13.05.2020.

Seminarul 12

Geometrie analitică euclidiană. Izometrii

Ex1 $(\mathcal{E}_3, (E_3, \langle \cdot,\cdot \rangle), \varphi)$

Fie dreptele $D_1: \begin{cases} x_1+x_3=0 \\ x_2-x_3-1=0 \end{cases}$, $D_2: \begin{cases} x_2=0 \\ x_3=0. \end{cases}$

a) Să se determine ecuația perpendicularei comune a dreptelor D_1, D_2.

b) Să se afle $dist(D_1, D_2)$.

SOL

a) • $D_1: \begin{cases} x_1+x_3=0 \\ x_2-x_3-1=0 \end{cases} \Rightarrow \begin{cases} x_1=-t \\ x_2=1+t,\ x_3=t \end{cases}$

$D_1: \frac{x_1}{-1} = \frac{x_2-1}{1} = \frac{x_3}{1} = t$, $A_1(0,1,0)$, $u=(-1,1,1)$

• $D_2: \begin{cases} x_2=0 \\ x_3=0. \end{cases}$

$D_2: \frac{x_1}{1} = \frac{x_2}{0} = \frac{x_3}{0} = s \Rightarrow \begin{cases} x_1=s \\ x_2=0 \\ x_3=0. \end{cases}$ $A_2(0,0,0)$, $v=(1,0,0)$

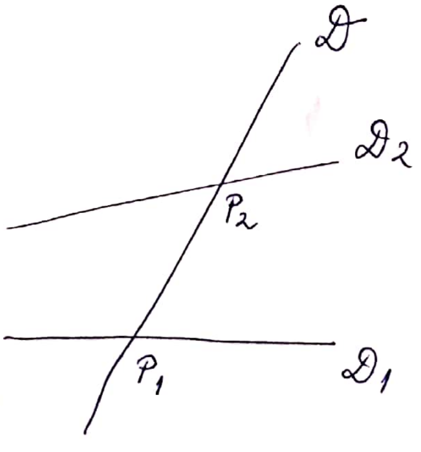
OBS · $D_1: \begin{cases} \pi_1: x_1+x_3=0 \\ \pi_2: x_2-x_3-1=0 \end{cases}$ $N_1=(1,0,1)$, $N_2=(0,1,-1)$.

$$u = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} = (-1,1,1)$$

Fie. $x_3=0 \Rightarrow x_1=0$, $x_2=1$. $\Rightarrow A_1(0,1,0) \in D_1$

• D_1 și D sunt drepte necoplanare; $\overrightarrow{A_1A_2}=(0,-1,0)$

$$\Delta_c = \begin{vmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} \neq 0$$



(M1) $P_1(-t, 1+t, t) \in D_1 \cap D$

$P_2(s,0,0) \in D_2 \cap D$.

$\overrightarrow{P_1P_2} = (s+t, -1-t, -t)$, $u=(-1,1,1)$, $v=(1,0,0)$

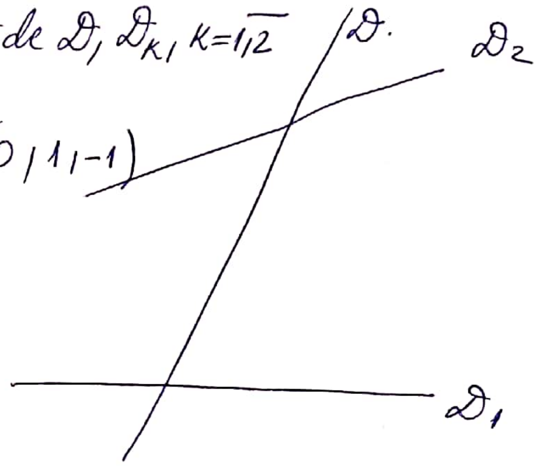
$$\begin{cases} \langle \overrightarrow{P_1P_2}, u \rangle = 0 \Rightarrow -s-t-1-t-t=0 \\ \langle \overrightarrow{P_1P_2}, v \rangle = 0 \Rightarrow s+t=0 \end{cases} \Rightarrow \begin{cases} t=-\frac{1}{2} \\ s=\frac{1}{2} \end{cases}$$

