Probleme de conectivitate în grafuri neorientate

Muchii critice

Muchii critice

- G – graf neorientat
- $e \in E(G)$ **critică** (**punte**, **muchie de articulaţie**) = prin eliminarea ei crește numărul de componente conexe ale grafului

  nr componente (G – e) > nr. componente (G)

- Un graf conex fără punți se numește **2-muchie conex.**

Muchii critice

- O muchie este critică $\Leftrightarrow$ nu este conținută într-un ciclu

Muchii critice

- O muchie este critică $\Leftrightarrow$ nu este conținută într-un ciclu

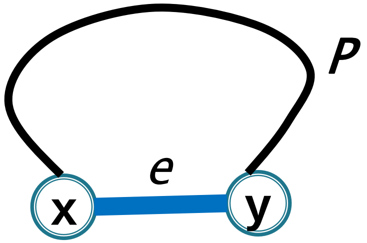
Demonstrație

Muchii critice

- O muchie este critică ⇔ nu este conținută într-un ciclu

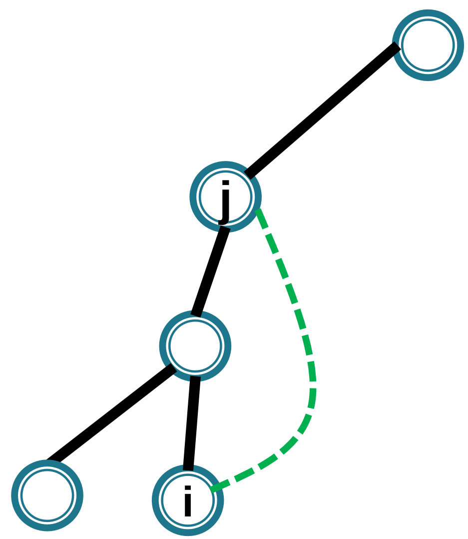
Demonstrație

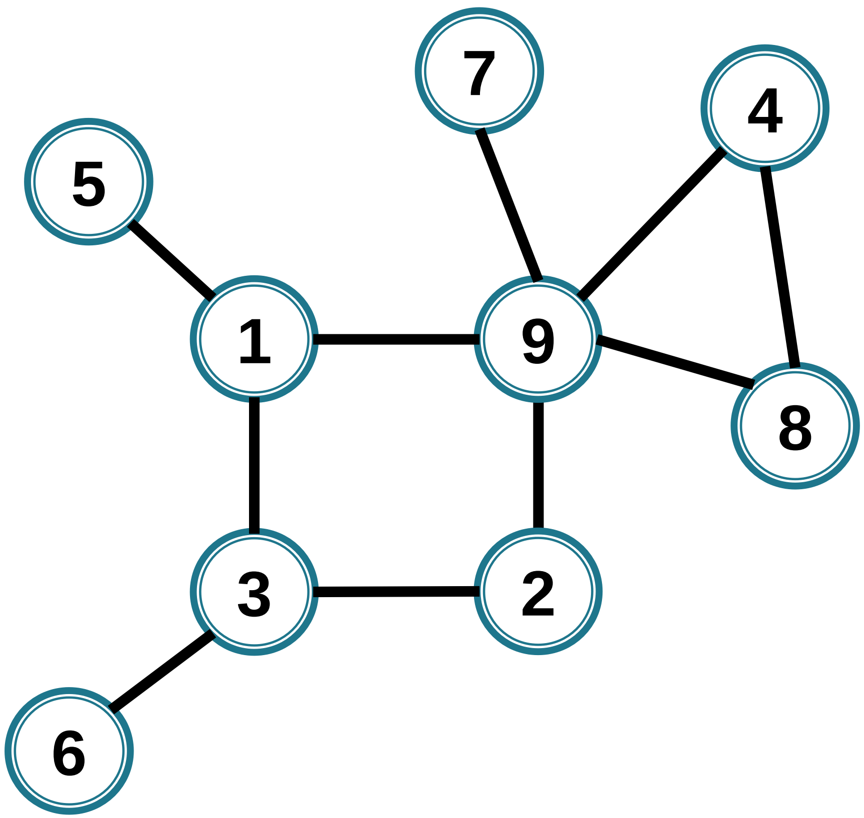
O muchie nu este critică ⇔ este conținută într-un ciclu



Muchii critice

- Găsirea unui ciclu - parcurgere DF
  - **muchii de avansare** - ale arborelui DF (memorat cu vector tata), prin care se descoperă vârfuri noi
  - **muchii de întoarcere** - închid ciclu, nu pot fi critice

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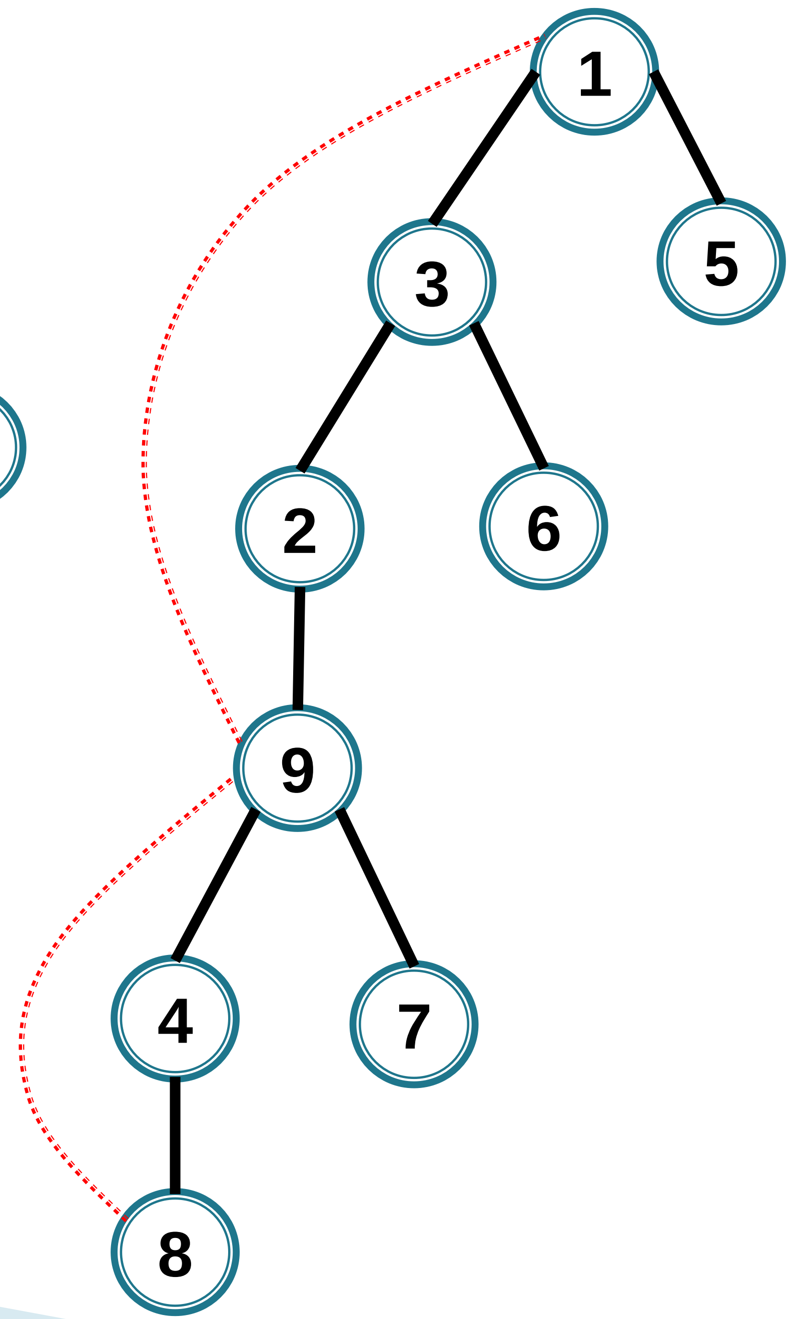
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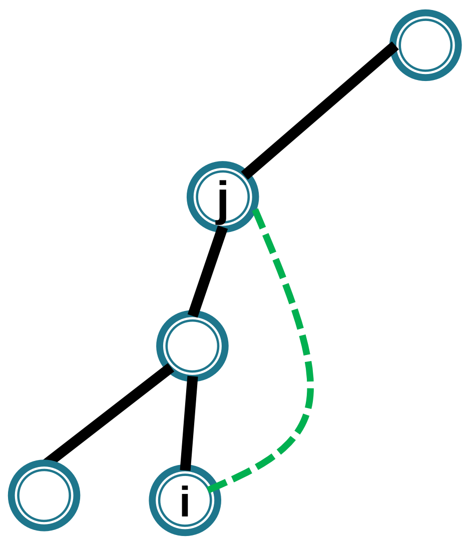
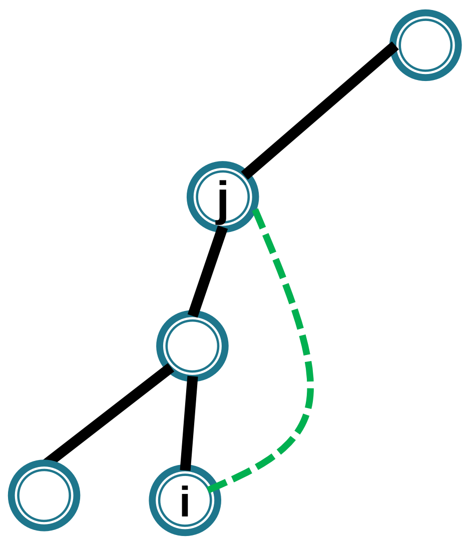
Muchii critice

- Găsirea unui ciclu – parcurgere DF – **O(n)**
  - **muchii de avansare** – ale arborelui DF (memorat cu vector tata), prin care se descoperă vârfuri noi
  - **muchii de întoarcere** – închid ciclu



Muchii critice

- Găsirea unui ciclu – parcurgere DF – **O(n)**
  - **muchii de avansare** – ale arborelui DF (memorat cu vector tata), prin care se descoperă vârfuri noi
  - **muchii de întoarcere** – închid ciclu, nu pot fi critice



Doar muchiile de avansare pot fi critice

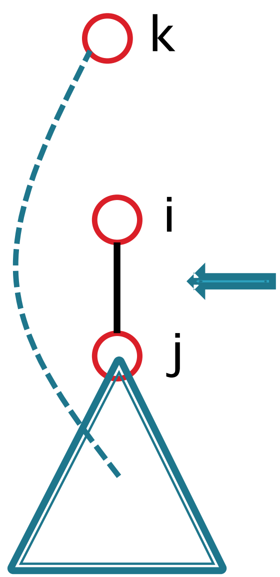
Muchii critice

Cum testăm dacă o muchie de avansare (i,j) este critică?

Muchii critice

Cum testăm dacă o muchie de avansare (i,j) este critică?

- nu este conținută într-un ciclu închis de o muchie de întoarcere



Muchii critice

O muchie de avansare (i,j) este critică

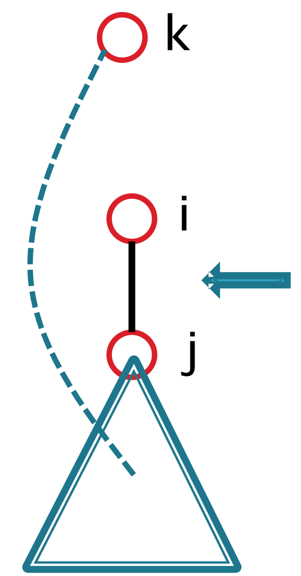
⇔

nu este conținută într-un ciclu închis de o muchie de întoarcere

⇔

nu există nicio muchie de întoarcere cu

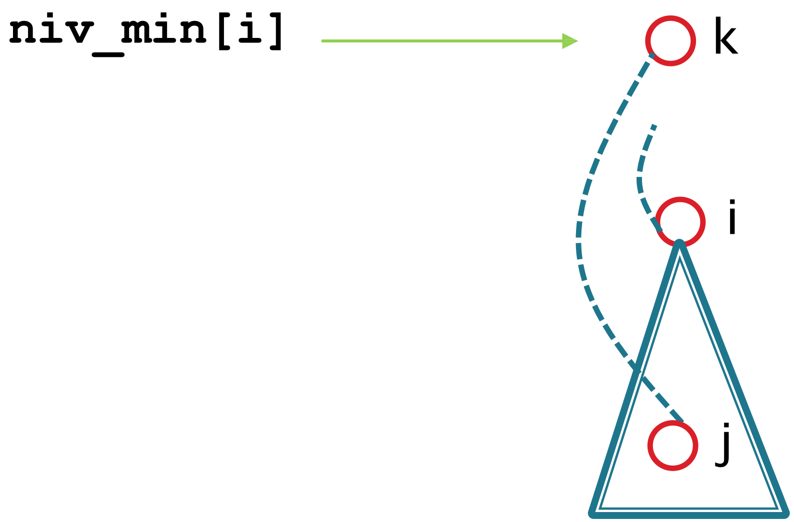
- o extremitate **în j sau într-un descendent al lui j** și
- cealaltă extremitate **în i sau într-un ascendent al lui i** (într-un vârf de **pe un nivel mai mic sau egal cu nivelul lui i**)



Muchii critice

Memorăm pentru fiecare vârf i:

`niv_min[i]` = *intuitiv: cât de sus putem ajunge din i mergând în sensul parcurgeii DF*

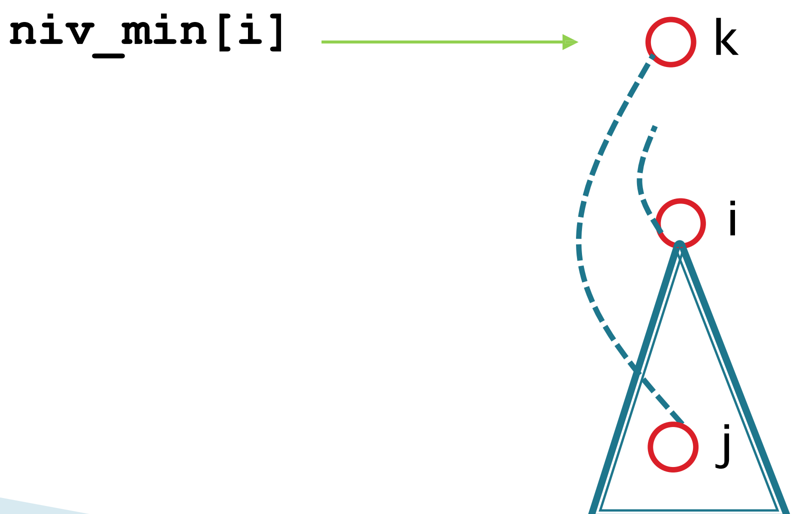


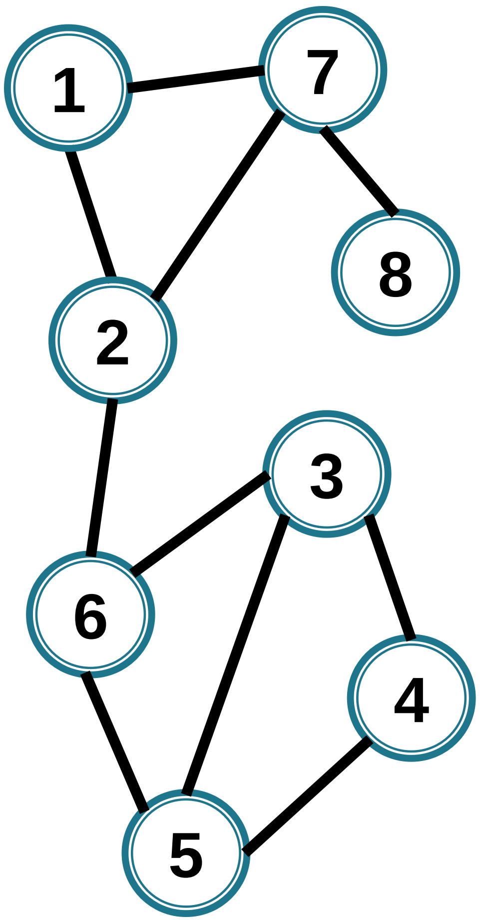
Muchii critice

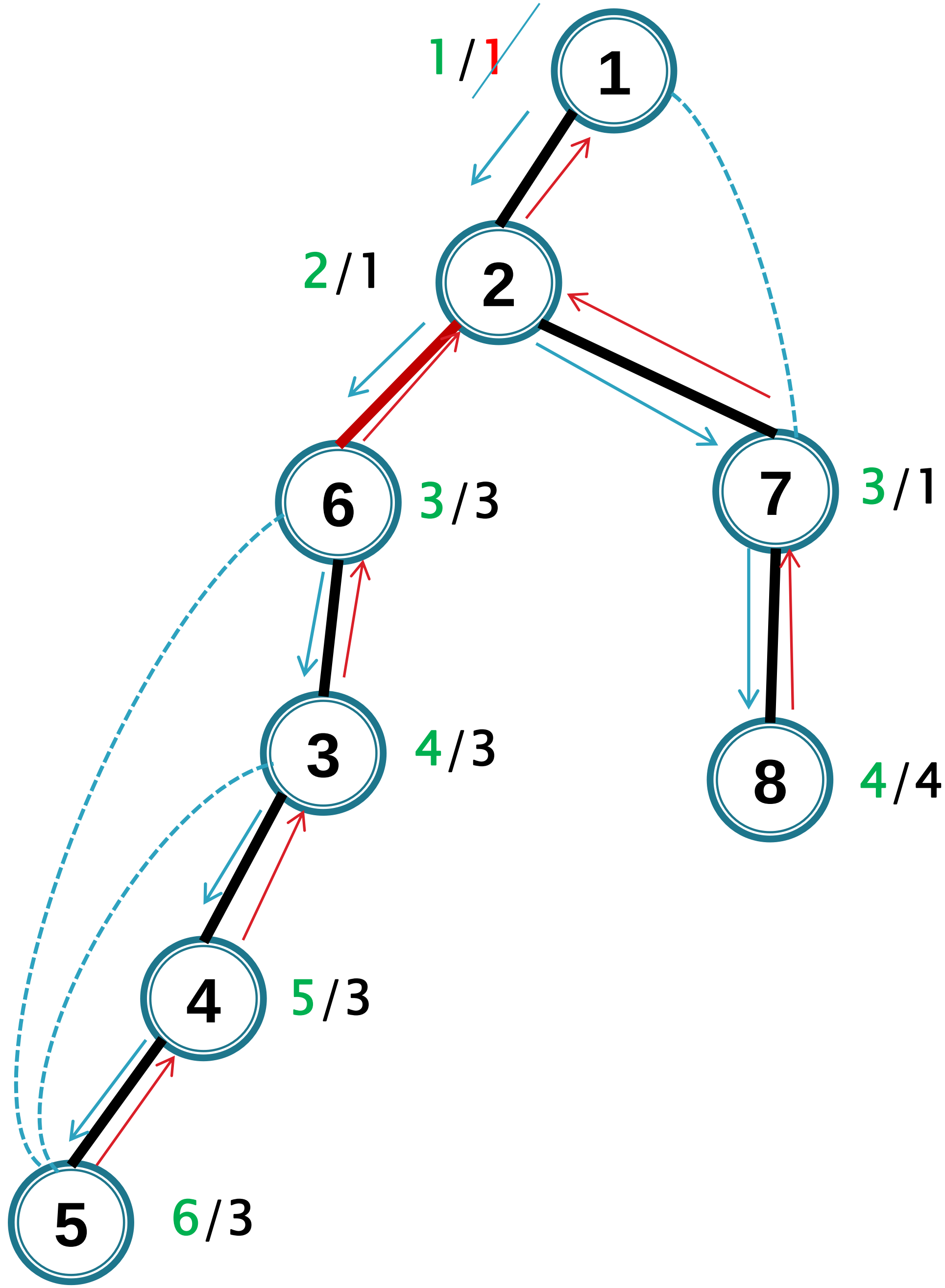
Memorăm pentru fiecare vârf i:

`niv_min[i]` = nivelul minim al unui vârf care este extremitate a unei muchii de întoarcere din i sau dintr-un descendent al lui i

= nivelul minim la care se închide un ciclu elementar care conține vârful i (printr-o muchie de întoarcere)






1/1
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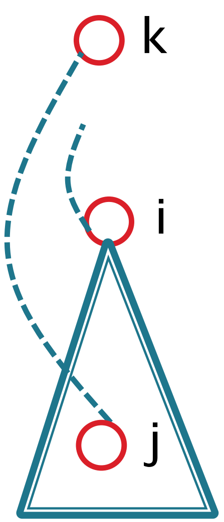
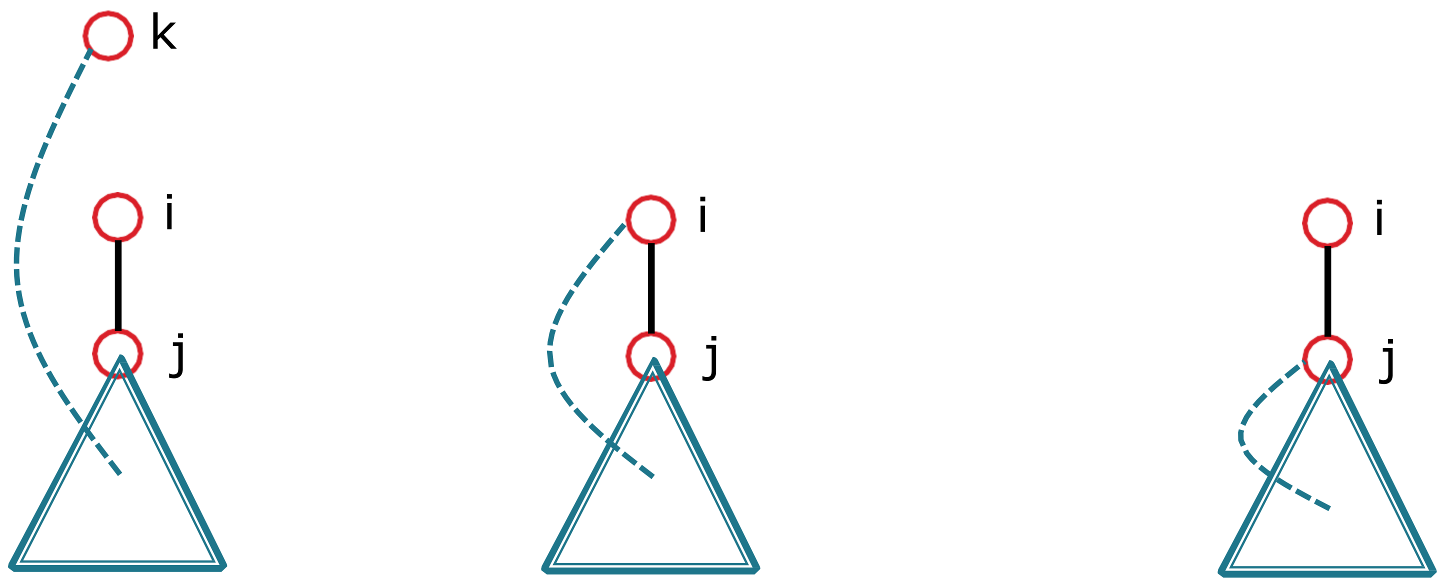
Muchii critice

- **`nivel[i]`** = nivelul lui i în arborele DF
- **`niv_min[i]`** = min { **`nivel[i]`**, A, B}
  - A = min {**`nivel[k]`** | **`ik`** muchie de întoarcere}
  - B = min {**`nivel[k]`** | **`j`** <u>descendent</u> al lui **`i`**,
    `jk` muchie de întoarcere}



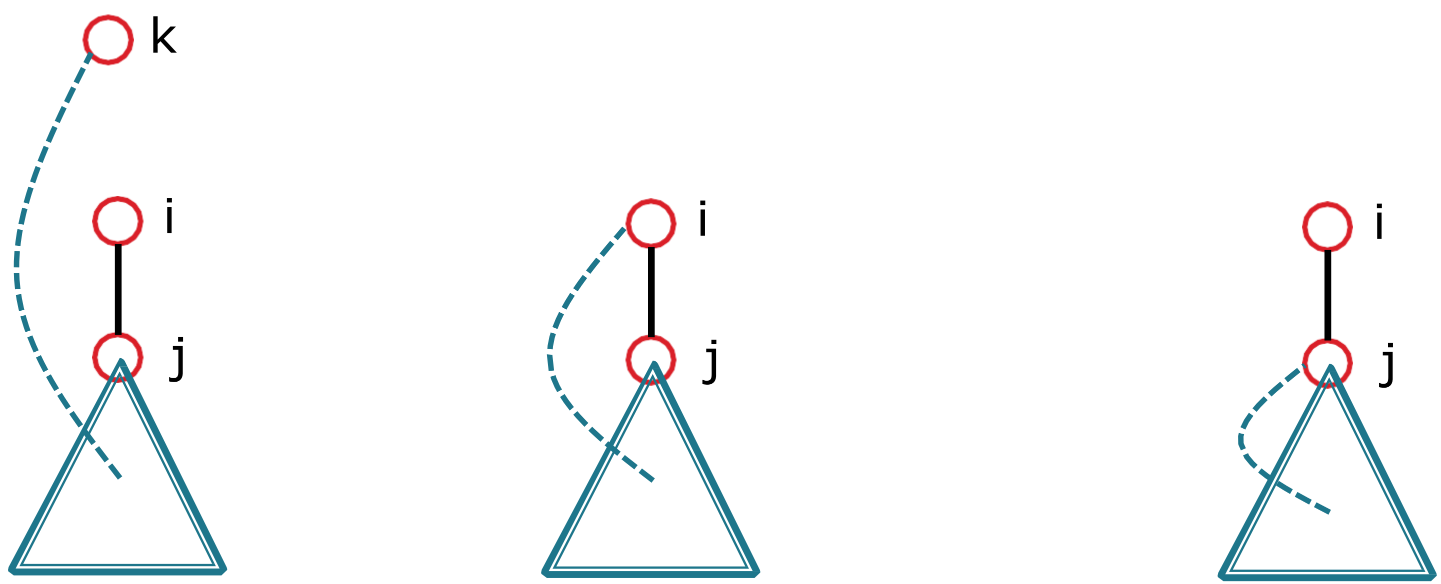
Muchii critice

O **muchie de avansare** ij este critică ⇔ ?



Muchii critice

O **muchie de avansare** ij este critică ⇔ ?



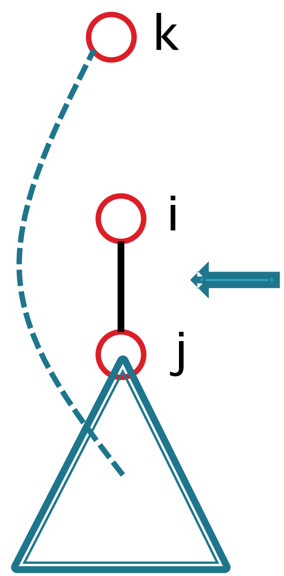
NU este critică
`niv_min[j]<nivel[i]`

NU este critică
`niv_min[j]=nivel[i]`

ESTE critică
`niv_min[j]>nivel[i]`

Muchii critice

O **muchie de avansare** ij este critică ⇔ **`niv_min[j] > nivel[i]`**



Muchii critice

Cum calculăm eficient **`niv_min[i]`** ?

`niv_min[i]` = min { **`nivel[i]`**, A, B}

A = min {**`nivel[k]`** | **`ik`** muchie de întoarcere}

B = min {**`nivel[k]`** | **`j`** <u>descendent</u> al lui **`i`**,
`jk` muchie de întoarcere}

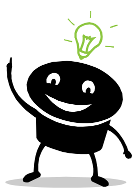
Muchii critice

Cum calculăm eficient `niv_min[i]` ?

`niv_min[i]` = min { `nivel[i]`, A, B}

A = min {`nivel[k]` | `ik` muchie de întoarcere}

B = min {`nivel[k]` | `j` <u>descendent</u> al lui `i`,
`jk` muchie de întoarcere}



B se poate calcula recursiv

Muchii critice

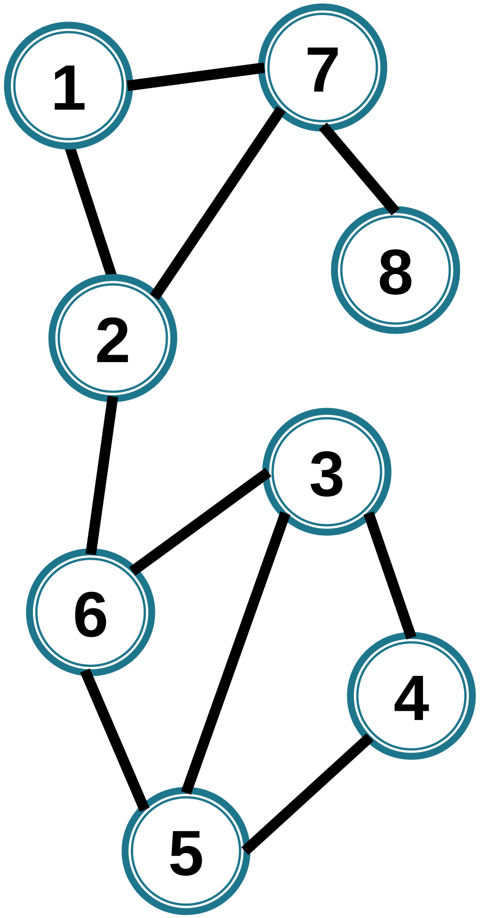
Cum calculăm eficient `niv_min[i]` ?

`niv_min[i]` = min { `nivel[i]`, A, B}

A = min {`nivel[k]` | `ik` muchie de întoarcere}

B = min {`nivel[k]` | `j` <u>descendent</u> al lui `i`,
`jk` muchie de întoarcere}

B = min {`niv_min[j]` | j <u>fiu</u> al lui i}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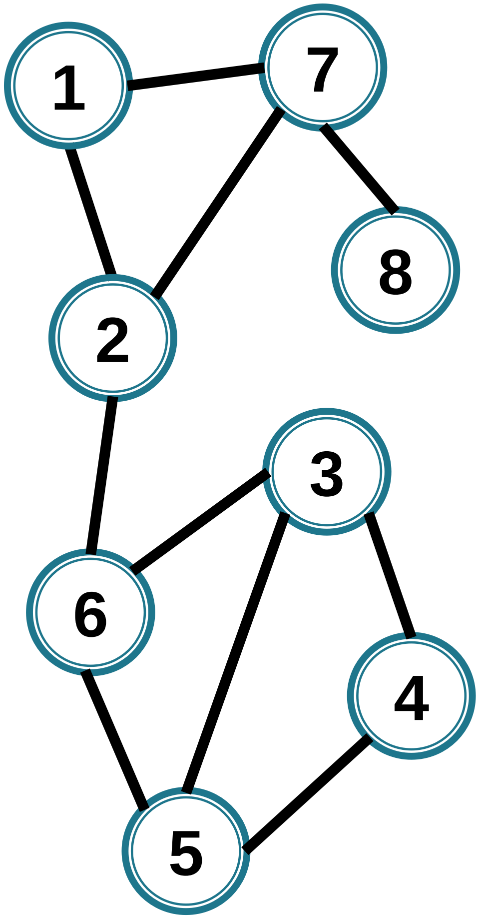
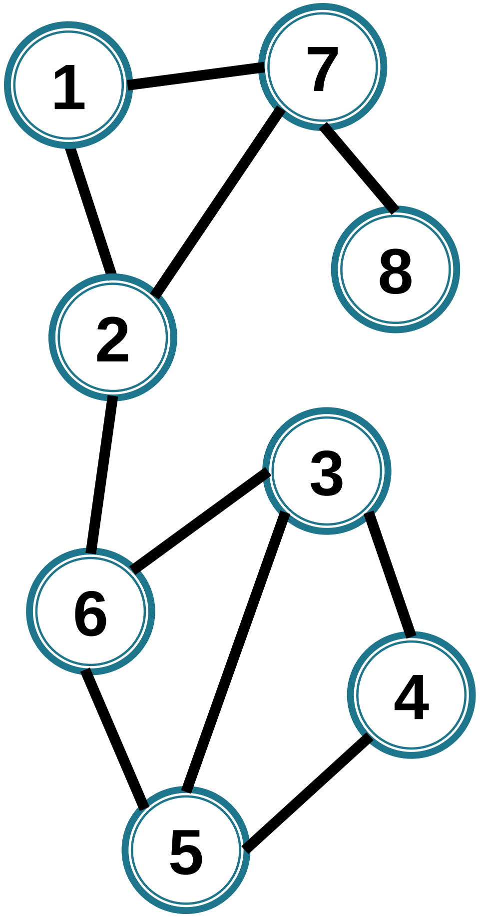
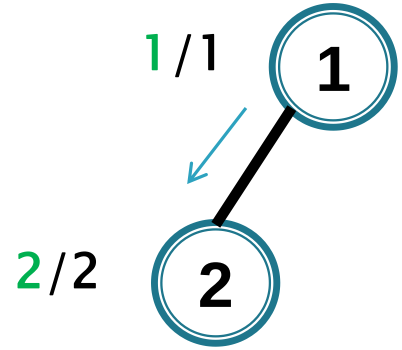


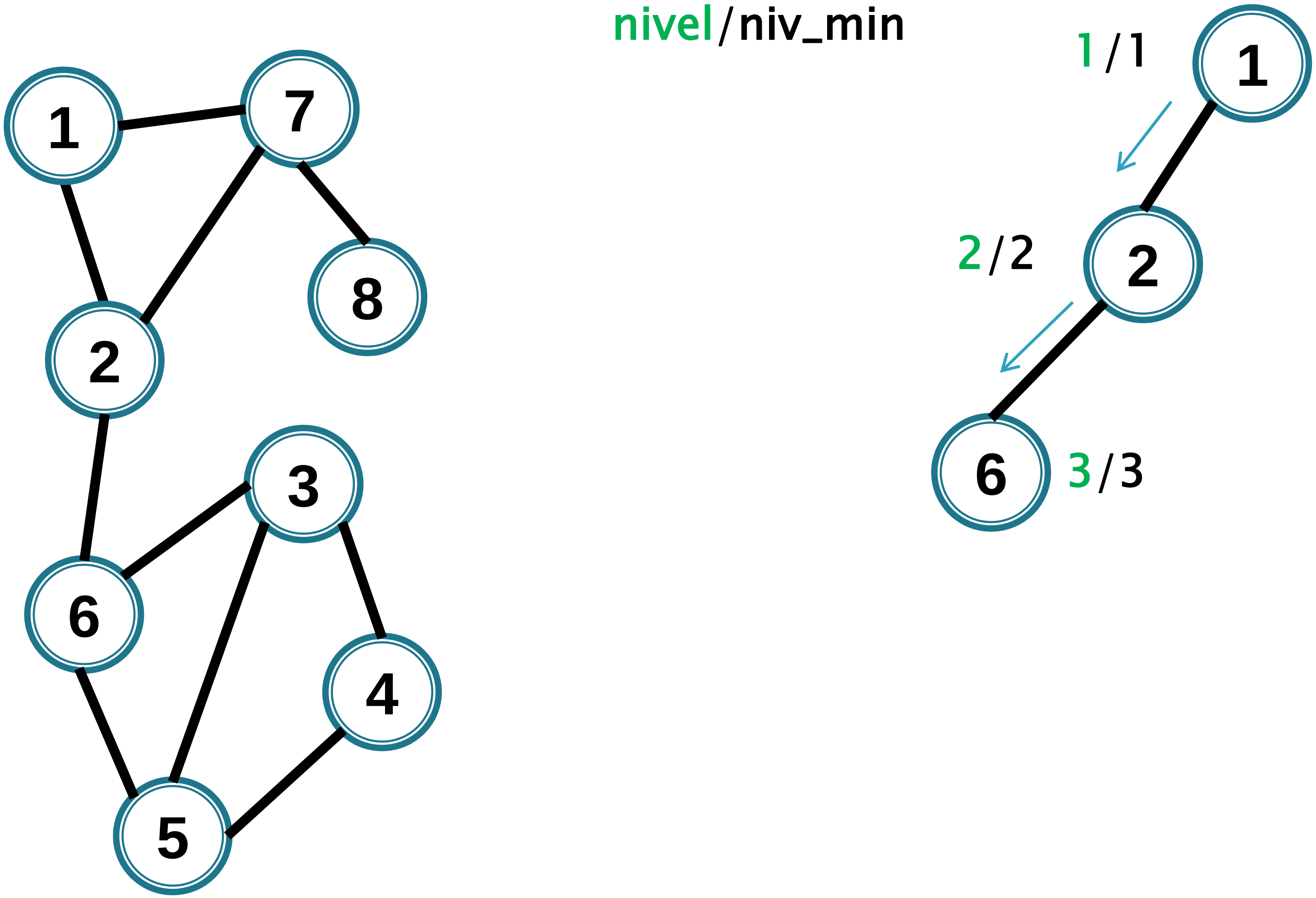
nivel/niv_min

1/1



nivel/niv_min




nivel/niv_min
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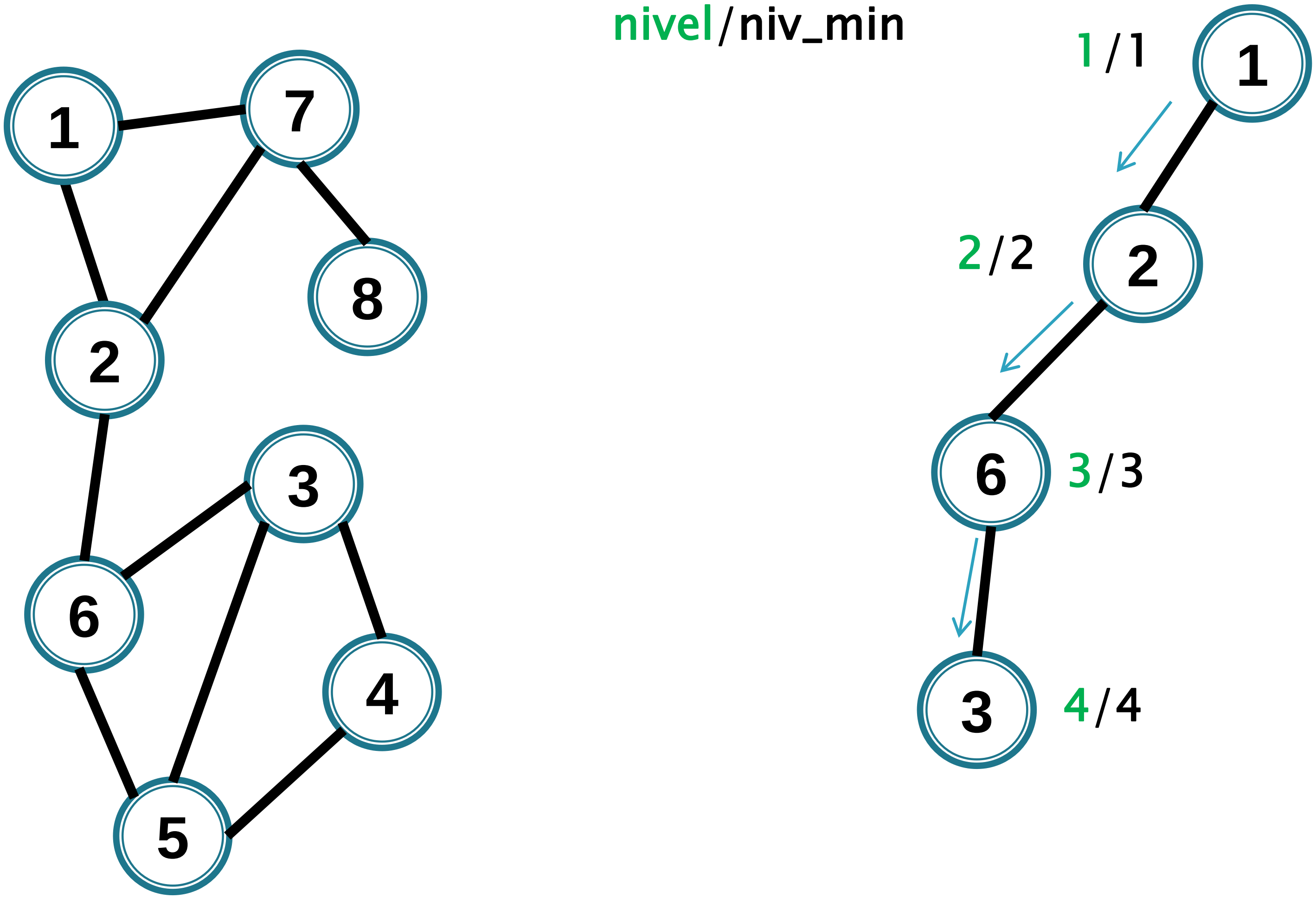

