

**Bachelor of Science (B.Sc.) (I.T.) Semester-II (C.B.S.) Examination****APPLIED MATHEMATICS-II****Paper—VI**

Time : Three Hours]

[Maximum Marks : 50

**N.B. :—** (1) All questions are compulsory and carry equal marks.

(2) Assume suitable data wherever necessary.

**EITHER**

1. (A) Prove that

$$A - B = A \cap \overline{B}$$

for the set A and B.

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- (B) Let
- $A = \{a, b, c, d\}$
- and let R be the relation on A that has the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the diagram of R and list in-degrees and out-degrees of all vertices.

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**OR**

- (C) Show that

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

for the set A and B.

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- (D) Define :

(i) Partial order relations

(ii) Equivalence relations.

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**EITHER**

2. (A) Using mathematical induction, prove that

$$1 + 2^n < 3^n, n \geq 2.$$

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- (B) Write the permutation

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$$

of the set  $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$  as a product of disjoint cycles.

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**OR**

- (C) Use backtrack to find explicit formula for the sequence defined by recurrence relation

$$b_n = 2b_{n-1} + 1 \text{ with the initial condition } b_1 = 7.$$

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- (D) Prove that :

If  $A = (a_1, a_2, \dots, a_n)$  is finite set with n elements,  $n \geq 2$  then there are  $\frac{n!}{2}$  even permutationsand  $\frac{n!}{2}$  odd permutations.

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**EITHER**

3. (A) Let  $L$  be a distributive lattice. Show that if there exists an  $a$  with  $a \wedge x = a \wedge y$  and  $a \vee x = a \vee y$ , then  $x = y$ . 5
- (B) Prove that if  $H$  and  $K$  are two normal subgroups of group  $G$ , then  $H \cap K$  is normal subgroup of  $G$ . 5

**OR**

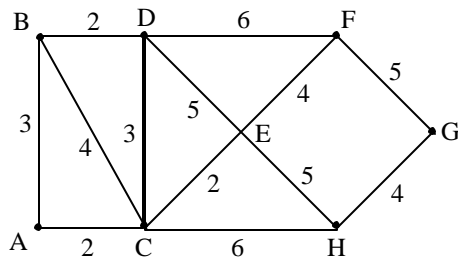
- (C) Define the following :  
 (i) Binary operation  
 (ii) Semigroup  
 (iii) Monoid  
 (iv) Isomorphism. 5
- (D) Let  $L$  be a bounded distributive lattice. Show that if a complement exists then it is unique. 5

**EITHER**

4. (A) Define :  
 (i) Graph  
 (ii) Connected graph  
 (iii) Digraph  
 (iv) Complete graph. 5
- (B) Prove that :  
 A tree with  $n$  vertices has  $n - 1$  edges. 5

**OR**

- (C) Find the Hamiltonian circuit for the graph



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- (D) Show that if a graph  $G$  has more than two vertices of odd degree then there can be no Euler path in  $G$ . 5

5. Attempt all :

- (A) Define :  
 (i) Power set  
 (ii) Symmetric difference. 2½
- (B) Let  $A = \{1, 2, 3, 4, 5, 6\}$   
 Compute :  $(4, 1, 3, 5) \circ (5, 6, 3)$  2½
- (C) Let  $G$  be a group and let  $a, b$  and  $c$  be elements of  $G$ . Then prove that  $ab = ac$  implies that  $b = c$ . 2½
- (D) Let  $(T, V_0)$  be a rooted tree. Prove that there are no cycles in tree  $T$ . 2½