$\boxed{\text{Seminar 8}}$ $\boxed{8.04.2020}$

Forme pătratice. Formă canonică

Ex1 Fie $Q:\mathbb{R}^3 \to \mathbb{R}$, $Q(x) = x_1^2+x_2^2+x_3^2+x_1x_2+x_1x_3+x_2x_3$

a) Să se determine G matricea asociată formei pătratice Q, în raport cu reperul canonic.

b) Să se determine forma polară $g:\mathbb{R}^3\times\mathbb{R}^3 \to \mathbb{R}$, asociată lui Q

c) Să se aducă Q la o formă canonică, utilizând metoda Gauss, metoda Jacobi. Precizați signatura. Este Q pozitiv definită?

SOL

a) $Q:\mathbb{R}^3 \to \mathbb{R}$, $Q(x) = \sum_{i,j=1}^{3} g_{ij} x_i x_j$

$2g_{12}=1;\ 2g_{13}=1;\ 2g_{23}=1$

$$G = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}.$$

b) $g:\mathbb{R}^3\times\mathbb{R}^3 \to \mathbb{R}$ forma polară asociată lui Q.

$g(x,y) = \frac{1}{2}\left[Q(x+y) - Q(x) - Q(y)\right]$

$g(x,y) = \sum_{i,j=1}^{3} g_{ij} x_i y_j = x_1y_1 + x_2y_2 + x_3y_3 + \frac{1}{2}x_1y_2 + \frac{1}{2}x_2y_1 + \frac{1}{2}x_1y_3 + \frac{1}{2}x_3y_1 + \frac{1}{2}x_2y_3 + \frac{1}{2}x_3y_2.$

c) Metoda Gauss

$$Q(x) = x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_1x_3 + x_2x_3$$

$$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 - \frac{1}{4}x_2^2 - \frac{1}{4}x_3^2 - \frac{1}{2}x_2x_3 + x_2^2 + x_3^2 + x_2x_3 =$$

$$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}x_2^2 + \frac{1}{2}x_2x_3 + \frac{3}{4}x_3^2 =$$

$$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}\left(x_2^2 + \frac{2}{3}x_2x_3\right) + \frac{3}{4}x_3^2 =$$

$$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}\left(x_2 + \frac{1}{3}x_3\right)^2 - \frac{1}{12}x_3^2 + \frac{3}{4}x_3^2$$

$$= \left(x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3\right)^2 + \frac{3}{4}\left(x_2 + \frac{1}{3}x_3\right)^2 + \frac{2}{3}x_3^2$$

Fie schimbarea de reper:

$$\begin{cases} x_1' = x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 \\ x_2' = x_2 + \frac{1}{3}x_3 \\ x_3' = x_3 \end{cases} \Rightarrow Q(x) = x_1'^2 + \frac{3}{4}x_2'^2 + \frac{2}{3}x_3'^2$$

$(3,0)$ signatura $\Rightarrow Q$ poz. definită.

$$\begin{cases} x_1 = x_1' - \frac{1}{2}\left(x_2' - \frac{1}{3}x_3'\right) - \frac{1}{2}x_3' \\ x_2 = x_2' - \frac{1}{3}x_3' \\ x_3 = x_3' \end{cases} \Rightarrow \begin{cases} x_1 = x_1' - \frac{1}{2}x_2' - \frac{1}{3}x_3' \\ x_2 = x_2' - \frac{1}{3}x_3' \\ x_3 = x_3' \end{cases}$$

$$\mathcal{R} \xrightarrow{C} \mathcal{R}' \qquad e_k' = \sum_{i=1}^{3} c_{ik} e_i$$

$$x = \sum_{i=1}^{3} x_i e_i = \sum_{k=1}^{3} x_k' \left(\sum_{i=1}^{3} c_{ik} e_i \right) = \sum_{i,k=1}^{3} c_{ik} x_k' e_i$$

$$x_i = \sum_{k=1}^{3} c_{ik} x_k' \Rightarrow X = CX'$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$$

$\mathcal{R}' = \{ e_1' = e_1 ;\ e_2' = -\frac{1}{2} e_1 + e_2 ;\ e_3' = -\frac{1}{3} e_1 - \frac{1}{3} e_2 + e_3 \}$.

