

Assignment 1 Guide

BIOL4062/5062: Analysis of Biological Data

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Introduction

Welcome to the assignment guide for BIOL4062/5062: Analysis of Biological Data.

This website is designed to walk you through the assignments for this class. It is a resource to help you figure out how to code for your assignments, and bring attention to key questions to ask yourself as you interpret your results, both statistically and biologically. Keep in mind that there are always different ways to get to the right answer with coding. The content here is not a monolith. You don't need to follow it if you don't want to (more in Assignment Guidelines), but make sure what you are doing is clear and sufficiently analogous to this guide, else you lose marks for being unclear, or running the wrong analyses.

Getting Help

Don't suffer in silence! If you need help on the assignments, there are multiple options available to you:

- Assignment Drop-In Sessions:
 - TA-run in-person help sessions the week before each assignment is due.
 - Optional, but recommended
 - * Even if you don't have questions, you may benefit from hearing the questions of others
- BrightSpace Discussion Board:
 - Feel free to ask questions on the BrightSpace discussion board, or peruse questions already asked
 - * You may find your answer without even asking!
 - Make sure to start your question with the assignment number
- Email
 - Feel free to ask questions, or set up an appointment

- Direct Assignment 1 questions to the TA, and Assignment 2 and/or class administration (e.g. extension requests) to the instructors

Without further ado, let's get into it!

Assignment Guidelines

This section provide general advice on how to approach your assignments, and also describes how to format them:

General Advice

1. Read the grading rubric carefully!

- It is designed to be as objective as possible. There is little latitude for part marks if you are missing things that are listed.

2. You don't need to describe the statistical theory (unless it's relevant to your answers)

- All you have to do is answer the questions in the assignment. Anything you write outside of that is just eating up your page limit.

3. However, biological interpretations are important: put them together at the end

- Your interpretations are more likely to make sense and be easier to mark if you put them at the end, including all of your results together in them, rather than inserting them throughout piecemeal.
- **Also, make sure your biological interpretations are consistent with your data/results! They don't have to be correct, but they do have to match your data.**

4. Make sure your figures are readable

- We can't tell if your interpretation of your figures is correct if we can't interpret your figures.
- All text on figures should be legible.
- If you use color, make sure that the colors you use are clearly distinguishable (and consider colour-blind safe palettes)

5. You're not alone!

- If you have questions, come to the drop-in sessions, read the Brightspace discussion boards, or email the TA to ask questions or set up a meeting if those options don't work for you.
- Do the first two (drop-in sessions, Brightspace discussion boards) even if you don't have questions: you may find the answer to questions you didn't know you had.
- You're also welcome to ask questions after an assignment has been graded, if you want to know why you were graded the way you were, or if you have questions about the comments provided or what you may have done wrong.

6. Ask if you need an extension

- We're pretty reasonable.

Submission Formatting

Please format your submission following these guidelines: it will make both our lives easier.

1. Hand in your assignment in 3 parts:

a) Your assignment text

- All assignment 1s have a 2 page limit. Put all your text first, with figures and tables together separately at the end. It is easier for the TA to tell how many pages you wrote this way. You're not going to lose marks if you're slightly over (this is not a writing class).
- Should be a word doc or PDF file so it can be opened in BrightSpace. It doesn't matter whether it's produced through word, markdown, etc, as long as it's in one of those two formats.

b) Your script, submitted as a .txt file

- Submitting as a .txt file allows us to open the script in BrightSpace rather than having to download it if it's a .R file. Please don't paste your script in your assignment document.

c) The data you read into your script

- This is just to make it easy to run your script if there is a mistake.

Note: If you do your assignments in markdown, you can combine a) and b)

2. Don't put your name on your assignments, in your scripts, or in any of your file names

- The BrightSpace system of using Banner ID numbers is anonymous so your assignments are marked blind by the TA. That doesn't work if you write your name.
- Delete your file paths in your script for submission if they have your name in them.
- **You do need to put your B0 number, data code number, and whether you're a graduate student or an undergraduate student.**

3. Follow the assignment guides on this site

- You don't have to follow this guide to get full marks on the assignments (as always, there are many correct answers when it comes to statistics and coding). That said, it's easier to follow what you're doing if you're doing the same thing as everyone else.
- It's OK to do your own thing, but if you make a mistake, it's going to be much harder to help you out, and it's going to take significantly more effort to mark.

Part I

Assignment 1

1 Assignment 1a: Principal Components Analysis

Assignment 1a focuses on Principal Components Analysis (PCA). Think of PCA as a method of finding associations between data series.

For this tutorial, we're going to use the dataset in `fishcatch.csv`.

1.1 Looking at the Data

With any data analysis, step 1 is always to look at your data:

```
# Load in data
data = read.csv('fishcatch.csv')

# View data structure
head(data)
```

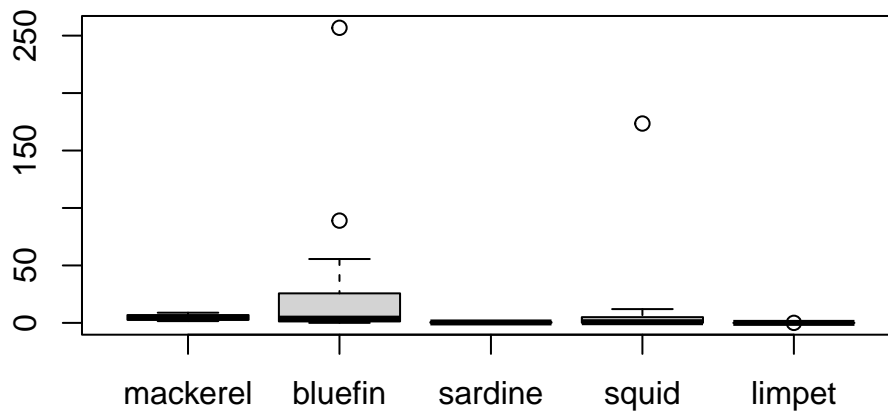
| | Hauls | mackerel | bluefin | sardine | squid | limpet |
|---|-------|----------|---------|---------|-------|--------|
| 1 | 1 | 1.851 | 55.60 | 0.058 | 6.00 | 0.0004 |
| 2 | 2 | 1.925 | 1.20 | 0.252 | 0.08 | 0.0027 |
| 3 | 3 | 2.506 | 1.56 | 0.133 | 0.06 | 0.0015 |
| 4 | 4 | 1.537 | 30.00 | 0.064 | 9.35 | 0.0013 |
| 5 | 5 | 1.795 | 0.04 | 0.086 | 4.70 | 0.0022 |
| 6 | 6 | 3.371 | 45.00 | 0.078 | 7.66 | 0.0006 |

```
dim(data)
```

```
[1] 25  6
```

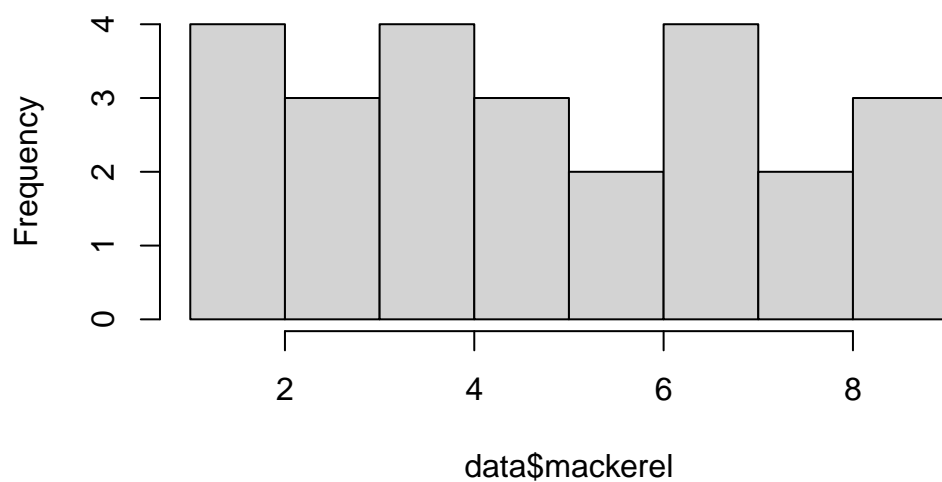
Our data is a 25 row, 6 column data frame, describing catch of 5 different fisheries species (columns 2-6) caught across 25 hauls (column 1). We want to know if certain species are associated with each other. Lets look a little deeper at the data:

```
# Generate boxplots  
boxplot(data[,-1]) # Exclude haul
```



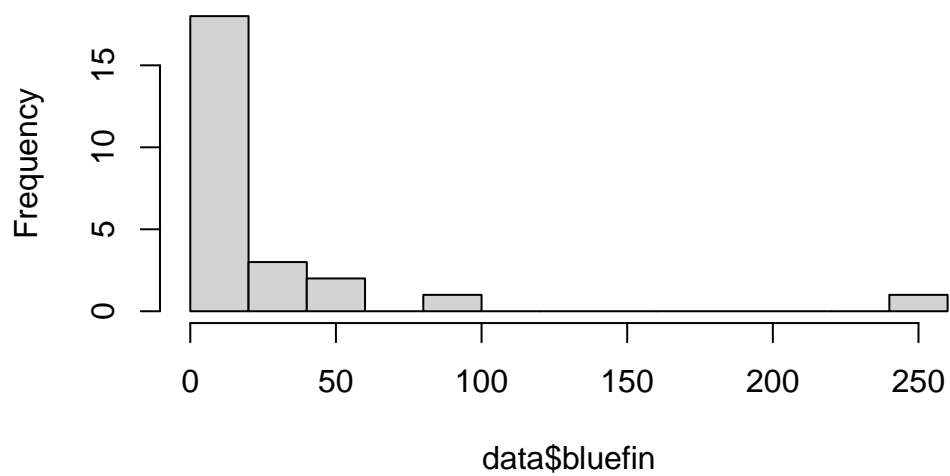
```
# look at data distribution  
# par(mfrow = c(3,2)) # 1 column 5 row grid plot  
hist(data$mackerel, breaks = 10)
```

Histogram of data\$mackerel



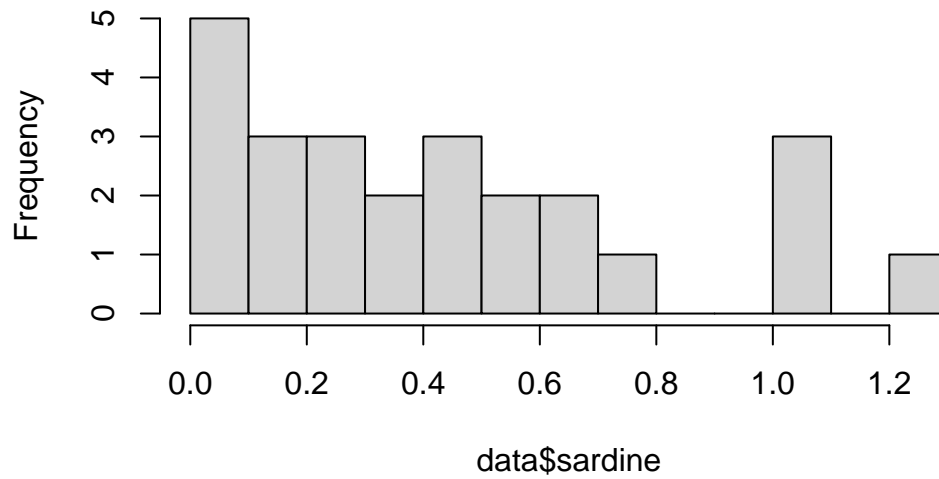
```
hist(data$bluefin, breaks = 10)
```

Histogram of data\$bluefin



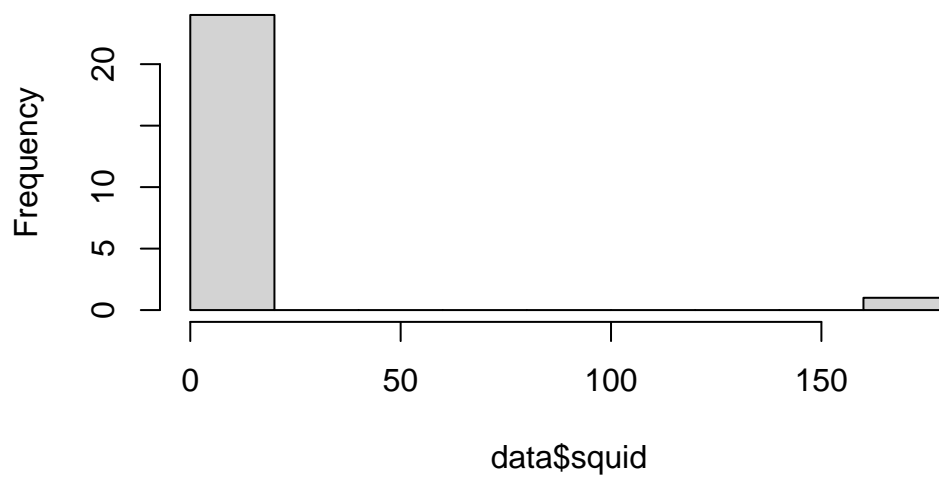
```
hist(data$sardine, breaks = 10)
```

Histogram of data\$sardine

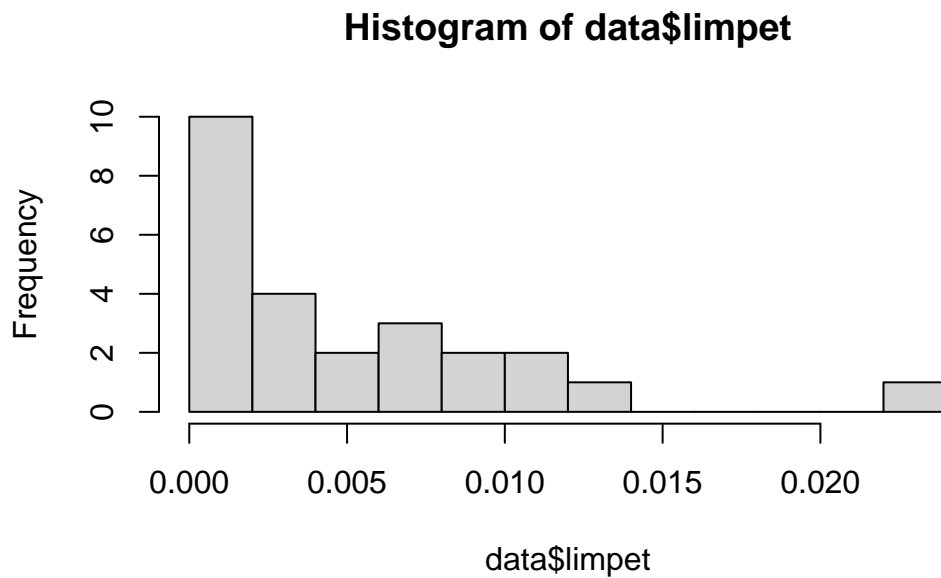


```
hist(data$squid, breaks = 10)
```

Histogram of data\$squid



```
hist(data$limpet, breaks = 10)
```



A few things are immediately obvious from looking at our data:

1. There are some large outliers
2. The data scales vary greatly across species
3. The species all have relatively different distributions, none of which look normal.

Are these issues? How do we fix them?

1.2 Transformations

Look back at the PCA lecture. What are potential problems with PCA?

1. Covariance Matrix PCA requires data to be in the same units
2. Normality is desirable, but not essential
3. Precision is desirable, but not essential
4. Many zeroes in the data

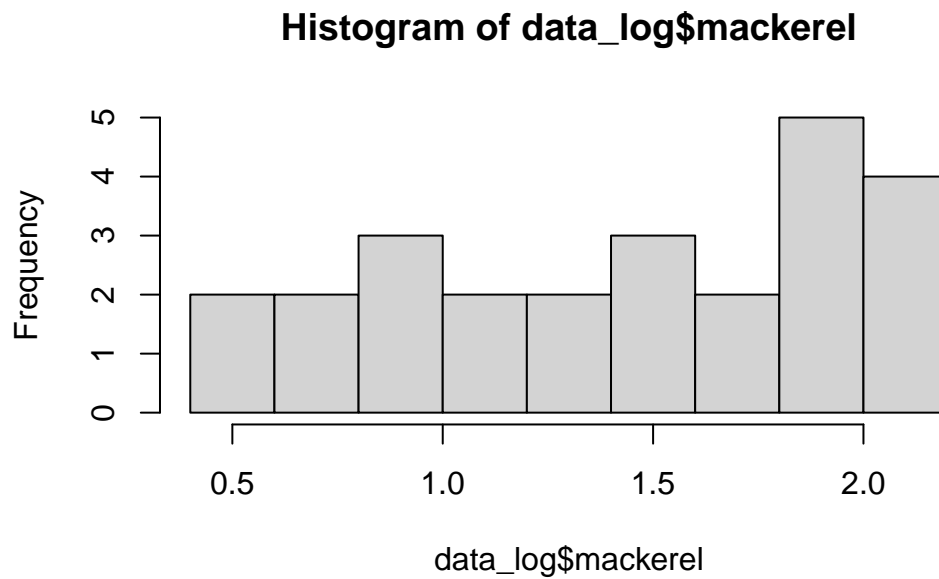
We can fix issue 1 by logging our data:

```
# Create a new data object so we can log the data
data_log = data

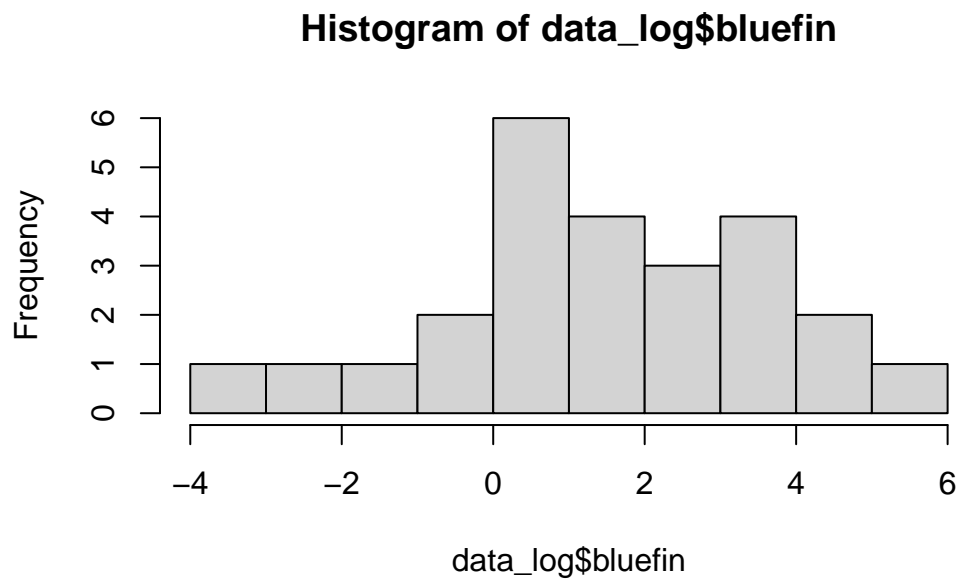
# Log data
data_log[, -1] = log(data_log[, -1]) # Remember to exclude haul
```

Now that we've transformed the data, let's check for normality again:

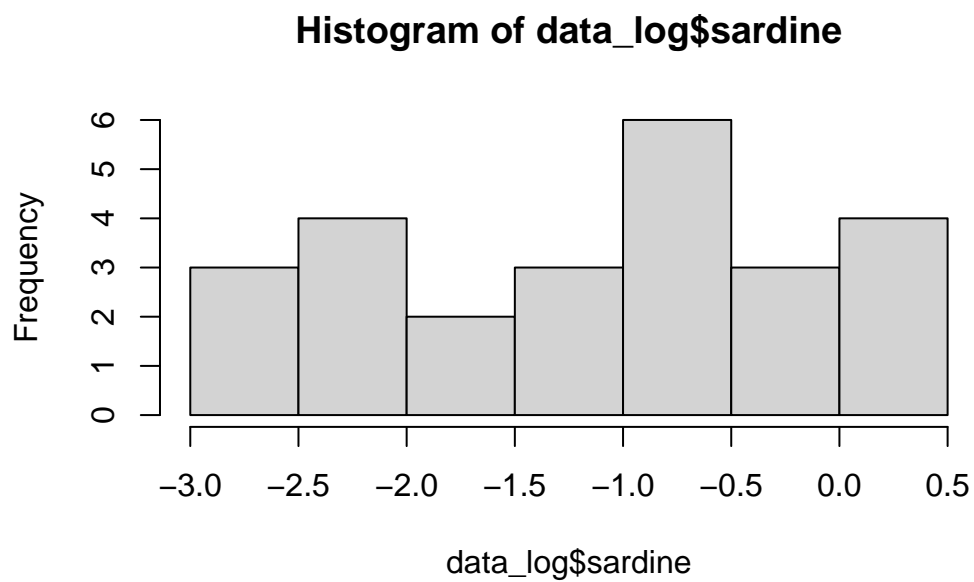
```
# Generate histograms
# par(mfrow = c(3,2)) # 1 column 5 row grid plot
hist(data_log$mackerel, breaks = 10)
```



```
hist(data_log$bluefin, breaks = 10)
```

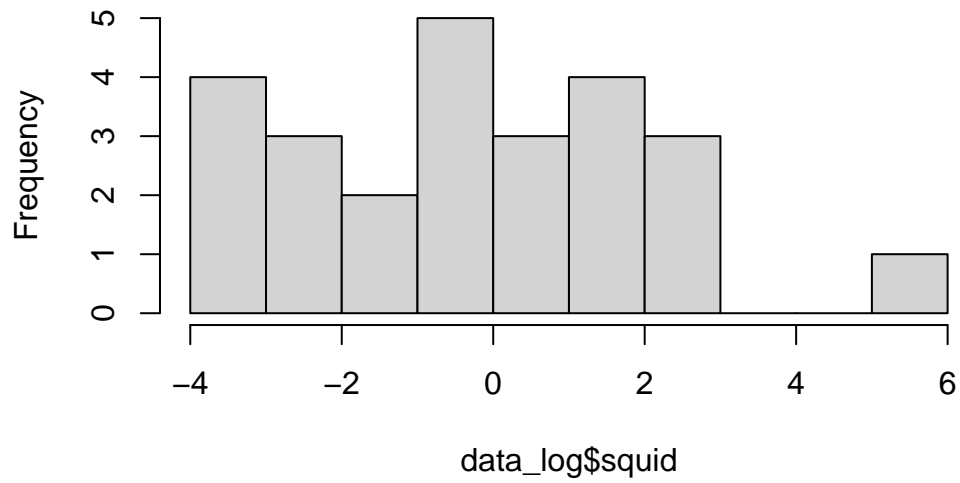


```
hist(data_log$sardine, breaks = 10)
```



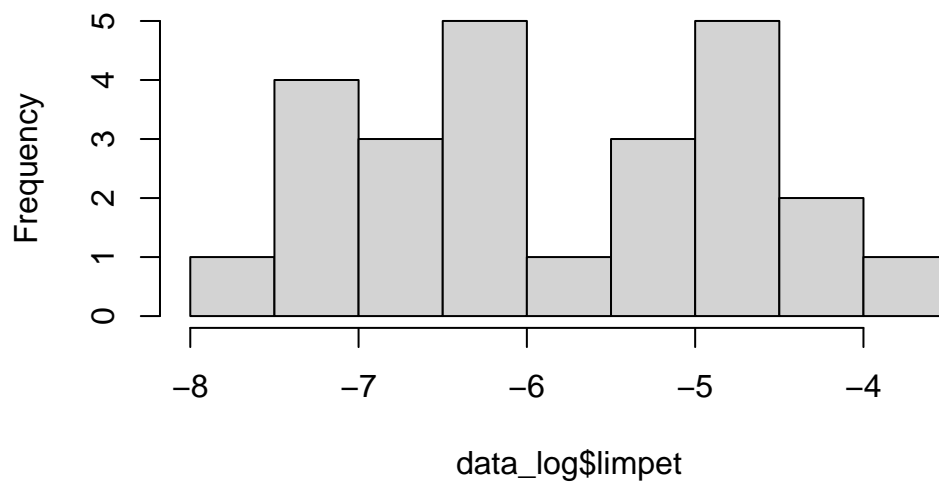

```
hist(data_log$squid, breaks = 10)
```

Histogram of data_log\$squid



```
hist(data_log$limpet, breaks = 10)
```

Histogram of data_log\$limpet



These look much better. We can also confirm this statistically:

```
# Generate histograms  
shapiro.test(data_log$mackerel)
```

Shapiro-Wilk normality test

```
data: data_log$mackerel  
W = 0.9425, p-value = 0.1691
```

```
shapiro.test(data_log$bluefin)
```

Shapiro-Wilk normality test

```
data: data_log$bluefin  
W = 0.98186, p-value = 0.9193
```

```
shapiro.test(data_log$sardine)
```

Shapiro-Wilk normality test

```
data: data_log$sardine  
W = 0.94113, p-value = 0.1572
```

```
shapiro.test(data_log$squid)
```

Shapiro-Wilk normality test

```
data: data_log$squid  
W = 0.96226, p-value = 0.4613
```

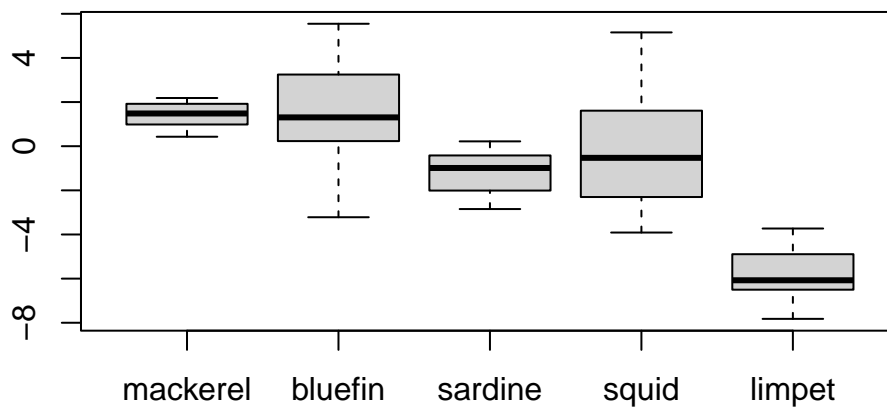
```
shapiro.test(data_log$limpet)
```

Shapiro-Wilk normality test

```
data: data_log$limpet  
W = 0.96437, p-value = 0.5082
```

All 5 species fail to reject the null hypothesis that the data are normally distributed. Logging the data also helps deal with the outliers:

```
# Generate boxplots  
boxplot(data_log[, -1])
```



Note that we can only log the data if there are no zeroes:

```
# Generate test data  
data_test = data; data_test[1,6] = 0 # Change the first limpet value to 0  
  
# Try to log the data  
data_test[1,] # Print first row
```

```
Hauls mackerel bluefin sardine squid limpet  
1      1    1.851    55.6    0.058      6      0
```

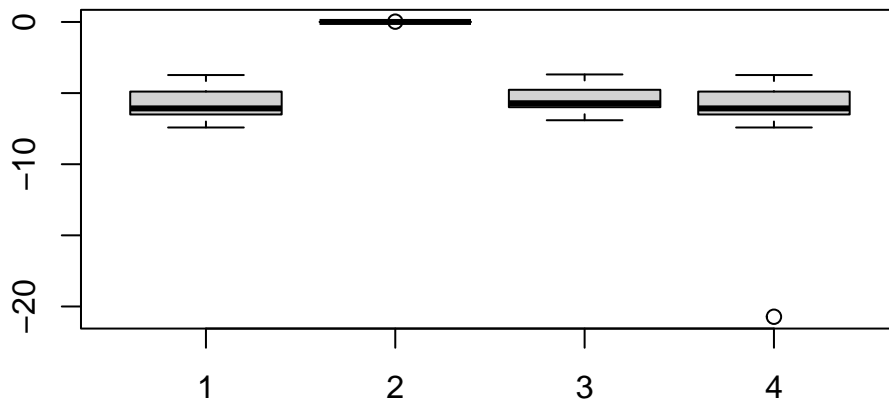
```
log(data_test[,-1])[1,] # Print logs of the first row
```

```
   mackerel bluefin  sardine  squid limpet  
1 0.615726 4.018183 -2.847312 1.791759  -Inf
```

`log(0)` returns negative infinity. That's going to be a problem later in our analysis. We can fix that by adding a small increment before taking the log. Keep in mind though that each species has a different magnitude in this dataset, and adding an inappropriate increment could cause us trouble later:

```
# Test boxplots of different increments  
boxplot(log(data_test$limpet), # Warning because of -Inf  
        log(data_test$limpet + 1),  
        log(data_test$limpet + 0.001),  
        log(data_test$limpet + 0.000000001))
```

```
Warning in bplt(at[i], wid = width[i], stats = z$stats[, i], out =  
z$out[z$group == : Outlier (-Inf) in boxplot 1 is not drawn
```



If the increment is too big, we eliminate the variance in our data. If the increment is too small, we create an outlier.

1.3 Running PCA

Now that we've checked and transformed our data, we're ready to run PCA. There are two kinds of PCA: We can run PCA on the Covariance Matrix, or the Correlation Matrix.

1.3.1 Covariance Matrix

We can run PCA on the covariance matrix as follows:

```
# Run PCA - Covariance
pca_1 = princomp(data_log[, -1]) # We don't want haul in our PCA!
summary(pca_1)
```

Importance of components:

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|------------------------|-----------|-----------|-----------|------------|-------------|
| Standard deviation | 2.8055354 | 1.3857803 | 1.3351790 | 0.55247629 | 0.182937780 |
| Proportion of Variance | 0.6607195 | 0.1612035 | 0.1496458 | 0.02562199 | 0.002809263 |
| Cumulative Proportion | 0.6607195 | 0.8219229 | 0.9715687 | 0.99719074 | 1.000000000 |

Running a summary on our PCA gives us the standard deviation of each principal component, the proportion of variance explained by each principal component, and the cumulative variance explained as we add each component.

Each principal component is an **eigenvector** of the correlation/covariance matrix (remember from lecture that the j th principal component is the j th eigenvector of the correlation/covariance matrix). The **eigenvalues** are the variance of each individual principal component. The principal components are organized by their eigenvalues - the first principal component is the eigenvector with the largest eigenvalue, the second principal component is the eigenvector with the second largest eigenvalue, and so on.

The `princomp()` function gives us the standard deviation of each principal component; We can square these to get the eigenvalues:

```
# Calculate and print eigenvalues
eigenvalues = pca_1$sdev^2
eigenvalues
```

| Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|------------|------------|------------|------------|------------|
| 7.87102905 | 1.92038712 | 1.78270294 | 0.30523005 | 0.03346623 |

We can use the eigenvalues to reproduce the rest of the `princomp()` output. The proportion of variance is the value of each eigenvalue divided by the sum of all the eigenvalues. The cumulative proportion is the cumulative sum of the proportions of variance. Since there are 5 components, the cumulative proportion of component 5 is 1 (i.e. all of the variance).

```
# Calculate and print proportion of variance
prop_var = eigenvalues/sum(eigenvalues); prop_var
```

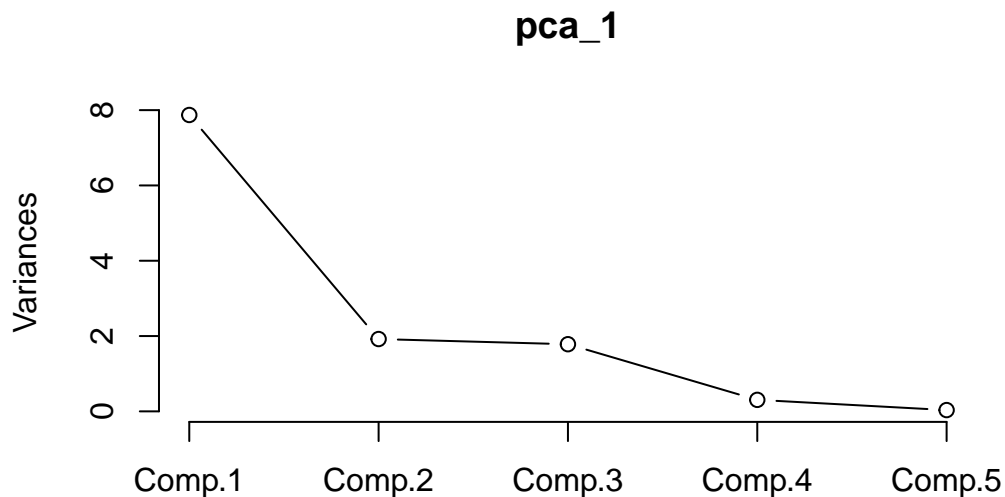
| Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|-------------|-------------|-------------|-------------|-------------|
| 0.660719468 | 0.161203465 | 0.149645813 | 0.025621991 | 0.002809263 |

```
# Calculate cumulative proportion of variance
cum_prop = cumsum(prop_var); cum_prop
```

| Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|-----------|-----------|-----------|-----------|-----------|
| 0.6607195 | 0.8219229 | 0.9715687 | 0.9971907 | 1.0000000 |

Here, we see the first principal component explains 66% of the variance. The second explains 16%, which adds up to 82% with the first component, and so on up to component 5. We can visualize the cumulative variance explained with a scree plot:

```
# Generate scree plot
plot(pca_1, type = 'l') # Scree is built into the plot for PCA
```



We see most of the variance is explained by component 1, then a similar lesser amount is explained by 2 and 3, followed by another drop to 4 and 5.

```
# Print loadings
print(loadings(pca_1), cutoff=0.00) #all loadings!
```

Loadings:

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|----------|--------|--------|--------|--------|--------|
| mackerel | 0.018 | 0.294 | 0.047 | 0.527 | 0.796 |
| bluefin | -0.654 | 0.136 | 0.739 | -0.089 | -0.020 |
| sardine | 0.060 | 0.626 | -0.015 | 0.520 | -0.577 |
| squid | -0.745 | 0.049 | -0.664 | 0.029 | 0.018 |
| limpet | 0.116 | 0.707 | -0.102 | -0.665 | 0.183 |

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|----------------|--------|--------|--------|--------|--------|
| SS loadings | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| Proportion Var | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Cumulative Var | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |

The PCA loadings are the correlations between the variables and each component. Here, we see bluefin and squid are strongly negatively correlated with component 1, while mackerel, sardine, and limpet are weakly positively correlated with component 1. We can continue this type of interpretation through the other components as well.