nivel/niv_min
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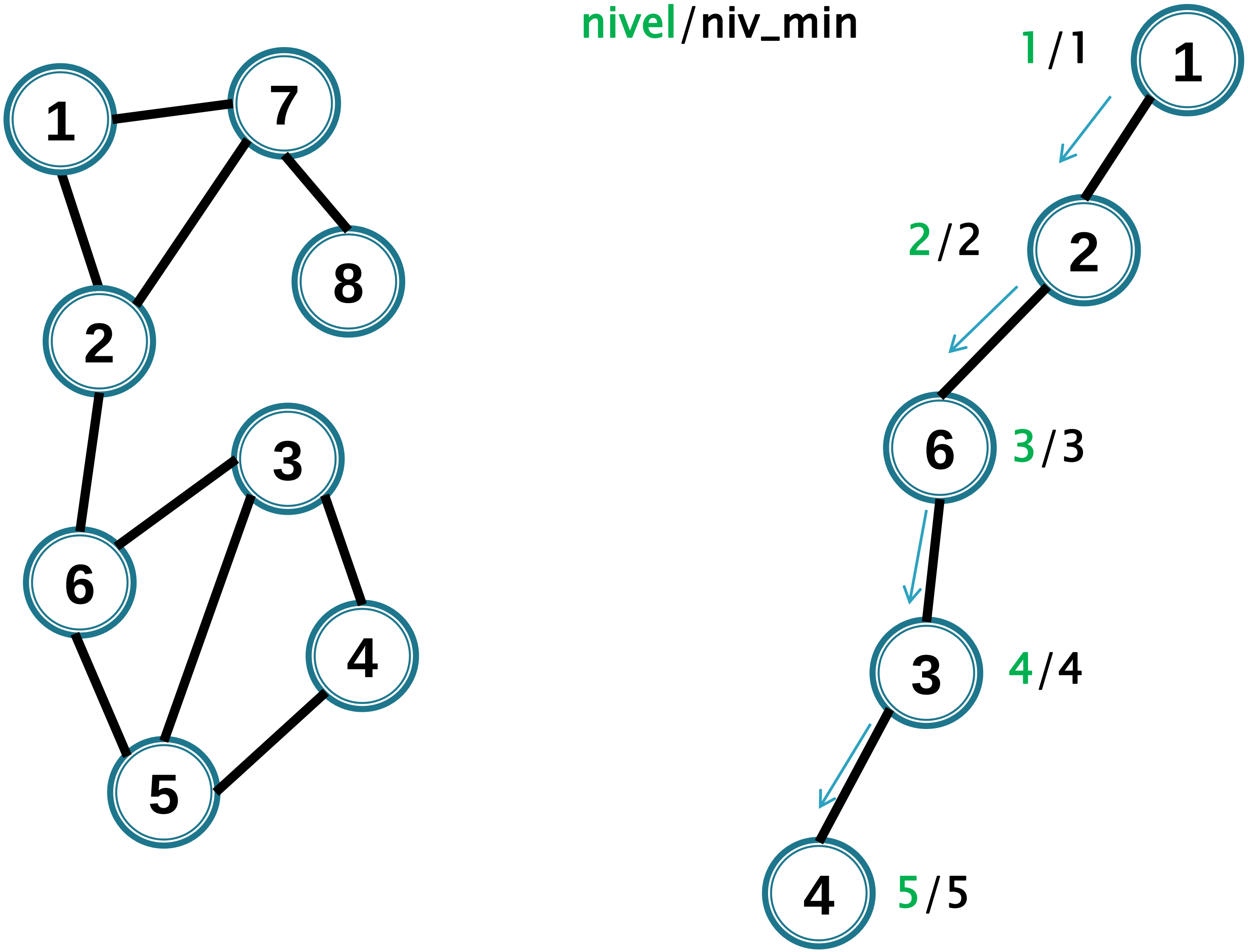
nivel/niv_min
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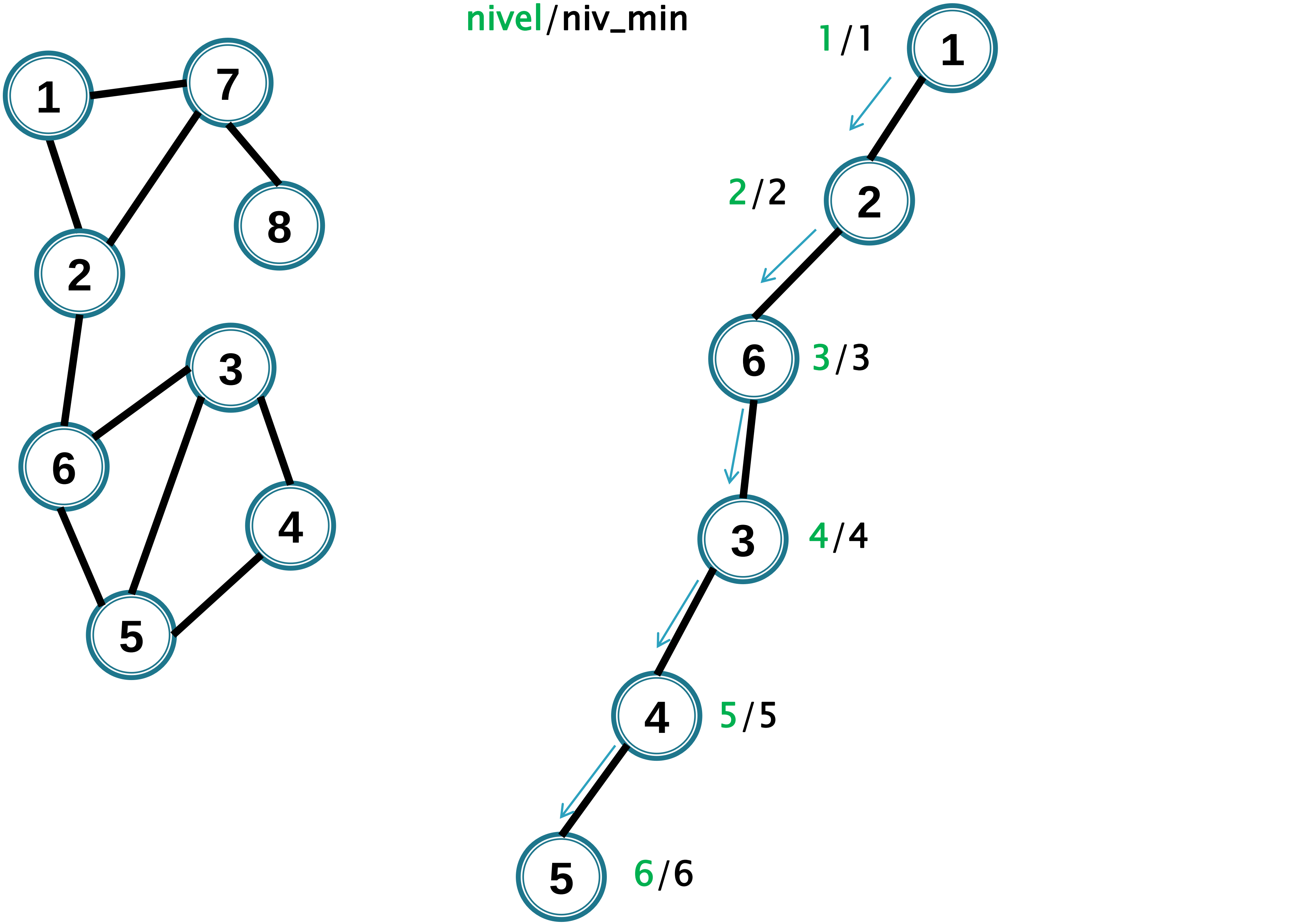
nivel/niv_min
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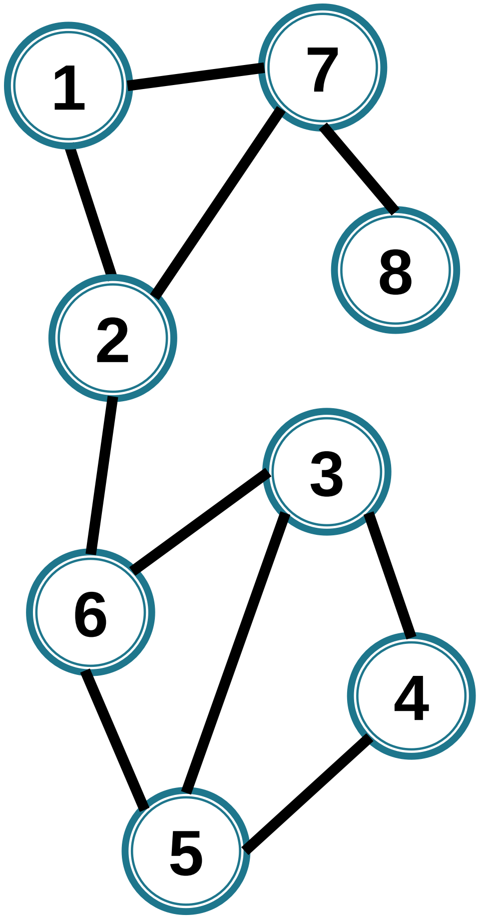
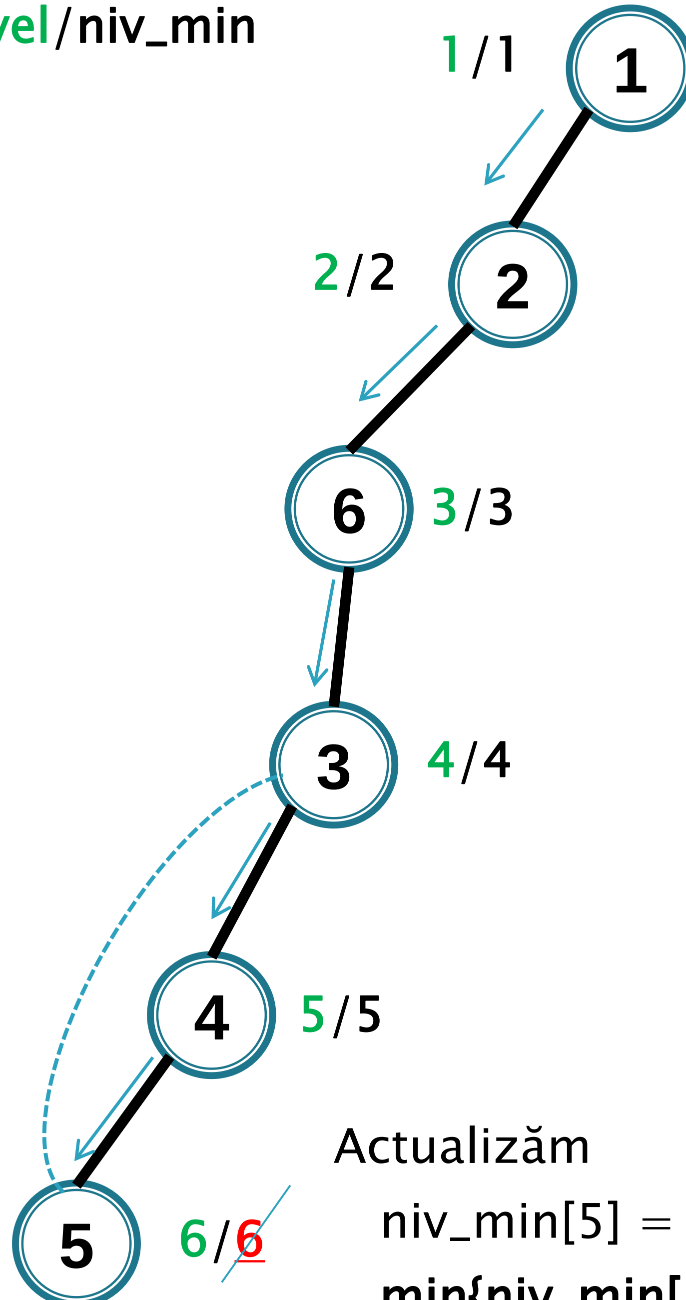


nivel/niv_min

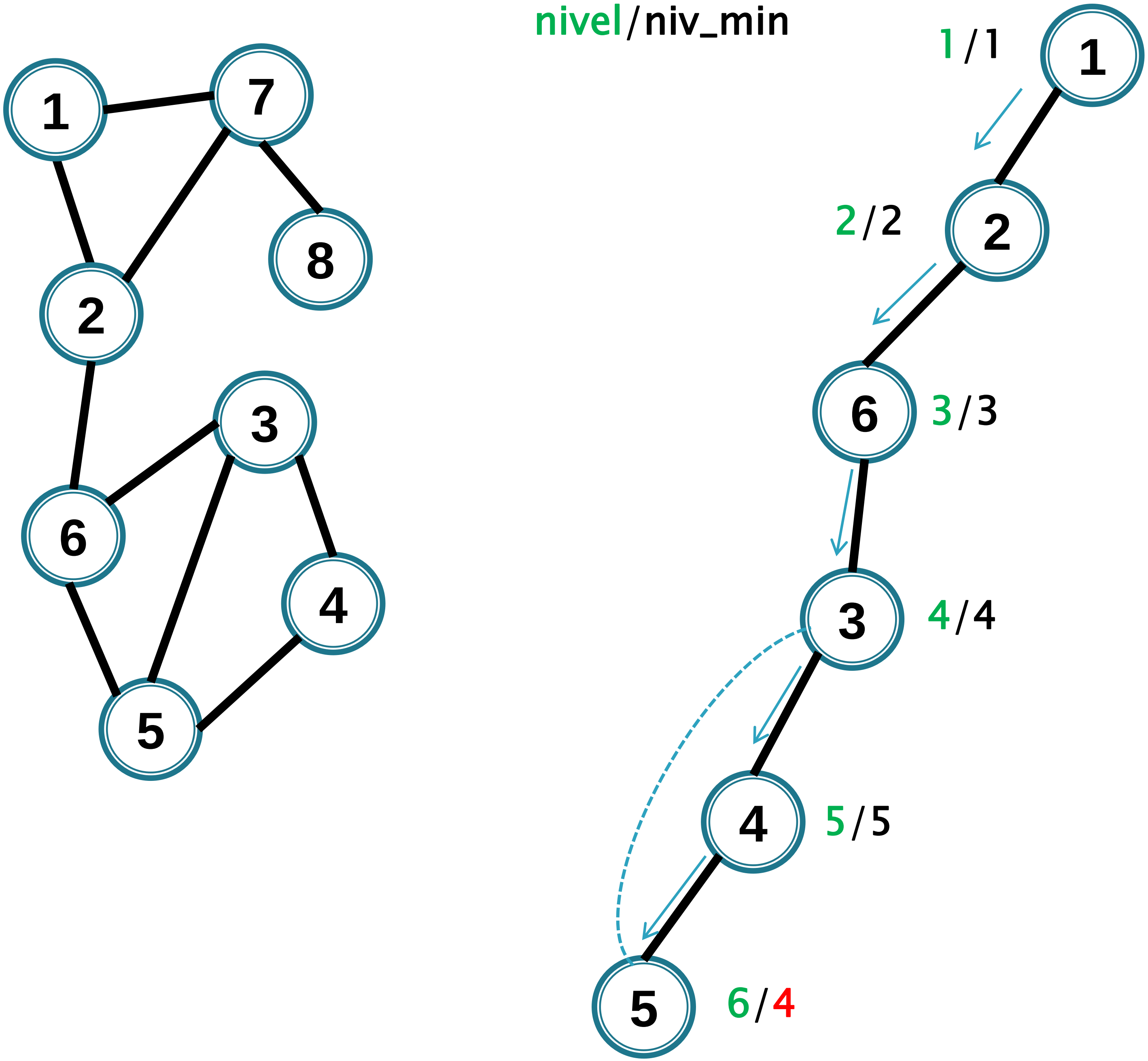


Actualizăm

niv_min[5] =

min{niv_min[5], nivel[3] }

(cazul A)


nivel/niv_min
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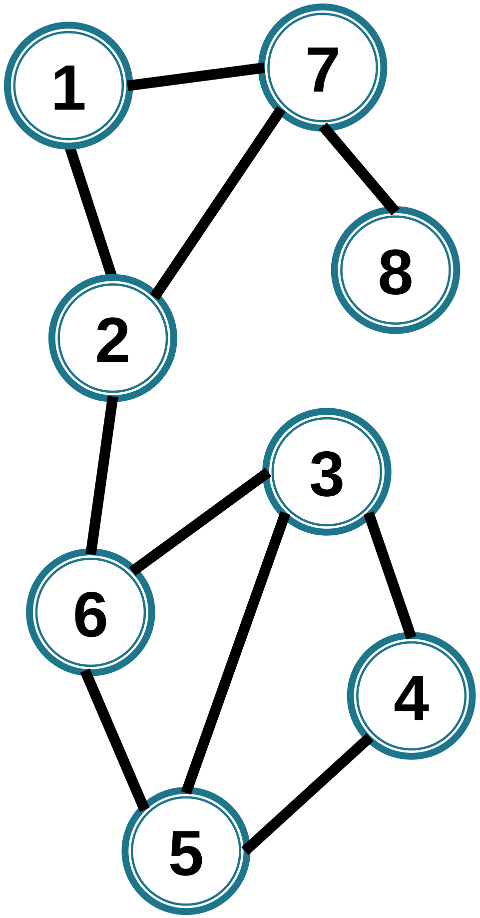
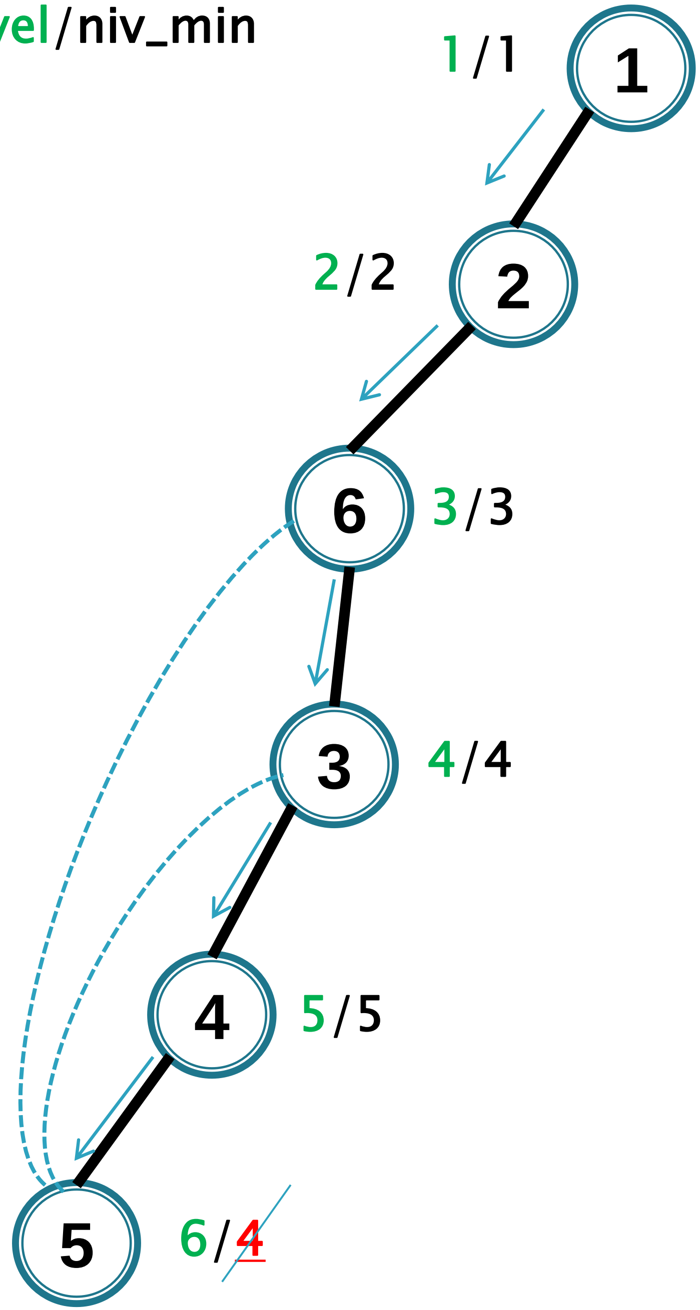
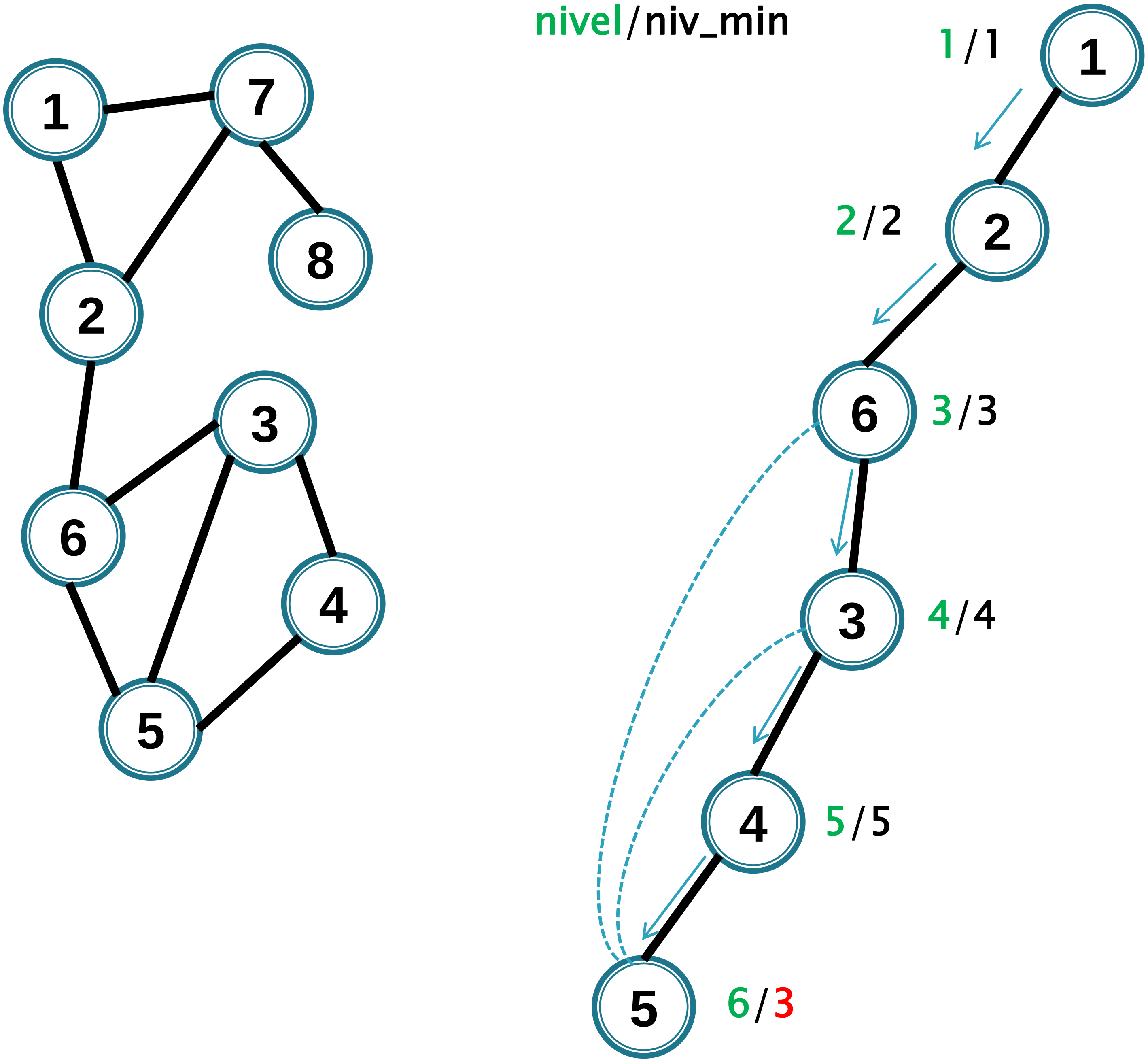
nivel/niv_min






nivel/niv_min
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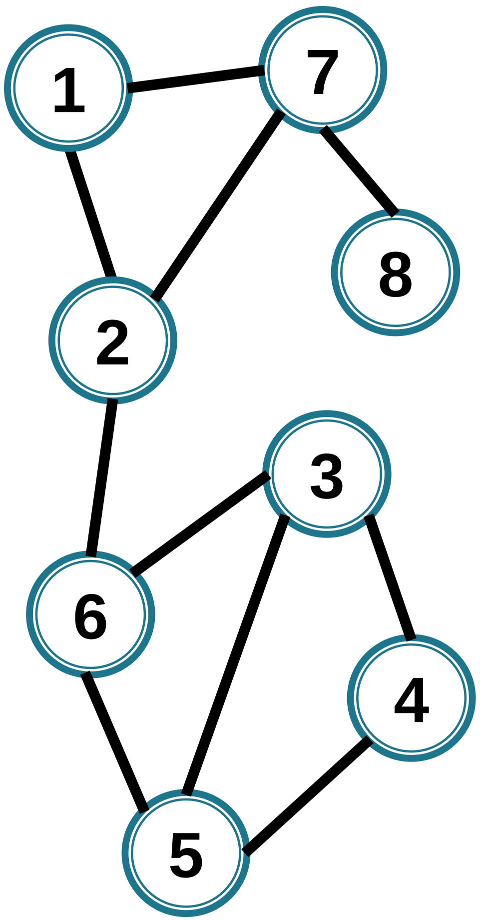
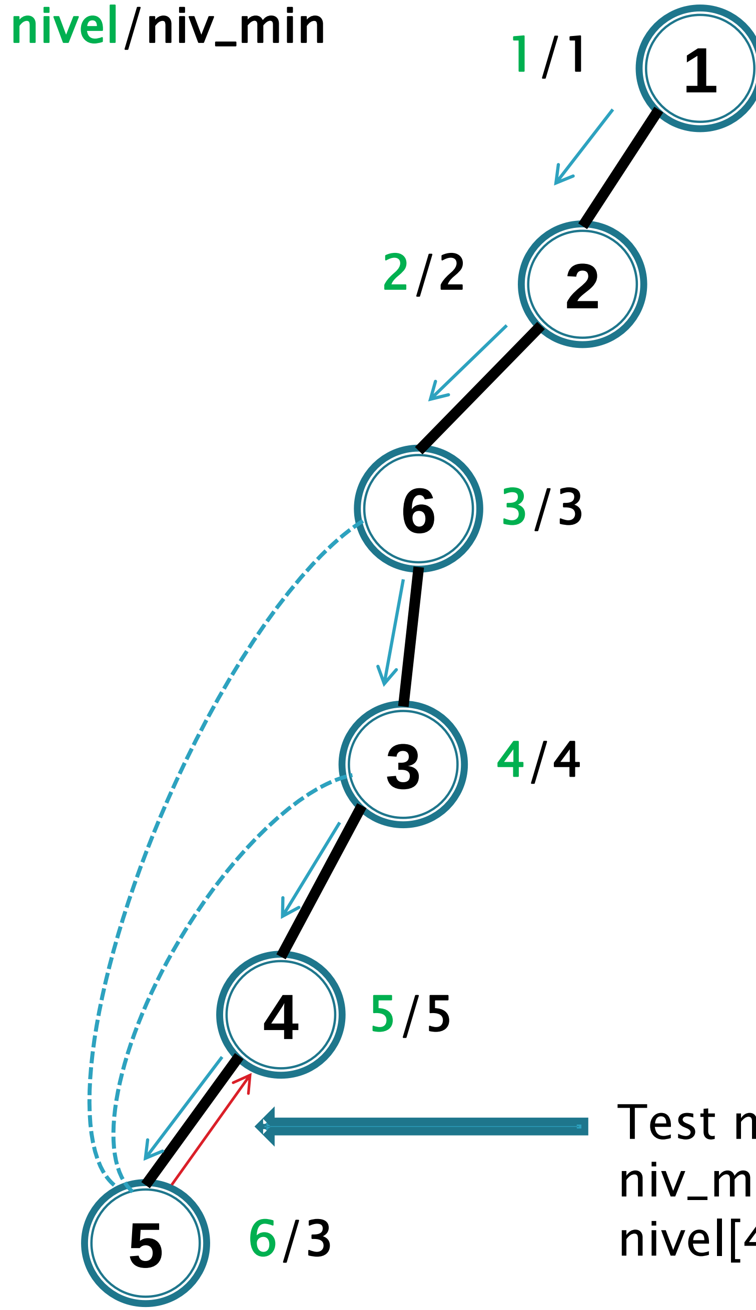
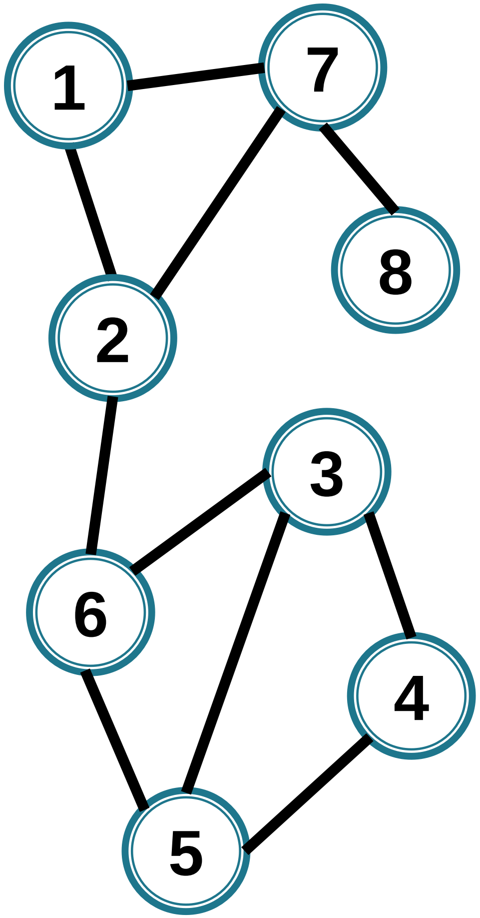
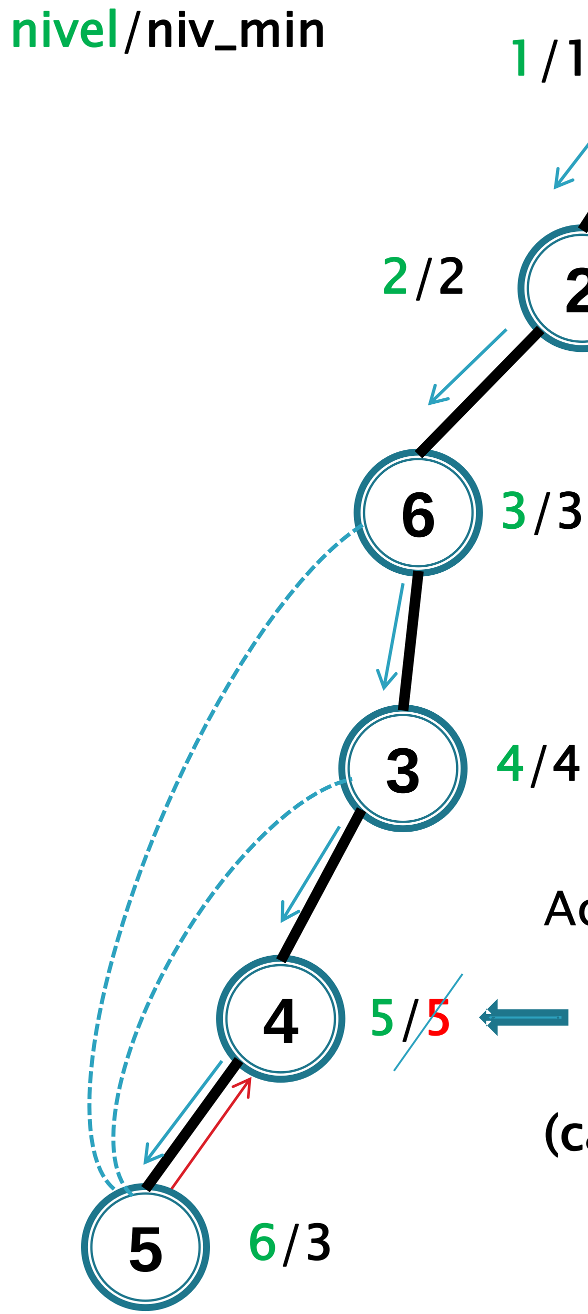




Test muchie critică:
niv_min[5] $=3 <$
nivel[4] $=5 \Rightarrow$ **NU**

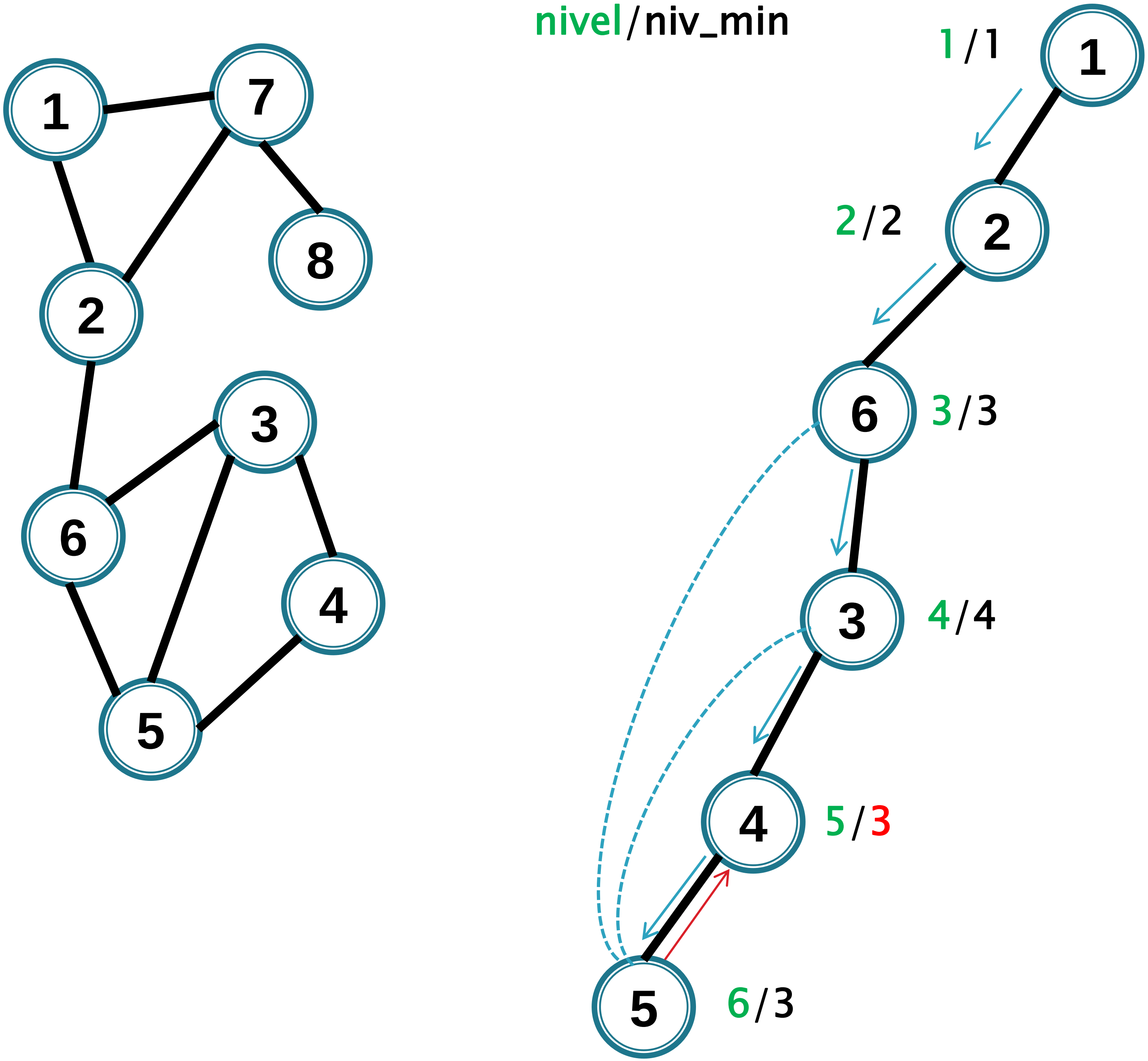




Actualizam niv_min[4] =

min{niv_min[4], niv_min[5]}

(cazul B)


nivel/niv_min
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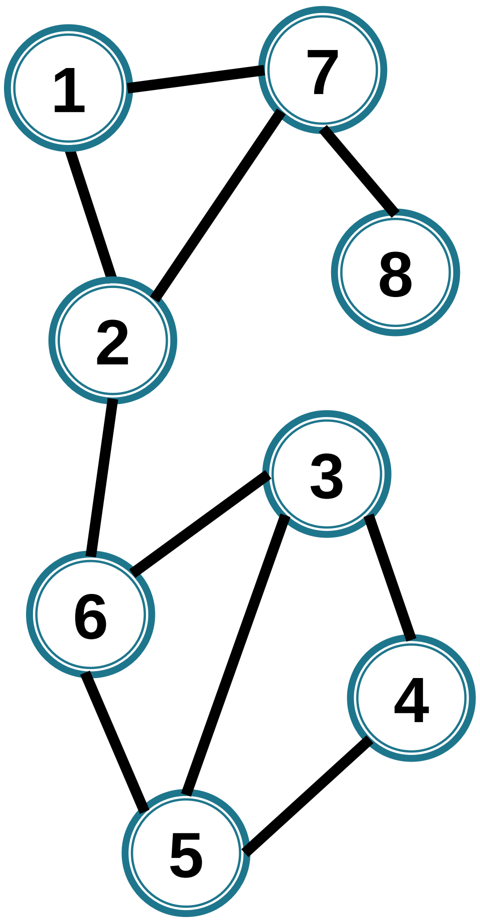
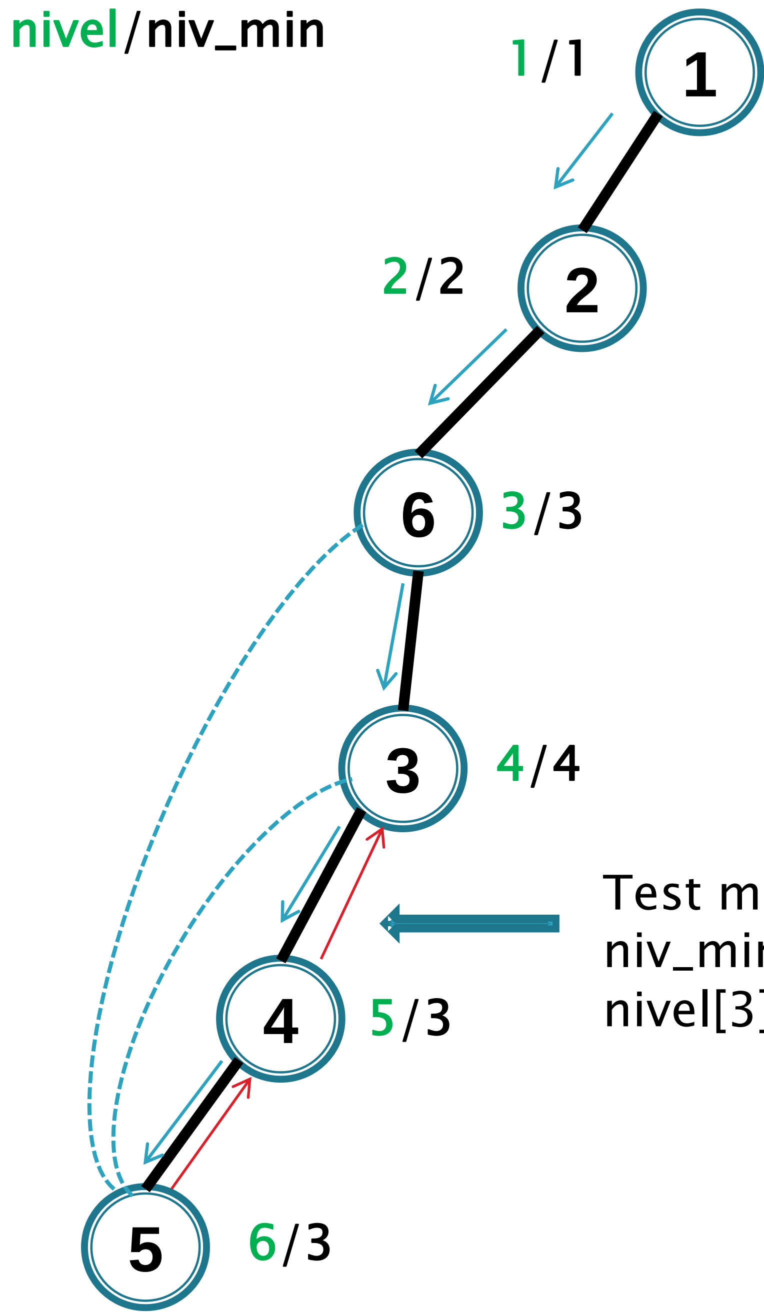
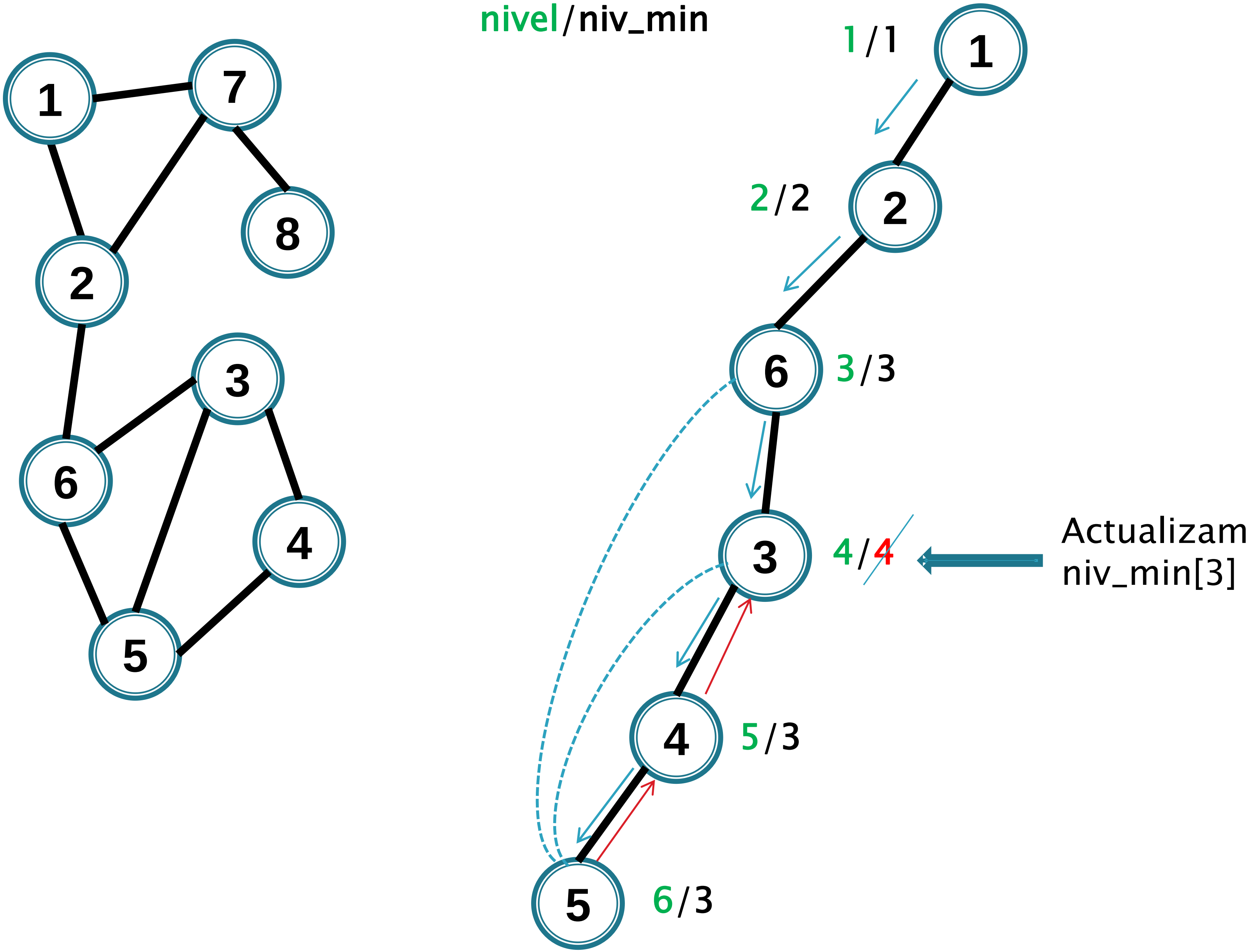




