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Lucrare I

3. $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $f(x) = (x_1, x_1, x_1 + x_2 + x_3)$

a) $\text{Ker } f = \{x \in \mathbb{R}^3 \mid f(x) = (0, 0, 0)\}$

$$\Rightarrow \begin{cases} x_1 = 0 \\ x_1 = 0 \\ x_1 + x_2 + x_3 = 0 \end{cases} \Rightarrow x_2 = -x_3$$

$$\Rightarrow \text{Ker } f = \{(0, x_2, -x_2) \mid x_2 \in \mathbb{R}\} \\ = \{x_2(0, 1, -1) \mid x_2 \in \mathbb{R}\}$$

b) $\text{Im } f = \{v \in \mathbb{R}^3 \mid \exists y \in \mathbb{R}^3 \text{ a.n. } f(y) = v\}$

fi $y = (y_1, y_2, y_3)$ si $v = (v_1, v_2, v_3)$

~~$f(y) = v$~~ $f(y) = v \Leftrightarrow$

$$\Leftrightarrow \begin{cases} y_1 = v_1 \\ y_1 = v_2 \\ y_1 + y_2 + y_3 = v_3 \end{cases} \Rightarrow \bar{M} = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & v_1 \\ 1 & 0 & 0 & v_2 \\ 1 & 1 & 1 & v_3 \end{array} \right)$$

$\det M = 0$

$$\Delta C = \left(\begin{array}{ccc|ccc} 1 & 0 & v_1 & v_1 \\ 1 & 0 & v_2 & v_2 \\ 1 & 1 & v_3 & v_3 \end{array} \right) = v_1 - v_2$$