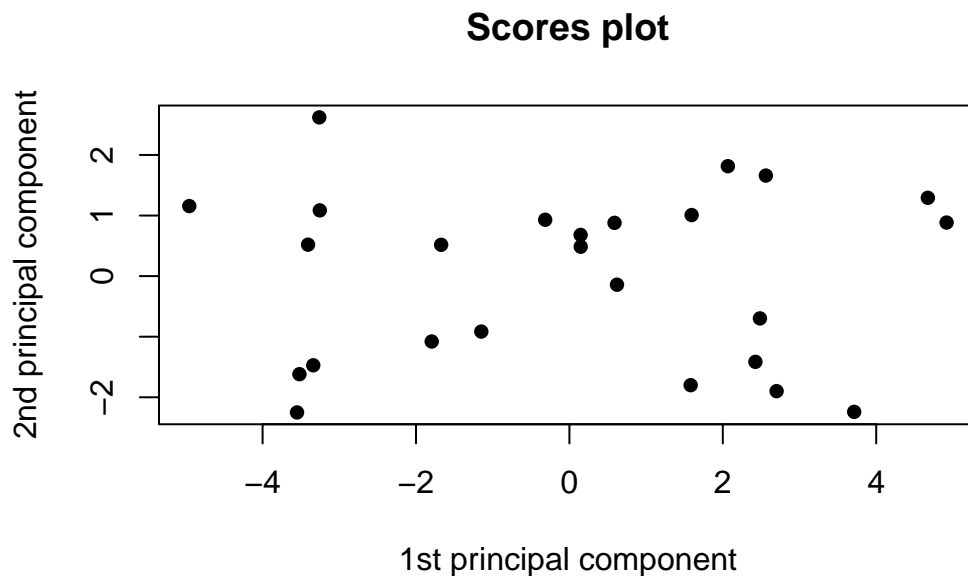
Our PCA object also contains the PCA scores for each individual data point:

```
# Print PCA scores
head(pca_1$scores)
```

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|------|-----------|------------|------------|------------|-------------|
| [1,] | -3.551007 | -2.2507718 | 0.6725187 | -0.1223707 | -0.06754585 |
| [2,] | 2.484116 | -0.6980669 | 0.4886052 | -0.3932653 | -0.53609582 |
| [3,] | 2.425206 | -1.4149241 | 0.9556963 | -0.2274184 | -0.07560834 |
| [4,] | -3.339469 | -1.4723306 | -0.2089590 | -0.8854067 | -0.03598265 |
| [5,] | 1.580988 | -1.8002844 | -4.6958830 | -0.4313924 | 0.13590020 |
| [6,] | -3.519487 | -1.6187995 | 0.3362833 | 0.1040827 | 0.32134755 |

Scores are the value of each data point on each principal component. Lets try plotting them:

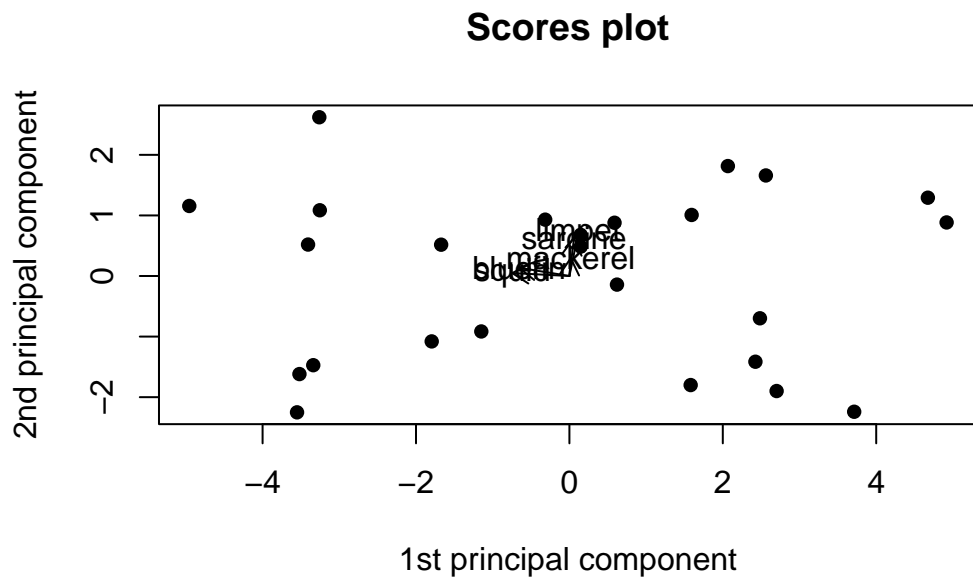
```
# Plot scores - components 1 and 2
plot(pca_1$scores[,1], # Scores on component 1
     pca_1$scores[,2], # Scores on component 3
     pch=16, # Point 16 (colored circle)
     xlab="1st principal component",ylab="2nd principal component",main="Scores plot") # Axis
```



This generates a scatterplot showing us the value of each data point in principal components 1 (x) and 2 (y).

```
# Plot scores - components 1 and 2
plot(pca_1$scores[,1], # Scores on component 1
     pca_1$scores[,2], # Scores on component 3
     pch=16, # Point 16 (colored circle)
     xlab="1st principal component",ylab="2nd principal component",main="Scores plot") # Axis

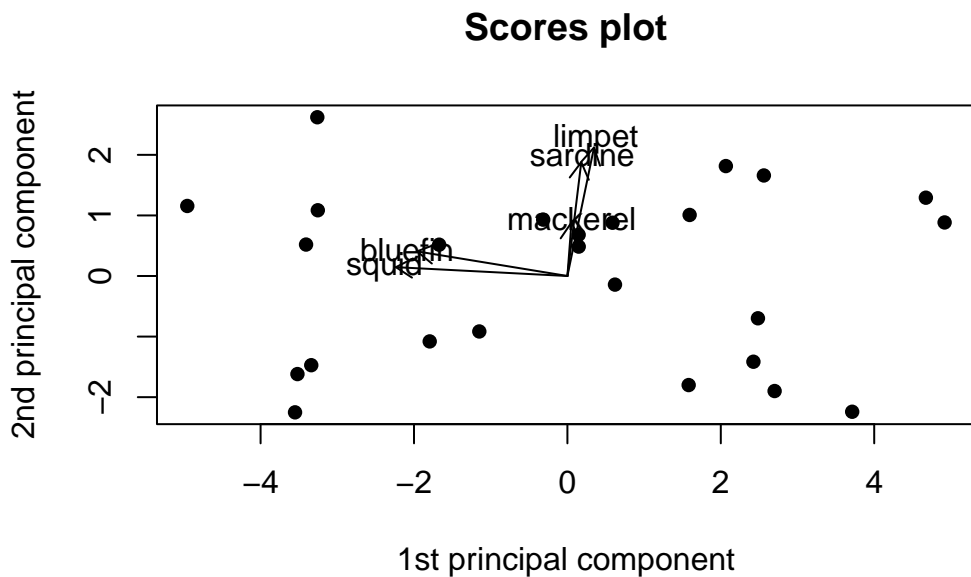
# Add loadings to plot
arrows(0,0, # Draw arrows from zero
      pca_1$loadings[,1], # Draw to PC1 loading in X
      pca_1$loadings[,2], # Draw to PC2 loading in Y
      col="black", length = 0.1) # Arrow color and arrowhead length
text(pca_1$loadings[,1],pca_1$loadings[,2],names(data_log[,-1]),cex=1.0 ,col="black") # Add t
```

The arrows are a little small, so let's add a scaling factor:

```
# Plot scores - components 1 and 2
plot(pca_1$scores[,1], # Scores on component 1
     pca_1$scores[,2], # Scores on component 3
     pch=16, # Point 16 (colored circle)
     xlab="1st principal component", ylab="2nd principal component", main="Scores plot") # Axis

# Add loadings to plot
sf = 3 # Scaling factor
sft = 3.2 # Scaling factor for text
arrows(0,0, # Draw arrows from zero
      pca_1$loadings[,1]*sf, # Draw to PC1 * scaling factor loading in X
      pca_1$loadings[,2]*sf, # Draw to PC2 * scaling factor loading in Y
      col="black", length = 0.1) # Arrow color and arrowhead length
text(pca_1$loadings[,1]*sft, pca_1$loadings[,2]*sft, names(data_log[,-1]), cex=1.0, col="black")
```

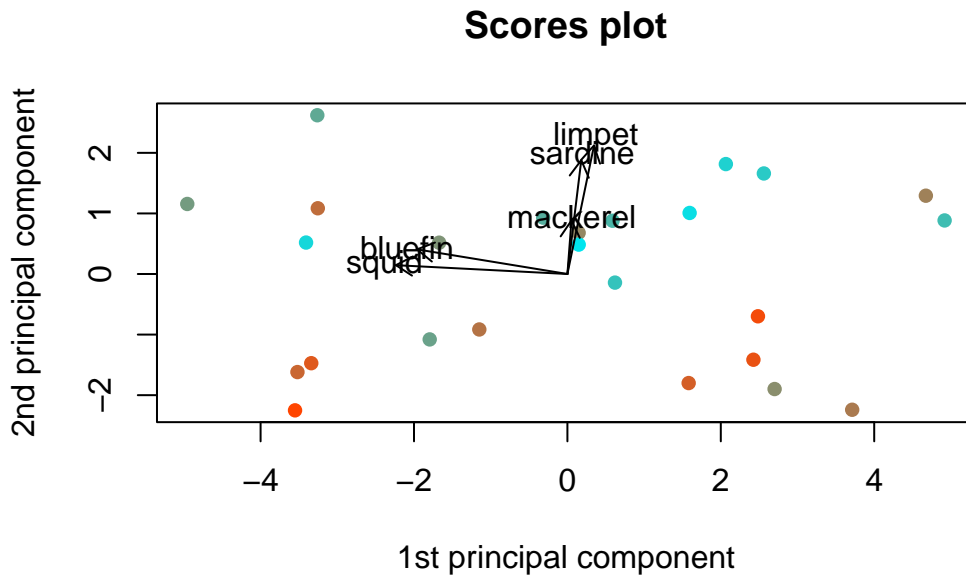


What about the haul number? Does that have an effect? Let's try adding that on as well:

```
# Create a color palette
colfunc = colorRampPalette(c('orangered1', 'turquoise2'))

# Plot scores - components 1 and 2
plot(pca_1$scores[,1], # Scores on component 1
     pca_1$scores[,2], # Scores on component 3
     pch=16, # Point 16 (colored circle)
     col = colfunc(nrow(pca_1$scores)), # Color points by haul using our color palette
     xlab="1st principal component", ylab="2nd principal component", main="Scores plot") # Axis

# Add loadings to plot
sf = 3 # Scaling factor
sft = 3.2 # Scaling factor for text
arrows(0,0, # Draw arrows from zero
       pca_1$loadings[,1]*sf, # Draw to PC1 * scaling factor loading in X
       pca_1$loadings[,2]*sf, # Draw to PC2 * scaling factor loading in Y
       col="black", length = 0.1) # Arrow color and arrowhead length
text(pca_1$loadings[,1]*sft, pca_1$loadings[,2]*sft, names(data_log[, -1]), cex=1.0, col="black")
```



Since we used color for haul, we need to add a legend:

```
# Set plot layout
layout(matrix(1:2,ncol=2), # 1 row, 2 columns
        width = c(2,1), # Width
        height = c(1,1)) # Height

# Create a color palette
colfunc = colorRampPalette(c('orangered1', 'turquoise2'))

# Plot scores - components 1 and 2
plot(pca_1$scores[,1], # Scores on component 1
     pca_1$scores[,2], # Scores on component 3
     pch=16, # Point 16 (colored circle)
     col = colfunc(nrow(pca_1$scores)), # Color points by haul using our color palette
     xlab="1st principal component", ylab="2nd principal component", main="Scores plot") # Axis

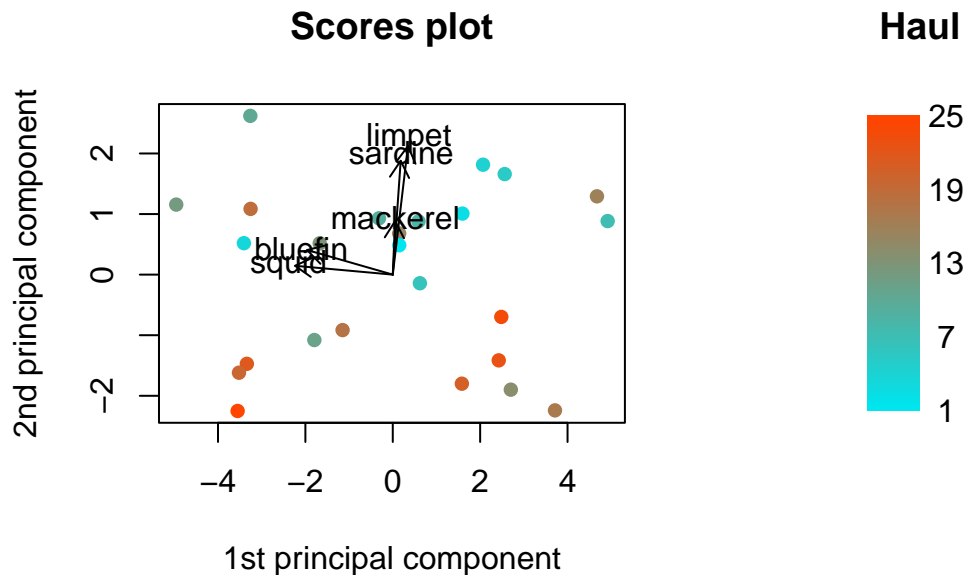
# Add loadings to plot
sf = 3 # Scaling factor
sft = 3.2 # Scaling factor for text
arrows(0,0, # Draw arrows from zero
       pca_1$loadings[,1]*sf, # Draw to PC1 * scaling factor loading in X
       pca_1$loadings[,2]*sf, # Draw to PC2 * scaling factor loading in Y
       label=c("limpet", "sardine", "mackerel", "bluefin", "squid"),
       angle=90, # Angle of text
       sft=sft)
```

```

col="black", length = 0.1) # Arrow color and arrowhead length
text(pca_1$loadings[,1]*sft,pca_1$loadings[,2]*sft, names(data_log[,-1]), cex=1.0, col="black")

# Generate legend
legend_image <- as.raster(matrix(colfunc(nrow(pca_1$scores))), ncol=1))
plot(c(0,2),c(0,1),type = 'n', axes = F,xlab = '', ylab = '', main = 'Haul')
text(x=1.5, y =seq(0,1,l=5), labels = seq(1,25,l=5))
rasterImage(legend_image, 0, 0, 1,1)

```



Now we have a completed scores plot with loadings arrows. How would you interpret this plot?

1.3.2 Correlation Matrix

Now let's try the correlation matrix. The correlation matrix performs the same analysis, but on standardized data. The `princomp()` function does this for us if we set `cor = T`:

```

# Run PCA - Correlation
pca_2 = princomp(data_log[, -1], cor = T)
summary(pca_2)

```

Importance of components:

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|------------------------|----------|-----------|------------|------------|------------|
| Standard deviation | 1.595782 | 1.2503536 | 0.70708572 | 0.57220519 | 0.25041296 |
| Proportion of Variance | 0.509304 | 0.3126768 | 0.09999404 | 0.06548376 | 0.01254133 |
| Cumulative Proportion | 0.509304 | 0.8219809 | 0.92197491 | 0.98745867 | 1.00000000 |

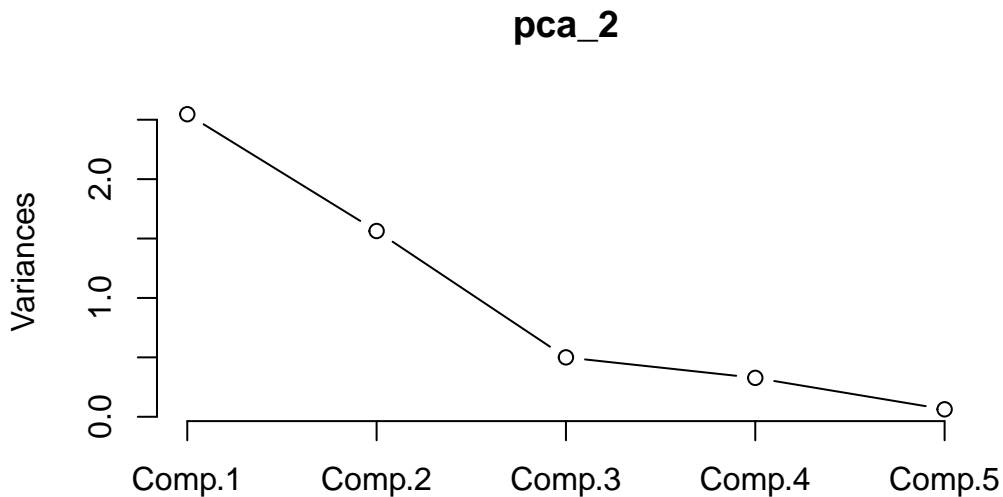
```
# In case you don't believe me, heres the covariance matrix if we pre-standardize the data
pca_test = princomp(scale(data_log[-1]))
summary(pca_test)
```

Importance of components:

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|------------------------|----------|-----------|------------|------------|------------|
| Standard deviation | 1.563541 | 1.2250914 | 0.69279969 | 0.56064429 | 0.24535359 |
| Proportion of Variance | 0.509304 | 0.3126768 | 0.09999404 | 0.06548376 | 0.01254133 |
| Cumulative Proportion | 0.509304 | 0.8219809 | 0.92197491 | 0.98745867 | 1.00000000 |

Now we can go through the same pattern of analyses as we did for covariance:

```
# Generate scree plot
plot(pca_2, type = 'l') # Scree is built into the plot for PCA
```



```
# Print loadings
print(loadings(pca_2),cutoff=0.00) #all loadings!
```

Loadings:

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|----------|--------|--------|--------|--------|--------|
| mackerel | 0.524 | 0.272 | 0.527 | 0.297 | 0.535 |
| bluefin | -0.198 | 0.682 | 0.264 | -0.651 | -0.050 |
| sardine | 0.591 | 0.209 | 0.025 | 0.109 | -0.771 |
| squid | -0.233 | 0.645 | -0.472 | 0.550 | 0.059 |
| limpet | 0.532 | 0.036 | -0.655 | -0.416 | 0.338 |

| | Comp.1 | Comp.2 | Comp.3 | Comp.4 | Comp.5 |
|----------------|--------|--------|--------|--------|--------|
| SS loadings | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| Proportion Var | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Cumulative Var | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |

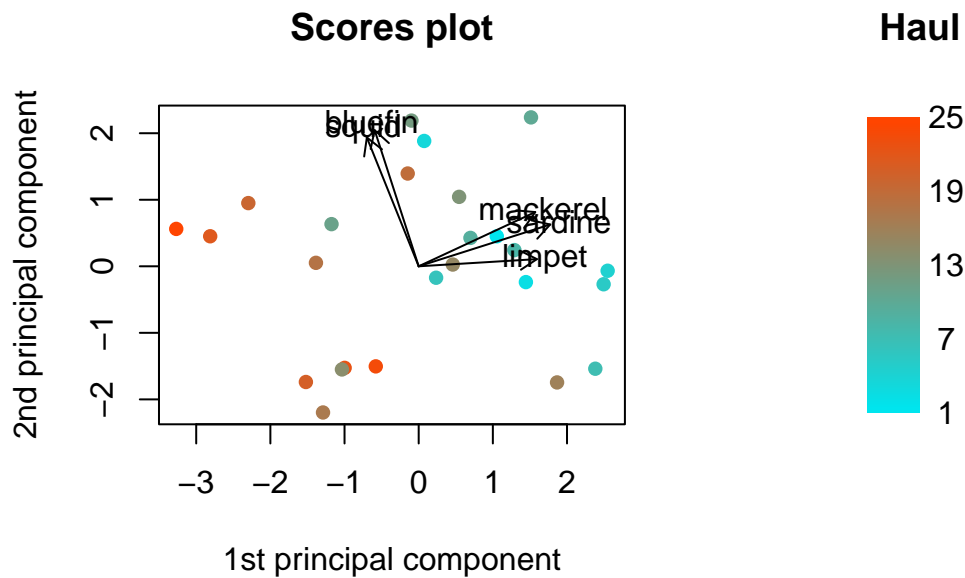
```
# Set plot layout
layout(matrix(1:2,ncol=2), # 1 row, 2 columns
        width = c(2,1), # Width
        height = c(1,1)) # Height

# Create a color palette
colfunc = colorRampPalette(c('orangered1', 'turquoise2'))

# Plot scores - components 1 and 2
plot(pca_2$scores[,1], # Scores on component 1
     pca_2$scores[,2], # Scores on component 3
     pch=16, # Point 16 (colored circle)
     col = colfunc(nrow(pca_2$scores)), # Color points by haul using our color palette
     xlab="1st principal component",ylab="2nd principal component",main="Scores plot") # Axis

# Add loadings to plot
sf = 3 # Scaling factor
sft = 3.2 # Scaling factor for text
arrows(0,0, # Draw arrows from zero
       pca_2$loadings[,1]*sf, # Draw to PC1 * scaling factor loading in X
       pca_2$loadings[,2]*sf, # Draw to PC2 * scaling factor loading in Y
       col="black", length = 0.1) # Arrow color and arrowhead length
text(pca_2$loadings[,1]*sft,pca_2$loadings[,2]*sft, names(data_log[,-1]), cex=1.0, col="black")
```

```
# Generate legend
legend_image <- as.raster(matrix(colfunc(nrow(pca_2$scores)), ncol=1))
plot(c(0,2),c(0,1),type = 'n', axes = F,xlab = '', ylab = '', main = 'Haul')
text(x=1.5, y =seq(0,1,l=5), labels = seq(1,25,l=5))
rasterImage(legend_image, 0, 0, 1,1)
```



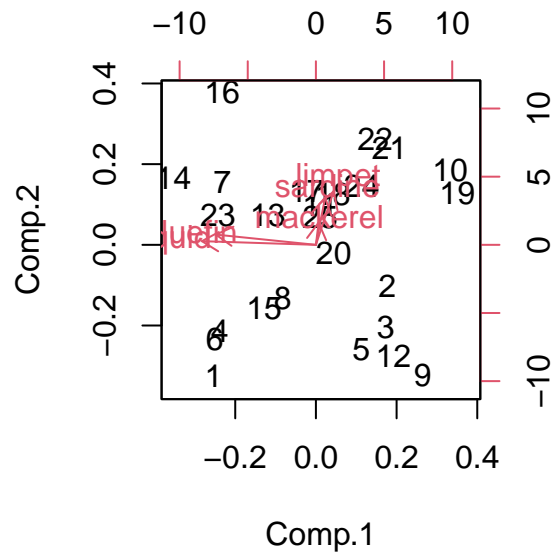
How would you interpret this plot? Does it differ from the covariance plot?

1.3.3 Alternative Methods

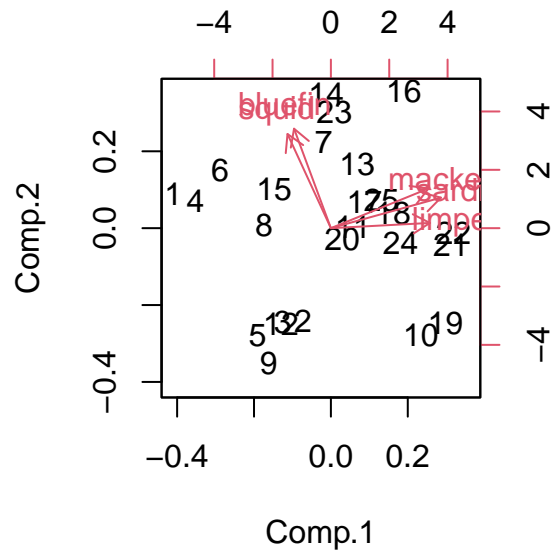
There are a few other ways you can generate, and/or plot your PCAs if you prefer.

1.3.3.1 Biplot

```
# Exploring biplot
biplot(pca_1) # Covariance
```



```
biplot(pca_2) # Correlation
```



1.3.3.2 ggplot

```
library(ggplot2)

# ggplot version - Covariance

# turn PCA scores into data frame
pca_1_plot = data.frame(Haul = data_log$Haul, pca_1$scores)

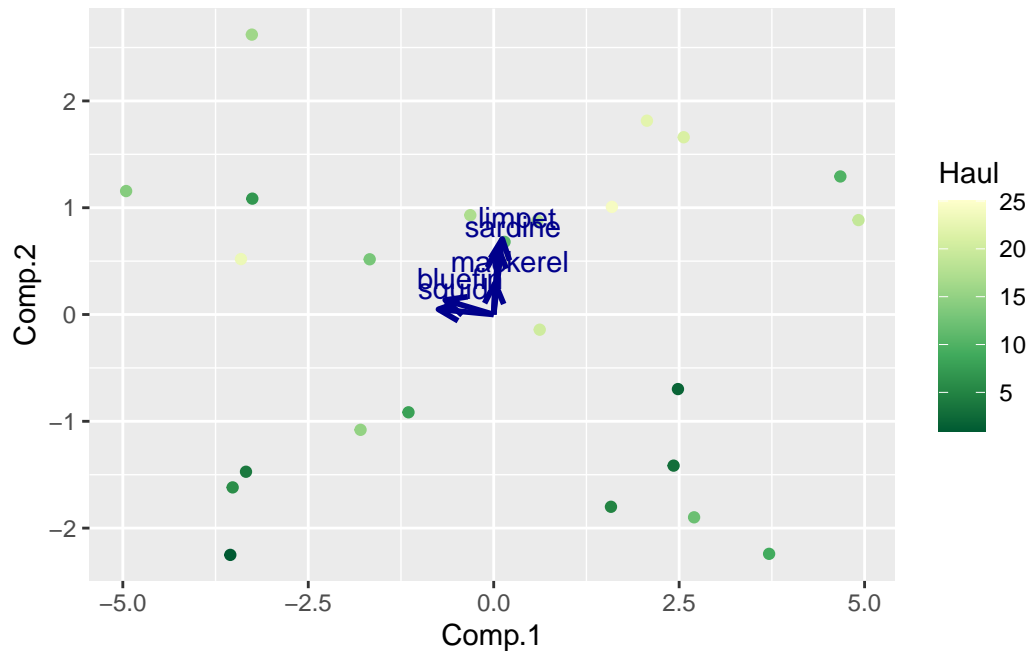
# Turn PCA loadings into data frame (This gets a little complicated)
pca_1_loadings = as.data.frame(matrix(as.numeric(pca_1$loadings),
                                         dim(pca_1$loadings)[1], dim(pca_1$loadings)[2]))
colnames(pca_1_loadings) = colnames(pca_1_plot)[-1]

# Plot
ggplot(pca_1_plot, aes(x = Comp.1, y = Comp.2, color = Haul)) +

  # Scores
  geom_point() + scale_colour_distiller(palette = 15) +

  # Loadings
  geom_segment(data = pca_1_loadings, aes(x = 0, y = 0, xend = Comp.1, yend = Comp.2),
              arrow = arrow(length = unit(0.3, "cm"), type = "open", angle = 25),
              linewidth = 1, color = "darkblue") +

  # Labels
  geom_text(data = pca_1_loadings, color = 'darkblue', nudge_x = 0.2, nudge_y = 0.2, # Label
            aes(x = Comp.1, y = Comp.2, label = colnames(data_log)[-1]))
```



```
# ggplot version - Correlation

# turn PCA scores into data frame
pca_2_plot = data.frame(Haul = data_log$Haul, pca_2$scores)

# Turn PCA loadings into data frame
pca_2_loadings = as.data.frame(matrix(as.numeric(pca_2$loadings),
                                       dim(pca_2$loadings)[1], dim(pca_2$loadings)[2]))
colnames(pca_2_loadings) = colnames(pca_2_plot)[-1]

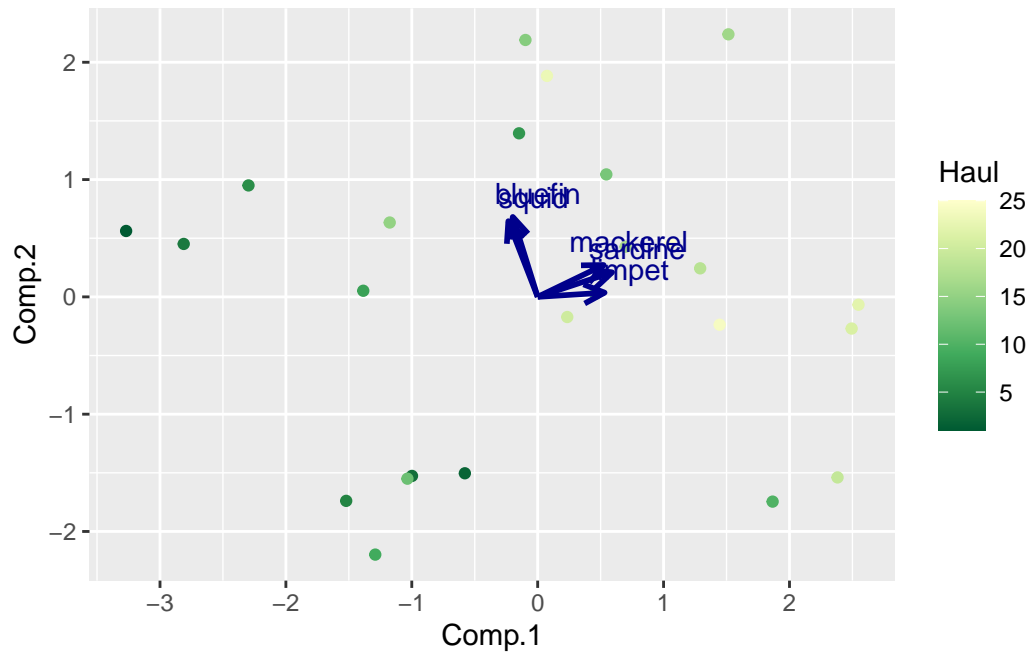
# Plot
ggplot(pca_2_plot, aes(x = Comp.1, y = Comp.2, color = Haul)) +

  # Scores
  geom_point() + scale_colour_distiller(palette = 15) +

  # Loadings
  geom_segment(data = pca_2_loadings, aes(x = 0, y = 0, xend = Comp.1, yend = Comp.2),
              arrow = arrow(length = unit(0.3, "cm"), type = "open", angle = 25),
              linewidth = 1, color = "darkblue") +

  # Labels
  geom_text(data = pca_2_loadings, color = 'darkblue', nudge_x = 0.2, nudge_y = 0.2, # Label.
```

```
aes(x = Comp.1, y = Comp.2, label = colnames(data_log)[-1]))
```

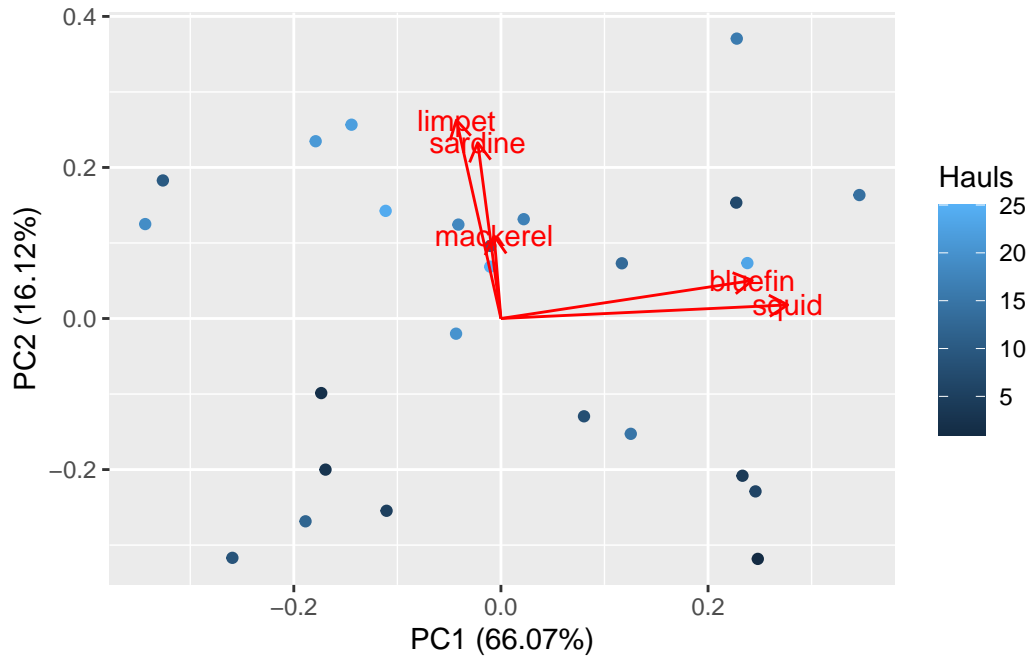


You can also run PCA using the `prcomp()` function instead of `princomp()`, setting `scale = T` if you want the correlation matrix. You can then use `autoplot()` with the `ggfortify` package to plot the results.

```
# ggplot v2
library(ggfortify)

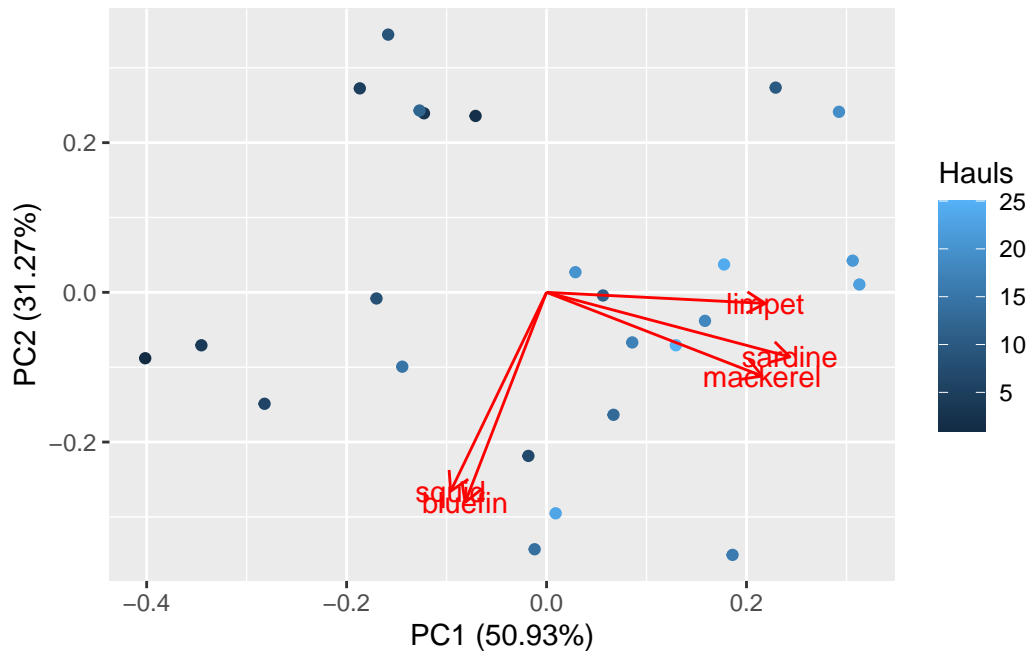
# Run PCA - Covariance
pca_1a = prcomp(data_log[, -1])

# Run autoplot
autoplot(pca_1a, data = data_log, color = 'Hauls', loadings = T, loadings.label = T)
```



```
# Run PCA - Correlation
pca_2a = prcomp(data_log[,-1], scale = T)

# Run autoplot
autoplot(pca_2a, data = data_log, color = 'Hauls', loadings = T, loadings.label = T)
```



1.4 Varimax Rotation (Optional)

Varimax rotation attempts to improve the interpretability of PCA results by lining up loadings with the axes. This can be useful, particularly with large numbers of variables.

