Report: Model of the Longitudinal Dynamics of Hang-gliders on Matlab

by Kevin Garcia in April 2013

This model is based on Gary Valle study written in 1979 and called: “A Preliminary Analysis of the Longitudinal Dynamics of Ultralight Gliders”.

The aim of the model development described below was to reproduce his model and put in evidence the influence of the Cmo in the response of a glider and the influence of the damping to understand more the ‘tumbling’ phenomenon.

The purpose of this paper is to describe how the model is made of and to explain how to use it. Also, to present the landmark used to allow people to improve it after me and to present the plot we can have showing the different responses we can get by changing the glider, the initial conditions or the pilot behavior.

The main model consists of “Program.m” and “Motion.m”. To work, the model needs that both files are placed in the same repertory.

There is also a Matlab file called “Static.m” which is not necessary to use the model but necessary to do some adjustment or to improve this model.

These 3 Matlab files are open-source as well as this report.

I. Glossary

Symbol	Description	Dimension
Alpha stable	The last value of the simulation, supposed to be stable if the simulation lasts enough (exists for Gamma and V)	deg
α	(Alpha) Angle of Attack	deg
γ	(Gamma) Flight path angle	deg
θ	(Theta) Pitch Attitude	deg
ξ	(Xi) Angle between the control bar and the CGP	deg
φ	(Phi) Angle between the downtube and the perpendicular of the glider	deg
$\lambda_{c/4}$	(Lambda) Sweep Angle (used in Roskam formulas)	deg
mpilot	Weight of pilot+harness (Hook-in-weight)	kg
mwing	Weight of the wing	kg
I	Inertia of the glider and the pilot about the CGT. Equal to 102 according to Gary Valle report.	$kg.m^2$
ρ	(Rhô) Density of the air	kg/m^3
c	Mean chord	m
cgdist	Distance between the TP and de CGW	m
d	Distance between the AC and the CGT	m
Lb	Distance between the CGP and the downtubes (cf figure Landmark) Positive if forward. Negative if pilot backward.	m
tpdist	Distance between the AC and the TP	m
xCGT	X-axis value of CGT	m
yCGT	Y-axis value of CGT	m
Xw	Correspond to xCGT. (cf Roskam p51) . Distance d projected on the X-axis.	m
Glide angle	Glide angle taken on the two lasts point of the simulation (Output)	m/m
Vx	Horizontal Airspeed	m/s
Vy	Vertical Airspeed	m/s
S	Area of the Wing	m^2
Scx	Area time the drag coefficient of the pilot alone (depending on his posture)	m^2
Dcdg	Drag of the wing alone	N
Dpilot	Drag of the pilot alone	N
Drag	Drag of the system total (pilot + wing)	N
Lift	Lift of the wing alone	N
Pp	Force due to the weight of Pilot+Harness	N
Pw	Force due to the weight of the Wing alone	N
dyn	Dynamic pressure ($1/2 \times \rho \times S \times V^2$)	N
Mq	Damping due to the relative camber and the larger displacement of the wing chords due to wing sweep (cf Gary Valle report)	N.m or J
Mq2	Damping due the rotation of the wing around the low center of gravity (cf Gary Valle report)	N.m or J
Tr5	Tr5% : Time necessary to reach the range of +/-5% of the final value of alpha in the simulation (Output only)	sec
AC	Aerodynamic Center	
AR	Aspect Ratio	
Cdg	Drag coefficient of the wing alone	
CGP	Center of gravity of the Pilot	
CGT	Center of gravity of the system total (Pilot+Wing)	
CGW	Center of gravity of the Wing	
Claw	Spanwise average value of the wing section lift curve slope (slope of CL between 5 and 15 degrees of alpha)	
Clift	Lift coefficient of the Wing alone	
Cmo	Static Coefficient taken about the AC	
Cmq	Damping coefficient of Mq (cf Roskam p47)	
Cx	Projection of Dcdg in the absolute basis	
Cy	Projection of Lift in the absolute basis	
K	Correction Constant for Wing contribution to Cmq (Roskam)	
TP	Tether Point	

z(1)	U (Position)	m
z(2)	θ (Pitch Attitude)	deg
z(3)	V (Airspeed)	m/s
z(4)	γ (Flight Path Angle)	deg
z(5)	θ_dot (Pitch Rate)	deg/s
dz(1)	V (Airspeed)	m/s
dz(2)	θ_dot (Pitch Rate)	deg/s
dz(3)	Γ (Acceleration)	$m.s^{-2}$
dz(4)	γ_dot (Flight Path Angle Rate)	deg/s
dz(5)	θ_dot_dot (Pitch Acceleration)	$deg.s^{-2}$
alph0	Initial Angle of Attack Step 1	deg
u0	Initial Position (0) Step 1	m
a0	Initial Flight Path Angle Step 1	deg
b0	Initial Pitch Rate Step 1	deg/s
v0	Initial Airspeed Step 1	m/s
alph0_2	Initial Angle of Attack Step 2	deg
u0_2	Initial Position (0) Step 2	m
a0_2	Initial Flight Path Angle Step 2	deg
b0_2	Initial Pitch Rate Step 2	deg/s
v0_2	Initial Airspeed Step 2	m/s
T1	Duration of the simulation Step 1	sec
DT2	Duration of the simulation Step 2 if "Next"	sec
alphamin	Defines the domain we know Lift and Drag as a function of Alpha (degrees)	deg
alphamax	Defines the domain we know Lift and Drag as a function of Alpha (degrees)	deg

II. Equations of motions

$$\Gamma = -g * \sin(\gamma) - \frac{Drag}{M} \qquad (Eq.1)$$

$$\dot{\gamma} = \frac{1}{V}\left[-g * \cos(\gamma) + \frac{Lift}{M}\right] \qquad (Eq.2)$$

$$\ddot{\theta} = \frac{\Sigma Moments/CGT}{Inertia} + \frac{\Sigma Damping}{Inertia} \qquad (Eq.3)$$

ΣMoments about CGT includes : Moments from : **Cmo** , weight of the wing , weight of the pilot , Lift , Drag of the wing and Drag of the pilot.

ΣDamping includes : Damping due to the relative camber and the larger displacement of the wing chords due to wing sweep (**Mq**) , Damping due the rotation of the wing around the low center of gravity (**Mq2**)

Drag includes : Drag of the glider alone (**Dcg**) and the Drag of the pilot (**Dpilot**).