Test muchie critică:
niv_min[4] =3 < nivel[3] =4 ⇒ **NU**


nivel/niv_min
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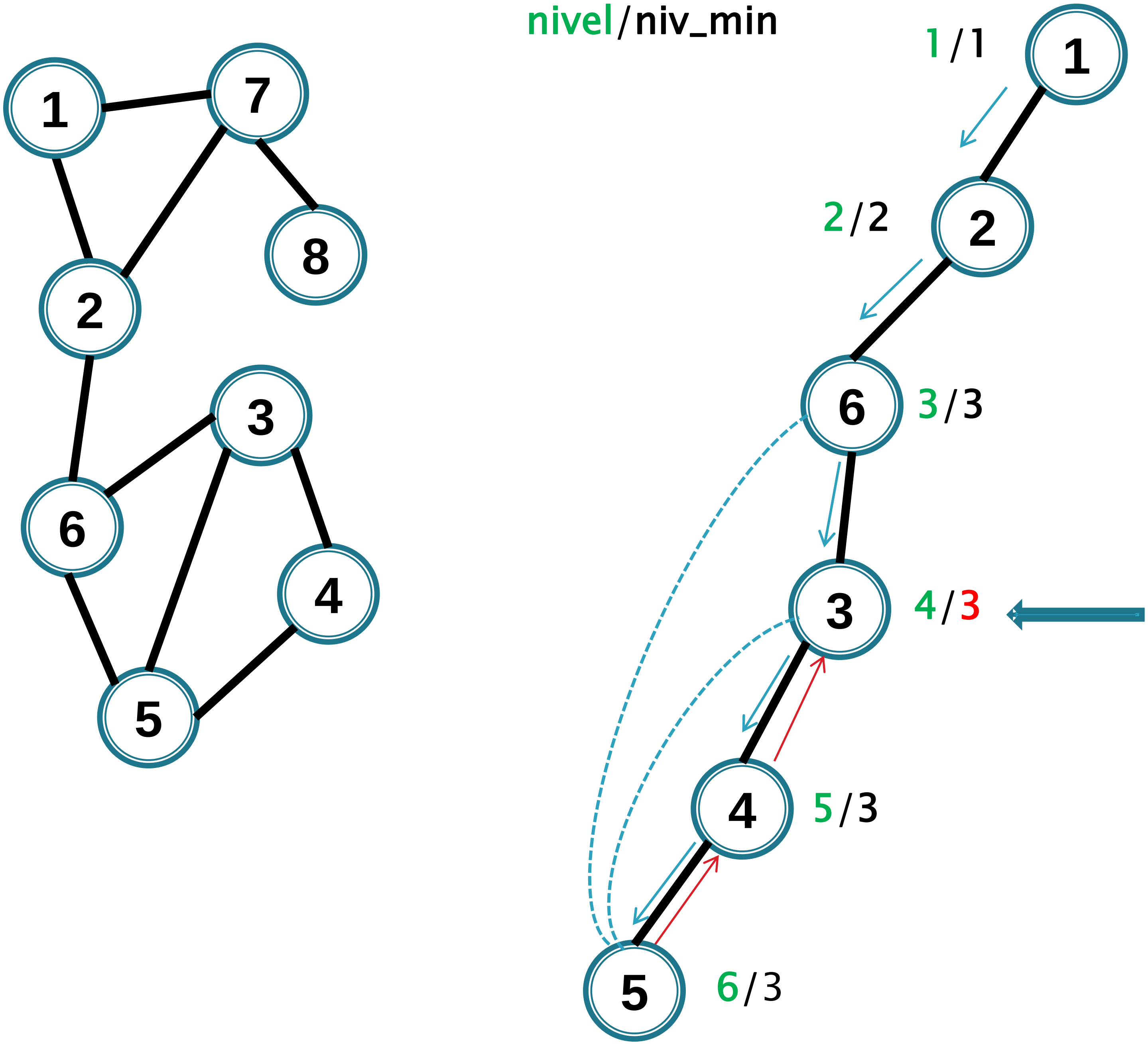
Actualizam niv_min[3]


nivel/niv_min
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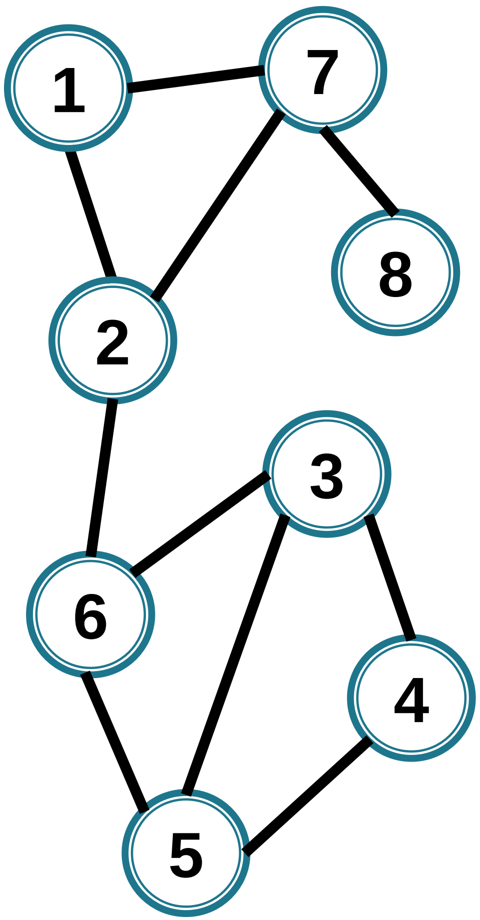

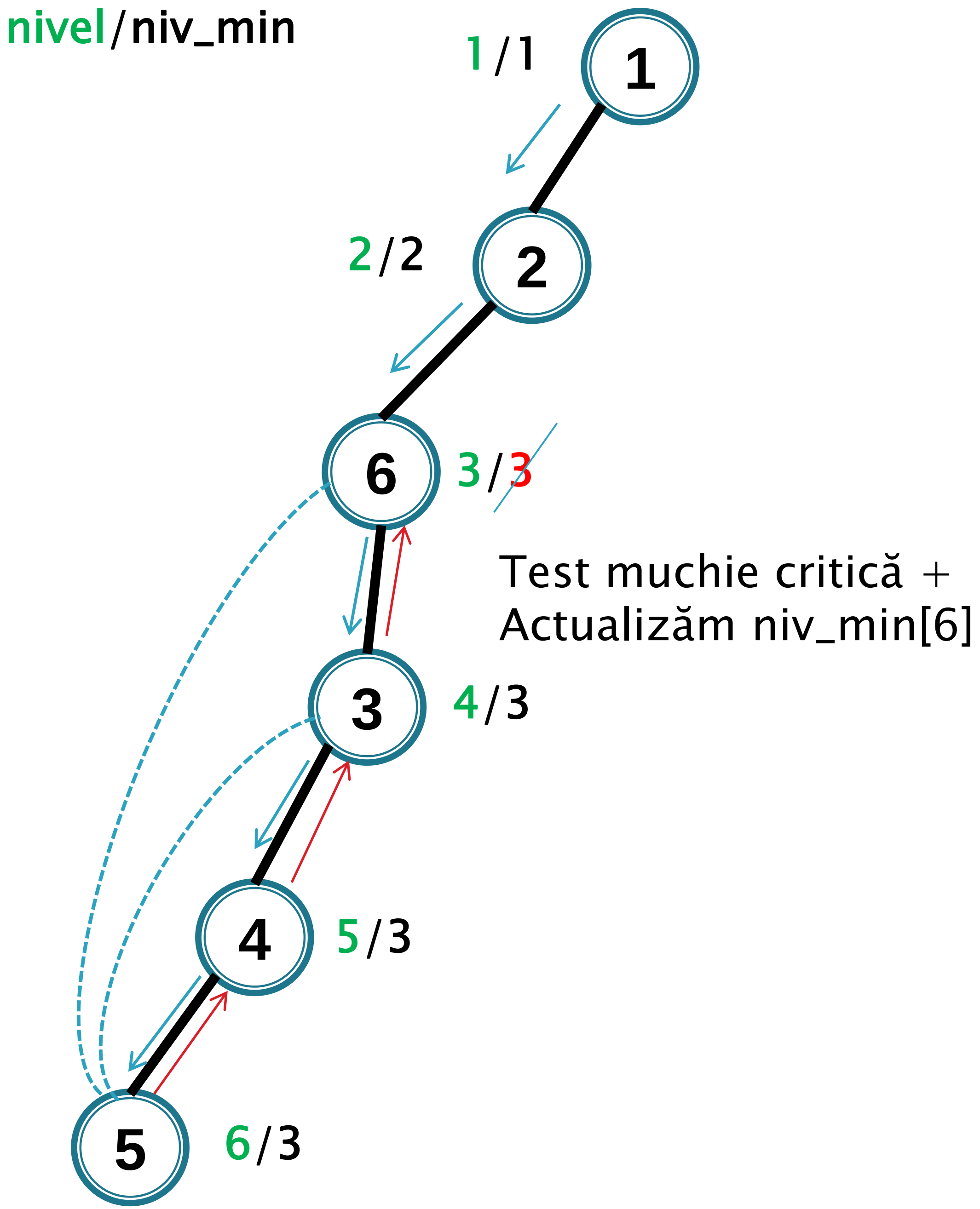

nivel/niv_min
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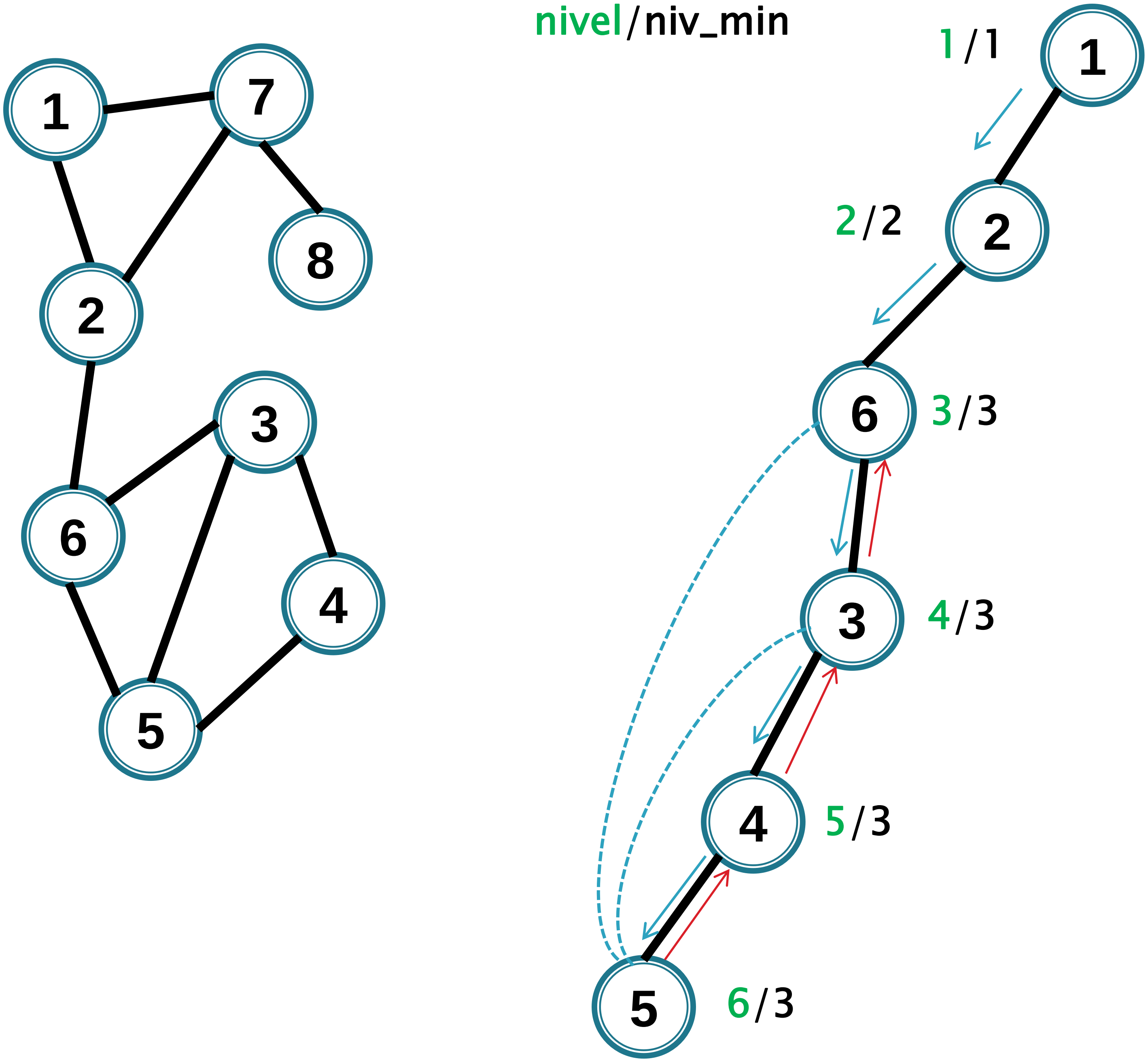
Test muchie critică +
Actualizăm niv_min[6]


nivel/niv_min
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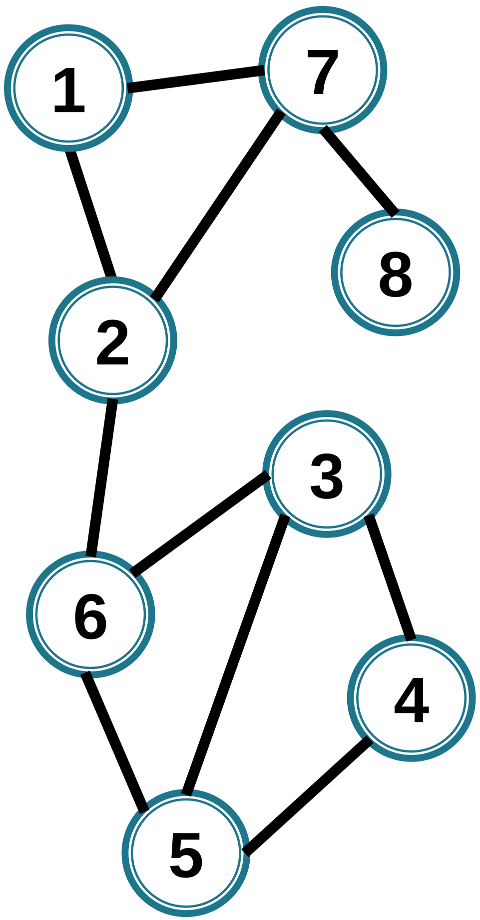
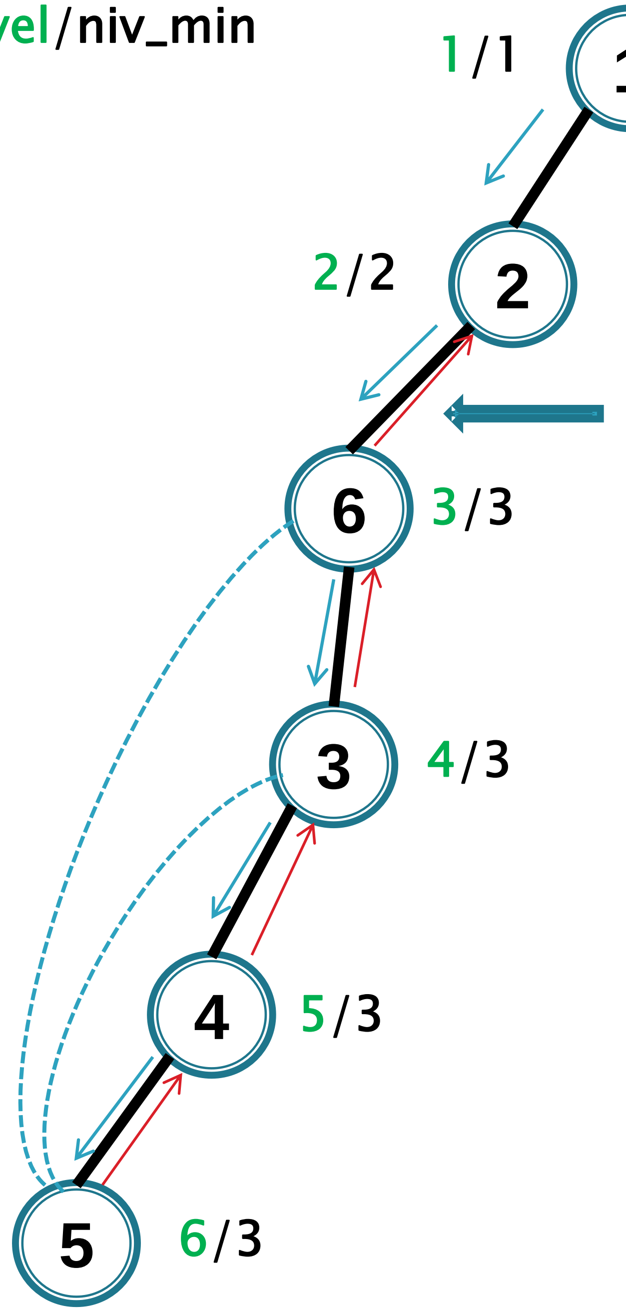
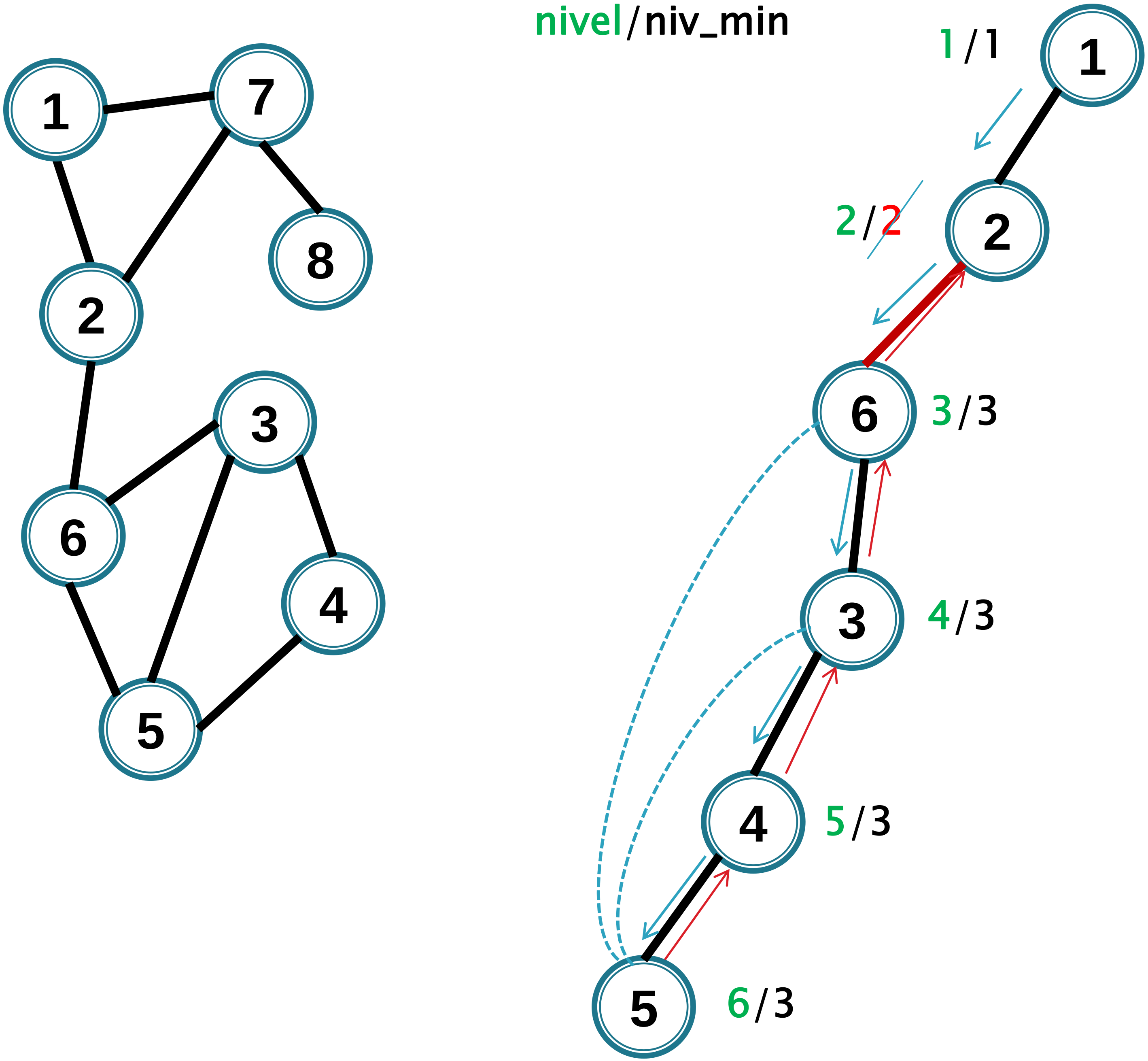
nivel/niv_min





Test muchie critică:
niv_min[6] =3 >
nivel[2] =2 ⇒ **DA**


nivel/niv_min
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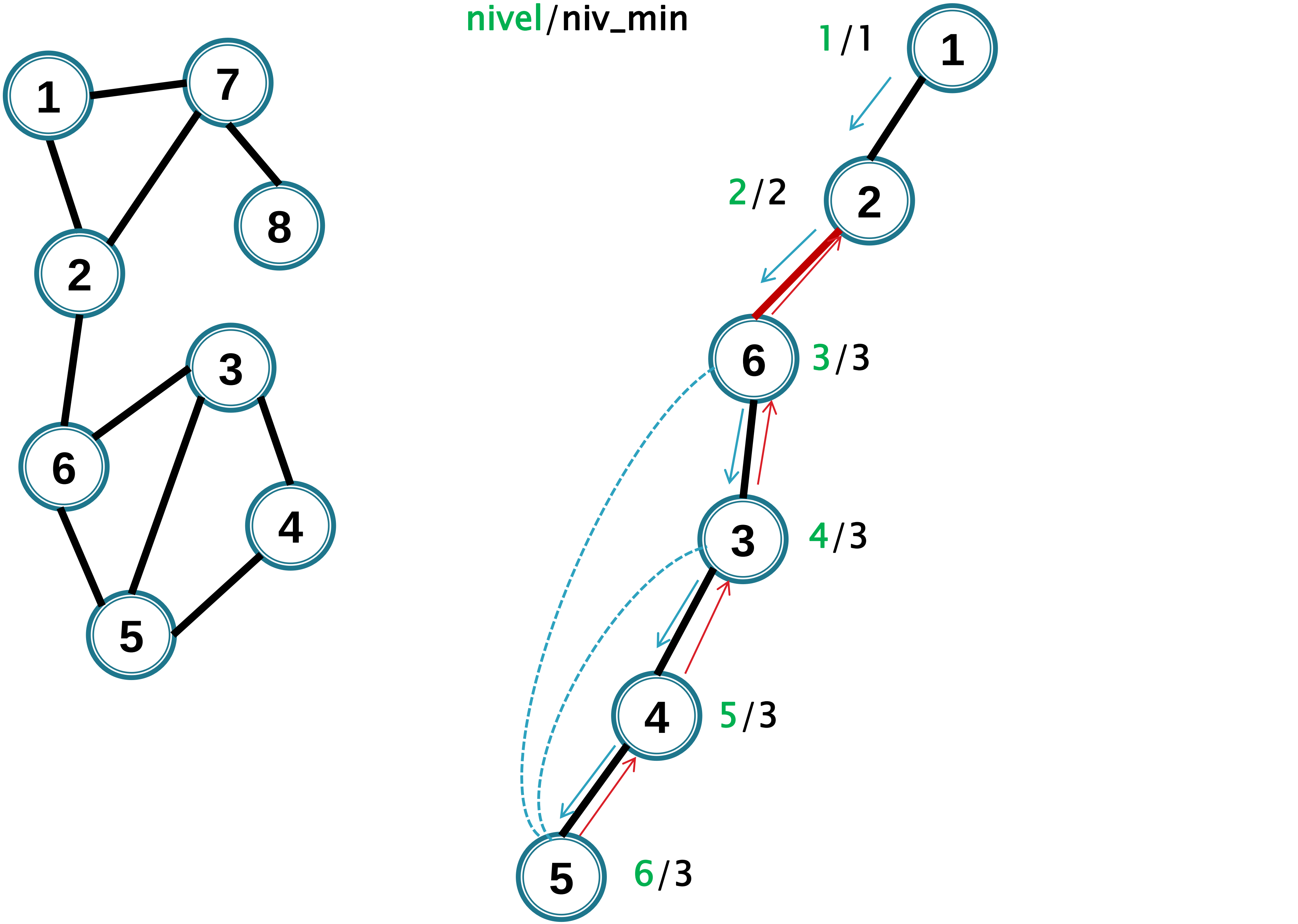

nivel/niv_min
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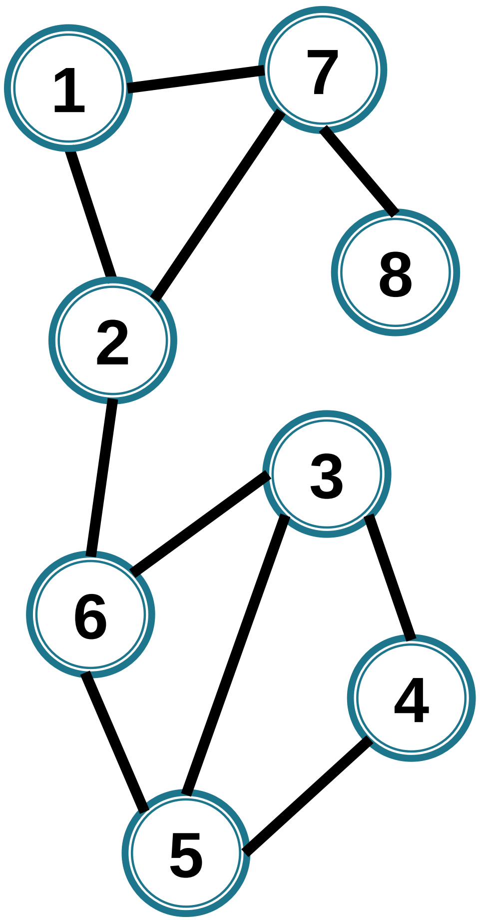
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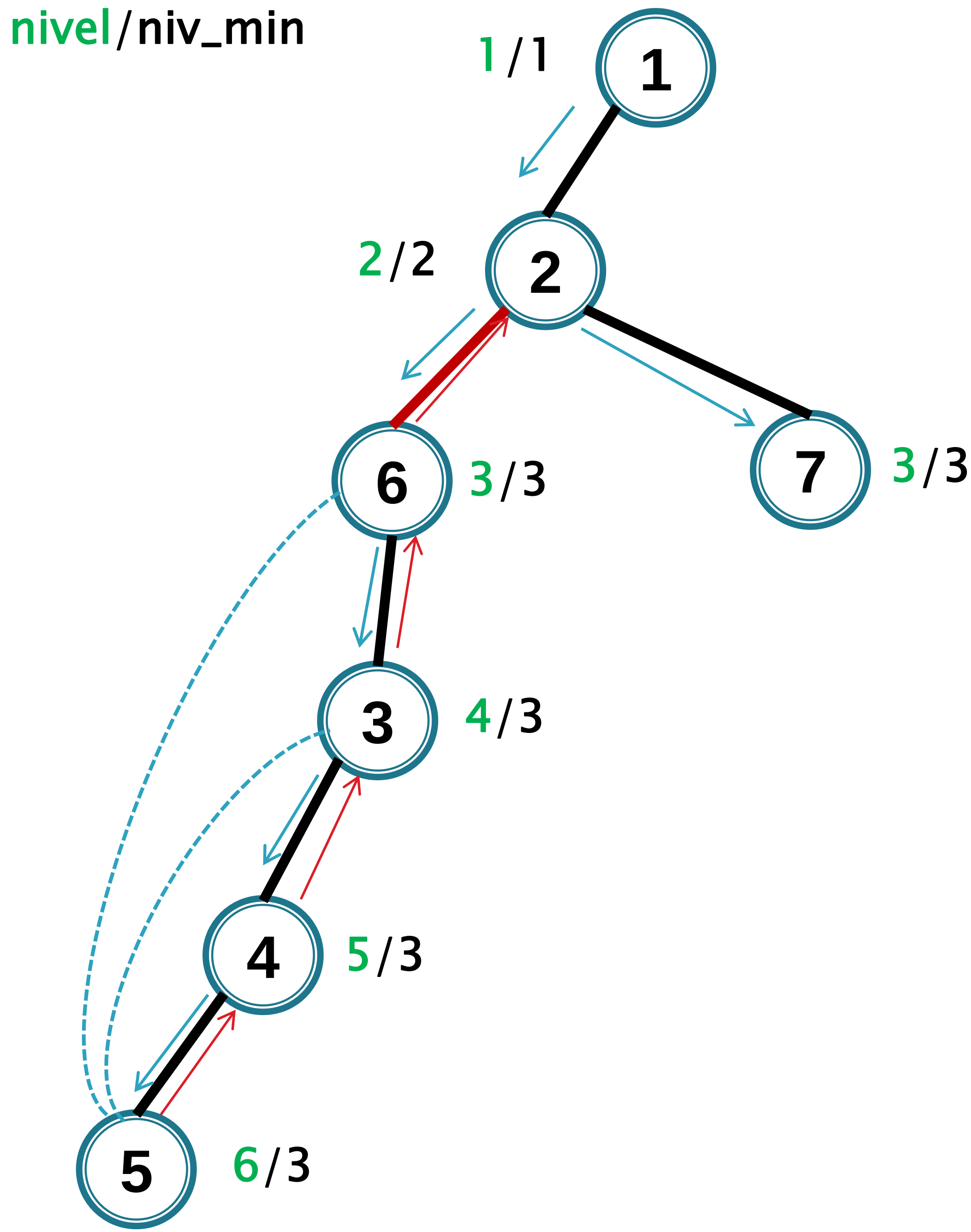
nivel/niv_min
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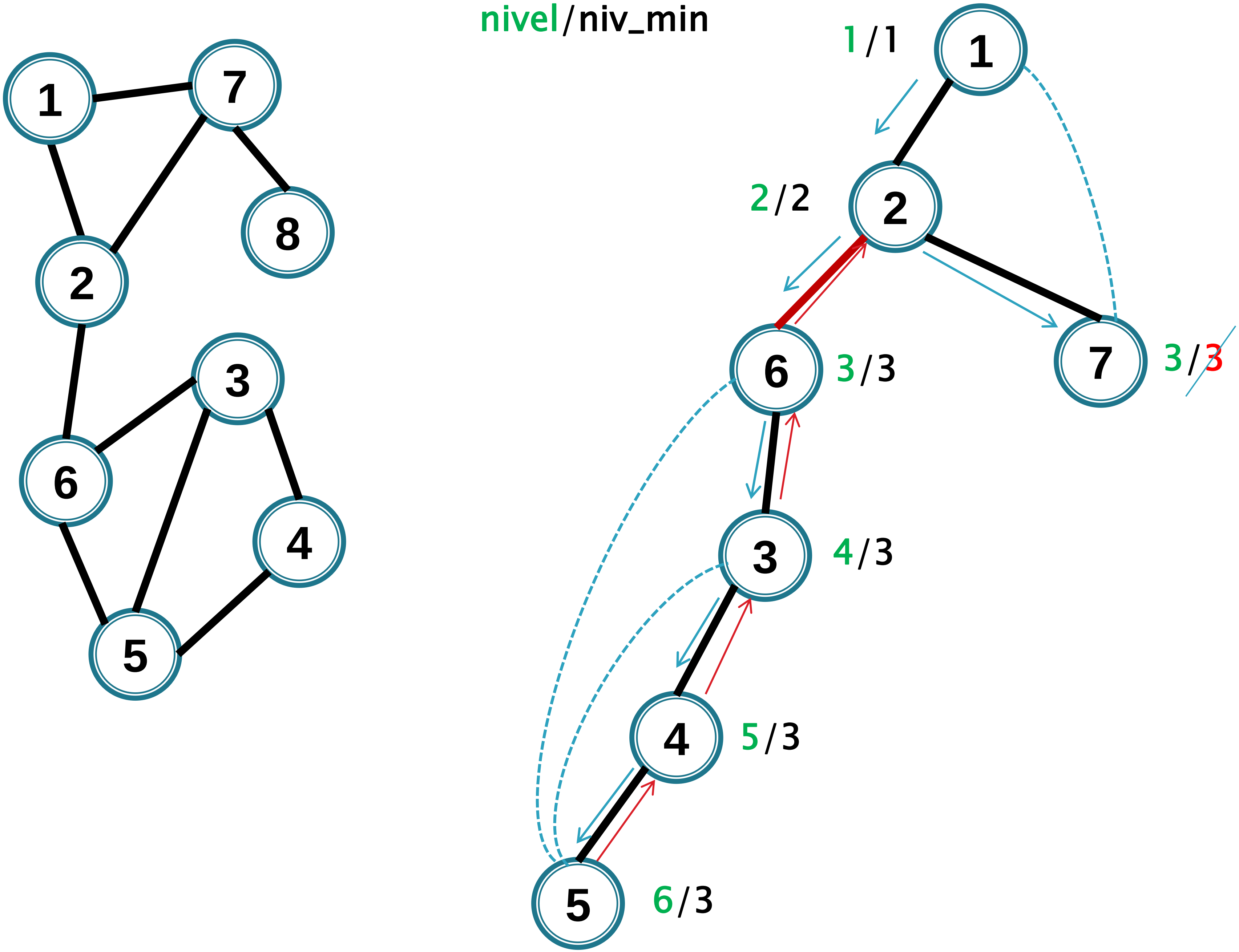

nivel/niv_min
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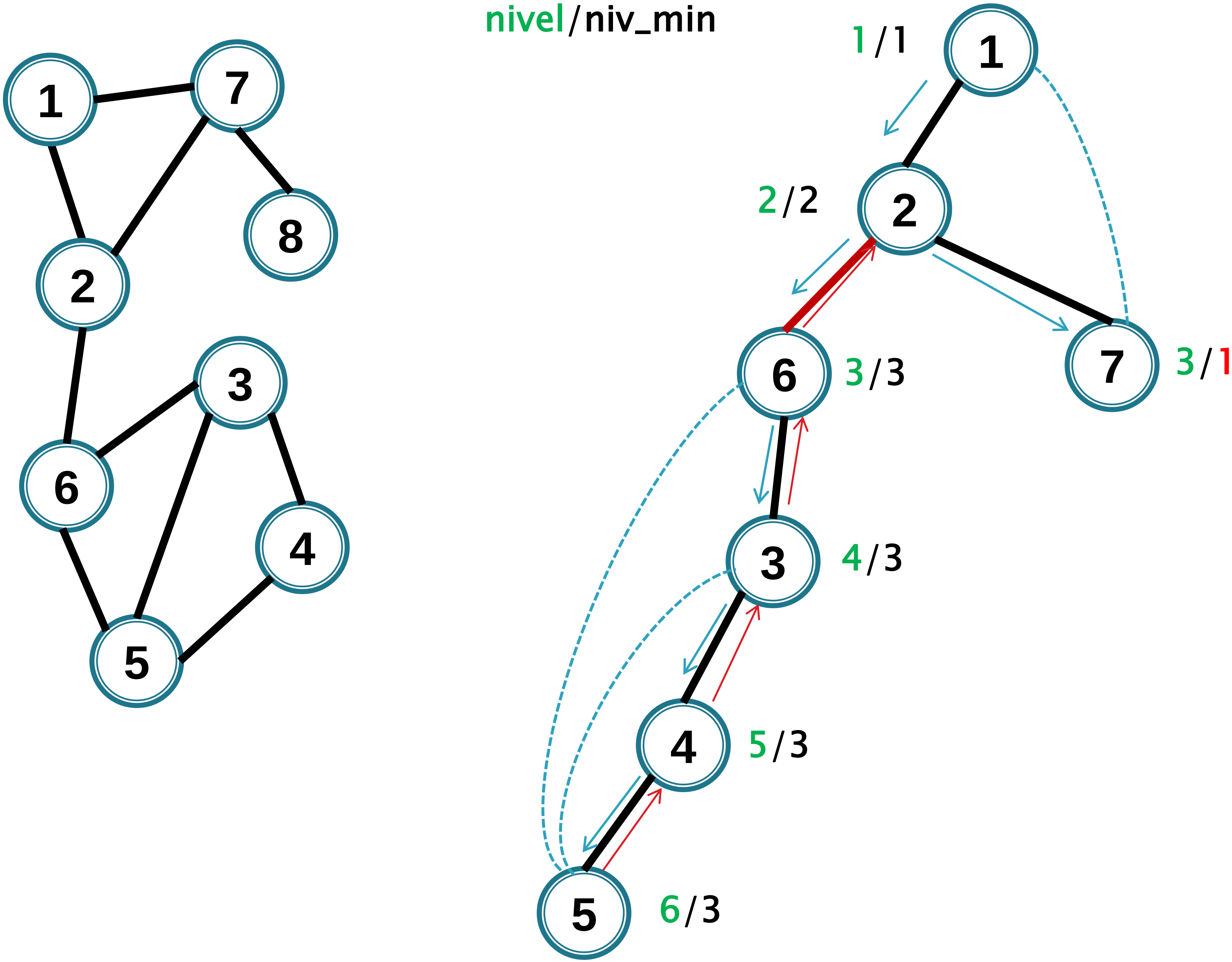
nivel/niv_min
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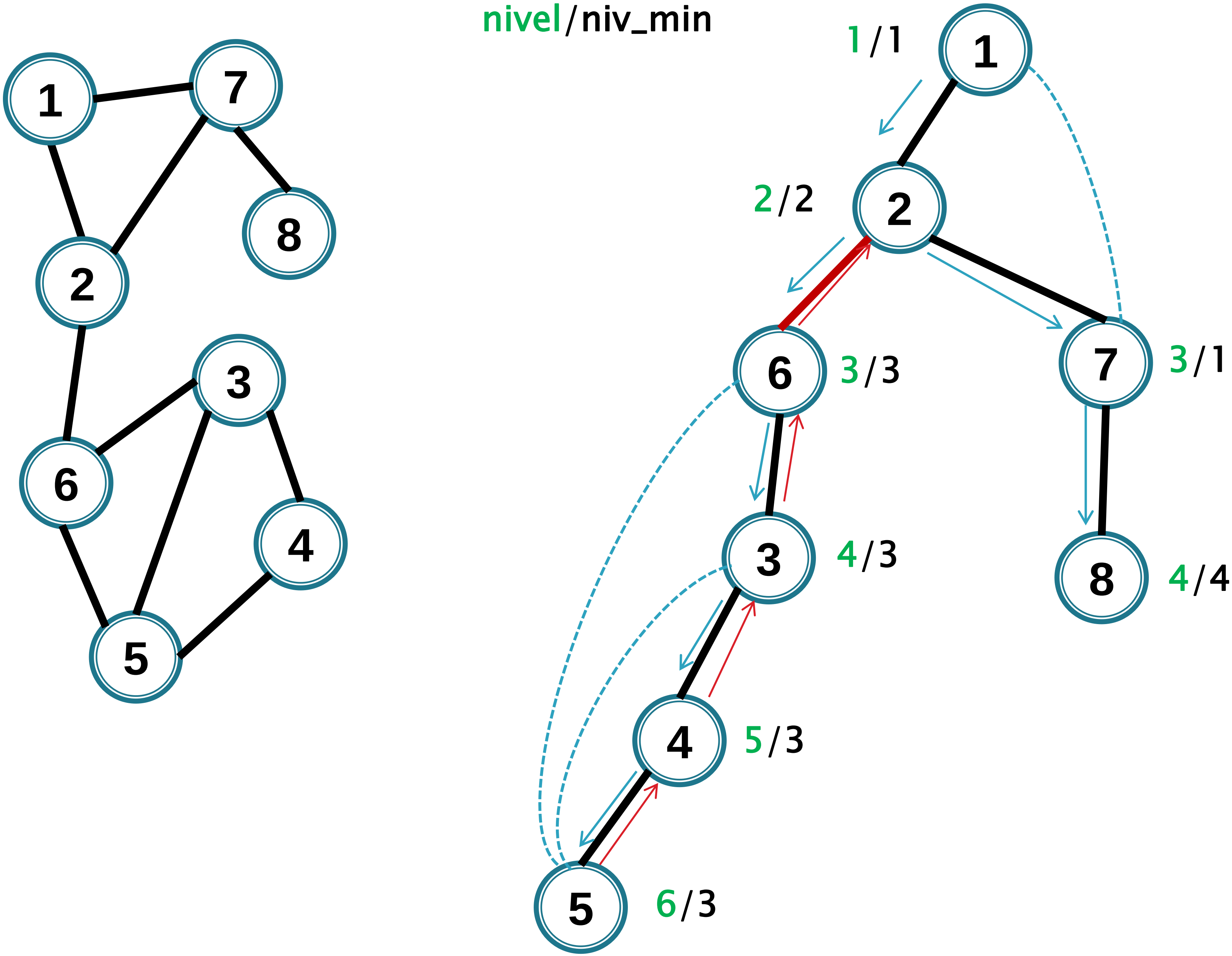
3/3
3/1
4/3
5/3
6/3


nivel/niv_min
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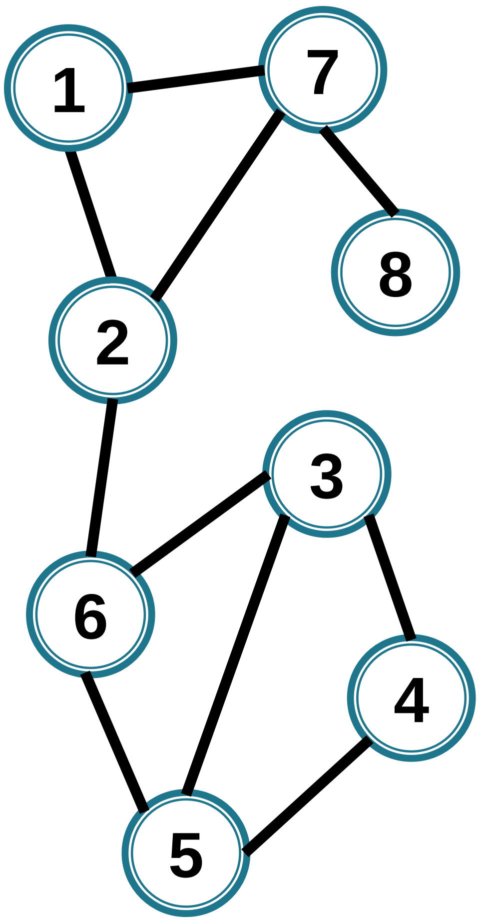
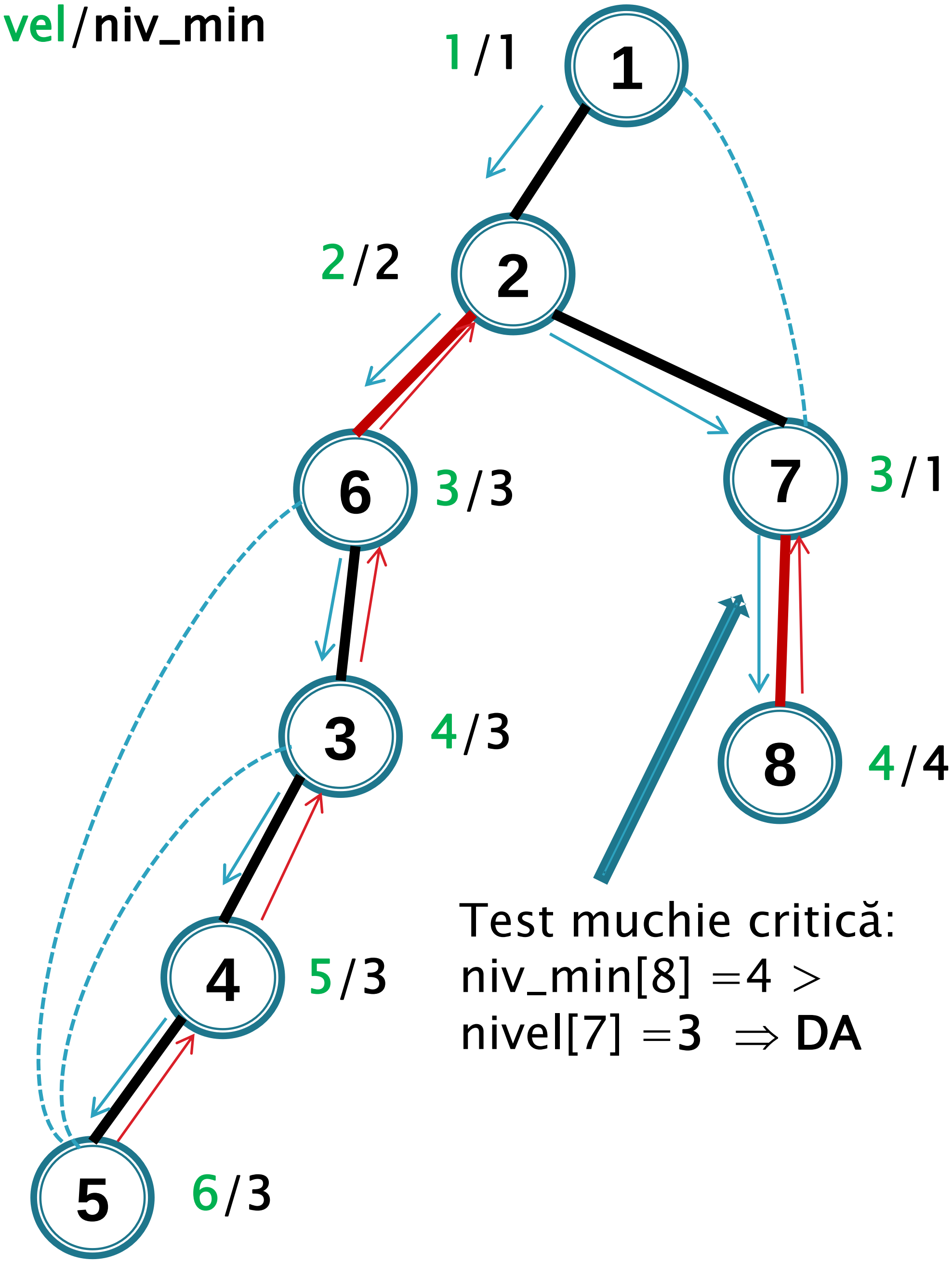
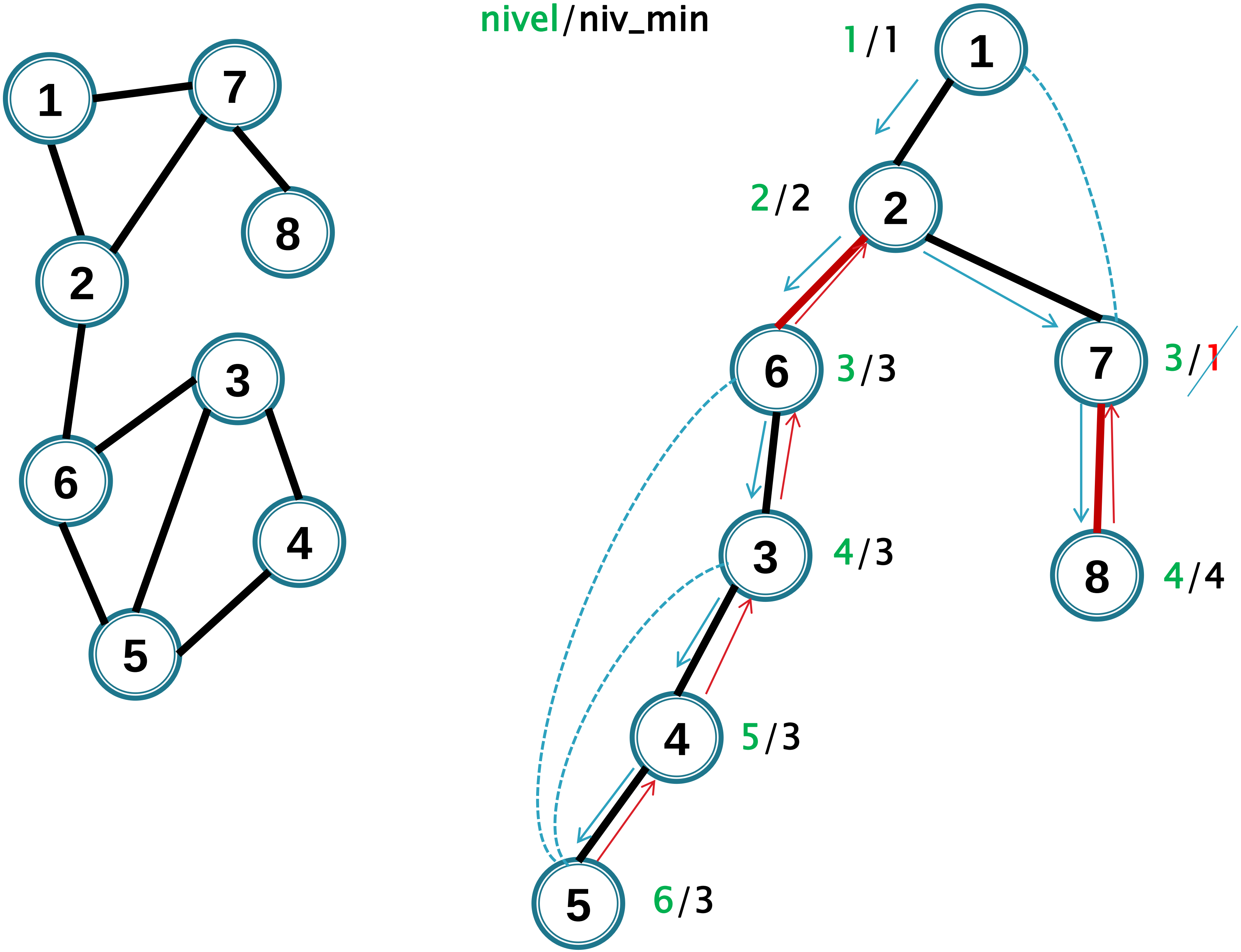
6/3