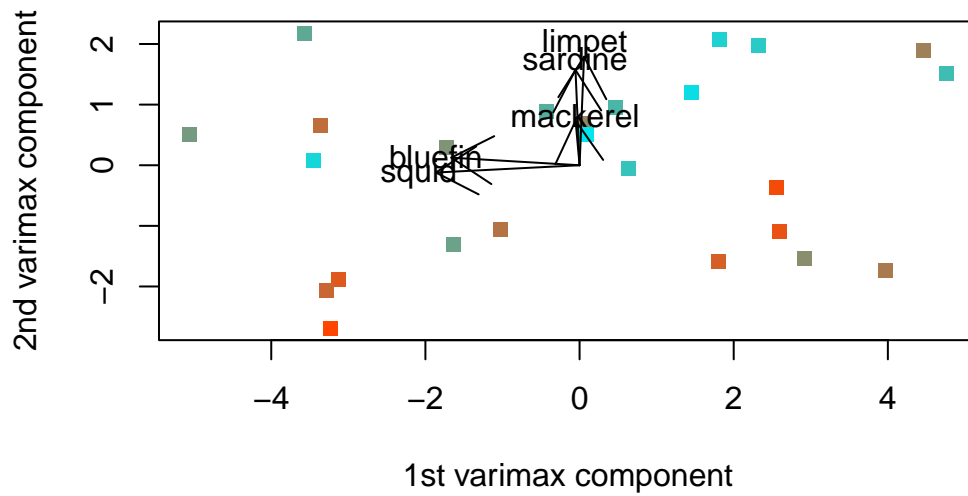
```
# Scaling factors
sf = 2.5
sft = 2.8

# Varimax rotation - Covariance
v1 = varimax(pca_1$loadings[,1:2])
v1_scores = pca_1$scores[,1:2] %>% v1$rotmat

# Plot scores - components 1 and 2
plot(v1_scores[,1], v1_scores[,2], pch=15, col = colfunc(nrow(v1_scores)),
     xlab="1st varimax component", ylab="2nd varimax component", main="varimax scores plot")

# Add loadings
arrows(0,0,v1$loadings[,1]*sf,v1$loadings[,2]*sf,col="black")
text(v1$loadings[,1]*sft,v1$loadings[,2]*sft,names(data_log[,-1]),asp=1,cex=1.0,col="black")
```

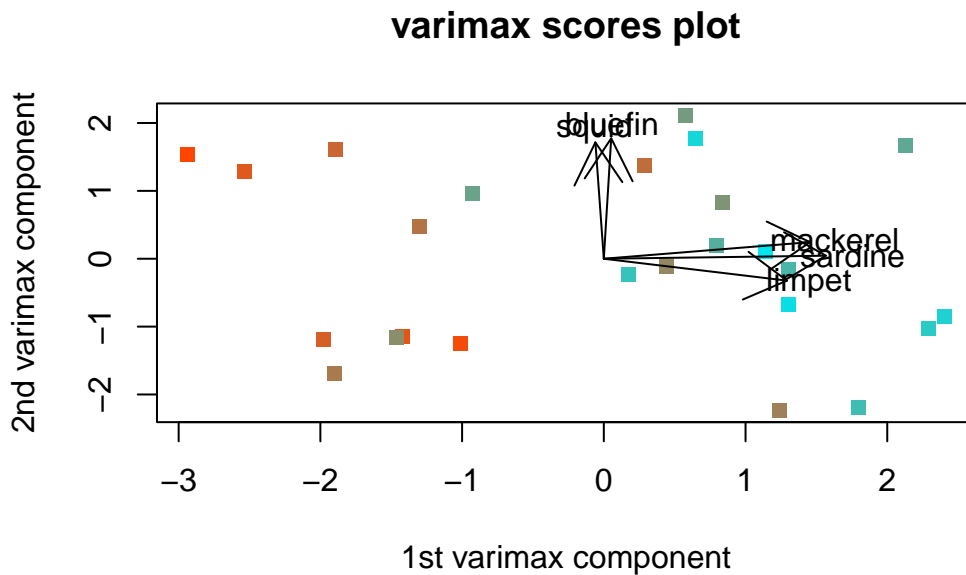
varimax scores plot



```
# Varimax rotation - Correlation
v2 = varimax(pca_2$loadings[,1:2])
v2_scores = pca_2$scores[,1:2]%%v2$rotmat

# Plot scores - components 1 and 2
plot(v2_scores[,1],v2_scores[,2],pch=15, col = colfunc(nrow(v2_scores)),
      xlab="1st varimax component",ylab="2nd varimax component",main="varimax scores plot")

# Add loadings
arrows(0,0,v2$loadings[,1]*sf,v2$loadings[,2]*sf,col="black")
text(v2$loadings[,1]*sft,v2$loadings[,2]*sft,names(data_log[,-1]),asp=1,cex=1.0 ,col="black")
```



Note that it's pretty hard to tell the hauls apart using this color scale. Make sure your plots are always clear and readable.

1.5 Tips for your assignment:

Some things you may want to think about for your assignment:

1. Do your covariance and correlation plots differ? Do you think one is better suited to answering your research question? Why? Is your answer conceptual, or does it have to do with the results? Both?
2. How would you quantitatively examine the effect of haul on the PCA scores above? Is it associated with any of the principal components?
3. How would you interpret your statistical results biologically? You don't have to be right, but don't be vague, and don't contradict your results.

2 Assignment 1b: Linear Discriminant Analysis

Assignment 1b focuses on Linear Discriminant Analysis (LDA), also known as Canonical Variate Analysis. LDA is used to disclose relationships between groups, create models to differentiate between groups based on data, and discern the contribution of different variables to a model's ability to discriminate between groups.

For this tutorial, we'll be using `snake.csv`.

2.1 Looking at the data

```
# Load in data
snake = read.csv('snake.csv')

# Look at data
head(snake)
```

| | Species | M1 | M2 | M3 | M4 | M5 | M6 |
|---|---------|------|------|------|------|------|------|
| 1 | A | 41.6 | 6.7 | 8.2 | 12.2 | 24.7 | 27.0 |
| 2 | A | 40.2 | 8.5 | 9.2 | 15.5 | 27.1 | 30.3 |
| 3 | A | 40.4 | 12.6 | 14.2 | 19.6 | 46.9 | 26.8 |
| 4 | A | 26.4 | 9.0 | 8.6 | 14.0 | 37.6 | 32.2 |
| 5 | A | 34.4 | 7.0 | 12.1 | 11.1 | 31.0 | 35.8 |
| 6 | A | 38.8 | 8.2 | 10.2 | 12.4 | 42.2 | 33.6 |

```
dim(snake)
```

```
[1] 35  7
```

Our data is a 35 row, 7 column data frame. The first column identifies the species of snake (A or B). The other columns are morphological measurements of each individual snake. We want to know if we can use the morphological measurements of the snakes to determine their species. Let's keep examining the data:


```
# Make a boxplot
library(tidyverse)
```

```
-- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
```

```
v dplyr      1.1.4      v readr      2.1.5
v forcats    1.0.0      v stringr    1.5.1
v ggplot2    3.4.4      v tibble     3.2.1
v lubridate  1.9.3      v tidyr      1.3.0
v purrr      1.0.2
```

```
-- Conflicts ----- tidyverse_conflicts() --
```

```
x dplyr::filter() masks stats::filter()
```

```
x dplyr::lag()     masks stats::lag()
```

```
i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become
```

```
# Convert the data to long format so we can use ggplot
```

```
snake_long = pivot_longer(snake, # Enter data
```

```
                           colnames(snake)[-1], # Pivot all columns except species
```

```
                           names_to = 'Measurement', values_to = 'Value') # Feed labels to new
```

```
# Lets take a look at the new data frame
```

```
head(snake_long)
```

```
# A tibble: 6 x 3
```

| | Species | Measurement | Value |
|---|---------|-------------|-------|
| | <chr> | <chr> | <dbl> |
| 1 | " A | " M1 | 41.6 |
| 2 | " A | " M2 | 6.7 |
| 3 | " A | " M3 | 8.2 |
| 4 | " A | " M4 | 12.2 |
| 5 | " A | " M5 | 24.7 |
| 6 | " A | " M6 | 27 |

```
# We've converted from wide format to long format,
```

```
# now all the data values are contained in a single column
```

```
# which is described by a metadata column
```

```
# You can also do this with melt from reshape2
```

```
library(reshape2)
```

Attaching package: 'reshape2'

The following object is masked from 'package:tidyr':

smiths

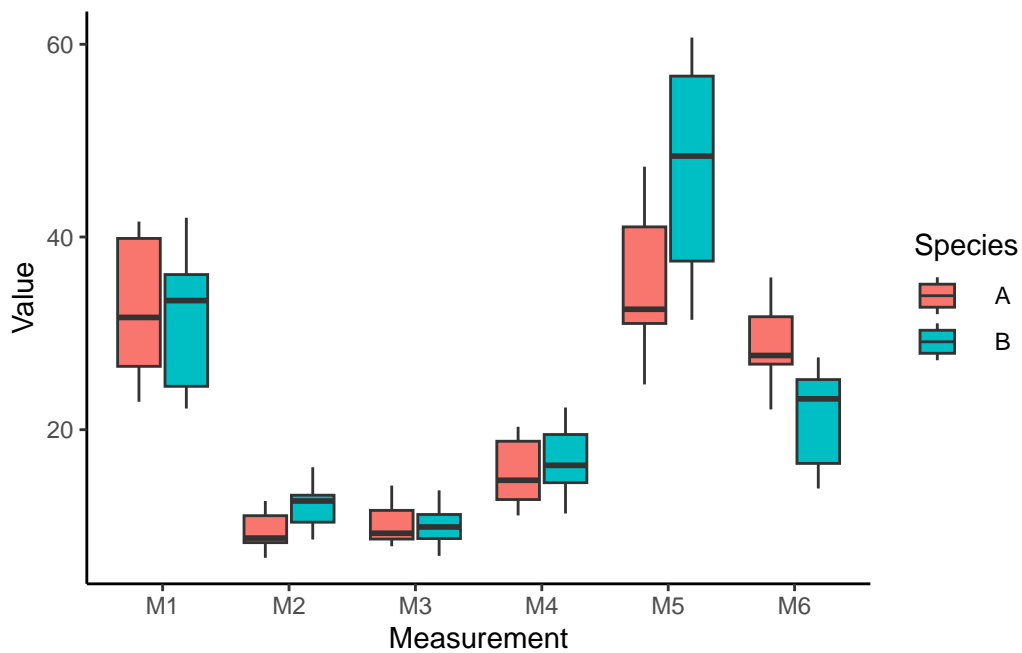
```
head(melt(snake))
```

Using Species as id variables

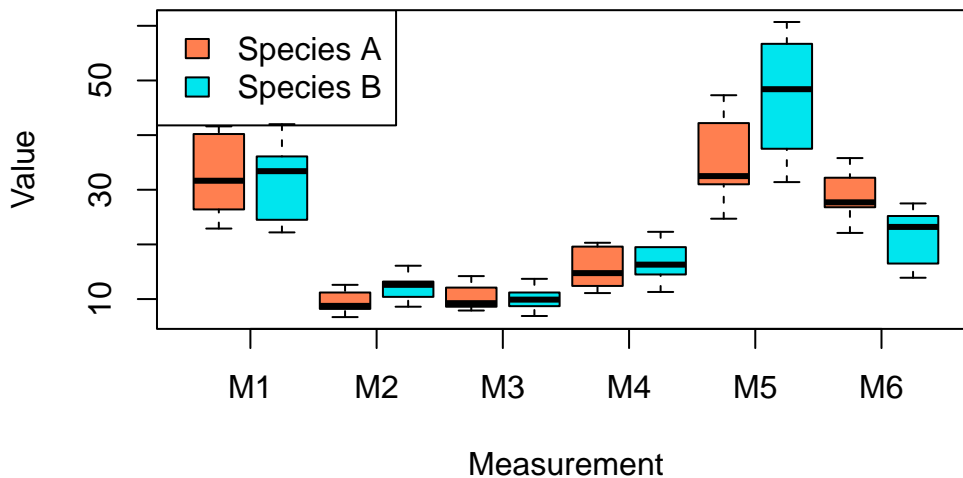
| | Species | variable | value |
|---|---------|----------|-------|
| 1 | A | M1 | 41.6 |
| 2 | A | M1 | 40.2 |
| 3 | A | M1 | 40.4 |
| 4 | A | M1 | 26.4 |
| 5 | A | M1 | 34.4 |
| 6 | A | M1 | 38.8 |

```
# Let's make a boxplot
```

```
ggplot(snake_long, aes(x = Measurement, y = Value, fill = Species)) +  
  geom_boxplot() + theme_classic()
```



```
# We can do this in R base plot too
boxplot(Value ~ Species*Measurement, # Plot value by species and measurement
        data = snake_long, col = c('coral', 'turquoise2'), # Color by species
        xaxt = 'n', xlab = 'Measurement') # Remove and label x axis
legend('topleft', legend = c('Species A', 'Species B'), fill = c('coral', 'turquoise2')) # Add legend
axis(1, at = seq(1.5,11.5,2), labels = colnames(snake)[-1]) # Add x axis back in with appropriate labels
```



Some of our measurements are very similar across species, and others are quite different. Do they differ statistically as a whole?

2.2 MANOVA

The purpose of LDA is to try to discriminate our snakes into species based on their measurements. However, that only makes sense to do if our two species of snake actually differ across the measurements. Our first step then is to discern whether our snake species differ as a multivariate whole. We'll do this using a MANOVA.

```
# Run MANOVA
sm = manova(cbind(M1,M2,M3,M4,M5,M6) ~ Species, data = snake)
summary(sm, test = 'Hotelling')
```

```

          Df Hotelling-Lawley approx F num Df den Df    Pr(>F)
Species    1          1.2263    5.7229      6    28 0.000552 ***
Residuals 33
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

```
summary(sm, test = 'Wilks')
```

```

          Df   Wilks approx F num Df den Df    Pr(>F)
Species    1 0.44917    5.7229      6    28 0.000552 ***
Residuals 33
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

By both the Hotelling's and Wilks' tests, our MANOVA is significant, indicating the snake species vary as a multivariate whole.

What about our assumptions though? Our MANOVA assumptions are independence, normality, linearity, and homogeneity of covariances. You've been told to assume the latter, so let's skip that one. Independence states that measurements of each snake are independent from all others. For example, it would be violated if our data were related to each other - for example, if some of our snakes were closely related, or if the graduate students measuring them were using different methods. We don't have information about how this data was collected, so we cannot assess independence. We'll skip that one as well.

Let's start by testing for normality:

```

# Testing normality
library(mvnormtest)
mshapiro.test(t(sm$residuals))

```

Shapiro-Wilk normality test

```

data:  Z
W = 0.91571, p-value = 0.01075

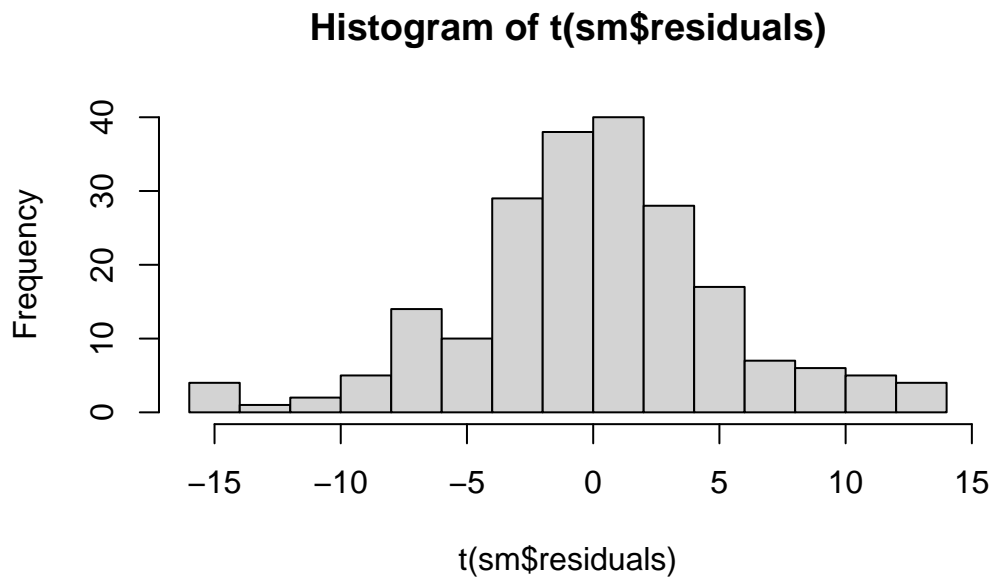
```

Uh oh, the residuals are significantly non-normal. Let's take a look at them visually:

```

# Residual histogram
hist(t(sm$residuals), breaks = 20)

```



Visually, our residuals actually look quite close to normal. There may be some slight skew, or outliers that are forcing our residuals to statistical non-normality. We might be able to fix this by removing multivariate outliers, or by transforming some of our data (feel free to play around with these ideas!), but based on the shape of our residuals, it is unlikely that our model is fatally biased, and we may end up doing more harm than good. Based on this, we can conclude that our two species have significantly different morphometries given the measurements provided.

2.3 Linear Discriminant Analysis

Now that we've confirmed our species differ as a multivariate whole, we can try to use LDA to build a model to predict which species each snake belongs to based on its measurements.

```
# LDA
library(MASS)
```

Attaching package: 'MASS'

The following object is masked from 'package:dplyr':

```
select
```

```
ldaf1 <- lda(Species ~ M1+M2+M3+M4+M5+M6, snake)
ldaf1
```

Call:

```
lda(Species ~ M1 + M2 + M3 + M4 + M5 + M6, data = snake)
```

Prior probabilities of groups:

| | A | B |
|--|-----------|-----------|
| | 0.2857143 | 0.7142857 |

Group means:

| | M1 | M2 | M3 | M4 | M5 | M6 |
|---|--------|--------|-------|-------|--------|--------|
| A | 32.700 | 9.410 | 10.16 | 15.54 | 35.290 | 28.950 |
| B | 31.496 | 12.128 | 10.18 | 16.78 | 47.356 | 21.752 |

Coefficients of linear discriminants:

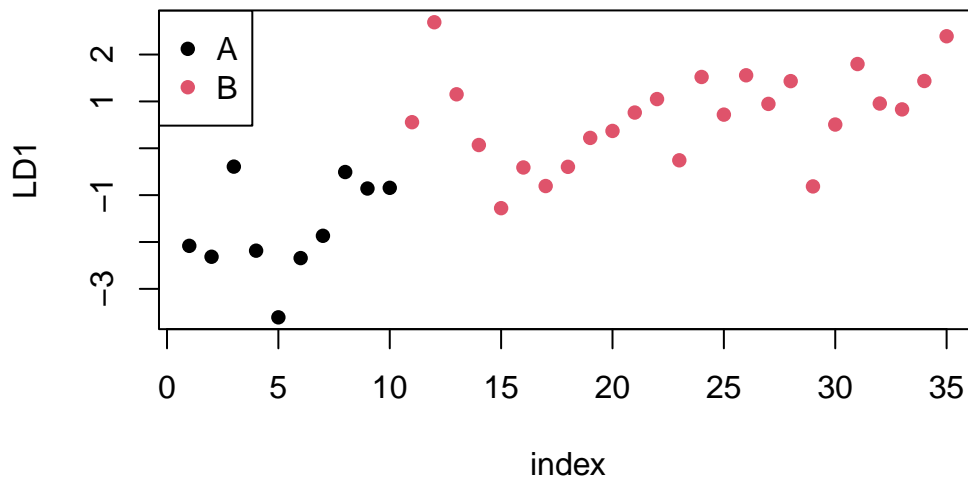
| | LD1 |
|----|-------------|
| M1 | 0.01428023 |
| M2 | 0.29104494 |
| M3 | -0.07327616 |
| M4 | -0.05544769 |
| M5 | 0.03629586 |
| M6 | -0.17208517 |

Running our LDA object tells us the prior probabilities used for each species (the proportion of each species in the data), the group means for each measure on each species, and the linear discriminant (LD1) for each measure. We can then plot the LD1 value for each individual:

```
# Plot discriminant function analysis

# Create a data frame to plot
ldaf_plot = cbind(snake, # Data
                  predict(ldaf1)$x, # LD1 value for each individual given its measurements
                  index = seq(1,nrow(snake), 1)) # Row/Individual number

# Plot
plot(LD1 ~ index, data = ldaf_plot, col = as.factor(snake$Species), pch = 16)
legend('topleft', legend = c('A', 'B'), col = c(1, 2), pch = 16) # Add legend
```



Here we can see higher LD1 values are associated with species B, while lower LD1 values are associated with species A. This is just based on model fit however; how do we know we aren't overfitting? One way to avoid overfitting is by jackknifing (AKA leave-one-out cross validation in this context). This method runs the model once without each point in the dataset, then calculates the posterior probability that the left out point belongs to each species. Let's try it out:

```
# LDA 2, CV = T
ldaf2 = lda(Species ~ M1+M2+M3+M4+M5+M6, snake, CV = T)

# Gather posteriors
as.data.frame(cbind(ldaf2$posterior, # Pull posteriors from ldaf2
                    ResultantSpp=as.character(ldaf2$class))) # Pull predicted species (i.e. s
```

| | A | B | ResultantSpp |
|---|---------------------|----------------------|--------------|
| 1 | 0.897801237948676 | 0.102198762051324 | A |
| 2 | 0.957033498274347 | 0.0429665017256533 | A |
| 3 | 0.00486396795570823 | 0.995136032044292 | B |
| 4 | 0.939579607872302 | 0.0604203921276979 | A |
| 5 | 0.999020574119129 | 0.000979425880871101 | A |
| 6 | 0.958283942083953 | 0.0417160579160473 | A |
| 7 | 0.859914694048175 | 0.140085305951825 | A |
| 8 | 0.0790276689479016 | 0.920972331052098 | B |

| | | | |
|----|----------------------|-------------------|---|
| 9 | 0.250711809994119 | 0.749288190005881 | B |
| 10 | 0.277233534989757 | 0.722766465010243 | B |
| 11 | 0.0654339037846644 | 0.934566096215336 | B |
| 12 | 8.1304517568464e-05 | 0.999918695482432 | B |
| 13 | 0.00857331675606218 | 0.991426683243938 | B |
| 14 | 0.119793120831736 | 0.880206879168264 | B |
| 15 | 0.868897347918874 | 0.131102652081126 | A |
| 16 | 0.291404395123415 | 0.708595604876585 | B |
| 17 | 0.580893601645516 | 0.419106398354484 | A |
| 18 | 0.407526292222817 | 0.592473707777183 | B |
| 19 | 0.0971393472407663 | 0.902860652759234 | B |
| 20 | 0.0629455676122025 | 0.937054432387798 | B |
| 21 | 0.0262176442553783 | 0.973782355744622 | B |
| 22 | 0.0110464412594654 | 0.988953558740534 | B |
| 23 | 0.379676706769171 | 0.620323293230829 | B |
| 24 | 0.00300786068222622 | 0.996992139317774 | B |
| 25 | 0.0331152011340239 | 0.966884798865976 | B |
| 26 | 0.00270158005931191 | 0.997298419940688 | B |
| 27 | 0.0161164609849136 | 0.983883539015087 | B |
| 28 | 0.00346528534198871 | 0.996534714658011 | B |
| 29 | 0.761253716426844 | 0.238746283573156 | A |
| 30 | 0.0597294571669357 | 0.940270542833064 | B |
| 31 | 0.00139900114299067 | 0.998600998857009 | B |
| 32 | 0.014630451548708 | 0.985369548451292 | B |
| 33 | 0.0215114427320869 | 0.978488557267913 | B |
| 34 | 0.00359029891803414 | 0.996409701081966 | B |
| 35 | 8.92739861715472e-05 | 0.999910726013828 | B |

How does this differ from the predictions from our first model?

```
# Pull ldaf1 model predictions
ldaf_pred = predict(ldaf1)$class

# Gather Predictions
ldaf_diff = data.frame(ldaf1 = as.character(ldaf_pred), ldaf2 = as.character(ldaf2$class))

# Add match column
ldaf_diff$match = (ldaf_diff$ldaf1 == ldaf_diff$ldaf2)

# Which ones are different?
ldaf_diff[which(ldaf_diff$match == F),]
```

```
ldaf1  ldaf2 match
```



```
17    B    A    FALSE
29    B    A    FALSE
```

Individuals 17 and 29 both differed in species prediction between the model fit and the jackknife posterior probability. Now let's check the accuracy of our model fit:

```
# Calculate error
ldaf_wrong = length(which(ldaf_pred != snake$Species)) # Number of incorrect predictions
ldaf_err = ldaf_wrong/nrow(snake) # Divide by number of individuals for error

# Print error
ldaf_wrong
```

```
[1] 5
```

```
ldaf_err
```

```
[1] 0.1428571
```

Our model classified 5 out of 35 (~14.3%) of the snakes as the incorrect species, meaning 30/35 were correct (~85.7%). Not bad, but can we do better?

2.4 Model Selection

Our previous model used all 6 measurements, but do we really need all of them, or are some of them unhelpful (or even detrimental)? To test this, we can run model selection using the `stepclass()` function:

```
# stepclass package
library(klaR)

# Model selection (forward)
ms_f = stepclass(Species ~ M1+M2+M3+M4+M5+M6, data=snake,
                 method="lda", fold=35, direction="forward")
```

```
`stepwise classification', using 35-fold cross-validated correctness rate of method lda'.
```

```
35 observations of 6 variables in 2 classes; direction: forward
```

stop criterion: improvement less than 5%.

correctness rate: 0.85714; in: "M6"; variables (1): M6

| hr.elapsed | min.elapsed | sec.elapsed |
|------------|-------------|-------------|
| 0.000 | 0.000 | 0.644 |

```
# Print model selection result
ms_f
```

```
method      : lda
final model  : Species ~ M6
<environment: 0x562b5814a2e0>
```

correctness rate = 0.8571

After model selection, we end up with a model using only M6 to predict species, with a correctness rate of 85.7%. This model has the same correctness as the full model, using only one measurement. In other words, this model is more **efficient** - it gets to the same accuracy using less information.

This model was generated using forward model selection, meaning the selection process works exclusively by adding variables to the model. We can also do the opposite:

```
# stepclass package
library(klaR)

# Model selection (forward)
ms_b = stepclass(Species ~ M1+M2+M3+M4+M5+M6,data=snake,
                 method="lda", fold=35, direction="backward")
```

`stepwise classification', using 35-fold cross-validated correctness rate of method lda'.

35 observations of 6 variables in 2 classes; direction: backward

stop criterion: improvement less than 5%.

correctness rate: 0.8; starting variables (6): M1, M2, M3, M4, M5, M6
correctness rate: 0.85714; out: "M5"; variables (5): M1, M2, M3, M4, M6

| hr.elapsed | min.elapsed | sec.elapsed |
|------------|-------------|-------------|
| 0.000 | 0.000 | 0.618 |

```
# Print model selection result
ms_b
```

```
method      : lda
final model  : Species ~ M1 + M2 + M3 + M4 + M6
<environment: 0x562b53eaeca8>

correctness rate = 0.8571
```

Backwards model selection works by removing variables from the full model. This means backwards selection usually returns a model with equal or more variables than forwards selection.

Lastly, we can run both:

```
# stepclass package
library(klaR)

# Model selection (forward)
ms_d = stepclass(Species ~ M1+M2+M3+M4+M5+M6,data=snake,
                 method="lda", fold=35, direction="both")
```

`stepwise classification', using 35-fold cross-validated correctness rate of method lda'.

35 observations of 6 variables in 2 classes; direction: both

stop criterion: improvement less than 5%.

correctness rate: 0.85714; in: "M6"; variables (1): M6

| hr.elapsed | min.elapsed | sec.elapsed |
|------------|-------------|-------------|
| 0.000 | 0.000 | 0.494 |

```
# Print model selection result
ms_d
```

```
method      : lda
final model  : Species ~ M6
<environment: 0x562b564efbd8>

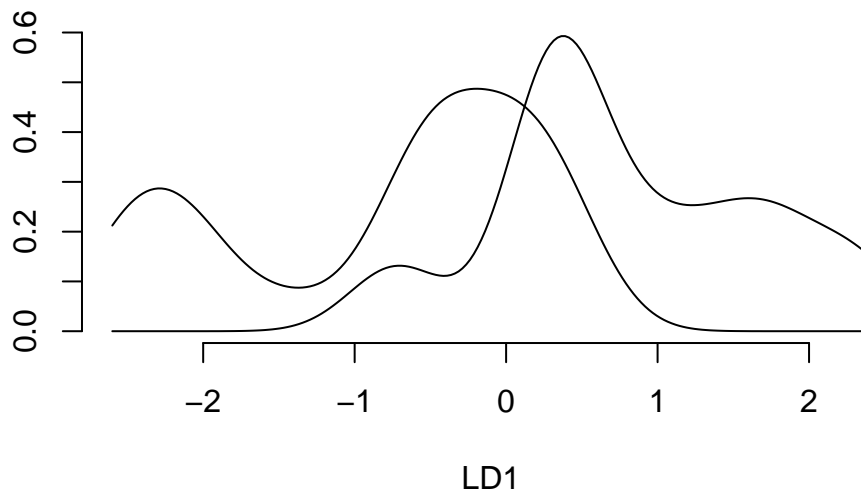
correctness rate = 0.8571
```

2.5 Plotting Probabilities

Lets finish off by making some plots to visualize our LDA model results.

```
# Pick a model to plot
ldaf3 = lda(Species ~ M6, data = snake)

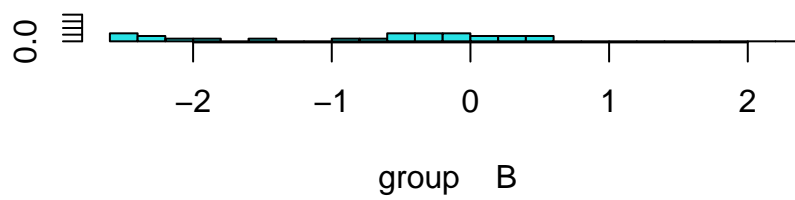
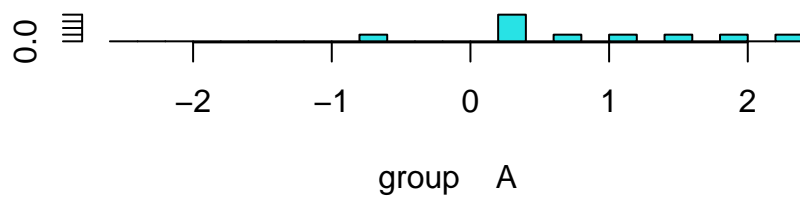
# Plot density curve
plot(ldaf3, dimen = 1, type = 'dens')
```



This plots the posterior probabilities of an individual belonging to either species given its LD1 value. Remember from earlier that species A is associated with lower LD1 values.

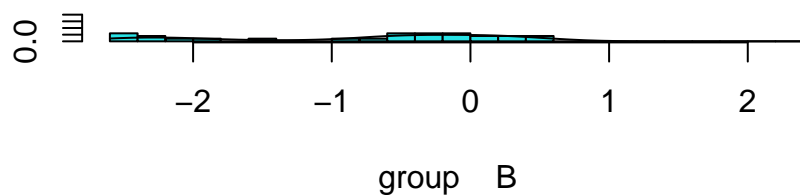
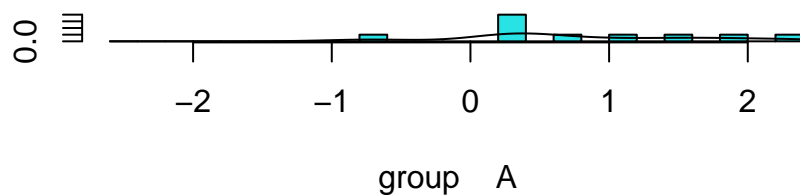
We can also make this plot as a histogram:

```
# Plot density curve
par(mar = c(4,4,4,4))
plot(ldaf3, dimen = 1, type = 'hist')
```



Or combine both plots:

```
# Plot density curve
par(mar = c(4,4,4,4))
plot(ldaf3, dimen = 1, type = 'both')
```

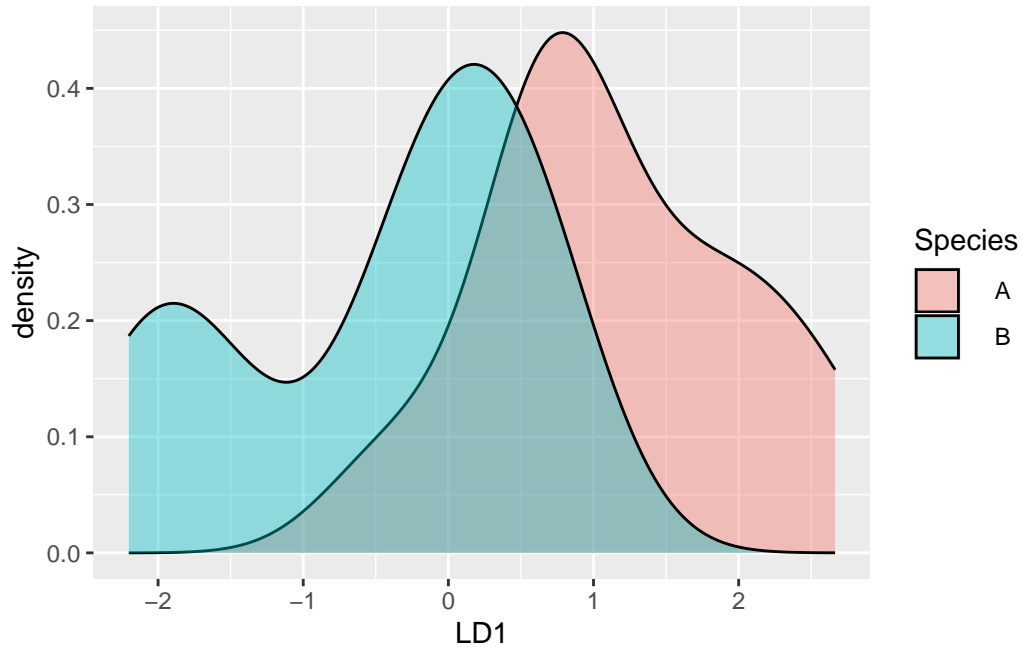


As always, we can also do this with ggplot too:

```
# Predict species
ldaf3_pred = predict(ldaf3)

# Plot
pred_species = as.data.frame(ldaf3_pred$x) # Gather LD1 values
pred_species$Species = snake$Species # Gather true species from data

# Plot
ggplot(pred_species, aes(x = LD1, fill = Species))+
  geom_density(alpha = 0.4) # alpha tells you how transparent the plots will be
```



2.6 Tips for your assignment

Some things you may want to think about for your assignment:

1. How would you pick which model you think is best? What factors would you consider? Are there any factors you would consider other than those discussed in this tutorial?
2. How would you interpret your statistical results biologically (can be in terms of the snakes, how you would study them, or both)? You don't have to be right, but don't be vague, and don't contradict your results.