$P_1\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)$, $P_2\left(\frac{1}{2}, 0, 0\right)$; $\overrightarrow{P_1P_2} = \left(0, -\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{2}(0,-1,1)$

$$\mathcal{D}: \frac{x_1-\frac{1}{2}}{0} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$dist(\mathcal{D}_1, \mathcal{D}_2) = dist(P_1, P_2) = \|\overrightarrow{P_1P_2}\| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$$

M2 π_k = planul determinat de $\mathcal{D}, \mathcal{D}_k$, $k=\overline{1,2}$



$$N = u \times v = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = (0,1,-1)$$

$$\pi_1: \begin{vmatrix} x_1 & -1 & 0 \\ x_2-1 & 1 & 1 \\ x_3 & 1 & -1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x_1 & -1 & 0 \\ x_2+x_3-1 & 2 & 0 \\ x_3 & 1 & -1 \end{vmatrix} = 0$$

$$\pi_1: 2x_1 + x_2 + x_3 - 1 = 0$$

$$\pi_2: \begin{vmatrix} x_1 & 1 & 0 \\ x_2 & 0 & 1 \\ x_3 & 0 & -1 \end{vmatrix} = 0 \Rightarrow \pi_2: x_2 + x_3 = 0$$

$$\mathcal{D} \begin{cases} 2x_1 + x_2 + x_3 - 1 = 0 \\ x_2 + x_3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2} \\ x_2 = -t \\ x_3 = t \end{cases} \Rightarrow \mathcal{D}: \frac{x_1-\frac{1}{2}}{0} = \frac{x_2}{-1} = \frac{x_3}{1}$$

$$dist(\mathcal{D}_1, \mathcal{D}_2) = \frac{|\langle N, \overrightarrow{A_1A_2} \rangle|}{\|N\|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\overrightarrow{A_1A_2} = (0,-1,0)$; $N = (0,1,-1)$

Ex 2. Fie dreptele: $\mathcal{D}_1: \frac{x_1-1}{1} = \frac{x_2-2}{-1} = \frac{x_3+2}{2}$

$\mathcal{D}_2: \begin{cases} 2x_1 - x_3 - 1 = 0 \\ 2x_2 + x_3 + 3 = 0. \end{cases}$

a) Să se arate că $\mathcal{D}_1$ și $\mathcal{D}_2$ sunt drepte coplanare și să se scrie ecuația planului determinat de $\mathcal{D}_1$ și $\mathcal{D}_2$.

b) $dist(\mathcal{D}_1, \mathcal{D}_2) = ?$

SOL

a) $\mathcal{D}_1: \begin{cases} x_1 = 1+t \\ x_2 = 2-t \\ x_3 = -2+2t \end{cases}$ $A_1(1,2,-2) \in \mathcal{D}_1$; $u = (1,-1,2)$

$\mathcal{D}_2: \begin{cases} 2x_1 - x_3 - 1 = 0 \\ 2x_2 + x_3 + 3 = 0 \end{cases} \Rightarrow \begin{cases} 2x_1 = t+1 \\ 2x_2 = -3-t, \ x_3 = t \end{cases} \Rightarrow \begin{cases} x_1 = \frac{1}{2} + \frac{1}{2}t \\ x_2 = -\frac{3}{2} - \frac{1}{2}t \\ x_3 = t \end{cases}$

$A_2\left(\frac{1}{2}, \frac{-3}{2}, 0\right) \in \mathcal{D}_2$, $\tilde{v} = \left(\frac{1}{2}, -\frac{1}{2}, 1\right)$

sau $= \frac{1}{2}(1,-1,2)$, $v = (1,-1,2)$

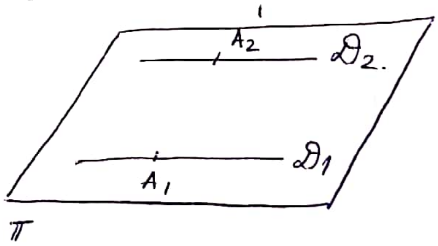
$A_2'(1,-2,1)$

$u = v \Rightarrow V_{\mathcal{D}_1} = V_{\mathcal{D}_2} \Rightarrow \mathcal{D}_1 \parallel \mathcal{D}_2 \Rightarrow$ drepte coplanare.

