TGLF to IMAS data conversion

A. Najlaoui

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Contents

Chapter 1

Preamble

This document describes how to transform inputs and outputs from a TGLF simulation to match the standard IMAS format.
The reader is assumed to have some knowledge of TGLF [1], [2] and to have read the documentation of IMAS [3].

Chapter 2

Conventions and normalisations

2.1 Coordinate systems

In TGLF the toroidal direction is defined such as (R, φ, Z) is right-handed, similarly to IMAS, see Fig.2.1. In practice, it means that:

$$\varphi^{\texttt{IMAS}} = \varphi^{\texttt{TGLF}} \tag{2.1}$$

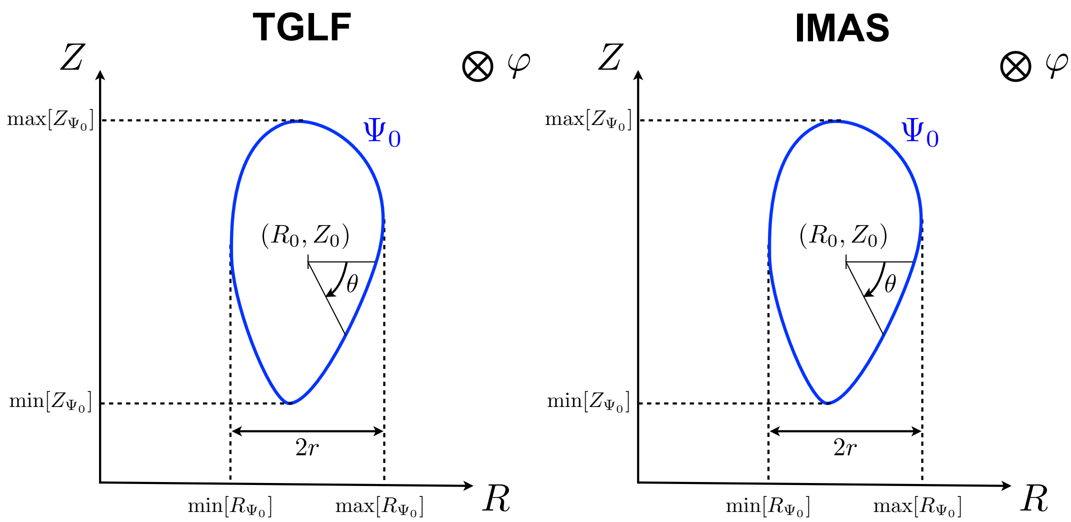

Figure 2.1: Cylindrical coordinate system used in TGLF (left) and in imas (right).

For `miller` geometry, the definitions of r, R_0 and Z_0 is identical to the one used in IMAS:

$$R_0^{\texttt{TGLF}} = R_0^{\texttt{IMAS}} = \frac{1}{2}[R_{\min} + R_{\max}] \qquad Z_0^{\texttt{TGLF}} = Z_0^{\texttt{IMAS}} = \frac{1}{2}[Z_{\min} + Z_{\max}] \tag{2.2}$$

and:

$$r^{\texttt{TGLF}} = r^{\texttt{IMAS}} = \frac{1}{2}[R_{\max} - R_{\min}] \tag{2.3}$$

2.2 Reference quantities

In TGLF and IMAS, all quantities are normalised and made dimensionless by making use of reference quantities. In what follows, normalised quantities are denoted with a "N" subscript. For instance, the normalised version of an arbitrary quantity $\mathcal{A}$ with the dimension of a length will be $\mathcal{A}_N^{\texttt{TGLF}} = \mathcal{A}/L_{\text{ref}}^{\texttt{TGLF}}$ in

Quantity	Notation	Description
macro-length	a	minor radius (r at the last closed flux surface)
micro-length	$\rho_s = c_s/\Omega_i$ $\Omega_i = eB_{unit}/m_D$	Larmor radius
mass	m_D	Deuterium mass $= 3.345 \times 10^{-27} kg$
density	n_e	electron density
temperature	T_e	electron temperature
velocity	$c_s = \sqrt{T_e/m_D}$	deuterium sound speed
time	a/c_s	minor radius over sound speed

Table 2.1: TGLF Normalization

TGLF and $\mathcal{A}_N^{\mathtt{IMAS}} = \mathcal{A}/L_{\mathrm{ref}}^{\mathtt{IMAS}}$ in IMAS.
TGLF normalizations are listed in table 2.1

The electron density and temperature are assumed constant on a flux surface.

