Revisiting the Origin of the Universe and the Arrow of Time.

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Abstract

We reconsider the old but yet unsolved problems, origin of the universe and the arrow of time. We show that only the closed universe is free from the singularity with the arrow of time symmetric with respect to the maximal size of the cosmic scale. The Wheeler-DeWitt equation is explicitly solved to obtain the local dynamical times. Correspondingly to these local dynamical times, the thermodynamic arrow of time is proved to coincide with the arrows of dynamical time and of expanding universe (cosmological time). The proof is explicitly done in two-dimensional spacetime.

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1 Introduction

What is the origin of the universe ? Can it be free from the singularity [1, 2] ? What is time ? These are the long-standing problems. Vilenkin proposed the model that the universe appears from nothing [3, 4] in the closed universe, which seems to be nicely fitted with the noboundary condition of the universe [5] and free from the singularity. After the birth of the universe it just leads to the inflation, which may solve the problem why the early universe is in low entropy state [6]. These facts come from the Wheeler-Dewitt equation [7, 8, 9], which shows that the universe does not include the extrinsic time. Then, under the closed universe, it gives rise to the problems how to derive the intrinsic time and how to determine the arrow of time in the expanding and contracting phases. The early universe can be interpretted only after we solve the above mentioned problems comprehensively and consistently. This is the purpose of this paper. The outline of this paper is as follows. Section 2 is devoted to the short review of the birth of the universe in Vilenkin's scenario. Then we derive the dynamical time from the Wheeler-DeWitt equation in Section 3. Thermodynamic arrow of time is discussed in Section 4. Section 5 is devoted to discussions.

2 Creation of the Universe from Nothing

This section is a review of the creation of the universe from nothing by Vilenkin [3, 4], which is necessary to understand the motivation of this paper and a key ingredient of this paper. Let us start with the following system of gravity plus scalar matter,

$$S = \int \sqrt{-g}d^4x \left[-\frac{1}{16\pi G}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) \right]. \tag{1}$$

If we consider the closed Friedman-Robertson-Walker (FRW) universe, the invariant line element is

$$ds^2 = dt^2 - a^2(t) \left(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2) \right), \tag{2}$$

where physical distance $r = a\sin\chi$. In this paper we adopt $\hbar = c = 1$ units. Lagrangin of (1) in the above metric takes the form of

$$\mathcal{L} = \frac{3\pi}{4G}(1 - \dot{a}^2)a + \pi^2 a^3 \dot{\phi}^2 - 2\pi^2 a^3 V(\phi). \tag{3}$$

Here and hereafter dot indicates the derivative with respect t. The Wheeler-DeWit equation of this system is

$$\mathcal{H}\Psi = 0, \tag{4}$$

where

$$\mathcal{H} = -\frac{G}{3\pi a}P_a^2 + \frac{1}{4\pi^2 a^3}P_\phi^2 - \frac{3\pi}{4G}a\left(1 - \frac{8\pi}{3}Ga^2 V(\phi)\right). \tag{5}$$

P_a and P_ϕ are the canonical conjugates of a and ϕ, respectively. Then (4) indicates

$$\left(\frac{\partial^2}{\partial a^2} + \frac{p}{a}\frac{\partial}{\partial a} - \frac{1}{a^2}\frac{\partial^2}{\partial\tilde{\phi}^2} - U(a, \phi)\right)\Psi = 0. \tag{6}$$