3 Assignment 1c: Cluster Analysis and Multidimensional Scaling

This assignment is centered on cluster analysis and multidimensional scaling (MDS), which are both methods of measuring associations within a group (e.g. associations between individuals within a population).

For this tutorial, we'll be using `monkey.csv`.

3.1 Looking at the data

You know the drill by now:

```
# Load in data
data = read.csv('monkey.csv', row.names = 1) # First column is row names
data # Print data
```

| | ind1 | ind2 | ind3 | ind4 | ind5 | ind6 | ind7 | ind8 | ind9 | ind10 | ind11 | ind12 | ind13 |
|-------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|
| ind1 | 21 | 2 | 2 | 10 | 2 | 2 | 8 | 0 | 0 | 8 | 14 | 12 | 4 |
| ind2 | 2 | 21 | 16 | 2 | 16 | 8 | 2 | 2 | 4 | 4 | 4 | 0 | 2 |
| ind3 | 2 | 16 | 21 | 0 | 10 | 16 | 2 | 0 | 2 | 4 | 4 | 0 | 2 |
| ind4 | 10 | 2 | 0 | 21 | 2 | 2 | 16 | 2 | 2 | 8 | 12 | 8 | 4 |
| ind5 | 2 | 16 | 10 | 2 | 21 | 10 | 2 | 4 | 0 | 2 | 4 | 4 | 2 |
| ind6 | 2 | 8 | 16 | 2 | 10 | 21 | 4 | 2 | 0 | 0 | 0 | 4 | 4 |
| ind7 | 8 | 2 | 2 | 16 | 2 | 4 | 21 | 4 | 2 | 16 | 8 | 8 | 4 |
| ind8 | 0 | 2 | 0 | 2 | 4 | 2 | 4 | 21 | 0 | 2 | 0 | 0 | 0 |
| ind9 | 0 | 4 | 2 | 2 | 0 | 0 | 2 | 0 | 21 | 0 | 4 | 0 | 0 |
| ind10 | 8 | 4 | 4 | 8 | 2 | 0 | 16 | 2 | 0 | 21 | 14 | 14 | 2 |
| ind11 | 14 | 4 | 4 | 12 | 4 | 0 | 8 | 0 | 4 | 14 | 21 | 12 | 4 |
| ind12 | 12 | 0 | 0 | 8 | 4 | 4 | 8 | 0 | 0 | 14 | 12 | 21 | 2 |
| ind13 | 4 | 2 | 2 | 4 | 2 | 4 | 4 | 0 | 0 | 2 | 4 | 2 | 21 |

Our data is a matrix containing the number of social interactions observed between individuals in a group of monkeys at the zoo. The matrix is symmetrical - the top/right half is identical to the bottom/left half.

3.2 Calculating Dissimilarity

For this assignment we'll be using 3 R functions: `hclust`, `metaMDS` (from the `vegan` package), `isoMDS` (from the `MASS` package), and `cmdscale()`. Let's see what type of input data those functions need:

```
# Check help functions
library(vegan)
library(MASS)
?hclust()
?metaMDS()
?isoMDS()
?cmdscale()
```

You'll notice all of these functions require a **dissimilarity matrix** produced by `dist`. Let's start by running `dist()`.

```
# Convert data to a dist object
dist = as.dist(data)
dist # Print dist
```

| | ind1 | ind2 | ind3 | ind4 | ind5 | ind6 | ind7 | ind8 | ind9 | ind10 | ind11 | ind12 |
|-------|------|------|------|------|------|------|------|------|------|-------|-------|-------|
| ind2 | 2 | | | | | | | | | | | |
| ind3 | 2 | 16 | | | | | | | | | | |
| ind4 | 10 | 2 | 0 | | | | | | | | | |
| ind5 | 2 | 16 | 10 | 2 | | | | | | | | |
| ind6 | 2 | 8 | 16 | 2 | 10 | | | | | | | |
| ind7 | 8 | 2 | 2 | 16 | 2 | 4 | | | | | | |
| ind8 | 0 | 2 | 0 | 2 | 4 | 2 | 4 | | | | | |
| ind9 | 0 | 4 | 2 | 2 | 0 | 0 | 2 | 0 | | | | |
| ind10 | 8 | 4 | 4 | 8 | 2 | 0 | 16 | 2 | 0 | | | |
| ind11 | 14 | 4 | 4 | 12 | 4 | 0 | 8 | 0 | 4 | 14 | | |
| ind12 | 12 | 0 | 0 | 8 | 4 | 4 | 8 | 0 | 0 | 14 | 12 | |
| ind13 | 4 | 2 | 2 | 4 | 2 | 4 | 4 | 0 | 0 | 2 | 4 | 2 |

Now our data is in a `dist` object. All of the redundant entries in the data have been removed.

Right now, our data reflects **similarity** (i.e. **high numbers reflect greater association between individuals**). We need to convert it to **dissimilarity**. Dissimilarity is simply the opposite of similarity. We can convert similarity to dissimilarity by subtracting each data value from the maximum of the data.

```
# Convert to dissimilarity
dist = max(dist) - dist
dist # Print dist
```

| | ind1 | ind2 | ind3 | ind4 | ind5 | ind6 | ind7 | ind8 | ind9 | ind10 | ind11 | ind12 |
|-------|------|------|------|------|------|------|------|------|------|-------|-------|-------|
| ind2 | 14 | | | | | | | | | | | |
| ind3 | 14 | 0 | | | | | | | | | | |
| ind4 | 6 | 14 | 16 | | | | | | | | | |
| ind5 | 14 | 0 | 6 | 14 | | | | | | | | |
| ind6 | 14 | 8 | 0 | 14 | 6 | | | | | | | |
| ind7 | 8 | 14 | 14 | 0 | 14 | 12 | | | | | | |
| ind8 | 16 | 14 | 16 | 14 | 12 | 14 | 12 | | | | | |
| ind9 | 16 | 12 | 14 | 14 | 16 | 16 | 14 | 16 | | | | |
| ind10 | 8 | 12 | 12 | 8 | 14 | 16 | 0 | 14 | 16 | | | |
| ind11 | 2 | 12 | 12 | 4 | 12 | 16 | 8 | 16 | 12 | 2 | | |
| ind12 | 4 | 16 | 16 | 8 | 12 | 12 | 8 | 16 | 16 | 2 | 4 | |
| ind13 | 12 | 14 | 14 | 12 | 14 | 12 | 12 | 16 | 16 | 14 | 12 | 14 |

Now we're ready to run our analyses!

3.3 Hierarchical Cluster Analysis

Remember from lecture there are 4 types of hierarchical cluster analysis:

1. Single linkage
2. Average linkage
3. Complete linkage
4. Ward linkage

Let's run through them one by one:

3.3.1 Single linkage

We can run all 4 types of cluster analysis using the `hclust()` R function:

```
# run single linkage cluster analysis
clust_1 = hclust(dist, method = 'single')
clust_1 # print object
```

Call:

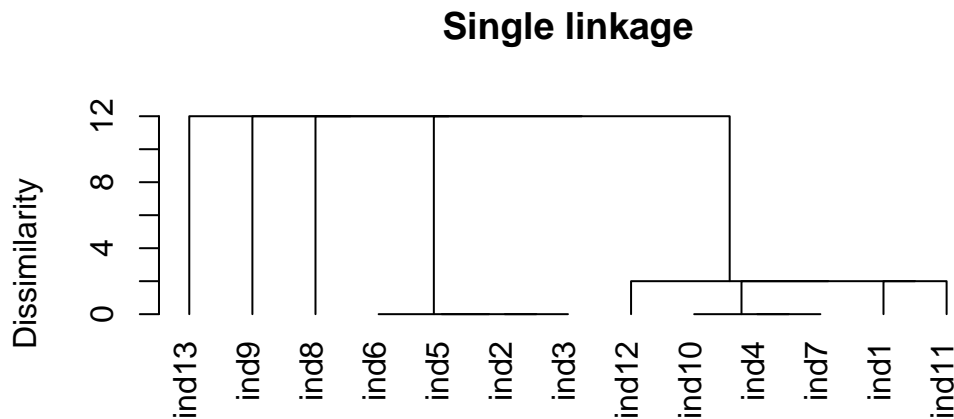
```
hclust(d = dist, method = "single")
```

Cluster method : single

Number of objects: 13

Printing the `hclust` object doesn't really tell us much. For more detail, we're going to have to plot it:

```
# Plot single linkage tree
plot(clust_1, hang = -1, main = 'Single linkage',
     ylab = 'Dissimilarity', # Label y axis
     xlab = '', sub = '') # Remove x-axis label
```



This outputs a tree showing the associations between our individual monkeys. dissimilarity is on the y-axis. The greater the distance between individuals on the y-axis, the greater their dissimilarity. Our tree has grouped the monkeys according to how frequently they interact with each other. For example. individuals 2, 3, 5, and 6 interact often, as evidenced by their low dissimilarity.

But how well does this tree fit the data? To answer that question, we need to calculate the cophenetic correlation coefficient (CCC):

```
# Calculate CCC
coph_1 = cophenetic(clust_1) # Get cophenetic
ccc_1 = cor(coph_1, dist) # Calculate correlation of the cophenetic with the data
ccc_1 # Print CCC
```

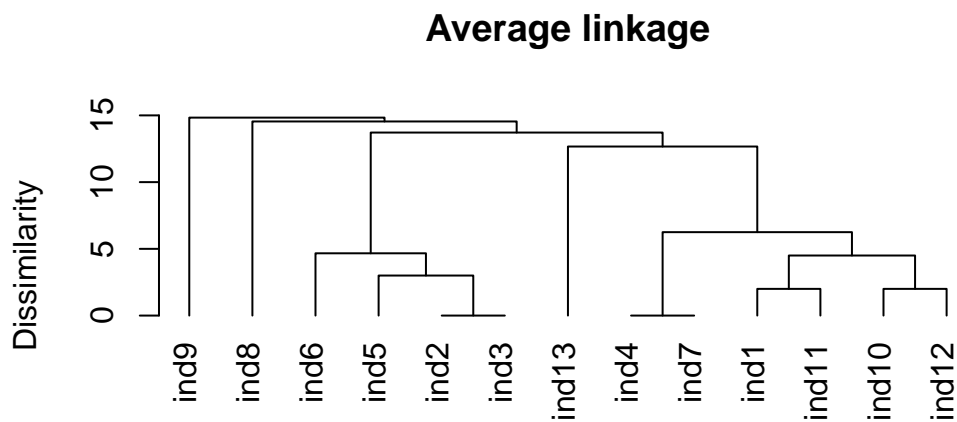
```
[1] 0.9036043
```

That's a pretty high correlation coefficient, indicating our dendrogram represented the structure in the original data very well. Let's try some other methods:

3.3.2 Average Linkage

```
# run cluster analysis
clust_2 = hclust(dist, method = 'average')

# Plot
plot(clust_2, hang = -1, main = 'Average linkage', ylab = 'Dissimilarity', xlab = '', sub =
```



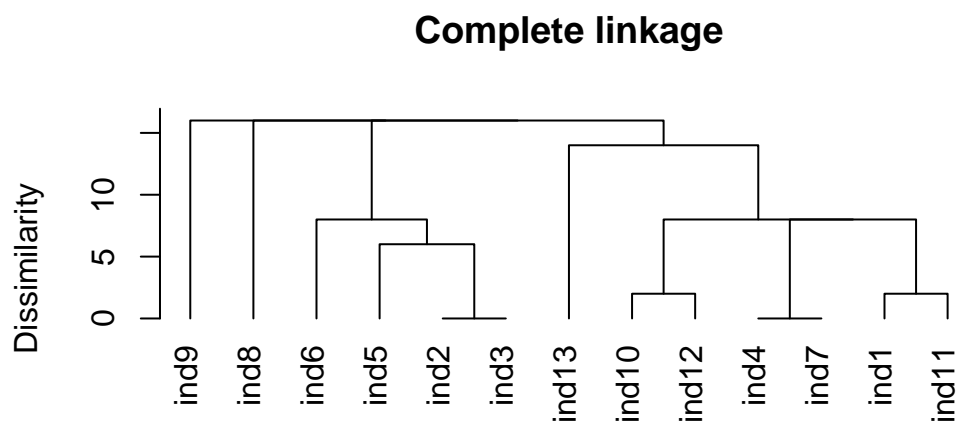
```
# Calculate CCC
coph_2 = cophenetic(clust_2)
ccc_2 = cor(coph_2, dist)
ccc_2
```

```
[1] 0.9288949
```

3.3.3 Complete Linkage

```
# run cluster analysis
clust_3 = hclust(dist, method = 'complete')

# Plot
plot(clust_3, hang = -1, main = 'Complete linkage', ylab = 'Dissimilarity', xlab = '', sub =
```



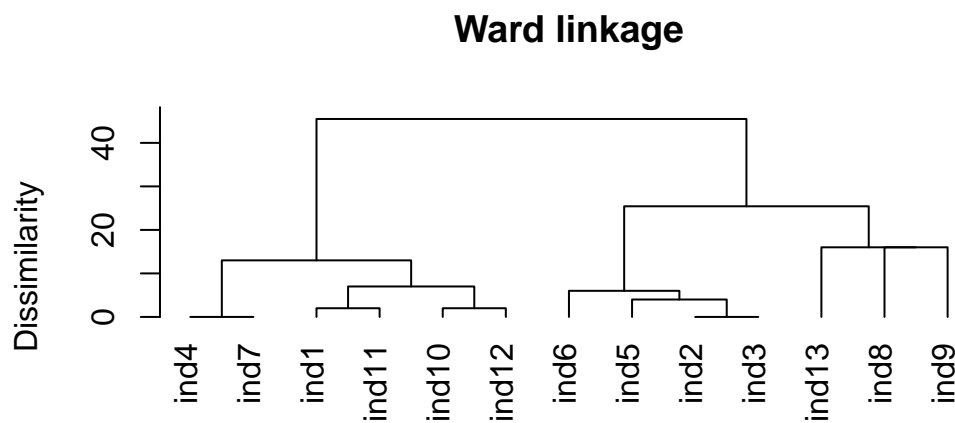
```
# Calculate CCC
coph_3 = cophenetic(clust_3)
ccc_3 = cor(coph_3, dist)
ccc_3
```

```
[1] 0.9141956
```

3.3.4 Ward Linkage

```
# run cluster analysis
clust_4 = hclust(dist, method = 'ward.D')

# Plot
plot(clust_4, hang = -1, main = 'Ward linkage', ylab = 'Dissimilarity', xlab = '', sub = '')
```



```
# Calculate CCC
coph_4 = cophenetic(clust_4)
ccc_4 = cor(coph_4, dist)
ccc_4
```

```
[1] 0.7633159
```

Each method gives a slightly different tree and CCC value. Where are they similar? Where do they differ? Which one(s) would you trust? Why?

3.4 Multidimensional Scaling

Another method we can use to test for associations between our monkeys is multidimensional scaling (MDS). There are two types of MDS: non-metric, and metric MDS. Let's start with non-metric MDS.

3.4.1 Non-Metric MDS

```
# Run non-metric MDS - metaMDS
mds1 = metaMDS(dist, wascores = F)
```

```
Run 0 stress 0.07592385
Run 1 stress 0.07875788
Run 2 stress 0.0757299
... New best solution
... Procrustes: rmse 0.008580763  max resid 0.02329352
Run 3 stress 0.07366297
... New best solution
... Procrustes: rmse 0.1937486  max resid 0.565543
Run 4 stress 0.08055018
Run 5 stress 0.07575137
Run 6 stress 0.07875788
Run 7 stress 0.08055015
Run 8 stress 0.08568213
Run 9 stress 0.08055013
Run 10 stress 0.07239302
... New best solution
... Procrustes: rmse 0.1707907  max resid 0.515529
Run 11 stress 0.0757299
Run 12 stress 0.07575137
Run 13 stress 0.07575137
Run 14 stress 0.1426486
Run 15 stress 0.0757299
Run 16 stress 0.08055013
Run 17 stress 0.07366297
Run 18 stress 0.07305571
Run 19 stress 0.072393
... New best solution
... Procrustes: rmse 0.0001192849  max resid 0.0002606965
... Similar to previous best
```

```
Run 20 stress 0.0757299
*** Best solution repeated 1 times
```

```
# Print mds results
mds1
```

```
Call:
metaMDS(comm = dist, wascores = F)

global Multidimensional Scaling using monoMDS

Data:      dist
Distance:  user supplied

Dimensions: 2
Stress:     0.072393
Stress type 1, weak ties
Best solution was repeated 1 time in 20 tries
The best solution was from try 19 (random start)
Scaling: centring, PC rotation
Species: scores missing
```

By default, metaMDS has two dimensions. This MDS has a stress value of 0.072. Remember from lecture that stress < 0.10 is a “good representation”, so this MDS result is pretty good. If we want, we can test different numbers of dimensions (k) and create a scree plot to find the best one:

```
# Create a container object
scree = data.frame(k = 1:5, stress = NA)

# Loop through k 1 to 5
for(k in 1:5){

  # Run MDS
  mds = metaMDS(dist, wascores = F, k = k) # Set k to our loop index

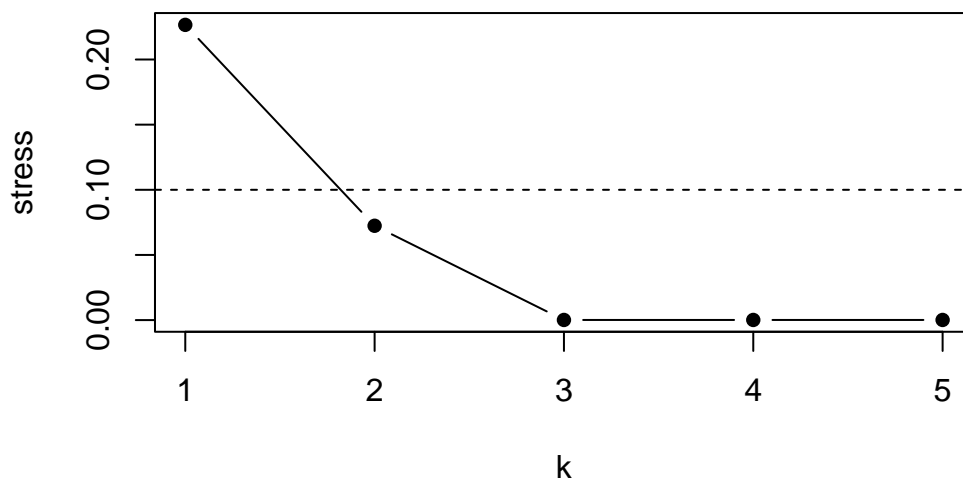
  # Pull out stress
  scree[k, 'stress'] = mds$stress # Fill kth row of the column 'stress' in scree
} # End loop
```



```
# Print results
scree
```

```
      k      stress
1 1 2.266009e-01
2 2 7.239300e-02
3 3 9.111961e-05
4 4 9.891182e-05
5 5 9.443829e-05
```

```
# Make scree plot
plot(stress ~ k, data = scree, # Plot stress against k
     type = 'b', # Lines and points
     pch = 16) # Point 16 (filled circle)
abline(h = 0.1, lty = 'dashed') # Plot a dashed line at 0.1
```

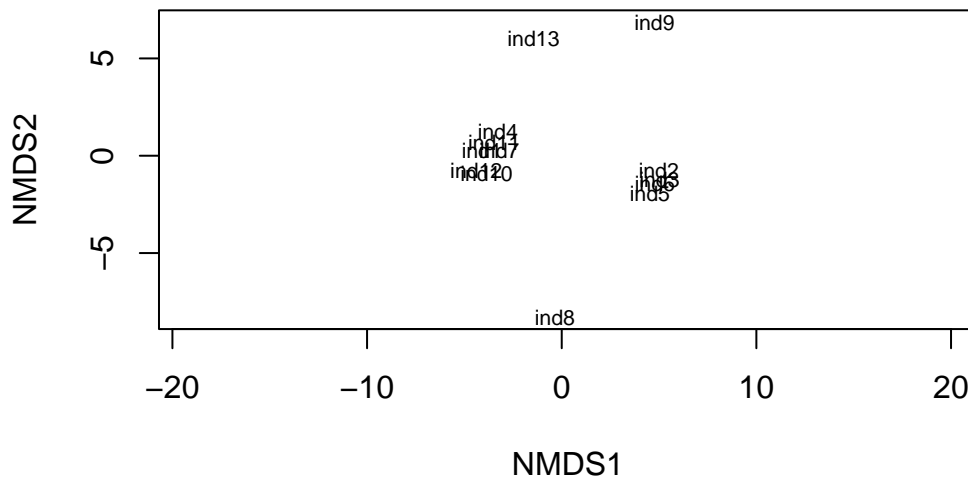


We have an elbow at $k=3$, but we also get warnings that our dataset may be too small using $k=3$. The stress at $k=2$ is low enough that we can stick to using that.

Let's plot our results:

```
# Plot result
plot(mds1, type = 't')
```

species scores not available



Here we've plotted the values of our two MDS dimensions against each other for each individual. Similar to the cluster analysis, we see certain individuals are grouped together. Is it the same groups of individuals? What does that tell you about your results?

Let's try a different non-metric MDS function:

```
# Run non-metric MDS - isoMDS
mds2 = isoMDS(dist)
```

Error in isoMDS(dist): zero or negative distance between objects 2 and 3

Uh oh. This function doesn't like zeroes in the data. Let's fix that by translating our data to proportions, and adding a small increment.

```
# Translate to proportions
dist2 = dist/max(dist)

# Add an increment
dist2 = dist2 + 0.0001

# Print new dist
dist2
```

```
      ind1  ind2  ind3  ind4  ind5  ind6  ind7  ind8  ind9  ind10
ind2 0.8751
ind3 0.8751 0.0001
ind4 0.3751 0.8751 1.0001
ind5 0.8751 0.0001 0.3751 0.8751
ind6 0.8751 0.5001 0.0001 0.8751 0.3751
ind7 0.5001 0.8751 0.8751 0.0001 0.8751 0.7501
ind8 1.0001 0.8751 1.0001 0.8751 0.7501 0.8751 0.7501
ind9 1.0001 0.7501 0.8751 0.8751 1.0001 1.0001 0.8751 1.0001
ind10 0.5001 0.7501 0.7501 0.5001 0.8751 1.0001 0.0001 0.8751 1.0001
ind11 0.1251 0.7501 0.7501 0.2501 0.7501 1.0001 0.5001 1.0001 0.7501 0.1251
ind12 0.2501 1.0001 1.0001 0.5001 0.7501 0.7501 0.5001 1.0001 1.0001 0.1251
ind13 0.7501 0.8751 0.8751 0.7501 0.8751 0.7501 0.7501 1.0001 1.0001 0.8751
      ind11  ind12
ind2
ind3
ind4
ind5
ind6
ind7
ind8
ind9
ind10
ind11
ind12 0.2501
ind13 0.7501 0.8751
```

Let's make sure this doesn't mess with our results:

```
# Run non-metric MDS - metaMDS
mds1 = metaMDS(dist2, wascores = F)
```

```

Run 0 stress 0.07575137
Run 1 stress 0.07239301
... New best solution
... Procrustes: rmse 0.2253594  max resid 0.6639861
Run 2 stress 0.1396354
Run 3 stress 0.08569059
Run 4 stress 0.072393
... New best solution
... Procrustes: rmse 0.0001059207  max resid 0.0002256804
... Similar to previous best
Run 5 stress 0.07358653
Run 6 stress 0.08055026
Run 7 stress 0.072393
... New best solution
... Procrustes: rmse 8.467439e-05  max resid 0.0001816759
... Similar to previous best
Run 8 stress 0.0757299
Run 9 stress 0.07358653
Run 10 stress 0.0757299
Run 11 stress 0.1183846
Run 12 stress 0.07875788
Run 13 stress 0.07572991
Run 14 stress 0.1180402
Run 15 stress 0.120356
Run 16 stress 0.07358653
Run 17 stress 0.07366297
Run 18 stress 0.08055013
Run 19 stress 0.07358653
Run 20 stress 0.08055013
*** Best solution repeated 1 times

```

```

# Print mds results
mds1

```

```

Call:
metaMDS(comm = dist2, wascores = F)

global Multidimensional Scaling using monoMDS

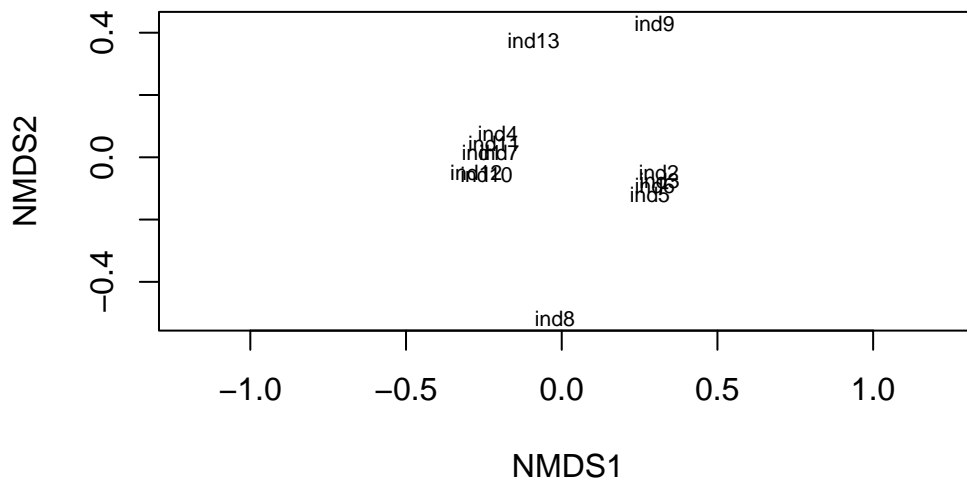
Data:      dist2
Distance: user supplied

```

Dimensions: 2
Stress: 0.072393
Stress type 1, weak ties
Best solution was repeated 1 time in 20 tries
The best solution was from try 7 (random start)
Scaling: centring, PC rotation
Species: scores missing

```
# Plot result  
plot(mds1, type = 't')
```

species scores not available



The values have shifted around a bit but the structure and interpretation of the plot is the same. Let's continue on:

```
# Run non-metric MDS - isoMDS  
mds2 = isoMDS(dist2)
```

```
initial value 24.760322  
iter 5 value 14.153502
```

```
iter 10 value 12.254154
iter 15 value 11.639473
iter 20 value 11.360460
final value 11.341572
converged
```

```
# Print output
mds2
```

```
$points
      [,1]      [,2]
ind1 0.5060798 -0.103253718
ind2 -0.6157097  0.285522725
ind3 -0.6133549  0.310223762
ind4  0.5008471 -0.144443835
ind5 -0.6264450  0.279477605
ind6 -0.6228134  0.325577873
ind7  0.4940123 -0.147103149
ind8 -0.6783876 -0.680257989
ind9 -0.2575043 -1.075686777
ind10 0.5482152 -0.033286470
ind11 0.5478840 -0.082682890
ind12 0.5895070 -0.001005794
ind13 0.2276697  1.066918657

$stress
[1] 11.34157
```

The modelling algorithms seems to be a little different, and we end up with a different stress result - in this case, one that is above the 10% threshold (note that stress is in % in this function, unlike metaMDS where it is in proportion). Let's try another scree plot:

```
# Create a container object
scree = data.frame(k = 1:5, stress = NA)

# Loop through k 1 to 5
for(k in 1:5){

  # Run MDS
  mds = isoMDS(dist2, k = k) # Set k to our loop index

  # Pull out stress
```

```

    scree[k,'stress'] = mds$stress # Fill kth row of the column 'stress' in scree
  } # End loop

```

```

# Print results
scree

```

```

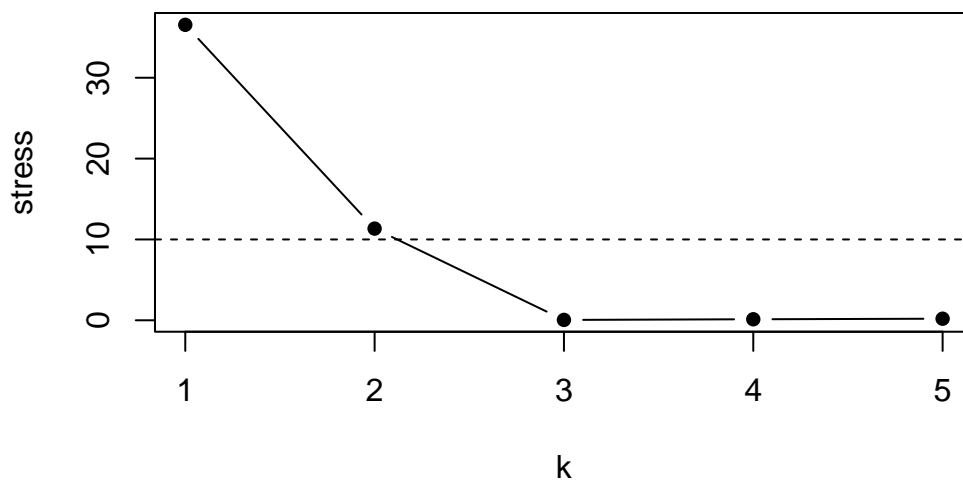
      k      stress
1 1 36.54857293
2 2 11.34157190
3 3  0.04614441
4 4  0.12630298
5 5  0.19439990

```

```

# Make scree plot
plot(stress ~ k, data = scree, # Plot stress against k
     type = 'b', # Lines and points
     pch = 16) # Point 16 (filled circle)
abline(h = 10, lty = 'dashed') # Plot a dashed line at 0.1

```



In this case, it seems we're better off using 3 dimensions:

```
# Run non-metric MDS - isoMDS
mds2 = isoMDS(dist2, k = 3)
```

```
initial value 18.960422
iter 5 value 11.725940
iter 10 value 6.417141
iter 15 value 4.149185
iter 20 value 1.466748
iter 25 value 0.764657
iter 30 value 0.449114
iter 35 value 0.302911
iter 40 value 0.156116
iter 45 value 0.087536
iter 50 value 0.046144
final value 0.046144
stopped after 50 iterations
```

```
# Print output
mds2
```

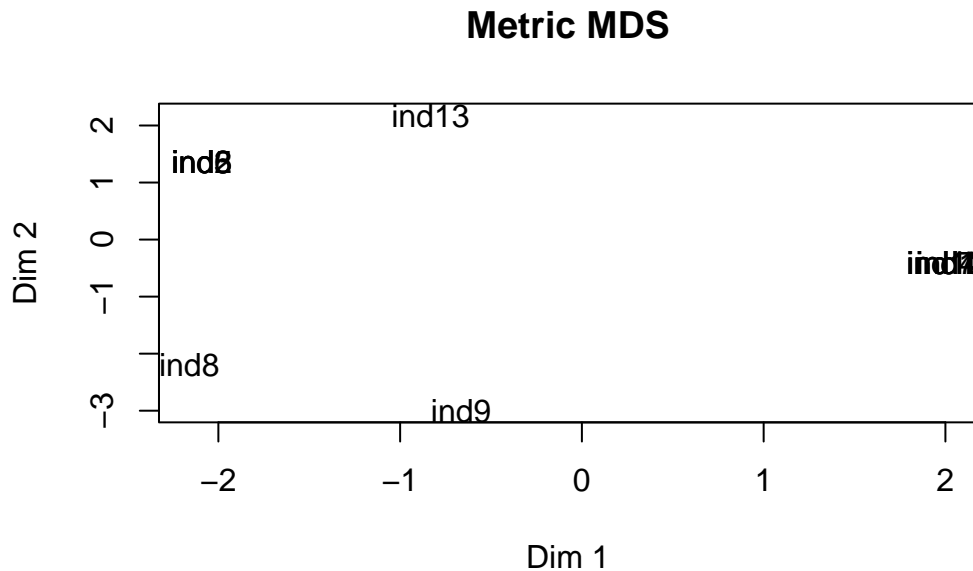
```
$points
      [,1]      [,2]      [,3]
ind1  2.0028765 -0.4079841 -0.4202376
ind2 -2.0901193  1.3657278  1.2720375
ind3 -2.0910832  1.3666943  1.2729356
ind4  2.0066696 -0.4042391 -0.4161698
ind5 -2.0896284  1.3631915  1.2769256
ind6 -2.0921681  1.3637290  1.2724192
ind7  2.0032691 -0.4111298 -0.4185160
ind8 -2.1600174 -2.2033842 -1.8938700
ind9 -0.6641520 -2.9972846  2.5707229
ind10 2.0009095 -0.4046552 -0.4185896
ind11 2.0041166 -0.4044773 -0.4197885
ind12 2.0019583 -0.4023552 -0.4223419
ind13 -0.8326313  2.1761669 -3.2555274

$stress
[1] 0.04614441
```

Let's plot our results:


```
# Plot isoMDS
plot(mds2$points[,1], mds2$points[,2], # MDS dimension 1 and 2 values
     type = 'n', # Don't plot any points
     xlab = 'Dim 1', ylab = 'Dim 2', main = 'Metric MDS') # Labelling

# Plot individual names
text(mds2$points[,1], mds2$points[,2], rownames(data))
```



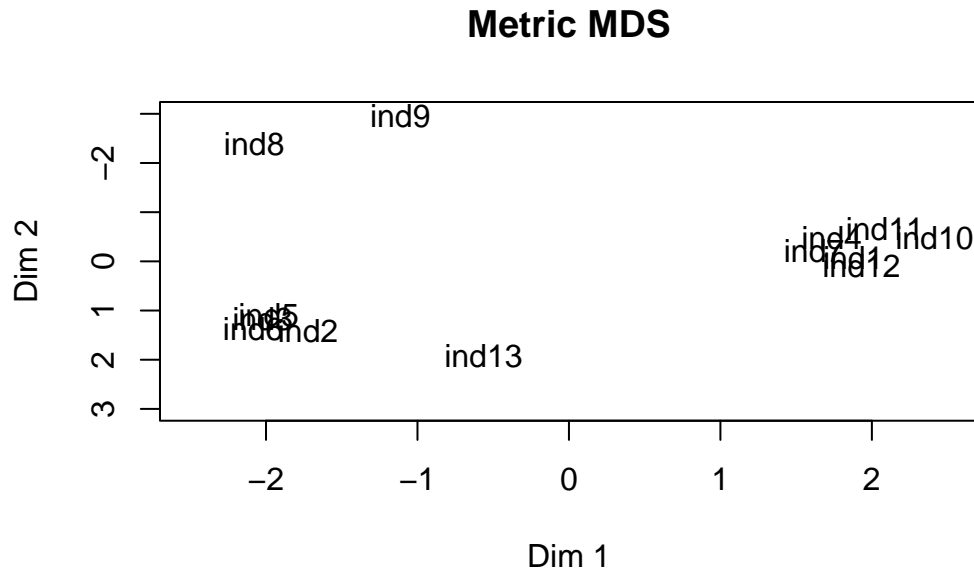
All of our grouped individuals are plotted on top of each other. Let's try adding some random jiggle so we can see them

```
# Plot isoMDS
plot(mds2$points[,1], mds2$points[,2], # MDS dimension 1 and 2 values
     type = 'n', # Don't plot any points
     xlab = 'Dim 1', ylab = 'Dim 2', main = 'Metric MDS', # Labelling
     xlim = c(-2.5, 2.5), ylim = c(3, -3)) # Set axis limits

# Set random seed for consistency
set.seed(1212)

# Plot individual names
text(mds2$points[,1] + rnorm(13, 0, 0.2), # Add random values pulled from a
```

```
mds2$points[,2] + rnorm(13, 0, 0.2), # normal distribution with mean 0, sd 0.2
rownames(data)) # Add names
```



That's a bit better. We can also add some color to this plot if we want - say, individuals 6 to 9 are juveniles:

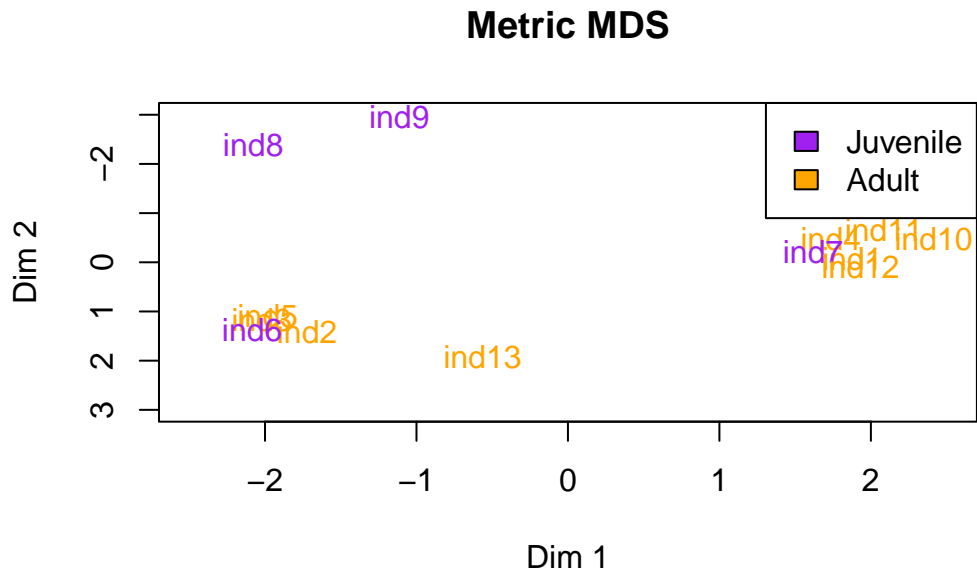
```
# Plot isoMDS
plot(mds2$points[,1], mds2$points[,2], # MDS dimension 1 and 2 values
     type = 'n', # Don't plot any points
     xlab = 'Dim 1', ylab = 'Dim 2', main = 'Metric MDS', # Labelling
     xlim = c(-2.5, 2.5), ylim = c(3, -3)) # Set axis limits

# Set random seed for consistency
set.seed(1212)

# Juvenile identifier
ad = c(rep(1,5), rep(0,4), rep(1,4)) # ad is 1 for first 5 and last 4
ad
```

```
[1] 1 1 1 1 1 0 0 0 0 1 1 1 1
```

```
# Plot individual names
text(mds2$points[,1] + rnorm(13, 0, 0.2), # Add random values pulled from a
     mds2$points[,2] + rnorm(13, 0, 0.2), # normal distribution with mean 0, sd 0.2
     rownames(data), # Add names
     col = ifelse(ad == 0, 'purple', 'orange')) # color
# Add a legend
legend('topright', legend = c('Juvenile', 'Adult'), fill = c('purple', 'orange'))
```



Does this plot match the previous one, and/or the cluster analyses?

