1 Elementary algebra

Problem 1.1

$$\frac{x^{n+2}}{x^{n-2}} = \frac{x^n \cdot x^2}{\frac{x^n}{x^{-2}}} = x^n \cdot x^2 \cdot \frac{x^{+2}}{x^n} = x^4 \frac{x^n}{x^n} = \underline{1 \cdot x^4}$$

Problem 1.2

$$x^{-1} \cdot 8 = 2 \quad / 8$$

$$x^{-1} = \frac{2}{8}$$

$$x^{-1} = \frac{1}{4}$$

$$x^{-1} = 4^{-1}$$

$$\underline{x = 4}$$

Problem 1.3

$$a = 5 \quad b = 10$$

$$(a^b)^0 = (5^{10})^0 = 5^{10 \cdot 0} = 5^0 = \underline{1}$$

Problem 1.4

$$X = \mathbb{R}^+$$

$$\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2\sqrt{x}}{\sqrt{x}} = 2 \cdot 1 = \underline{2}$$

Problem 1.5

$$x^2 + (x+1)^2 = (x+2)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4 \quad / -x^2$$

$$x^2 + 2x + 1 = 4x + 4 \quad / -4x - 4$$

$$x^2 - 2x - 3 = 0$$

~~$(x+1)(x-3) = 0 \quad / (x+1)$~~ ~~$x$~~

$$(x-3)(x+1) = 0$$

$$\boxed{x_1 = 3}$$

$$\boxed{x_2 = -1}$$

Problem 1.6

$$2^x > 1024$$

$$2^x > 2^{10}$$

$$\underline{x > 10 \in \mathbb{R}}$$

2 Functions of one variable

Problem 2.1

$F = a + b \cdot C$

if $C = 0$ then $F = 32$

if $C = 100$ then $F = 212$

$32 = a + b \cdot 0$

$\underline{a = 32}$

$212 = a + b\,100$

$212 = 32 + b \cdot 100 \quad /-32$

$180 = b \cdot 100$

$\frac{180}{100} = b$

$\underline{b = 1.8}$

$F = 32 + 1.8C \quad /-32$

$F - 32 = 1.8C \quad /:1.8$

$\frac{F}{1.8} - \frac{32}{1.8} = C$

$F = C \rightarrow C = 32 + 1.8C \quad /-32 \; -C$

$-32 = 0.8C \quad /:0.8$

$\boxed{C = -40}$

$\boxed{C° = -40° = F°}$

Problem 2.2

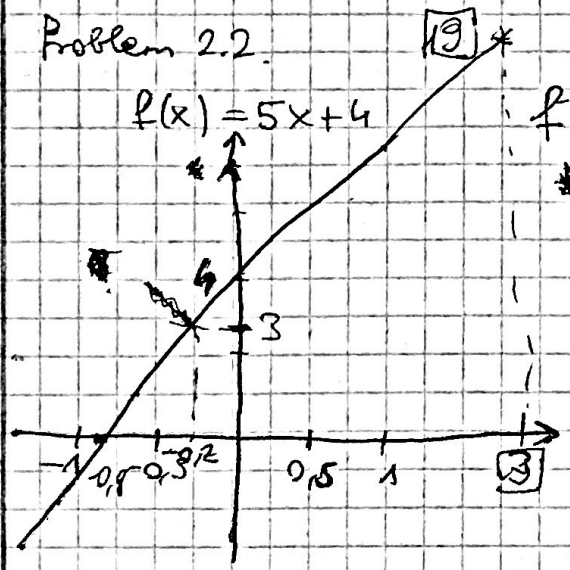
$f(x) = 5x + 4$

$f(3) = y \rightarrow f(3) = 5(3) + 4$

$y = 15 + 4$

$\boxed{y = 19}$

$x = 3$



Problem 2.3

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$\boxed{x_1 = 3}$

$\boxed{x_2 = 1}$

Problem 2.4

$f(x) = a \cdot x^t$

$a = 10$

$t = 90$

$x = 1.02$

$10 \cdot 1.02^{90} = 10 \cdot 5.9431 = \underline{59.43\ HUF}$

Problem 2.5

$e^{\ln 5} \quad e^{\ln a} = a$

$e^{\ln 5} = \underline{5}$

3 Calculus

Problem 3.1

$\sum_{i=2}^{\infty} \frac{12}{6^i}$ $\quad \sum_{i=1}^{\infty} a_i = \frac{a}{1-b}$