Here p is an order one parameter relevant to the order of a and P_a, $\tilde{\phi}^2 \equiv 4\pi G\phi^2/3$, and

$$U(a,\phi) = \left(\frac{3\pi}{2G}\right)^2 a^2 \left(1 - \frac{8\pi}{3}Ga^2 V(\phi_0)\right)$$

$$= \left(\frac{3\pi}{2G}\right)^2 a^2 (1 - H^2 a^2) \tag{7}$$

with $H \equiv \frac{8\pi G}{3}|V(\phi_0)|$. $V(\phi$ is

$$V(\phi) = \rho_v - \frac{1}{2}m^2\phi^2 \tag{8}$$

with $V'(\phi_0) = 0$, having a spontaneous symmetry breaking with a wrong mass sign. The scale factor satisfies the Friedman equation,

$$\dot{a}^2 - H^2 a^2 = -k, \quad (k = 1, 0, -1 \text{ for closed, flat, and open universes, respectively}) \tag{9}$$

with k=1 in this case. This potential is depicted in Fig.1. It is very important that, in the region
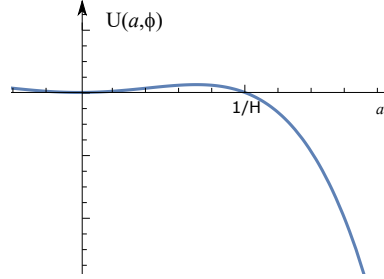

Figure 1:   The birth of the inflating universe from nothing [3], which is possible only in the closed universe. The horizontal axis is the FRW scale factor a and the vertical axis is $U(a, \phi)$ given in (7). In the open universe, $U(a, \phi)$ is obtained by changing the signature of $H^2 a^2$ in (7) and has no barrier to the singular point $a = 0$.

$0 < a < 1/H$ (region I), $\dot{a}^2 < 0$ and the metric is Euclidean (time parameter t is imaginary) and that, in the region $a > 1/H$ (region II), the metric is Lorenzian.

Before discussing furthermore on this potential energy, let me make a comment on the implication of the closed universe. We can consider the case of open FRW universe simply by changing $a \to ia$ in Eq.(7) or $k = -1$ in Eq.(9). Its invariant length is

$$ds^2 = dt^2 - a^2(t)\left(d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)\right). \tag{10}$$

In this case, the physical distance $r = a\sinh\chi$ increases without limit. As is easily seen from Eq.(7), the whole region is Lorenzian and the universe shrinks into singularity in the early stage [1, 2].

We proceed to discuss on the closed FRW universe in more detail. From (9), Lorenzian region is restricted in $a > 1/H$ region. In region I the time is imaginary ($t = i\tau$), which means that $a = 1/H$ region is connected with $a = 0$ point by tunneling [3]. $a = 0$ has no space-time and no energy, called "nothing". From (9),

$$a = H^{-1}\cos(H\tau). \tag{11}$$

After the tunneling in the real time, the scale factor a behaves as

$$a = H^{-1}\cosh Ht, \tag{12}$$

which implies the inflation. Thus the universe is free from the singularity and has no bounadary [5]. Due to the potential of (8), this inflation phase ends when ϕ reaches $\frac{m}{H}\phi \approx 1$ and the Big Bang begins. This also explains why the early universe is in low entropy state [6].

3   The Dynamical Arrow of Time

We have reviewed that the closed universe is free from the singularity. However, it gives rise another problem whether $t = 0$ is really the beginning of the universe over the whole history of it, leading to the question to which direction the arrow of time is directed in the contracting phase.

So far t is really a parameter. (4) indicates that there is no time variable outside of the dynamical system, the universe. We must introduce a *"time"* variable intrinsic to the dynamical system. In this and next sections, we consider two different time arrows: dynamical time arrow and thermodynamic one. These two are mutually related as we will show.

In these two sections we consider the two dimensional gravity, called Jackiw-Teitelboim gravity [10, 11] which is amenable but reserves the universality of FRW metric in four dimension.

$$S_{grav} = \int \sqrt{-g}(R - 2\Lambda)Nd^2x. \tag{13}$$

Here R and 2Λ are scalar curvature and cosmological constant, respectively. It should be remarked that R is total derivative in two dimensins. This action was formulated as $O(2,1)$ gauge theory by Fukuyama-Kamimura [12]. The usual four dimensional gravity is formulated as $O(4,1)$ or $O(3,2)$ gravity, which will be argued in Discuusions. In general, $g_{\mu\nu}$ is expressed as

$$g_{\mu\nu} = e^{2\chi}\begin{pmatrix} \eta_1^2 - \eta_\perp^2 & \eta_1 \\ \eta_1 & 1 \end{pmatrix}. \tag{14}$$