3.4.2 Metric MDS

We can run metric MDS using the `cmdscale()` function:

```
# run metric MDS
mds3 = cmdscale(dist, eig = T)
mds3
```

```
$points
      [,1]      [,2]
ind1  5.84977914  2.7981654
ind2 -7.46899061 -0.4452479
```

```

ind3  -7.87360160  2.4742095
ind4   6.04970828 -1.1964346
ind5  -6.77887791  2.8518073
ind6  -7.21161499  3.9150940
ind7   4.79289905 -0.9896933
ind8  -2.46317365 -5.3304368
ind9  -1.86216019 -9.7770302
ind10  5.45394235  1.1317599
ind11  5.27120921 -0.1097836
ind12  6.20052885  3.6063451
ind13  0.04035206  1.0712450

$eig
 [1] 4.150450e+02  1.794715e+02  1.684147e+02  1.309366e+02  6.931537e+01
 [6] 5.811388e+01  3.306204e+01  1.780496e+01 -5.684342e-14 -8.643935e+00
[11] -2.208342e+01 -4.468321e+01 -7.952271e+01

$x
NULL

$ac
[1] 0

$GOF
[1] 0.4844901 0.5545014

```

For metric MDS, we look at goodness of fit (GOF) instead of stress to assess how well the analysis worked. GOF is similar to an R^2 value, where numbers closer to 1 indicate a better fit (though be wary of overfitting!). There are two different GOF values for each metric MDS.

As with the other MDS functions, k defaults to 2. We can make another scree plot:

```

# Create a container object
scree = data.frame(k = 1:5, GOF1 = NA, GOF2 = NA)

# Loop through k 1 to 5
for(k in 1:5){

  # Run MDS
  mds = cmdscale(dist, eig = T, k = k) # Set k to our loop index

  # Pull out stress
  scree[k,c(2,3)] = mds$GOF # Fill kth row of the GOF columns in scree
}

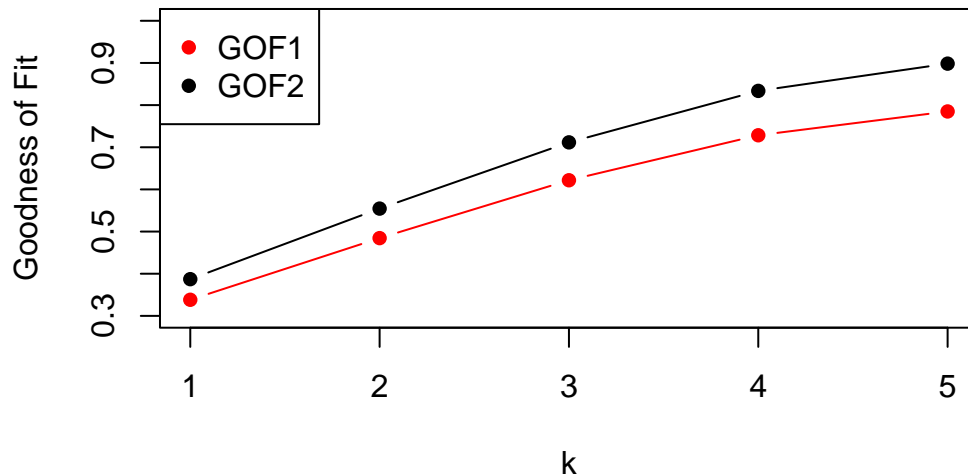
```

```
} # End loop
```

```
# Print results  
scree
```

```
      k      GOF1      GOF2  
1 1 0.3382331 0.3871096  
2 2 0.4844901 0.5545014  
3 3 0.6217365 0.7115806  
4 4 0.7284408 0.8337043  
5 5 0.7849281 0.8983543
```

```
# Make scree plot  
plot(GOF2 ~ k, data = scree, # Plot stress against k  
      type = 'b', # Lines and points  
      pch = 16, # Point 16 (filled circle)  
      ylab = 'Goodness of Fit', ylim = c(0.3, 1))  
points(GOF1 ~ k, data = scree, type = 'b', pch = 16, col = 'red') # Add second GOF value  
abline(h = 0.1, lty = 'dashed') # Plot a dashed line at 0.1  
legend('topleft', pch = 16, legend = c('GOF1', 'GOF2'), col = c('red', 'black')) # Add legend
```



Goodness of fit scales linearly, so what k to use is more of a judgement call.

```
# run metric MDS
mds3 = cmdscale(dist, k=4, eig = T)
mds3
```

\$points

| | [,1] | [,2] | [,3] | [,4] |
|-------|-------------|------------|------------|------------|
| ind1 | 5.84977914 | 2.7981654 | 1.2685787 | -0.6738375 |
| ind2 | -7.46899061 | -0.4452479 | 2.6496404 | 2.1091477 |
| ind3 | -7.87360160 | 2.4742095 | 3.4326421 | 1.2339175 |
| ind4 | 6.04970828 | -1.1964346 | -0.9306802 | -1.2319775 |
| ind5 | -6.77887791 | 2.8518073 | -1.2465315 | 2.3989342 |
| ind6 | -7.21161499 | 3.9150940 | -1.9699687 | -2.4369605 |
| ind7 | 4.79289905 | -0.9896933 | -2.7067201 | -0.2043847 |
| ind8 | -2.46317365 | -5.3304368 | -9.4034890 | 2.0126521 |
| ind9 | -1.86216019 | -9.7770302 | 5.2905629 | -1.1532342 |
| ind10 | 5.45394235 | 1.1317599 | 0.3565324 | 4.1657272 |
| ind11 | 5.27120921 | -0.1097836 | 4.1678975 | 1.4238728 |
| ind12 | 6.20052885 | 3.6063451 | -0.2995254 | 1.5298300 |
| ind13 | 0.04035206 | 1.0712450 | -0.6089393 | -9.1736869 |

\$eig

| | | | | | |
|------|---------------|---------------|---------------|---------------|---------------|
| [1] | 4.150450e+02 | 1.794715e+02 | 1.684147e+02 | 1.309366e+02 | 6.931537e+01 |
| [6] | 5.811388e+01 | 3.306204e+01 | 1.780496e+01 | -5.684342e-14 | -8.643935e+00 |
| [11] | -2.208342e+01 | -4.468321e+01 | -7.952271e+01 | | |

\$x

NULL

\$ac

[1] 0

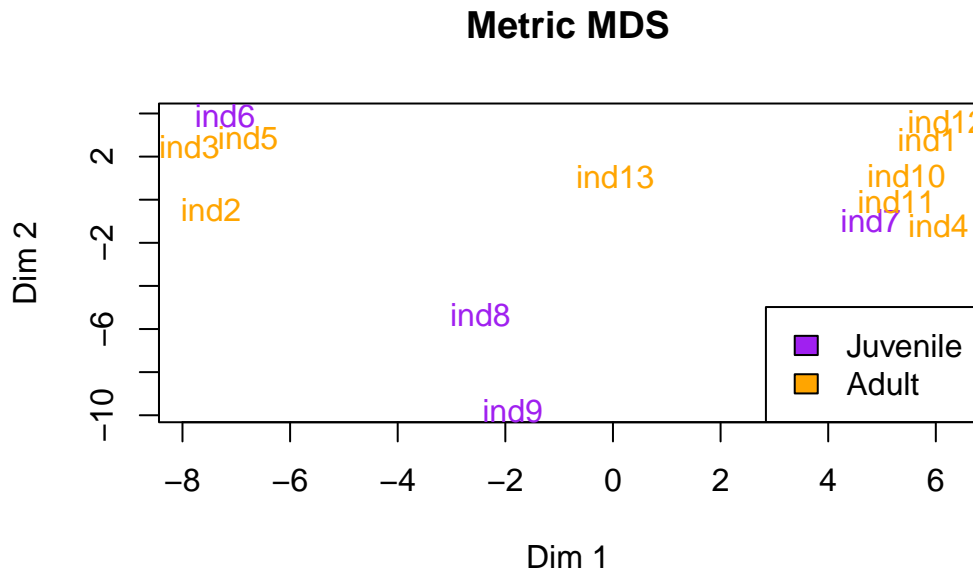
\$GOF

[1] 0.7284408 0.8337043

Let's plot the first two dimensions:

```
# Plot metric MDS
plot(mds3$points[,1], mds3$points[,2], # MDS dimension 1 and 2 values
     type = 'n', # Don't plot any points
     xlab = 'Dim 1', ylab = 'Dim 2', main = 'Metric MDS') # Labelling
```

```
# Plot individual names
text(mds3$points[,1], # Add random values pulled from a
     mds3$points[,2], # normal distribution with mean 0, sd 0.2
     rownames(data), # Add names
     col = ifelse(ad == 0, 'purple', 'orange')) # color
# Add a legend
legend('bottomright', legend = c('Juvenile', 'Adult'), fill = c('purple', 'orange'))
```



3.4.3 3D Plotting (Optional)

It may not be necessary, but if your MDS has more than 2 dimensions, you can try plotting it in three dimensions and see if it helps:

```
library(plot3D)
```

Warning: no DISPLAY variable so Tk is not available

```
# Prepare data to plot
x = mds3$points[,1]
y = mds3$points[,2]
```

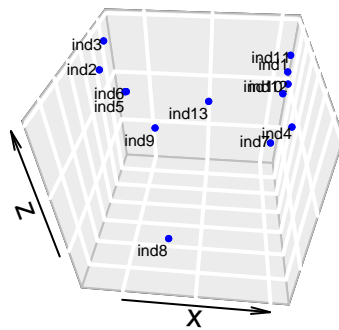
```

z = mds3$points[,3]

# Create 3D plot
scatter3D(x,y,z, colvar = NULL, col = 'blue',
          pch = 16, cex = 0.5, bty = 'g', theta = 5)

# Add text
text3D(x,
       # Add some jiggle to the labels
       y+rnorm(13, mean = 0, sd = 0.5), z + rnorm(13, mean = 0, sd = 0.5),
       labels = names(mds3$points[,1]), add = T, colkey = F,
       cex = 0.5, adj = 1, d = 2)

```



3.5 Mantel Test (Graduate Students Only)

We can infer to some extent whether juveniles and adults preferentially associate with each other from our colored MDS plots, but we can also test it statistically using a Mantel test. To run the Mantel test, we need to convert our adult index into a `dist` object:


```
# Create dist matrix for adults
ad_dist = dist(ad)
ad_dist
```

```
      1 2 3 4 5 6 7 8 9 10 11 12
2    0
3    0 0
4    0 0 0
5    0 0 0 0
6    1 1 1 1 1
7    1 1 1 1 1 0
8    1 1 1 1 1 0 0
9    1 1 1 1 1 0 0 0
10   0 0 0 0 0 0 1 1 1 1
11   0 0 0 0 0 0 1 1 1 1 0
12   0 0 0 0 0 0 1 1 1 1 0 0
13   0 0 0 0 0 0 1 1 1 1 0 0 0
```

Note this is dissimilarity: adult-juvenile pairs are assigned 1, and same-class pairs are assigned 0.

The Mantel test looks for correlation between this matrix and our original dissimilarity matrix, and statistically tests if the associations are different from what we would expect due to chance.

```
# Run mantel test
library(ade4)
mantel.rtest(ad_dist, dist, nrepet = 999)
```

```
Warning in is.euclid(m1): Zero distance(s)
```

```
Warning in is.euclid(m2): Zero distance(s)
```

```
Monte-Carlo test
```

```
Call: mantelnoneuclid(m1 = m1, m2 = m2, nrepet = nrepet)
```

```
Observation: 0.1686576
```

```
Based on 999 replicates
```

```
Simulated p-value: 0.073
```

```
Alternative hypothesis: greater
```

| Std.Obs | Expectation | Variance |
|-------------|--------------|-------------|
| 1.369210491 | -0.001062026 | 0.015364686 |

It's very close, but we don't have statistically significant evidence that juveniles and adults associate preferentially with each other in this case.

3.6 Tips for your Assignment:

Some things you may want to think about for your assignment:

1. How would you pick which cluster analyses and MDS analyses are best for your data? Are they conceptual, or do they have to do with the results? Do they agree?
2. How would you interpret your statistical results biologically? You don't have to be right, but don't be vague, and don't contradict your results.

4 Assignment 1d: Multiple Linear Regression

This assignment is all about multiple linear regression. Linear regression is used to model relationships between a dependent (response) variable and one or more independent (predictor) variables. Multiple linear regression involves multiple predictor variables.

For this tutorial we're going to use `Schoenemann.csv`, derived from the data in [this](#) paper.

4.1 Looking at the data

```
# Read in data
data = read.csv('Schoenemann.csv')

# View data structure
head(data)
```

| | Order | Family | Genus | Species | Location | Mass | Fat | FFWT | CNS |
|---|-----------|-------------|---------|------------|----------|--------|--------|--------|--------|
| 1 | Carnivora | Felidae | Felis | canadensis | Alaska | 7688.0 | 1120.0 | 6568.0 | 105.09 |
| 2 | Carnivora | Felidae | Felis | rufus | Virginia | 6152.0 | 738.0 | 5414.0 | 81.75 |
| 3 | Carnivora | Mustelidae | Gulo | luscus | Alaska | 9362.0 | 562.0 | 8800.0 | 85.36 |
| 4 | Carnivora | Mustelidae | Mustela | erminea | Alaska | 183.3 | 3.1 | 180.2 | 6.69 |
| 5 | Carnivora | Mustelidae | Mustela | vison | Virginia | 1032.0 | 66.0 | 966.0 | 18.06 |
| 6 | Carnivora | Procyonidae | Procyon | lotor | Virginia | 6040.0 | 1013.0 | 5027.0 | 58.31 |
| | HEART | MUSCLE | BONE | | | | | | |
| 1 | 27.59 | 4341.45 | 631.18 | | | | | | |
| 2 | 25.45 | 3600.31 | 552.23 | | | | | | |
| 3 | 80.96 | 5271.20 | 879.12 | | | | | | |
| 4 | 1.87 | 104.70 | 21.98 | | | | | | |
| 5 | 7.63 | 581.53 | 80.27 | | | | | | |
| 6 | 36.19 | 2920.69 | 517.78 | | | | | | |

```
dim(data)
```

```
[1] 39 12
```

The Schoenemann dataset contains 39 observations of 12 variables, describing to the morphometry of different species of mammals, along with taxonomic and location information. Let's start by getting rid of the non-morphological data. We won't need it for this assignment.

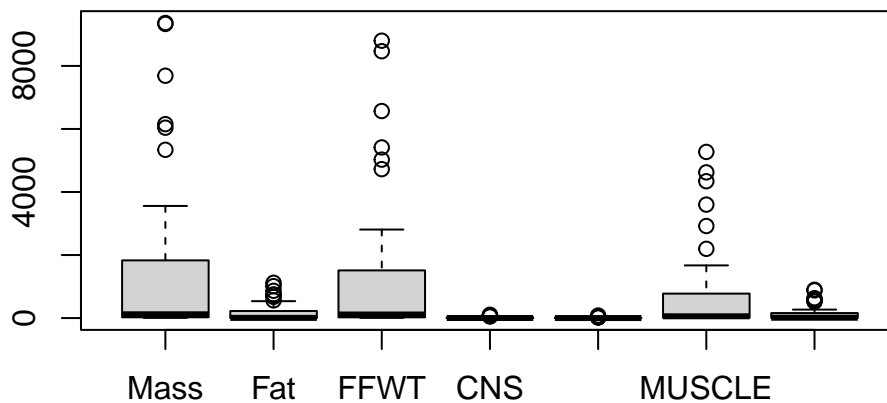
```
# Remove metadata
data = data[,which(colnames(data) == 'Mass'):ncol(data)] # I do it this way to avoid hard co

# check if it worked
head(data)
```

| | Mass | Fat | FFWT | CNS | HEART | MUSCLE | BONE |
|---|--------|--------|--------|--------|-------|---------|--------|
| 1 | 7688.0 | 1120.0 | 6568.0 | 105.09 | 27.59 | 4341.45 | 631.18 |
| 2 | 6152.0 | 738.0 | 5414.0 | 81.75 | 25.45 | 3600.31 | 552.23 |
| 3 | 9362.0 | 562.0 | 8800.0 | 85.36 | 80.96 | 5271.20 | 879.12 |
| 4 | 183.3 | 3.1 | 180.2 | 6.69 | 1.87 | 104.70 | 21.98 |
| 5 | 1032.0 | 66.0 | 966.0 | 18.06 | 7.63 | 581.53 | 80.27 |
| 6 | 6040.0 | 1013.0 | 5027.0 | 58.31 | 36.19 | 2920.69 | 517.78 |

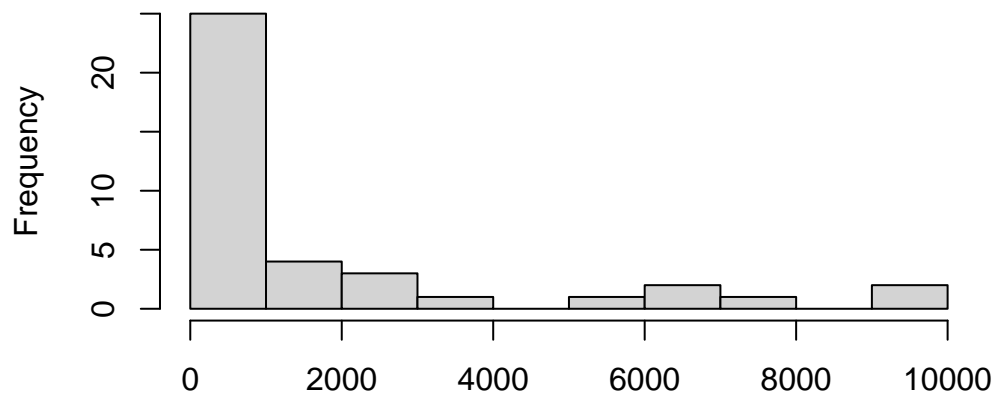
Now we only have the numeric morphological data left. Let's take a look at the data graphically.

```
# Visualize the data
boxplot(data)
```

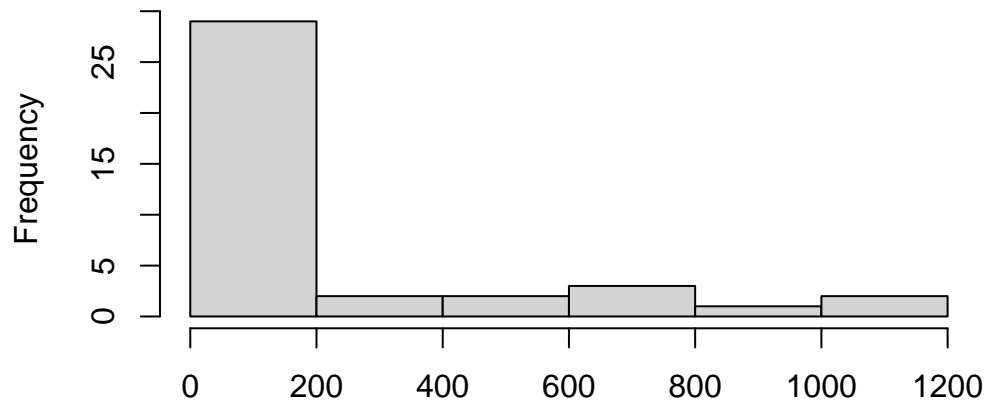


```
# Loop through columns to create histograms
for(i in 1:ncol(data)){hist(data[,i], main = colnames(data)[i], xlab = "")} # Name histogram
```

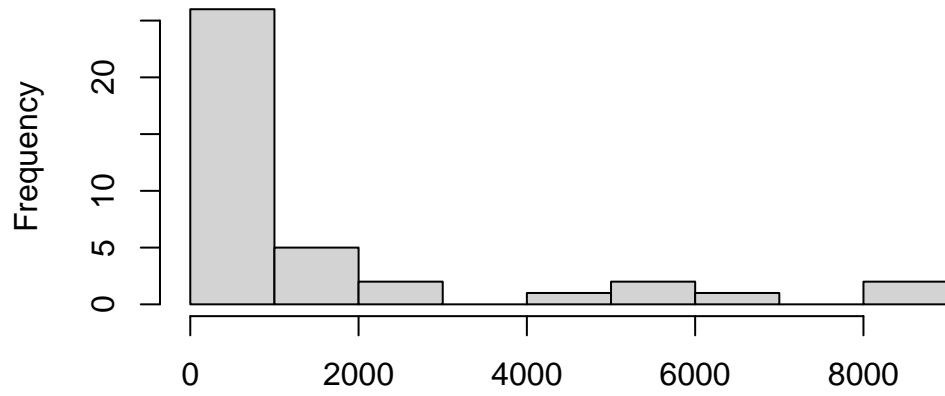
Mass



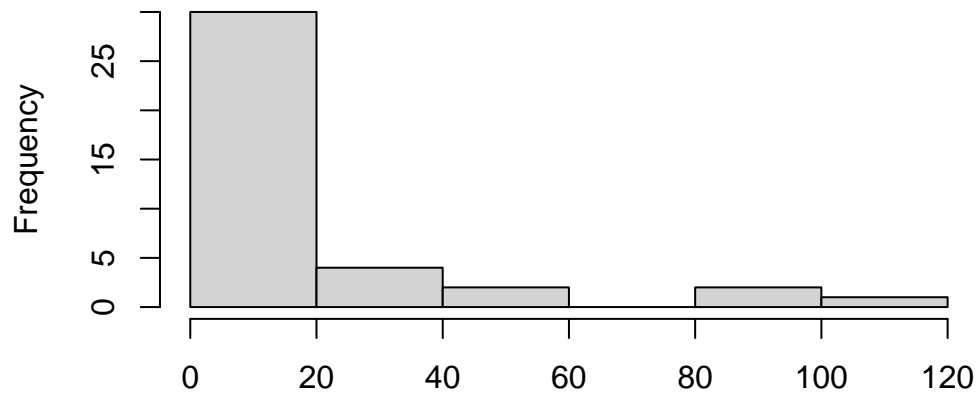
Fat



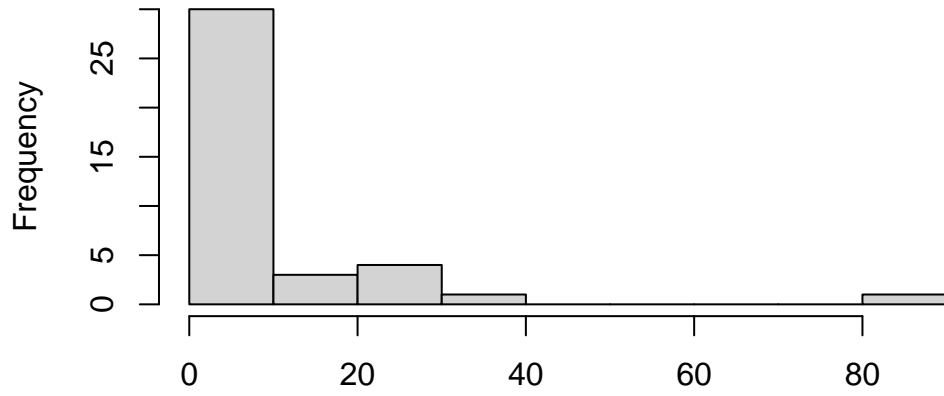
FFWT



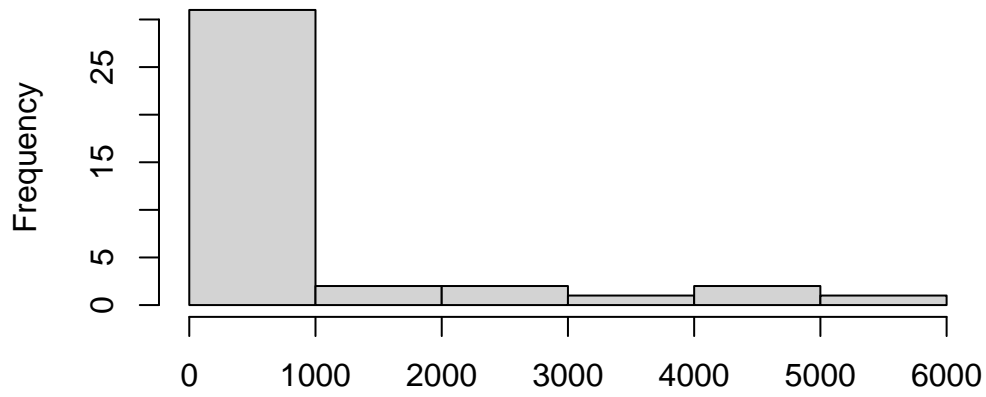
CNS

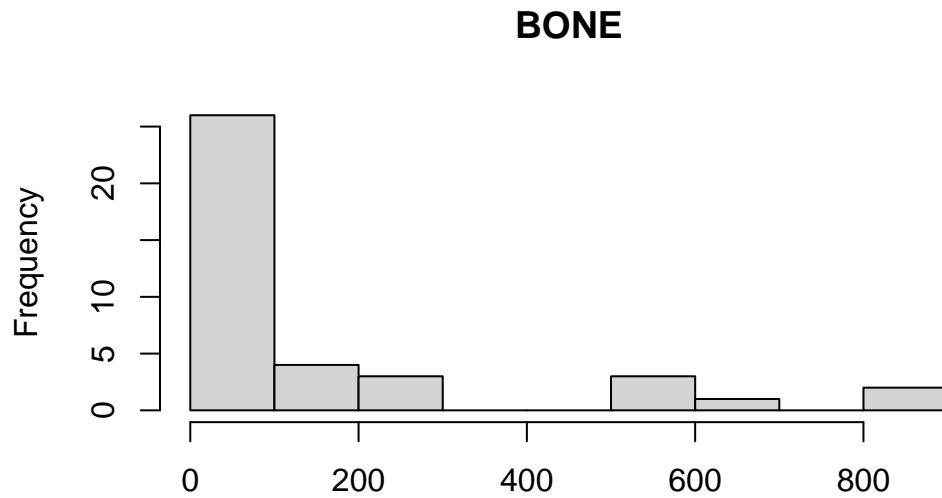


HEART



MUSCLE





4.2 Considering Transformations

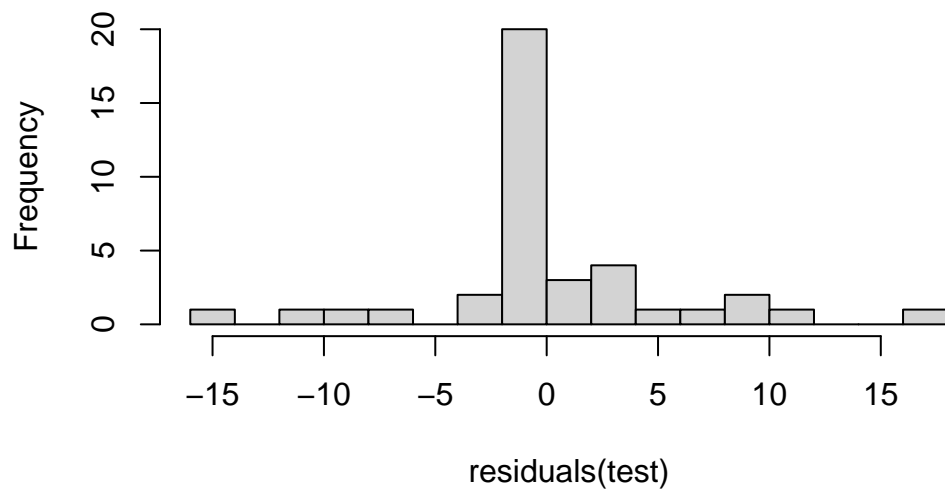
We can see that these data have many more small values, and the data at higher values has higher variance [try e.g. `plot(data$Mass, data$Fat)` to see this]. If we run our regressions on these data, the assumption of heteroscedasticity is going to be violated:

```
# Run a test model and check assumptions
test = lm(CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE, data = data)

# Check for normality as an example

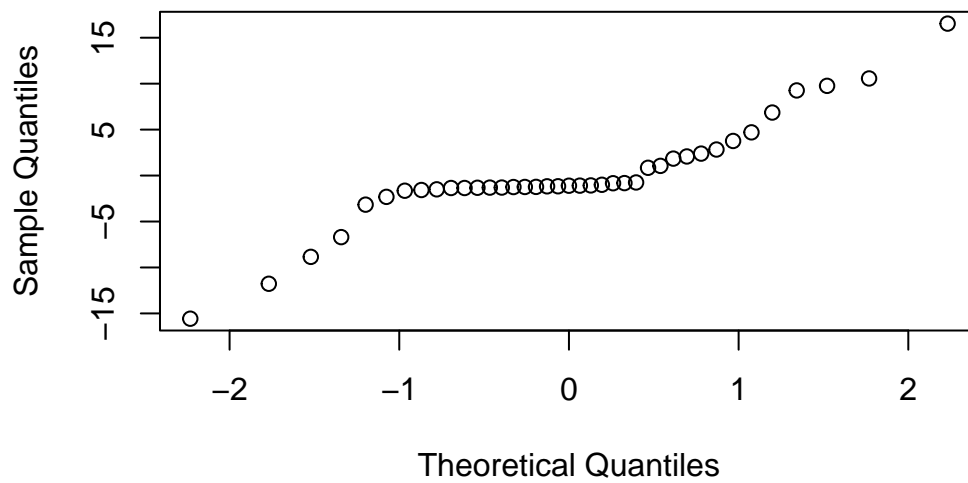
# Residual histogram
hist(residuals(test), 20)
```


Histogram of residuals(test)



```
# QQplot  
qqnorm(residuals(test))
```

Normal Q-Q Plot



```
# Statistical test for normality
shapiro.test(residuals(test))
```

Shapiro-Wilk normality test

```
data: residuals(test)
W = 0.87473, p-value = 0.0004494
```

These diagnostics look... less than ideal.