În raport cu acest reper Q are matricea asociată

$$G' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{4} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix}$$

Metoda Jacobi

$$G = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

$\Delta_1 = 1 \neq 0$

$$\Delta_2 = \begin{vmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{vmatrix} = 1 - \frac{1}{4} = \frac{3}{4} \neq 0$$

$$\Delta_3 = \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = 2 \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{vmatrix} =$$

$$= 2 \cdot \frac{1}{4} = \frac{1}{2} \neq 0$$

$$\frac{1}{\Delta_1} = 1 ;\quad \frac{\Delta_1}{\Delta_2} = \frac{1}{\frac{3}{4}} = \frac{4}{3} ;\quad \frac{\Delta_2}{\Delta_3} = \frac{\frac{3}{4}}{\frac{1}{2}} = \frac{3}{2}.$$

$\exists$ un reper $\mathcal{R}'$ în $\mathbb{R}^3$ a.î. $Q(x) = \frac{1}{\Delta_1} x_1'^2 + \frac{\Delta_1}{\Delta_2} x_2'^2 + \frac{\Delta_2}{\Delta_3} x_3'^2$

$$Q(x) = x_1'^2 + \frac{4}{3} x_2'^2 + \frac{3}{2} x_3'^2.$$

$(3,0)$ signatura

Ex 2 $\quad Q : \mathbb{R}^3 \to \mathbb{R},\ Q(x) = 2x_1x_2 - 6x_1x_3 - 6x_2x_3$

a) Să se afle matricea G asociată în rap. cu reperul canonic

b) Să se afle forma polară $g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ asociată

c) Să se aducă Q la o formă canonică.

SOL a)

$$G = \begin{pmatrix} 0 & 1 & -3 \\ 1 & 0 & -3 \\ -3 & -3 & 0 \end{pmatrix}$$

b) $g(x,y) = \sum_{i,j=1}^{3} g_{ij} x_i y_j$

$= x_1y_2 + x_2y_1 - 3x_1y_3 - 3x_3y_1 - 3x_2y_3 - 3x_3y_2$.

-) $Q(x) = 2x_1x_2 - 6x_1x_3 - 6x_2x_3$

Metoda Gauss

Fie schimbarea de reper.

$$\begin{cases} x_1' = x_1 + x_2 \\ x_2' = x_1 - x_2 \\ x_3' = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2}(x_1' + x_2') \\ x_2 = \frac{1}{2}(x_1' - x_2') \\ x_3 = x_3' \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$$

$$Q(x) = 2 \cdot \frac{1}{4}(x_1'^2 - x_2'^2) - 6x_3'x_1' = \frac{1}{2}x_1'^2 - \frac{1}{2}x_2'^2 - 6x_1'x_3'$$

$$= 2\left(\frac{1}{4}x_1'^2 - 3x_1'x_3'\right) - \frac{1}{2}x_2'^2 =$$

$$= 2\left(\frac{1}{2}x_1' - 3x_3'\right)^2 - 18x_3'^2 - \frac{1}{2}x_2'^2$$

Fie schimbarea de reper:

$$\begin{cases} x_1'' = \frac{1}{2}x_1' - 3x_3' \\ x_2'' = x_2' \\ x_3'' = x_3' \end{cases} \Rightarrow \begin{cases} x_1' = 2(x_1'' + 3x_3'') \\ x_2' = x_2'' \\ x_3' = x_3'' \end{cases} \Rightarrow \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 2 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1'' \\ x_2'' \\ x_3'' \end{pmatrix}$$

$\Rightarrow Q(x) = 2x_1''^2 - \frac{1}{2}x_2''^2 - 18x_3''^2$; $(1,2)$ signatura

Q nu e ~~poz~~ pozitiv def.

$$R \xrightarrow{C} R' \xrightarrow{D} R''$$

$$X = CX' \ ; \ X' = DX''$$

$$C = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}; \ R': \begin{cases} e_1' = \frac{1}{2}e_1 + \frac{1}{2}e_2 \\ e_2' = \frac{1}{2}e_1 - \frac{1}{2}e_2 \\ e_3' = e_3 \end{cases}$$

$$D = \begin{pmatrix} 2 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R'': \begin{cases} e_1'' = 2e_1' \\ e_2'' = e_2' \\ e_3'' = 6e_1' + e_3' \end{cases}$$

$R = \{e_1 = (1,0,0), e_2 = (0,1,0), e_3 = (0,0,1)\}$ reperul canonic.

