Seminar 7 (142)

Ex1 $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (-3x_1 + 2x_2, -5x_1 + 4x_2, 2x_1 - 2x_2 - x_3)$

Precizați un reper în $\mathbb{R}^3$ a.î. matricea asociată lui f este diagonală.

SOL $R_0 = \{e_1^0, e_2^0, e_3^0\}$ reperul canonic din $\mathbb{R}^3$

$$A = [f]_{R_0, R_0} = \begin{pmatrix} -3 & 2 & 0 \\ -5 & 4 & 0 \\ 2 & -2 & -1 \end{pmatrix}$$

Determinăm valorile proprii

$$P(\lambda) = \det(A - \lambda I_3) = \begin{vmatrix} -3-\lambda & 2 & 0 \\ -5 & 4-\lambda & 0 \\ 2 & -2 & -1-\lambda \end{vmatrix}$$

$$= (-1-\lambda) \underbrace{\begin{vmatrix} -3-\lambda & 2 \\ -5 & 4-\lambda \end{vmatrix}}_{\lambda^2 - \lambda - 12 + 10} = (-\lambda - 1)\underbrace{(\lambda^2 - \lambda - 2)}_{(\lambda+1)(\lambda-2)} =$$

$$= -(\lambda+1)^2(\lambda - 2) = 0 \Rightarrow \begin{cases} \lambda_1 = -1, & m_1 = 2 \\ \lambda_2 = 2, & m_1 = 1. \end{cases}$$

$\circledast$ $\lambda_1, \lambda_2 \in \mathbb{R}$

Det. subsp. proprii

$V_{\lambda_1} = \{x \in \mathbb{R}^3 \mid f(x) = -x\} = \{(x_1, x_1, x_3) \mid x_1, x_3 \in \mathbb{R}\}$

$= \langle \{(1,1,0), (0,0,1)\} \rangle$

$$\begin{cases} -3x_1 + 2x_2 = -x_1 \\ -5x_1 + 4x_2 = -x_2 \\ 2x_1 - 2x_2 - x_3 = -x_3 \end{cases} \Rightarrow \left\{ x_1 = x_2 \right. ; \quad rg\begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} = 2 = \max$$

$\underline{R_1} = \{(1,1,0), (0,0,1)\}$ reper în V_{λ_1}.

- $\dim V_{\lambda_1} = 2 = m_1$

$V_{\lambda_2} = \{x \in \mathbb{R}^3 \mid f(x) = 2x\} = \{(x_1, \frac{5}{2}x_1, -x_1) \mid x_1 \in \mathbb{R}\}$

$\overset{"}{} \frac{x_1}{2}(2, 5, -2)$

$$\begin{cases} -3x_1 + 2x_2 = 2x_1 \\ -5x_1 + 4x_2 = 2x_2 \\ 2x_1 - 2x_2 - x_3 = 2x_3 \end{cases} \Rightarrow \begin{cases} -5x_1 + 2x_2 = 0 \Rightarrow x_2 = \frac{5}{2}x_1 \\ -5x_1 + 2x_2 = 0 \\ 2x_1 - \underset{5x_1}{2x_2} = 3x_3 \Rightarrow x_3 = -x_1 \end{cases}$$

$V_{\lambda_2} = \langle \{(2,5,-2)\} \rangle$, $\underline{R_2} = \{(2,5,-2)\}$, $\dim V_{\lambda_2} = 1 = m_2$

$$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 = \{\underbrace{(1,1,0)}_{e_1}, \underbrace{(0,0,1)}_{e_2}, \underbrace{(2,5,-2)}_{e_3}\} \text{ reper în } \mathbb{R}^3$$

$$A' = [f]_{\mathcal{R},\mathcal{R}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad \begin{cases} f(e_1) = -e_1 \\ f(e_2) = -e_2 \\ f(e_3) = 2e_3. \end{cases}$$

Ex2 $f: \mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (4x_1 + x_2 + x_3, x_1 + 4x_2 + x_3, x_1 + x_2 + 4x_3)$

Precizați un reper în raport cu care matricea asociată lui f este diagonală.