$\overrightarrow{A_1A_2'} = (0,-4,3)$

π = planul determinat de $\mathcal{D}_1$, $\mathcal{D}_2$.

$\pi: \begin{vmatrix} x_1-1 & 1 & 0 \\ x_2-2 & -1 & -4 \\ x_3+2 & 2 & 3 \end{vmatrix} = 0.$



$N_\pi = u \times \overrightarrow{A_1A_2'} = \begin{vmatrix} i & j & k \\ 1 & -1 & 2 \\ 0 & -4 & 3 \end{vmatrix} = (-3+8, -(3+0), -4)$
$= (5,-3,-4)$

$\pi: 5(x_1-1) - 3(x_2-2) - 4(x_3+2) = 0$

$\pi: 5x_1 - 3x_2 - 4x_3 - 5 + 6 - 8 = 0$

$\pi: 5x_1 - 3x_2 - 4x_3 - 7 = 0.$

b) $dist(\mathcal{D}_1, \mathcal{D}_2) = dist(A_1, \mathcal{D}_2) = \frac{\| v \times \overrightarrow{A_1A_2'} \|}{\|v\|} = \frac{\|N_\pi\|}{\|u\|}$

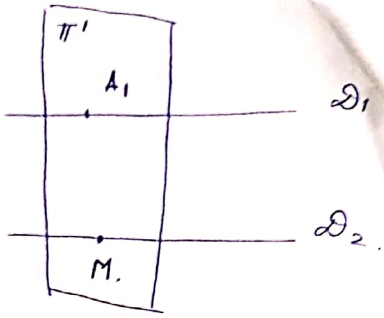
$N = (5,-3,-4)$, $v = u = (1,-1,2)$

$dist(\mathcal{D}_1, \mathcal{D}_2) = \frac{\sqrt{25+9+16}}{\sqrt{1+1+4}} = \frac{\sqrt{50}}{\sqrt{6}} = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

OBS $\pi' \perp \mathcal{D}_2$, $A_1 \in \pi'$

$A_1(1,2,-2)$

$u = N_{\pi'} = (1,-1,2)$



$\pi': 1\cdot(x_1-1)-(x_2-2)+2(x_3+2)=0$

$\pi': x_1-x_2+2x_3-1+2+4=0$

$\pi': x_1-x_2+2x_3+5=0$

$\mathcal{D}_2: \begin{cases} x_1 = \frac{1}{2}+\frac{1}{2}t \\ x_2 = -\frac{3}{2}-\frac{1}{2}t \\ x_3 = t \end{cases}$ $\quad \mathcal{D}_2 \cap \pi' = \{M\}$

$dist(A_1, \mathcal{D}_2) = dist(A_1, M)$.

Ex3. Fie dreapta $\mathcal{D}_1: \frac{x_1-1}{2} = \frac{x_2-1}{-1} = \frac{x_3}{3}$, planele

$\pi_1: x_1+x_2+x_3-1=0$

$\pi_2: x_1-x_2+x_3=0$ și punctul $M(1,2,-1)$.

a) Să se determine ec. dreptei $\mathcal{D}_2 = \pi_1 \cap \pi_2$.

b) Să se afle $\sphericalangle(\mathcal{D}_1, \mathcal{D}_2)$

c) Să se afle $\sphericalangle(\pi_1, \pi_2)$

d) Să se afle coordonatele simetricului lui M față de π_1

SOL

a) $\pi_1 \not\parallel \pi_2$ $\quad N_1 = (1,1,1)$

$N_2 = (1,-1,1)$

$\mathcal{D}_2: \begin{cases} x_1+x_2+x_3-1=0 \\ x_1-x_2+x_3=0 \end{cases} \Rightarrow$

$u_2 = N_1 \times N_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = (2,0,-2) = 2(1,0,-1)$.