M : Masse total of the system glider+pilot (with harness)

$$Mq = Cmq * \frac{\dot{\theta} * c^2 * \rho * V * S}{4}$$

$$Mq2 = \frac{1}{2} * \rho * Cdg * S * (-2 * \dot{\theta} * d^2 * V + \dot{\theta}^2 * d^3)$$

$$Cmq = -K * Clwa * \cos(sweep) * \left[\frac{AR * \left(2 * \left(\frac{Xw}{c}\right)^2 + \frac{1}{2} * \left(\frac{Xw}{c}\right)\right)}{AR + 2 * \cos(sweep)} + \frac{1}{24} * \frac{A^3 * \tan^2(sweep)}{AR + 6 * \cos(sweep)}) + \frac{1}{8}\right]$$

K , Cmq , Clwa are defined in *Methods for Estimating Stability and Control Derivatives for Standard Subsonic Airplanes* (1973) from Roskam , pages 47 and 51.

All the angles in input and output of the model are expressed in degrees, but they are used in radians in the equations of motions by Matlab. That is why they are transformed just after the input phase and just before the output phase in the Matlab program.

Lift and Drag laws :

Lift and Drag laws of the wings come from 1-g curves approximations (made on Excel).

To try to take the aeroelasticity of the wing into account, during a test vehicle, we don't just make change the angle of attack and note the **Clift** and **Cdrag** corresponding. We extract a lot of points with different airspeeds and angles of attack and the **Clift** and **Cdrag** coefficients.

Then, we only consider points where the Resultant (Drag + Lift) is equal to 1-g .For a pilot+wing with a total mass of 120 kg , this resultant must equal 120*9.81 to consider the measure of **Cdrag** or **Clift** according to **Alpha**. Only a few points remain after this sorting. We plot **Clift** (1-g)/**Alpha** and **Cdrag** (1-g)/**Alpha** on Excel and we ask him to do a polynomial approximation. The polynomials we get are the laws entered in the gliders data in **Motions.m** file.

Cmo :

We assume that the **Cmo** in this model is constant in the Aerodynamic center. This assuming doesn't take into account the variation of the **Cmo** due to variation of airspeed (so loads).

Inertia:

The inertia **I** of 102 kg.m² has been established by Gary Valle. It's hard to define it experimentally and several simulations lead us to think that the influence of this parameter is neglectable in a first approach. So, the same inertia is considered for all gliders in this model but it can be easily modified in "Motions.m" (Matlab File).

Pilot postures:

The estimations of the **Scx** due to the pilot in prone and stand position come from: "Manuel du vol libre" from Alain Jacques and Paul Menegoz (page 25).

The differences between the model developed here and the model developed by Gary Valle in 1979 are :

-In this model , the drag of the pilot is taken into account in the Drag resultant (Eq.1)and in the moment generated by this force in the Pitch Rate equation (Eq.3) because this force is applied on the Pilot (CGP).
-Lift and Drag laws of the gliders used are different from those used by Gary Valle.
-The Cmq from Mq , is calculated here in real time. It seemed to be a constant in Gary Valle study.
-In the calculation of Cmq, he seemed to consider Xw equal to 0. Here, we calculate it according to the Roskam formulas (the same as the ones used by Gary Valle).

These are the differences we are almost sure of. A lot of things he assumed remains unknown like the exact landmark, the position of CGT (also CGP and CGW) considered, CGT variable or constant, or the hang strap length considered.