Ex $\mathcal{R}_0 = \{e_1^0, e_2^0, e_3^0\}$, $A = [f]_{\mathcal{R}_0,\mathcal{R}_0} = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{pmatrix}$

$$P(\lambda) = \det(A - \lambda I_3) = \begin{vmatrix} 4-\lambda & 1 & 1 \\ 1 & 4-\lambda & 1 \\ 1 & 1 & 4-\lambda \end{vmatrix} = (6-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4-\lambda & 1 \\ 1 & 1 & 4-\lambda \end{vmatrix} =$$

$c_1' = c_1 + c_2 + c_3$

$$= (6-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 1 & 3-\lambda & 0 \\ 1 & 0 & 3-\lambda \end{vmatrix} = (6-\lambda)(3-\lambda)^2$$

$\lambda_1 = 6$, $m_1 = 1$; $\lambda_2 = 3$, $m_2 = 2$

$V_{\lambda_1} = \{x \in \mathbb{R}^3 / f(x) = 6x\} = \{(x_3, x_3, x_3) / x_3 \in \mathbb{R}\} = \langle\{(1,1,1)\}\rangle$

$$\begin{cases} 4x_1 + x_2 + x_3 = 6x_1 \\ x_1 + 4x_2 + x_3 = 6x_2 \\ x_1 + x_2 + 4x_3 = 6x_3 \end{cases} \Rightarrow \begin{cases} -2x_1 + x_2 + x_3 = 0 \\ x_1 - 2x_2 + x_3 = 0 \\ x_1 + x_2 - 2x_3 = 0 \end{cases} \qquad M = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$\det M = 0$

$$\begin{cases} -2x_1 + x_2 = -x_3 \quad | \cdot 2 \\ x_1 - 2x_2 = -x_3 \end{cases} \oplus$$

$-3x_1 \quad / = -3x_3$

$x_1 = x_3$

$x_2 = -x_3 + 2x_3 = x_3$

$\mathcal{R}_1 = \{(1,1,1)\}$ reper în V_{λ_1}, $\dim V_{\lambda_1} = 1 = m_1$

$V_{\lambda_2} = \{x \in \mathbb{R}^3 / f(x) = 3x\} = \{(x_1, x_2, -x_1 - x_2) / x_1, x_2 \in \mathbb{R}\}$

$$\begin{cases} 4x_1 + x_2 + x_3 = 3x_1 \\ x_1 + 4x_2 + x_3 = 3x_2 \\ x_1 + x_2 + 4x_3 = 3x_3 \end{cases} \Rightarrow x_1 + x_2 + x_3 = 0 \Rightarrow x_3 = -x_1 - x_2$$

$V_{\lambda_2} = \langle\{(1,0,-1), (0,1,-1)\}\rangle$, $rg\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{pmatrix} = 2 = \max$

$\mathcal{R}_2 = \{(1,0,-1), (0,1,-1)\}$ reper în V_{λ_2}, $\dim V_{\lambda_2} = m_2 = 2$.

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$\mathcal{R} = \mathcal{R}_1 \cup \mathcal{R}_2 = \{(1,1,1), (1,0,-1), (0,1,-1)\}$ reper în $\mathbb{R}^3$ a.î.

$$A' = [f]_{\mathcal{R},\mathcal{R}} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Ex $f \in End(\mathbb{R}^3)$

Fie $\lambda_1 = 3$, $\lambda_2 = -2$, $\lambda_3 = 1$ valorile proprii ale lui f, și $v_1 = (-3,2,1)$, $v_2 = (-2,1,0)$, $v_3 = (-6,3,1)$ sunt vectori proprii coresp. valorilor proprii λ_1, λ_2, resp λ_3.

Să se determine $A = [f]_{\mathcal{R}_0,\mathcal{R}_0}$, $\mathcal{R}_0$ = reperul canonic din $\mathbb{R}^3$.

SOL

$f(v_1) = \lambda_1 v_1$; $f(v_2) = \lambda_2 v_2$, $f(v_3) = \lambda_3 v_3$.

v_1, v_2, v_3 sunt vect pr. coresp. la valori pr. dist $\Rightarrow$

$$\left.\begin{array}{l} \Rightarrow \mathcal{R} = \{v_1, v_2, v_3\} \text{ SLI} \\ \text{dar } \dim \mathbb{R}^3 = |\mathcal{R}| \end{array}\right\} \Rightarrow \mathcal{R} \text{ reper în } \mathbb{R}^3.$$

$$A' = [f]_{\mathcal{R},\mathcal{R}} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{R}_0 = \{e_1, e_2, e_3\} \xrightarrow{C} \mathcal{R} = \{v_1, v_2, v_3\}$$