În raport cu $R'' = \{e_1'', e_2'', e_3''\}$ matricea asociată lui Q este $G'' = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -18 \end{pmatrix}$

OBS $\begin{cases} y_1 = \sqrt{2}x_1'' \\ y_2 = \frac{1}{\sqrt{2}}x_2'' \\ y_3 = 3\sqrt{2}x_3'' \end{cases} \Rightarrow Q(x) = y_1^2 - y_2^2 - y_3^2$ (forma normală)

OBS Metoda Jacobi nu se poate aplica: $\Delta_1 = 0$ $\quad G = \begin{pmatrix} 0 & 1 & -3 \\ 1 & 0 & -3 \\ -3 & -3 & 0 \end{pmatrix}$

(Ex3) Fie $g, g_s, g_a : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ forme biliniare.

$$G = \begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}, \quad G_s = \frac{1}{2}(G + G^T), \quad G_a = \frac{1}{2}(G - G^T)$$

matricele asociate lui g, g_s, respectiv g_a, în raport cu reperul canonic.

a) Să se determine g, g_s, g_a

b) Fie $Q : \mathbb{R}^3 \to \mathbb{R}$ forma pătratică asociată lui g_s. Să se aducă la o formă canonică. Este Q pozitiv definită?

Sol a) $G_s = \frac{1}{2}(G + G^T) = \frac{1}{2}\left(\begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} 2 & -1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}\right) \Rightarrow$

$$G_s = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$$

$G_a = \frac{1}{2}(G - G^T) = \frac{1}{2}\left(\begin{pmatrix} 2 & 1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 0 \\ 1 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}\right) \Rightarrow$

$$G_a = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}, \; g(x,y) = 2x_1y_1 - x_2y_2 - 2x_3y_3 + x_1y_2 - x_2y_1 - x_2y_3 - x_3y_2.$

$g_a : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}, \; g_a(x,y) = x_1y_2 - x_2y_1.$

$g_s : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}, \; g_s(x,y) = 2x_1y_1 - x_2y_2 - 2x_3y_3 - x_2y_3 - x_3y_2.$

b) $Q : \mathbb{R}^3 \to \mathbb{R}, \; Q(x) = \underline{2x_1^2} - \underline{\underline{x_2^2}} - 2x_3^2 - \underline{\underline{2x_2x_3}}.$

Met Gauss

$Q(x) = 2x_1^2 - (x_2^2 + 2x_2x_3 + x_3^2) - x_3^2 = 2x_1^2 - (x_2 + x_3)^2 - x_3^2$

Fie schimbarea de reper: $\quad R \xrightarrow{C} R'$

$$\begin{cases} x_1' = x_1 \\ x_2' = x_2 + x_3 \\ x_3' = x_3 \end{cases} \Rightarrow \begin{cases} x_1 = x_1' \\ x_2 = x_2' - x_3' \\ x_3 = x_3' \end{cases} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}}_{C} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$$

$Q(x) = 2x_1'^2 - x_2'^2 - x_3'^2$ în reperul $R' = \{e_1' = e_1, e_2' = e_2, e_3' = -e_2 + e_3\}$.

(1,2) signatura

Metoda Jacobi

$$G_S = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix}$$

$\Delta_1 = 2$; $\Delta_2 = \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2$

$$\Delta_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -2 \end{vmatrix} = 2\begin{vmatrix} -1 & -1 \\ -1 & -2 \end{vmatrix} = 2(2-1) = 2$$

$\frac{1}{\Delta_1} = \frac{1}{2}$; $\frac{\Delta_1}{\Delta_2} = \frac{2}{-2} = -1$; $\frac{\Delta_2}{\Delta_3} = \frac{-2}{2} = -1$

$\exists$ un reper R' a.î. Q are formă canonică

$Q(x) = \frac{1}{2}x_1'^2 - x_2'^2 - x_3'^2$ (1,2) signatura (invar)

Ex4

Fie $g : \mathcal{M}_2(\mathbb{R}) \times \mathcal{M}_2(\mathbb{R}) \to \mathbb{R}$,

$g(X,Y) = 2Tr(X\cdot Y) - Tr(X)\cdot Tr(Y)$, $\forall X, Y \in \mathcal{M}_2(\mathbb{R})$

a) g este formă biliniară simetrică

b) Să se precizeze G matricea asociată lui g în raport cu reperul canonic

$R = \{E_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, E_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, E_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, E_{22} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\}$.

c) Fie $Q : \mathcal{M}_2(\mathbb{R}) \to \mathbb{R}$ forma pătratică asociată lui g. Este Q pozitiv definită?

