# Problema de nota 10

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## 1 Maximum factorization

Some basic definitions:

**Definition 1.** *A factorization of $w$ is a tuple of strings $(u_1, u_2, \ldots, u_k)$ such that $w = u_1 u_2 \cdots u_k$. For example, one of the several factorizations of the word $w = abaab$ is $(ab, a, ab)$ since $w = ab \cdot a \cdot ab$ (where $\cdot$ is the symbol for concatenation).*

**Definition 2.** *An equality-free factorization is a factorization with all distinct factors. For example, given the word $w = abaab$ the factorization $(ab, a, ab)$* is NOT *equality-free since the first and the third factor are equal. However, $(a, baab)$, $(ab, aa, b)$ and $(aba, ab)$ are all equality-free factorizations.*

**Definition 3.** *A gapped equality factorization of string $w$ over alphabet $\Sigma$ is a tuple $(u_1, u_2, \ldots, u_k)$ such that $w = \alpha_0 u_1 \alpha_1 u_2 \alpha_2 \cdots \alpha_{k-1} u_k \alpha_k$, with $u_i \in \Sigma^+$ and $\alpha_i \in \Sigma^*$ and $u_i$ are all distinct. For example, a gapped equality free factorization of $w = abaab$ is: $(a, ab)$, where $\alpha_0$ and $\alpha_2$ are the empty strings, $u_1 = a$, $\alpha_1 = ba$, $u_2 = ab$.*

**Definition 4.** *The size of a factorization represents the number of factors.*

The problem is the following:

*Problem 1.* We are given a word $w$ together with an equality-free gapped factorization of size $k$ $w = \alpha_0 u_1 \alpha_1 u_2 \alpha_2 \cdots \alpha_{k-1} u_k \alpha_k$.

Find in polynomial time an equality-free factorization of $w$ *without gaps* of size at least $k$, if such a factorization exists. If such a factorization does not exist, your algorithm should output "Impossible".

*Example 1.* For example if $w = aaab$ and we are given the gapped factorization $(a, ab)$, a factorization with the same size without gaps is $(aa, ab)$.

*Example 2.* For example if $w = aaba$ and we are given the gapped factorization $(a, ab)$, a factorization with the same size without gaps is $(aa, ba)$.

A weaker version of the problem is:

*Problem 2.* We are given a word $w$ together with an equality-free gapped factorization of size $k$ $w = \alpha_0 u_1 \alpha_1 u_2 \alpha_2 \cdots \alpha_{k-1} u_k \alpha_k$.

Find in polynomial time an equality-free factorization of $w$ *without gaps* of size at least $k/1000$, if such a factorization exists. If such a factorization does not exist, your algorithm should output "Impossible".