III. Landmarks used

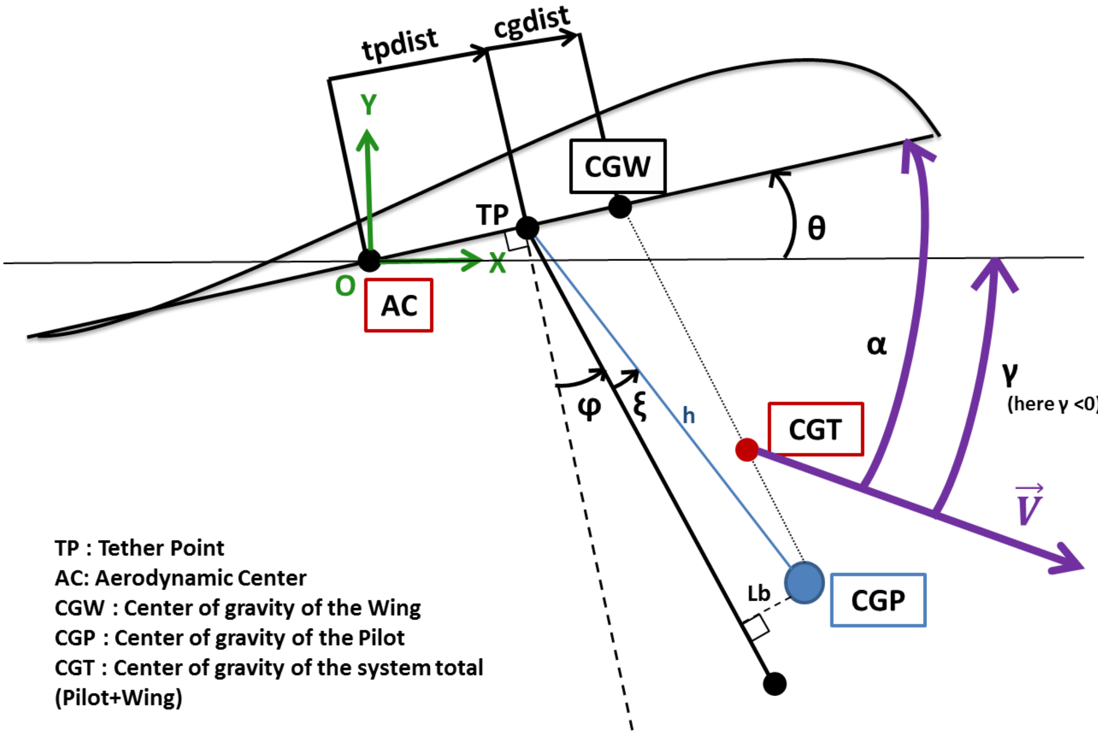

Figure 1 : Global Landmark

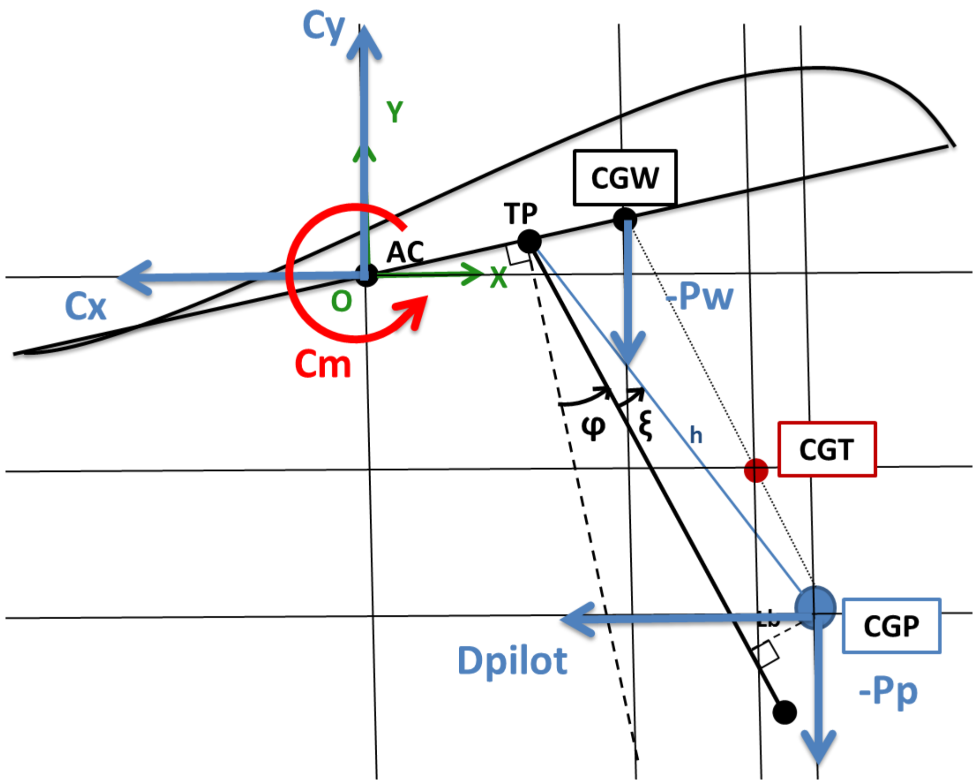

Figure 2: Forces and moments considered

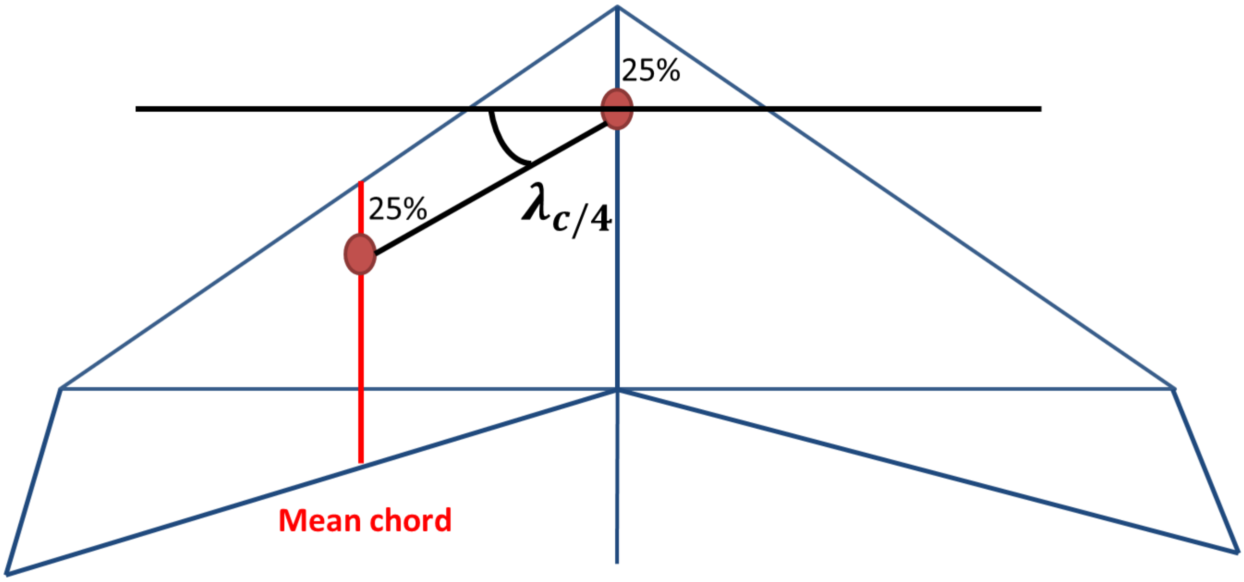

Figure 3 : Sweep angle estimation

IV. How to use the model

The model is made of "Program.m" and "Motion.m". To work, the model needs that both files are placed in the same repertory.

In **Program.m**: we have the initial conditions, the inputs about the Pilot and the glider chosen, the plot commands and the command to export into spreadsheets (Excel).

In **Motion.m**: we have the laws of behavior of the different gliders known, their parameters, the equations of motion (dz). So, all the forces and moments are calculated in this file

Until you need to change the Wing parameters, all you need to modify to do a simulation is in Program.m.

Trim means that the pilot remove his hands from the control bar, just hung. If Trim is selected, the model doesn't take into account the length Lb and the angle ξ induced.

Active means that the pilot maintains the distance Lb between his center of gravity and the downtubes (perpendicularly).

Prone means the pilot is in prone position, which provides the lowest drag from the pilot.

Stand means the pilot is standing up, which provides the highest drag from the pilot.

We can consider the stability reached once the Flight Path Angle, the Airspeed and the Angle of Attack are all constant.

A. 1 Step Mode

This mode is the simplest form. It can be used to do a simulation in one configuration with one glider.

This mode is active if: **Step2 = 'N'** (line 42 of Program)

Adapt the initial conditions and pilot parameters in this order:

```
%Initial conditions (Step 1)
        alph0=5;%angle of attack initial en degrees (alph=a+f)
        u0 =0 ; %initial position (0)
        a0 =-80; %flight path angle in degrees Gamma
        b0=0 ;%Pitch rate attitude in degrees/sec² Theta dot
        v0=5;%initial airspeed in m/s

timespan=0.1;
T0 =0;
T1 =10;% in seconds
```

T1 defines the duration of the simulation.

There is no limit of duration of the simulation. Nevertheless, except in **Trim** position, the stability is reached before 15 seconds.

```
%Parameters
        %Pilot
        mpilot=90;% kg (including harness and clothing = Hook-in weight)
        h=1.2; %in m (hang strap length)
        Lb=0.4; %0.4 if forward , -0.74 if backward :distance between th
        Pil_act='Active';% Choose Trim , or Active to make the pilot con
        Pil_pos='Prone'; % Choose Stand or Prone

        Glider='Falcon';%Falcon or T2C
```

The **Lb=0.4 m** for full forward position and **Lb=-0.74 m** for full backward position are given for information purposes. They can vary with the pilot and the glider.