$a = 12 \quad b = \frac{1}{6}$

$$\sum_{i=1}^{\infty} a_i = \frac{12 \cdot \frac{1}{6}}{1 - \frac{1}{6}} = \frac{\frac{12}{6}}{\frac{5}{6}} = \frac{12}{6} \cdot \frac{6}{5} = \frac{12}{5} = \underline{2.4}$$

Problem 3.2

$$\lim_{x \to 1} \frac{6^{1-x}}{x} = \lim_{x \to 1} \frac{6^{1-x}}{x} \cdot \frac{6^x}{6^x} = \lim_{x \to 1} \frac{6^1}{x \cdot 6^x} = \underline{1}$$

Problem 3.3

$f(x) = x^5 - 8$ $\quad$ slope at $x = -3$

$(f(x))' = 5x^4 \quad 5x^4 \Rightarrow x = -3$

$(f(-3))' = 5(-3)^4 = 5 \cdot 81 = \underline{405}$

Problem 3.4

$$\frac{d}{dx} \frac{x^3 + 2x - 1}{x - 2} = \frac{(3x^2+2)(x-2) - (x^3+2x-1)(1)}{(x-2)^2}$$

$$\frac{d}{dx} = \frac{3x^3 - 6x^2 + 2x - 4 - x^3 - 2x + 1}{(x-2)^2} = \frac{2x^3 - 6x^2 - 3}{(x-2)^2} = \frac{2x(x^2 - 3x - 1.5}{(x-2)^2}$$

$$= \underline{\frac{2x^3 - 6x^2 - 3}{x^2 - 4x + 4}}$$

Problem 3.5

$\frac{d^2}{dx^2} \quad 4x^4 + 4x^2$

$\frac{d}{dx} = 4 \cdot 4x^3 + 4 \cdot 2x = 16x^3 + 8x$

$\frac{d^2}{dx^2} = 16 \cdot 3x^2 + 8 = \underline{48x^2 + 8 = 8(6x^2 + 1)}$

Problem 3.6

$$\frac{d}{dx}\frac{\ln x}{e^x} = \frac{\frac{1}{x}\cdot e^x - e^x \cdot \ln x}{(e^x)^2} = \frac{\frac{1}{x} - \ln x}{e^x}$$

Problem 3.7

$f(x) = 3x^2 - 5x + 2$

$f'(x) = 6x - 5$ $\quad 6x - 5 = 0 \mid +5$

$f''(x) = 6$ $\quad 6x = 5 \mid :6$

$x = \frac{5}{6}$

$$x_{1/2} = \frac{+5 \pm \sqrt{25-24}}{6} = \frac{5 \pm 1}{6} \begin{cases} 1 = x_1 \\ \frac{2}{3} = x_2 \end{cases}$$

	$-\infty$	$\frac{2}{3}$	$\frac{2}{3} < x < \frac{5}{6}$	$\frac{5}{6}$	$\frac{5}{6} < x < 1$	1	$1 < x < 6$	6
$f(x)$	+	0	−	local min.	−	0	+	+
$f'(x)$	−	−	−	0	+	+	+	+
$f''(x)$	+	+	+	+	+	+	+	+
slope	−	−	−	0	+	+	+	+
convexity	⌣	⌣	⌣	⌣	⌣	⌣	⌣	⌣

Problem 3.8

$f(x,y) = x^2 + y^3$. Calculate $f(2,3)$

$f(2,3) = 2^2 + 3^3 = 4 + 27 = 31$ $\quad \boxed{f(2,3) = 31}$

Problem 3.9

$f(x,y) = \ln(x - y)$ $\quad x - y > 0$

$\boxed{x > y \in \mathbb{Z}}$

Problem 3.10.

$$\frac{\partial}{\partial x} x^5 + xy^3 = 5x^4 + y^3$$

Problem 3.11.

