6.05.2020

Seminar 11

Geometrie analitică euclidiană

$(\mathcal{E}_3, E_3, \varphi)$ spațiu punctual euclidian canonic

Ex1

$A(3,-1,3)$, $B(5,1,-1)$, $u=(-3,5,-6)$

a) Să se scrie ecuația dreptei d ai $A\in d$, $V_d = \langle\{u\}\rangle$.

b) Să scrie ecuația dreptei AB

c) Să se afle punctele de intersecție ale dreptei d cu planele de coordonate.

Sol

a) $d: \dfrac{x_1-3}{-3} = \dfrac{x_2+1}{5} = \dfrac{x_3-3}{-6} = t$; Ec. carteziană

$$d: \begin{cases} x_1 = 3-3t \\ x_2 = -1+5t \\ x_3 = 3-6t \end{cases}$$ Ec. parametrice.

b) $\overrightarrow{AB} = (5-3, 1+1, -1-3) = (2,2,-4) = 2(1,1,-2)$

$V_{AB} = \langle\{\overrightarrow{AB}\}\rangle$, $A\in AB$

$$AB: \frac{x_1-3}{1} = \frac{x_2+1}{1} = \frac{x_3-3}{-2} = t$$

$$AB: \begin{cases} x_1 = 3+t \\ x_2 = -1+t \\ x_3 = 3-2t \end{cases}$$

c) • Ox_1x_2: $x_3 = 0$

$d\cap Ox_1x_2 = \{P_3\}$ $\quad P_3: 3-6t=0 \Rightarrow t=\frac{1}{2}$

$P_3\left(3-\frac{3}{2}, -1+\frac{5}{2}, 3-\frac{6}{2}\right) \Rightarrow P_3\left(\frac{3}{2}, \frac{3}{2}, 0\right)$

• Ox_1x_3: $x_2 = 0$

$d\cap Ox_1x_3 = \{P_2\}$ $\quad P_2: -1+5t=0 \Rightarrow t=\frac{1}{5}$

$P_2\left(3-\frac{3}{5}, -1+\frac{5}{5}, 3-\frac{6}{5}\right) \Rightarrow P_2\left(\frac{12}{5}, 0, \frac{9}{5}\right)$

• Ox_2x_3: $x_1 = 0$

$d\cap Ox_2x_3 = \{P_1\}$ $\quad P_1: 3-3t=0 \Rightarrow t=1$

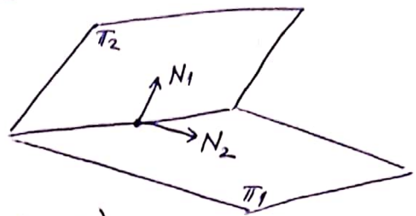
$P_1(0, 4, -3)$

Ex2 Să se scrie ecuația dreptei d a.î. $A(2,-5,3) \in d$; $d \parallel d'$, d': $\begin{cases} 2x_1 - x_2 + 3x_3 + 1 = 0 & \pi_1 \\ 5x_1 + 4x_2 - x_3 + 1 = 0 & \pi_2 \end{cases}$

SOL

$u_{d'} = N_1 \times N_2$; $N_1 = (2,-1,3)$

$N_2 = (5,4,-1)$



$$u_{d'} = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 5 & 4 & -1 \end{vmatrix} = (1-12,\ 17,\ 8+5)$$

$u_{d'} = (-11, 17, 13)$

$$d: \frac{x_1-2}{-11} = \frac{x_2+5}{17} = \frac{x_3-3}{13} = t \Rightarrow \begin{cases} x_1 = 2 - 11t \\ x_2 = -5 + 17t \\ x_3 = 3 + 13t \end{cases}$$

OBS

$$d': \begin{cases} 2x_1 - x_2 = -1 - 3t & \mid \cdot 4 \\ 5x_1 + 4x_2 = -1 + t & \end{cases}, \quad x_3 = t$$

$$\oplus$$

$$13x_1 \quad / \quad = -5 - 11t \Rightarrow x_1 = -\frac{5}{13} - \frac{11}{13}t$$

$$x_2 = 2x_1 + 1 + 3t = -\frac{10}{13} + 1 - \frac{22}{13}t + 3t = \frac{3}{13} + \frac{39-22}{13}t$$

$$x_2 = \frac{3}{13} + \frac{17}{13}t$$

$$d': \begin{cases} x_1 = -\frac{5}{13} - \frac{11}{13}t \\ x_2 = \frac{3}{13} + \frac{17}{13}t \\ x_3 = t \end{cases}$$

$$\tilde{u}_{d'} = \left(-\frac{11}{13}, \frac{17}{13}, 1\right) = \frac{1}{13}(\underbrace{-11, 17, 13})$$