Then, S_{grav} reads

$$S_{grav} = \int (P_\chi\dot{\chi} + P_N\dot{N} - \eta_\perp\mathcal{H}_\perp - \eta_1\mathcal{H}_1)d^2x \tag{15}$$

with

$$\mathcal{H}_\perp = P_N P_\chi + N'\chi' - N'' - \Lambda N e^{2\chi}, \tag{16}$$

and dffeomorphism constraint,

$$\mathcal{H}_1 = P_\chi \chi' + P_N N' - P_\chi'. \tag{17}$$

Thus, $\eta_\perp$ and η_1 act as Lagrange multipliers and we have the Hamiltonian constraint (the Wheeler-DeWitt equation) $(16) \approx 0$ and the diffeomorphism constraint $(17) \approx 0$.

We consider two dimensional space-time since it makes possible to solve the Wheeler-DeWitt equation explicitly and to induce the "*time*" variable as will be shown [13]. We diagonalize $\mathcal{H}_\perp$ by using

$$\sigma_\pm \equiv \frac{\chi \pm N}{\sqrt{2}}, \tag{18}$$

and their canonically conjugates

$$\pi_\pm = \frac{P_\chi \pm P_N}{\sqrt{2}} \tag{19}$$

as

$$\mathcal{H}_\perp = \frac{1}{2}(\pi_+^2 - \pi_-^2) + \frac{1}{2}(\sigma_+'^2 - \sqrt{2}\sigma_+'') - \frac{1}{2}(\sigma_-'^2 - \sqrt{2}\sigma_-'') - \frac{\Lambda}{\sqrt{2}}(\sigma_+ - \sigma_-)e^{\sqrt{2}(\sigma_+ + \sigma_-)}. \tag{20}$$

Here P_χ and P_N are variables canonically conjugate to χ and N, respectively. Dot (dash) implies the derivative with respect to t (x). However, if we use the FWR metric

$$ds^2 = dt^2 - a^2(t)dx^2, \tag{21}$$

then the metrics are reduced to

$$\eta_\perp = e^{-\chi} \equiv \frac{1}{a}, \quad \eta_1 = 0. \tag{22}$$

Here

$$\chi = \chi(t), \quad N = N(t), \tag{23}$$

being independent on x, and $(17) \approx 0$ is automatically satisfied. Equations of motion are

$$\ddot{\chi} + \dot{\chi} = \Lambda, \tag{24}$$
$$\ddot{N} + \dot{\chi}\dot{N} - 2\Lambda N = 0. \tag{25}$$

When

$$\Lambda = -\alpha^2 < 0, \tag{26}$$

with the initial condition,

$$a(t = 0) = 0, \tag{27}$$

(24) gives the closed universe solution

$$a = a_{max} \sin \alpha t, \tag{28}$$

and (25) does

$$N = A\cos\alpha t. \tag{29}$$

Here A is an intgral constant which is assumed to be positive. It should be remarked that N, overall factor of Lagrangian, changes its signature at the maximum radius (a_{max}) and that a variable with negative (positive) metric is transformed to that with positive (negative) metric. Correspnding to it, the dynamical time variable is induced localy in phases and not globally. Amenable point of two dimensional gravity is that the Wheeler-DeWitt equation in the FRW metric,

$$\frac{\partial^2 \psi}{\partial N \partial \chi} + \Lambda N e^{2\chi} \psi = 0, \tag{30}$$

can be solved explicitly as

$$\psi = c_0 \exp\left(\frac{c_1}{2} N^2 + \frac{\alpha^2}{2c_1} e^{2\chi}\right), \tag{31}$$

where $\Lambda = -\alpha^2$. c_0 and c_i are complex numbers.