The conversion from TGLF to IMAS involves the ratio of reference quantities. For the example above:

$$\mathcal{A}_N^{\mathtt{IMAS}} = \frac{R_{\mathrm{ref}}^{\mathtt{TGLF}}}{R_{\mathrm{ref}}^{\mathtt{IMAS}}} \mathcal{A}_N^{\mathtt{TGLF}} \tag{2.4}$$

The ratio of the various reference quantities used in TGLF and IMAS are:

$$q_{\mathrm{rat}} = \frac{q_{\mathrm{ref}}^{\mathtt{TGLF}}}{q_{\mathrm{ref}}^{\mathtt{IMAS}}} = \frac{e^{\mathtt{TGLF}}}{e^{\mathtt{IMAS}}} = \frac{1.6021746 \times 10^{-19} C}{1.602176634 \times 10^{-19} C} = 0.99999873347 \qquad L_{\mathrm{rat}} = \frac{L_{\mathrm{ref}}^{\mathtt{TGLF}}}{L_{\mathrm{ref}}^{\mathtt{IMAS}}} = \frac{\mathtt{a}}{R_0^{\mathtt{IMAS}}} = \frac{1}{\mathtt{RMAJ\_LOC}^{\mathtt{TGLF}}}$$

$$m_{\mathrm{rat}} = \frac{m_{\mathrm{ref}}^{\mathtt{TGLF}}}{m_{\mathrm{ref}}^{\mathtt{IMAS}}} = 1.00042356576 \qquad B_{\mathrm{rat}} = \frac{B_{\mathrm{ref}}^{\mathtt{TGLF}}}{B_{\mathrm{ref}}^{\mathtt{IMAS}}} = \frac{B_{\mathtt{unit}}^{\mathtt{TGLF}}}{B_t^{\mathtt{IMAS}}(R_0)}$$

$$T_{\mathrm{rat}} = \frac{T_{\mathrm{ref}}^{\mathtt{TGLF}}}{T_{\mathrm{ref}}^{\mathtt{IMAS}}} = \frac{T_e}{T_e(\theta = 0)} = 1 \qquad n_{\mathrm{rat}} = \frac{n_{\mathrm{ref}}^{\mathtt{TGLF}}}{n_{\mathrm{ref}}^{\mathtt{IMAS}}} = \frac{n_e}{n_e(\theta = 0)} = 1$$

where the e subscript denotes the electron species.
With the definition of $B_{\mathtt{unit}}^{\mathtt{TGLF}}$ [4] and equation 3.20, we obtain the following value for B_{rat}:

$$B_{\mathrm{rat}} = \frac{B_{\mathrm{ref}}^{\mathtt{TGLF}}}{B_{\mathrm{ref}}^{\mathtt{IMAS}}} = \frac{R_0}{2\pi r} \int_0^L \frac{dl'}{R|\nabla r|} \tag{2.5}$$

Using the value of $v_{\mathrm{thref}}^{\mathtt{IMAS}} = \sqrt{\frac{2T_{\mathrm{ref}}^{\mathtt{IMAS}}}{m_{\mathrm{ref}}^{\mathtt{IMAS}}}}$ we can derive the reference thermal velocity ratio:

$$v_{\mathrm{thrat}} = \frac{c_s^{\mathtt{TGLF}}}{v_{\mathrm{thref}}^{\mathtt{IMAS}}} = \frac{1}{\sqrt{2}} \sqrt{\frac{T_{\mathrm{rat}}}{m_{\mathrm{rat}}}} \tag{2.6}$$

Using the value of $\rho_{\mathrm{ref}}^{\mathtt{IMAS}} = \frac{m_{\mathrm{ref}}^{\mathtt{IMAS}} v_{\mathrm{ref}}^{\mathtt{IMAS}}}{q_{\mathrm{ref}}^{\mathtt{IMAS}} B_{\mathrm{ref}}^{\mathtt{IMAS}}}$ we can derive the reference Larmor radius ratio:

$$\rho_{\mathrm{rat}} = \frac{m_{\mathrm{rat}} v_{\mathrm{thrat}}}{q_{\mathrm{rat}} B_{\mathrm{rat}}} \tag{2.7}$$