SOL

a) $g(X,Y) = g(Y,X)$, $\forall X, Y \in \mathcal{M}_2(\mathbb{R})$, deoarece $Tr(XY) = Tr(YX)$

Arătăm că g este liniară în primul argument

$g(aX+bY, Z) = ag(X,Z) + bg(Y,Z)$, $\forall X, Y, Z \in \mathcal{M}_2(\mathbb{R})$ $\forall a, b \in \mathbb{R}$

$g(aX+bY, Z) = 2Tr((aX+bY)Z) - Tr(aX+bY)\,Tr Z =$

$= \underline{2aTr(XZ)} + \underline{\underline{2bTr(YZ)}} - \underline{aTrX\,TrZ} - \underline{\underline{bTrY\,TrZ}}$

$= ag(X,Z) + bg(Y,Z)$

$\Rightarrow$ g este formă biliniară simetrică.

b) $X = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} = x_1E_{11} + x_2E_{12} + x_3E_{21} + x_4E_{22}$.

$\wr$

$\mathbb{R}^4 \ni (x_1, x_2, x_3, x_4) = x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4$

$X\cdot Y = \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}\begin{pmatrix} y_1 & y_2 \\ y_3 & y_4 \end{pmatrix} = \begin{pmatrix} x_1y_1 + x_2y_3 & x_1y_2 + x_2y_4 \\ x_3y_1 + x_4y_3 & x_3y_2 + x_4y_4 \end{pmatrix}$

$2Tr(XY) = 2(x_1y_1 + x_2y_3 + x_3y_2 + x_4y_4)$

$\operatorname{Tr}X \operatorname{Tr}Y = (x_1+x_4)(y_1+y_4) = x_1y_1 + x_1y_4 + x_4y_1 + x_4y_4$

$g(X,Y) = 2\operatorname{Tr}(X\cdot Y) - \operatorname{Tr}(X)\operatorname{Tr}Y =$

$= 2(x_1y_1 + x_2y_3 + x_3y_2 + x_4y_4) - (x_1y_1 + x_1y_4 + x_4y_1 + x_4y_4) =$

$g(X,Y) = \underline{x_1y_1} + 2x_2y_3 + 2x_3y_2 + \underline{x_4y_4} - x_1y_4 - x_4y_1$.

$= \sum_{i,j=1}^{4} g_{ij}x_iy_j$

$$G = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

g formă biliniară simetrică $(G = G^T)$

$Q : \mathcal{M}_2(\mathbb{R}) \to \mathbb{R}$

$Q(x) = \sum_{i,j=1}^{4} g_{ij}x_ix_j = \underline{x_1^2} + x_4^2 - \underline{2x_1x_4} + 4x_2x_3$

Met Gauss

$Q(x) = (x_1 - x_4)^2 + 4x_2x_3$

Fie sch. de reper: $\Rightarrow Q(x) = x_1'^2 + 4\underline{x_2'x_3'}$

$\begin{cases} x_1' = x_1 - x_4 \\ x_j' = x_j, \ j = \overline{2,4} \end{cases}$

$\begin{cases} x_2'' = x_2' + x_3' \\ x_3'' = x_2' - x_3' \\ x_j'' = x_j', \ j \in \{1,4\} \end{cases} \Rightarrow \begin{cases} x_2' = \frac{1}{2}(x_2'' + x_3'') \\ x_3' = \frac{1}{2}(x_2'' - x_3'') \\ x_j' = x_j'', \ j \in \{1,4\} \end{cases}$

$Q(x) = x_1''^2 + x_2''^2 - x_3''^2$; $(2,1)$ signatura

Q nu e poz. def.