Ex3 $\pi: x_1 + x_2 + x_3 - 1 = 0$, $M(1,2,-1)$

$$d: \frac{x_1-1}{2} = \frac{x_2-1}{-1} = \frac{x_3}{3} = t \Rightarrow \begin{cases} x_1 = 1 + 2t \\ x_2 = 1 - t \\ x_3 = 3t \end{cases}$$

a) Să se scrie ecuația dreptei d' a.î. $M \in d'$; $d' \perp \pi$

b) —"— planului π' a.î. $M \in \pi'$ și $\pi' \perp d$

c) —"— planului π'' a.î. $M \in \pi''$, $d \subset \pi''$

d) $pr_d(M) = ?$

e) $pr_\pi(M) = ?$

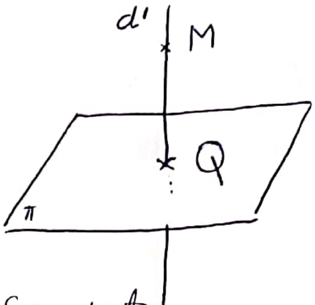
SOL

a) $\pi: x_1+x_2+x_3-1=0$

$N_\pi=(1,1,1)=u_{d'}$

$M(1,2,-1)$

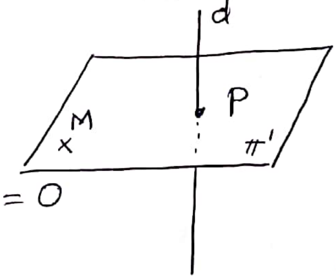
$M\in d'$, $d'\perp\pi$



$$d': \frac{x_1-1}{1}=\frac{x_2-2}{1}=\frac{x_3+1}{1}=t \Rightarrow \begin{cases} x_1=1+t\\ x_2=2+t\\ x_3=-1+t\end{cases}$$

b) $d: \frac{x_1-1}{2}=\frac{x_2-1}{-1}=\frac{x_3}{3}$

$d\perp\pi' \Rightarrow N_{\pi'}=u_d=(2,-1,3)$

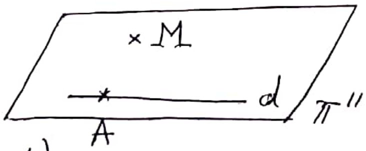


$\pi': 2(x_1-1)+(-1)(x_2-2)+3(x_3+1)=0$

$\pi': 2x_1-x_2+3x_3+3=0.$

c) $A(1,1,0)\in d.$

$V_{\pi''}=\langle\{u_d, \overrightarrow{AM}\}\rangle$



$\overrightarrow{AM}=(1-1,2-1,-1-0)=(0,1,-1)$

$$\pi'': \begin{vmatrix} x_1-1 & 2 & 0\\ x_2-2 & -1 & 1\\ x_3+1 & 3 & -1\end{vmatrix}=0 \Rightarrow \begin{vmatrix} x_1-1 & 2 & 0\\ x_2+x_3-1 & 2 & 0\\ x_3+1 & 3 & -1\end{vmatrix}=0$$

$$2\begin{vmatrix} x_1-1 & 1\\ x_2+x_3-1 & 1\end{vmatrix}=0 \Rightarrow x_1-1-x_2-x_3+1=0$$

$\pi'': x_1-x_2-x_3=0$

OBS. $N_{\pi''}=u_d\times\overrightarrow{AM}=\begin{vmatrix} i & j & k\\ 2 & -1 & 3\\ 0 & 1 & -1\end{vmatrix}=(-2,2,2)$
$=-2\underline{(1,-1,-1)}$

$M\in\pi''$

$\pi'': 1(x_1-1)+(-1)(x_2-2)+(-1)(x_3+1)=0$

$\pi'': x_1-x_2-x_3=0.$

d) $pr_d(M)=P$, $d\cap\pi'=\{P\}.$

$$\left.\begin{array}{l} d: \begin{cases} x_1=1+2t\\ x_2=1-t\\ x_3=3t\end{cases}\\ \pi': 2x_1-x_2+3x_3+3=0\end{array}\right\} \Rightarrow \begin{array}{l} 2(1+2t)-(1-t)+3\cdot 3t+3=\\ =0\Rightarrow\\ 14t+4=0\Rightarrow t=-\frac{2}{7}\end{array}$$

$P\left(1-\frac{4}{7}, 1+\frac{2}{7}, -\frac{6}{7}\right)\Rightarrow P\left(\frac{3}{7},\frac{9}{7},-\frac{6}{7}\right)$