$v_1 = (-3,2,1) = -3e_1 + 2e_2 + e_3$

$v_2 = (-2,1,0) = -2e_1 + e_2$

$v_3 = (-6,3,1) = -6e_1 + 3e_2 + e_3$

$$C = \begin{pmatrix} -3 & -2 & -6 \\ 2 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A = [f]_{\mathcal{R}_0,\mathcal{R}_0} \rightsquigarrow A' = [f]_{\mathcal{R},\mathcal{R}}.$$

(M1) $A' = C^{-1} A C \Rightarrow A = C A' C^{-1}$

(M2)
$$\begin{cases} f(v_1) = f(-3e_1 + 2e_2 + e_3) = -3f(e_1) + 2f(e_2) + f(e_3) = 3(-3,2,1) \\ f(v_2) = f(-2e_1 + e_2) = -2f(e_1) + f(e_2) = -2(-2,1,0) \\ f(v_3) = f(-6e_1 + 3e_2 + e_3) = -6f(e_1) + 3f(e_2) + f(e_3) = 1(-6,3,1) \end{cases}$$

ec1 - ec3 :
$$\begin{cases} 3f(e_1) - f(e_2) = (-9+6, 6-3, 3-1) = (-3,3,2) \\ -2f(e_1) + f(e_2) = (4,-2,0) \end{cases} \quad \oplus$$

$f(e_1) = (1,1,2)$

$$\begin{cases} f(e_1) = (1,1,2) \\ f(e_2) = (4,-2,0) + 2(1,1,2) = (6,0,4) \\ f(e_3) = (-9,6,3) + 3(1,1,2) - 2(6,0,4) \\ \quad = (-9+3-12,\ 6+3,\ 3+6-8) = (-18,9,1) \end{cases}$$

$$\begin{cases} f(e_1) = e_1 + e_2 + 2e_3 \\ f(e_2) = 6e_1 \quad + 4e_3 \\ f(e_3) = -18e_1 + 9e_2 + e_3 \end{cases} \qquad A = [f]_{R_0,R_0} = \begin{pmatrix} 1 & 6 & -18 \\ 1 & 0 & 9 \\ 2 & 4 & 1 \end{pmatrix}$$

Ex $f:\mathbb{R}^3 \to \mathbb{R}^3$, $f(x) = (x_2, x_3, 2x_1 - 5x_2 + 4x_3)$

Să se arate că f nu este diagonalizabil

Sol

R_0 reper canonic $\quad A = [f]_{R_0,R_0} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{pmatrix}$

$$P(\lambda) = \det(A - \lambda I_3) = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 2 & -5 & 4-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \sigma_1\lambda^2 + \sigma_2\lambda - \sigma_3 = 0$$

$$\sigma_1 = \operatorname{Tr} A = 4, \quad \sigma_2 = \begin{vmatrix} 0 & 1 \\ -5 & 4 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 2 & 4 \end{vmatrix} + \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 5$$

$$\sigma_3 = \det(A) = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{vmatrix} = -\begin{vmatrix} 0 & 1 \\ 2 & 4 \end{vmatrix} = 2$$

$$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0 \Rightarrow \lambda^3 - \lambda^2 - 3\lambda^2 + 3\lambda + 2\lambda - 2 = 0$$

$$(\lambda-1)\underbrace{(\lambda^2 - 3\lambda + 2)}_{(\lambda-1)(\lambda+2)} = 0 \Rightarrow (\lambda-1)^2(\lambda+2) = 0$$

$$\lambda_1 = 1,\ m_1 = 2; \quad \lambda_2 = -2,\ m_1 = 1$$

$$V_{\lambda_1} = \{x \in \mathbb{R}^3 \mid f(x) = x\} = \{(x_2, x_2, x_2) \mid x_2 \in \mathbb{R}\} = \langle\{(1,1,1)\}\rangle$$

$$\begin{cases} x_2 = x_1 \\ x_3 = x_2 \\ 2x_1 - 5x_2 + 4x_3 = x_3 \Rightarrow \underbrace{2x_1 - 5x_2}_{-3x_3} + 3x_3 = 0 \end{cases}$$

$$\dim V_{\lambda_1} = 1 \neq \underset{2}{m_1} \Rightarrow f \text{ nu e diagonalizabil}$$

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Forme biliniare

Ex1 $g: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ formă biliniară antisimetrică
$R_0 = \{e_1, e_2\}$ reper canonic și $g(e_1, e_2) = 5$
Care este matricea asociată lui g în raport cu reperul canonic?

