5) dist$(M_0(1,1,0), \pi\ \ 2x+2y\ldots$

Lucrare II (141)

① $f\in End(\mathbb{R}^2)$, $A=[f]_{R_0,R_0}=\begin{pmatrix}1 & -2\\ 2 & -4\end{pmatrix}$, R_0 = reperul canonic

a) f se poate diagonaliza; b) f nu se poate diagonaliza, c) valorile proprii sunt egale; d) pol. caract. are răd $\in\mathbb{C}\setminus\mathbb{R}$

② $(\mathbb{R}^3, g_0)$. Fie reperul $R=\{f_1=(1,-1,1), f_2=(0,1,0), f_3=(1,1,0)\}$

Reperul ortonormat obținut cu Gram-Schmidt este

a) $\left\{\frac{1}{\sqrt3}(1,-1,1), \frac{1}{\sqrt6}(1,2,1), \frac{1}{\sqrt2}(1,0,-1)\right\}$; b) $\left\{\frac{1}{\sqrt2}(1,0,0), \frac{1}{\sqrt2}(1,-1,1), \frac{1}{\sqrt3}(1,1,1)\right\}$;

c) $\left\{\frac{1}{\sqrt6}(2,1,-1), \frac{1}{\sqrt2}(1,0,1), \frac{1}{\sqrt3}(1,1,1)\right\}$; d) $\left\{\frac{1}{\sqrt2}(1,-1,0), \frac{1}{\sqrt2}(1,0,1), \frac{1}{\sqrt3}(-1,-1,1)\right\}$.

③ $(\mathbb{R}^3, g_0)$, $u=(1,0,1)$

$s\in End(\mathbb{R}^3)$ simetria ortogonală față de $\langle\{u\}\rangle^{\perp}$

a) $s(x)=(x_1,-x_3,-x_2)$, b) $s(x)=(-x_3,x_2,-x_1)$

c) $s(x)=(x_1,x_2,-x_3)$, d) $s(x)=(-x_1,x_2,-x_3)$.

④ $(\mathbb{R}^3, g_0)$, $u=(1,1,-1)$. Complementul ortogonal $\langle\{u\}\rangle^{\perp}$ este

a) $\{x\in\mathbb{R}^3 \mid x_1-x_2-x_3=0\}$; b) $\{x\in\mathbb{R}^3 \mid x_1+x_2+x_3=0\}$;

c) $\{x\in\mathbb{R}^3 \mid x_1+x_2-x_3=0\}$; d) $\{x\in\mathbb{R} \mid x_1+x_2=0\}$.

⑤ $Q:\mathbb{R}^3\to\mathbb{R}$ formă pătratică, $A=\begin{pmatrix}4 & -2 & 0\\ -2 & 4 & 1\\ 0 & 1 & 0\end{pmatrix}$ matricea asociată. Signatura lui Q este:

a) $(3,0)$; b) $(1,2)$; c) $(1,1)$; d) $(2,1)$

⑥ $(\mathbb{R}^3, g_0)$, $u=(1,-1,2)$, $f\in End(\mathbb{R}^3)$, $f(x)=\langle x,u\rangle u$

$g_0=\langle;\rangle$ produs scalar canonic

a) $\dim Ker f=2$, b) $\dim Ker f=1$; c) $f\in Aut(\mathbb{R}^3)$, d) $f\in Sim(\mathbb{R}^3)$

⑦ $(\mathbb{R}^3, g_0)$, $f\in End(\mathbb{R}^3)$, $[f]_{R_0,R_0}=A=\begin{pmatrix}3 & 2 & 2\\ 2 & 3 & 2\\ 2 & 2 & 3\end{pmatrix}$

$\exists$ un reper ortonormat în $\mathbb{R}^3$ ai matricea lui f are forma diagonală:

a) $\begin{pmatrix}7 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$; b) $\begin{pmatrix}7 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}$; c) $\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 7\end{pmatrix}$; d) $\begin{pmatrix}-7 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix}$.

8) $(\mathbb{R}^3, g_0)$, $f \in End(\mathbb{R}^3)$, $[f]_{R_0,R_0} = \frac{1}{7}\begin{pmatrix} -3 & -2 & 6 \\ 6 & -3 & 2 \\ 2 & 6 & 3 \end{pmatrix}$

a) $f = R_\varphi$, $\cos\varphi = \frac{5}{7}$; b) $f = R_\varphi$, $\cos\varphi = -\frac{5}{7}$;

c) $f = s \circ R_\varphi$, $\cos\varphi = -\frac{7}{5}$, c) $f = s \circ R_\varphi$, $\cos\varphi = \frac{1}{7}$,

unde R_φ = rotație de $\pm\varphi$, axă $\langle\{e_1\}\rangle$,

s = simetrie ortogonală față de $\langle\{e_1\}\rangle^\perp$.

9) $Q : \mathbb{R}^3 \longrightarrow \mathbb{R}$, $Q(x) = x_1^2 + 2x_1x_2$

$g : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ forma polară asociată

a) $g(x,y) = x_1y_1 + 2x_1y_2 + 2x_2y_1$; b) $g(x,y) = x_1y_1 + x_2y_1 + x_2y_2$

c) $g(x,y) = x_1y_1 + x_2y_2 + x_3y_3$; d) $g(x,y) = x_1y_1 + x_1y_2 + x_2y_1$.