Chapter 3

Inputs

For the conversion, the TGLF input values will be found in the file `input.tglf.gen`

3.1 Magnetic equilibrium

3.1.1 Radial coordinate

We have:

$$r_N^{\texttt{IMAS}} = \frac{r}{L_{\mathrm{ref}}^{\texttt{IMAS}}} \qquad \text{and} \qquad r_N^{\texttt{TGLF}} = \frac{r}{L_{\mathrm{ref}}^{\texttt{TGLF}}} \tag{3.1}$$

Then it comes:

$$r_N^{\texttt{IMAS}} = r_N^{\texttt{TGLF}} \cdot L_{\mathrm{rat}} = \texttt{RMIN\_LOC} \cdot L_{\mathrm{rat}} \tag{3.2}$$

3.1.2 Toroidal field and current direction

We have:

$$s_b^{\texttt{IMAS}} = \mathrm{sign}[\mathbf{B} \cdot \nabla\varphi] \qquad \text{and} \qquad s_j^{\texttt{IMAS}} = \mathrm{sign}[\mathbf{j} \cdot \nabla\varphi] \tag{3.3}$$

In IMAS and TGLF, the toroidal field and plasma current are positive when counter-clockwise.

$$s_b^{\texttt{IMAS}} = s_b^{\texttt{TGLF}} = \texttt{SIGN\_BT} \qquad \text{and} \qquad s_j^{\texttt{IMAS}} = s_j^{\texttt{TGLF}} = \texttt{SIGN\_IT} \tag{3.4}$$

3.1.3 Safety factor

We have:

$$q^{\texttt{IMAS}} = \frac{1}{2\pi}\int_0^{2\pi} \frac{\mathbf{B} \cdot \nabla\varphi}{\mathbf{B} \cdot \nabla\theta}\,\mathrm{d}\theta = s_b^{\texttt{IMAS}} s_j^{\texttt{IMAS}} |q| \qquad \text{and} \qquad \texttt{Q\_LOC} = |q| \tag{3.5}$$

then it comes:

$$q^{\texttt{IMAS}} = s_b^{\texttt{TGLF}} s_j^{\texttt{TGLF}} q^{\texttt{TGLF}} = \texttt{SIGN\_BT} \cdot \texttt{SIGN\_IT} \cdot \texttt{Q\_LOC} \tag{3.6}$$

3.1.4 Magnetic shear

We have:

$$\hat{s}^{\texttt{IMAS}} = \frac{r}{q}\frac{\partial q}{\partial r} \qquad \text{and} \qquad \texttt{Q\_PRIME\_LOC} = \frac{q^2 a^2}{r^2}\frac{r}{q}\frac{\partial q}{\partial r} \tag{3.7}$$

Then it comes:

$$\hat{s}^{\texttt{IMAS}} = \left(\frac{r_N^{\texttt{TGLF}}}{q^{\texttt{TGLF}}}\right)^2 \cdot \texttt{Q\_PRIME\_LOC} = \left(\frac{\texttt{RMIN\_LOC}}{\texttt{Q\_LOC}}\right)^2 \cdot \texttt{Q\_PRIME\_LOC} \tag{3.8}$$

3.1.5 Pressure gradient (entering the curvature drift)

We have:

$$p'^{\texttt{IMAS}}_N = -\frac{2\mu_0 L^{\texttt{IMAS}}_{\text{ref}}}{B^{\texttt{IMAS}\,2}_{\text{ref}}}\frac{\partial p}{\partial r} \qquad \text{and} \qquad \texttt{P\_PRIME\_LOC} = -\frac{\mu_0}{4\pi}\frac{qa^2}{rB^{\texttt{TGLF}\,2}_{\text{ref}}}\frac{\partial p}{\partial r} \tag{3.9}$$

Then it comes:

$$p'^{\texttt{IMAS}}_N = -8\pi\frac{B^2_{\text{rat}}}{L_{\text{rat}}}\frac{r^{\texttt{TGLF}}_N}{s_b s_j q}p'^{\texttt{TGLF}}_N = -8\pi\frac{B^2_{\text{rat}}}{L_{\text{rat}}}\frac{\texttt{RMIN\_LOC}}{\texttt{SIGN\_BT}\cdot\texttt{SIGN\_IT}\cdot\texttt{Q\_LOC}}\texttt{P\_PRIME\_LOC} \tag{3.10}$$

3.1.6 Magnetic equilibrium

In IMAS, the magnetic equilibrium is described by using the Miller eXtended Harmonics (MXH) parametrisation [5]. For the IMAS-TGLF coupling, we will therefore use the Miller parameterisation implemented in TGLF. In the future, when the MXH or Fourier parametrisation becomes available in TGLF, we could switch to it.