$x_3 = 0 \Rightarrow \begin{cases} x_1+x_2=1 \\ x_1-x_2=0 \end{cases}$ $\quad x_1 = \frac{1}{2} = x_2$. $\quad A_2\left(\frac{1}{2}, \frac{1}{2}, 0\right)$

$\mathcal{D}_2: \frac{x_1-\frac{1}{2}}{1} = \frac{x_2-\frac{1}{2}}{0} = \frac{x_3}{-1}$

b) $\measuredangle(\mathcal{D}_1, \mathcal{D}_2) = \measuredangle(u_1, u_2) - \measuredangle\varphi$

$\mathcal{D}_1 : \frac{x_1 - 1}{2} = \frac{x_2 - 1}{-1} = \frac{x_3}{3}$, $u_1 = (2, -1, 3)$

$u_2 = (1, 0, -1)$

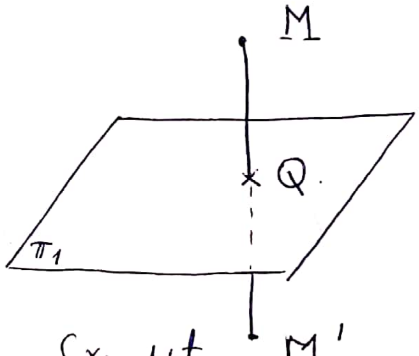
$\cos\varphi = \frac{\langle u_1, u_2 \rangle}{\|u_1\| \cdot \|u_2\|} = \frac{-1}{\sqrt{14}\cdot\sqrt{2}} = \frac{-1}{2\sqrt{7}} = \frac{-\sqrt{7}}{14}.$

$\varphi \in [0, \pi]$, $\varphi = \arccos\left(-\frac{\sqrt{7}}{14}\right) = \pi - \arccos\left(\frac{\sqrt{7}}{14}\right).$

c) $\measuredangle(\pi_1, \pi_2) = \measuredangle(N_1, N_2) = \measuredangle\theta$ $\quad N_1 = (1, 1, 1)$

$\cos\theta = \frac{\langle N_1, N_2 \rangle}{\|N_1\| \cdot \|N_2\|} = \frac{1}{3}$ $\quad N_2 = (1, -1, 1)$

$\theta = \arccos \frac{1}{3}$



d) $\pi_1 : x_1 + x_2 + x_3 - 1 = 0$

$MM' \perp \pi_1 \Rightarrow u_{MM'} = N_1 = (1, 1, 1)$

$M(1, 2, -1)$

$MM' : \frac{x_1 - 1}{1} = \frac{x_2 - 2}{1} = \frac{x_3 + 1}{1} = t \Rightarrow \begin{cases} x_1 = 1 + t \\ x_2 = 2 + t \\ x_3 = -1 + t \end{cases}$

$\{Q\} = MM' \cap \pi : 1 + t + \cancel{2} + t - \cancel{1} + t - \cancel{1} = 0 \Rightarrow 3t = -1$

$t = -\frac{1}{3} \Rightarrow Q\left(\frac{2}{3}, \frac{5}{3}, -\frac{4}{3}\right)$

Q mijlocul $[MM'] \Rightarrow \begin{cases} \frac{2}{3} = \frac{1}{2}(1 + x_1') \\ \frac{5}{3} = \frac{1}{2}(2 + x_2') \\ -\frac{4}{3} = \frac{1}{2}(-1 + x_3') \end{cases}$

$x_1' = \frac{4}{3} - 1 = \frac{1}{3}$

$x_2' = \frac{10}{3} - 2 = \frac{4}{3}$

$x_3' = -\frac{8}{3} + 1 = \frac{-5}{3}$ $\Rightarrow M'\left(\frac{1}{3}, \frac{4}{3}, -\frac{5}{3}\right).$

Izometrii

Ex4. Fie $\tau : \mathcal{E}_2 \to \mathcal{E}_2$, $\tau : X' = AX + B$.