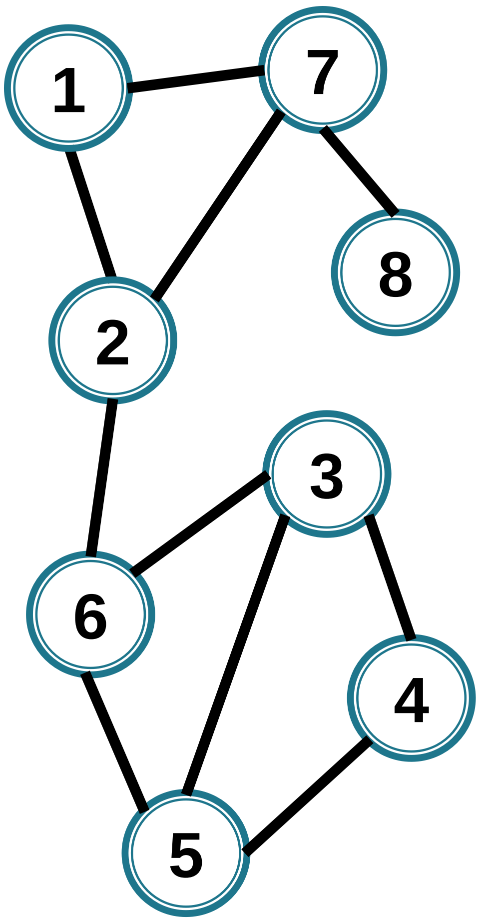




Test muchie critică:
niv_min[8] =4 >
nivel[7] =**3** ⇒ **DA**



nivel/niv_min
1/1
2/2
3/3
3/~~1~~
4/3
4/4
5/3
6/3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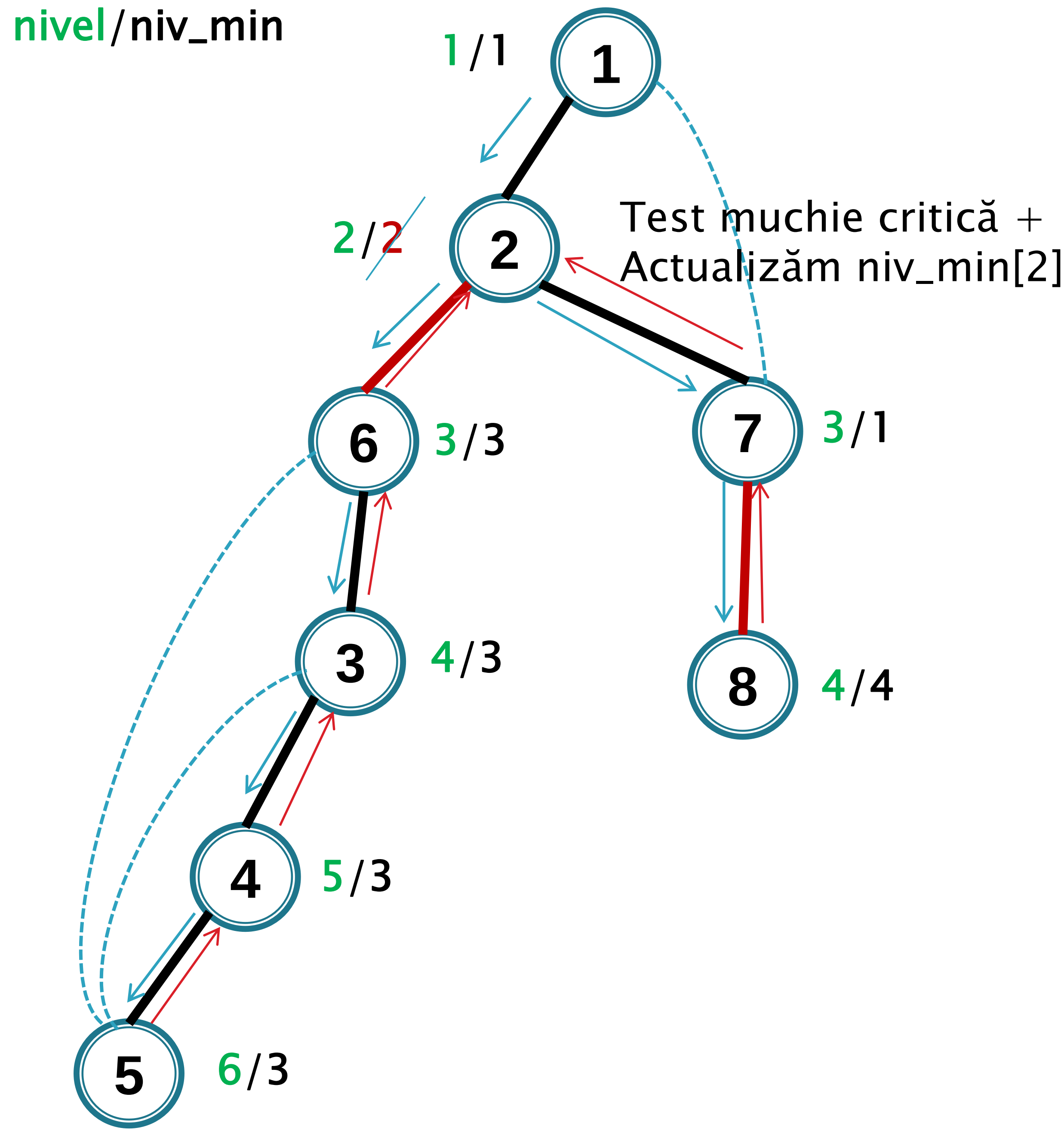
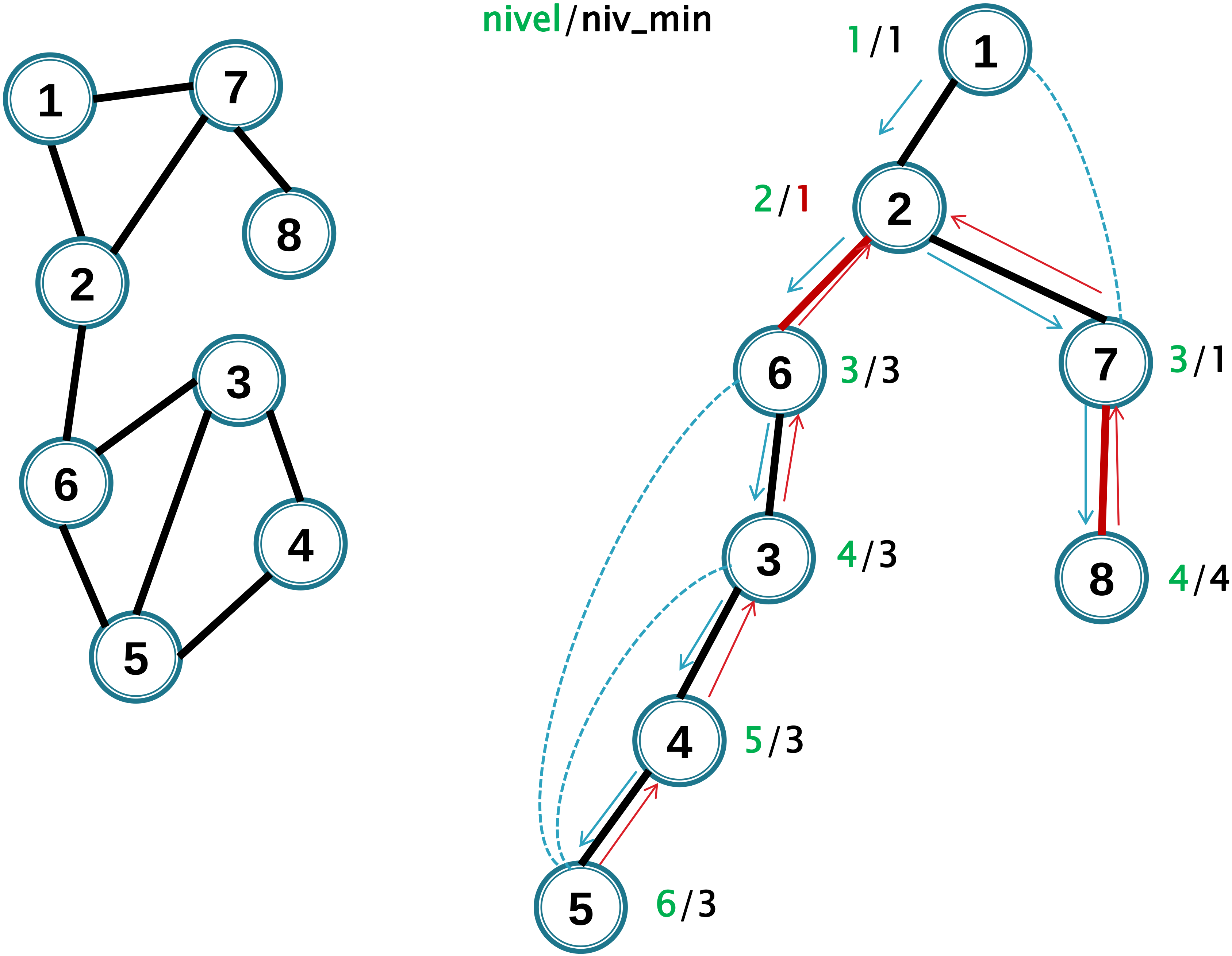
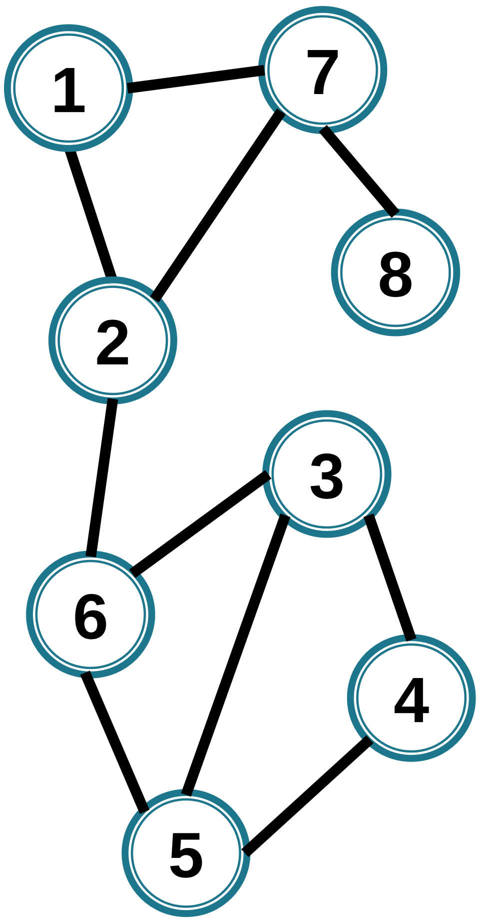
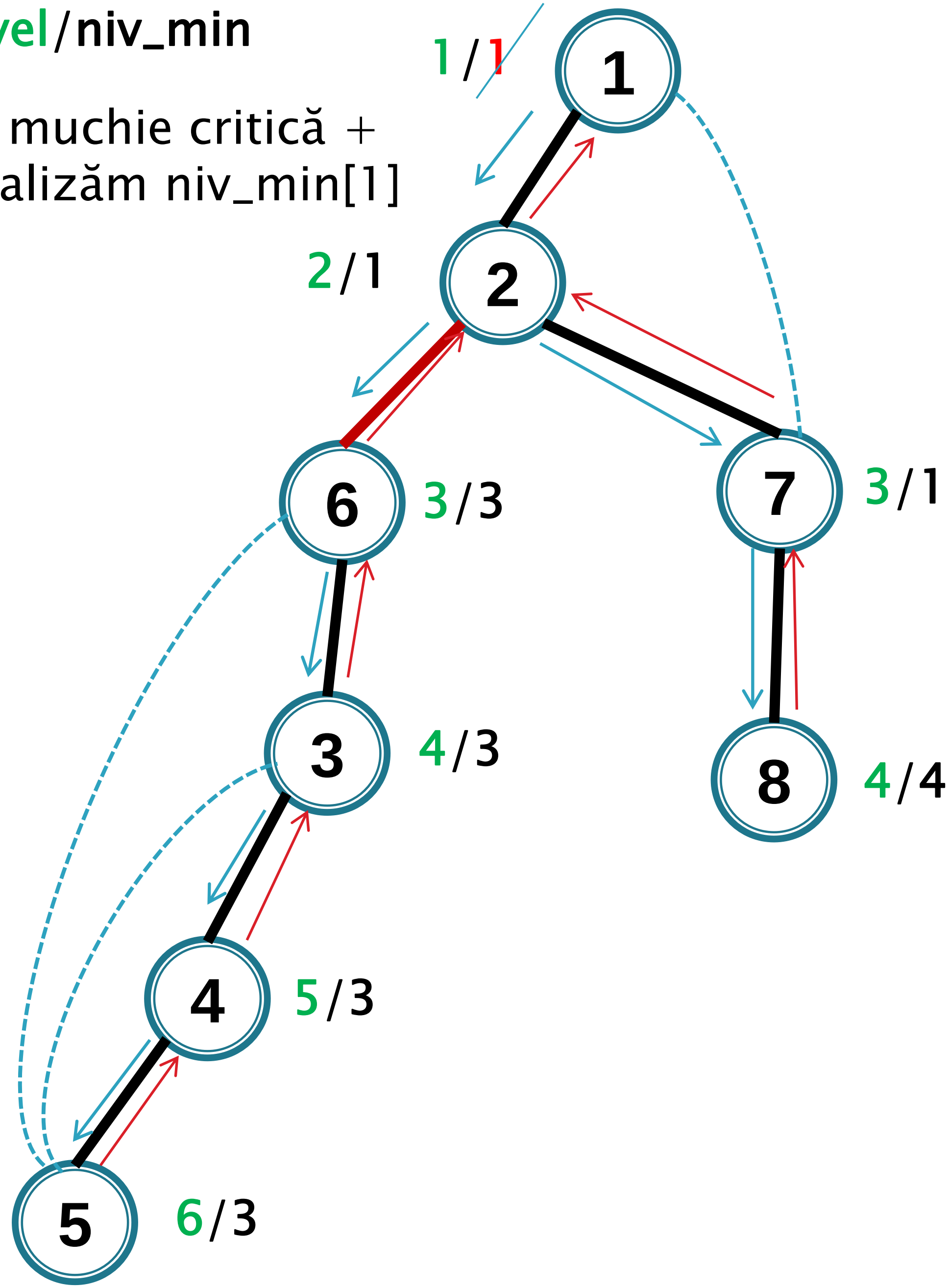

nivel/niv_min
1/1
1
2/2
2
Test muchie critică +
Actualizăm niv_min[2]
6
3/3
7
3/1
3
4/3
8
4/4
4
5/3
5
6/3

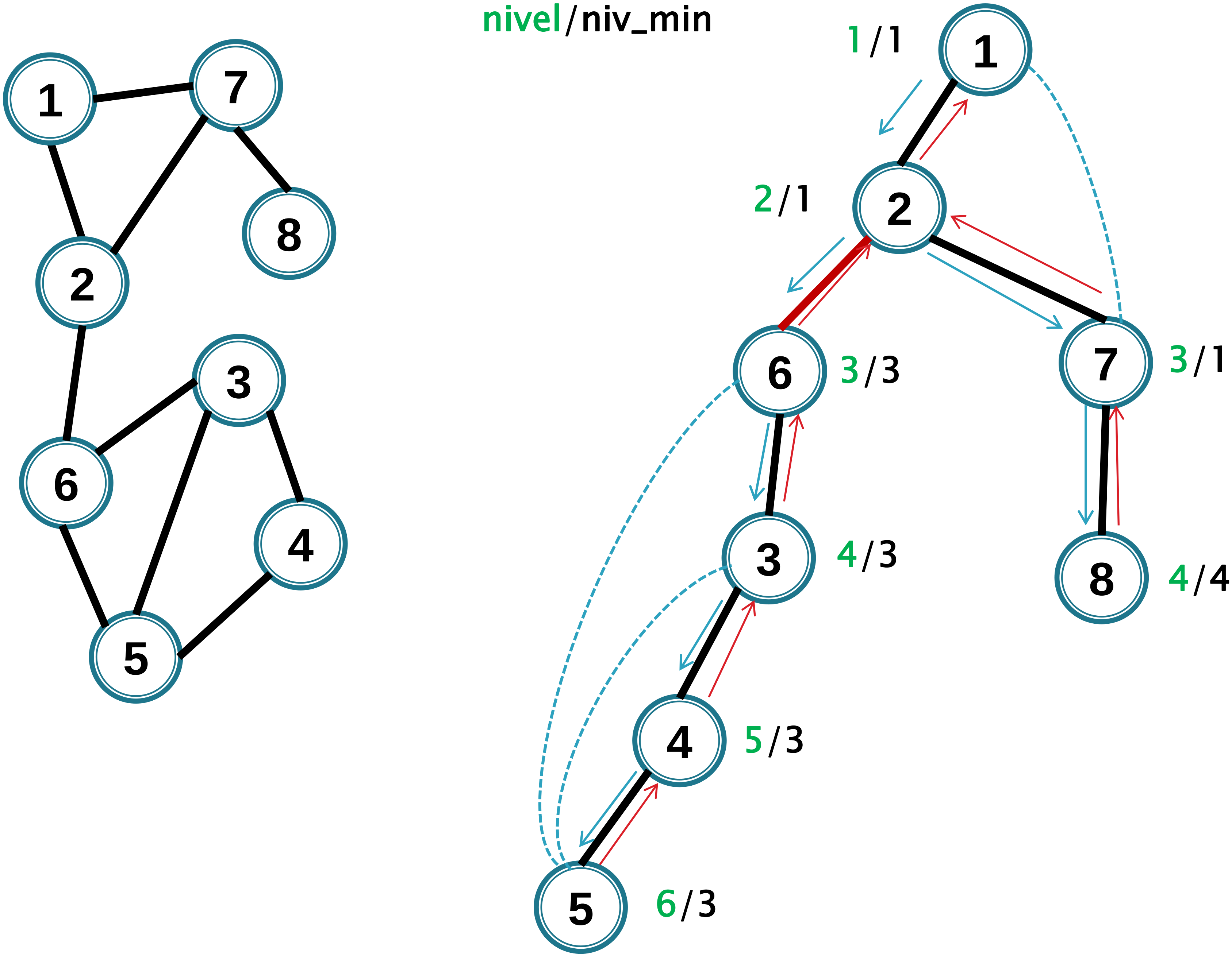

nivel/niv_min
1
7
8
2
3
6
4
5
1/1
2/1
6
3/3
7
3/1
3
4/3
8
4/4
4
5/3
5
6/3

nivel/niv_min

Test muchie critică +
Actualizăm niv_min[1]







nivel/niv_min
1/1
2/1
3/3
3/1
4/3
4/4
5/3
6/3

Indicații implementare

```
void df(int i){
     viz[i] = 1;
     niv_min[i] = nivel[i];
     for(j vecin al lui i)
           if(viz[j]==0){ //ij muchie de avansare
                nivel[j] = nivel[i]+1;
                df(j);

                //actualizare niv_min[i]- formula B
                ...

                //test ij este muchie critica
                ...
           }
           else
              if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
                  //actualizare niv_min[i]- formula A
                  ...
 }
```

Indicații implementare

```
void df(int i){
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     niv_min[i] = nivel[i];
     for(j vecin al lui i)
           if(viz[j]==0){ //ij muchie de avansare
                nivel[j] = nivel[i]+1;
                df(j);

                //actualizare niv_min[i]- formula B
                niv_min[i] = min{niv_min[i], niv_min[j] }

                //test ij este muchie critica
                ...
           }
           else
              if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
                  //actualizare niv_min[i]- formula A
                  ...
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```

Indicații implementare

```
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     niv_min[i] = nivel[i];
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           if(viz[j]==0){ //ij muchie de avansare
                nivel[j] = nivel[i]+1;
                df(j);

                //actualizare niv_min[i]- formula B
                niv_min[i] = min{niv_min[i], niv_min[j] }

                //test ij este muchie critica
                if (niv_min[j]>nivel[i]) scrie muchia ij
           }
           else
              if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
                  //actualizare niv_min[i]- formula A
                   ...
 }
```

Indicații implementare

```
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           if(viz[j]==0){ //ij muchie de avansare
                nivel[j] = nivel[i]+1;
                df(j);

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                niv_min[i] = min{niv_min[i], niv_min[j] }

                //test ij este muchie critica
                if (niv_min[j]>nivel[i]) scrie muchia ij
           }
           else
              if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
                  //actualizare niv_min[i]- formula A
                  niv_min[i] = min{niv_min[i], nivel[j] }
 }
```

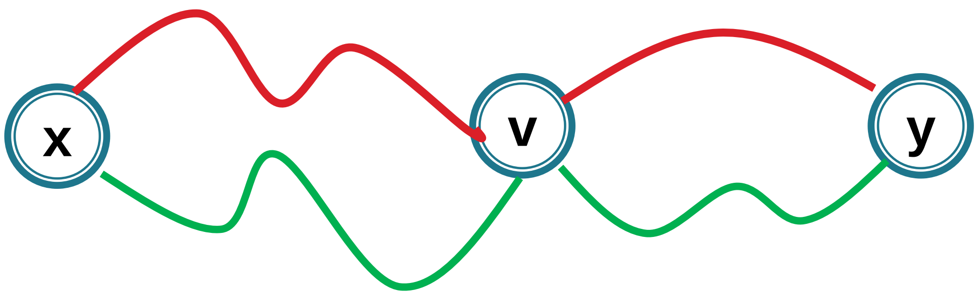
Puncte critice

Puncte critice

- Un vârf v este punct critic $\Leftrightarrow$

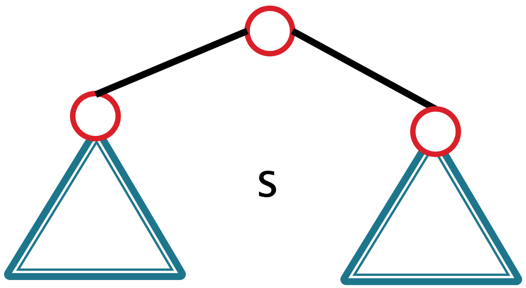
  există două vârfuri x,y $\neq$ v astfel

  încât v aparține oricărui x,y-lanț



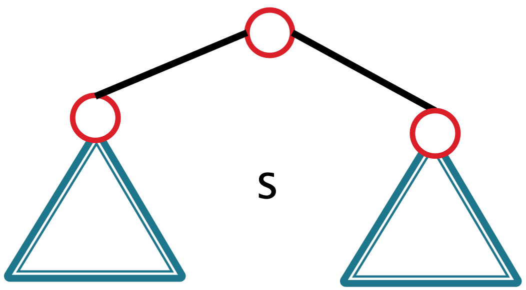
Puncte critice

- Arborele DF
  - **rădăcina s este punct critic $\Leftrightarrow$?**



Puncte critice

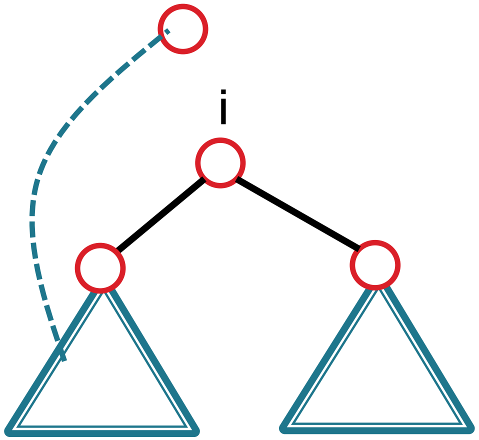
- Arborele DF
  - rădăcina s este punct critic ⇔ ?



are cel puțin 2 fii în arborele DF

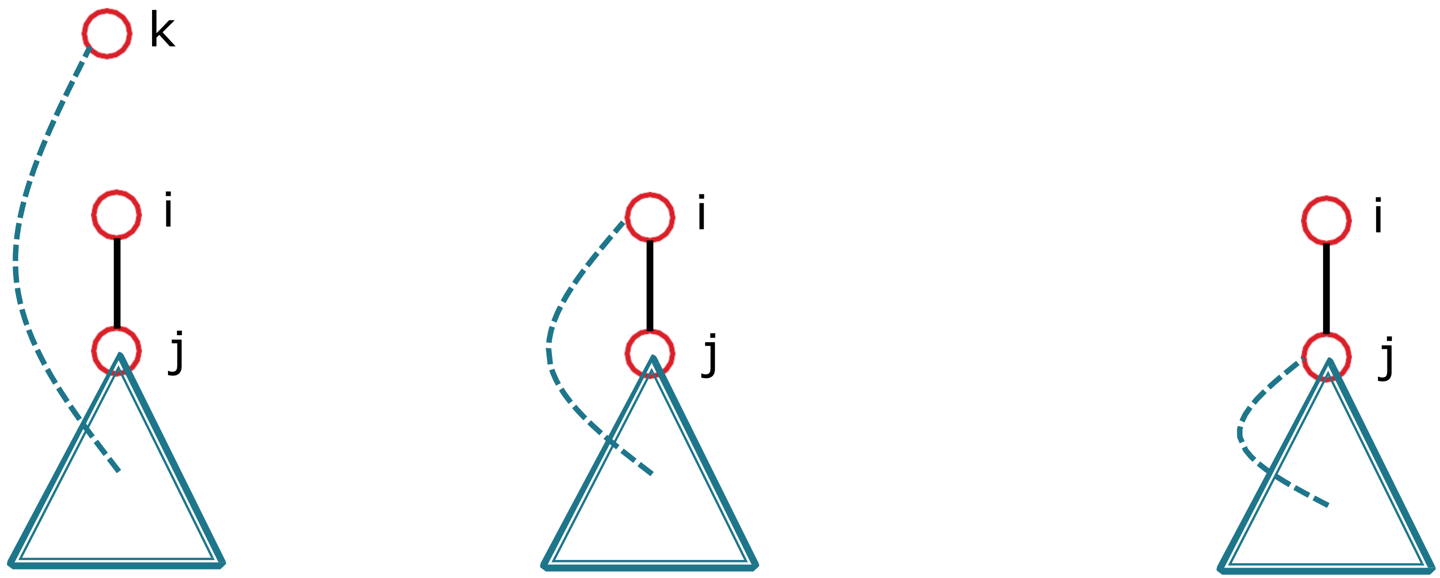
Puncte critice

- Arborele DF
  - **un alt vârf i din arbore este critic ⇔ ?**



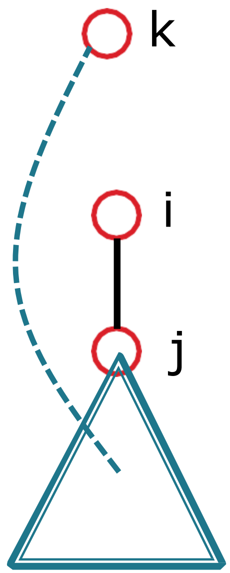
Puncte critice

Pentru $i \neq s$



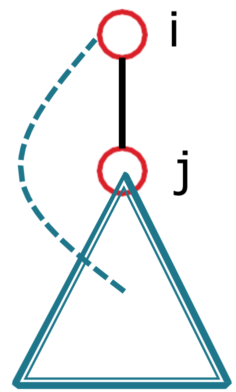
Puncte critice

Pentru i ≠ s



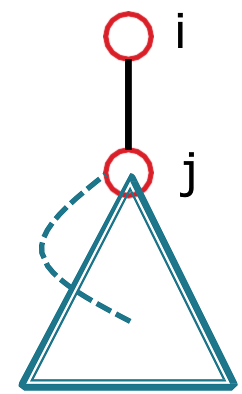
i NU este critic

`niv_min[j]<nivel[i]`



i ESTE critic

`niv_min[j]=nivel[i]`



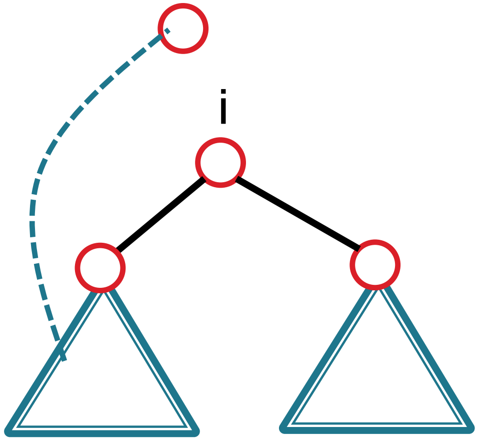
i ESTE critic

`niv_min[j]>nivel[i]`

Puncte critice

- Arborele DF
  - **un alt vârf i din arbore este critic ⇔**

    are cel puțin un <u>fiu</u> j cu
    niv_min[j] ≥ nivel[i]

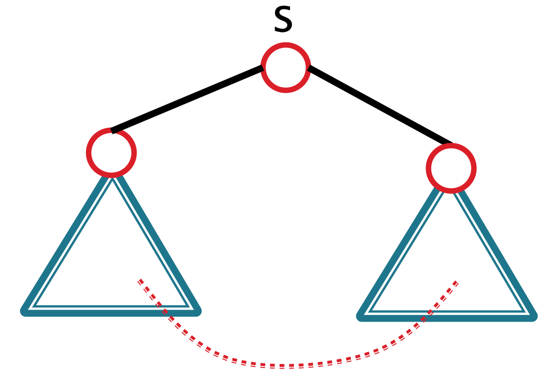


Puncte critice

- Un vârf v este punct critic ⇔ există două vârfuri x,y ≠ v astfel încât v aparține oricărui x,y-lanț

- Arborele DF
  - rădăcina s este punct critic ⇔

    are cel puțin 2 fii în arborele DF

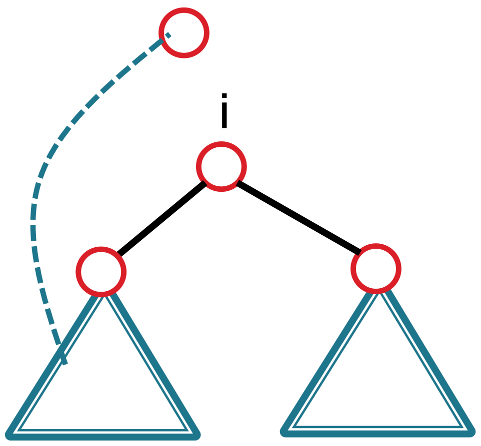


nu există muchii între subarbori
(de traversare)

  - un alt vârf i din arbore este critic ⇔

    `are cel puțin un fiu j cu`

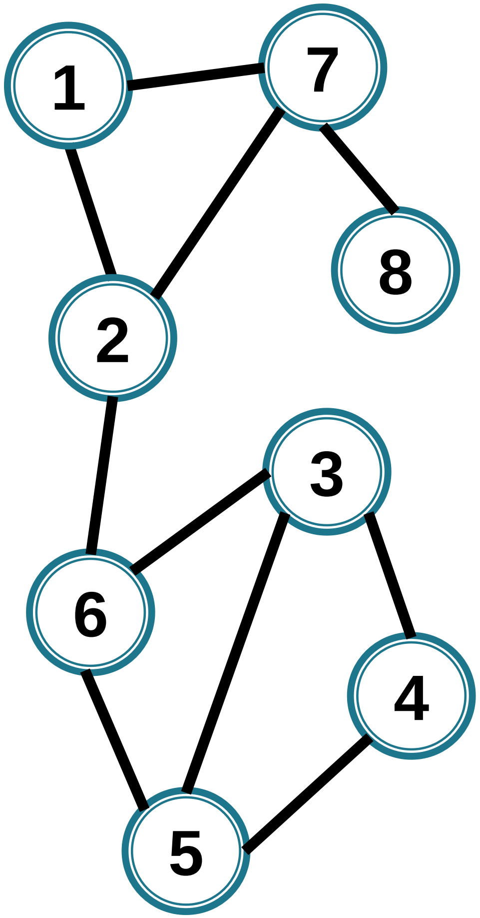
    `niv_min[j] ≥ nivel[i]`



Componente biconexe

Componente biconexe

- G – graf neorientat
- G=(V,M) biconex = **nu are puncte de articulaţie.**
- **Componentă biconexă** (**bloc**) a lui G = subgraf biconex maximal.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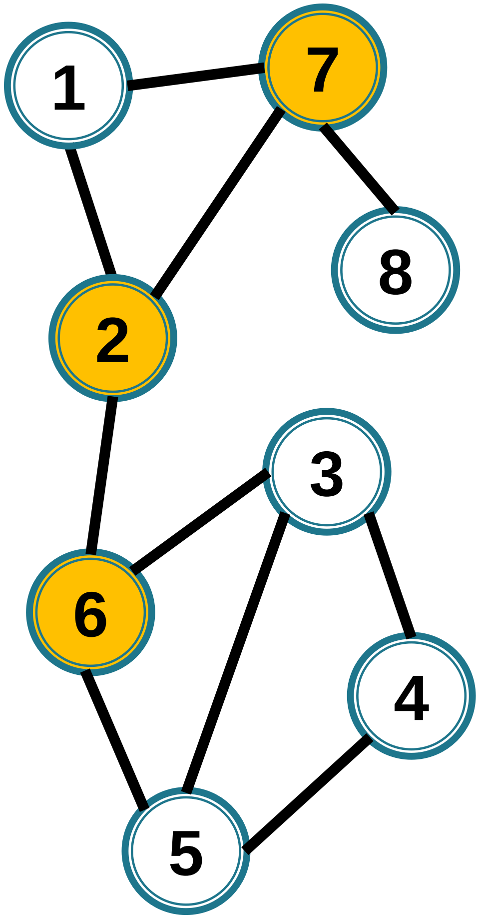
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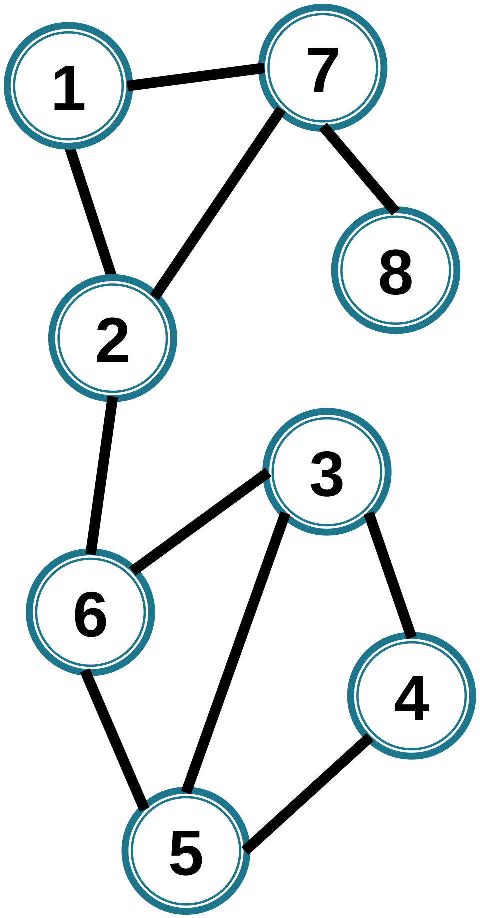
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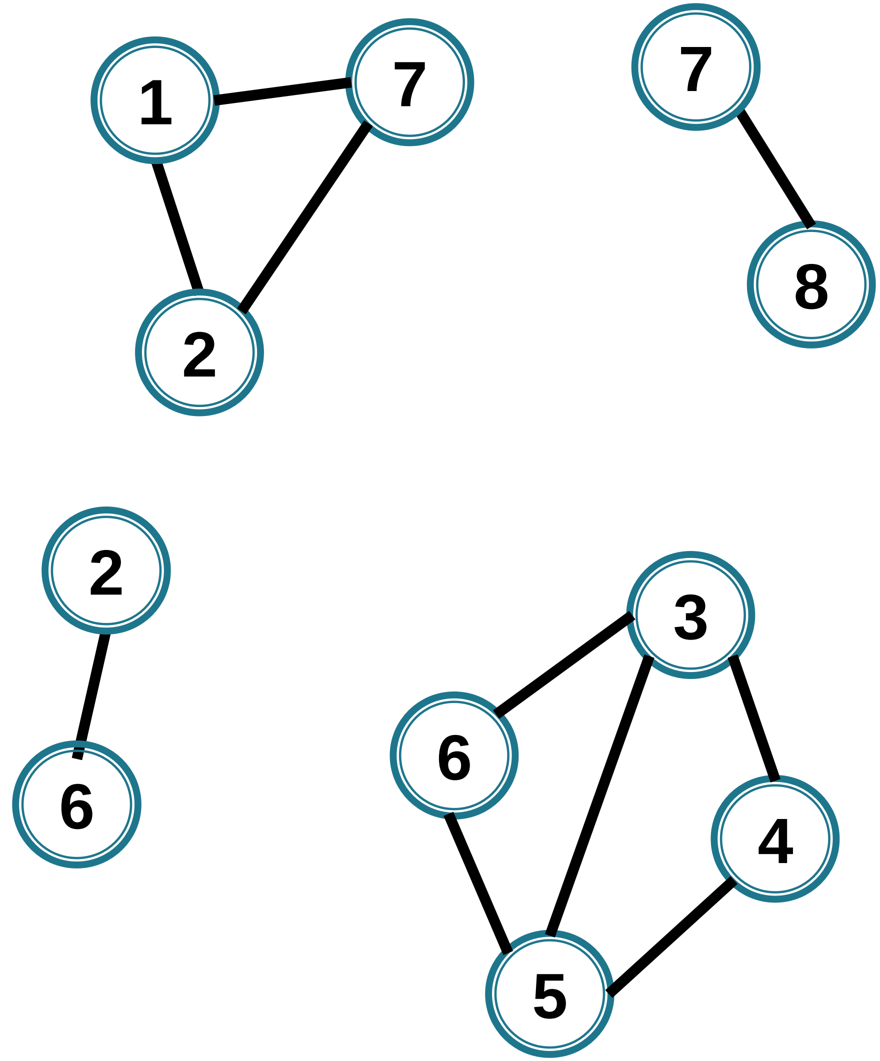
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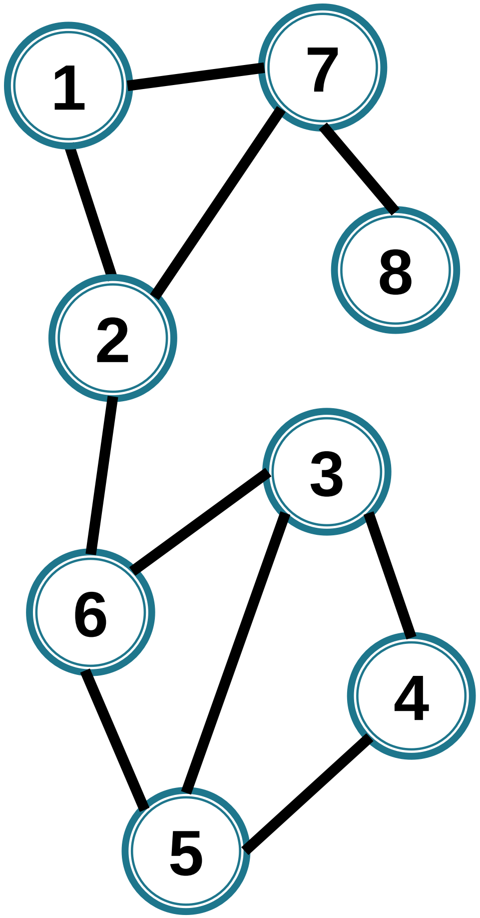
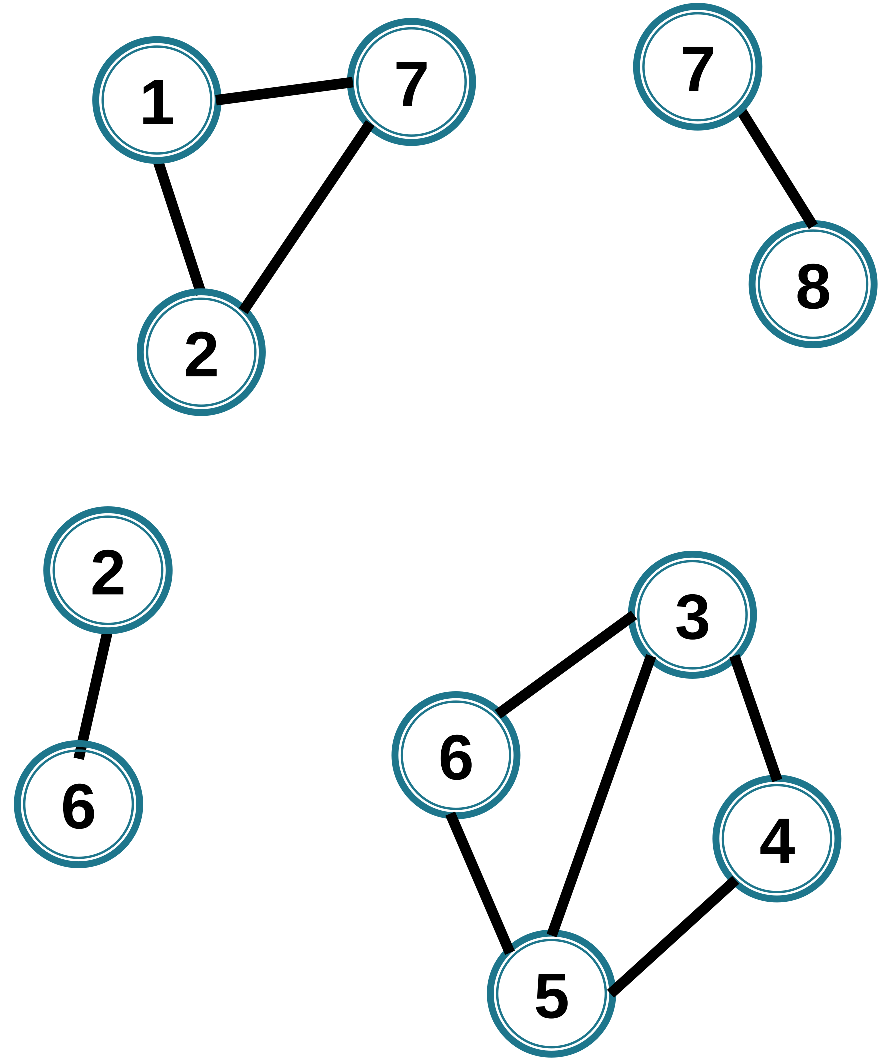


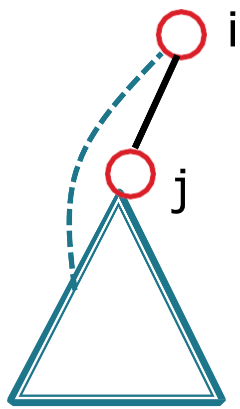


Muchie-disjuncte, nu vârf-disjuncte

Componente biconexe

- **Algoritm**

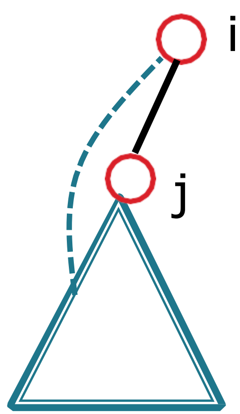
Intuitiv: când un fiu j semnalează că vârful curent i este critic, se poate afișa componenta care conține ij (care "se rupe" din vârful j)



Componente biconexe

- **Algoritm**

Intuitiv: când un fiu j semnalează că vârful curent i este critic, se poate afișa componenta care conține ij (care “se rupe” din vârful j)

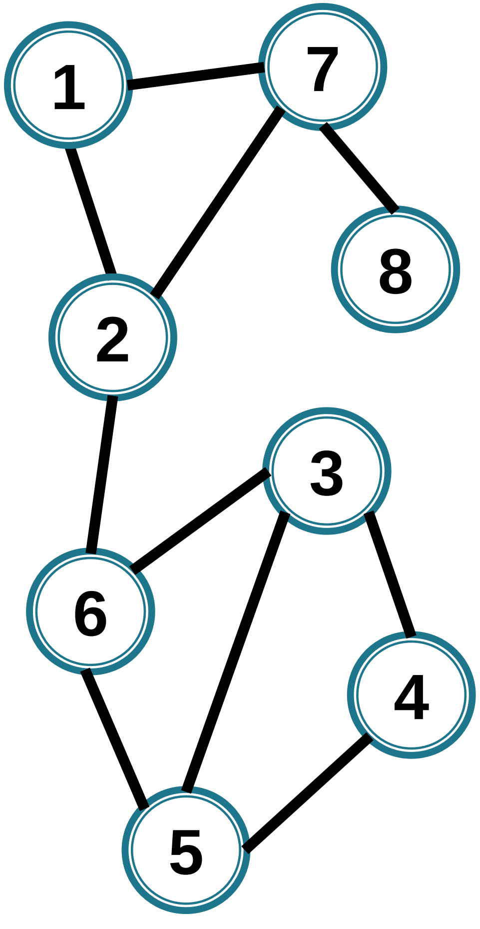


Memorăm muchiile într-o **stivă**

Când detectăm o componentă biconexă – scoatem muchiile din stivă până la muchia ij => acestea formează o componentă

Indicații implementare