SOL

OBS $g: V \times V \to \mathbb{K}$ s.n. formă biliniară $\Leftrightarrow$ liniară în ambele argumente.

simetrică: $g(x,y) = g(y,x) \Leftrightarrow G = G^T$

antisimetrică: $g(x,y) = -g(y,x) \Leftrightarrow G = -G^T$

$R = \{e_1, \dots, e_n\}$ reper în V

$G = (g_{ij})_{i,j=\overline{1,n}}$, $g_{ij} = g(e_i, e_j)$; $g(x,y) = \sum_{i,j=1}^{n} g_{ij} x_i y_j$

$$G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = -G^T \Rightarrow G = \begin{pmatrix} 0 & g_{12} \\ -g_{12} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$$

$g_{12} = g(e_1, e_2) = 5$

$$g(x,y) = \sum_{i,j=1}^{2} g_{ij} x_i y_j = 5x_1y_2 - 5x_2y_1$$

Ex2 $g: \mathbb{R}^3 \times \mathbb{R}^3 \to \mathbb{R}$, $g(x,y) = x_1y_1 - x_2y_2 - x_1y_3 - x_3y_1 + 2x_2y_3 + 2x_3y_2$

a) $g \in L^s(\mathbb{R}^3, \mathbb{R}^3; \mathbb{R})$ (formă biliniară, simetrică)

b) Precizați matricea G asociată lui g în raport cu reperul canonic R_0

c) $\operatorname{Ker} g = ?$ Este g nedegenerată?

d) Să se afle matricea G' asociată lui g în raport cu reperul $R' = \{e_1' = (1,1,1), e_2' = (1,2,1), e_3' = (0,0,1)\}$.

e) Să se afle forma pătratică $Q: \mathbb{R}^3 \to \mathbb{R}$ asociată lui g.

SOL

a) • g simetrică : $g(x,y)=g(y,x)$, $\forall x,y\in\mathbb{R}^3$

$g(x,y)=x_1y_1-x_2y_2-x_1y_3-x_3y_1+2x_2y_3+2x_3y_2$

• g liniară în primul arg.

$$g(ax+by,z)\overset{?}{=}a\,g(x,z)+b\,g(y,z)$$

$$(ax_1+by_1)\cdot z_1-(ax_2+by_2)z_2-(ax_1+by_1)z_3-(ax_3+by_3)z_1+$$
$$+2(ax_2+by_2)z_3+2(ax_3+by_3)z_2=$$
$$=a\,g(x,z)+b\,g(y,z)$$

$\Rightarrow g$ liniari și în al doilea arg.

$\Rightarrow g\in L^1(\mathbb{R}^3,\mathbb{R}^3;\mathbb{R})$

b) $G=\begin{pmatrix}1&0&-1\\0&-1&2\\-1&2&0\end{pmatrix}$ $\quad g(x,y)=X^TGY$, $X=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$, $Y=\begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}$

$g(e_i,e_j)=g_{ij}$.

$g_{11}=g(e_1,e_1)=g((1,0,0),(1,0,0))=1,\ \dots\ g_{33}=0$

c) $\mathrm{Ker}\,g=\{x\in\mathbb{R}^3\ |\ g(x,y)=0,\ \forall y\in\mathbb{R}^3\}$

g nedegenerată $\Leftrightarrow \mathrm{Ker}\,g=\{0_{\mathbb{R}^3}\}\Leftrightarrow G\in GL(3,\mathbb{R})$

$$\det G=\begin{vmatrix}1&0&-1\\0&-1&2\\-1&2&0\end{vmatrix}=\begin{vmatrix}1&0&0\\0&-1&2\\-1&2&-1\end{vmatrix}=1-4=-3\neq 0$$

SAU

$$\begin{cases}g(x,e_1)=0 & \Rightarrow g((x_1,x_2,x_3),(1,0,0))=x_1-x_3=0\\ g(x,e_2)=0 & \Rightarrow g((x_1,x_2,x_3),(0,1,0))=-x_2+2x_3=0\\ g(x,e_3)=0 & g((x_1,x_2,x_3),(0,0,1))=-x_1+2x_2=0\end{cases}$$