$A = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$, $B = \begin{pmatrix} -1 + \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$

a) Să se arate că $\tau \in Iso(\mathcal{E}_2)$

b) Precizați care este speța, care este mulțimea punctelor fixe și determinați τ.

SOL a) $A = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \cos\frac{\pi}{6} & -\sin\frac{\pi}{6} \\ \sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{pmatrix}$

$A \cdot A^T = I_2 \Rightarrow A \in O(2) \Rightarrow \tau \in Iso(\mathcal{E}_2)$

b) $\det A = \frac{3}{4} + \frac{1}{4} = 1 \Rightarrow A \in SO(2) \Rightarrow \tau$ de speță 1

Mulțimea de pcte fixe : $X' = X \Rightarrow$

$X = AX + B \Rightarrow (J_2 - A)X = B$ $(*)$

$J_2 - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 1-\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & 1-\frac{\sqrt{3}}{2} \end{pmatrix}$

$\det(J_2 - A) = \left(1-\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{4} > 0 \Rightarrow (*)$ are soluție unică (un pct fix Ω)

$$\begin{cases} \left(1-\frac{\sqrt{3}}{2}\right)x_1 + \frac{1}{2}x_2 = -1+\frac{\sqrt{3}}{2} \\ -\frac{1}{2}x_1 + \left(1-\frac{\sqrt{3}}{2}\right)x_2 = \frac{1}{2} \end{cases} \Rightarrow \begin{cases} x_1 = -1 \\ x_2 = 0 \end{cases}$$

$\Omega(-1, 0)$

τ izometrie de speță 1 cu 1 pct fix.

$\tau = R_{\Omega,\varphi}$, $\varphi = \frac{\pi}{6}$

(rotație de centru Ω și $\measuredangle\varphi = \frac{\pi}{6}$).

Ex 5. $\tau : \mathcal{E}_2 \to \mathcal{E}_2$, $\tau : X' = AX + B$, $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

a) Să se arate că $\tau \in Iso(\mathcal{E}_2)$

b) Precizați care este speța, care este mulțimea punctelor fixe și determinați τ.

c) Fie $\mathcal{D}_1 : 2x_1 + 2x_2 = 3$. Să se determine ecuația dreptei $\mathcal{D}_1' = \tau(\mathcal{D}_1)$. Să se arate că $V_{\mathcal{D}_1} = V_{\mathcal{D}_1'}$

d) Fie $\mathcal{D}_2$ $x_1 - x_2 = 1$. Să se arate că $\mathcal{D}_2$ este o dreaptă invariantă a lui τ.

e) Fie $\mathcal{D}_3$ $x_1 + 2x_2 = 1$. Să se determine ec. dreptei $\mathcal{D}_3' = \tau(\mathcal{D}_3)$. Să se arate că $\mathcal{D}_3$ și $\mathcal{D}_3'$ sunt concurente.

SOL

a) $\tau : X' = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} X + \begin{pmatrix} -1 \\ -1 \end{pmatrix}$

$A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$, $A \cdot A^T = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = I_2 \Rightarrow A \in O(2)$

$T : E_2 \longrightarrow E_2$, $T : X' = AX$ transformare ortogonală

(T e urma lui τ)

$\Rightarrow \tau \in Izo(\mathcal{E}_2)$

b) $\det A = -1 \Rightarrow T$ de speță $2 \Rightarrow \tau$ de speță 2.