The export the result matrix (zdeg) to spreadsheets is possible if :

export='Y'

And don't forget to choose a filename:

filename=['Forward_fly','.xls']

The Excel file created is in the repertory of Matlab model.

B. 2 Steps Mode

This mode is active if: **Step2 = 'Y'** (line 42 of Program) and allows you to do a simulation in two different modes: **Parallel** and **Next** Modes

a. Parallel Mode

Active if **Conf_step2='Parallel'**

In this mode, the export to spreadsheet is not available.

Give a name to the steps to see their name on the plots and distinguish them.

```
Name_step1='Falcon Forward recovery';
Name_step2='Falcon Backward recovery';
```

Step 1 will be displayed in Blue. Step 2 will be displayed in Red.

This mode is used to plot two configurations with the same initial conditions (by default) but not necessarily observe the difference between two gliders or two pilots' behaviors.
By default, the both parallels simulations last the same time. The time defined by changing T1 in Step 1.

In Initial conditions of Step 2 / case 'Parallel' (line 68):

```
alph0_2=alph0;%angle of attack initial in degrees (alph=a+f)
u0_2 =u0 ; %initial position (0)
a0_2 =a0; %flight path angle in degrees Gamma
b0_2=b0 ;%Pitch rate attitude in degrees/sec² Theta dot
v0_2=v0;%initial airspeed in m/s
```

b. Next Mode (One after the other)

Active if **Conf_step2='Next'**

This mode is used to do a simulation where the behavior of the pilot changes, or the conditions of flight are modified after a certain lap of time. So, the Step 1 is simulated, and the lasts conditions of this simulation are reuse as initial conditions of the Step 2 except the value of **Lb** or initial conditions you changed (a gust from the rear would make decrease the airspeed).

The initial conditions to change for the Step 2 / case 'Next' are from line 90 of Program. You can change all the parameters of flight below.

```
case 'Next'

    alph0_2=(zdeg_1(r_1,2)-zdeg_1(r_1,4));%angle of attack initial in
    u0_2 =zdeg_1(r_1,1) ; %initial position (0)
    a0_2 =zdeg_1(r_1,4); %flight path angle in degrees Gamma
    b0_2=zdeg_1(r_1,5) ;%Pitch rate attitude in degrees/sec² Theta dot
    %b0_2=-90; %Pitch rate if gust
    v0_2=zdeg_1(r_1,3);%initial airspeed in m/s
    %v0_2=5;  % Airspeed if rear gust

            %Transformation in radians
            alphr0_2=alph0_2.*pi./180;
            ar0_2=(a0_2.*pi)/180;%flight path angle in radians
            fr0_2=alphr0_2+ar0_2;% pitch attitude in radians/sec Theta
            br0_2=b0_2.*pi./180;%Pith rate attitude in radians/sec²

    DT2 =8;   %How long Step 2 lasts
    T0_2=T1;
    T1_2=T0_2+DT2;

    Lb=-0.5; %0.4 if forward , -0.74 if backward :distance between the
    Pil_act='Active';% Choose Trim , or Active to make the pilot contr
    Pil_pos='Prone'; % Choose Stand or Prone
```

V. Plots

3 figures are produced for every simulation, whatever the mode (1 Step, 2 Steps Parallel or 2 Steps Next).

The first called Dashboard is an overview of the evolution of all useful parameters.

The second presents the evolution of Alpha as a function of time and display the $Tr_{5\%}$ calculated about the last value of alpha of the simulation. We can consider this value of $Tr_{5\%}$ only if the stability is reached at the end of the simulation.

The third is the view of the trajectory where X and Y-axis have the same scale.

The results are reliable if the following message is displayed in the command windows on Matlab: **"Alpha remains in the domain we know the behavior law in Step x"**

If: **"Warning: During the simulation, Alpha from Step x is out of the domain we knows the behavior law"** The results need to be analyzed seriously before concluding anything. This message may reveal a pitchover phenomenon or a stall for instance.

The simulations below are examples of what we can get in every simulation mode.