For the parametrisation of Miller [6], the flux surfaces are defined with:

$$R(r,\theta) = R_0(r) + r\cos\left(\theta + \arcsin\delta\sin\theta\right) \tag{3.11}$$

$$Z(r,\theta) = Z_0(r) + \kappa r\sin\left(\theta + \zeta\sin 2\theta\right) \tag{3.12}$$

In this section, we will start from theses relations to obtain the value of B_{rat} and to convert Miller input parameters to MXH inputs.

We have:

$$\frac{\partial R(r,\theta)}{\partial\theta} = -r\sin\left(\theta + \arcsin\delta\sin\theta\right)(1 + \arcsin\delta\cos\theta) \tag{3.13}$$

$$\frac{\partial Z(r,\theta)}{\partial\theta} = \kappa r\cos\left(\theta + \zeta\sin 2\theta\right)(1 + 2\zeta\cos 2\theta) \tag{3.14}$$

and

$$\frac{\partial R(r,\theta)}{\partial x} = \frac{\partial R_0}{\partial x} + \frac{\partial r}{\partial x}\cos\left(\theta + \arcsin\delta\sin\theta\right) - \sin\left(\theta + \arcsin\delta\sin\theta\right)r\frac{\partial\delta}{\partial x}\sin\theta\frac{1}{\sqrt{1-\delta^2}} \tag{3.15}$$

$$\frac{\partial Z(r,\theta)}{\partial x} = \frac{\partial Z_0}{\partial x} + \kappa\sin\left(\theta + \zeta\sin 2\theta\right)\left(\frac{\partial r}{\partial x} + \frac{r}{\kappa}\frac{\partial\kappa}{\partial x}\right) + \kappa\cos\left(\theta + \zeta\sin 2\theta\right)r\frac{\partial\zeta}{\partial x}\sin 2\theta \tag{3.16}$$

We call:

$$J_r = -R * \left(\frac{\partial R}{\partial x}\frac{\partial Z}{\partial\theta} - \frac{\partial R}{\partial\theta}\frac{\partial Z}{\partial x}\right) \tag{3.17}$$

and

$$\frac{\partial l}{\partial\theta} = \sqrt{\left(\frac{\partial R}{\partial\theta}\right)^2 + \left(\frac{\partial Z}{\partial\theta}\right)^2} \tag{3.18}$$

Then it comes:

$$\nabla r = \frac{R}{J_r}\frac{\partial l}{\partial\theta} \tag{3.19}$$

Finally, we obtain for the value of B_{rat}:

$$B_{rat} = \frac{B_{\texttt{unit}}}{B_t(R_0)} = \frac{B_{\texttt{unit}}}{|f|}R_0 = \frac{R_0}{2\pi r}\int_0^L \frac{dl'}{R|\nabla r|} \tag{3.20}$$

For the conversion to the MXH parametrisation, we will start from the (R,Z) description of a flux surface and two adjacent neighbours from its Miller parametrisation (equations 3.11 and 3.12).

To obtain the two adjacent neighbours flux surfaces we will use:

$$dr = 0.01 \cdot r \tag{3.21}$$

Where r_0 is the minor radius at the central flux surface.
We then obtain:

$$\begin{aligned} r_\pm &= r \pm dr & R_{0\pm} &= R_0 \pm \frac{\partial R}{\partial r} \cdot dr \\ \kappa_\pm &= \kappa \pm sk \cdot \kappa/r \cdot dr & Z_{0\pm} &= Z_0 \pm \frac{\partial Z}{\partial r} \cdot dr \\ \delta &= \delta \pm sd/r \cdot dr & \zeta &= \zeta \pm sd/r \cdot dr \end{aligned}$$

So the two adjacent neighbours flux surfaces are obtained as:

$$R(r,\theta)_\pm = R_{0\pm} + r_\pm \cos(\theta + \arcsin \delta_\pm \sin\theta) \tag{3.22}$$

$$Z(r,\theta)_\pm = Z_{0\pm} + \kappa_\pm r \sin(\theta + \zeta_\pm \sin 2\theta) \tag{3.23}$$

Then, we can compute the Miller extended harmonic parametrisation of flux surfaces from thoses three flux surfaces.