Then, we have the current conservation

$$\partial_- j^- + \partial_+ j^+ = 0. \tag{32}$$

Here

$$j^- \equiv i(\psi^* \partial_- \psi - \psi \partial_- \psi^*), \quad j^+ \equiv -i(\psi^* \partial_+ - \psi \partial_+ \psi^*), \tag{33}$$

and

$$\partial_- \equiv \frac{\partial}{\partial \sigma_-} \quad \text{and} \quad \partial_+ \equiv \frac{\partial}{\partial \sigma_+}. \tag{34}$$

Writing $c_1 \equiv -\beta + i\gamma$, (31) becomes

$$\psi = c_0 \exp\left[\frac{i\gamma}{2}\left(N^2 - \frac{\alpha^2}{|c_1|^2} e2\chi\right) - \frac{\beta}{2}\left(N^2 + \frac{\alpha^2}{|c_1|^2} e^{2\chi}\right)\right]. \tag{35}$$

This equation implies that the width of the wave packet is

$$\Delta N \approx \frac{1}{\sqrt{\beta}} \tag{36}$$

in the early universe and

$$\Delta e^\chi \approx \frac{|c_1|}{\alpha\sqrt{\beta}} \tag{37}$$

around a_{max}. (36) and (37) indicate that the space-time relation,

$$\sigma_+ - \sigma_- \approx \frac{1}{\sqrt{\beta}} \tag{38}$$

and

$$\sigma_+ + \sigma_- \approx \ln\left(\frac{|c_1|}{\alpha\sqrt{\beta}}\right), \tag{39}$$

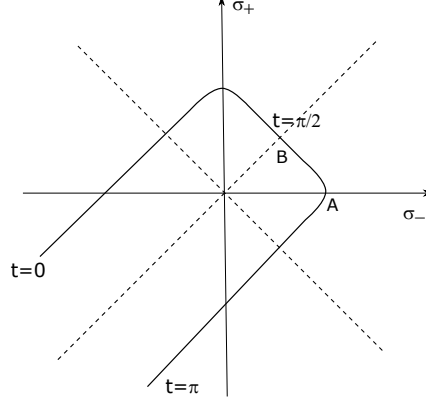respectively. These asymptotiuc behaviours are depicted in Fig.2. In the semi-classical limit,

Figure 2: Classical trajectory in the $\sigma_- - \sigma_+$ [13]. B is the point at the maximal scale a_{max}. At the point A and further, the condition $\dot{\sigma}_- > 0$ is broken.

$$\psi = ce^{iS}, \tag{40}$$

where S is the classical action (13) in FRW metric. In this case, j^- takes the form

$$j^- = -2|c|^2\partial_- S = 2|c|^2 e^\chi \dot{\sigma}_-. \tag{41}$$

Positive definitenass of the probability density is thus equivalent to that of $\dot{\sigma}_-(= -\partial_- S)$. In the contracting phase, we should replace ∂_- by ∂_+ in j^-,

$$j^{(-)} \equiv j^-(\partial_- \to \partial_+) = -2|c|^2\partial_+ S = -2|c|^2 e^\chi \dot{\sigma}_+. \tag{42}$$

Taking the behaviour of Fig.2 into consideration, the positive definiteness of $j^{(-)}$ is satisfied in the from $t = 0$ point to the point A. Over the point A, $\dot{\sigma}_- < 0$, then "time" is replaced by σ_+ there.