```
void df(int i){
     viz[i] = 1;
     niv_min[i] = nivel[i];
     for(j vecin al lui i)
          if(viz[j]==0){ //ij muchie de avansare
                nivel[j] = nivel[i]+1;
               adauga(S,ij)
               df(j);
               niv_min[i] = min{niv_min[i], niv_min[j] }
               if (niv_min[j]> = nivel[i])
                       elimina din S toate muchiile pana la ij
           }
          else
             if(nivel[j]<nivel[i]-1) //ij muchie de intoarcere
                 //actualizare niv_min[i]- formula A
               niv_min[i] = min{niv_min[i], nivel[j]
               adauga(S,ij)
              }
 }
```


1
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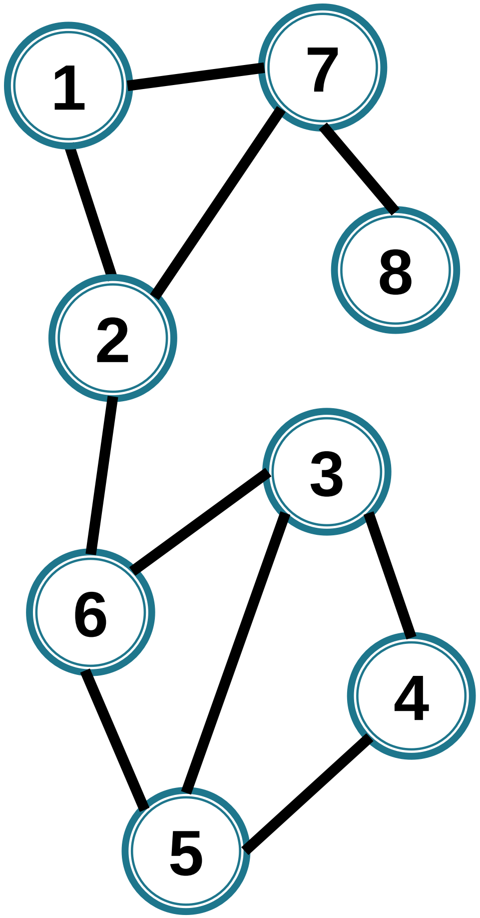
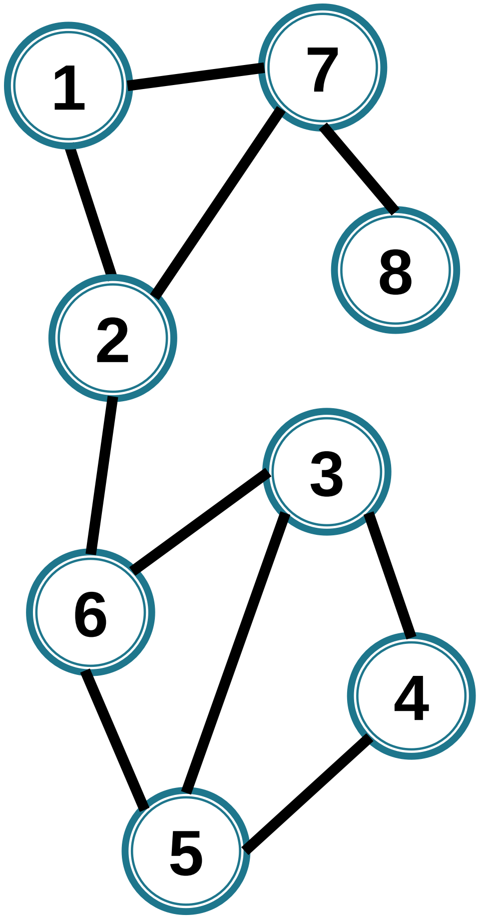
4
5

nivel/niv_min

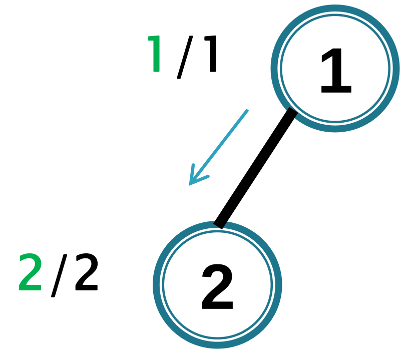




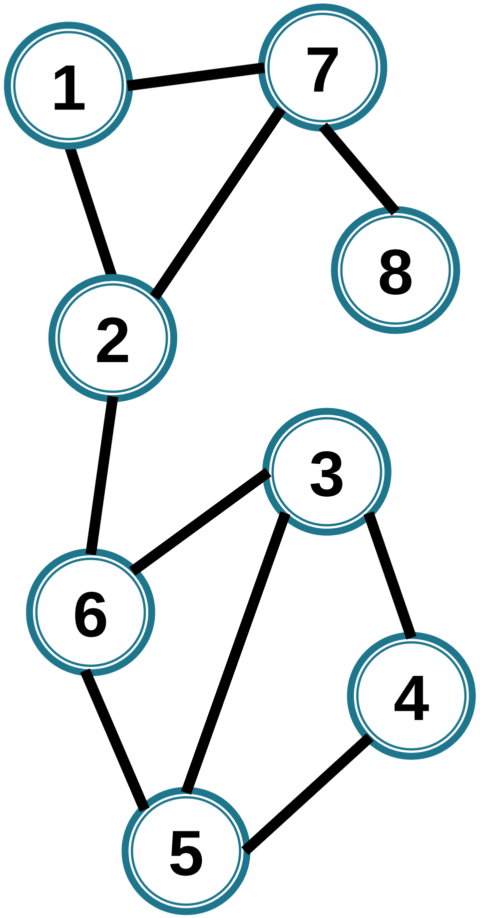
nivel/niv_min

S:

1 2



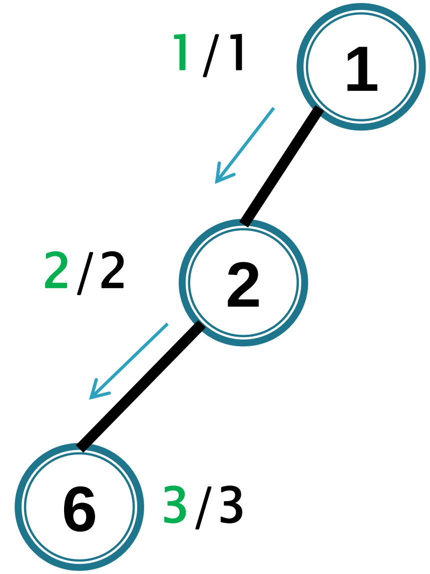
nivel/niv_min



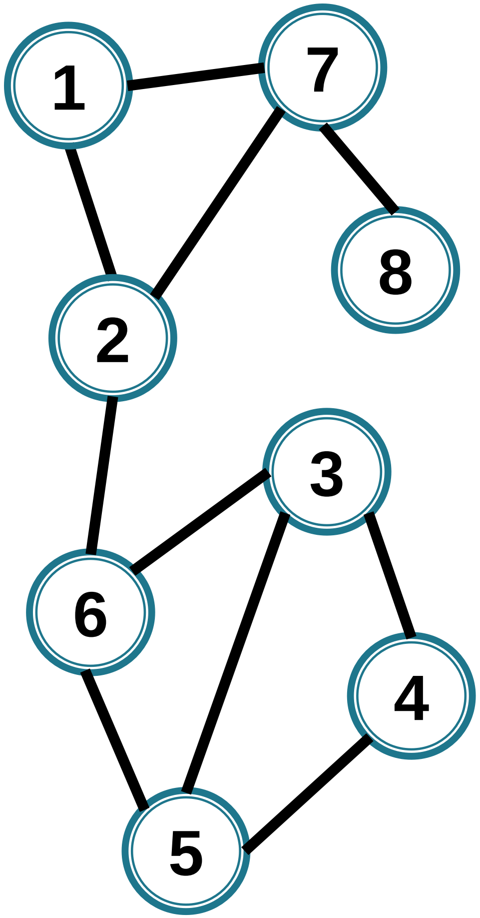
S:

1 2

2 6



nivel/niv_min

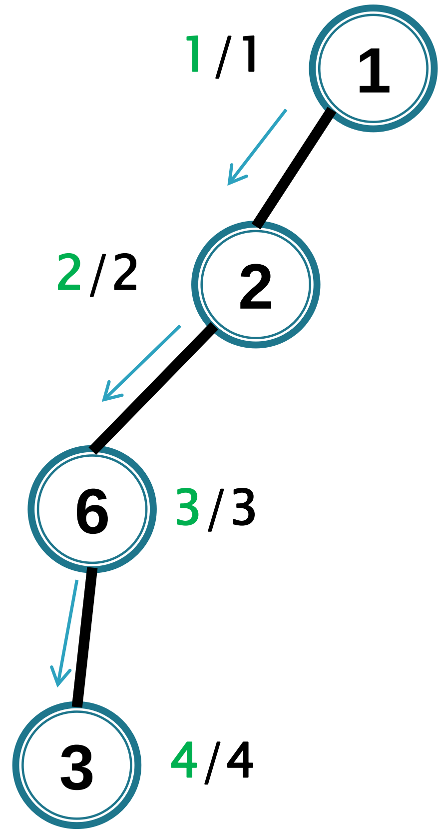


S:

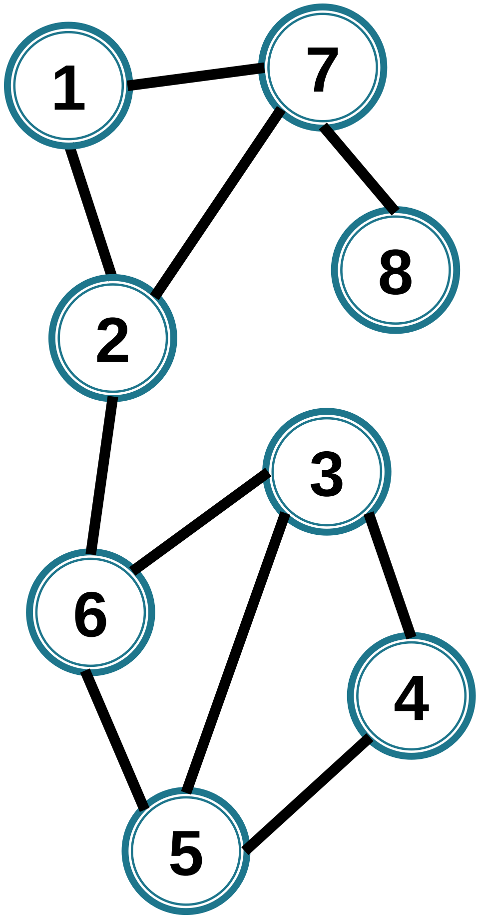
1 2

2 6

6 3



nivel/niv_min



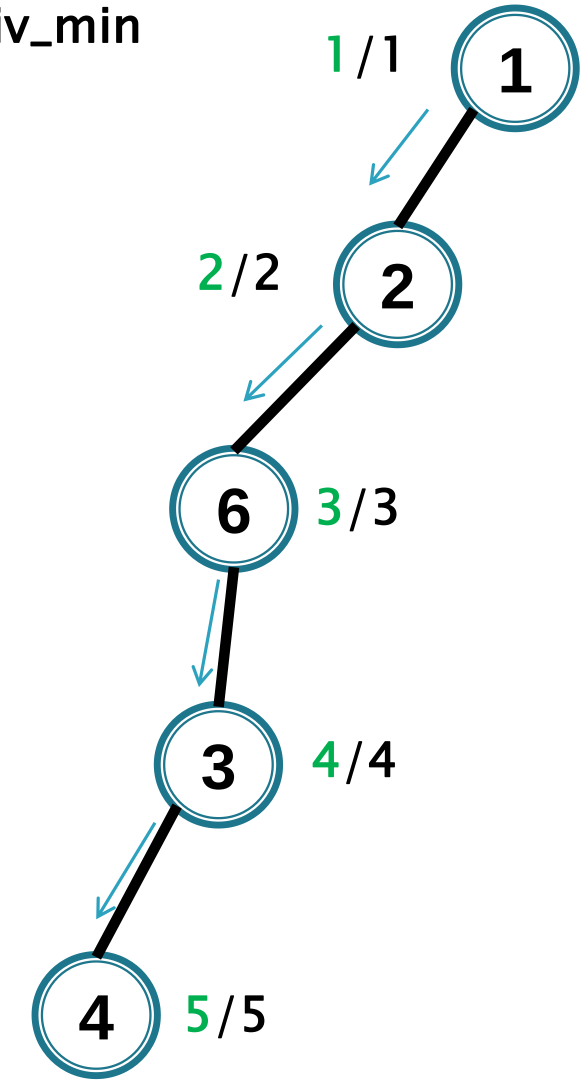
S:

1 2

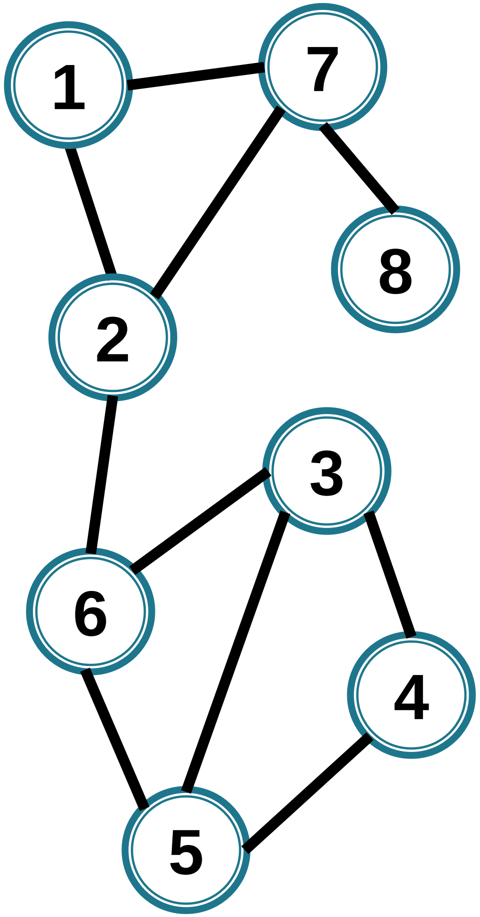
2 6

6 3

3 4



nivel/niv_min



S:

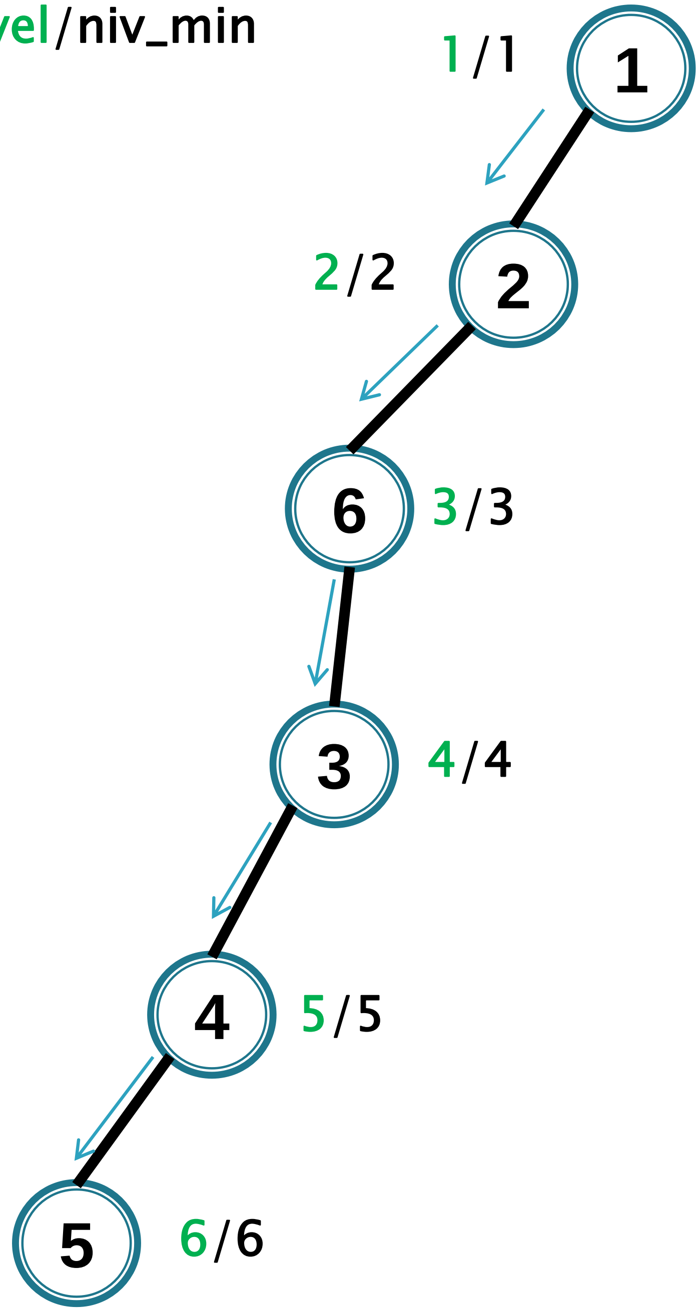
1 2

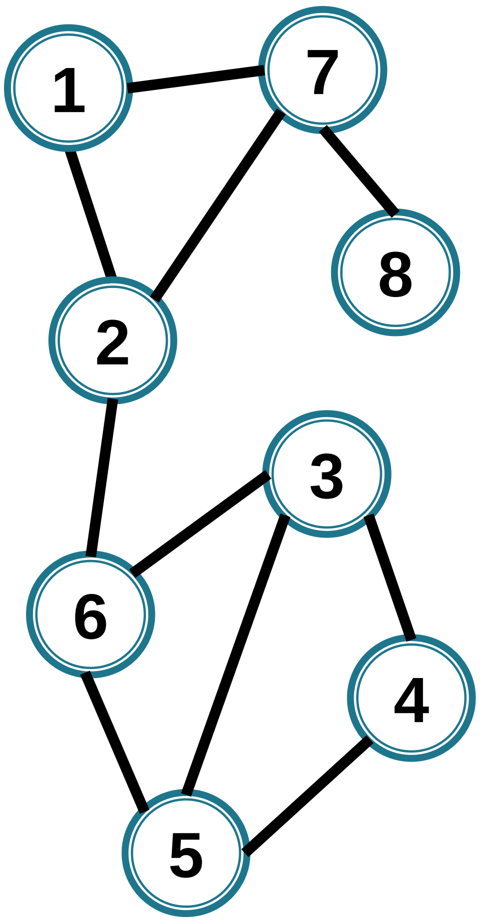
2 6

6 3

3 4

4 5





nivel/niv_min

S:

1 2

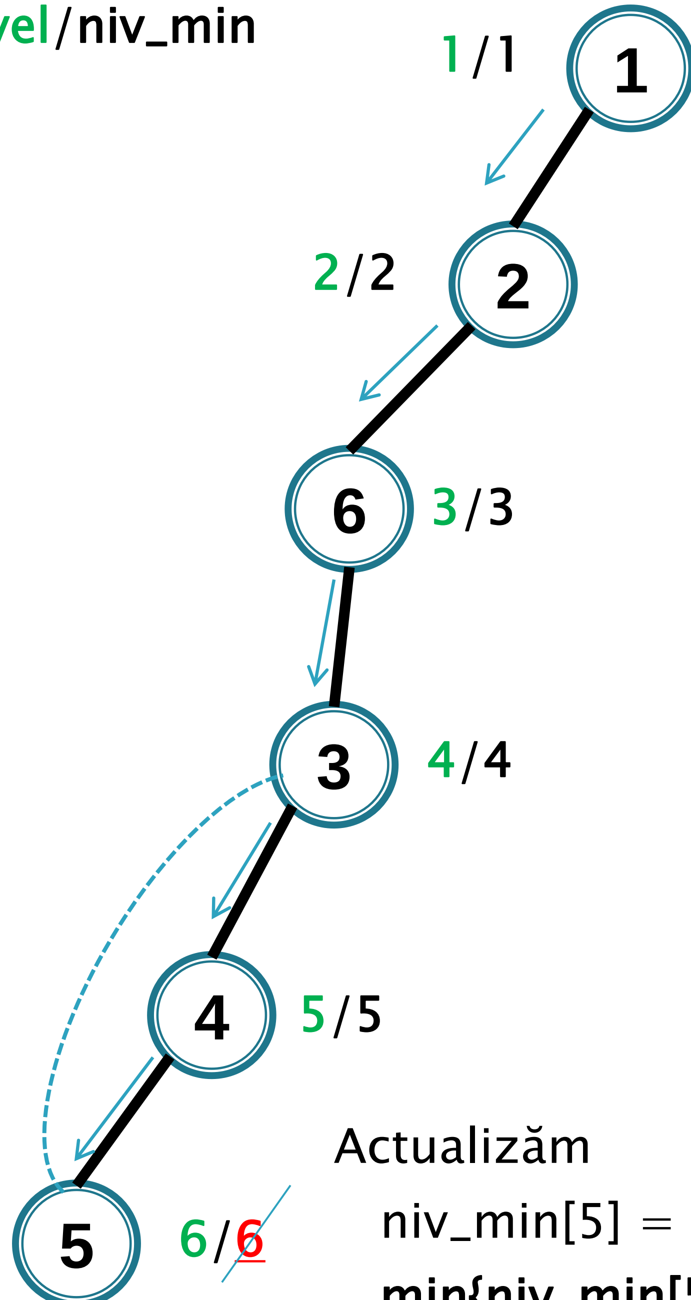
2 6

6 3

3 4

4 5

5 3

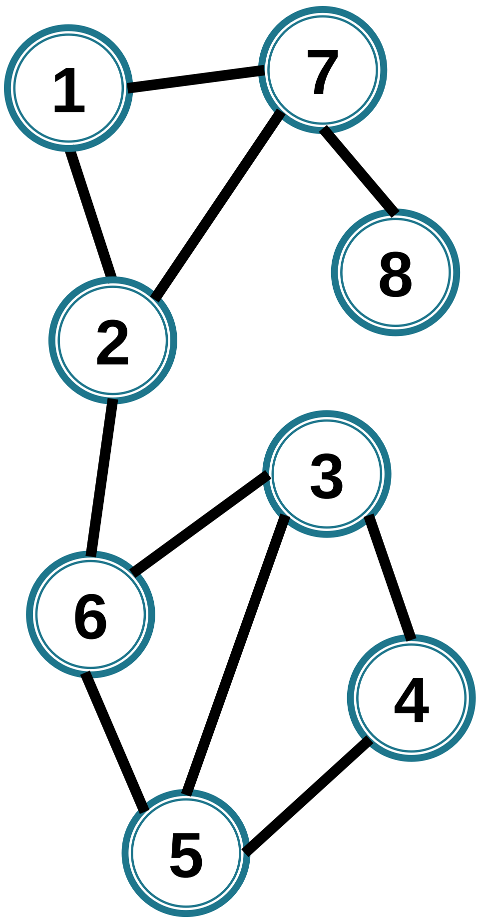


Actualizăm

niv_min[5] =

min{niv_min[5], nivel[3] }

(cazul A)



nivel/niv_min

S:

1 2

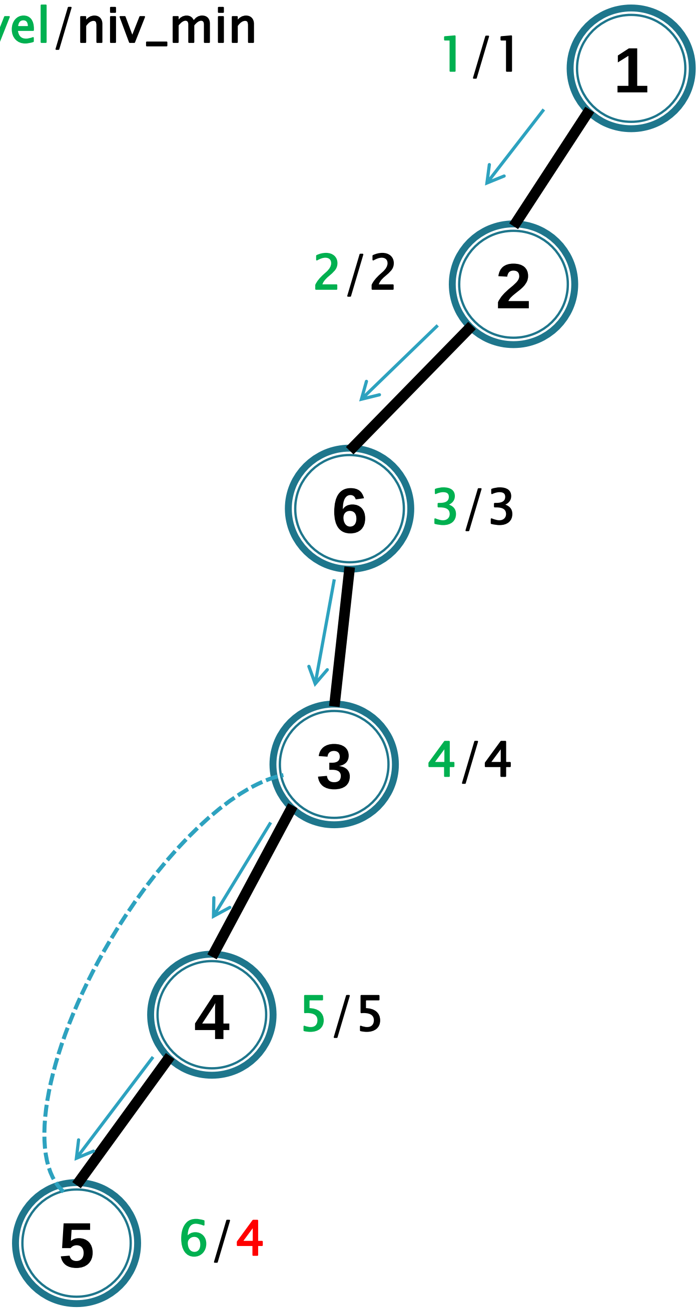
2 6

6 3

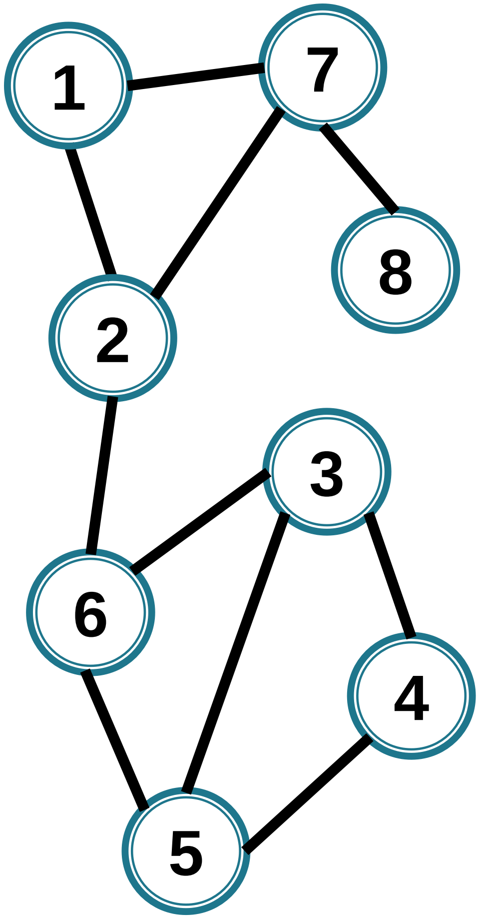
3 4

4 5

5 3



nivel/niv_min



S:

1 2

2 6

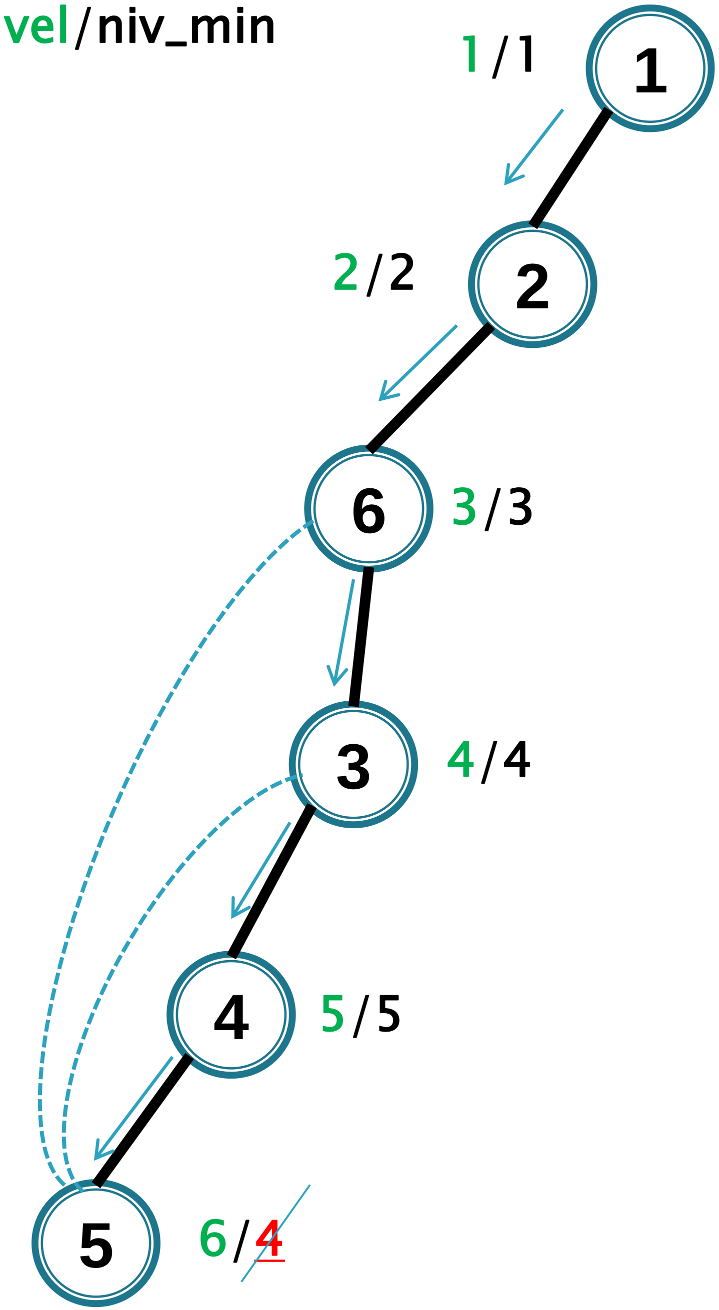
6 3

3 4

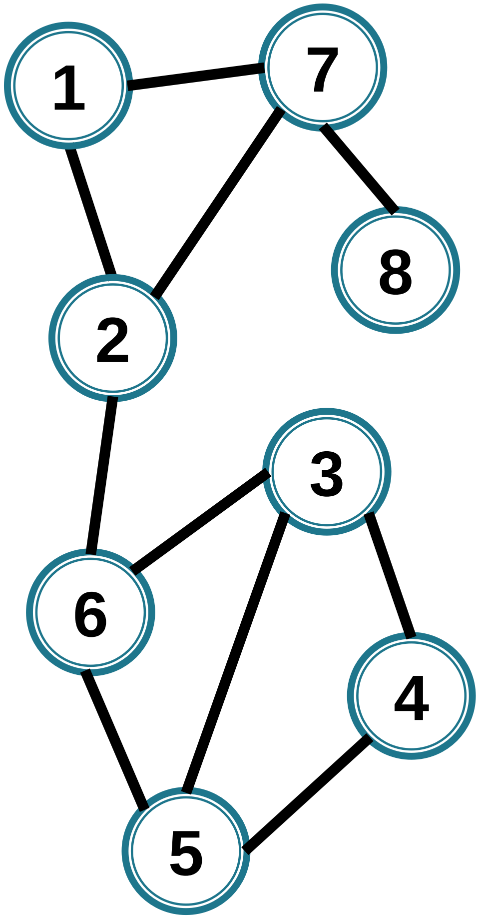
4 5

5 3

5 6

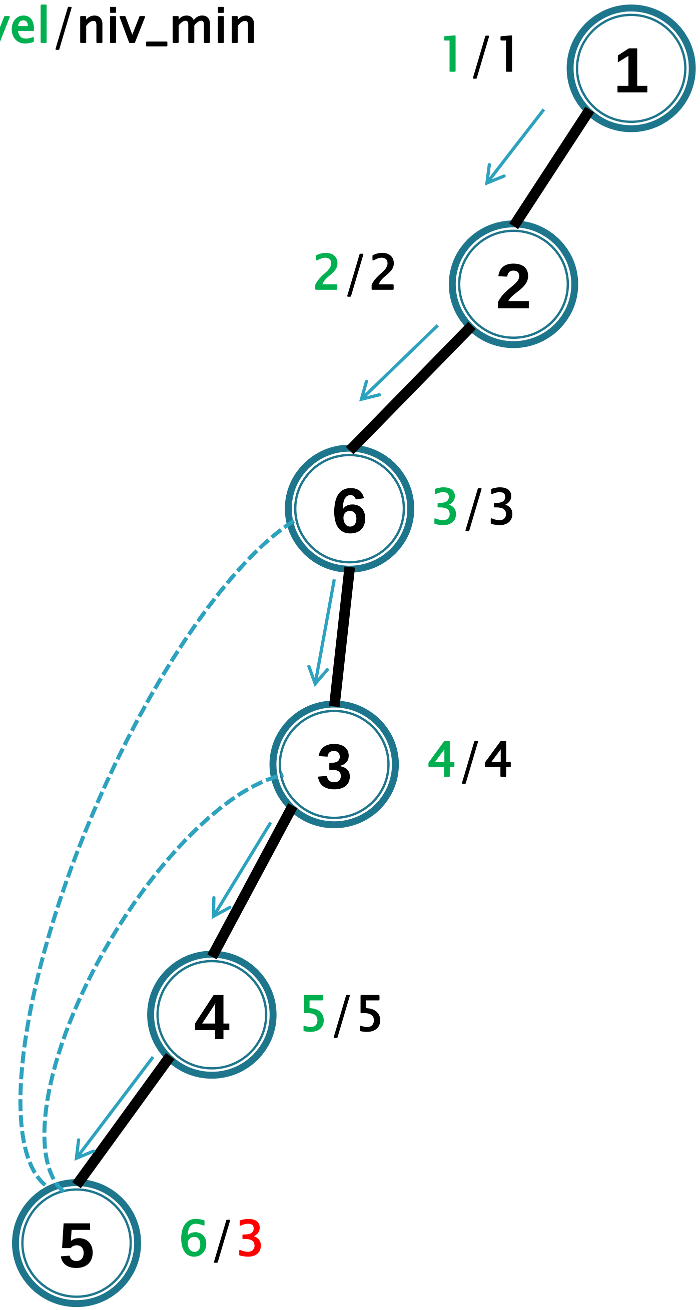


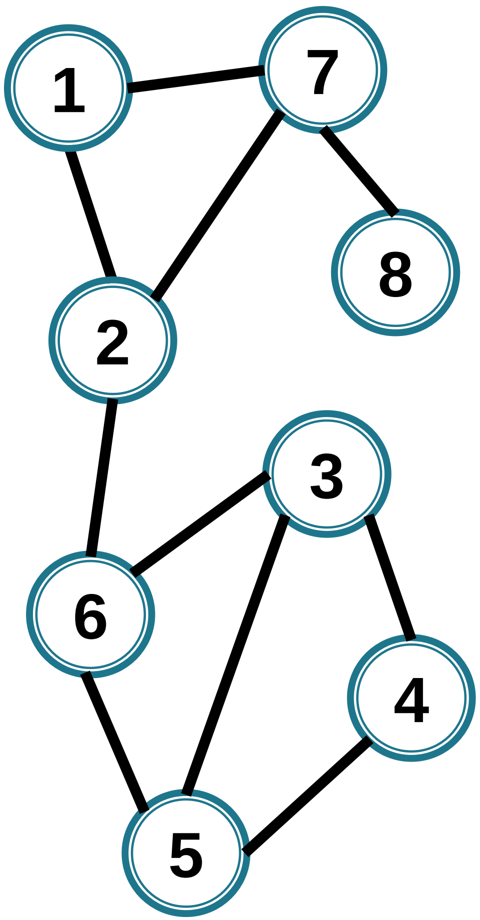
nivel/niv_min



S:

1 2
2 6
6 3
3 4
4 5