For the MXH parametrisation, the flux surfaces are defined with:

$$R(r,\theta) = R_0(r) + r\cos\theta_R \tag{3.24}$$

$$Z(r,\theta) = Z_0(r) + \kappa r \sin\theta \tag{3.25}$$

Where:

$$\theta_R = \theta + c_0(r) + \sum_{n=1}^{N} [c_n(r)\cos n\theta + s_n(r)\sin n\theta] \tag{3.26}$$

So we can obtain the values of θ_R using: $\cos\theta_R = (R - R_0)/r$ and the values of θ using: $\sin\theta = (Z - Z_0)/\kappa r$. We then perform the fourier transform on the angle $\theta_R - \theta$ and we obtain the coefficients c_n and s_n. Finnaly, we use the adjacent flux surfaces to interpolate the shaping coefficients and compute the radial derivatives.

3.2 Species

In this section, the subscript `x` is a number that correspond to a species.

3.2.1 Charge

$$Z_{sN}^{\texttt{IMAS}} = Z_{sN}^{\texttt{TGLF}} \cdot q_{\mathrm{rat}} = \texttt{ZS\_x} \cdot q_{\mathrm{rat}} \tag{3.27}$$

3.2.2 Mass

$$m_{sN}^{\texttt{IMAS}} = m_{sN}^{\texttt{TGLF}} \cdot m_{\mathrm{rat}} = \texttt{MASS\_x} \cdot m_{\mathrm{rat}} \tag{3.28}$$

3.2.3 Density

$$n_{sN}^{\texttt{IMAS}} = n_{sN}^{\texttt{TGLF}} \cdot n_{\mathrm{rat}} = \texttt{AS\_x} \cdot n_{\mathrm{rat}} \tag{3.29}$$

3.2.4 Logarithmic density gradient

We have:

$$\frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{L_{n_s}^{\texttt{IMAS}}} = -\frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{n_s}\frac{\partial n_s}{\partial r} \quad \text{and} \quad \frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{L_{n_s}^{\texttt{TGLF}}} = -\frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{n_s}\frac{dn_s}{dr} \tag{3.30}$$

Then it comes:

$$\frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{L_{n_s}^{\texttt{IMAS}}} = \frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{L_{n_s}^{\texttt{TGLF}}} \cdot \frac{1}{L_{\mathrm{rat}}} = \texttt{RLNS\_x} \cdot \frac{1}{L_{\mathrm{rat}}} \tag{3.31}$$

3.2.5 Temperature

$$T_{sN}^{\texttt{IMAS}} = T_{sN}^{\texttt{TGLF}} \cdot T_{\mathrm{rat}} = \texttt{TAUS\_x} \cdot T_{\mathrm{rat}} \tag{3.32}$$

3.2.6 Logarithmic temperature gradient

We have:

$$\frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{L_{T_s}^{\texttt{IMAS}}} = -\frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{T_s}\frac{\partial T_s}{\partial r} \qquad \text{and} \qquad \frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{L_{T_s}^{\texttt{TGLF}}} = -\frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{T_s}\frac{dT_s}{dr} \tag{3.33}$$

Then it comes:

$$\frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{L_{T_s}^{\texttt{IMAS}}} = \frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{L_{T_s}^{\texttt{TGLF}}} \cdot \frac{1}{L_{\mathrm{rat}}} = \texttt{RLTS\_x} \cdot \frac{1}{L_{\mathrm{rat}}} \tag{3.34}$$

3.2.7 Plasma beta

we have:

$$\beta_{eN}^{\texttt{IMAS}} = 2\mu_0 \frac{n_{\mathrm{ref}}^{\texttt{IMAS}} T_{\mathrm{ref}}^{\texttt{IMAS}}}{B_{\mathrm{ref}}^{2\ \texttt{IMAS}}} \qquad \text{and} \qquad \beta_{N}^{\texttt{TGLF}} = 4\mu_0 \frac{n_{\mathrm{ref}}^{\texttt{TGLF}} T_{\mathrm{ref}}^{\texttt{TGLF}}}{B_{\mathrm{ref}}^{2\ \texttt{TGLF}}} \tag{3.35}$$