Ex. Fie $Q : \mathbb{R}^3 \to \mathbb{R}$, $Q(x) = x_1^2 + 3x_2^2 + 4x_2x_3$

Să se aducă Q la o formă canonică, utilizând metoda valorilor proprii

SOL

$P(\lambda) = \det(G - \lambda I_3) = 0$.

$$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 3-\lambda & 2 \\ 0 & 2 & -\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(\underbrace{\lambda^2 - 3\lambda - 4}_{(\lambda+1)(\lambda-4)}) = 0$

$\lambda_1 = 1$, $m_1 = 1$

$\lambda_2 = 4$, $m_2 = 1$

$\lambda_3 = -1$, $m_3 = 1$.

• $V_{\lambda_1} = \{x \in \mathbb{R}^3 \mid GX = 1 \cdot X\} = \{(x_1,0,0) \mid x_1 \in \mathbb{R}\} = \langle \{(1,0,0)\} \rangle.$

$(G - I_3)X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 2x_2 + 2x_3 = 0 \\ 2x_2 - x_3 = 0 \end{cases}$

$\Rightarrow x_2 = x_3 = 0$

$\dim V_{\lambda_1} = 3 - 2 = 1.$

• $V_{\lambda_2} = \{x \in \mathbb{R}^3 \mid GX = 4X\} = \{(0,x_2,0) \mid x_2 \in \mathbb{R}\} = \langle \{(0,1,0)\} \rangle$

$(G - 4I_3)X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} -3 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad x_1 = 0$, $-x_1 + 2x_3 = 0$, $x_3 = 0.$

$\dim V_{\lambda_2} = 3 - 2 = 1.$

• $V_{\lambda_3} = \{x \in \mathbb{R}^3 \mid GX = -X\} = \{(0,x_2,-2x_2) \mid x_2 \in \mathbb{R}\} = \langle \{(0,1,-2)\} \rangle$

$(G + I_3)X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\quad x_1 = 0.$ $\quad 4x_2 + 2x_3 = 0$, $x_3 = -2x_2$

$\dim V_{\lambda_2} = 3 - 2 = 1.$

$\exists\, R' = \{(1,0,0), (0,1,0), (0,1,-2)\}$ ai matricea asoc. lui Q este $G' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

$Q(x) = x_1'^2 + 4x_2'^2 - x_3'^2$; $(2,1)$ signatura ; nu e ~~poz~~ def.

<u>OBS</u> <u>Met Gauss</u> $\quad Q(x) = x_1^2 + 3x_2^2 + 4x_2x_3$

$Q(x) = x_1^2 + 3\left(x_2^2 + \frac{4}{3}x_2x_3\right) = x_1^2 + 3\left(x_2 + \frac{2}{3}x_3\right)^2 - \frac{4}{3}x_3^2$

$\begin{cases} x_1' = x_1 \\ x_2' = x_2 + \frac{2}{3}x_3 \\ x_3' = x_3 \end{cases}$ $\quad Q(x) = x_1'^2 + 3x_2'^2 - \frac{4}{3}x_3'^2$ $\quad (2,1)$ sign.

• <u>Met Jacobi</u>

$G = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 0 \end{pmatrix}$

$\Delta_1 = 1$; $\Delta_2 = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 3$; $\Delta_3 = \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} = -4.$

$\frac{1}{\Delta_1} = 1$; $\frac{\Delta_1}{\Delta_2} = \frac{1}{3}$; $\frac{\Delta_2}{\Delta_3} = -\frac{3}{4}.$

$\exists R'$ in $\mathbb{R}^3$ ai $Q(x) = x_1'^2 + \frac{1}{3}x_2'^2 - \frac{3}{4}x_3'^2$ $\quad (2,1)$ sign.

Tema 4 (seminar)

① $f:\mathbb{R}^3 \longrightarrow \mathbb{R}^3$, $f(x) = (x_1 - 2x_2, -2x_1 + 2x_2 - 2x_3, -2x_2 + 3x_3)$

Este f un endomorfism diagonalizabil?

② Fie $G = \begin{pmatrix} 3 & -2 & -4 \\ -2 & 6 & 2 \\ -4 & 2 & 3 \end{pmatrix}$

matricea asociată formei pătratice $Q : \mathbb{R}^3 \longrightarrow \mathbb{R}$ în raport cu reperul canonic.

a) Să se determine Q.

b) — " — $g : \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$ forma polară asociată și $\mathrm{Ker}(g)$

c) Să se aducă Q la o formă canonică, utilizând metoda Gauss, metoda Jacobi, metoda valorilor proprii. Este Q poz. definită?