# 1 Fields

**Definition:** A field is a set with two operations:

$+ : F \times F \longrightarrow F, \qquad (a, b) \longmapsto a + b$

$* : F \times F \longrightarrow F, \qquad (a, b) \longmapsto ab$

**Axioms of a field:**

- (F1) $a + b = b + a, \quad ab = ba \quad \forall a, b \in F$
- (F2) $(a + b) + c = a + (b + c), \quad (ab)c = a(bc) \quad \forall a, b, c \in F$
- (F3) There are $0_F,\ 1_F \in F\ s.t.\ 0_F + a = a,\ 1_F * a = a \quad \forall a \in F$
- (F4) $\forall a,\ b \neq 0 \in F,\ \exists c,\ d \in F\ s.t.\ a + c = 0,\ bd = 1$
- (F5) $a(b + c) = ab + bc \quad \forall a,\ b,\ c \in F$

**Examples:** $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Z}/p\mathbb{Z}$ where $p$ is prime

**Non-examples:** $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}^{2\times 2}$, $\mathbb{Z}/p\mathbb{Z}$