Then it comes:

$$\beta_{eN}^{\texttt{IMAS}} = \beta_{N}^{\texttt{TGLF}} \cdot \frac{B_{\mathrm{rat}}^2}{2 n_{\mathrm{rat}} T_{\mathrm{rat}}} = \frac{B_{\mathrm{rat}}^2}{n_{\mathrm{rat}} T_{\mathrm{rat}}} \cdot \texttt{BETAE} \tag{3.36}$$

3.2.8 Collisionality

In IMAS, the collisionality for each species couple a/b is defined as:

$$\nu_N^{a/b\,\texttt{IMAS}} = \frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{v_{\mathrm{thref}}^{\texttt{IMAS}}} \frac{n_b Z_a^2 Z_b^2 e^4 \ln \Lambda^{a/b}}{4\pi\epsilon_0 m_a^2 v_{\mathrm{tha}}^3} \tag{3.37}$$

In TGLF, only electron-ion collision frequency is included. In fact we have:

$$\nu_{e/iN}^{\texttt{TGLF}} = \frac{a}{c_s}\nu_{e/i} \tag{3.38}$$

We can then define:

$$\nu_{e/iN}^{\texttt{IMAS}} = \nu_{e/iN}^{\texttt{TGLF}} \cdot \frac{v_{\mathrm{thrat}}}{L_{\mathrm{rat}}} = \texttt{XNUE} \cdot \frac{v_{\mathrm{thrat}}}{L_{\mathrm{rat}}} \tag{3.39}$$

3.3 Wave vector

3.3.1 Binormal wave vector

We have:

$$k_{\theta *}\rho_{\mathrm{ref}}{}^{\texttt{IMAS}} = k_y \sqrt{g^{yy}(\theta=0)}\rho_{\mathrm{ref}}^{\texttt{IMAS}} \qquad \text{and} \qquad \texttt{KY} = k_y \rho_{\mathrm{ref}}{}^{\texttt{TGLF}} = \frac{nq}{r}\rho_{s,\mathrm{unit}}^{\texttt{TGLF}} \tag{3.40}$$

Using the catalogue of shape functions defined in [4] we can relate the two definitions as:

$$k_{\theta *}\rho_{\mathrm{ref}}{}^{\texttt{IMAS}} = \texttt{KY} \cdot G_q(\theta=0) \cdot \frac{1}{\rho_{\mathrm{rat}}} \tag{3.41}$$

with

$$G_q = \frac{1}{q}\frac{rB}{RB_p} \tag{3.42}$$

3.3.2 Radial wave vector

We have:

$$k_{r*}\rho_{\mathrm{ref}}{}^{\texttt{IMAS}} = k_x \sqrt{g^{xx}(\theta=0)}\rho_{\mathrm{ref}}^{\texttt{IMAS}} \qquad \text{and} \qquad \texttt{KX0\_LOC} = \frac{k_x}{k_y} \tag{3.43}$$

We can then define:

$$k_{r*}\rho_{\mathrm{ref}}{}^{\texttt{IMAS}} = \texttt{KX0\_LOC} \cdot \texttt{KY} \cdot \frac{1}{\rho_{\mathtt{rat}}} \sqrt{g^{xx}(\theta=0)} = \texttt{KX0\_LOC} \cdot \texttt{KY} \cdot \frac{1}{\rho_{\mathtt{rat}}} \cdot |\nabla r|_{\theta=0} \tag{3.44}$$

Chapter 4

Outputs

4.1 Fields

The fields in IMAS are normalised as follows:

$$A_{\parallel N}^{\texttt{IMAS}} = \frac{1}{L_{\mathrm{ref}}^{\texttt{IMAS}} B_{\mathrm{ref}}^{\texttt{IMAS}}} \left(\frac{R_0^{\texttt{IMAS}}}{\rho_{\mathrm{ref}}^{\texttt{IMAS}}} \right)^2 \cdot A_\parallel \qquad \text{and} \qquad B_{\parallel N}^{\texttt{IMAS}} = \frac{1}{B_{\mathrm{ref}}^{\texttt{IMAS}}} \frac{R_0^{\texttt{IMAS}}}{\rho_{\mathrm{ref}}^{\texttt{IMAS}}} \cdot B_\parallel \tag{4.1}$$