5 3
5 6





nivel/niv_min

S:

1 2

2 6

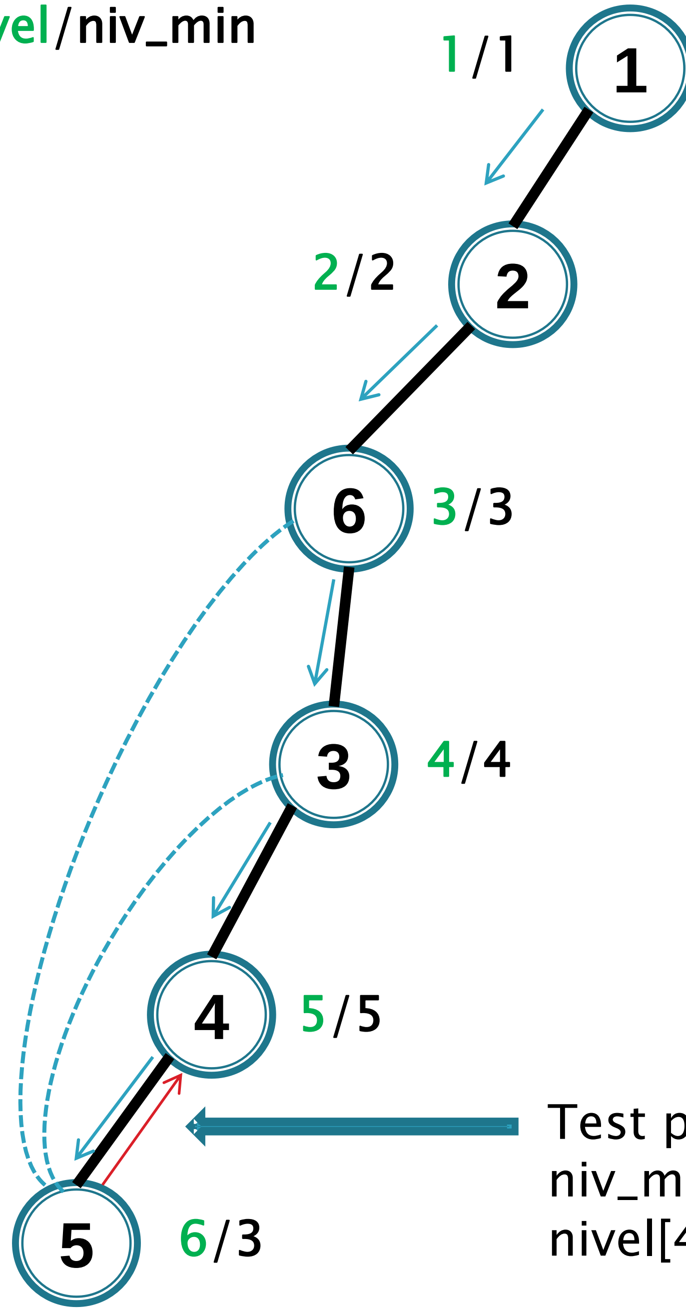
6 3

3 4

4 5

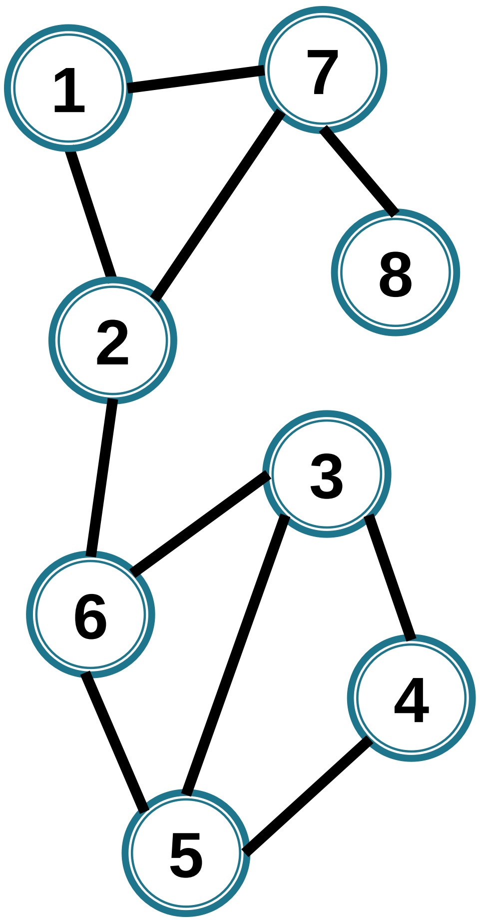
5 3

5 6



Test punct critic:
niv_min[5] =3 <
nivel[4] =5 $\Rightarrow$ **NU**

nivel/niv_min



S:

1 2

2 6

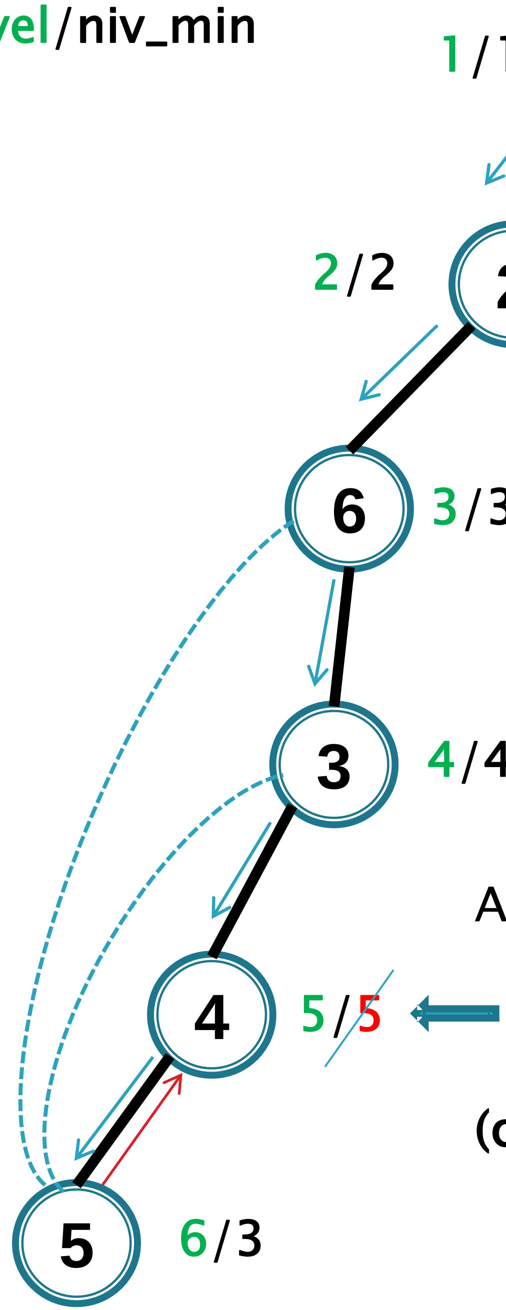
6 3

3 4

4 5

5 3

5 6

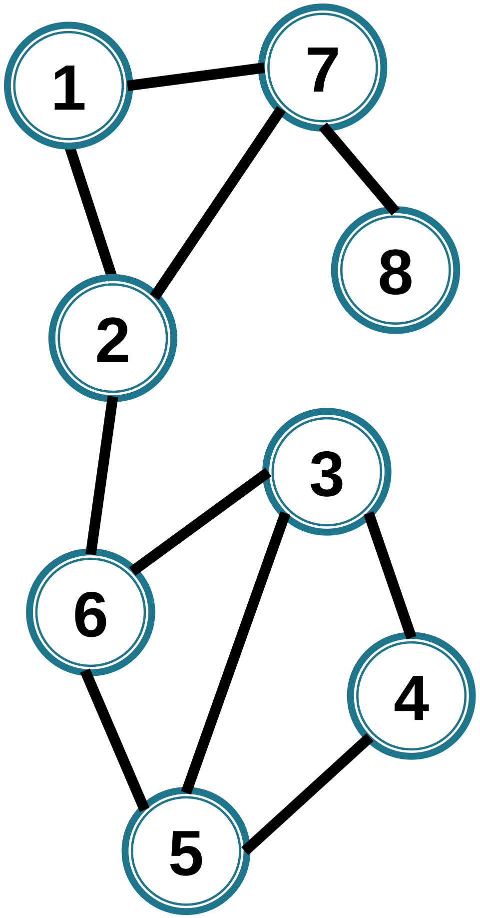


Actualizam

niv_min[4] =

min{niv_min[4], niv_min[5]}

(cazul B)



nivel/niv_min

S:

1 2

2 6

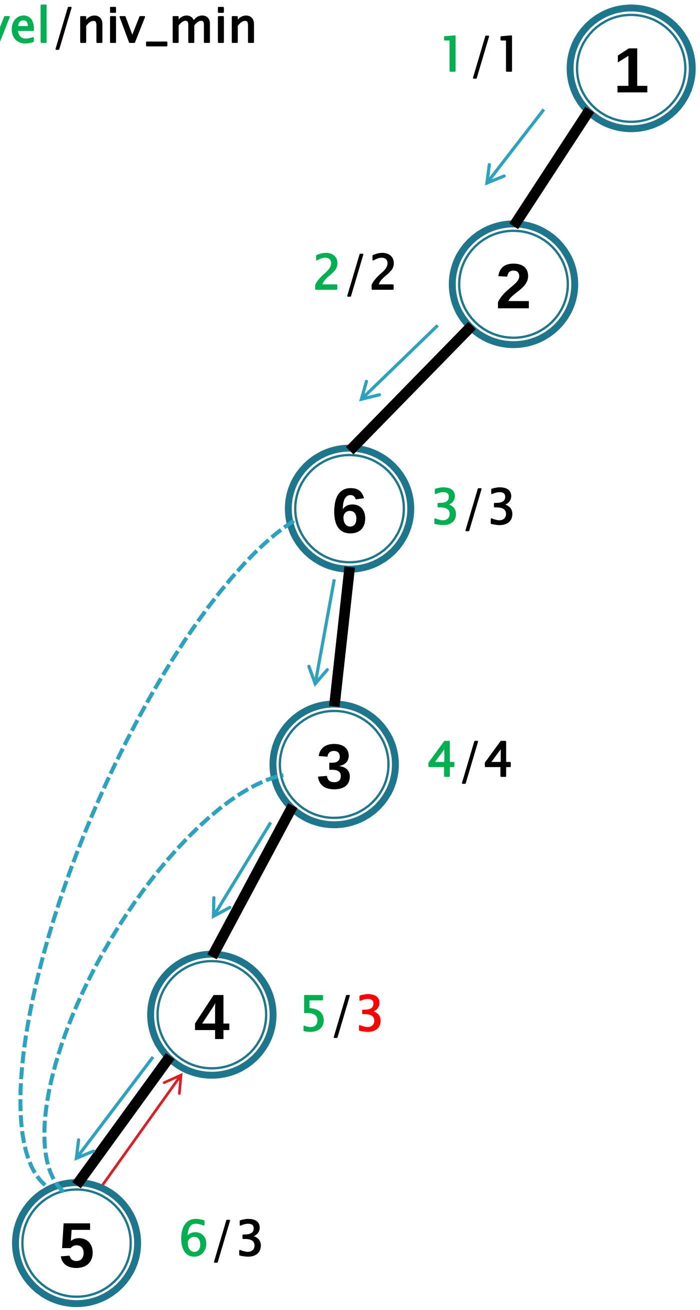
6 3

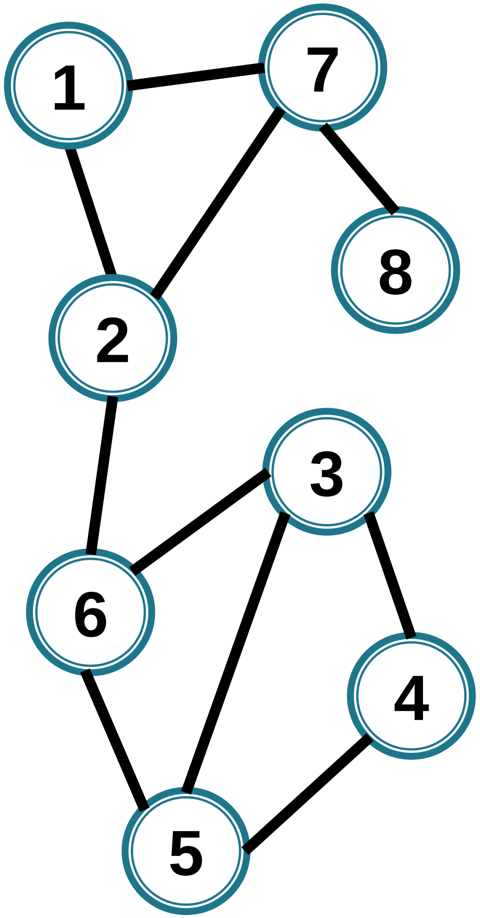
3 4

4 5

5 3

5 6





nivel/niv_min

S:

1 2

2 6

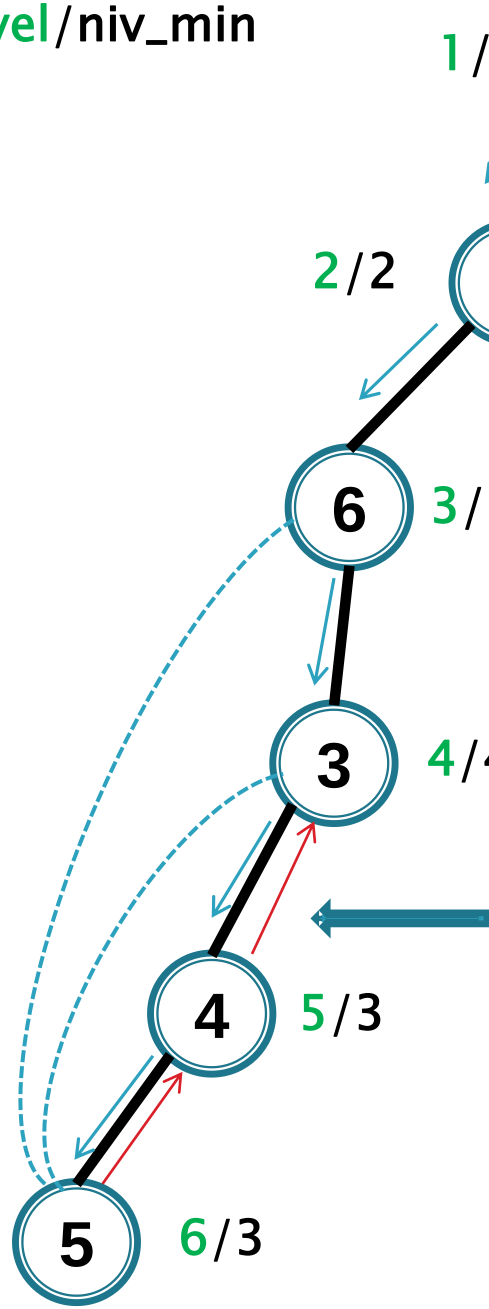
6 3

3 4

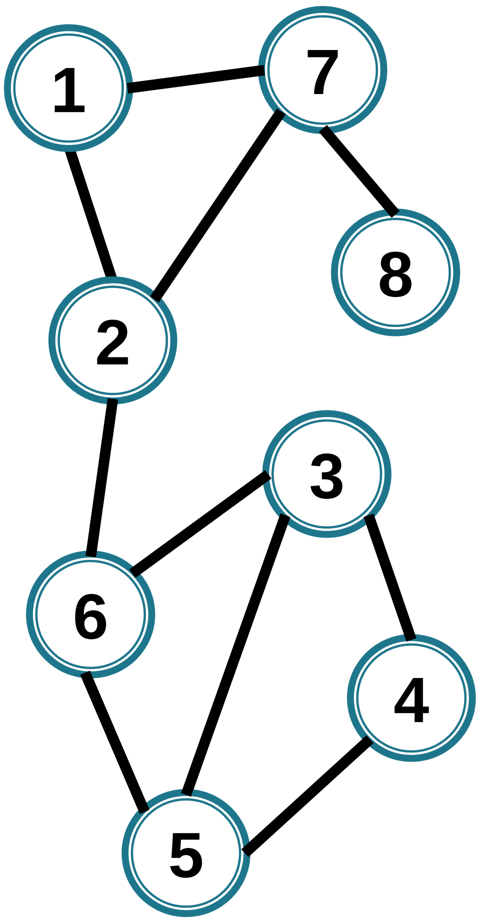
4 5

5 3

5 6



Test punct critic:
niv_min[4] =3 <
nivel[3] =4 $\Rightarrow$ **NU**



S:

1 2

2 6

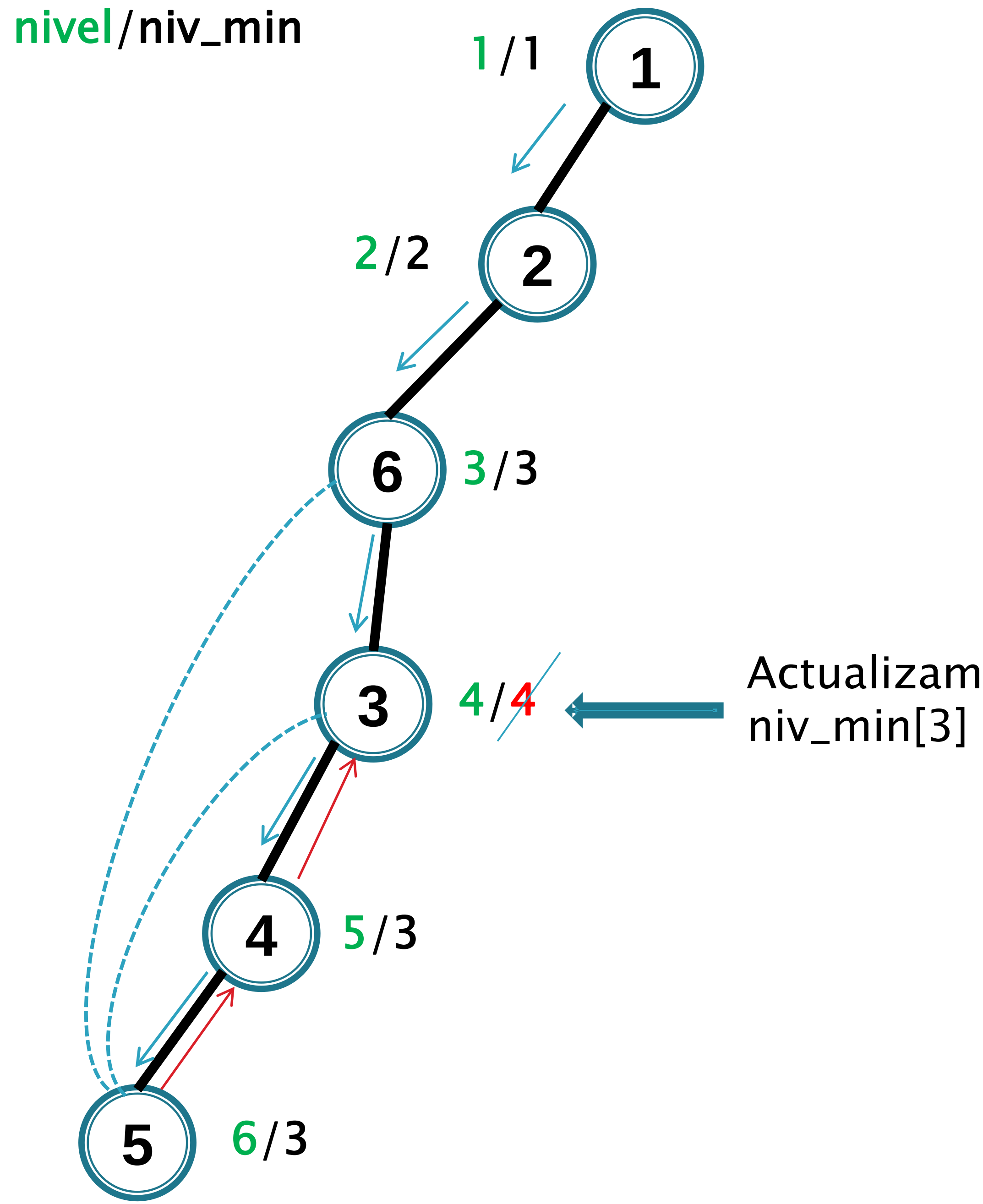
6 3

3 4

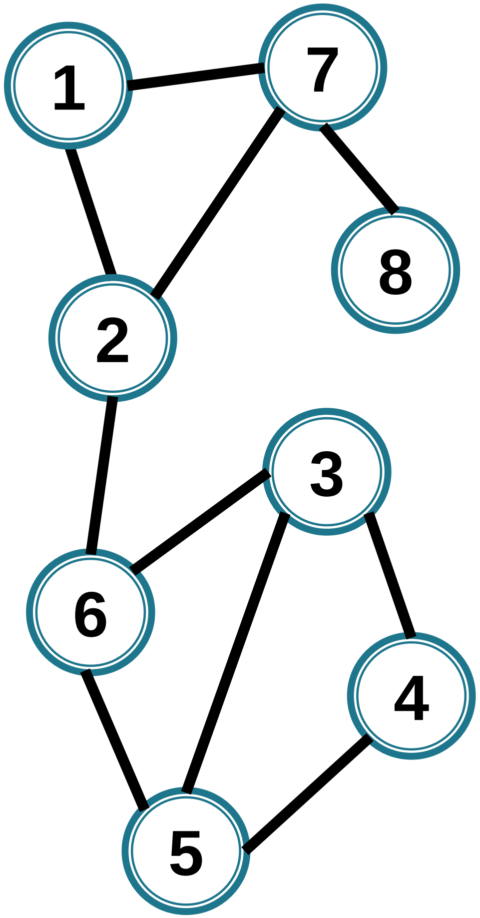
4 5

5 3

5 6



nivel/niv_min



S:

1 2

2 6

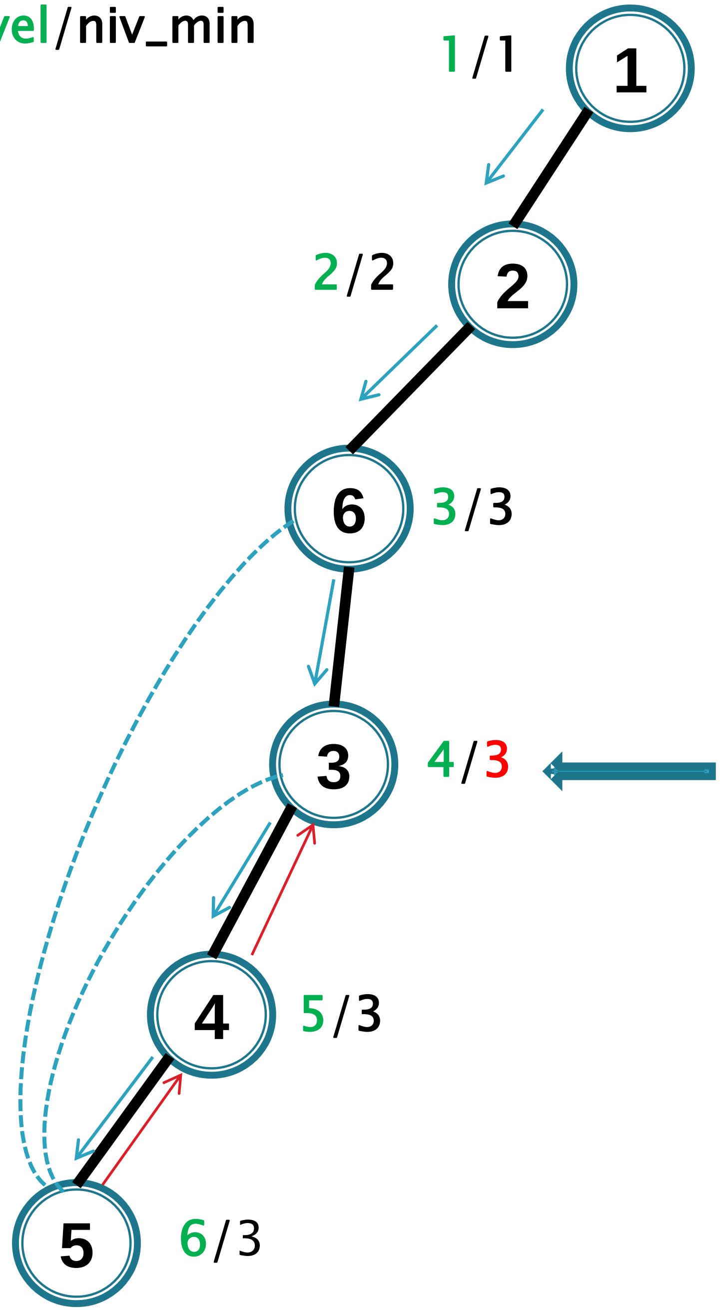
6 3

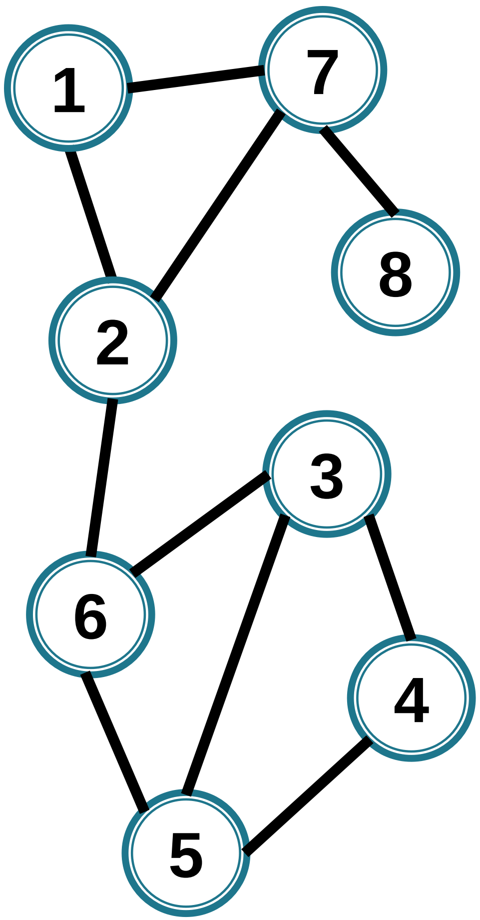
3 4

4 5

5 3

5 6





S:

1 2

2 6

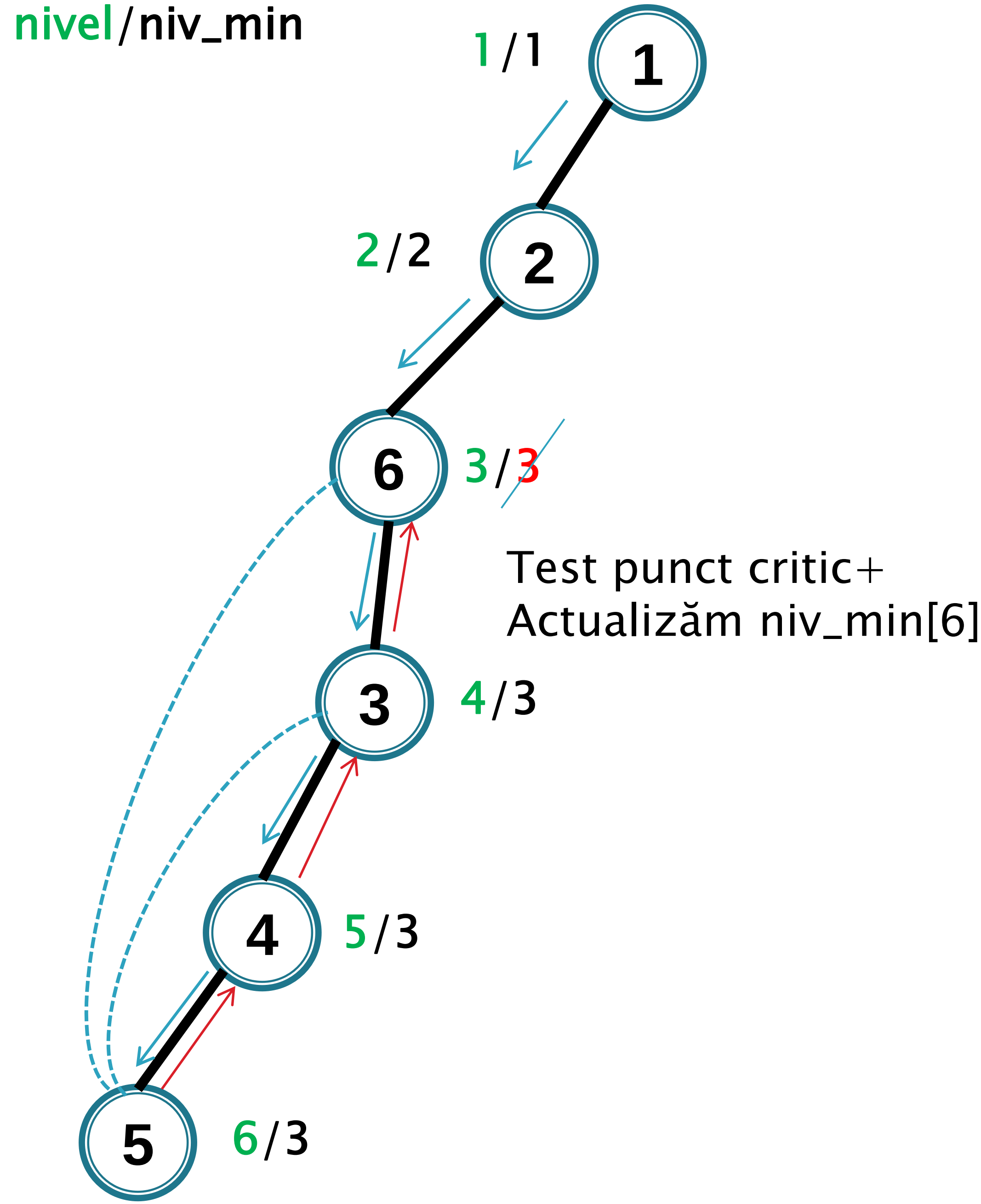
6 3

3 4

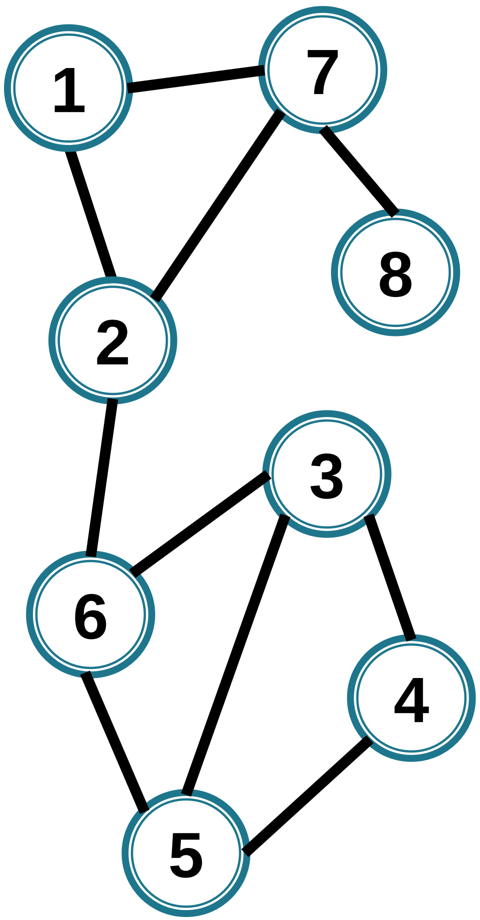
4 5

5 3

5 6



nivel/niv_min



S:

1 2

2 6

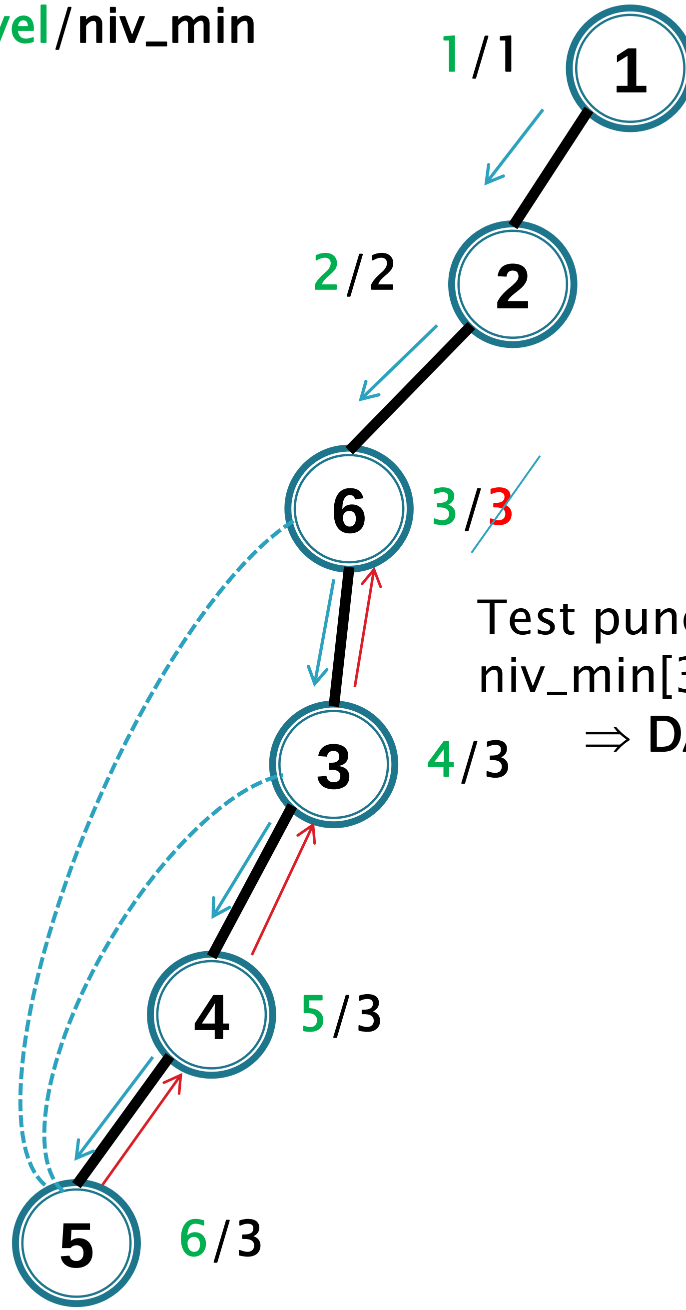
6 3

3 4

4 5

5 3

5 6

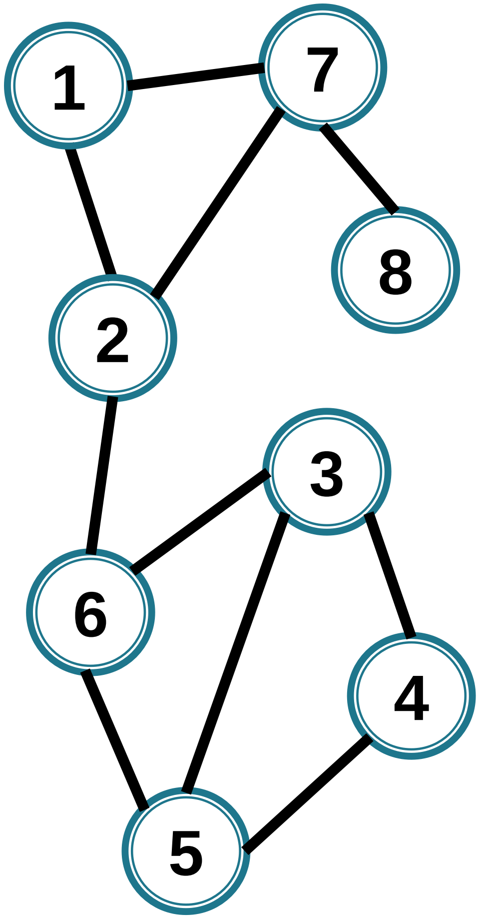


Test punct critic:

niv_min[3] =3 = nivel[6]

$\Rightarrow$ **DA**

nivel/niv_min



S:

1 2

2 6

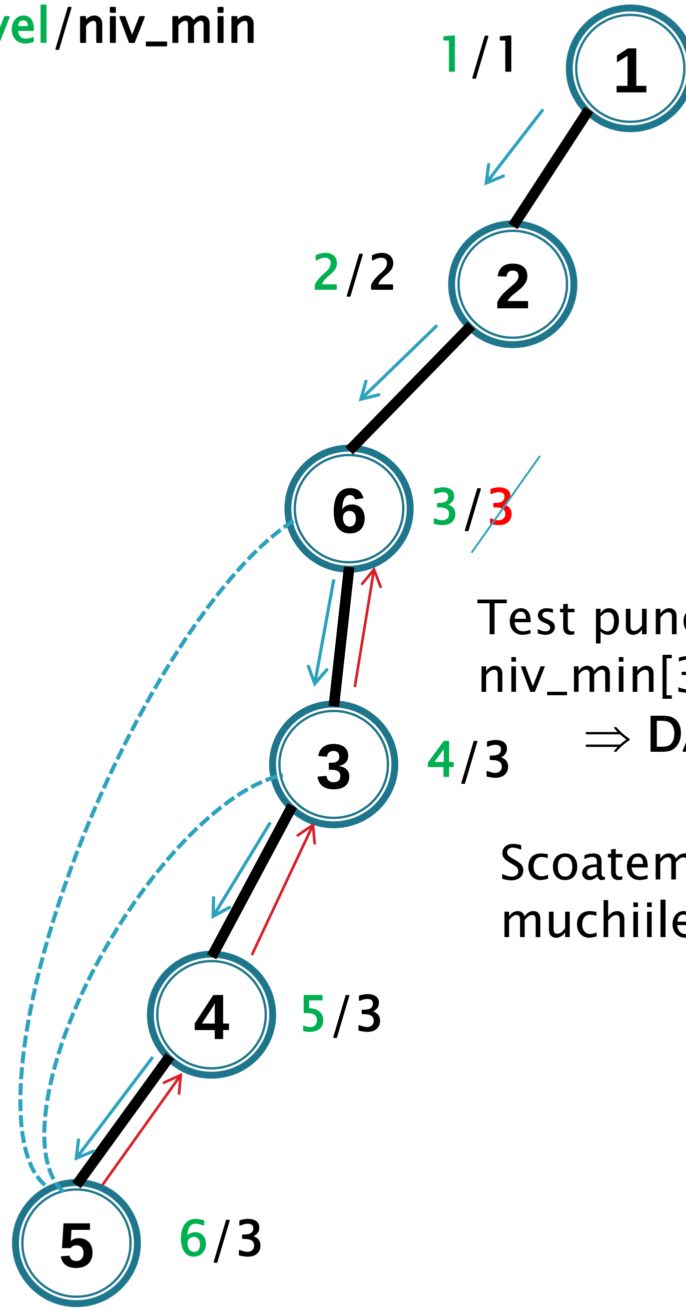
6 3

3 4

4 5

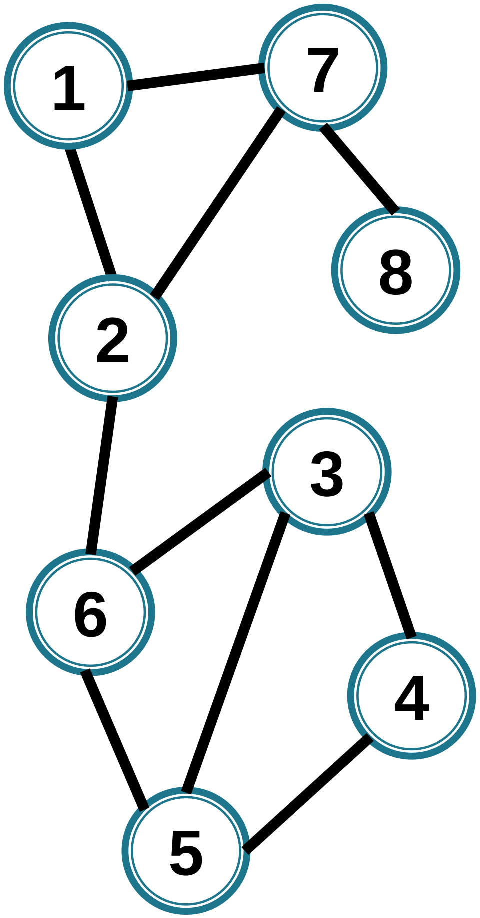
5 3

5 6



Test punct critic:
niv_min[3] =3 = nivel[6]
⇒ **DA**

Scoatem din stiva toate muchiile pana la 3 6 =>

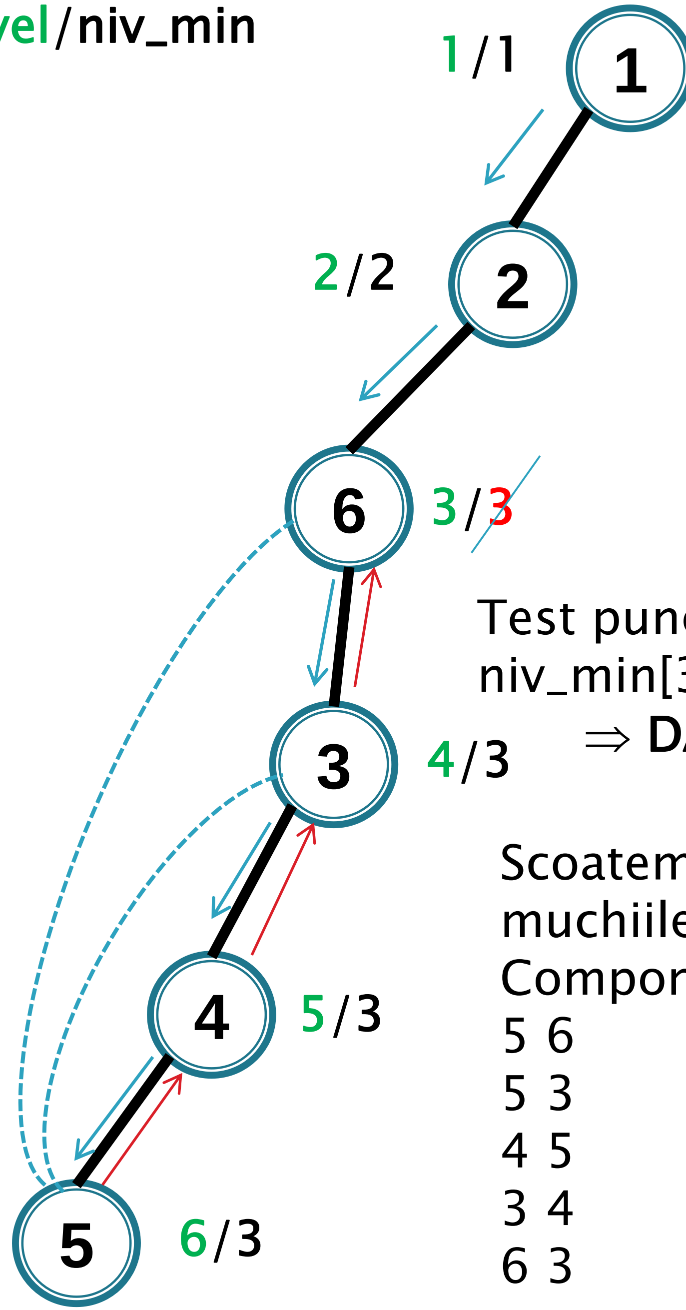


nivel/niv_min

S:

1 2

2 6



Test punct critic:

niv_min[3] =3 = nivel[6]

⇒ **DA**

Scoatem din stiva toate muchiile pana la 3 6 => Componenta cu muchiile

5 6

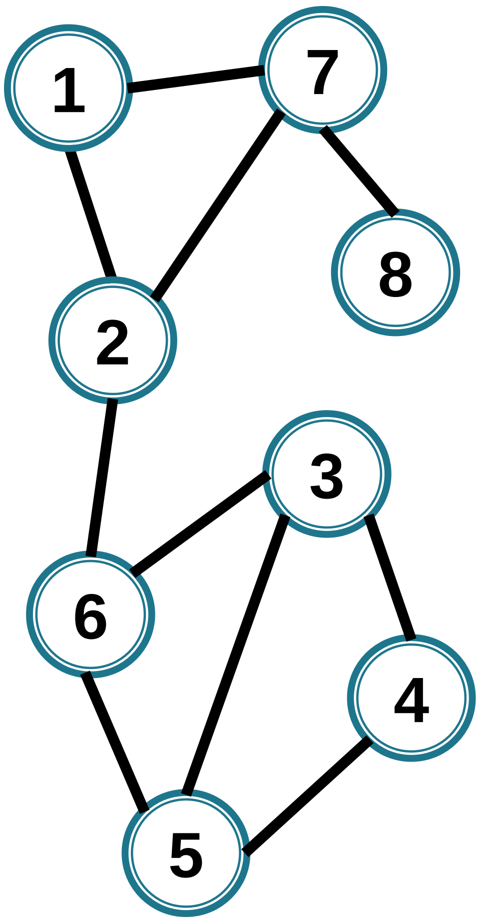
5 3

4 5

3 4

6 3

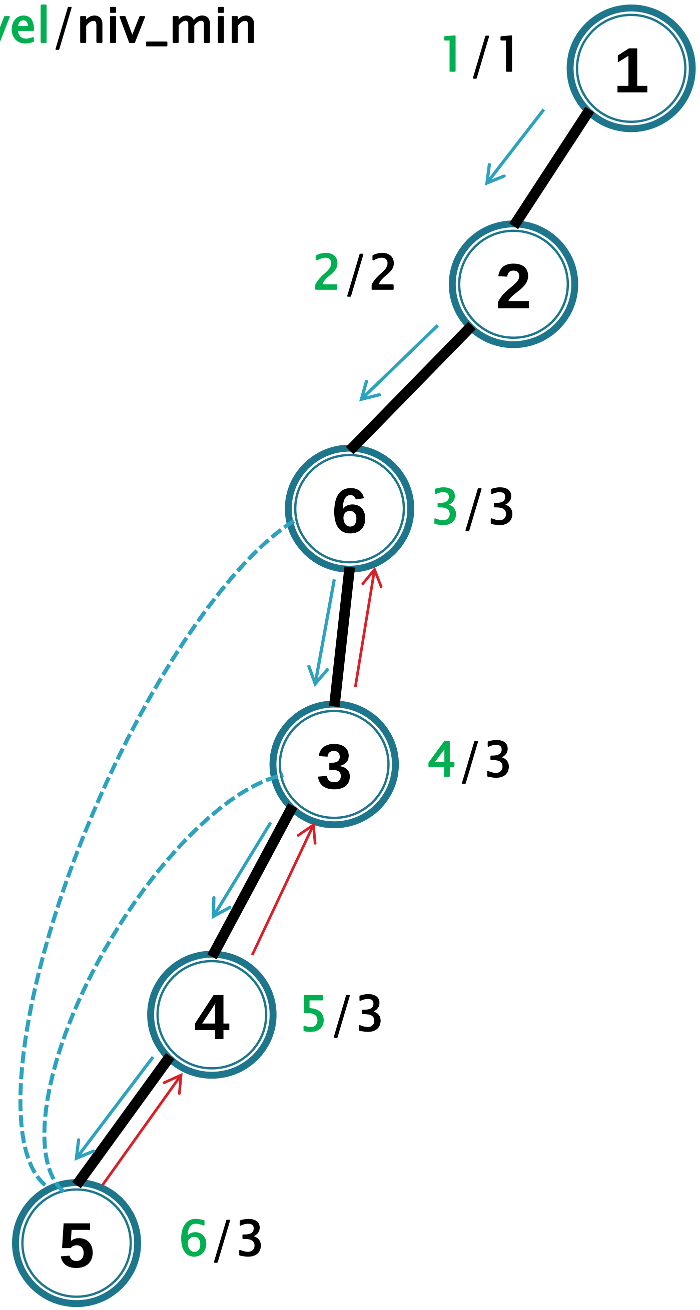
nivel/niv_min

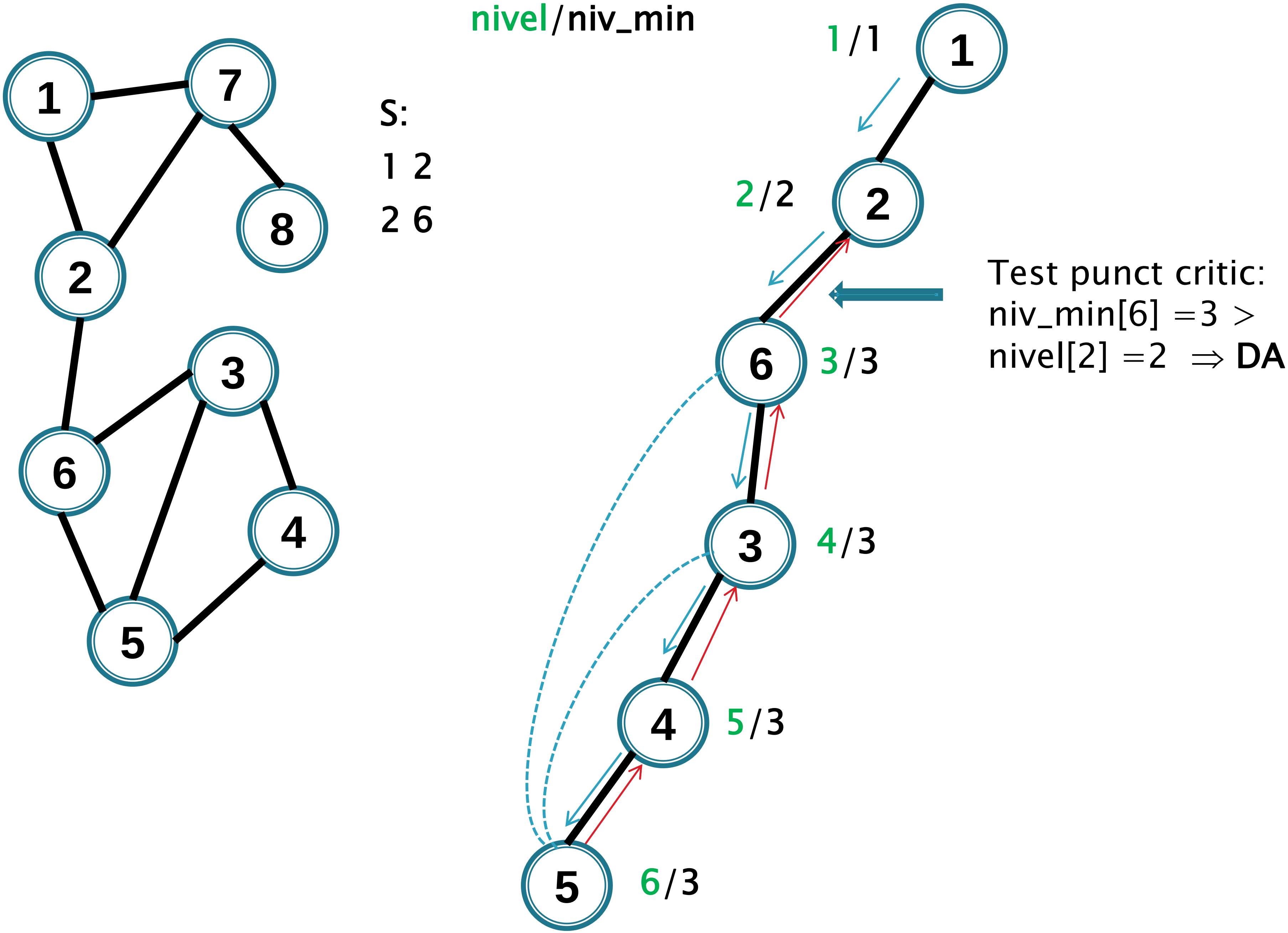