Determinăm mulțimea de puncte fixe:

$X' = X \Rightarrow X = AX + B \Rightarrow (I_2 - A)X = B$ $(*)$

$I_2 - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

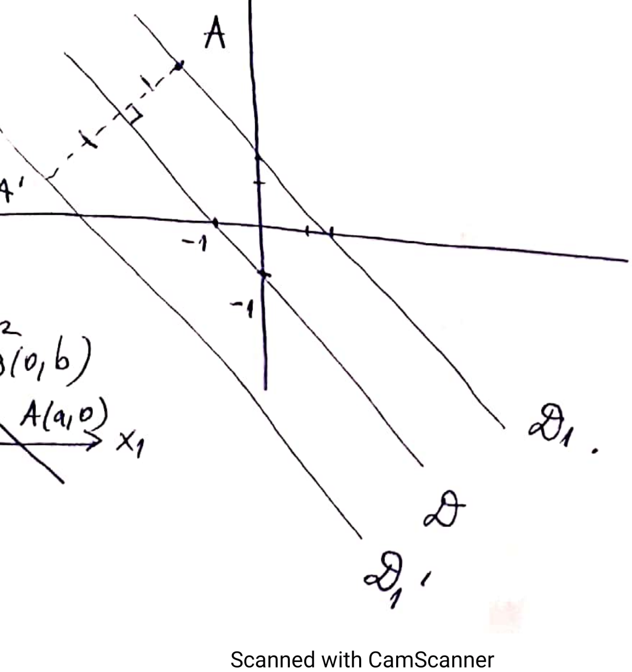
$\det(I_2 - A) = 0$.

$(*) \Rightarrow \mathcal{D} : x_1 + x_2 = -1$ ($\mathcal{D}$ = dreaptă de pcte fixe)

$\tau = S_{\mathcal{D}}$ (simetrie axială)

c) $\mathcal{D}_1 : 2x_1 + 2x_2 = 3$

$\mathcal{D} : x_1 + x_2 = -1$

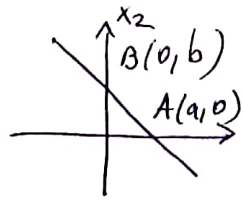


Ec. prin tăieturi:

$\mathcal{D}_1 : \dfrac{x_1}{\frac{3}{2}} + \dfrac{x_2}{\frac{3}{2}} = 1$

$\mathcal{D} : \dfrac{x_1}{-1} + \dfrac{x_2}{-1} = 1$

$\dfrac{x_1}{a} + \dfrac{x_2}{b} = 1$



$\tau: X' = AX + B \Rightarrow \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -1 \\ -1 \end{pmatrix} \Rightarrow$

$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -x_2 - 1 \\ -x_1 - 1 \end{pmatrix} \Rightarrow \begin{cases} x_1 = -x_2' - 1 \\ x_2 = -x_1' - 1 \end{cases}$

$\mathcal{D}_1: 2x_1 + 2x_2 = 3$

$\tau(\mathcal{D}_1) = \mathcal{D}_1': 2(-x_2' - 1) + 2(-x_1' - 1) = 3 \Rightarrow$

$2x_1 + 2x_2 + 7 = 0 \Rightarrow \dfrac{x_1}{-\frac{7}{2}} + \dfrac{x_2}{-\frac{7}{2}} = 1.$