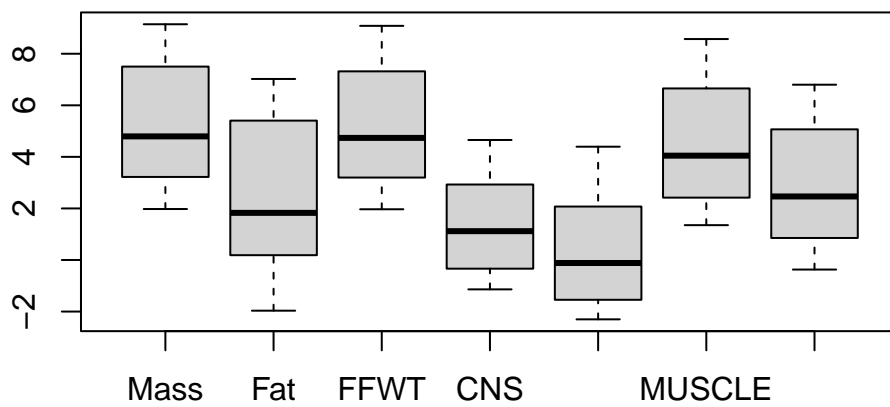
This is a textbook case of when to apply a log transformation to standardize variance - remember logging is the opposite of exponentiating:

```
# Apply log transformation
data_l = log(data)

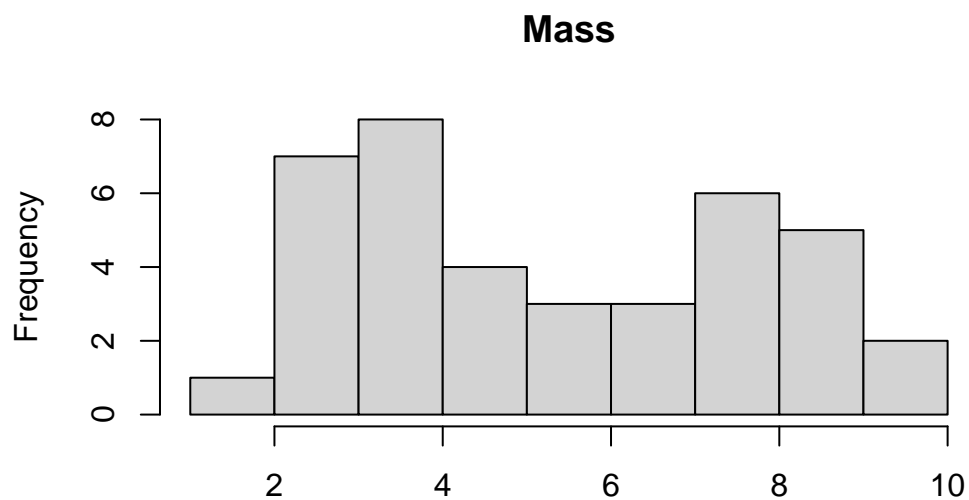
# Check out the new data
head(data_l)
```

| | Mass | Fat | FFWT | CNS | HEART | MUSCLE | BONE |
|---|----------|----------|----------|----------|-----------|----------|----------|
| 1 | 8.947416 | 7.021084 | 8.789965 | 4.654817 | 3.3174534 | 8.375964 | 6.447591 |
| 2 | 8.724533 | 6.603944 | 8.596743 | 4.403666 | 3.2367157 | 8.188775 | 6.313965 |
| 3 | 9.144414 | 6.331502 | 9.082507 | 4.446878 | 4.3939552 | 8.570013 | 6.778921 |
| 4 | 5.211124 | 1.131402 | 5.194067 | 1.900614 | 0.6259384 | 4.651099 | 3.090133 |
| 5 | 6.939254 | 4.189655 | 6.873164 | 2.893700 | 2.0320878 | 6.365663 | 4.385396 |
| 6 | 8.706159 | 6.920672 | 8.522579 | 4.065774 | 3.5887828 | 7.979575 | 6.249550 |

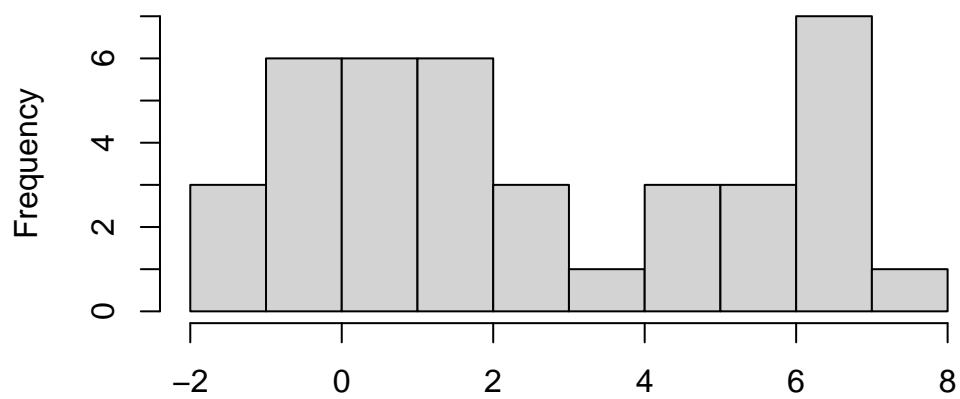
```
# Looking at the data
boxplot(data_l)
```



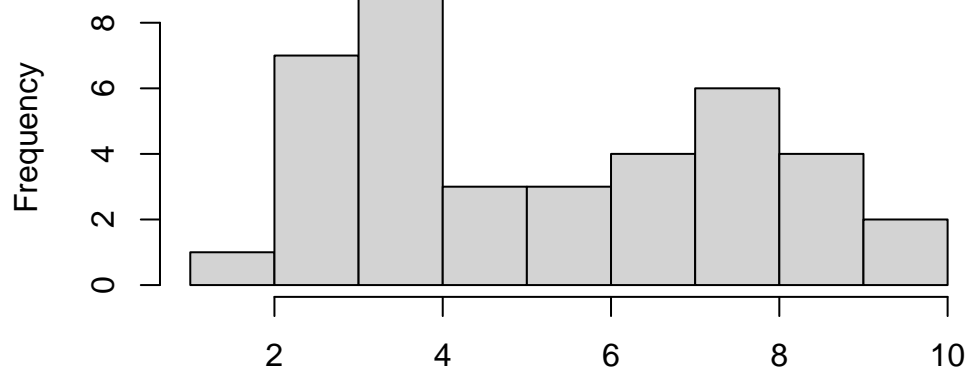
```
# Loop through columns to create histograms
for(i in 1:ncol(data_1)){hist(data_1[,i], main = colnames(data_1)[i], xlab = "")} # Name hist
```



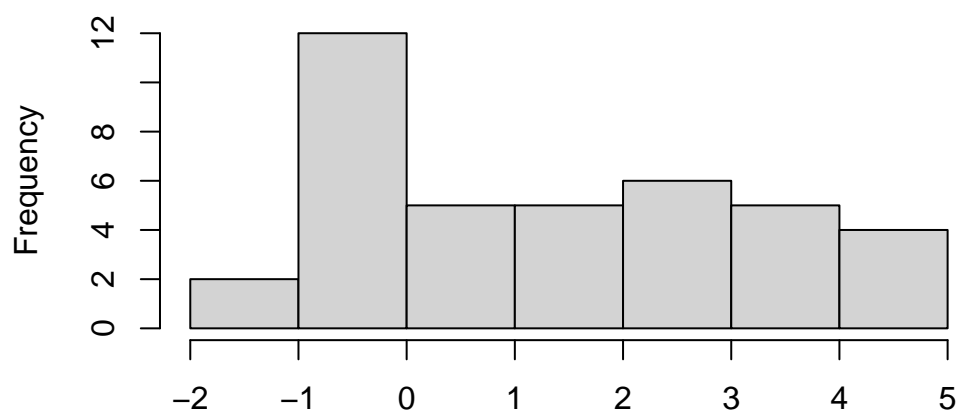
Fat



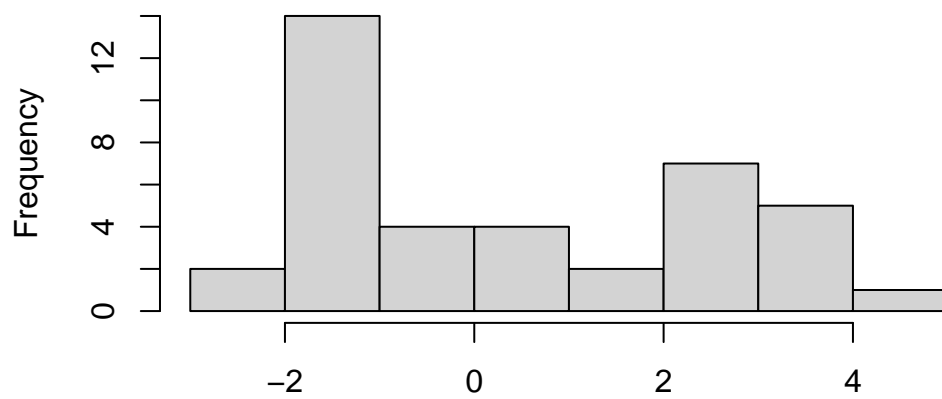
FFWT



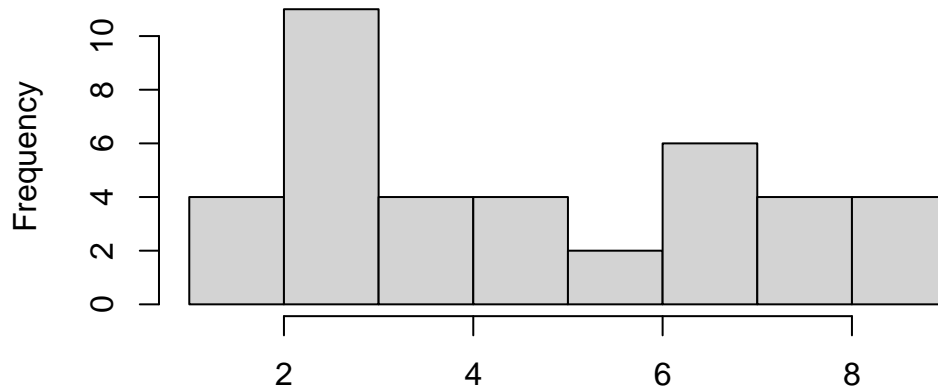
CNS



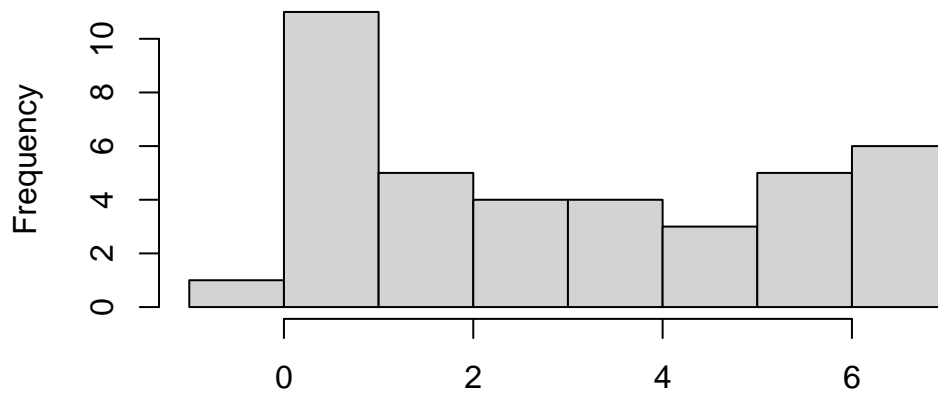
HEART



MUSCLE



BONE



Now our data looks much more uniform.

Always remember that transforming your data incorrectly or unnecessarily can do more harm than good. How do you decide if it is helpful to transform your data?

What is the purpose of transforming your data? Think carefully about these questions for your assignment when you're deciding whether to transform any data.

4.3 Simple Linear Regression

Now that our data is good to go, we're going to run some simple linear regressions on the logged data to predict central nervous system mass (CNS). Simple linear regressions only have one predictor variable, so we will run separate models for each predictor. Linear regression is run using the `lm()` command:

```
# Run simple linear regressions - Mass
m1 = lm(CNS ~ Mass, data = data_1) # run model
summary(m1) # model summary
```

Call:

```
lm(formula = CNS ~ Mass, data = data_1)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|----------|----------|---------|---------|
| -0.77785 | -0.20227 | -0.05439 | 0.19607 | 0.78453 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|------------|
| (Intercept) | -2.79097 | 0.14844 | -18.80 | <2e-16 *** |
| Mass | 0.77105 | 0.02556 | 30.16 | <2e-16 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3657 on 37 degrees of freedom

Multiple R-squared: 0.9609, Adjusted R-squared: 0.9599

F-statistic: 909.7 on 1 and 37 DF, p-value: < 2.2e-16

```
# Run simple linear regressions - Fat
m2 = lm(CNS ~ Fat, data = data_1) # run model
summary(m2) # model summary
```

Call:

```
lm(formula = CNS ~ Fat, data = data_1)
```

Residuals:

| | Min | 1Q | Median | 3Q | Max |
|--|----------|----------|---------|---------|---------|
| | -1.22918 | -0.41510 | 0.01431 | 0.36008 | 1.38000 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|------------|
| (Intercept) | -0.15712 | 0.13223 | -1.188 | 0.242 |
| Fat | 0.59903 | 0.03518 | 17.028 | <2e-16 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6223 on 37 degrees of freedom

Multiple R-squared: 0.8868, Adjusted R-squared: 0.8838

F-statistic: 289.9 on 1 and 37 DF, p-value: < 2.2e-16

```
# Run simple linear regressions - FFWT
m3 = lm(CNS ~ FFWT, data = data_1) # run model
summary(m3) # model summary
```

Call:

```
lm(formula = CNS ~ FFWT, data = data_1)
```

Residuals:

| | Min | 1Q | Median | 3Q | Max |
|--|---------|---------|---------|--------|--------|
| | -0.8057 | -0.2112 | -0.0535 | 0.1907 | 0.7654 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|------------|
| (Intercept) | -2.80193 | 0.14382 | -19.48 | <2e-16 *** |
| FFWT | 0.78494 | 0.02515 | 31.20 | <2e-16 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3539 on 37 degrees of freedom

Multiple R-squared: 0.9634, Adjusted R-squared: 0.9624

F-statistic: 973.7 on 1 and 37 DF, p-value: < 2.2e-16

```
# Run simple linear regressions - HEART
m4 = lm(CNS ~ HEART, data = data_1) # run model
summary(m4) # model summary
```



```
Call:
lm(formula = CNS ~ HEART, data = data_1)

Residuals:
    Min       1Q   Median       3Q      Max
-0.75646 -0.16000 -0.03248  0.15018  0.85234

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.99514    0.05853   17.00  <2e-16 ***
HEART        0.88201    0.02872   30.71  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3594 on 37 degrees of freedom
Multiple R-squared:  0.9622,    Adjusted R-squared:  0.9612
F-statistic:  943 on 1 and 37 DF,  p-value: < 2.2e-16
```

```
# Run simple linear regressions - MUSCLE
m5 = lm(CNS ~ MUSCLE, data = data_1) # run model
summary(m5) # model summary
```

```
Call:
lm(formula = CNS ~ MUSCLE, data = data_1)

Residuals:
    Min       1Q   Median       3Q      Max
-0.82059 -0.15588 -0.00489  0.17331  0.80475

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -2.20579    0.11856  -18.61  <2e-16 ***
MUSCLE       0.76488    0.02296   33.31  <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3323 on 37 degrees of freedom
Multiple R-squared:  0.9677,    Adjusted R-squared:  0.9669
F-statistic: 1109 on 1 and 37 DF,  p-value: < 2.2e-16
```

```
# Run simple linear regressions - BONE
m6 = lm(CNS ~ BONE, data = data_1) # run model
summary(m6) # model summary
```

Call:

```
lm(formula = CNS ~ BONE, data = data_1)
```

Residuals:

| | Min | 1Q | Median | 3Q | Max |
|--|----------|----------|---------|---------|---------|
| | -1.10309 | -0.24611 | 0.01155 | 0.25195 | 0.63931 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|--------------|
| (Intercept) | -1.0497 | 0.1042 | -10.07 | 3.75e-12 *** |
| BONE | 0.7856 | 0.0277 | 28.36 | < 2e-16 *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3879 on 37 degrees of freedom

Multiple R-squared: 0.956, Adjusted R-squared: 0.9548

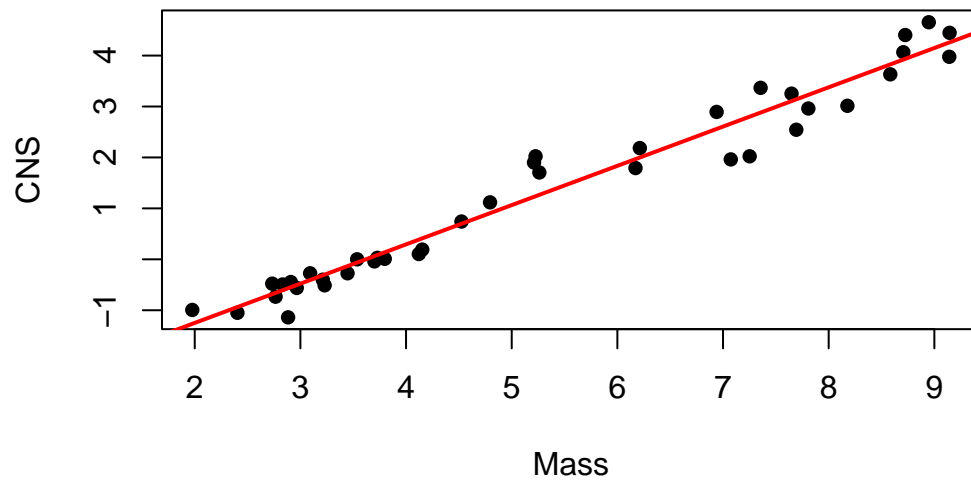
F-statistic: 804.4 on 1 and 37 DF, p-value: < 2.2e-16

In this case, it looks like all of our variables are strong, significant predictors with high R^2 values.

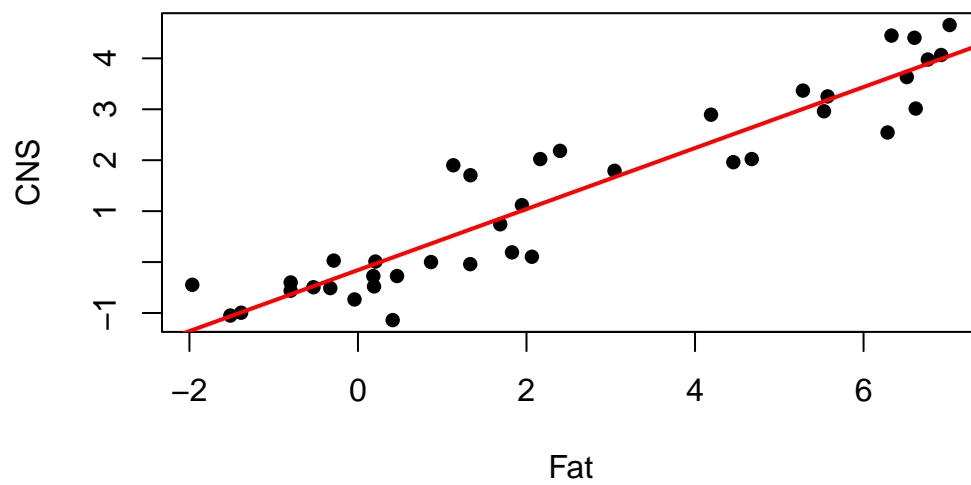
Let's plot all of these regressions:

```
# Plot regressions

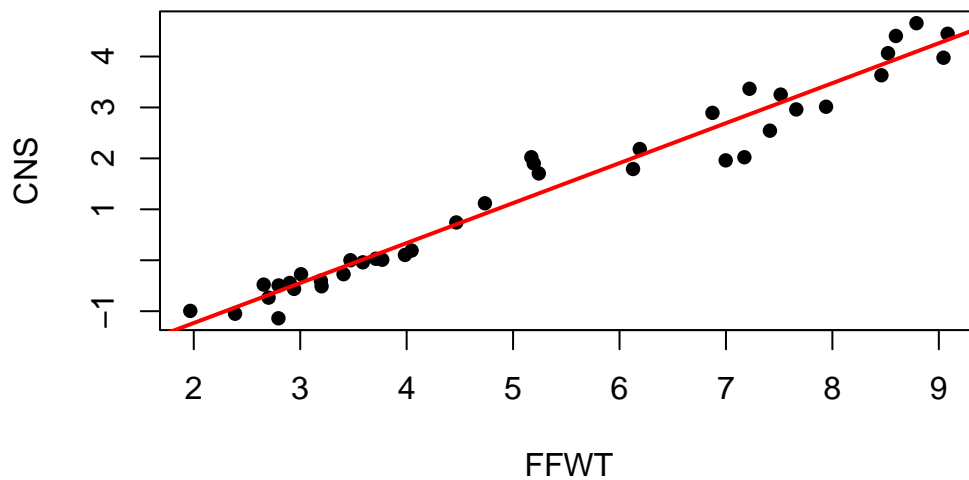
# Plot simple linear regressions - Mass
plot(CNS ~ Mass, data = data_1, pch = 16) # plot points
abline(m1, lwd = 2, col = 'red') # Plot model
```



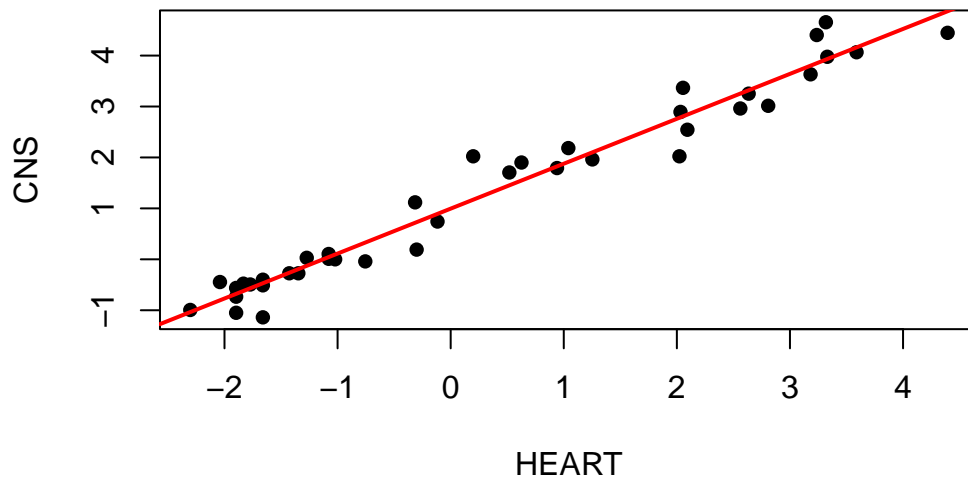
```
# Plot simple linear regressions - Fat
plot(CNS ~ Fat, data = data_1, pch = 16) # plot points
abline(m2, lwd = 2, col = 'red') # Plot model
```



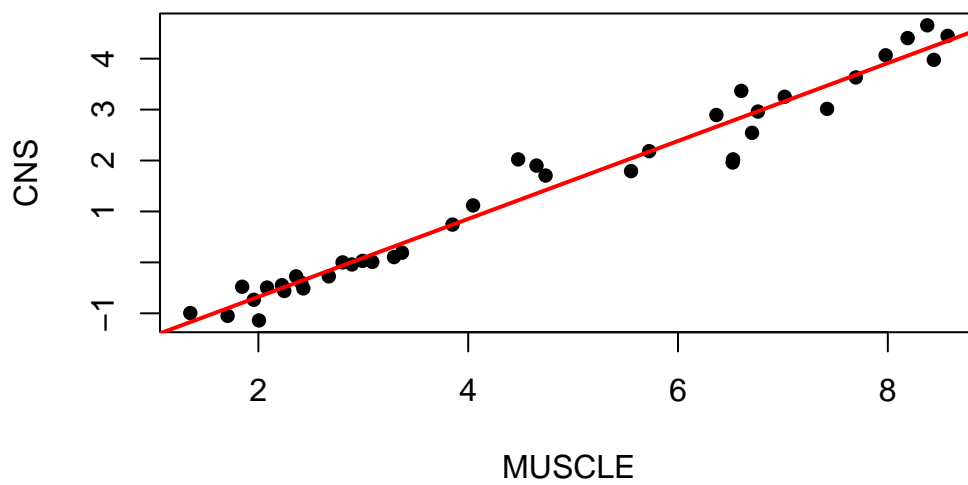
```
# Plot simple linear regressions - FFWT
plot(CNS ~ FFWT, data = data_1, pch = 16) # plot points
abline(m3, lwd = 2, col = 'red') # Plot model
```



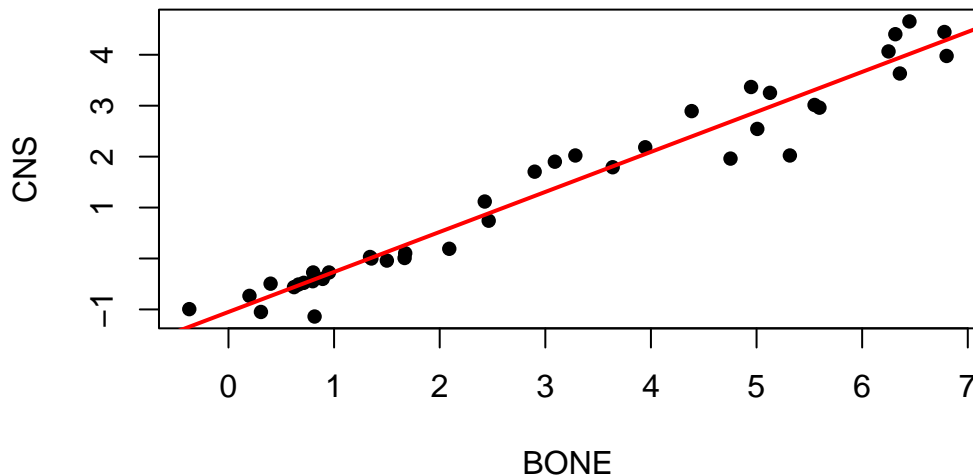
```
# Plot simple linear regressions - HEART
plot(CNS ~ HEART, data = data_1, pch = 16) # plot points
abline(m4, lwd = 2, col = 'red') # Plot model
```



```
# Plot simple linear regressions - MUSCLE
plot(CNS ~ MUSCLE, data = data_1, pch = 16) # plot points
abline(m5, lwd = 2, col = 'red') # Plot model
```



```
# Plot simple linear regressions - BONE
plot(CNS ~ BONE, data = data_1, pch = 16) # plot points
abline(m6, lwd = 2, col = 'red') # Plot model
```



All of the regression slopes are positive. This makes sense - larger animals tend to have larger brains. Remember to always think about whether your results make biological sense.

4.4 Multiple Linear Regression

We've made 6 models using 1 variable. Now, let's try making 1 model with 6 variables:

```
# Run full model
m7 = lm(CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE, data = data_1)
summary(m7)
```

Call:

```
lm(formula = CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE,
    data = data_1)
```

Residuals:

| | Min | 1Q | Median | 3Q | Max |
|--|----------|----------|---------|---------|---------|
| | -0.72690 | -0.12073 | 0.00376 | 0.08672 | 0.85638 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|----------|
| (Intercept) | -0.61138 | 1.18496 | -0.516 | 0.6094 |
| Mass | -0.46867 | 2.14250 | -0.219 | 0.8282 |
| Fat | -0.06818 | 0.15489 | -0.440 | 0.6628 |
| FFWT | -0.02606 | 2.30347 | -0.011 | 0.9910 |
| HEART | 0.41894 | 0.21913 | 1.912 | 0.0649 |
| MUSCLE | 1.03524 | 0.61123 | 1.694 | 0.1000 |
| BONE | -0.06339 | 0.32054 | -0.198 | 0.8445 |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3286 on 32 degrees of freedom

Multiple R-squared: 0.9727, Adjusted R-squared: 0.9676

F-statistic: 190.1 on 6 and 32 DF, p-value: < 2.2e-16

Our full model has a very(!) high R^2 value, and, in contrast to the simple linear regressions where every predictor was significant, none of our predictors are considered significant in the final model at $\alpha = 0.05$. Why do you think that is?

4.5 Checking Assumptions

Now that we've run our full model, it's time to check its assumptions. Those assumptions are **Independence, Linearity, Homoscedasticity, and Normality**. By now, you should be familiar with what these all mean, but let's run through them anyways:

4.5.1 Independence

The assumption of independence states that the value of each data point ('datum', if you will) is independent of all other data points. Some of the ways in which it could be violated may not be testable (e.g. if they have to do with how the data was collected), but what we *can* test for is **autocorrelation**. Autocorrelation translates to self correlation (auto = self). We can test for autocorrelation statistically using a Durbin-Watson test, and visually using an autocorrelation function on the residuals:

```
library(lmtest)
```

```
Loading required package: zoo
```

```
Attaching package: 'zoo'
```

```
The following objects are masked from 'package:base':
```

```
as.Date, as.Date.numeric
```

```
# Durbin-watson test  
dwtest(m7)
```

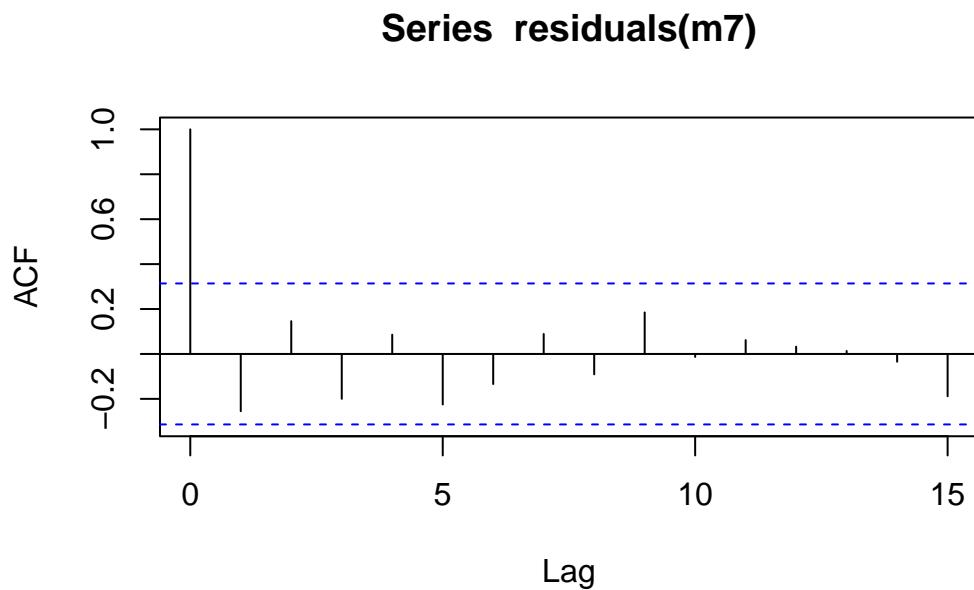
```
Durbin-Watson test
```

```
data: m7
```

```
DW = 2.4315, p-value = 0.8545
```

```
alternative hypothesis: true autocorrelation is greater than 0
```

```
# Autocorrelation function  
acf(residuals(m7))
```



The Durbin-Watson test returns an insignificant p-value, indicating no autocorrelation structure is present. The ACF plots the correlation coefficient of the data against itself using lags. Lag 0 correlates the data against itself, which is always 1. Lag 1 correlates each data point against the point after it, and so on. All of the correlation coefficients are between the blue lines, so again, we have no autocorrelation structure, and we can say independence is respected.

4.5.2 Linearity

The assumption of linearity states that the response variable consistently scales linearly with its predictors. We can test for linearity statistically using Ramsey's RESET test on our model:

```
# Run RESET test  
resettest(m7)
```

RESET test

```
data:  m7  
RESET = 0.12784, df1 = 2, df2 = 30, p-value = 0.8805
```

In this case, the p-value is not significant, meaning the assumption of linearity is respected.

4.5.3 Homoscedasticity

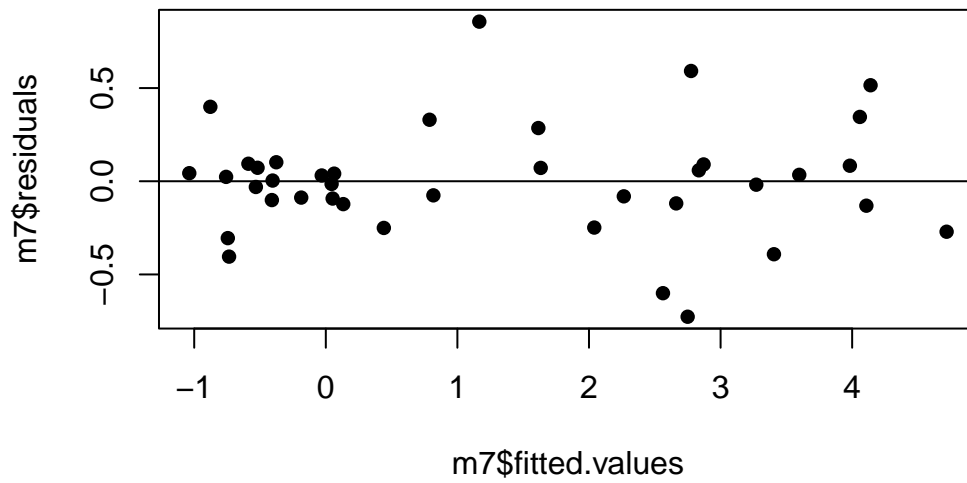
The assumption of homoscedasticity is that the variance in the data is independent of the value of the data - i.e. the variance in the data is consistent. We can test this statistically using the Breusch-Pagan test, and visually by plotting the model residuals against the fitted values.