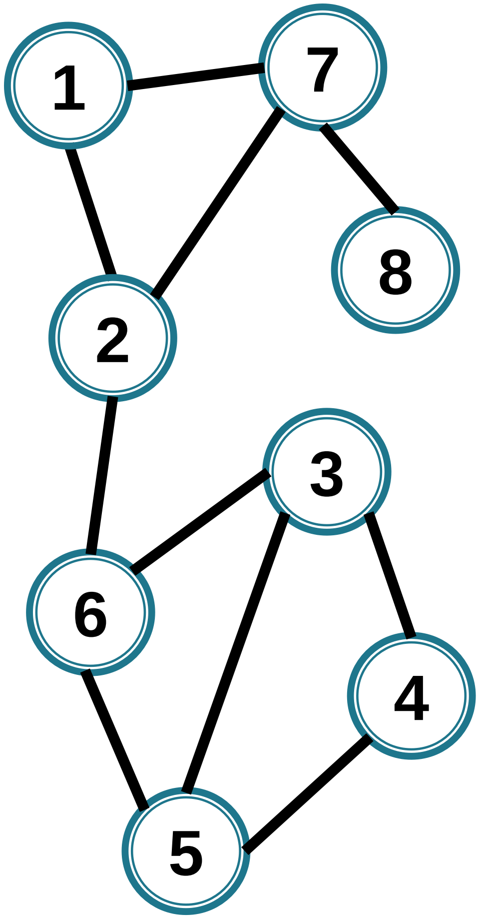
S:

1 2

2 6




nivel/niv_min
S:
1 2
2 6
1/1
2/2
3/3
4/3
5/3
6/3
1
7
8
2
3
6
4
5
Test punct critic:
niv_min[6] =3 >
nivel[2] =2 ⇒ DA

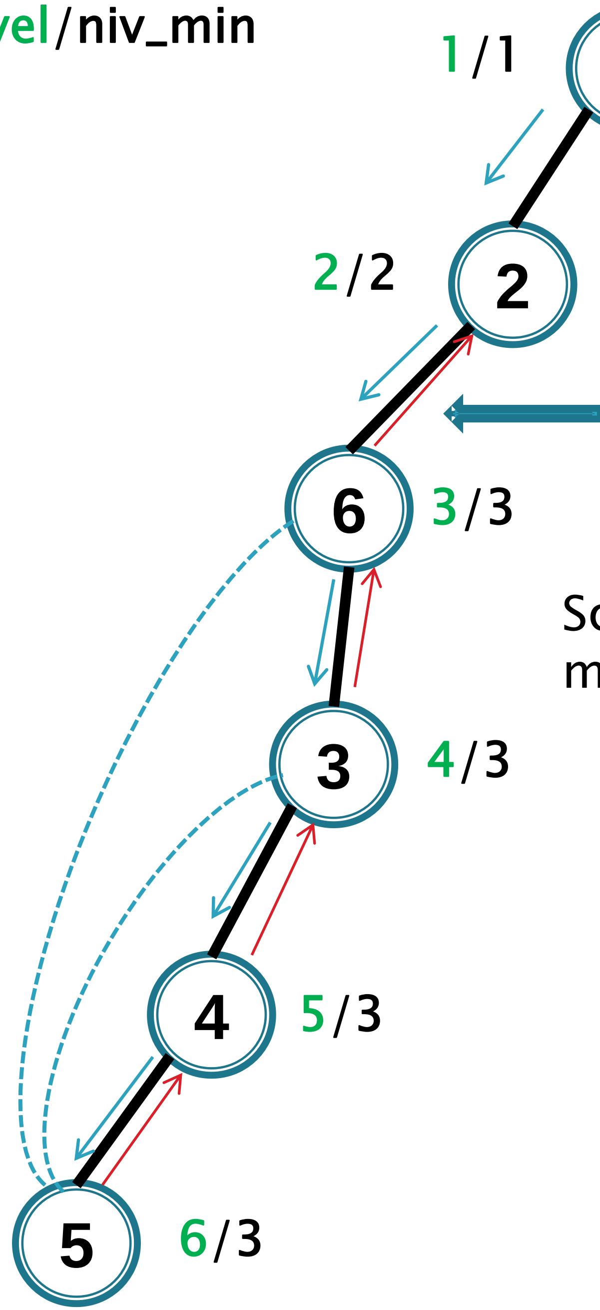


nivel/niv_min

S:

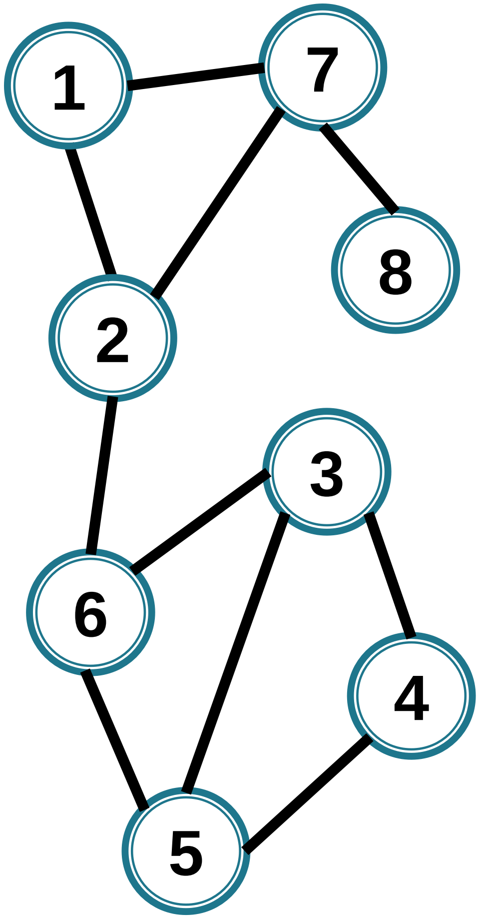
1 2

2 6



Test punct critic:
niv_min[6] =3 >
nivel[2] =2 ⇒ **DA**

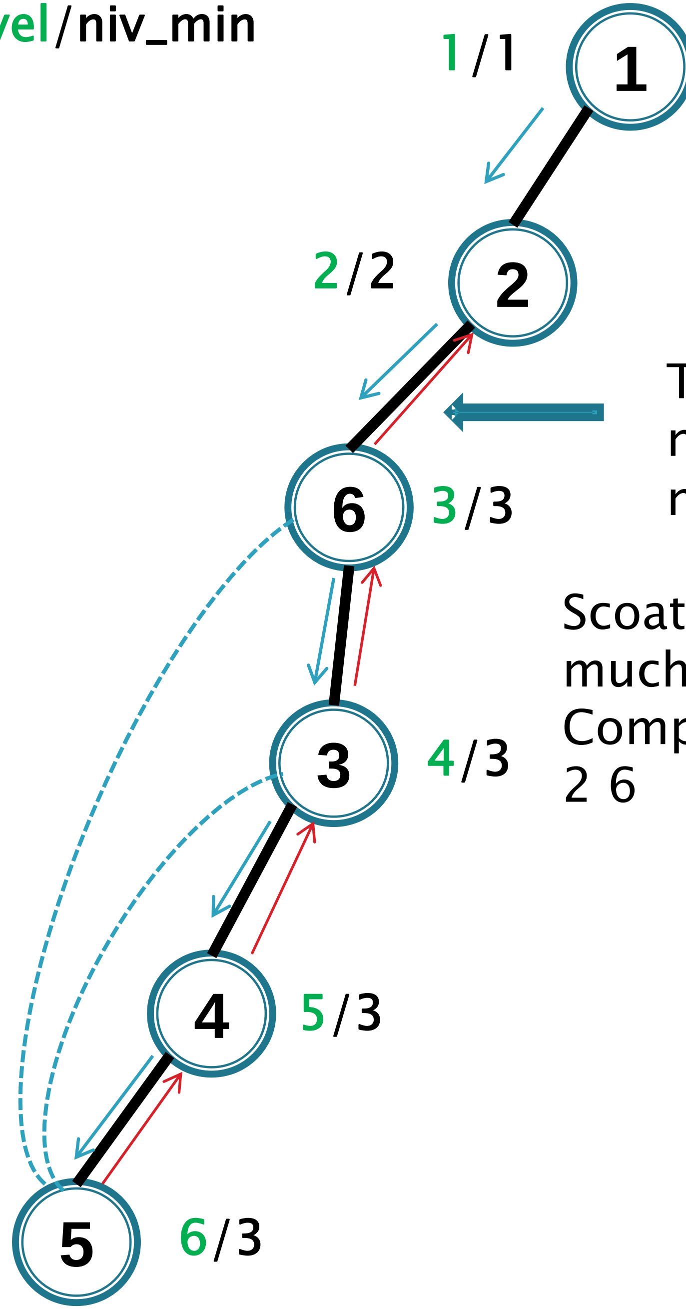
Scoatem din stiva toate muchiile pana la 2 6 =>



nivel/niv_min

S:

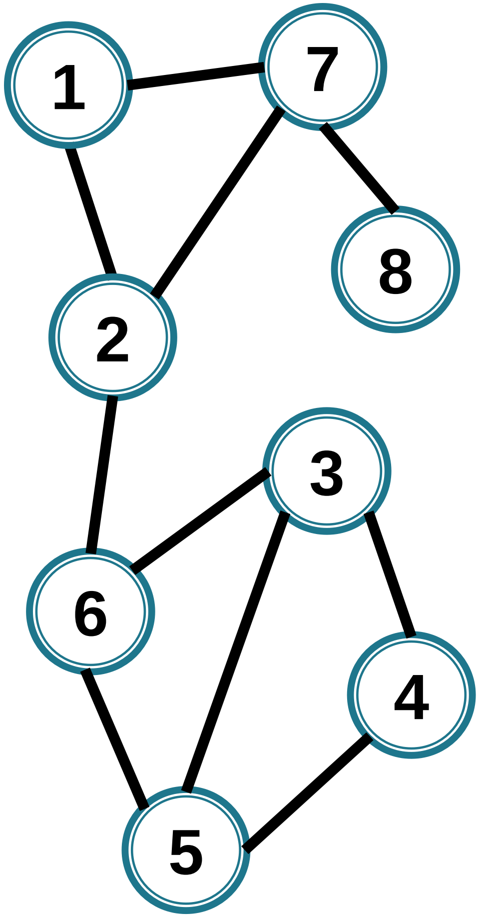
1 2



Test punct critic:
niv_min[6] =3 >
nivel[2] =2 ⇒ **DA**

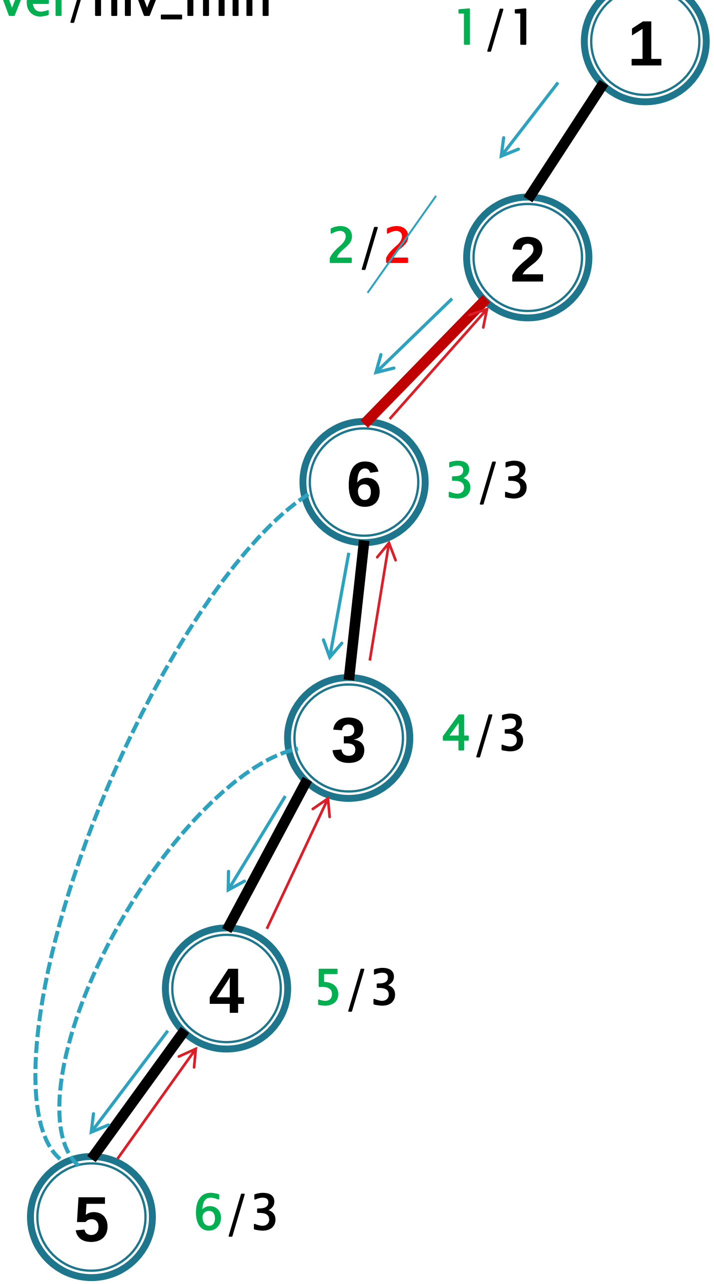
Scoatem din stiva toate
muchiile pana la 2 6 =>
Componenta cu muchiile
2 6

nivel/niv_min

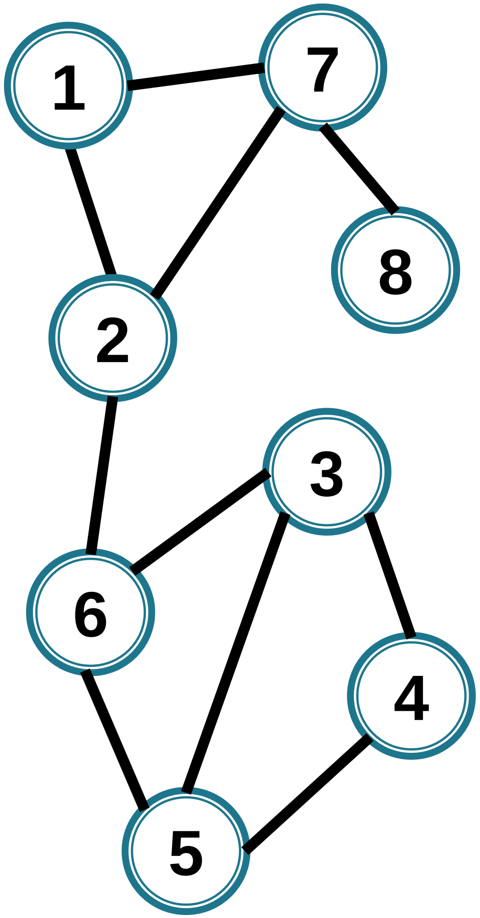


S:

1 2

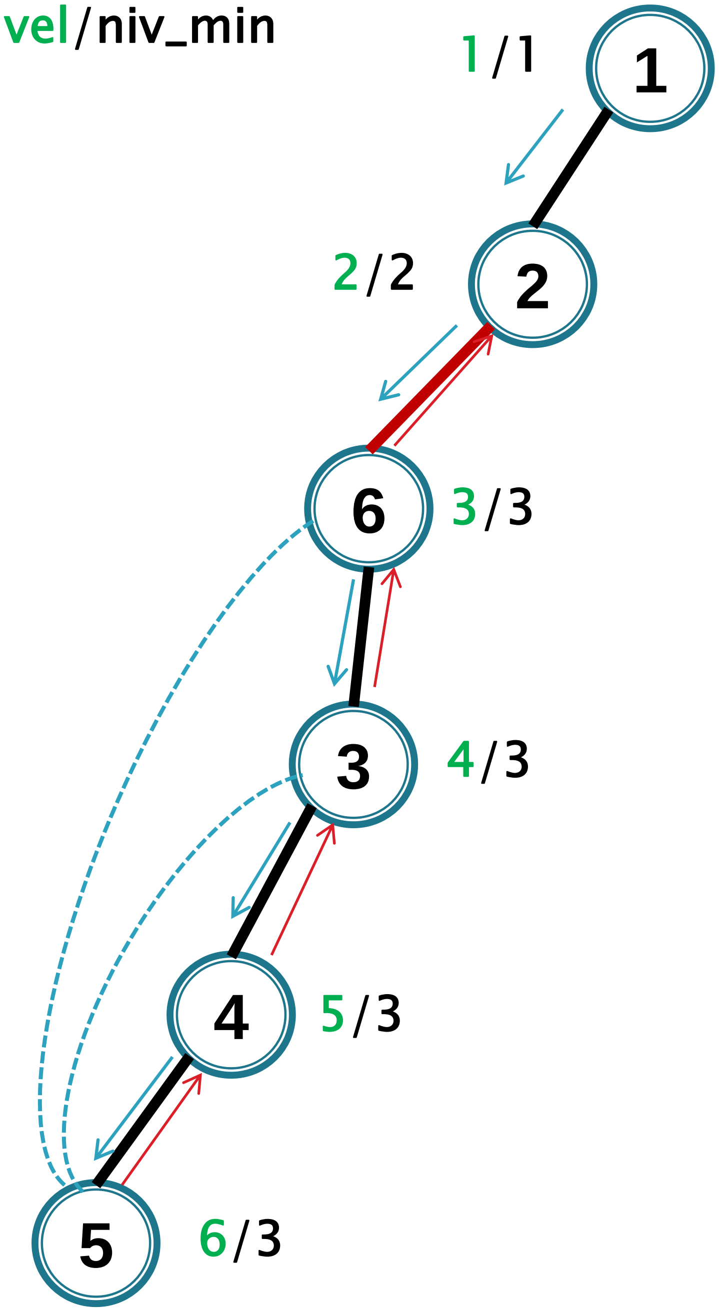


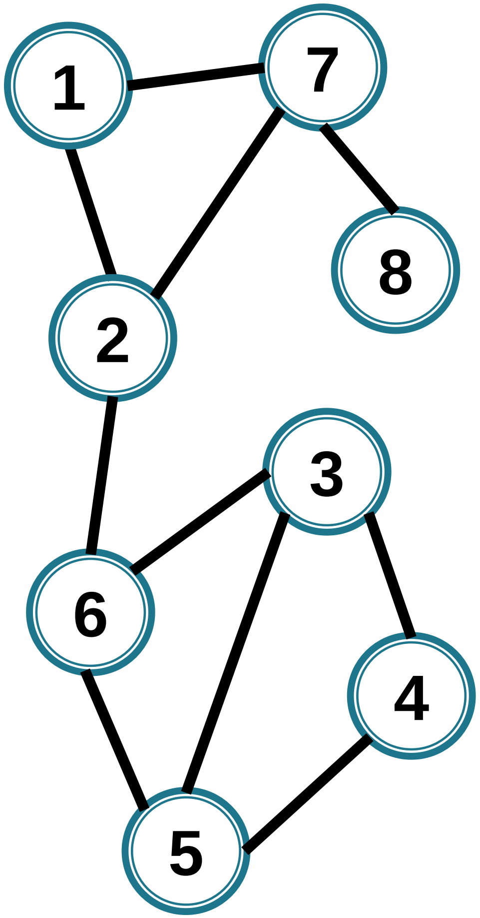
nivel/niv_min



S:

1 2



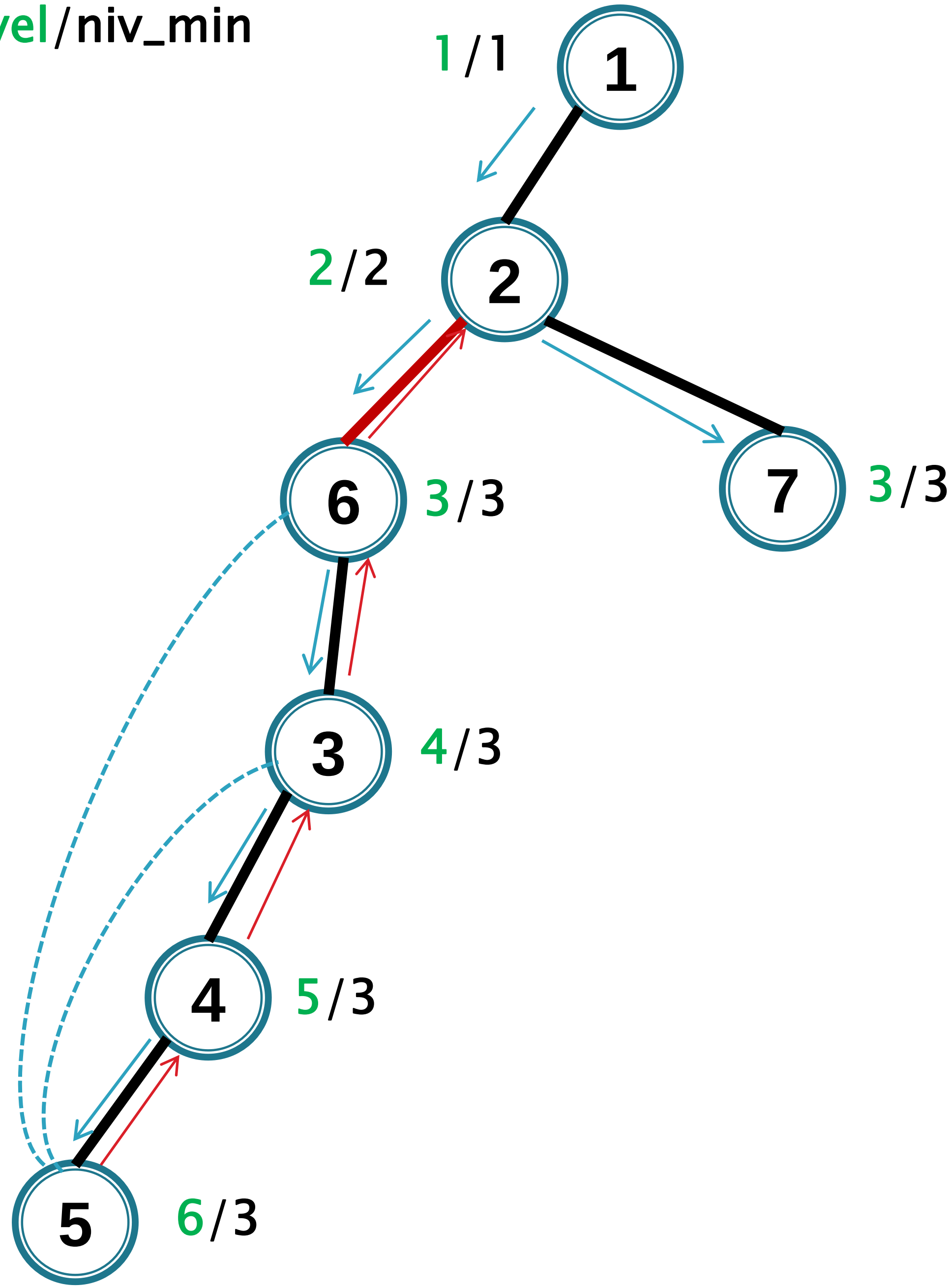


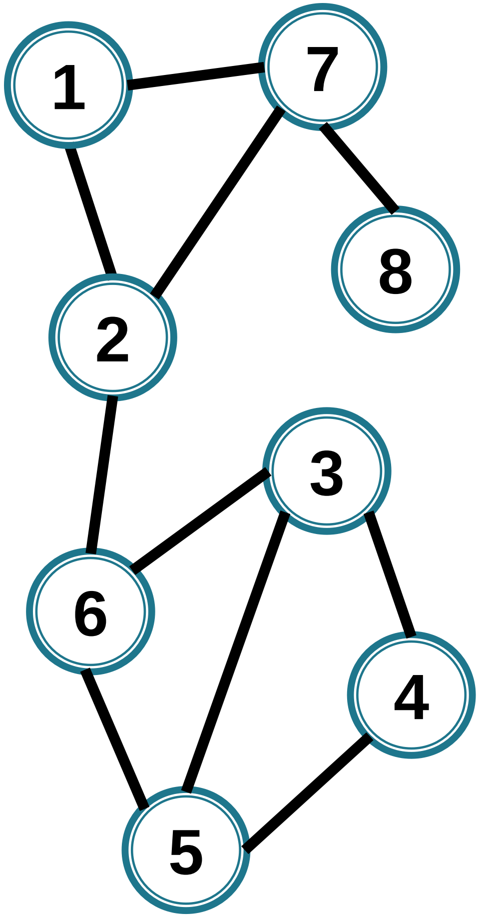
nivel/niv_min

S:

1 2

2 7





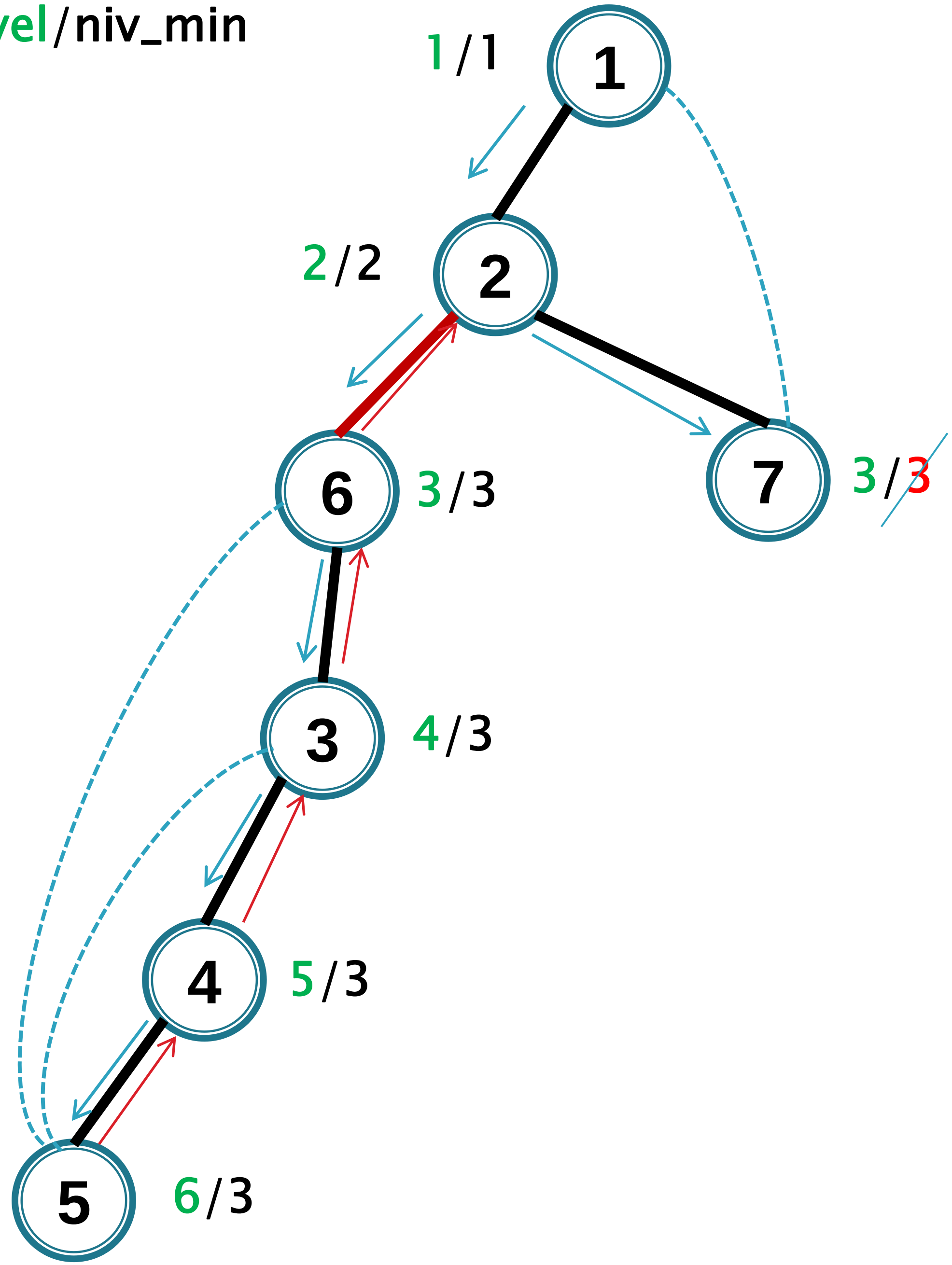
S:

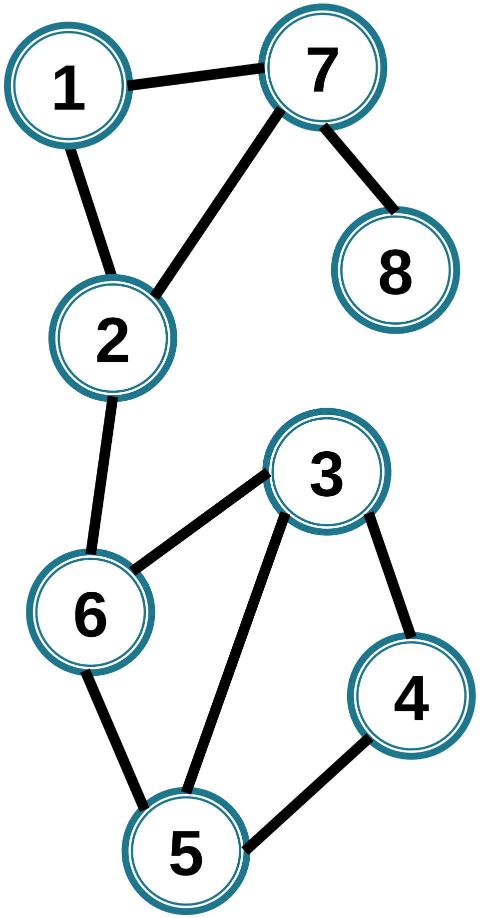
1 2

2 7

1 7

nivel/niv_min





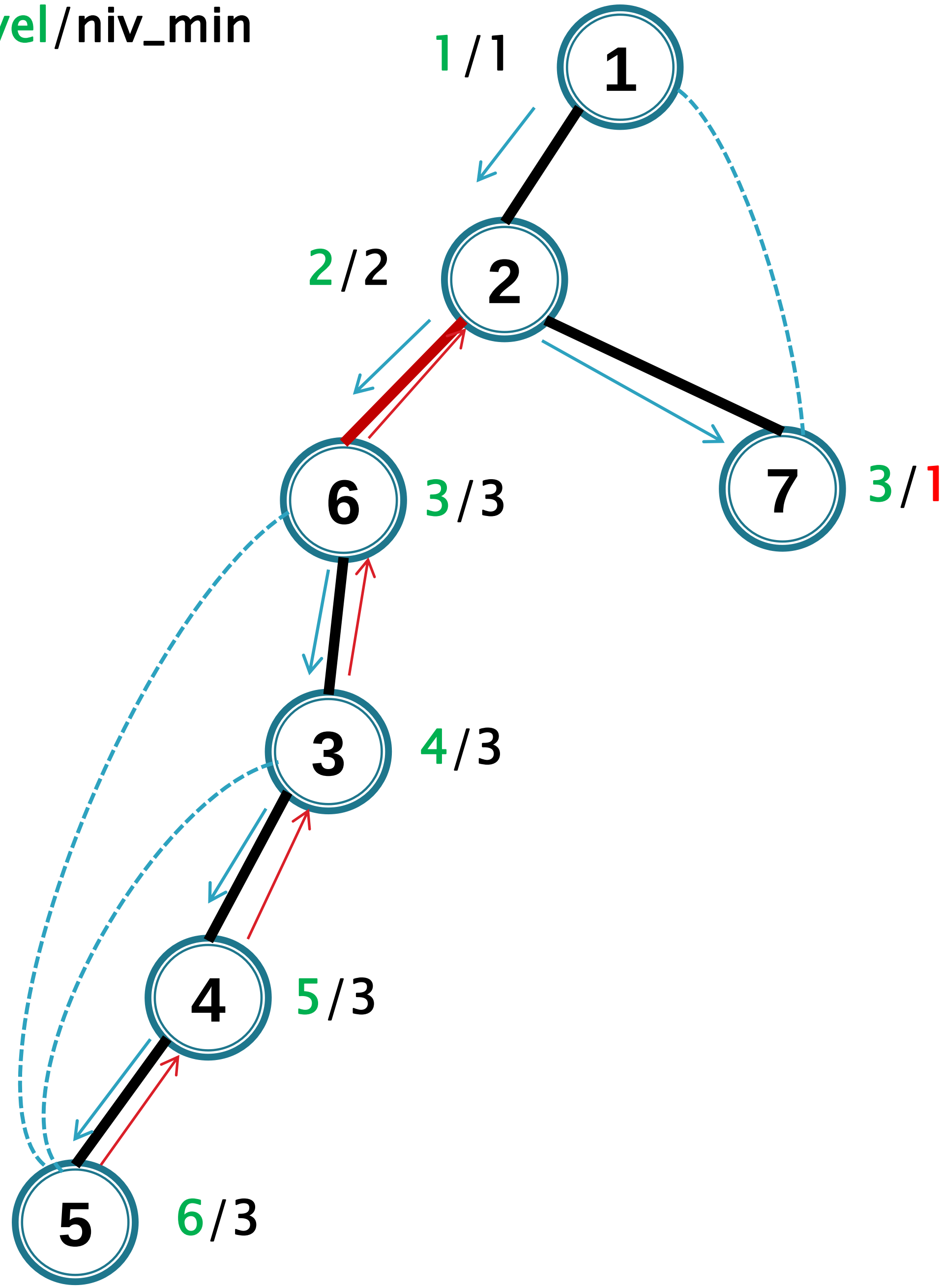
nivel/niv_min

S:

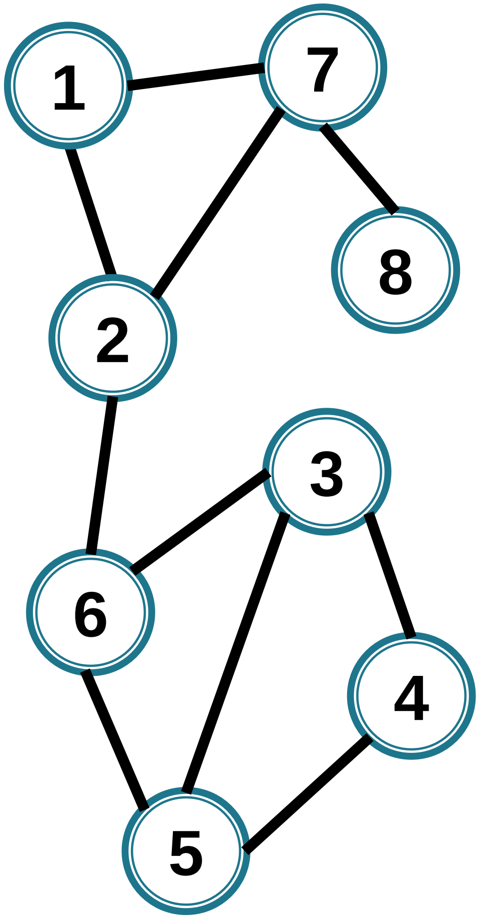
1 2

2 7

1 7



nivel/niv_min



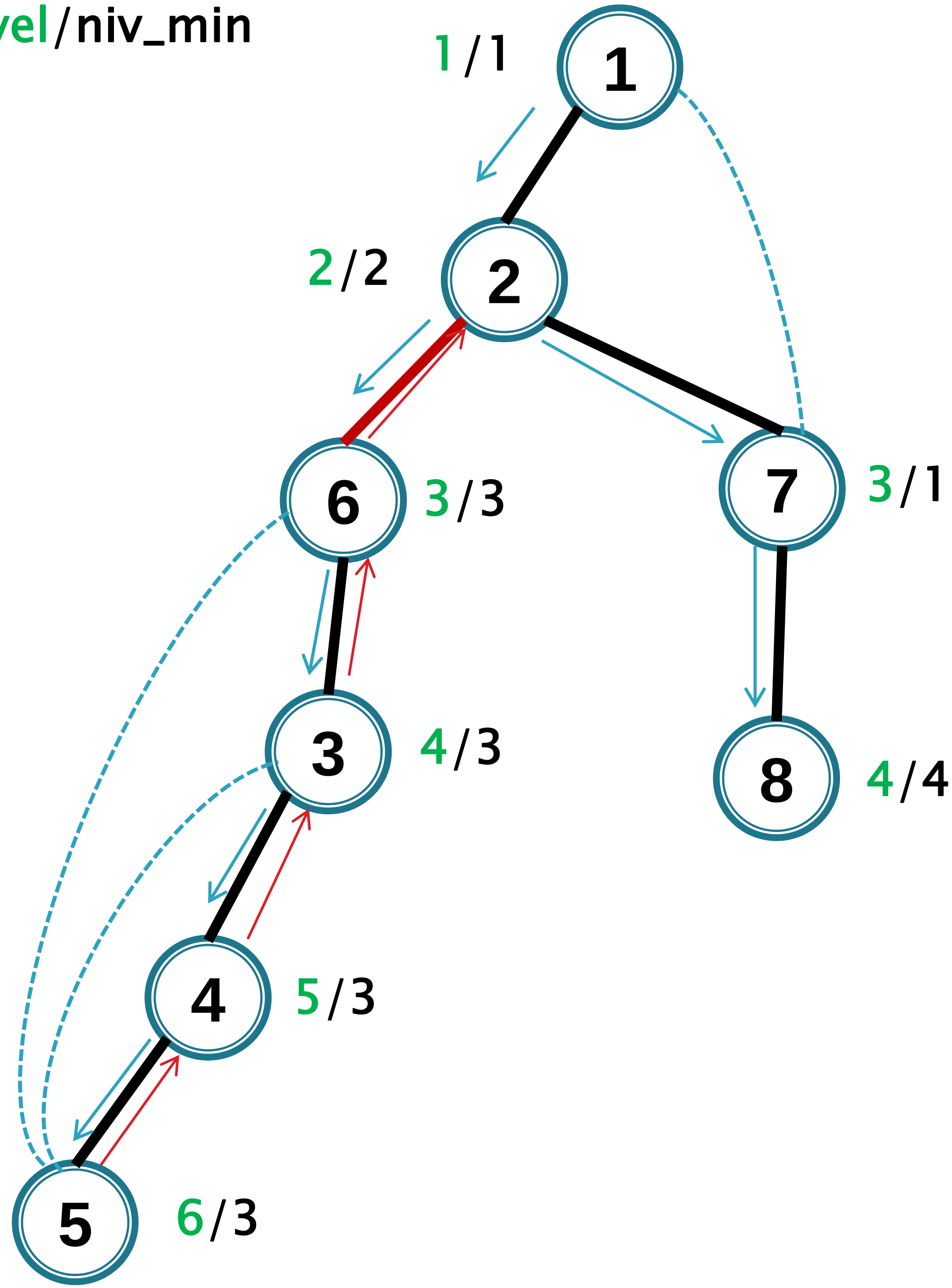
S:

1 2

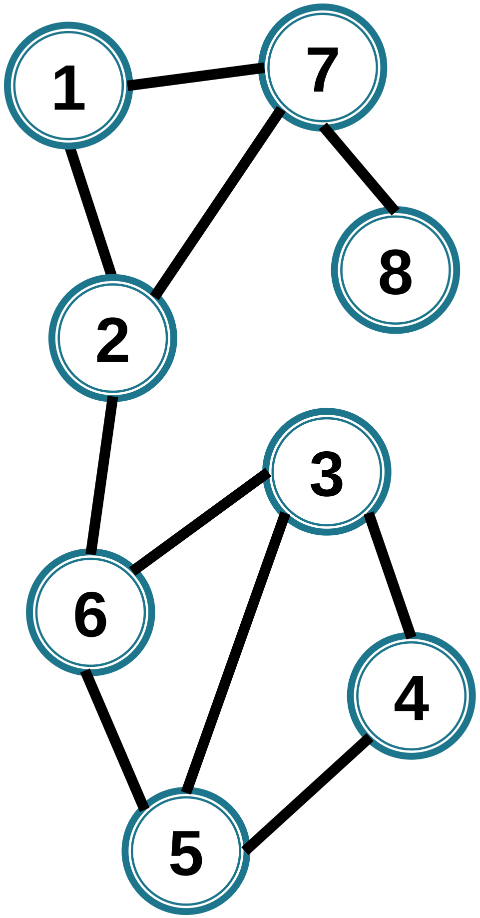
2 7

1 7

7 8



nivel/niv_min



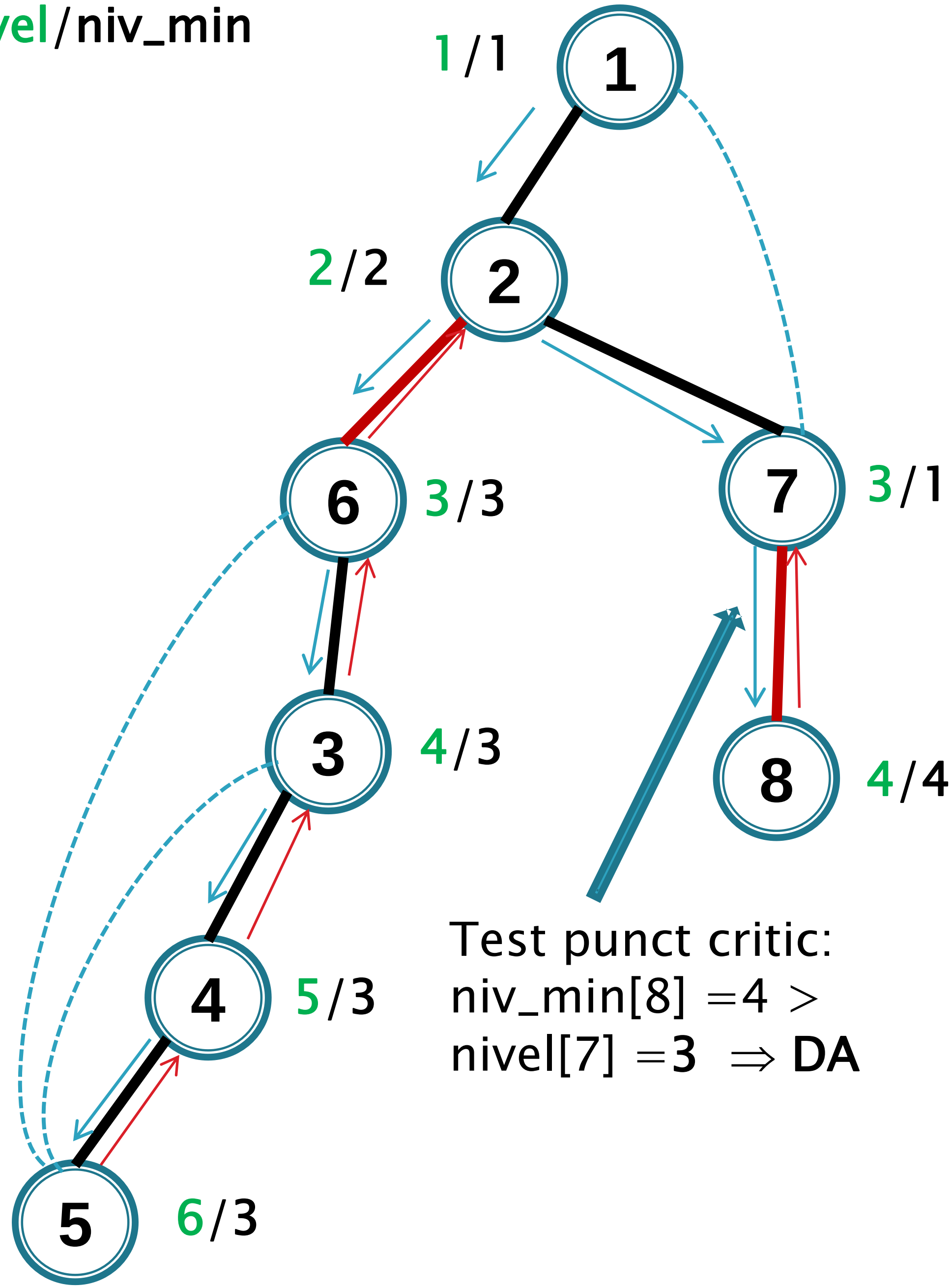
S:

1 2

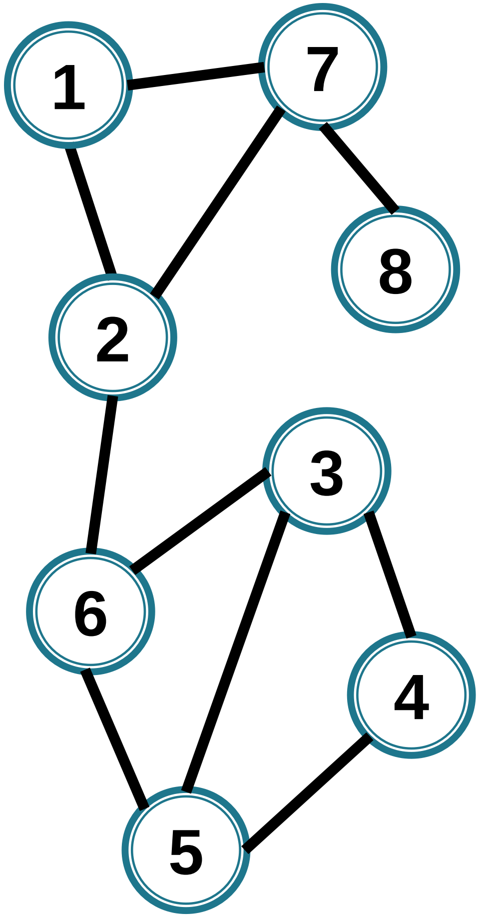
2 7

1 7

7 8



Test punct critic:
niv_min[8] = 4 >
nivel[7] = **3** ⇒ **DA**



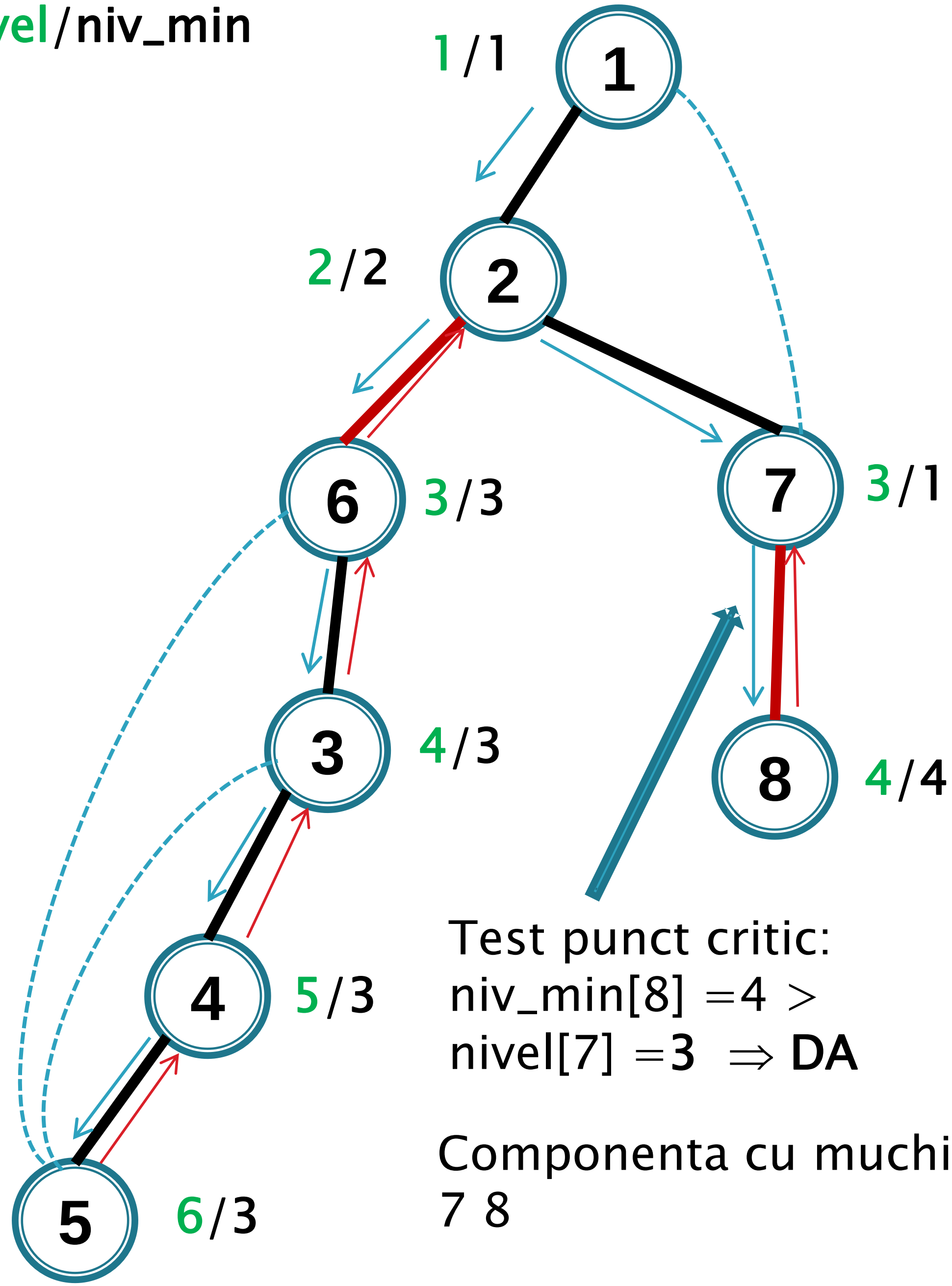
nivel/niv_min

S:

1 2

2 7

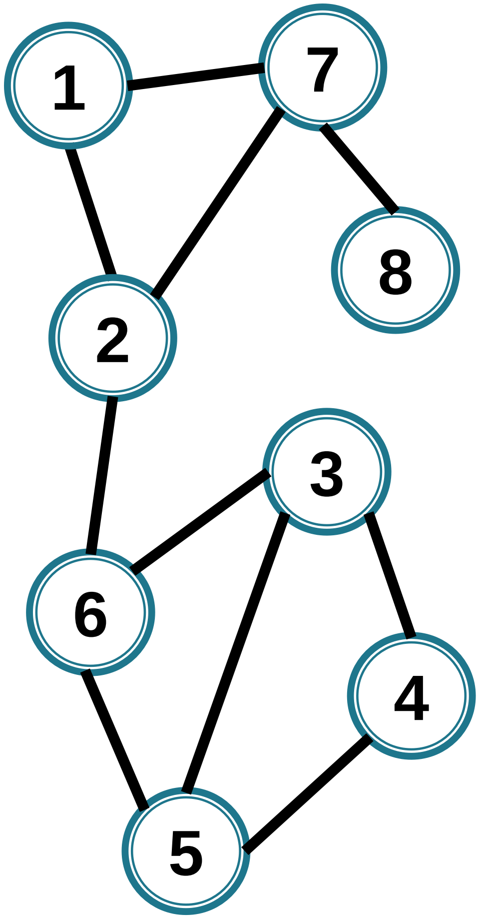
1 7



Test punct critic:
niv_min[8] =4 >
nivel[7] =**3** ⇒ **DA**

Componenta cu muchiile
7 8

nivel/niv_min

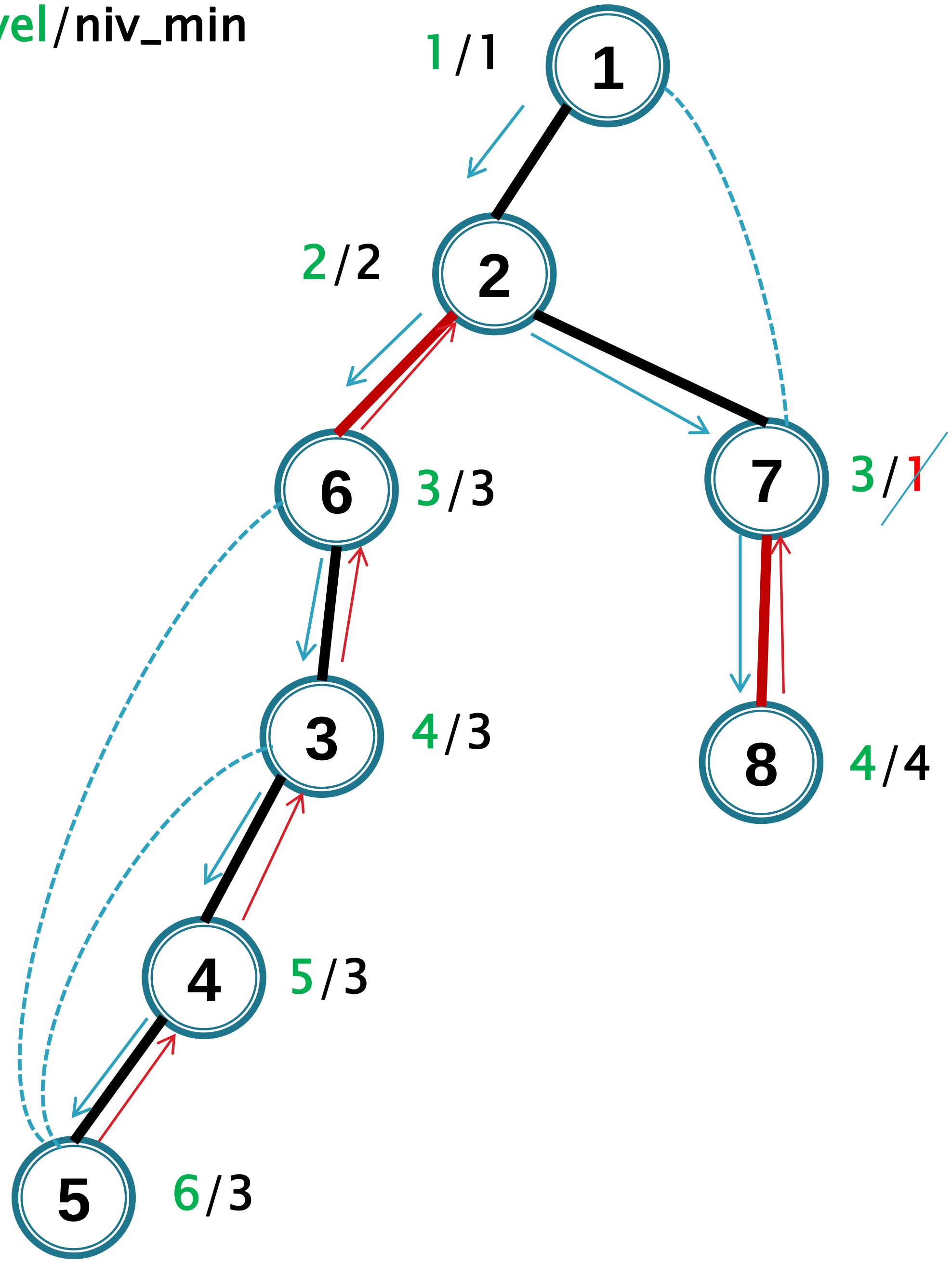


S:

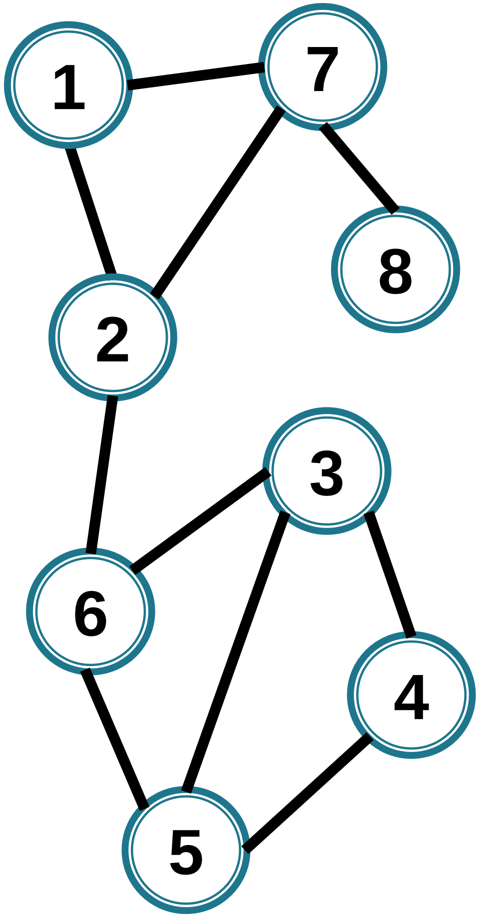
1 2

2 7

1 7



nivel/niv_min

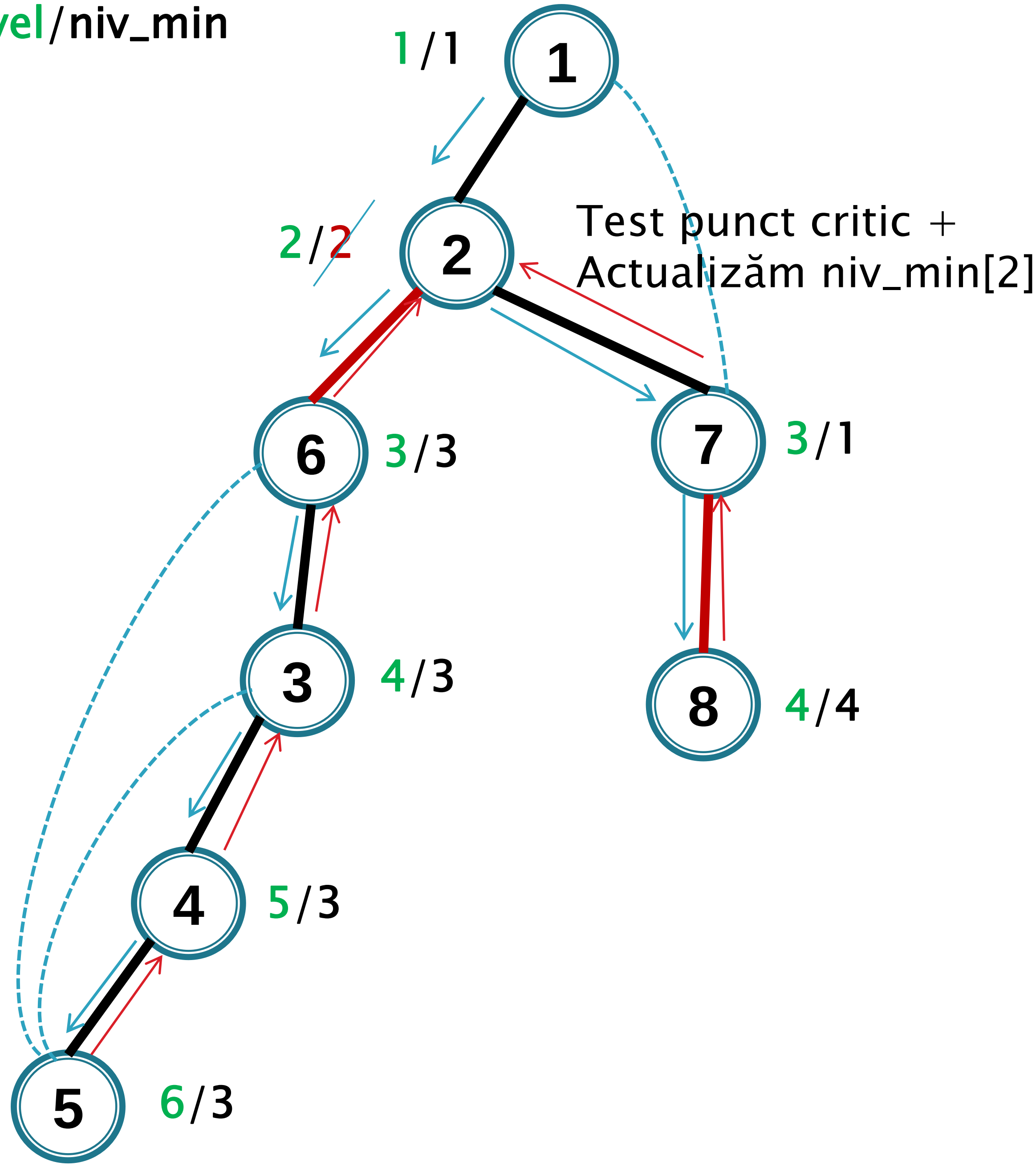


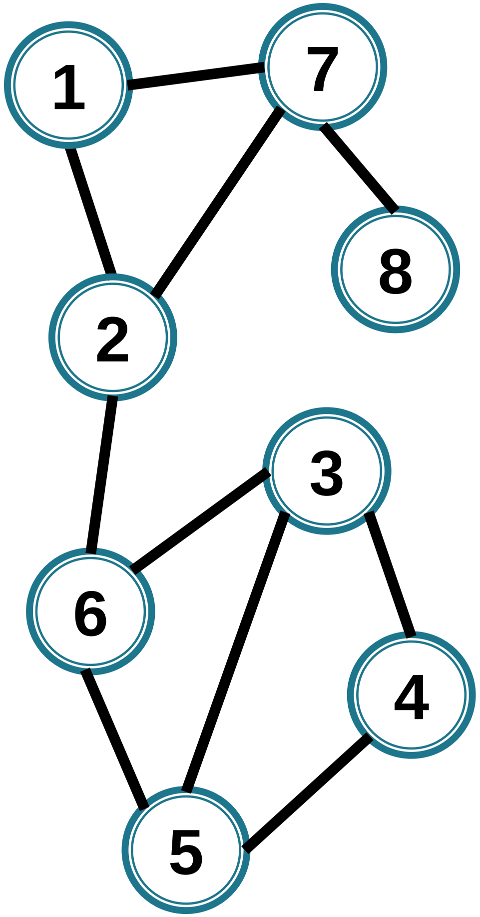
S:

1 2

2 7

1 7





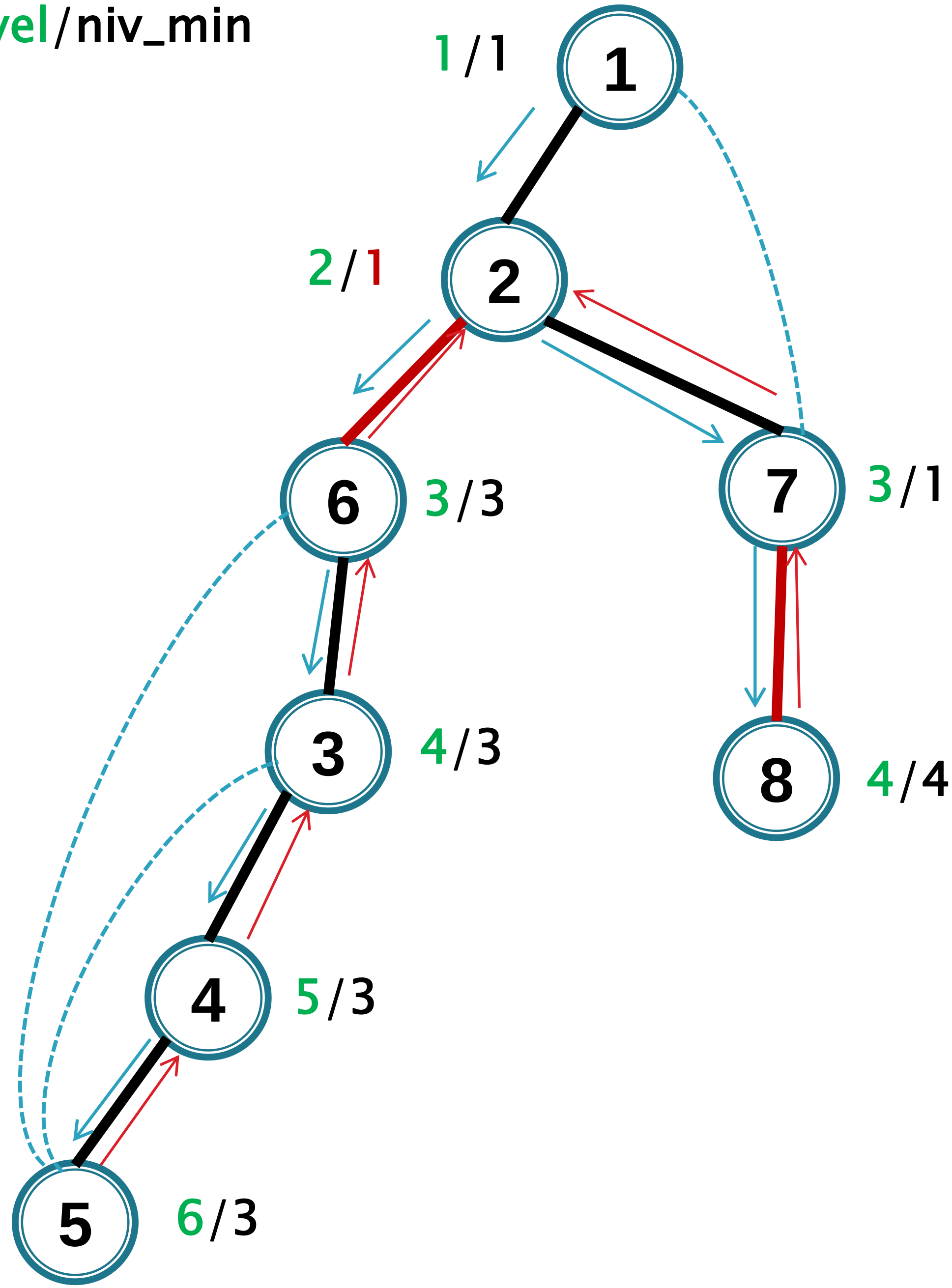