And

$$\phi_N^{\texttt{IMAS}} = \frac{q_{\mathrm{ref}}^{\texttt{IMAS}} R_0^{\texttt{IMAS}}}{T_{\mathrm{ref}}^{\texttt{IMAS}} \rho_{\mathrm{ref}}^{\texttt{IMAS}}} \cdot \phi \tag{4.2}$$

The mode amplitude used to normalise linear runs and compute the mode growth rate in IMAS is:

$$\mathcal{A}_f^{\texttt{IMAS}} = \sqrt{\frac{1}{2\pi} \int \left[|\hat{\phi}_N^{\texttt{IMAS}}|^2 + |\hat{A}_{\parallel N}^{\texttt{IMAS}}|^2 + |\hat{B}_{\parallel N}^{\texttt{IMAS}}|^2 \right] \mathrm{d}\theta} \tag{4.3}$$

And the ratio of the IMAS to GKW mode amplitudes is therefore given by:

$$\mathcal{A}_{\mathrm{rat}} = \frac{1}{\mathcal{A}_f^{\texttt{IMAS}}} \tag{4.4}$$

The Gyro-Bohm normalisation is used for the fields in TGLF, it then comes:

$$\hat{\phi}_N^{\texttt{IMAS}} = \hat{\phi}_N^{\texttt{TGLF}} \cdot \frac{T_{\mathrm{rat}} \rho_{\mathrm{rat}}}{q_{\mathrm{rat}} L_{\mathrm{rat}}} \cdot \mathcal{A}_{\mathrm{rat}}, \qquad \hat{A}_{\parallel N}^{\texttt{IMAS}} = -\hat{A}_{\parallel N}^{\texttt{TGLF}} \cdot \frac{B_{\mathrm{rat}} \rho_{\mathrm{rat}}^2}{L_{\mathrm{rat}}} \cdot \mathcal{A}_{\mathrm{rat}}, \qquad \hat{B}_{\parallel N}^{\texttt{IMAS}} = -\hat{B}_{\parallel N}^{\texttt{TGLF}} \cdot \frac{B_{\mathrm{rat}} \rho_{\mathrm{rat}}}{L_{\mathrm{rat}}} \cdot \mathcal{A}_{\mathrm{rat}} \tag{4.5}$$

Where $\hat{\phi}_N^{\texttt{TGLF}}$, $\hat{A}_{\parallel N}^{\texttt{TGLF}}$, $\hat{B}_{\parallel N}^{\texttt{TGLF}}$ are found in the file `out.tglf.wavefunction`

4.2 Mode growth rate and frequency

In IMAS, the normalised mode growth rate $\gamma^{\texttt{IMAS}}$ and frequency $\omega_r^{\texttt{IMAS}}$ are defined as:

$$\gamma_N^{\texttt{IMAS}} = \gamma \cdot \frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{v_{\mathrm{thref}}^{\texttt{IMAS}}} \qquad \text{and} \qquad \omega_{rN}^{\texttt{IMAS}} = \omega_r \cdot \frac{L_{\mathrm{ref}}^{\texttt{IMAS}}}{v_{\mathrm{thref}}^{\texttt{IMAS}}} \tag{4.6}$$

In TGLF we have:

$$\gamma_N^{\texttt{TGLF}} = \gamma \cdot \frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{v_{\mathrm{thref}}^{\texttt{TGLF}}} \qquad \text{and} \qquad \omega_{rN}^{\texttt{TGLF}} = \omega_r \cdot \frac{L_{\mathrm{ref}}^{\texttt{TGLF}}}{v_{\mathrm{thref}}^{\texttt{TGLF}}} \tag{4.7}$$

Then it comes:

$$\gamma_N^{\texttt{IMAS}} = \gamma_N^{\texttt{TGLF}} \cdot \frac{v_{\mathrm{thrat}}}{L_{\mathrm{rat}}} \qquad \text{and} \qquad \omega_{rN}^{\texttt{IMAS}} = \omega_{rN}^{\texttt{TGLF}} \cdot \frac{v_{\mathrm{thrat}}}{L_{\mathrm{rat}}} \tag{4.8}$$

Where $\gamma_N^{\texttt{TGLF}}$ and $\omega_{rN}^{\texttt{TGLF}}$ are found in the file `out.tglf.eigenvalue_spectrum`

