Bachelor of Science (B.Sc. I.T.) Semester–II (C.B.S.) Examination

APPLIED MATHEMATICS–II

Paper–VI

Time : Three Hours] [Maximum Marks : 50

N.B. :— ALL questions are compulsory and carry equal marks.

EITHER

1. (A) Define power set of a set.

If A = {2, 3, 9, 10},

(i) Find P(A)

(ii) What is |A| ? 5

(B) Let R be a relation from A to B and let A_1 and A_2 be subsets of A then prove that

$$R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$$ 5

OR

(C) Let A = Z and let

$R = \{(a, b) \in A \times A : a \equiv r \pmod 2 \text{ and } b \equiv r \pmod 2\}$

Show that the relation R is an equivalence relation. 5

(D) Prove that for all sets A and B,

$$A - (A \cap B) = A - B$$ 5

EITHER

2. (A) Prove by mathematical induction for all $n \geq 1$,

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$ 5

(B) Prove that if n pigeons are assigned to m pigeon holes and m < n, then at least one pigeon hole contains two or more pigeons. 5

OR

(C) Show that :

$$nC_r = nC_{n-r}$$ 5

(D) Show by mathematical induction that for all $n \geq 1$,

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$ 5

EITHER

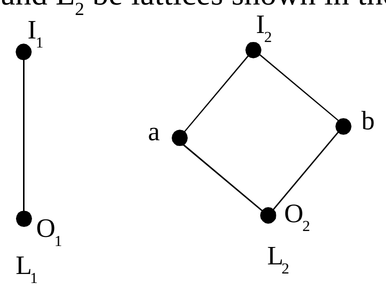
3. (A) Prove that :

let G be a group and let a and b be elements of G. Then,

(i) $(a^{-1})^{-1} = a$

(ii) $(ab)^{-1} = b^{-1}.a^{-1}$ 5

(B) Let L_1 and L_2 be lattices shown in the following figures :



(i) Find $L = L_1 \times L_2$

(ii) Draw Hasse diagram for L

(iii) Is L, a lattice ? 5

OR

(C) Show that in a complemented distributive lattice

$$a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$$ 5

(D) Show that if G is an abelian group, then every subgroup of G is a normal subgroup. 5

EITHER

4. (A) Let number of edges of graph G be m, then prove that G has a Hamiltonian circuit if

$m \geq \frac{1}{2}(n^2 - 3n + 6)$ where n is the number of vertices. 5

(B) Construct the tree of the algebraic expression

$$\left(\left(5 * \left(1 \quad x\right)\right) \div \left(\left(4 + \left(8 - \left(y + 3\right)\right)\right) * \left(7 + \left(x \div y\right)\right)\right)\right)$$ 5

OR

(C) Show that the maximum number of vertices in a binary tree of height n is $2^{n+1} - 1$. 5

(D) Prove that

If graph G is connected and has exactly two vertices of odd degree, then there is Euler path in G. Any Euler path in G must begin at one vertex of odd degree and end at the other. 5

5. Attempt **all** :

(A) Suppose that R and S are relations from A to B.

prove that :

(i) If $R \subseteq S$, then $R^{-1} \subseteq S^{-1}$

(ii) $(R \cap S)^{-1} = R^{-1} \cap S^{-1}$ 2½

(B) Let $f : A \to B$ and $g : B \to C$ be invertible functions then probe that

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ 2½

(C) Let G be the set of all non-zero real numbers and let $a * b = \frac{ab}{2}$. Show that $(G, *)$ is an abelian group. 2½

(D) Define :

(i) Graph

(ii) Connected grapdh. 2½