$\mathcal{D} \parallel \mathcal{D}_1 \parallel \mathcal{D}_1'$

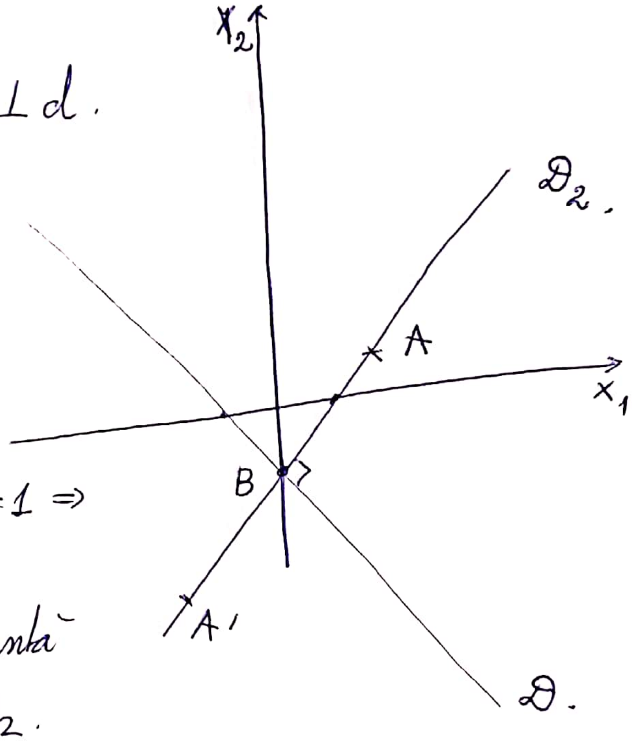
d) $\mathcal{D}_2: x_1 - x_2 = 1 \quad \Rightarrow \mathcal{D}_2 \perp d.$

$\mathcal{D}: x_1 + x_2 = -1$

$B(0,-1) \in \mathcal{D} \cap \mathcal{D}_2$

$\mathcal{D}_2: \dfrac{x_1}{1} + \dfrac{x_2}{-1} = 1.$

$\begin{cases} x_1 = -x_2' - 1 \\ x_2 = -x_1' - 1 \end{cases}$



$\mathcal{D}_2' = \tau(\mathcal{D}_2): -x_2' - 1 + x_1' + 1 = 1 \Rightarrow$

$x_1' - x_2' = 1$

$\mathcal{D}_2 = \mathcal{D}_2' =$ dreaptă invariantă

i.e. $\forall A \in \mathcal{D}_2 \Rightarrow \tau(A) \in \mathcal{D}_2$.

$A' =$ simetricul lui A față de B.

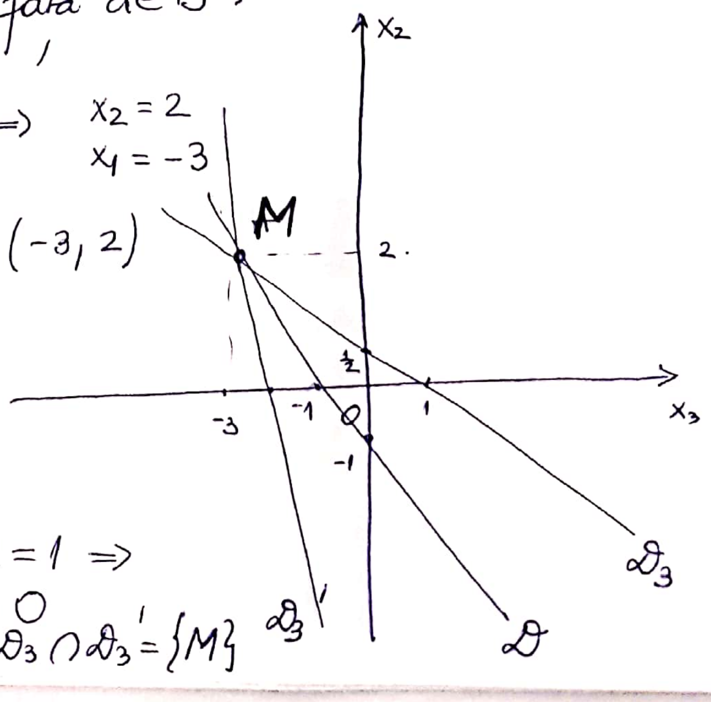
e) $\mathcal{D}_3: x_1 + 2x_2 = 1 \quad \Rightarrow \quad x_2 = 2,\ x_1 = -3$