4.3 Fluxes

The particle, energy and toroidal angular momentum fluxes in IMAS are defined as:

$$\Gamma_{sN}^{\texttt{IMAS}} = \frac{{L_{\text{ref}}^{\texttt{IMAS}}}^2}{n_s v_{\text{thref}}^{\texttt{IMAS}} \, {\rho_{\text{ref}}^{\texttt{IMAS}}}^2} \cdot \Gamma_s^{\texttt{IMAS}} \tag{4.9}$$

$$\Pi_{sN}^{\parallel} = \frac{{L_{\text{ref}}^{\texttt{IMAS}}}^2}{n_s m_{\text{ref}}^{\texttt{IMAS}} L_{\text{ref}}^{\texttt{IMAS}} \, {v_{\text{thref}}^{\texttt{IMAS}}}^2 \, {\rho_{\text{ref}}^{\texttt{IMAS}}}^2} \cdot \Pi_s^{\parallel} \tag{4.10}$$

$$\Pi_{sN}^{\perp} = \frac{{L_{\text{ref}}^{\texttt{IMAS}}}^2}{n_s m_{\text{ref}}^{\texttt{IMAS}} L_{\text{ref}}^{\texttt{IMAS}} \, {v_{\text{thref}}^{\texttt{IMAS}}}^2 \, {\rho_{\text{ref}}^{\texttt{IMAS}}}^2} \cdot \Pi_s^{\perp} \tag{4.11}$$

$$Q_{sN}^{\texttt{IMAS}} = \frac{{L_{\text{ref}}^{\texttt{IMAS}}}^2}{n_s T_{\text{ref}}^{\texttt{IMAS}} v_{\text{thref}}^{\texttt{IMAS}} \, {\rho_{\text{ref}}^{\texttt{IMAS}}}^2} \cdot Q_s^{\texttt{IMAS}} \tag{4.12}$$

In TGLF, the energy and particle fluxes are normalized to the gyro-reduced flux, $n_e^{\texttt{TGLF}} T_e^{\texttt{TGLF}} c_s^{\texttt{TGLF}} (\rho_s^{\texttt{TGLF}}/a)^2$ for energy and $n_e^{\texttt{TGLF}} c_s^{\texttt{TGLF}} (\rho_s^{\texttt{TGLF}}/a)^2$ for particle flux.

It then comes:

$$\Gamma_{sN}^{\texttt{IMAS}} = 2\Gamma_{sN}^{\texttt{TGLF}} \cdot \mathcal{A}_{\text{rat}}^2 \cdot \frac{n_{\text{rat}} v_{\text{thrat}} \, {\rho_{\text{rat}}}^2}{{L_{\text{rat}}}^2} \tag{4.13}$$

$$\Pi_{sN\parallel}^{\texttt{IMAS}} = 2\Pi_{sN\parallel}^{\texttt{TGLF}} \cdot \mathcal{A}_{\text{rat}}^2 \cdot \frac{m_{\text{rat}} L_{\text{rat}} n_{\text{rat}} v_{\text{thrat}}^2 \, {\rho_{\text{rat}}}^2}{{L_{\text{rat}}}^2} \tag{4.14}$$

$$\Pi_{sN\perp}^{\texttt{IMAS}} = 2\Pi_{sN\perp}^{\texttt{TGLF}} \cdot \mathcal{A}_{\text{rat}}^2 \cdot \frac{m_{\text{rat}} L_{\text{rat}} n_{\text{rat}} v_{\text{thrat}}^2 \, {\rho_{\text{rat}}}^2}{{L_{\text{rat}}}^2} \tag{4.15}$$

$$Q_{sN}^{\texttt{IMAS}} = 2Q_{sN}^{\texttt{TGLF}} \cdot \mathcal{A}_{\text{rat}}^2 \cdot \frac{n_{\text{rat}} T_{\text{rat}} v_{\text{thrat}} \, {\rho_{\text{rat}}}^2}{{L_{\text{rat}}}^2} \tag{4.16}$$

Where $\Gamma_{sN}^{\texttt{TGLF}}$, $\Pi_{sN\parallel}^{\texttt{TGLF}}$, $\Pi_{sN\perp}^{\texttt{TGLF}}$ and $Q_{sN}^{\texttt{TGLF}}$ are found in the file `out.tglf.QL_flux_spectrum`

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