A. 1 Step Mode

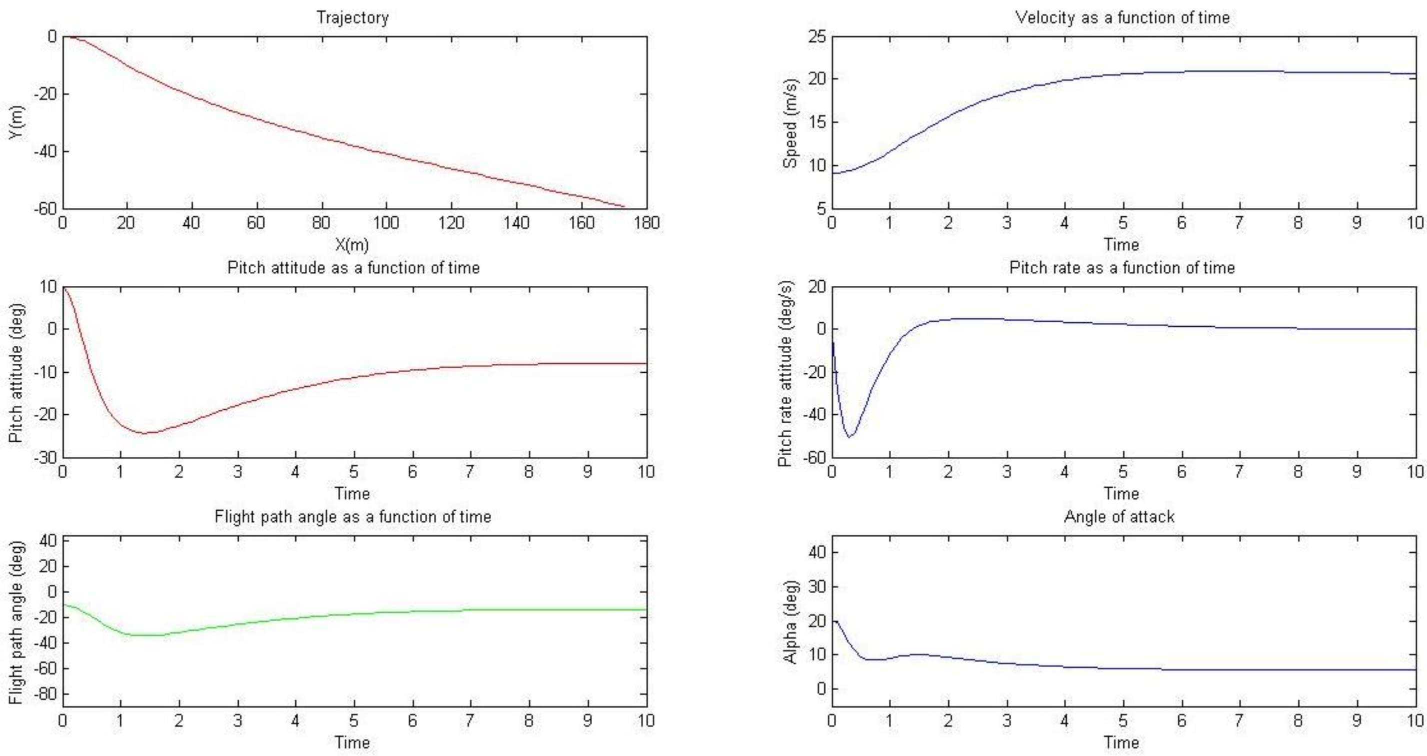

Figure 4 : 1 Step Mode example of Dashboard (Forward)

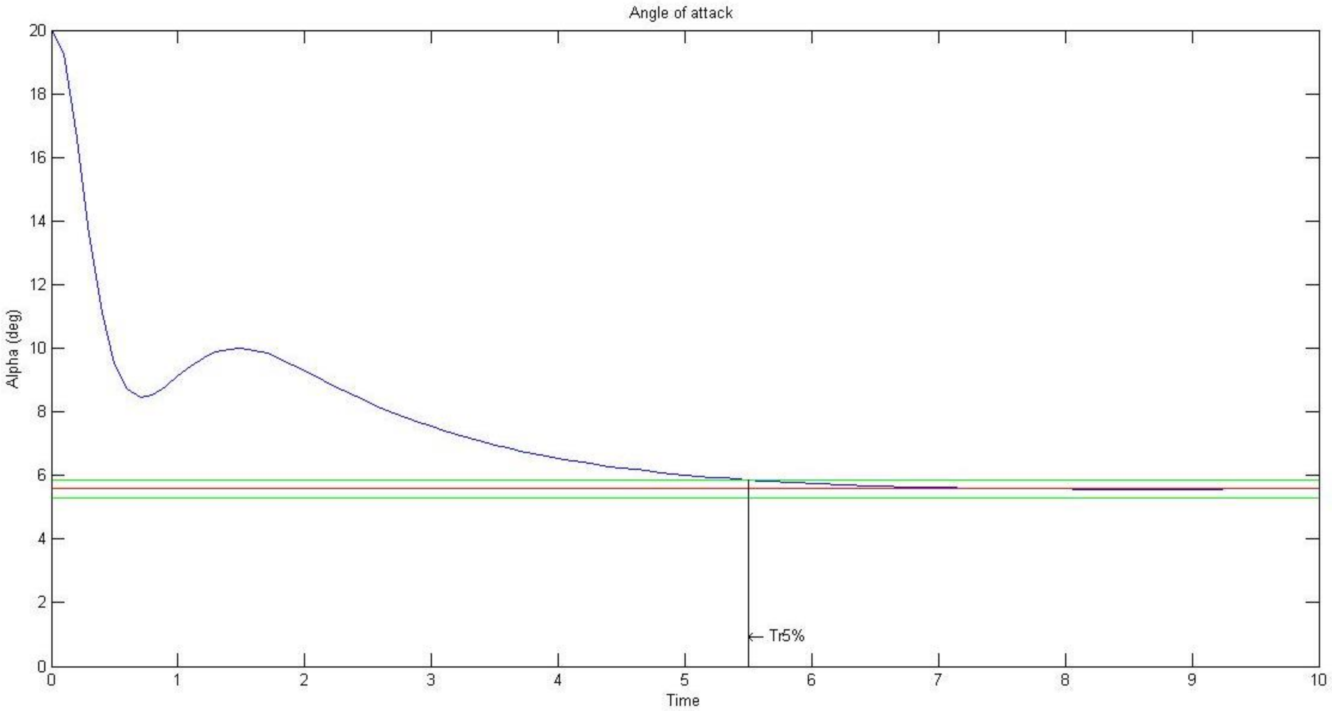

Figure 6 : 1 Step Mode example of Alpha as a function of time (Forward)