4　Thermodynamic Arrow of Time

We have considered the thermodynamic arrow of time [14] in the same system as (13). We consider that the universe was under quantum theory in the vicinity of birth, whereas at present there seems to be no quantum correlation which, if it exists, drastically destroys deterministic interpretation of cosmic observation. There must be a transition from an era which is fully quantum mechanical to an era which has no quantum coherence in the course of cosmological evolution. We call this direction from the quantum to the classical era the thermodynamic arrow of time. More concretely speaking, this direction is that of the decrease of the dispersion (the width of the quantum coherence) σ^2 of the density matrix

$$\overline{\rho}[a_+, \eta_{\perp+}, \phi_+; a_-, \eta_{\perp-}, \phi_-] \equiv \psi(a_+, \eta_{\perp+}, \phi_+)\psi^*(a_-, \eta_{\perp-}, \phi_-) \tag{43}$$

Here $g_{\mu\nu}$ of (14) is parametrised as

$$g_{\mu\nu} = a^2 \mathrm{diag}(-\eta_\perp^2, 1) \tag{44}$$

and ϕ ia a scalar matter. Based on the influencefunctional method [15, 16], the reduced density matrix is given by

$$\rho[a_+, \eta_{\perp+}; a_-, \eta_{\perp-}] \equiv \int d\phi^+ \int d\phi^- \overline{\rho}[a_+, \eta_{\perp+}, \phi_+; a_-, \eta_{\perp-}, \phi_-] \delta(\phi_+ - \phi_-). \tag{45}$$

Then, the dispersion σ is given by

$$\rho[a_+; a_-] \propto exp[-\frac{(a_+ - a_-)^2}{2\sigma^2}]. \tag{46}$$

The detailed calculations are given in [14]. The dispersion versus the scale facor a is depicted in Fig.3. Thus the dispersion is decreasing in the expanding phase but turns to increase at the contracting phase. Since the thermodynamic arrow of time is directed to that of the decreasing dispersion, it means that the arrow of the time changes its direction at a_{max} (that is, the arrow of time is symmetric with respect to a_{max}). It appears to decrease again beyond the point A as the parameter t increases. However it is fictitious since in this region $\dot{\sigma}_- < 0$ and the dynamical time should be replaced by σ_+ instead of σ_- as we explained at the end of the previous section. It should be remarked that in the contracting phase ($\frac{\pi}{2} < t < \pi$)

$$\sigma_+(t) = \sigma_-(\pi - t), \quad \sigma_-(t) = \sigma_+(\pi - t), \tag{47}$$

which implies that the direction of the dynamical time changes the signatute. Then the gradient of the dispersion changes its sign correspondingly and the dispersion increases in the region AO as the time σ^+ increses.
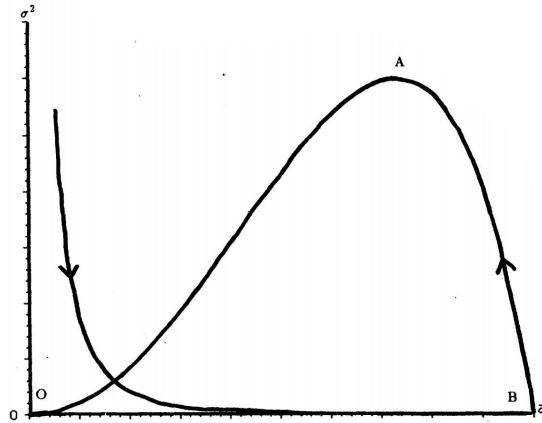

Figure 3: Quantum coherence width σ^2 versus the scale factor a is shown for the closed universe [14]. A and B points correspond to those of Fig.2.

On the contrary, Hawking et al. asserted that the universe does not change its arrow of thermodynamic time in the contracting phase, asymmetric with respect to the maximal radius, a_{max} [17]. In their work, they explicitly showed that one of the spatial metric components, a_n, continues to increases over the a_{max}. It should be noted in our case the dispersion σ^2 is decreasing again in AO region in Fig.3, seemingly leading to the same result as that of [17], except for the region BA. However, as we have mentioned in the previous section, it is fictitious. If we correctly replace the time variable from σ_- to σ_+ in the region AO, the dispersion σ^2 continues to increase as in the region of BA. The global use of the "$time$", σ_-, in the whole region leads to the negative probability in the region AO as was discussed in (41), (42). Also if the time was asymmetric with respect to a_{max}, the universe is bounced at $a = 1/H$ (since the tunneling is rare), and gave rise to another problem why we are not in one of such bouncing phases and why our universe was in low entropy state in the early universe.