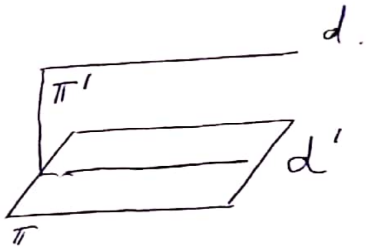
c) $pr_{\pi} M = Q$, $\{Q\} = d' \cap \pi$

$$d' \begin{cases} x_1 = 1+t \\ x_2 = 2+t \\ x_3 = -1+t \end{cases}, \quad \pi: x_1+x_2+x_3-1=0$$

$\Rightarrow 1+t+2+t-1+t-\cancel{1} = 0 \Rightarrow 3t = -1 \Rightarrow t = -\frac{1}{3}$

$Q\left(1-\frac{1}{3}, 2-\frac{1}{3}, -1-\frac{1}{3}\right) \Rightarrow Q\left(\frac{2}{3}, \frac{5}{3}, -\frac{4}{3}\right)$

<u>Ex4</u> $d: \frac{x_1-1}{2} = \frac{x_2}{3} = \frac{x_3-7}{-1}$



$\pi: 2x_1 - x_2 + x_3 - 2 = 0$

a) Să se arate că $d \parallel \pi$

b) $pr_{\pi} d$

<u>SOL</u>

a) $u_d = (2, 3, -1)$, $N_{\pi} = (2, -1, 1)$

$\langle u_d, N_{\pi} \rangle = 4 - 3 - 1 = 0 \Rightarrow u_d \perp N_{\pi} \Rightarrow d \parallel \pi$

b) $\pi' \perp \pi$, $d \subset \pi'$

$A(1, 0, 7) \in d \subset \pi'$

$V_{\pi'} = \langle \{ \vec{u_d}, \vec{N_{\pi}} \} \rangle$

$$\pi': \begin{vmatrix} x_1 - 1 & 2 & 2 \\ x_2 - 0 & 3 & -1 \\ x_3 - 7 & -1 & 1 \end{vmatrix} = 0$$

SAU

$$N_{\pi'} = u_d \times N_{\pi} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 2 & -1 & 1 \end{vmatrix} = (2, -4, -8) = 2(1, -2, -4)$$

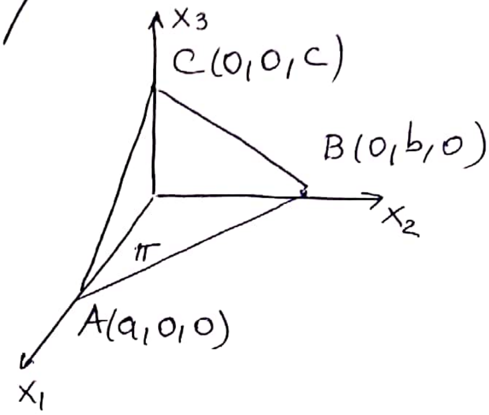
$\pi': 1 \cdot (x_1 - 1) - 2 \cdot (x_2 - 0) - 4(x_3 - 7) = 0$

$\pi': x_1 - 2x_2 - 4x_3 + 27 = 0.$

$$d': \begin{cases} x_1 - 2x_2 - 4x_3 + 27 = 0 \\ 2x_1 - x_2 + x_3 - 2 = 0. \end{cases}$$

OBS Ec. prin tăieturi a unui plan

$$\pi: \frac{x_1}{a} + \frac{x_2}{b} + \frac{x_3}{c} = 1.$$



Ex5. Fie familia de plane:

$$\pi_\lambda: \frac{x}{\lambda} + \frac{y}{\sqrt{a^2-\lambda^2}} + z - b = 0, \quad \lambda \in \mathbb{R} \setminus \{0, \pm a\}$$
$$a \neq 0, b \neq 0$$

Să se arate că π_λ, $\forall \lambda \in \mathbb{R}\setminus\{0, \pm a\}$, determină pe axele de coordonate segmente care au suma pătratelor lungimilor o const.

SOL

$$\pi_\lambda: \frac{x}{\lambda} + \frac{y}{\sqrt{a^2-\lambda^2}} + z = b \Rightarrow$$

$$\frac{x}{b\lambda} + \frac{y}{b\sqrt{a^2-\lambda^2}} + \frac{z}{b} = 1.$$

$A(b\lambda, 0, 0)$, $B(0, b\sqrt{a^2-\lambda^2}, 0)$, $C(0, 0, b)$

$$b^2\lambda^2 + b^2(a^2-\lambda^2) + b^2 = b^2(\cancel{\lambda^2} + a^2 - \cancel{\lambda^2} + 1) = b^2(a^2+1) = ct.$$