```
# Run Breusch-Pagan test  
bptest(m7)
```

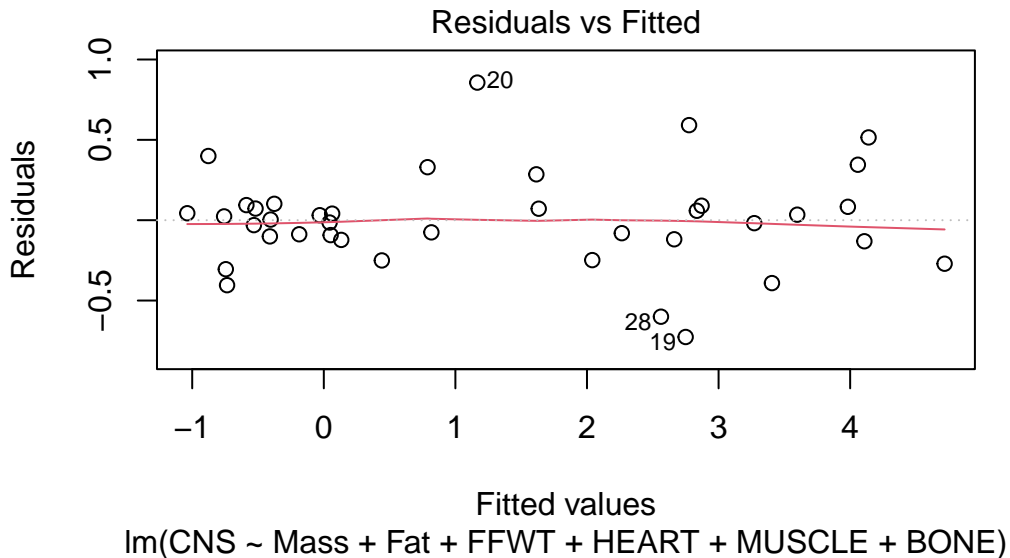
studentized Breusch-Pagan test

```
data:  m7  
BP = 15.974, df = 6, p-value = 0.01389
```

```
# Plot residuals vs fitted
plot(m7$residuals ~ m7$fitted.values, pch = 16); abline(h = 0)
```



```
# Can also be done using plot.lm, ?plot.lm for details
plot(m7, 1)
```



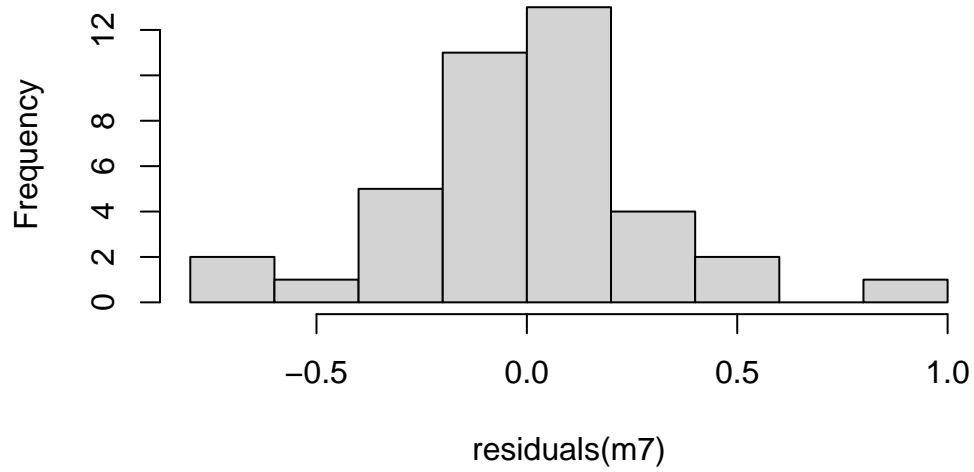
The Breusch-Pagan returns a significant p-value, indicating the assumption of homoscedasticity is violated. We can see in the residuals versus fitted plot that the variance in the data is smaller at low values than it is at higher values (the points on the left of the plot are clustered more closely than they are on the right). Let's come back to this later.

4.5.4 Normality

The assumption of normality states that the residuals of our model should be normally distributed. If they aren't, that would indicate that our model is biased towards overprediction or underprediction in some way. As we did earlier in the transformation section, we can check for normality visually by looking at histograms and QQ plots of our residuals, and statistically by running a Shapiro-Wilk test on the residuals.

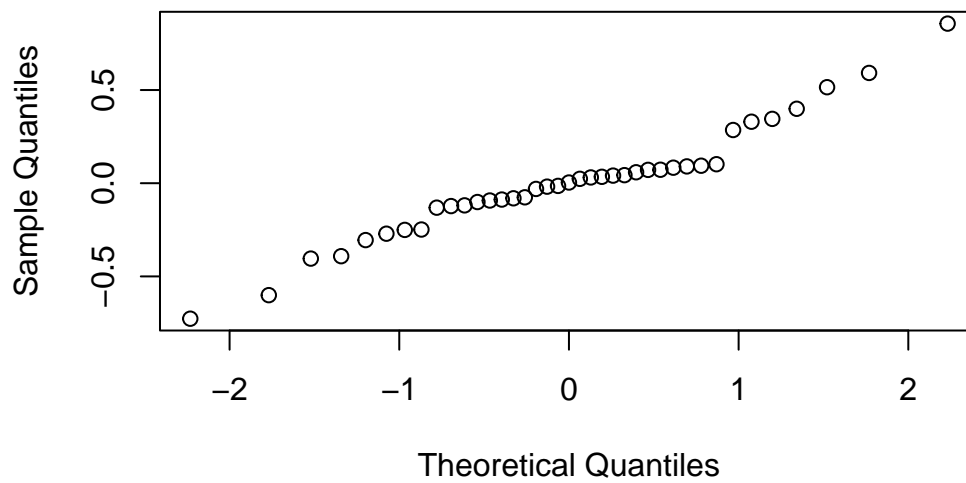
```
# Residual histogram
hist(residuals(m7))
```

Histogram of residuals(m7)

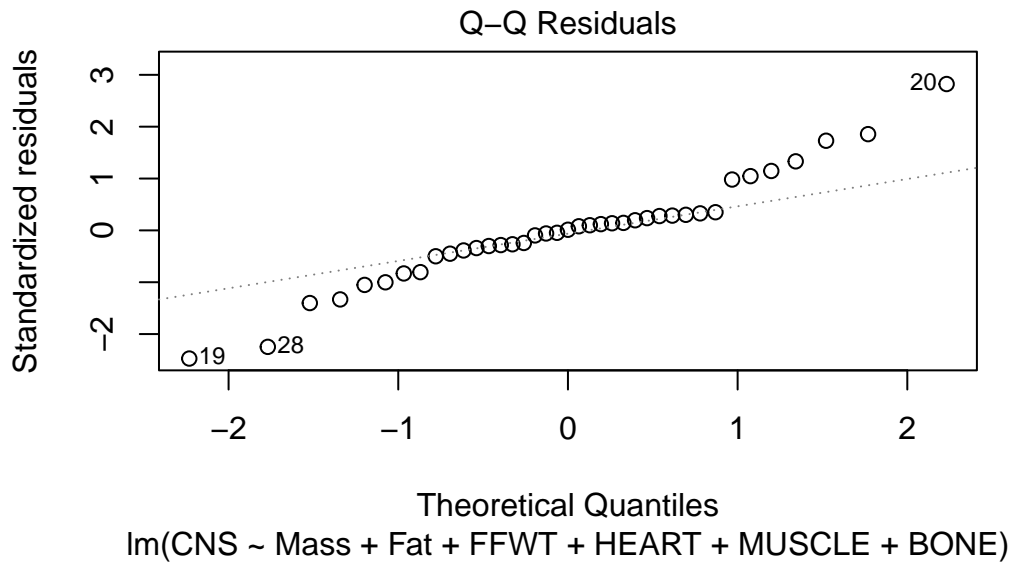


```
# QQplot  
qqnorm(residuals(m7))
```

Normal Q-Q Plot



```
# Can also use plot.lm for qqplot
plot(m7, 2)
```



```
# Statistical test for normality
shapiro.test(residuals(m7))
```

Shapiro-Wilk normality test

```
data: residuals(m7)
W = 0.95391, p-value = 0.1113
```

The Shapiro-Wilk test p-value is not significant (though it comes close), meaning the assumption of normality is respected. The residual histogram largely looks normal, and the QQ plot tails start to pull off the line at high and low values, possibly indicating outliers are causing us some trouble, but not enough to violate the assumption.

4.5.5 What if my assumptions aren't respected?

The typical fixes for violated assumptions are data transformations, and the removal of outliers. In our case, we pass all assumptions except for homoscedasticity. We've already transformed

our data to meet the assumption of normality, so further transformation is likely off the table, though we could potentially try different transformations. We could also try outlier removal - our model diagnostics using `plot.lm()` identify three outliers - 19, 20, and 28. Feel free to play around with removing outliers if you want, although in general it is good practice to only remove outliers where absolutely necessary, as they may contain important biological or other information.

Keep in mind that data transformations and removing outliers both represent trade-offs. Removing outliers may help meet your model assumptions, but you may also be removing data that reflects reality from your model. In that case, is it really helping you to remove outliers? Similarly, transforming your data may help you meet your assumptions, but in a case like this, transforming our data further or in a different way could end up violating other assumptions. Sometimes the best way to deal with violated assumptions is simply to state that they are violated and think about what that means for the interpretation of your model. Play around with all these different ideas, and come up with what you think is best. At the end of the day, a lot of statistical choices are judgement calls, with no perfect right answer.

4.6 Model Selection

In assignment 1b, we created 1 model with 6 variables, then tested if we could get a similarly effective model using fewer variables - i.e. a more **efficient** model. Let's do the same thing here:

```
# Stepwise model selection - forward
m8 = step(lm(CNS ~1, data = data_1),
          scope=(CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE),
          direction='forward')
```

Start: AIC=47.92

CNS ~ 1

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|---------|---------|
| + MUSCLE | 1 | 122.51 | 4.086 | -83.984 |
| + FFWT | 1 | 121.96 | 4.634 | -79.074 |
| + HEART | 1 | 121.82 | 4.780 | -77.869 |
| + Mass | 1 | 121.65 | 4.948 | -76.519 |
| + BONE | 1 | 121.03 | 5.567 | -71.921 |
| + Fat | 1 | 112.27 | 14.327 | -35.056 |
| <none> | | | 126.595 | 47.920 |

Step: AIC=-83.98

CNS ~ MUSCLE

| | Df | Sum of Sq | RSS | AIC |
|---------|----|-----------|--------|---------|
| + HEART | 1 | 0.233798 | 3.8522 | -84.282 |
| + Mass | 1 | 0.212536 | 3.8735 | -84.067 |
| <none> | | | 4.0860 | -83.984 |
| + Fat | 1 | 0.152823 | 3.9332 | -83.470 |
| + FFWT | 1 | 0.139313 | 3.9467 | -83.337 |
| + BONE | 1 | 0.021542 | 4.0645 | -82.190 |

Step: AIC=-84.28

CNS ~ MUSCLE + HEART

| | Df | Sum of Sq | RSS | AIC |
|--------|----|-----------|--------|---------|
| + Mass | 1 | 0.35155 | 3.5007 | -86.014 |
| + Fat | 1 | 0.29737 | 3.5549 | -85.415 |
| + FFWT | 1 | 0.23682 | 3.6154 | -84.756 |
| <none> | | | 3.8522 | -84.282 |
| + BONE | 1 | 0.10287 | 3.7494 | -83.337 |

Step: AIC=-86.01

CNS ~ MUSCLE + HEART + Mass

| | Df | Sum of Sq | RSS | AIC |
|--------|----|-----------|--------|---------|
| <none> | | | 3.5007 | -86.014 |
| + Fat | 1 | 0.040574 | 3.4601 | -84.468 |
| + FFWT | 1 | 0.018388 | 3.4823 | -84.219 |
| + BONE | 1 | 0.000458 | 3.5002 | -84.019 |

`summary(m8)`

Call:

`lm(formula = CNS ~ MUSCLE + HEART + Mass, data = data_1)`

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|----------|----------|---------|---------|
| -0.74267 | -0.11882 | -0.00818 | 0.10790 | 0.84702 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|----------|
| (Intercept) | -0.1840 | 0.8704 | -0.211 | 0.83383 |

| | | | | | |
|--------|---------|--------|--------|---------|----|
| MUSCLE | 1.2436 | 0.4254 | 2.923 | 0.00603 | ** |
| HEART | 0.3855 | 0.1997 | 1.931 | 0.06166 | . |
| Mass | -0.8197 | 0.4372 | -1.875 | 0.06919 | . |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3163 on 35 degrees of freedom

Multiple R-squared: 0.9723, Adjusted R-squared: 0.97

F-statistic: 410.2 on 3 and 35 DF, p-value: < 2.2e-16

Running forward model selection cuts the model down to 3 variables. As with 1b, we can also do backward:

```
# Stepwise model selection - backwards
m9 = step(m7, direction = 'backward')
```

Start: AIC=-80.52

CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| - FFWT | 1 | 0.00001 | 3.4552 | -82.523 |
| - BONE | 1 | 0.00422 | 3.4594 | -82.476 |
| - Mass | 1 | 0.00517 | 3.4604 | -82.465 |
| - Fat | 1 | 0.02092 | 3.4761 | -82.288 |
| <none> | | | 3.4552 | -80.523 |
| - MUSCLE | 1 | 0.30975 | 3.7650 | -79.175 |
| - HEART | 1 | 0.39468 | 3.8499 | -78.305 |

Step: AIC=-82.52

CNS ~ Mass + Fat + HEART + MUSCLE + BONE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| - BONE | 1 | 0.00487 | 3.4601 | -84.468 |
| - Fat | 1 | 0.04499 | 3.5002 | -84.019 |
| - Mass | 1 | 0.05110 | 3.5063 | -83.951 |
| <none> | | | 3.4552 | -82.523 |
| - MUSCLE | 1 | 0.38148 | 3.8367 | -80.439 |
| - HEART | 1 | 0.39621 | 3.8514 | -80.289 |

Step: AIC=-84.47

CNS ~ Mass + Fat + HEART + MUSCLE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| - Fat | 1 | 0.04057 | 3.5007 | -86.014 |
| - Mass | 1 | 0.09476 | 3.5549 | -85.415 |
| <none> | | | 3.4601 | -84.468 |
| - HEART | 1 | 0.40303 | 3.8631 | -82.171 |
| - MUSCLE | 1 | 0.41063 | 3.8707 | -82.095 |

Step: AIC=-86.01
CNS ~ Mass + HEART + MUSCLE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| <none> | | | 3.5007 | -86.014 |
| - Mass | 1 | 0.35155 | 3.8522 | -84.282 |
| - HEART | 1 | 0.37281 | 3.8735 | -84.067 |
| - MUSCLE | 1 | 0.85479 | 4.3555 | -79.493 |

`summary(m9)`

Call:
lm(formula = CNS ~ Mass + HEART + MUSCLE, data = data_1)

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|----------|----------|---------|---------|
| -0.74267 | -0.11882 | -0.00818 | 0.10790 | 0.84702 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|------------|
| (Intercept) | -0.1840 | 0.8704 | -0.211 | 0.83383 |
| Mass | -0.8197 | 0.4372 | -1.875 | 0.06919 . |
| HEART | 0.3855 | 0.1997 | 1.931 | 0.06166 . |
| MUSCLE | 1.2436 | 0.4254 | 2.923 | 0.00603 ** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3163 on 35 degrees of freedom
Multiple R-squared: 0.9723, Adjusted R-squared: 0.97
F-statistic: 410.2 on 3 and 35 DF, p-value: < 2.2e-16

And both:

```
# Stepwise model selection - both
m10 = step(m7, direction = 'both')
```

Start: AIC=-80.52

CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| - FFWT | 1 | 0.00001 | 3.4552 | -82.523 |
| - BONE | 1 | 0.00422 | 3.4594 | -82.476 |
| - Mass | 1 | 0.00517 | 3.4604 | -82.465 |
| - Fat | 1 | 0.02092 | 3.4761 | -82.288 |
| <none> | | | 3.4552 | -80.523 |
| - MUSCLE | 1 | 0.30975 | 3.7650 | -79.175 |
| - HEART | 1 | 0.39468 | 3.8499 | -78.305 |

Step: AIC=-82.52

CNS ~ Mass + Fat + HEART + MUSCLE + BONE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| - BONE | 1 | 0.00487 | 3.4601 | -84.468 |
| - Fat | 1 | 0.04499 | 3.5002 | -84.019 |
| - Mass | 1 | 0.05110 | 3.5063 | -83.951 |
| <none> | | | 3.4552 | -82.523 |
| + FFWT | 1 | 0.00001 | 3.4552 | -80.523 |
| - MUSCLE | 1 | 0.38148 | 3.8367 | -80.439 |
| - HEART | 1 | 0.39621 | 3.8514 | -80.289 |

Step: AIC=-84.47

CNS ~ Mass + Fat + HEART + MUSCLE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| - Fat | 1 | 0.04057 | 3.5007 | -86.014 |
| - Mass | 1 | 0.09476 | 3.5549 | -85.415 |
| <none> | | | 3.4601 | -84.468 |
| + BONE | 1 | 0.00487 | 3.4552 | -82.523 |
| + FFWT | 1 | 0.00067 | 3.4594 | -82.476 |
| - HEART | 1 | 0.40303 | 3.8631 | -82.171 |
| - MUSCLE | 1 | 0.41063 | 3.8707 | -82.095 |

Step: AIC=-86.01

CNS ~ Mass + HEART + MUSCLE

| | Df | Sum of Sq | RSS | AIC |
|----------|----|-----------|--------|---------|
| <none> | | | 3.5007 | -86.014 |
| + Fat | 1 | 0.04057 | 3.4601 | -84.468 |
| - Mass | 1 | 0.35155 | 3.8522 | -84.282 |
| + FFWT | 1 | 0.01839 | 3.4823 | -84.219 |
| - HEART | 1 | 0.37281 | 3.8735 | -84.067 |
| + BONE | 1 | 0.00046 | 3.5002 | -84.019 |
| - MUSCLE | 1 | 0.85479 | 4.3555 | -79.493 |

```
summary(m10)
```

Call:

```
lm(formula = CNS ~ Mass + HEART + MUSCLE, data = data_1)
```

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|----------|----------|---------|---------|
| -0.74267 | -0.11882 | -0.00818 | 0.10790 | 0.84702 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|----------|------------|---------|------------|
| (Intercept) | -0.1840 | 0.8704 | -0.211 | 0.83383 |
| Mass | -0.8197 | 0.4372 | -1.875 | 0.06919 . |
| HEART | 0.3855 | 0.1997 | 1.931 | 0.06166 . |
| MUSCLE | 1.2436 | 0.4254 | 2.923 | 0.00603 ** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3163 on 35 degrees of freedom

Multiple R-squared: 0.9723, Adjusted R-squared: 0.97

F-statistic: 410.2 on 3 and 35 DF, p-value: < 2.2e-16

If you want to get fancy, we can even look at every possible model

```
library(MuMIn)
```

```
# Set global options to avoid error
```

```
options('na.action' = na.fail)
```

```
# Run dredge to get full selection table
```

```
dredge(m7, rank = 'AIC')
```

Fixed term is "(Intercept)"

Global model call: `lm(formula = CNS ~ Mass + Fat + FFWT + HEART + MUSCLE + BONE, data = data_1)`

Model selection table

| | (Intrc) | BONE | Fat | FFWT | HEART | Mass | MUSCL | df | logLik | AIC |
|----|---------|-----------|-----------|----------|---------|---------|--------|----|---------|------|
| 57 | -0.1840 | | | | 0.3855 | -0.8197 | 1.2440 | 5 | -8.332 | 26.7 |
| 43 | -1.1710 | -0.124700 | | | 0.3924 | | 0.5758 | 5 | -8.631 | 27.3 |
| 45 | -0.2475 | | -0.85860 | 0.3612 | | | 1.2890 | 5 | -8.961 | 27.9 |
| 59 | -0.4891 | -0.061790 | | 0.4060 | -0.5708 | | 1.0530 | 6 | -8.104 | 28.2 |
| 47 | -0.5634 | -0.096360 | -0.57650 | 0.4140 | | | 1.0840 | 6 | -8.124 | 28.2 |
| 41 | -1.1430 | | | 0.2965 | | | 0.5107 | 4 | -10.198 | 28.4 |
| 61 | -0.3049 | | 0.58870 | 0.3869 | -1.2410 | | 1.0870 | 6 | -8.229 | 28.5 |
| 49 | -1.7220 | | | | -0.6191 | | 1.3760 | 4 | -10.305 | 28.6 |
| 58 | -0.2132 | -0.01926 | | 0.3888 | -0.8022 | | 1.2420 | 6 | -8.329 | 28.7 |
| 33 | -2.2060 | | | | | | 0.7649 | 3 | -11.347 | 28.7 |
| 44 | -1.2470 | -0.17370 | -0.114900 | | 0.4320 | | 0.6975 | 6 | -8.363 | 28.7 |
| 35 | -2.4620 | | -0.085950 | | | | 0.8664 | 4 | -10.603 | 29.2 |
| 42 | -1.2550 | -0.24820 | | 0.3639 | | | 0.6919 | 5 | -9.670 | 29.3 |
| 37 | -1.7080 | | -0.64360 | | | | 1.3900 | 4 | -10.670 | 29.3 |
| 46 | -0.3146 | -0.03584 | -0.80970 | 0.3672 | | | 1.2710 | 6 | -8.953 | 29.9 |
| 60 | -0.6121 | -0.06454 | -0.066880 | | 0.4188 | -0.4915 | 1.0320 | 7 | -8.077 | 30.2 |
| 63 | -0.4991 | | -0.070960 | -0.17140 | 0.4086 | -0.4111 | 1.0710 | 7 | -8.101 | 30.2 |
| 48 | -0.6664 | -0.05386 | -0.097010 | -0.50110 | 0.4234 | | 1.0550 | 7 | -8.106 | 30.2 |
| 15 | -1.3260 | | -0.141700 | 0.53690 | 0.4770 | | | 5 | -10.136 | 30.3 |
| 62 | -0.4490 | -0.07620 | | 0.72770 | 0.4004 | -1.2710 | 1.0430 | 7 | -8.195 | 30.4 |
| 29 | -0.9446 | | | 2.32300 | 0.4383 | -1.8930 | | 5 | -10.222 | 30.4 |
| 53 | -1.8390 | | 0.54460 | | | -1.0080 | 1.2310 | 5 | -10.226 | 30.5 |
| 50 | -1.4810 | 0.10920 | | | | -0.7284 | 1.3790 | 5 | -10.226 | 30.5 |
| 34 | -2.3590 | -0.10740 | | | | | 0.8684 | 4 | -11.244 | 30.5 |
| 51 | -1.9150 | | -0.030810 | | | -0.4897 | 1.2850 | 5 | -10.253 | 30.5 |
| 27 | -1.3680 | | -0.174800 | | 0.4968 | 0.5506 | | 5 | -10.291 | 30.6 |
| 39 | -2.0530 | | -0.062690 | -0.43960 | | | 1.2660 | 5 | -10.335 | 30.7 |
| 36 | -2.5030 | -0.03440 | -0.083230 | | | | 0.8963 | 5 | -10.593 | 31.2 |
| 38 | -1.4490 | 0.10160 | | -0.79230 | | | 1.4370 | 5 | -10.612 | 31.2 |
| 16 | -1.6470 | -0.20880 | -0.139600 | 0.71490 | 0.5072 | | | 6 | -9.877 | 31.8 |
| 30 | -1.3160 | -0.23860 | | 2.54100 | 0.4741 | -1.9060 | | 6 | -9.885 | 31.8 |
| 13 | -1.0500 | | | 0.42240 | 0.4110 | | | 4 | -11.885 | 31.8 |
| 31 | -1.1730 | | -0.087570 | 1.35400 | 0.4642 | -0.8572 | | 6 | -10.045 | 32.1 |
| 64 | -0.6114 | -0.06339 | -0.068180 | -0.02606 | 0.4189 | -0.4687 | 1.0350 | 8 | -8.077 | 32.2 |
| 28 | -1.6410 | -0.16770 | -0.182100 | | 0.5254 | 0.6983 | | 6 | -10.119 | 32.2 |

| | | | | | | | | | | |
|--------------|---------|----------|-----------|----------|--------|---------|--------|---|---------|-------|
| 54 | -1.6320 | 0.07991 | | 0.40060 | | -0.9854 | 1.2720 | 6 | -10.189 | 32.4 |
| 52 | -1.6650 | 0.09609 | -0.024700 | | | -0.6115 | 1.3050 | 6 | -10.194 | 32.4 |
| 55 | -1.8600 | | -0.005303 | 0.48770 | | -0.9454 | 1.2310 | 6 | -10.225 | 32.5 |
| 40 | -1.7890 | 0.10350 | -0.062910 | -0.59040 | | | 1.3130 | 6 | -10.273 | 32.5 |
| 21 | -2.8120 | | | 2.53800 | | -1.7250 | | 4 | -12.557 | 33.1 |
| 25 | -0.7492 | | | | 0.4796 | 0.3550 | | 4 | -12.592 | 33.2 |
| 14 | -1.4090 | -0.23040 | | 0.62070 | 0.4454 | | | 5 | -11.597 | 33.2 |
| 32 | -1.4890 | -0.22300 | -0.075840 | 1.68700 | 0.4941 | -1.0080 | | 7 | -9.751 | 33.5 |
| 5 | -2.8020 | | | 0.78490 | | | | 3 | -13.802 | 33.6 |
| 7 | -3.2380 | | -0.110900 | 0.92010 | | | | 4 | -12.820 | 33.6 |
| 56 | -1.6680 | 0.08319 | -0.011140 | 0.27490 | | -0.8521 | 1.2720 | 7 | -10.186 | 34.4 |
| 19 | -3.4510 | | -0.166800 | | | 0.9721 | | 4 | -13.253 | 34.5 |
| 9 | 0.9951 | | | | 0.8820 | | | 3 | -14.404 | 34.8 |
| 12 | 0.3485 | 0.33210 | -0.111800 | | 0.6667 | | | 5 | -12.415 | 34.8 |
| 10 | 0.3284 | 0.25560 | | | 0.5985 | | | 4 | -13.429 | 34.9 |
| 22 | -3.0100 | -0.08987 | | 2.62700 | | -1.7240 | | 5 | -12.513 | 35.0 |
| 23 | -2.8680 | | -0.014310 | 2.38200 | | -1.5540 | | 5 | -12.553 | 35.1 |
| 26 | -0.8465 | -0.06583 | | | 0.4905 | 0.4097 | | 5 | -12.568 | 35.1 |
| 6 | -3.0010 | -0.09063 | | 0.87490 | | | | 4 | -13.760 | 35.5 |
| 8 | -3.3620 | -0.05836 | -0.109700 | 0.97660 | | | | 5 | -12.802 | 35.6 |
| 11 | 1.1350 | | -0.071490 | | 0.9798 | | | 4 | -13.981 | 36.0 |
| 17 | -2.7910 | | | | | 0.7710 | | 3 | -15.079 | 36.2 |
| 20 | -3.4240 | 0.01165 | -0.166400 | | | 0.9601 | | 5 | -13.252 | 36.5 |
| 24 | -3.0350 | -0.08761 | -0.007846 | 2.53900 | | -1.6310 | | 6 | -12.511 | 37.0 |
| 18 | -2.5850 | 0.09433 | | | | 0.6790 | | 4 | -15.033 | 38.1 |
| 2 | -1.0500 | 0.78560 | | | | | | 3 | -17.378 | 40.8 |
| 4 | -1.1140 | 0.84970 | -0.052420 | | | | | 4 | -17.190 | 42.4 |
| 3 | -0.1571 | | 0.599000 | | | | | 3 | -35.811 | 77.6 |
| 1 | 1.3230 | | | | | | | 2 | -78.299 | 160.6 |
| delta weight | | | | | | | | | | |
| 57 | 0.00 | 0.104 | | | | | | | | |
| 43 | 0.60 | 0.077 | | | | | | | | |
| 45 | 1.26 | 0.056 | | | | | | | | |
| 59 | 1.55 | 0.048 | | | | | | | | |
| 47 | 1.58 | 0.047 | | | | | | | | |
| 41 | 1.73 | 0.044 | | | | | | | | |
| 61 | 1.79 | 0.042 | | | | | | | | |
| 49 | 1.95 | 0.039 | | | | | | | | |
| 58 | 1.99 | 0.038 | | | | | | | | |
| 33 | 2.03 | 0.038 | | | | | | | | |
| 44 | 2.06 | 0.037 | | | | | | | | |
| 35 | 2.54 | 0.029 | | | | | | | | |
| 42 | 2.68 | 0.027 | | | | | | | | |

| | | |
|----|------|-------|
| 37 | 2.68 | 0.027 |
| 46 | 3.24 | 0.021 |
| 60 | 3.49 | 0.018 |
| 63 | 3.54 | 0.018 |
| 48 | 3.55 | 0.018 |
| 15 | 3.61 | 0.017 |
| 62 | 3.73 | 0.016 |
| 29 | 3.78 | 0.016 |
| 53 | 3.79 | 0.016 |
| 50 | 3.79 | 0.016 |
| 34 | 3.82 | 0.015 |
| 51 | 3.84 | 0.015 |
| 27 | 3.92 | 0.015 |
| 39 | 4.01 | 0.014 |
| 36 | 4.52 | 0.011 |
| 38 | 4.56 | 0.011 |
| 16 | 5.09 | 0.008 |
| 30 | 5.11 | 0.008 |
| 13 | 5.11 | 0.008 |
| 31 | 5.43 | 0.007 |
| 64 | 5.49 | 0.007 |
| 28 | 5.57 | 0.006 |
| 54 | 5.71 | 0.006 |
| 52 | 5.72 | 0.006 |
| 55 | 5.79 | 0.006 |
| 40 | 5.88 | 0.005 |
| 21 | 6.45 | 0.004 |
| 25 | 6.52 | 0.004 |
| 14 | 6.53 | 0.004 |
| 32 | 6.84 | 0.003 |
| 5 | 6.94 | 0.003 |
| 7 | 6.98 | 0.003 |
| 56 | 7.71 | 0.002 |
| 19 | 7.84 | 0.002 |
| 9 | 8.14 | 0.002 |
| 12 | 8.17 | 0.002 |
| 10 | 8.19 | 0.002 |
| 22 | 8.36 | 0.002 |
| 23 | 8.44 | 0.002 |
| 26 | 8.47 | 0.002 |
| 6 | 8.86 | 0.001 |
| 8 | 8.94 | 0.001 |
| 11 | 9.30 | 0.001 |

| | | |
|----|--------|-------|
| 17 | 9.49 | 0.001 |
| 20 | 9.84 | 0.001 |
| 24 | 10.36 | 0.001 |
| 18 | 11.40 | 0.000 |
| 2 | 14.09 | 0.000 |
| 4 | 15.72 | 0.000 |
| 3 | 50.96 | 0.000 |
| 1 | 133.93 | 0.000 |

Models ranked by AIC(x)

Here, we're ranking models by AIC. AIC balances fit with model complexity. Lower values of AIC are considered better. Generally, 2 is used as a rule of thumb for delta AIC: if delta AIC is >2 , the model with the higher AIC value has little support. If delta AIC is <2 , there is at least some support for the model with the higher AIC.

4.7 Tips for your Assignment:

Some things you may want to think about for your assignment:

1. What role is collinearity playing in your assignment? Is it something you should be concerned about? Why or why not?
2. What does it mean if your assumptions are violated? How would you fix it? Is it worth fixing it? Why or why not?
3. How would you interpret your statistical results biologically? You don't have to be right, but don't be vague, and don't contradict your results.

5 Assignment 1e: Bayesian Data Analysis

Assignment 1e is an introduction to Bayesian data analysis, using Bayesian generalized linear models.

For this tutorial, we'll be using `cuse.csv` .

5.1 Looking at the Data

```
# Load in data
data = read.csv('cuse.csv')

# Look at the data structure
head(data)
```

| | X | age | education | wantsMore | notUsing | using |
|---|---|-------|-----------|-----------|----------|-------|
| 1 | 1 | <25 | low | yes | 53 | 6 |
| 2 | 2 | <25 | low | no | 10 | 4 |
| 3 | 3 | <25 | high | yes | 212 | 52 |
| 4 | 4 | <25 | high | no | 50 | 10 |
| 5 | 5 | 25-29 | low | yes | 60 | 14 |
| 6 | 6 | 25-29 | low | no | 19 | 10 |

```
dim(data)
```

```
[1] 16 6
```

Our data contains 16 observations of 5 variables - a two-column binomial matrix (`notUsing` and `using`) of the number of women using and not using birth control within 16 groups, and three categorical predictors - age, expressed as categories, education, and whether they want more children. The first column is a duplicate of our row names. We can get rid of that:


```
# Remove column 1
data = data[,-1]
head(data)
```

| | age | education | wantsMore | notUsing | using |
|---|-------|-----------|-----------|----------|-------|
| 1 | <25 | low | yes | 53 | 6 |
| 2 | <25 | low | no | 10 | 4 |
| 3 | <25 | high | yes | 212 | 52 |
| 4 | <25 | high | no | 50 | 10 |
| 5 | 25-29 | low | yes | 60 | 14 |
| 6 | 25-29 | low | no | 19 | 10 |

5.2 Binomial GLM

Binomial generalized linear models (logistic regression with a binary response variable and a logit link) are used to calculate the probability of a binomial response - in this case, whether someone is using or not using birth control. Binomial GLM responses can be fed in either as a true/false set, or as a two-column matrix of successes and failures. According to `?family`, we need to feed in the data with successes (whatever that means in each context) first and failures second. Let's create the matrix:

```
# create response matrix
resp = cbind(data$using, data$notUsing)
head(resp)
```

| | [,1] | [,2] |
|------|------|------|
| [1,] | 6 | 53 |
| [2,] | 4 | 10 |
| [3,] | 52 | 212 |
| [4,] | 10 | 50 |
| [5,] | 14 | 60 |
| [6,] | 10 | 19 |

In this case, all of our variables are categorical, and they are currently stored as characters:

```
# Check predictor classes
class(data$age)
```

```
[1] "character"
```

```
class(data$education)
```

```
[1] "character"
```

```
class(data$wantsMore)
```

```
[1] "character"
```

These should function fine as categorical variables. Let's make our GLM:

```
# Run GLM
m1 = glm(resp ~ age + education + wantsMore, family = 'binomial', data = data)
summary(m1) # Summary
```

Call:

```
glm(formula = resp ~ age + education + wantsMore, family = "binomial",
     data = data)
```

Coefficients:

| | Estimate | Std. Error | z value | Pr(> z) | |
|--------------|----------|------------|---------|----------|-----|
| (Intercept) | -0.8082 | 0.1590 | -5.083 | 3.71e-07 | *** |
| age25-29 | 0.3894 | 0.1759 | 2.214 | 0.02681 | * |
| age30-39 | 0.9086 | 0.1646 | 5.519 | 3.40e-08 | *** |
| age40-49 | 1.1892 | 0.2144 | 5.546 | 2.92e-08 | *** |
| educationlow | -0.3250 | 0.1240 | -2.620 | 0.00879 | ** |
| wantsMoreyes | -0.8330 | 0.1175 | -7.091 | 1.33e-12 | *** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

Null deviance: 165.772 on 15 degrees of freedom
Residual deviance: 29.917 on 10 degrees of freedom
AIC: 113.43

Number of Fisher Scoring iterations: 4

In our summary we see we have 5 predictor categories with parameter estimates: different age bins (values representing the difference from the age <25 bin), low education (representing the

difference from high), and wanting more kids (representing the difference from not wanting more kids). We can also see the values of our model coefficients, their standard errors, and the model AIC.

5.3 Making it Bayesian

The default GLM function is frequentist (that's why we have p-values). Now lets try a Bayesian approach:

```
# Stan  
library(rstanarm)
```

Loading required package: Rcpp

This is rstanarm version 2.32.1

- See <https://mc-stan.org/rstanarm/articles/priors> for changes to default priors!
- Default priors may change, so it's safest to specify priors, even if equivalent to the default
- For execution on a local, multicore CPU with excess RAM we recommend calling

```
options(mc.cores = parallel::detectCores())
```

```
library(bayesplot)
```

This is bayesplot version 1.11.1

- Online documentation and vignettes at mc-stan.org/bayesplot
- bayesplot theme set to `bayesplot::theme_default()`
 - * Does `_not_` affect other `ggplot2` plots
 - * See `?bayesplot_theme_set` for details on theme setting