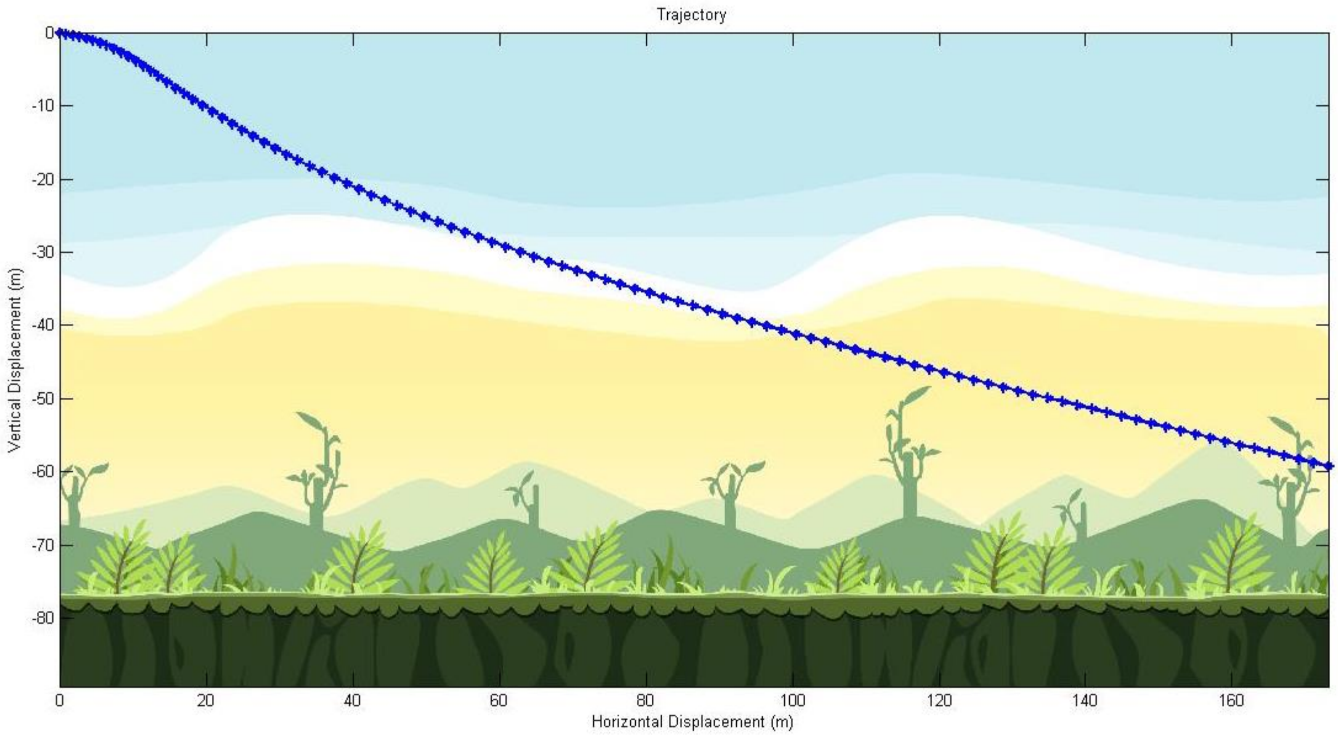

Figure 5 : 1 Step Mode example of Scaled Trajectory

B. 2 Steps Mode

a. Parallel Mode

The differences between the two steps simulated are:
Falcon Forward behavior: Lb= 0.4
Falcon Backward behavior: Lb=-0.4
It means that in Step 1 the pilot pulls full forward during all the simulation and in Step 2 he pushes half of full backward during all the simulation.