$G=\begin{pmatrix}1&0&-1\\0&-1&2\\-1&2&0\end{pmatrix}$ $\det G\neq 0 \Rightarrow x_1=x_2=x_3=0\Rightarrow x=0_{\mathbb{R}^3}$

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d) $\mathcal{R}_0 = \{e_1, e_2, e_3\} \xrightarrow[C]{-\gamma-} \mathcal{R}' = \{e_1' = (1,1,1), e_2' = (1,2,1), e_3' = (0,0,1)\}$

$G \qquad\qquad G' \; ; \; C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

$G' = C^T G C$

$$G' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 2 \\ -1 & 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 2 \\ 1 & 3 & 0 \end{pmatrix} \Rightarrow G' = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

$x = \sum_{i=1}^{3} x_i' e_i' \;, \; y = \sum_{j=1}^{3} y_j' e_j'$

$$g(x,y) = 2x_1'y_1' + 3x_1'y_2' + x_1'y_3' + 3x_2'y_1' + 3x_2'y_2' + 3x_2'y_3' + x_3'y_1' + 3x_3'y_2'$$

OBS $g_{ij}' = g(e_i', e_j') \;, \; \forall\, i,j = \overline{1,3}$

$g_{11}' = g(e_1', e_1') = g((1,1,1),(1,1,1)) = \cancel{1} - \cancel{1} - \cancel{1} - \cancel{1} + \cancel{2} + 2 = 2$

$g(x,y) = x_1y_1 - x_2y_2 - x_1y_3 - x_3y_1 + 2x_2y_3 + 2x_3y_2$

e) Def $Q : V \longrightarrow \mathbb{K}$ s.n. formă pătratică $\Longleftrightarrow$

$\exists\, g : V \times V \longrightarrow \mathbb{K}$ formă biliniară simetrică

a.î. $Q(x) = g(x,x)$

$Q(x) = g(x,x) = x_1^2 - x_2^2 - 2x_1x_3 + 4x_2x_3$

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(Ex) $f \in End(\mathbb{R}^3)$, $g : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ formă biliniară

Fie $g_f : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$, $g_f(x,y) = g(f(x),y)$, $\forall x,y \in \mathbb{R}^3$

a) g_f formă biliniară

b) Dacă $G = \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ -2 & -1 & -1 \end{pmatrix}$ este matricea asociată lui g în rap. cu R_0 și $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix} = [f]_{R_0,R_0}$, să se determine matricea asociată lui g_f în raport cu R_0.

SOL

a) $g_f : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$, $g_f(x,y) = g(f(x),y)$

- $g_f(ax+by, z) = g(f(ax+by), z) = g(\underline{af(x)+bf(y)}, z)$
  $= a\,g(f(x),z) + b\,g(f(y),z) = a\,g_f(x,z) + b\,g_f(y,z)$

- $g_f(x, ay+bz) = g(f(x), \underline{ay+bz}) = a\,g(f(x),y) + b\,g(f(x),z)$
  $= a\,g_f(x,y) + b\,g_f(x,z)$

b) $\underset{\tilde{g}_{ij}}{\underset{\parallel}{g_f(e_i,e_j)}} = g(\underline{f(e_i)}, e_j) = g\left(\sum_{k=1}^{3} a_{ki} e_k, e_j\right) =$

$$= \sum_{k=1}^{3} a_{ki}\, g(e_k, e_j) \Rightarrow$$

$$\tilde{g}_{ij} = \underbrace{\sum_{k=1}^{3} a_{ki}\, g_{kj}} \Rightarrow \tilde{G} = A^T G$$

$$= \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ -2 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ -2 & -2 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

OBS

- $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$, $f(x) = (x_1 - x_2 + x_3,\ x_2 - x_3,\ x_1 + x_3)$
- $g : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$, $g(x,y) = 2x_1y_1 + x_1y_2 - x_2y_2 - 2x_3y_1 - x_3y_2 - x_3y_3$
- $g_f : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$, $g_f(x,y) = -x_1y_3 - 2x_2y_1 - 2x_2y_2 + x_3y_2 - x_3y_3$.

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