```
library(shinystan)
```

Loading required package: shiny

This is shinystan version 2.6.0

```
library(ggplot2)
```

```
# Run glm
```

```
m2 = stan_glm(resp ~ age + education + wantsMore, family = 'binomial', data = data)
```

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 1).

Chain 1:

Chain 1: Gradient evaluation took 3.1e-05 seconds

Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.31 seconds.

Chain 1: Adjust your expectations accordingly!

Chain 1:

Chain 1:

Chain 1: Iteration: 1 / 2000 [0%] (Warmup)

Chain 1: Iteration: 200 / 2000 [10%] (Warmup)

Chain 1: Iteration: 400 / 2000 [20%] (Warmup)

Chain 1: Iteration: 600 / 2000 [30%] (Warmup)

Chain 1: Iteration: 800 / 2000 [40%] (Warmup)

Chain 1: Iteration: 1000 / 2000 [50%] (Warmup)

Chain 1: Iteration: 1001 / 2000 [50%] (Sampling)

Chain 1: Iteration: 1200 / 2000 [60%] (Sampling)

Chain 1: Iteration: 1400 / 2000 [70%] (Sampling)

Chain 1: Iteration: 1600 / 2000 [80%] (Sampling)

Chain 1: Iteration: 1800 / 2000 [90%] (Sampling)

Chain 1: Iteration: 2000 / 2000 [100%] (Sampling)

Chain 1:

Chain 1: Elapsed Time: 0.063 seconds (Warm-up)

Chain 1: 0.068 seconds (Sampling)

Chain 1: 0.131 seconds (Total)

Chain 1:

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 2).

Chain 2:

Chain 2: Gradient evaluation took 1.2e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration: 1 / 2000 [0%] (Warmup)
Chain 2: Iteration: 200 / 2000 [10%] (Warmup)
Chain 2: Iteration: 400 / 2000 [20%] (Warmup)
Chain 2: Iteration: 600 / 2000 [30%] (Warmup)
Chain 2: Iteration: 800 / 2000 [40%] (Warmup)
Chain 2: Iteration: 1000 / 2000 [50%] (Warmup)
Chain 2: Iteration: 1001 / 2000 [50%] (Sampling)
Chain 2: Iteration: 1200 / 2000 [60%] (Sampling)
Chain 2: Iteration: 1400 / 2000 [70%] (Sampling)
Chain 2: Iteration: 1600 / 2000 [80%] (Sampling)
Chain 2: Iteration: 1800 / 2000 [90%] (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%] (Sampling)
Chain 2:
Chain 2: Elapsed Time: 0.062 seconds (Warm-up)
Chain 2: 0.063 seconds (Sampling)
Chain 2: 0.125 seconds (Total)
Chain 2:

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 3).

Chain 3:
Chain 3: Gradient evaluation took 1.2e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration: 1 / 2000 [0%] (Warmup)
Chain 3: Iteration: 200 / 2000 [10%] (Warmup)
Chain 3: Iteration: 400 / 2000 [20%] (Warmup)
Chain 3: Iteration: 600 / 2000 [30%] (Warmup)
Chain 3: Iteration: 800 / 2000 [40%] (Warmup)
Chain 3: Iteration: 1000 / 2000 [50%] (Warmup)
Chain 3: Iteration: 1001 / 2000 [50%] (Sampling)
Chain 3: Iteration: 1200 / 2000 [60%] (Sampling)
Chain 3: Iteration: 1400 / 2000 [70%] (Sampling)
Chain 3: Iteration: 1600 / 2000 [80%] (Sampling)
Chain 3: Iteration: 1800 / 2000 [90%] (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%] (Sampling)
Chain 3:

```
Chain 3: Elapsed Time: 0.066 seconds (Warm-up)
Chain 3:           0.066 seconds (Sampling)
Chain 3:           0.132 seconds (Total)
Chain 3:
```

```
SAMPLING FOR MODEL 'binomial' NOW (CHAIN 4).
```

```
Chain 4:
Chain 4: Gradient evaluation took 1.2e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:    1 / 2000 [  0%] (Warmup)
Chain 4: Iteration:   200 / 2000 [ 10%] (Warmup)
Chain 4: Iteration:   400 / 2000 [ 20%] (Warmup)
Chain 4: Iteration:   600 / 2000 [ 30%] (Warmup)
Chain 4: Iteration:   800 / 2000 [ 40%] (Warmup)
Chain 4: Iteration:  1000 / 2000 [ 50%] (Warmup)
Chain 4: Iteration:  1001 / 2000 [ 50%] (Sampling)
Chain 4: Iteration:  1200 / 2000 [ 60%] (Sampling)
Chain 4: Iteration:  1400 / 2000 [ 70%] (Sampling)
Chain 4: Iteration:  1600 / 2000 [ 80%] (Sampling)
Chain 4: Iteration:  1800 / 2000 [ 90%] (Sampling)
Chain 4: Iteration:  2000 / 2000 [100%] (Sampling)
Chain 4:
Chain 4: Elapsed Time: 0.064 seconds (Warm-up)
Chain 4:           0.069 seconds (Sampling)
Chain 4:           0.133 seconds (Total)
Chain 4:
```

```
summary(m2) # Summary
```

```
Model Info:
function:    stan_glm
family:      binomial [logit]
formula:     resp ~ age + education + wantsMore
algorithm:   sampling
sample:      4000 (posterior sample size)
priors:      see help('prior_summary')
observations: 16
predictors:  6
```

Estimates:

| | mean | sd | 10% | 50% | 90% |
|---------------|------|-----|------|------|------|
| (Intercept) | -0.8 | 0.2 | -1.0 | -0.8 | -0.6 |
| age25-29 | 0.4 | 0.2 | 0.2 | 0.4 | 0.6 |
| age30-39 | 0.9 | 0.2 | 0.7 | 0.9 | 1.1 |
| age40-49 | 1.2 | 0.2 | 0.9 | 1.2 | 1.5 |
| educationlow | -0.3 | 0.1 | -0.5 | -0.3 | -0.2 |
| wantsMoreeyes | -0.8 | 0.1 | -1.0 | -0.8 | -0.7 |

Fit Diagnostics:

| | mean | sd | 10% | 50% | 90% |
|----------|------|-----|------|------|------|
| mean_PPD | 31.7 | 1.6 | 29.7 | 31.7 | 33.7 |

The mean_ppd is the sample average posterior predictive distribution of the outcome variable

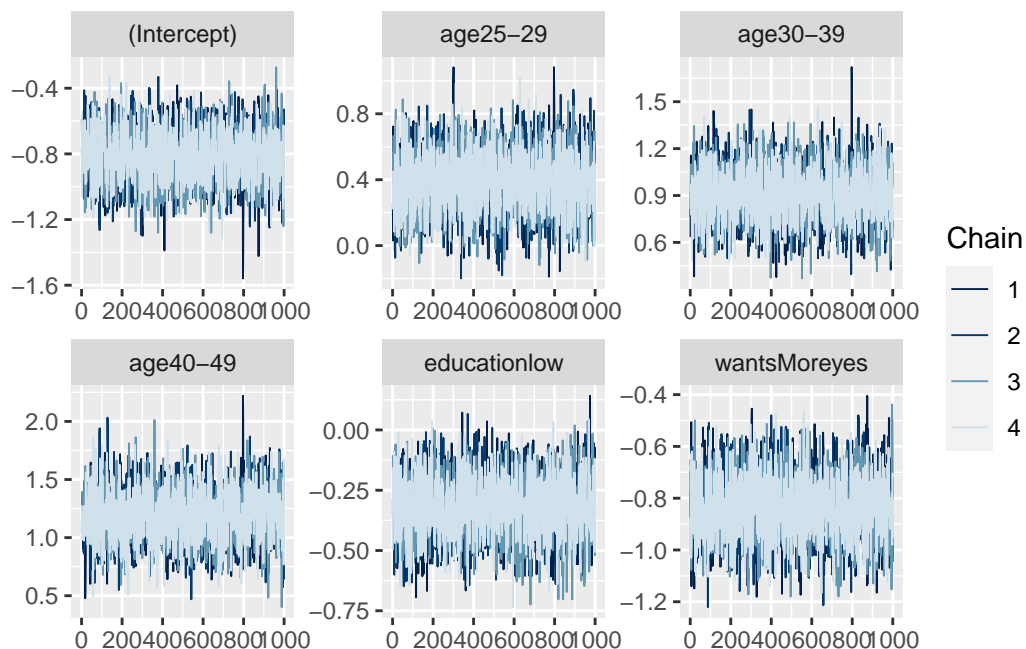
MCMC diagnostics

| | mcse | Rhat | n_eff |
|---------------|------|------|-------|
| (Intercept) | 0.0 | 1.0 | 1943 |
| age25-29 | 0.0 | 1.0 | 2185 |
| age30-39 | 0.0 | 1.0 | 1895 |
| age40-49 | 0.0 | 1.0 | 2327 |
| educationlow | 0.0 | 1.0 | 3152 |
| wantsMoreeyes | 0.0 | 1.0 | 2945 |
| mean_PPD | 0.0 | 1.0 | 3885 |
| log-posterior | 0.0 | 1.0 | 1923 |

For each parameter, mcse is Monte Carlo standard error, n_eff is a crude measure of effective

The `stan_glm` function automatically feeds our model into [Stan](#), which is a Hamiltonian Markov Chain Monte Carlo (MCMC) sampler. Running `summary` on our model gives us some model diagnostics - all our Rhat values are 1 and all our n_eff values are well into the thousands, both of which are a good sign. We can also do some visual checks and tests:

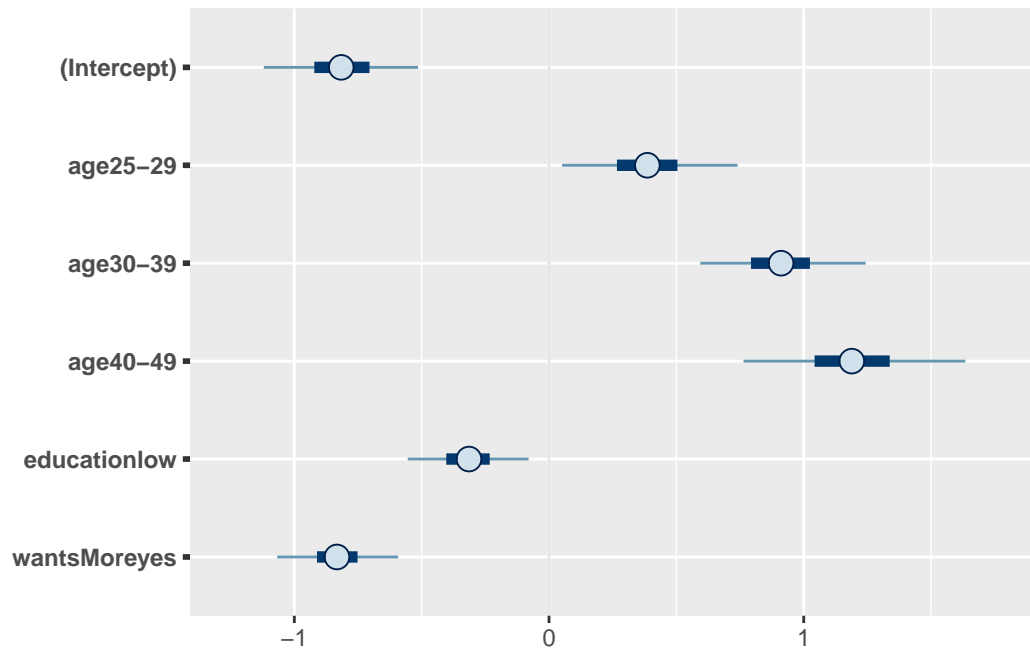
```
# Trace plot
plot(m2, 'trace')
```



These are trace plots, which show us the parameter values selected for each iteration of the MCMC chain. We want these to look “fuzzy” - that indicates the sampler is exploring the full range of possible values. If these lines were flat, that would indicate the sampler got “stuck” and didn’t sample the full posterior distributions. These look good.

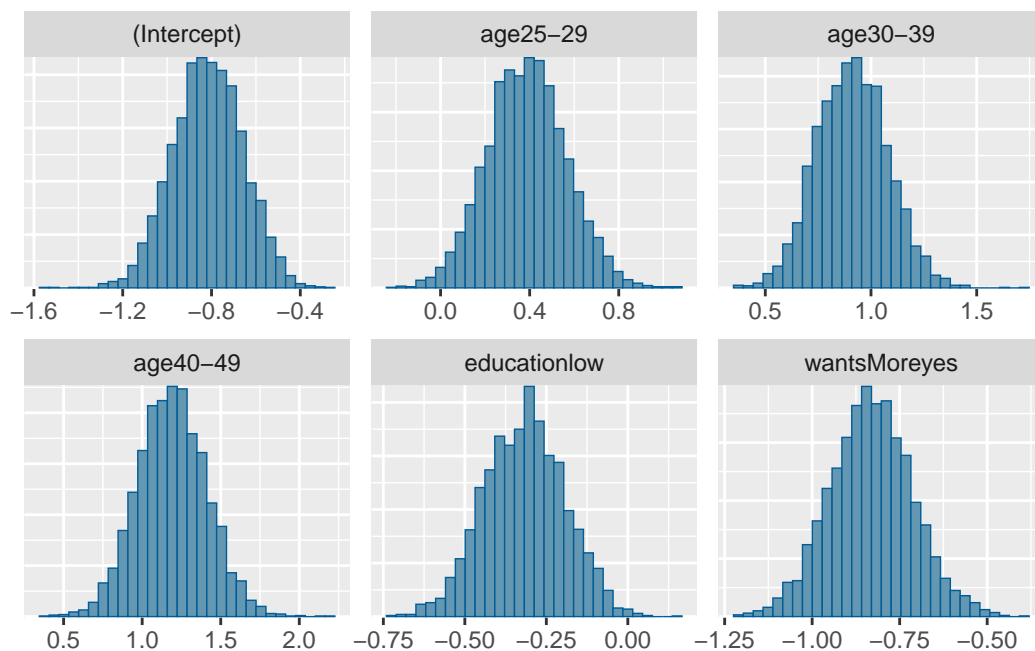
Lets look at our posteriors:

```
# Plot parameter values with uncertainties
plot(m2, probab_outer = 0.95)
```

```
# Plot posterior distributions  
plot(m2, 'mcmc_hist')
```

``stat_bin()`` using ``bins = 30``. Pick better value with ``binwidth``.



These plots both give us an idea of our parameter values and their posterior distributions. The former plot shows the median parameter estimates (circle), their 50% quantiles (dark blue box), and their 95% quantiles (thin blue line). The latter shows histograms of the posterior distributions of each of our parameters.

We can also pull out our coefficients and posteriors directly

```
# Model coefficients
m2$coefficients
```

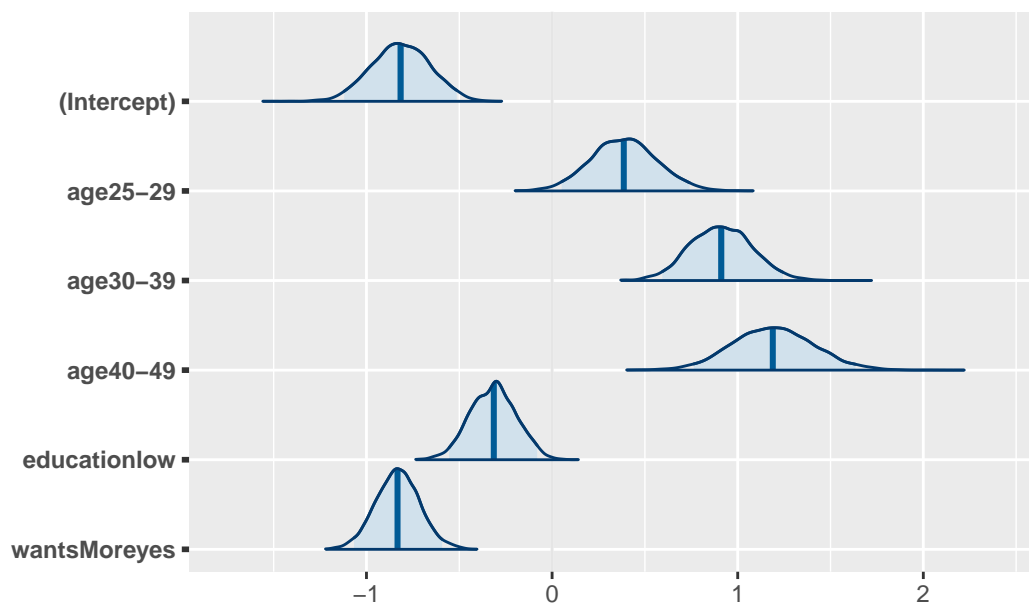
| (Intercept) | age25-29 | age30-39 | age40-49 | educationlow | wantsMoreyes |
|-------------|-----------|-----------|-----------|--------------|--------------|
| -0.8158023 | 0.3860991 | 0.9110547 | 1.1888319 | -0.3142539 | -0.8325785 |

```
# Model posteriors
posterior <- as.matrix(m2)

# Plot model posteriors (95% quantile)
plot_title <- ggtitle("Posterior distributions with medians and 95% credible intervals")

mcmc_areas(posterior, pars = names(m2$coefficients),
            prob = 0.95) + plot_title
```

Posterior distributions with medians and 95% credible int



How would you interpret these plots? Note the asymmetry in the histograms, in contrast to typical frequentist approaches

5.4 Adding Priors

Lets try adding some priors:

```
# Run glm with priors
m3 = stan_glm(resp ~ age + education + wantsMore, family = 'binomial', data = data,
              prior = normal(location = c(0.2, 1.5, 2, -1, -0.25), # Normal priors, means
                             scale = c(0.03, 0.03, 0.03, 0.03, 0.03))) # And standard deviat
```

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 1).

Chain 1:

Chain 1: Gradient evaluation took 2.3e-05 seconds

Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 0.23 seconds.

Chain 1: Adjust your expectations accordingly!

Chain 1:

Chain 1:

Chain 1: Iteration: 1 / 2000 [0%] (Warmup)

```

Chain 1: Iteration: 200 / 2000 [ 10%] (Warmup)
Chain 1: Iteration: 400 / 2000 [ 20%] (Warmup)
Chain 1: Iteration: 600 / 2000 [ 30%] (Warmup)
Chain 1: Iteration: 800 / 2000 [ 40%] (Warmup)
Chain 1: Iteration: 1000 / 2000 [ 50%] (Warmup)
Chain 1: Iteration: 1001 / 2000 [ 50%] (Sampling)
Chain 1: Iteration: 1200 / 2000 [ 60%] (Sampling)
Chain 1: Iteration: 1400 / 2000 [ 70%] (Sampling)
Chain 1: Iteration: 1600 / 2000 [ 80%] (Sampling)
Chain 1: Iteration: 1800 / 2000 [ 90%] (Sampling)
Chain 1: Iteration: 2000 / 2000 [100%] (Sampling)
Chain 1:
Chain 1: Elapsed Time: 0.075 seconds (Warm-up)
Chain 1:           0.045 seconds (Sampling)
Chain 1:           0.12 seconds (Total)
Chain 1:

```

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 2).

```

Chain 2:
Chain 2: Gradient evaluation took 1.2e-05 seconds
Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
Chain 2: Adjust your expectations accordingly!
Chain 2:
Chain 2:
Chain 2: Iteration: 1 / 2000 [ 0%] (Warmup)
Chain 2: Iteration: 200 / 2000 [ 10%] (Warmup)
Chain 2: Iteration: 400 / 2000 [ 20%] (Warmup)
Chain 2: Iteration: 600 / 2000 [ 30%] (Warmup)
Chain 2: Iteration: 800 / 2000 [ 40%] (Warmup)
Chain 2: Iteration: 1000 / 2000 [ 50%] (Warmup)
Chain 2: Iteration: 1001 / 2000 [ 50%] (Sampling)
Chain 2: Iteration: 1200 / 2000 [ 60%] (Sampling)
Chain 2: Iteration: 1400 / 2000 [ 70%] (Sampling)
Chain 2: Iteration: 1600 / 2000 [ 80%] (Sampling)
Chain 2: Iteration: 1800 / 2000 [ 90%] (Sampling)
Chain 2: Iteration: 2000 / 2000 [100%] (Sampling)
Chain 2:
Chain 2: Elapsed Time: 0.081 seconds (Warm-up)
Chain 2:           0.046 seconds (Sampling)
Chain 2:           0.127 seconds (Total)
Chain 2:

```

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 3).

```

Chain 3:
Chain 3: Gradient evaluation took 1.2e-05 seconds
Chain 3: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
Chain 3: Adjust your expectations accordingly!
Chain 3:
Chain 3:
Chain 3: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 3: Iteration:   200 / 2000 [ 10%]  (Warmup)
Chain 3: Iteration:   400 / 2000 [ 20%]  (Warmup)
Chain 3: Iteration:   600 / 2000 [ 30%]  (Warmup)
Chain 3: Iteration:   800 / 2000 [ 40%]  (Warmup)
Chain 3: Iteration:  1000 / 2000 [ 50%]  (Warmup)
Chain 3: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 3: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 3: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 3: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 3: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 3: Iteration: 2000 / 2000 [100%]  (Sampling)
Chain 3:
Chain 3: Elapsed Time: 0.081 seconds (Warm-up)
Chain 3:                  0.046 seconds (Sampling)
Chain 3:                  0.127 seconds (Total)
Chain 3:

```

SAMPLING FOR MODEL 'binomial' NOW (CHAIN 4).

```

Chain 4:
Chain 4: Gradient evaluation took 1.2e-05 seconds
Chain 4: 1000 transitions using 10 leapfrog steps per transition would take 0.12 seconds.
Chain 4: Adjust your expectations accordingly!
Chain 4:
Chain 4:
Chain 4: Iteration:    1 / 2000 [  0%]  (Warmup)
Chain 4: Iteration:   200 / 2000 [ 10%]  (Warmup)
Chain 4: Iteration:   400 / 2000 [ 20%]  (Warmup)
Chain 4: Iteration:   600 / 2000 [ 30%]  (Warmup)
Chain 4: Iteration:   800 / 2000 [ 40%]  (Warmup)
Chain 4: Iteration:  1000 / 2000 [ 50%]  (Warmup)
Chain 4: Iteration: 1001 / 2000 [ 50%]  (Sampling)
Chain 4: Iteration: 1200 / 2000 [ 60%]  (Sampling)
Chain 4: Iteration: 1400 / 2000 [ 70%]  (Sampling)
Chain 4: Iteration: 1600 / 2000 [ 80%]  (Sampling)
Chain 4: Iteration: 1800 / 2000 [ 90%]  (Sampling)
Chain 4: Iteration: 2000 / 2000 [100%]  (Sampling)

```

```
Chain 4:
Chain 4: Elapsed Time: 0.075 seconds (Warm-up)
Chain 4:           0.046 seconds (Sampling)
Chain 4:           0.121 seconds (Total)
Chain 4:
```

```
summary(m3) # Summary
```

Model Info:

```
function:      stan_glm
family:        binomial [logit]
formula:       resp ~ age + education + wantsMore
algorithm:     sampling
sample:        4000 (posterior sample size)
priors:        see help('prior_summary')
observations:  16
predictors:    6
```

Estimates:

| | mean | sd | 10% | 50% | 90% |
|--------------|------|-----|------|------|------|
| (Intercept) | -1.2 | 0.1 | -1.3 | -1.2 | -1.1 |
| age25-29 | 0.2 | 0.0 | 0.2 | 0.2 | 0.3 |
| age30-39 | 1.5 | 0.0 | 1.4 | 1.5 | 1.5 |
| age40-49 | 2.0 | 0.0 | 2.0 | 2.0 | 2.0 |
| educationlow | -1.0 | 0.0 | -1.0 | -1.0 | -0.9 |
| wantsMoreyes | -0.3 | 0.0 | -0.3 | -0.3 | -0.2 |

Fit Diagnostics:

| | mean | sd | 10% | 50% | 90% |
|----------|------|-----|------|------|------|
| mean_PPD | 31.7 | 1.5 | 29.8 | 31.7 | 33.8 |

The mean_ppd is the sample average posterior predictive distribution of the outcome variable

MCMC diagnostics

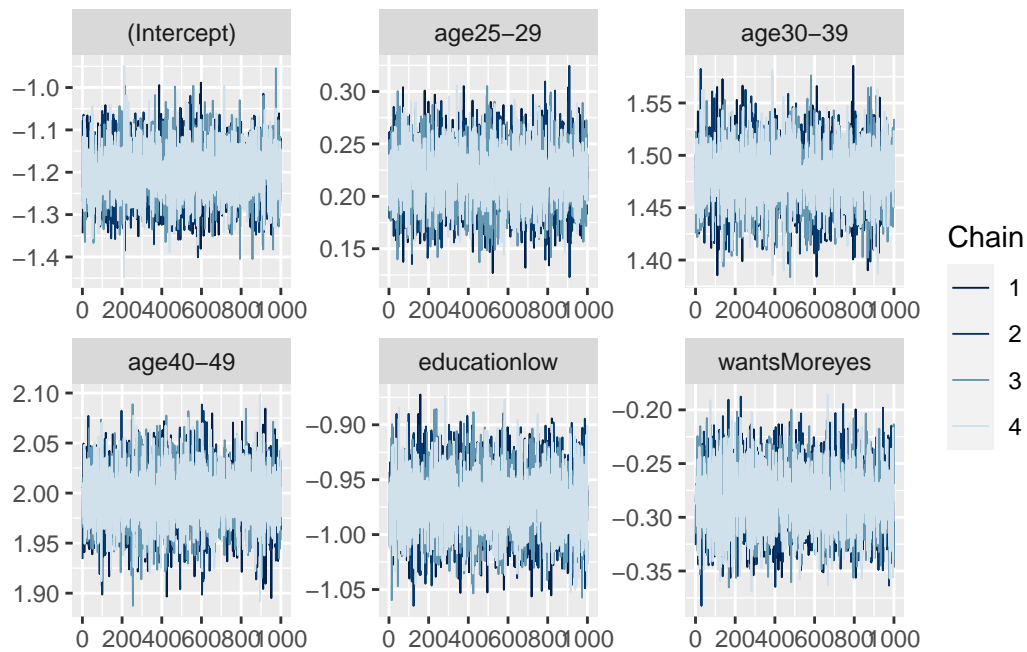
| | mcse | Rhat | n_eff |
|--------------|------|------|-------|
| (Intercept) | 0.0 | 1.0 | 7023 |
| age25-29 | 0.0 | 1.0 | 5830 |
| age30-39 | 0.0 | 1.0 | 7028 |
| age40-49 | 0.0 | 1.0 | 6388 |
| educationlow | 0.0 | 1.0 | 6131 |
| wantsMoreyes | 0.0 | 1.0 | 6479 |

```
mean_PPD      0.0  1.0  4621
log-posterior 0.0  1.0  1783
```

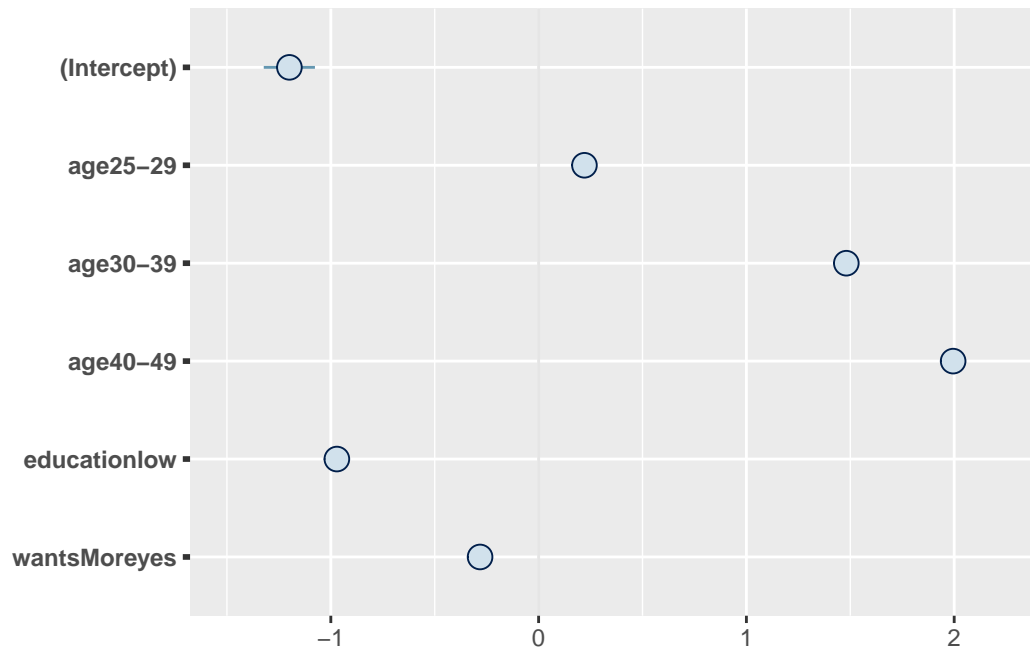
For each parameter, mcse is Monte Carlo standard error, n_eff is a crude measure of effective

Lets look at our plots again:

```
# Trace plot
plot(m3, 'trace')
```

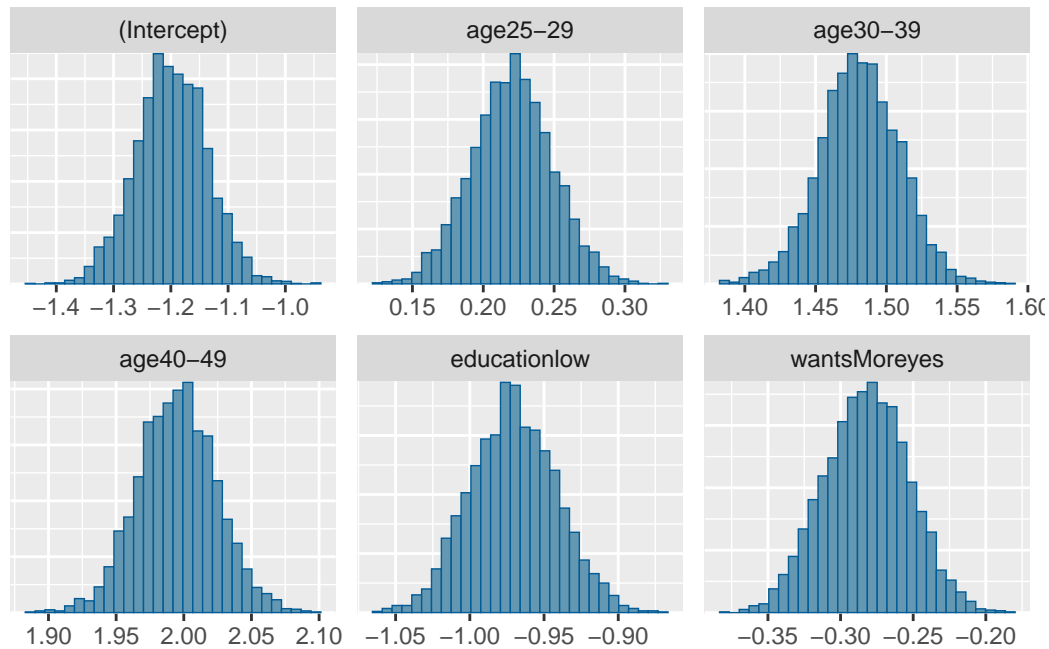


```
# Plot parameter values with uncertainties
plot(m3, prob_outer = 0.95)
```



```
# Plot posterior distributions  
plot(m3, 'mcmc_hist')
```

``stat_bin()`` using ``bins = 30``. Pick better value with ``binwidth``.



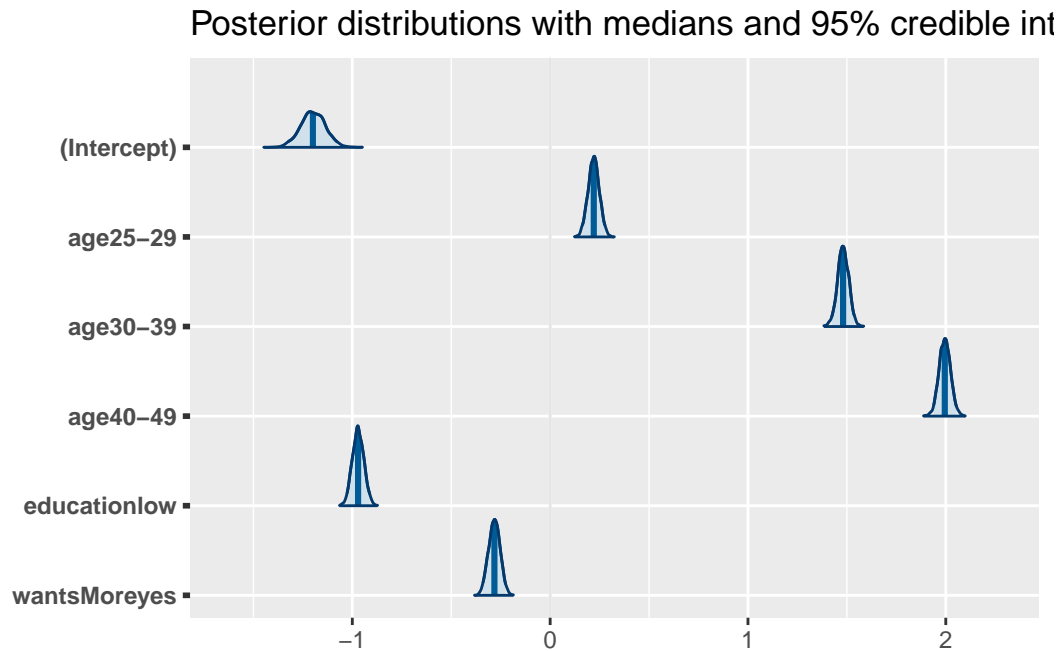
```
# Model coefficients
m3$coefficients
```

| (Intercept) | age25-29 | age30-39 | age40-49 | educationlow | wantsMoreyes |
|-------------|-----------|-----------|-----------|--------------|--------------|
| -1.1993324 | 0.2205816 | 1.4808672 | 1.9949537 | -0.9709552 | -0.2817410 |

```
# Model posteriors
posterior <- as.matrix(m3)

# Plot model posteriors (95% quantile)
plot_title <- ggtitle("Posterior distributions with medians and 95% credible intervals")

mcmc_areas(posterior, pars = names(m3$coefficients),
           prob = 0.95) + plot_title
```



What has changed versus the model without priors?

You can also look at all of your Stan model results using `shinystan` by running `launch_shinystan(model)`. Try it out on your end (it doesn't work in markdown)

5.5 Tips for your Assignment:

Some things you may want to think about for your assignment:

1. How do the results of these three models differ? Why do they or don't they?
2. Do you interpret certain models as being more or less correct? Why or why not?
3. How would you interpret your statistical results biologically? You don't have to be right, but don't be vague, and don't contradict your results.