$f(x,y) = x^2 y^2 + 10$

$f'_x(x,y) = 2xy^2$ $\quad f''_x(x,y) = 2y^2$

$f'_y(x,y) = 2x^2 y$ $\quad f''_y(x,y) = 2x^2$

Local minimum $\boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}}$

Problem 3.12 Lagrange's method $\max x^2y^2$ s.t. $x+y=10$

$$\mathcal{L} = f(x,y) - \lambda g(x,y)$$

$$\mathcal{L} = x^2y^2 - \lambda(x+y-10)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2xy^2 - \lambda = 0 \rightarrow 2xy^2 = \lambda$$

$$\frac{\partial \mathcal{L}}{\partial y} = 2yx^2 - \lambda = 0 \rightarrow 2yx^2 = \lambda$$

$\Big\}\ x = y$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x + y - 10 = 0$$

$$x + y = 10$$

$$y + y = 10$$

$$2y = 10$$

$$y = 5$$

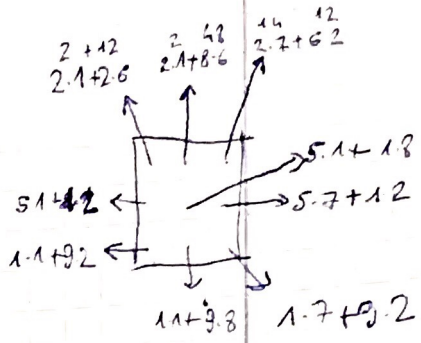
$$\boxed{x = y = 5}$$

4 Linear algebra

Problem 4.1.

$$A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$$

	1	1	7
	2	8	2
2 6	14	50	26
5 1	7	13	37
1 9	19	73	25

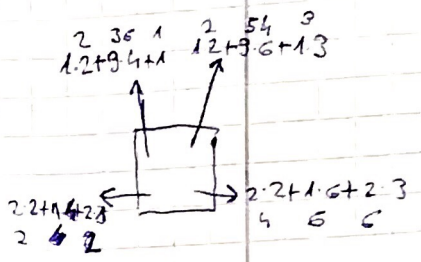


$$A \cdot B = \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

Problem 4.2.

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

	2	2
	4	6
	1	3
1 9 1	39	59
2 1 2	10	16



$$B \cdot A = \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

Problem 4.3.

$$A = \begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix} \qquad A^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

Problem 4.4

$$A = \begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix} \qquad \det(A) = 1 \cdot 8 - 9 \cdot 2 = 8 - 18 = \boxed{-10}$$

5 Probability theory

5.1 Problem

$$\Omega = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$\#\Omega = 36$$

2nd \ 1st	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

5.2 Problem

	Drug usage		Right	Wrong
+	1%	+	99%	1%
−	99%		99.5%	0,5% +

$$\mathbb{P} = 0.01 \cdot 0.99 + 0.99 \cdot 0{,}005 = 0.0099 + 0.00495 = 0.01485$$

1.485% $\mathbb{P} = 0.01485$

5.3 Problem

Drug usage + | Wrong 1% | Right 99% | − Wrong 0,5%

$$P = \frac{0.0099}{0.01485} = \frac{2}{3} = 0.6\dot{6}$$

~~Positive drug test already given therefore the correct answer is 99%.~~

0.01 · 0.99 + 0.99 · 0.005 = 0,00495 → 0.99 · 0.005

+wrong +right: 0.01 · 0.99

− false