5   Discussions

In the previous section, we have discussed on the arrow of time in two dimensional space-time, and not in real four dimensions. Though the arguments of Section 2 are those in minisuperspace, effectively in two dimensins, we needs some arguments depending on four dimensional space-time.

In four dimensions, Jackiw-Teitelboim gravity of (13) is generalised to [18, 19]

$$\mathcal{L}_{gravity} = \pm \epsilon^{abcd} \epsilon^{\mu\nu\rho\sigma} \hat{R}_{\mu\nu ab} \hat{R}_{\rho\sigma cd}/(16g^2). \tag{48}$$

Here Latin and Greek letters indicate world and local Lorentz coordinates, respectively. ϵ^{abcd} is fully antisymmetric tensor, and

$$\hat{R}_{\mu\nu ab} = R_{\mu\nu ab} + e_{[\mu a} e_{\nu] b}/l^2, \tag{49}$$

where

$$R_{\mu\nu ab} = \partial_{[\mu} \omega_{\nu] ab} - \omega_{[\mu a}{}^c \omega_{\nu] cb}. \tag{50}$$

Here $\omega_{\mu ab}$ and $e_{\mu a}$ are the spin connection and tetrad, respectively, and $e_{[\mu a} e_{\nu] b} \equiv e_{\mu a} e_{\nu b} - e_{\nu a} e_{\mu b}$. g and l are the gauge coupling constant and the length scale characterising (anti-) de-Sitter gauge group, respectively. The detail was shown in [19]. Substituting (49) and (50) into (48), we obtain

$$\mathcal{L}_{gravity} = \partial_\mu K^\mu - \frac{1}{16\pi G} e \left(R + \frac{6}{l^2} \right), \tag{51}$$

where

$$e = \det e_{\mu a} = \sqrt{-g}, \ R = e^{\mu a} e^{\nu b} R_{\mu\nu ab}, \ 16\pi G = g^2 l^2. \tag{52}$$

The first quadratic term of $R_{\mu\nu ab}$ in (48) is the topological invariant and a total derivative. Thus, gravity is formulated as the quadratic field-strength like the other gauge theories but dynamically survive the linear Einstein's actionon and cosmological constant due to the topological invariant. The scale parameter l or equivalently the cosmological constant $l^2/6$ comes from the breaking

of the conformal gauge symmetry $O(4,2)$ [19]. Thus, this Lagrangian leads us to the model of section 2 in the FRW metric (3) with the effective cosmological constant due to the homogeneous Higgs potential and a strict constant Λ. This cosmological constan inflates the universe and becomes the reason why the universe was born in the low entropy state [6]. Here the cosmological constant is exactly "constant " as the breaking scale of the $O(4,2)$ symmetry. Anyhow, we have not shown explicitly in this paper that the arguments of Section 3 is valid in four dimensional space-time. However, it is suggestive that the spectral dimension of the quantum universe as function of the diffusion time moves from two to four continuously under the definite arrow of time [20].

Lastly we have a comment on the so-called Aharanov-Bohm time variable [21],

$$T = \frac{1}{2} M \left(a \frac{1}{P_a} + \frac{1}{P_a} a \right),$$

(53)

which is the canonically conjugate to the Hamiltonian,

$$\mathcal{H} = \frac{P_a^2}{2M}$$

(54)

with the total mass of the universe, $M = \frac{3\pi a_{max}}{2G}$. This "time" variable has been discussed from the motivation different from that of this paper and leading to the weak measurement [22], which contradicts with the usual Copenhagen interpretarion of quantum mechanics [23]. Anyhow, it is suggestive that this "time" is not smoothly connected at $P_a \approx 0$ at a_{max}, unlike the asymmetric time with respect to a_{max}.

Acknowledgements

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