Fușneică Florentin-Cristian

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3. $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_1, x_1, x_1 + x_2 + x_3)$

a) $\ker f = \{ x \in \mathbb{R}^3 \mid f(x) = (0, 0, 0) \}$

$$\Rightarrow \begin{cases} x_1 = 0 \\ x_1 = 0 \\ x_1 + x_2 + x_3 = 0 \Rightarrow x_2 = -x_3 \end{cases}$$

$$\Rightarrow \ker f = \{ (0, x_2, -x_2) \mid x_2 \in \mathbb{R} \}$$
$$= \{ x_2 (0, 1, -1) \mid x_2 \in \mathbb{R} \}$$

b) $\operatorname{Im} f = \{ v \in \mathbb{R}^3 \mid \exists y \in \mathbb{R}^3 \text{ a.î. } f(y) = v \}$

Fie $y = (y_1, y_2, y_3)$ și $v = (v_1, v_2, v_3)$

$$\cancel{\qquad} f(y) = v \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y_1 = v_1 \\ y_1 = v_2 \\ y_1 + y_2 + y_3 = v_3 \end{cases} \qquad \Rightarrow \overline{M} = \begin{pmatrix} 1 & 0 & 0 & \downarrow_1 \\ 1 & 0 & 0 & \downarrow_2 \\ 1 & 1 & 1 & \downarrow_3 \end{pmatrix}$$

$$\det M = 0$$

$$\Delta_c = \begin{vmatrix} 1 & 0 & v_1 \\ 1 & 0 & v_2 \\ 1 & 1 & v_3 \end{vmatrix} = v_1 - v_2$$

$y_1 - y_2 = 0 \Rightarrow y_1 = y_2$

$\Rightarrow \text{Im } f = \{ y \in \mathbb{R}^3 \mid y_1 = y_2 \} =$

$= \{ (y_1, y_1, y_3) \mid y \in \mathbb{R}^3 \}$

$= \{ y_1(1,1,0) + y_3(0,0,1) \mid y \in \mathbb{R}^3 \}$

6. $U = \{ x \in \mathbb{R}^3 \mid x_1 + x_2 + x_3 = 0 \}$

a) $x_1 + x_2 + x_3 = 0 \Longleftrightarrow x_1 = -x_2 - x_3$

$U = \{ (-x_2 - x_3, x_2, x_3) \mid x_2, x_3 \in \mathbb{R} \}$

$U = \langle \{ (-1,1,0), (-1,0,1) \} \rangle$

$C = \begin{pmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

$rg\, C = 2 = |U| \Rightarrow U$ este SLI $\Big\} \Rightarrow U$ nu este reper

$|U| \neq \dim_{\mathbb{R}} \mathbb{R}^3 \Rightarrow U$ nu e bază

$C' = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$\det C' = 2 \neq 0 \Rightarrow rg\, C' = 3 = \dim_{\mathbb{R}} \mathbb{R}^3$

$R = \{ (-1,1,0), (-1,0,1), (1,1,0) \}$ reper

$S' = \{(1,2,3),(0,1,1)\}$ SLi maximal al lui S

b) $S' = \{(1,2,3),(0,1,1)\}$ SLi

$M' = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$

$\det M' = 1 \Rightarrow rg M' = 3$

$R = \{(1,2,3),(0,1,1),(0,0,1)\}$ SLi

$\dim \mathbb{R}^3 = |R| = 3 \Rightarrow R$ reper în $\mathbb{R}^3$