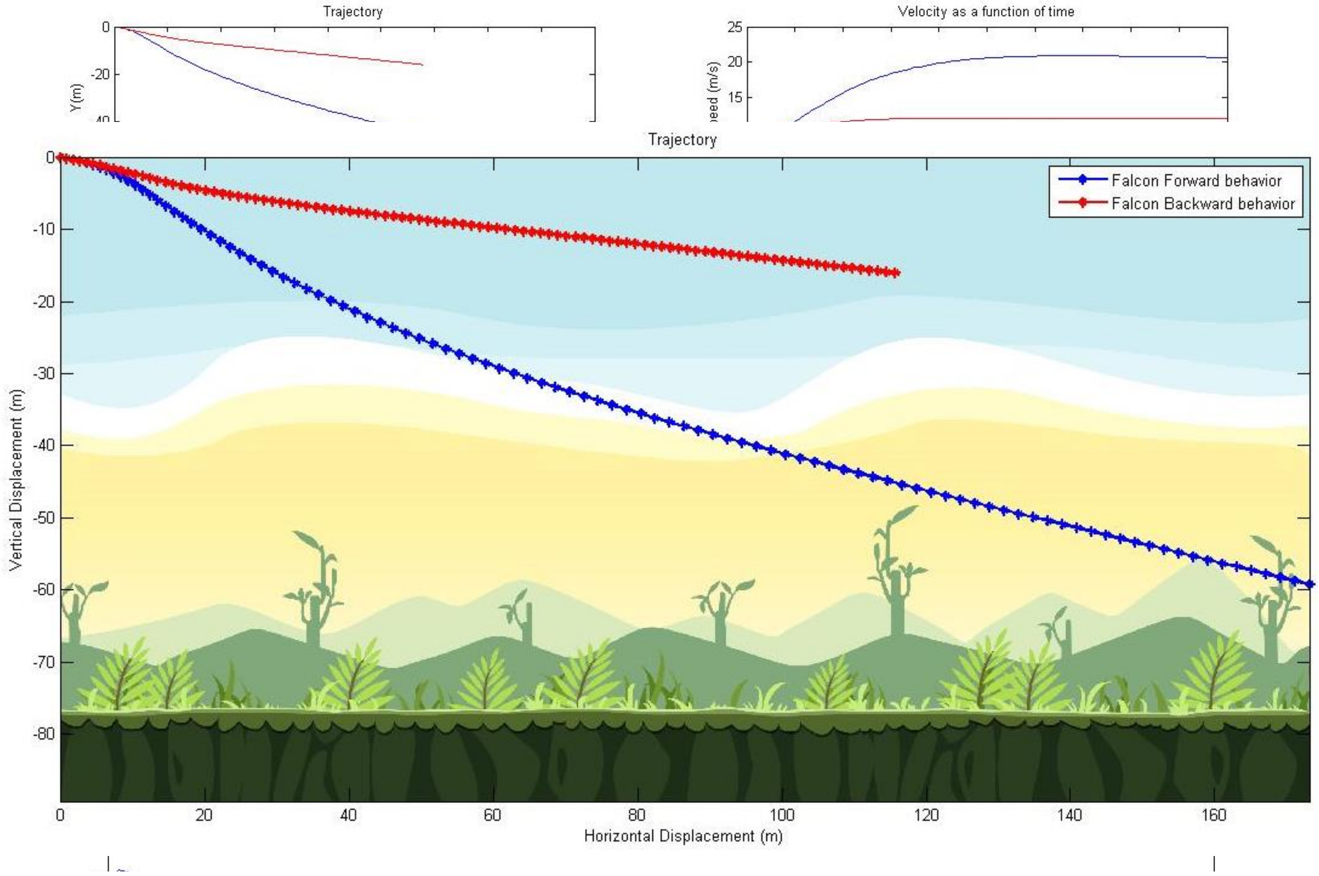

Figure 9 : 2 Steps Parallel Mode example of Scaled Trajectory

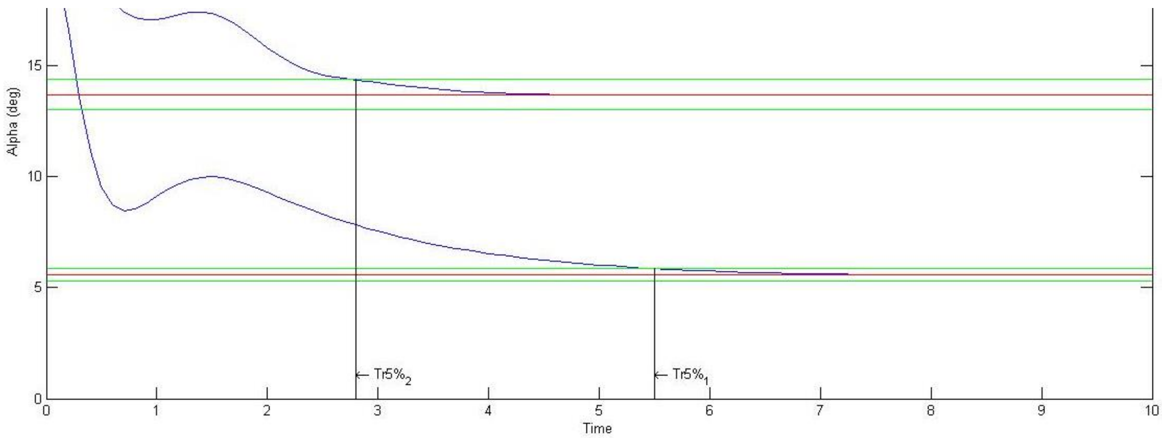

Figure 7 : 2 Steps Parallel Mode example of Alpha as a function of time

b. Next Mode

The differences between the two steps simulated are:
Falcon Forward behavior: Lb= 0.4
Falcon Backward behavior: Lb=-0.4 after the 8 seconds of Step 1 and lasts 8 seconds.

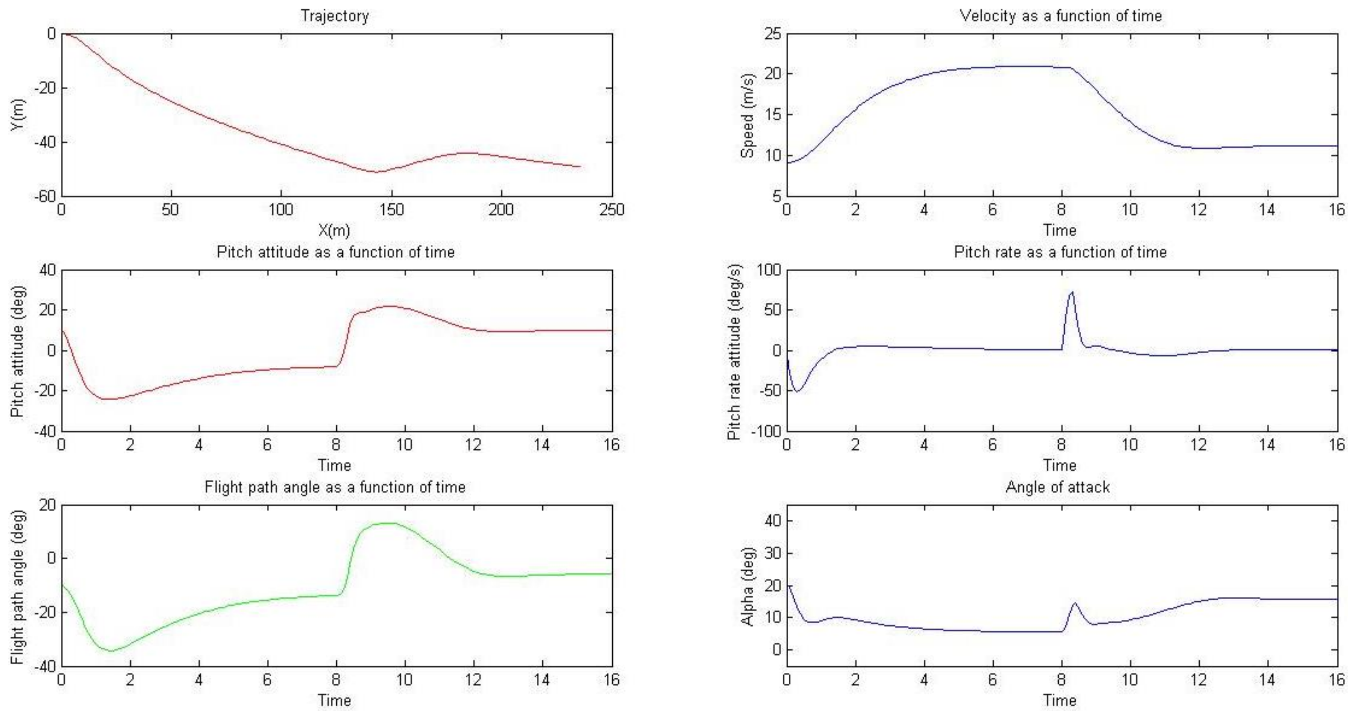

Figure 10 : 2 Steps Next Mode example of Dashboard.
8 seconds Full forward, then, 8 seconds half backward (Lb=-0.4)

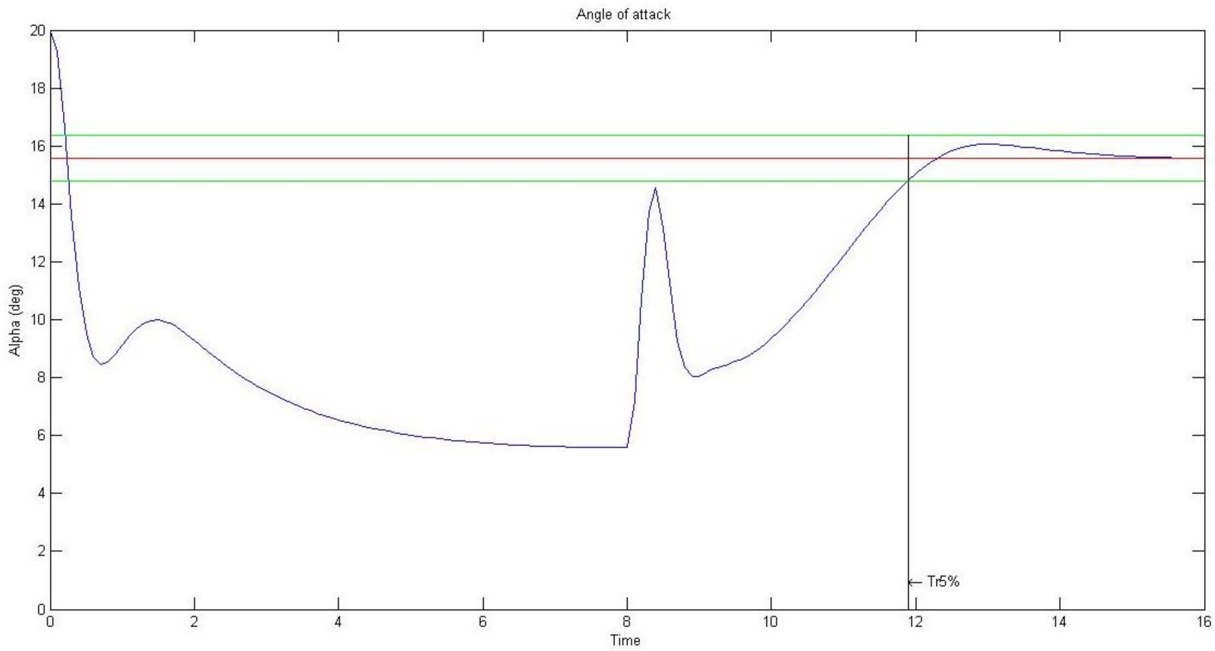

Figure 12 : 2 Steps Next Mode example of Alpha as a function of time.
8 seconds Full forward, then, 8 seconds half backward

