|A\cup B|} = \frac{|B\triangle A|}{|B\cup A|} = J_s(B,A)$

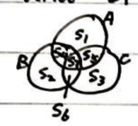
(c) If there are infinite set there in A, B, C, suppose is A

Then $J_s(A,B)=0$, $J_s(A,C)=0$, $J_s(B,C)\geq 0$ is clearly.

If A, B are both infinite, $J_s(A,B) = J_s(A,C) + J_s(B,C) = 0$

If A,B,C are all infinite, $J_s(A,B) = 1 \leq 2 = J_s(A,C) + J_s(B,C)$

If A, B, C are all finite. Suppose $|A\backslash(B\cup C)| = S_1$, $|B\backslash(A\cup C)| = S_2$, $|C\backslash(A\cup B)| = S_3$, $|(A\cap B)\backslash C| = S_4$, $|(A\cap C)\backslash B| = S_5$, $|(B\cap C)\backslash A| = S_6$, $|A\cap B\cap C| = S_7$ and $S_i \geq 0$ $(i=1,2,\cdots,7)$, $\sum_{i=1}^{7} S_i = S$



$$J_\delta(A,B) = \frac{S_1+S_2+S_5+S_6}{S-S_3}$$

$$J_\delta(A,C) + J_\delta(B,C) = \frac{S_1+S_3+S_4+S_6}{S-S_2} + \frac{S_2+S_3+S_4+S_5}{S-S_3}$$

$$J_\delta(A,B) \leq J_\delta(A,C) + J_\delta(B,C)$$

$$\Leftrightarrow (S-S_1)(S-S_2)(S_1+S_2+S_5+S_6) \leq (S-S_1)(S-S_3)(S_1+S_3+S_4+S_6) + (S-S_2)(S-S_3)(S_2+S_3+S_4+S_5)$$

$$\Leftrightarrow (S_1S_4+S_2S_4)S + S_1S_2S_3 + S_1S_3S_5 + S_1S_3S_7 + S_2S_3S_6 + S_2S_3S_7$$

$$\leq 2S_4S^2 + (S_1S_2+S_1S_3+S_1S_5+S_2S_3+S_2S_6+S_3S_5+S_3S_6+2S_3S_7)S + S_1S_2S_4 + S_1S_2S_7 \quad ①$$

Since $(S_1S_4+S_2S_4)S = S_4(S_1+S_2)S \leq S_4S^2$

$$S_1S_2S_3 \leq S_1S_2S$$

$$S_1S_3S_5 + S_1S_3S_7 \leq S_1S_3S$$

$$S_2S_3S_6 + S_2S_3S_7 \leq S_2S_3S$$

Hence ① holds, then $J_\delta(A,B) \leq J_\delta(A,C) + J_\delta(B,C)$ holds.