S:

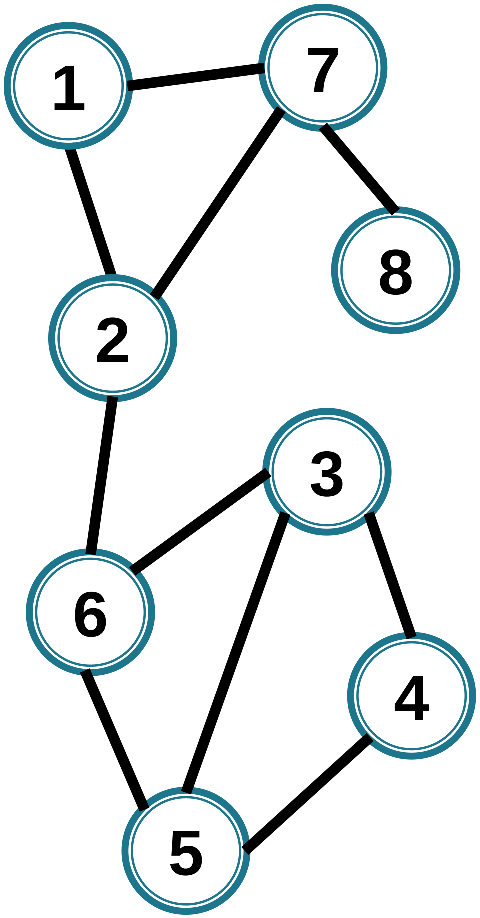
1 2

2 7

1 7

nivel/niv_min





S:

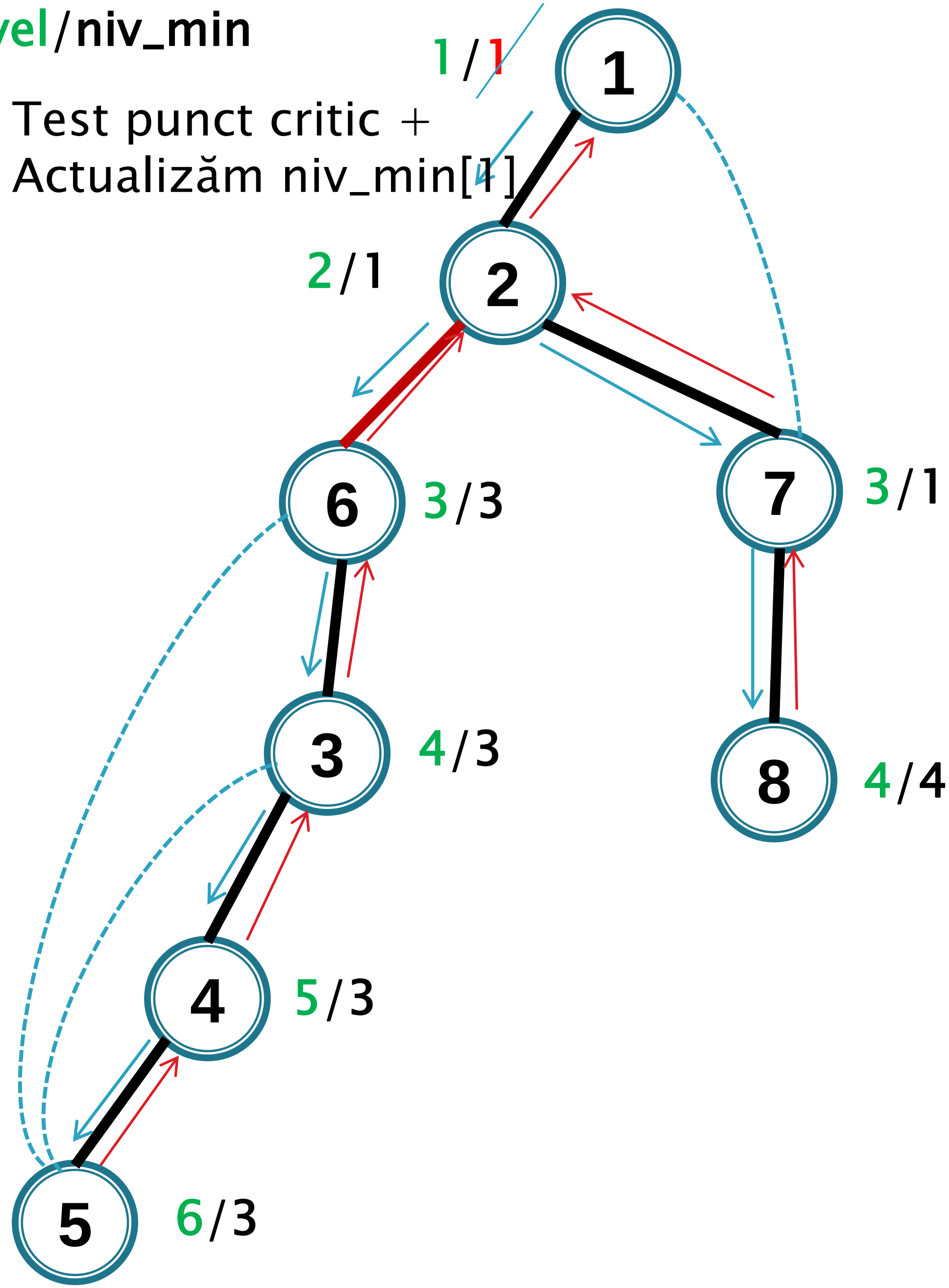
1 2

2 7

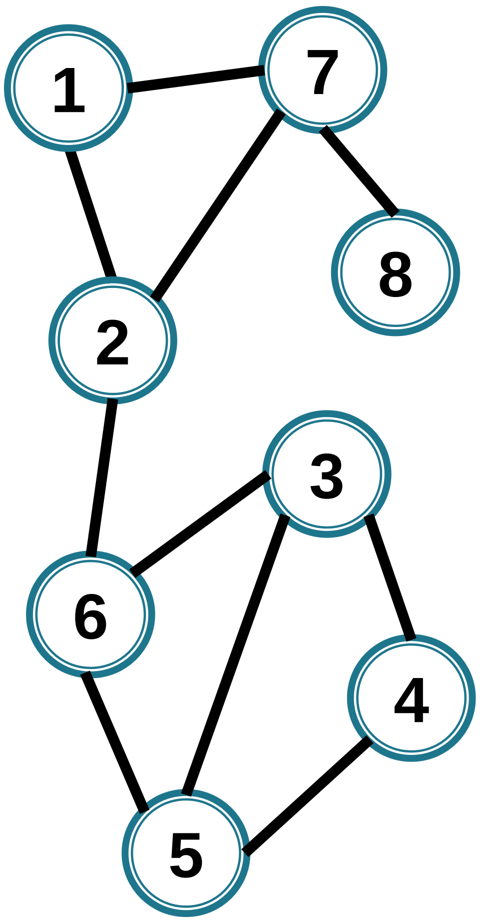
1 7

nivel/niv_min

Test punct critic +
Actualizăm niv_min[1]



nivel/niv_min

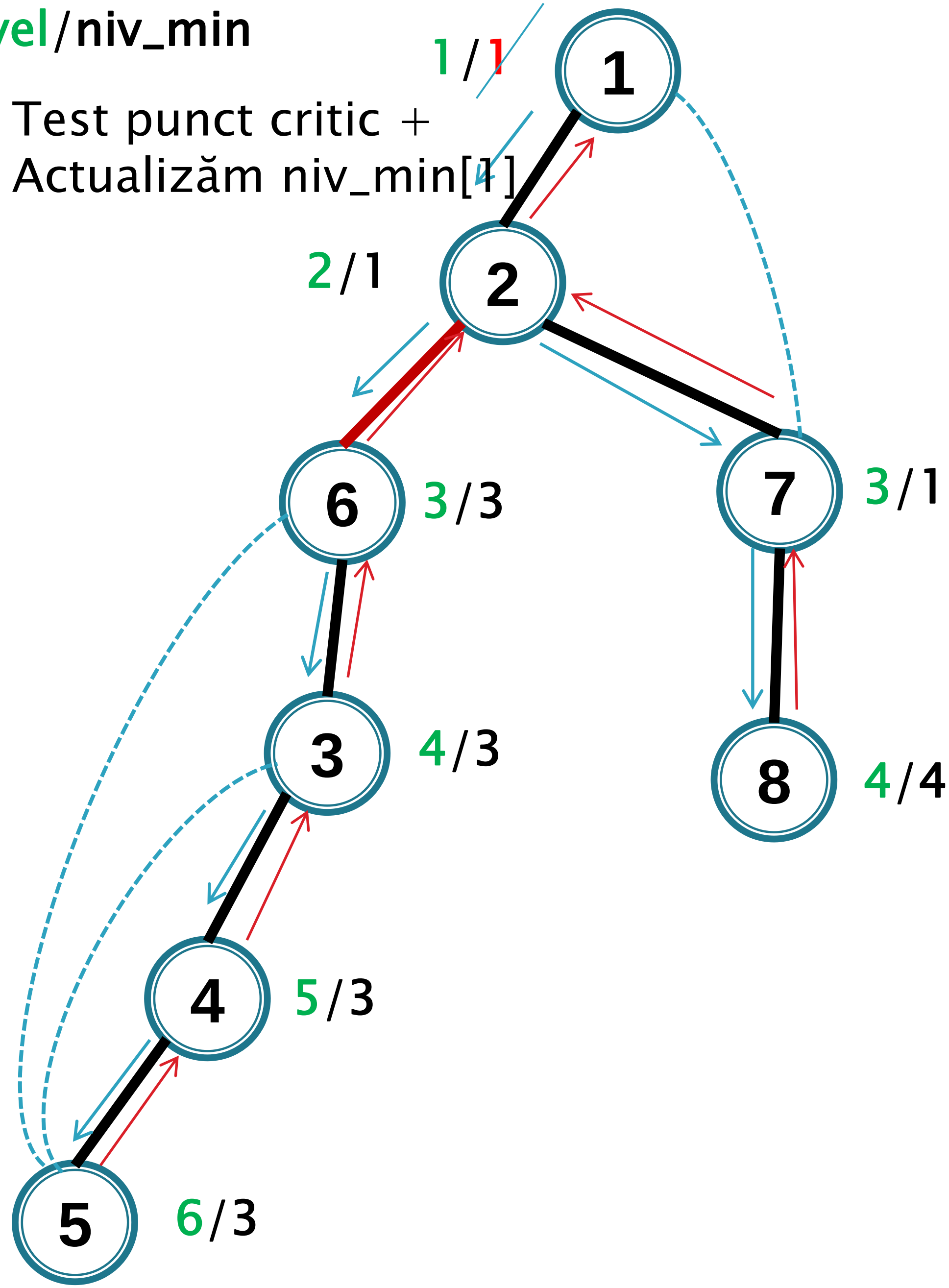


S:

1 2

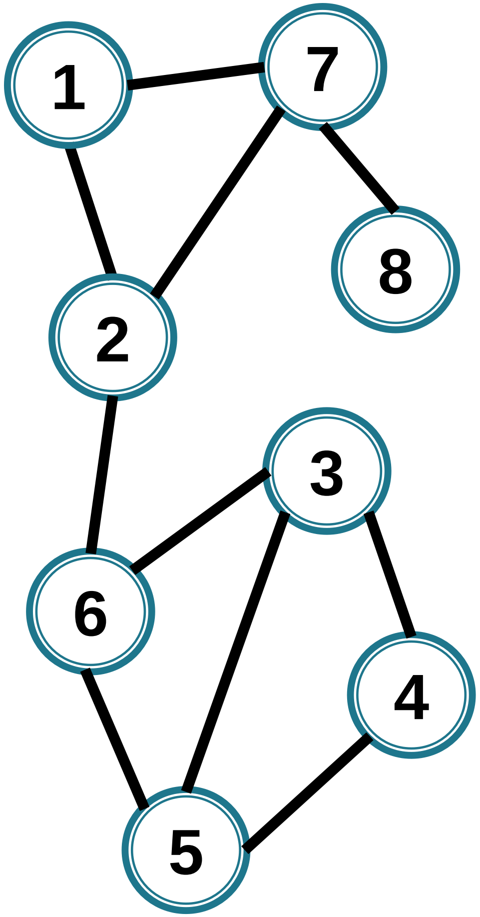
2 7

1 7



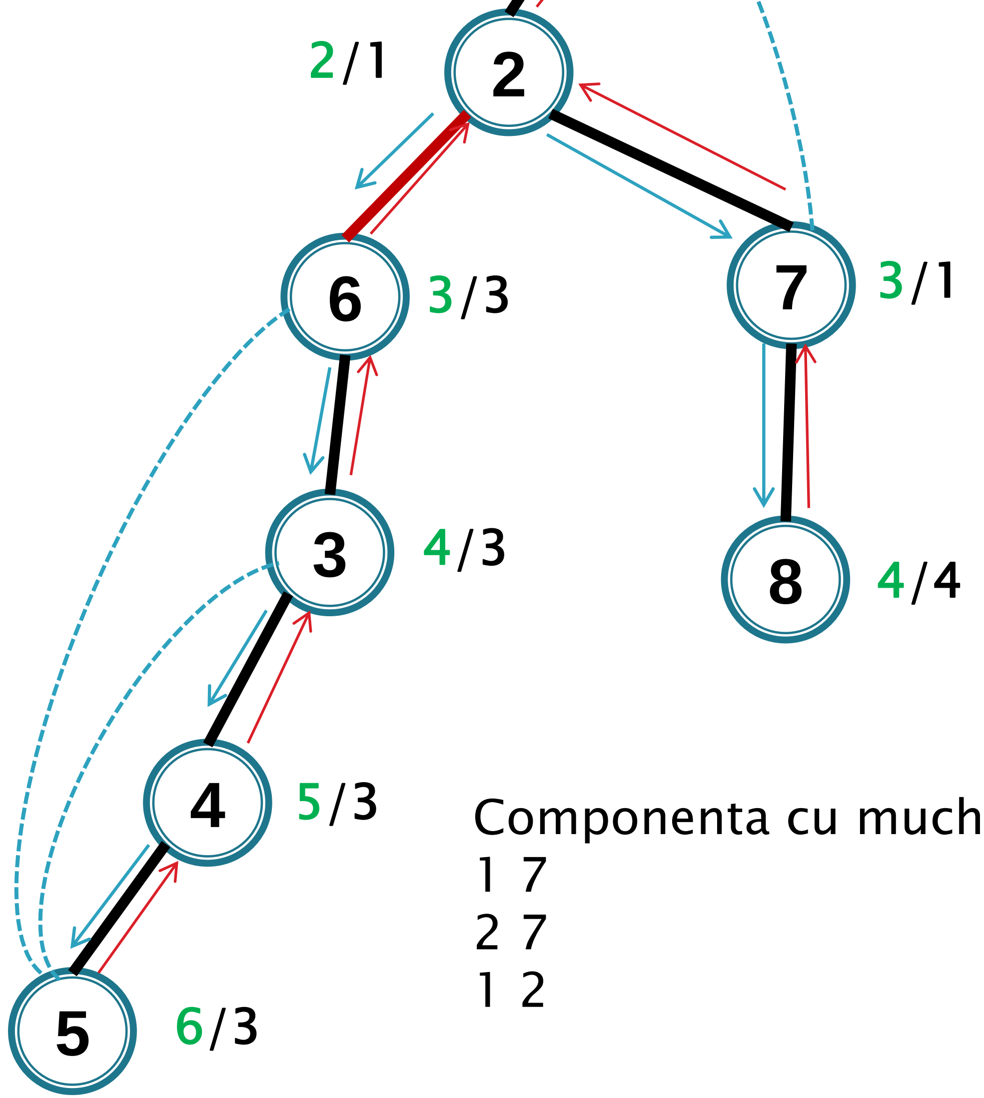
Rădăcina caz particular de test de punct critic – aici nu mai este necesar, se “rupe” o componenta biconexă chiar dacă are un singur fiu

nivel/niv_min



S:
1 2
2 7
1 7

Test punct critic +
Actualizăm niv_min[1]



Componenta cu muchiile
1 7
2 7
1 2

Rădăcina caz particular de test de punct critic – aici nu mai este necesar, se “rupe” o componenta biconexă chiar dacă are un singur fiu

Exerciții

Exerciții

1. Se dă un graf neorientat conex G = (V, E). Se consideră parcurgerile DFS și BFS care pornesc din nodul 1, iar vecinii unui nod sunt considerați în ordine crescătoare.

Ce proprietăți trebuie să respecte G pentru ca cele două parcurgeri să obțină același arbore parțial?

Exerciții

2. Fie G_1 si G_2 două grafuri cu proprietatea că ordinea în care sunt parcurse vârfurile în BF este aceeași pentru ambele grafuri, la fel și în DF (vecinii unui vârf sunt parcurși în ordine crescătoare).

Sunt G_1 și G_2 egale?

Exerciții

3. Fie G_1 si G_2 două grafuri cu proprietatea că arborii BF și DF ai celor două grafuri sunt egali (vecinii unui vârf sunt parcurși în ordine crescătoare).

Sunt G_1 și G_2 egale?

Exerciții

4. Se dă un graf neorientat conex G = (V, E) și un vârf s. Care este numărul minim de muchii ale unui graf parțial al lui G care conservă distanțele de la s la celelalte vârfuri

Exerciții

5. Fie T un graf neorientat cu n vârfuri.

Arătați că T este arbore $\Leftrightarrow$ T este conex și are n−1 muchii

Exerciții

6. Arătați că în orice grup de n persoane între care există relații de prietenie reciprocă există două persoane cu același număr de prieteni