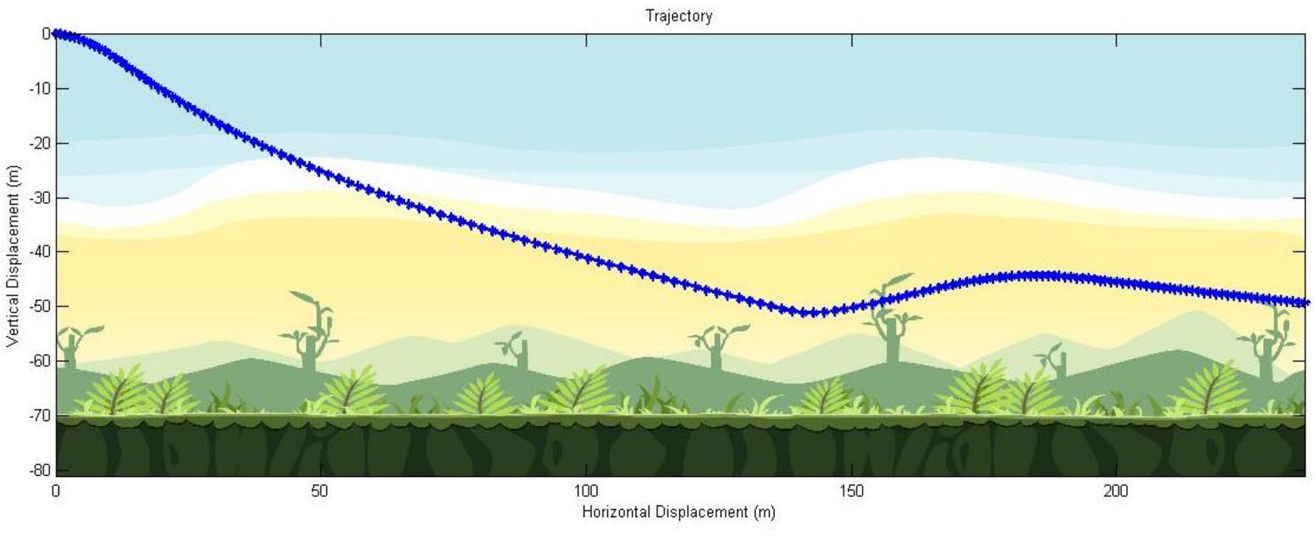

Figure 11 : 2 Steps Next Mode example of Scaled Trajectory.
8 seconds Full forward, then, 8 seconds half backward

VI. How to use Static.m

Static.m is necessary to test any modification in the equation of motion n°3 (called Sum of Moments).
If the system is stable in real, the Sum of Moments curve must cross the X-axis with a negative slope.

Whatever the improvement brought to this model, the sum of Moments provided by" Static.m" must present this shape at **Trim**. (cf below).

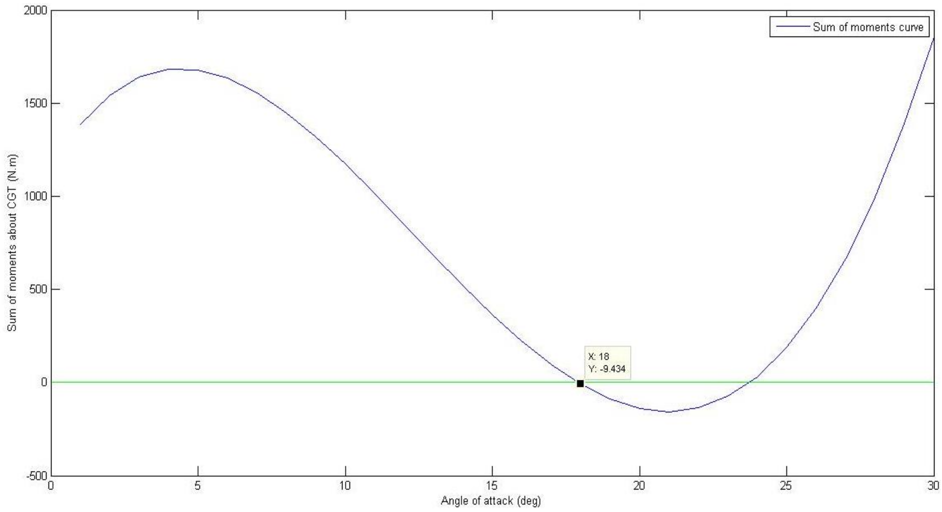

Figure 13 : Sum of Moments curve (Static) as a function of Alpha showing the Angle of Attack of the glider in stability at Trim and Prone position.

If you manage to get the laws **Clift** and **Cdrag** of a new glider and you know the distance between the Tether Point and the Center of Gravity of the Wing by localizing both points, but you don't know the position of the **Aerodynamic center**. You need to know the angle of attack of the glider at **Trim** position once the glider is **stable**. Then, by changing the parameter **tpdist** until the Sum of Moments curve crosses the X-axis at the value of $\boldsymbol{\alpha_{trim}}$ you know.

So, this curve shows the α_{stable} and if the model is well adjust to a glider and pilot , α_{stable} matches with $\boldsymbol{\alpha_{trim}}$ in a the same configuration (same posture of the pilot for instance)

The only way to know physically the position of the Aerodynamic Center of a rigid wing is to use a test vehicle and to change the position of the Tether Point until the **Cm** calculated from the Moment and the dynamic pressure becomes constant. On a flexible wing, because of the aeroelasticity this method becomes less right.

Reminder:

Trim means that the pilot remove his hands from the control bar, just hung. If **Trim** is selected, the model doesn't take into account the length **Lb** and the angle **ξ** induced.

Active means that the pilot maintains the distance **Lb** between his center of gravity and the downtubes (perpendicularly).

Prone means the pilot is in prone position, which provides the lowest drag from the pilot.

Stand means the pilot is standing up, which provides the highest drag from the pilot.

VII. Figure summary