$\mathcal{D}: x_1 + x_2 = -1.$

$\mathcal{D} \cap \mathcal{D}_3 = \{M\} \Rightarrow M(-3, 2)$

$\mathcal{D}_3: \dfrac{x_1}{1} + \dfrac{x_2}{\frac{1}{2}} = 1.$

$\begin{cases} x_1 = -x_2' - 1 \\ x_2 = -x_1' - 1 \end{cases}$



$\mathcal{D}_3' = \tau(\mathcal{D}_3): -x_2' - 1 - 2x_1' - 2 = 1 \Rightarrow$

$2x_1' + x_2' + 4 = 0$

$M(-3,2) \in \mathcal{D}_3'$

$\mathcal{D} \cap \mathcal{D}_3 \cap \mathcal{D}_3' = \{M\}$

Ex 6. Fie $\Omega(1,3)$, $u(1,2)$.

a) Să se scrie ec. rotației de centru Ω și $\measuredangle\varphi = -\frac{\pi}{3}$,
$\tau_1 = R_{\Omega, -\frac{\pi}{3}}$.

b) Să se scrie ec. lui $\tau_2 = T_u \circ \tau_1$,
T_u = translația de vector u.

SOL

a) $\tau = R_{O,\varphi}$: $X' = AX$, $A = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$

$\tau_1 = R_{\Omega,\varphi}$: $X' - X_0 = A(X - X_0)$, $X_0 = \begin{pmatrix} x_1^0 \\ x_2^0 \end{pmatrix}$, $\Omega(x_1^0, x_2^0)$

$$\tau_1 = R_{\Omega, -\frac{\pi}{3}} : X' = \begin{pmatrix} \cos\left(-\frac{\pi}{3}\right) & -\sin\left(-\frac{\pi}{3}\right) \\ \sin\left(-\frac{\pi}{3}\right) & \cos\left(-\frac{\pi}{3}\right) \end{pmatrix}\left(X - X_0\right) + X_0.$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x_1 - 1 \\ x_2 - 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \frac{1}{2}x_1 - \frac{1}{2} + \frac{\sqrt{3}}{2}x_2 - \frac{3\sqrt{3}}{2} + 1 \\ -\frac{\sqrt{3}}{2}x_1 + \frac{\sqrt{3}}{2} + \frac{1}{2}x_2 - \frac{3}{2} + 3 \end{pmatrix}$$

$$\tau_1 : \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -\frac{3\sqrt{3}}{2} + \frac{1}{2} \\ \frac{\sqrt{3}}{2} + \frac{3}{2} \end{pmatrix}$$

b) T_u : $X' = X + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \xrightarrow{\tau_1} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -\frac{3\sqrt{3}}{2} + \frac{1}{2} \\ \frac{\sqrt{3}}{2} + \frac{3}{2} \end{pmatrix} \xrightarrow{T_u}$$

$$\begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} -\frac{3\sqrt{3}}{2} + \frac{3}{2} \\ \frac{\sqrt{3}}{2} + \frac{7}{2} \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \xrightarrow{\tau_1} \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} \xrightarrow{T_u} \begin{pmatrix} x_1'' \\ x_2'' \end{pmatrix}$$

Ex 7 Fie $\mathcal{D}: x_1 + x_2 - 2 = 0$.

Să se determine ecuația dreptei $\mathcal{D}' = T_u(\mathcal{D})$, unde T_u = translația de vector $u = (2, 3)$

Sol.

$$T_u: \begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1' - 2 \\ x_2' - 3 \end{pmatrix}$$

$\mathcal{D}: x_1 + x_2 - 2 = 0$.

$T_u(\mathcal{D}) = \mathcal{D}': x_1' - 2 + x_2' - 3 - 2 = 0 \Rightarrow x_1' + x_2' - 7 = 0$.

$\mathcal{D}: \dfrac{x_1}{2} + \dfrac{x_2}{2} = 1$

$\mathcal{D}': \dfrac{x_1'}{7} + \dfrac{x_2'}{7} = 1$.

$\overrightarrow{AA'} = u$

$A' = T_u(A)$