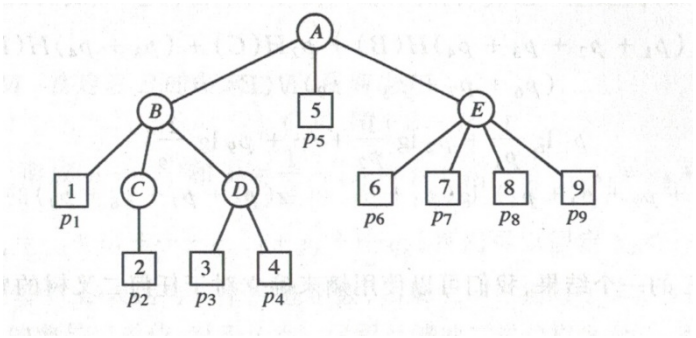
1. Suppose a set with n keys is stored in a hash table with size m.
Show that if all keys are from universal field U and |U|>nm, there is a subset of U with size n, so that the worst-case searching time for hashing with chaining is Θ(n).

2. 极小费用和称作熵的数学概念密切相关。如果p_1，p_2，p_3，…p_n是概率且$p_1+p_2+\ldots p_n=1$，由公式

$$H(p_1,\ p_2,\ p_3,\ldots p_n)=\sum_{k=1}^{n} p_k lg\frac{1}{p_k}$$

定义熵 $H(p_1,\ p_2,\ p_3,\ldots p_n)$，表示一个随机事件中预期的信息位数。如果第 k 个事件以概率p_k出现，我们就接收到了 $lg\frac{1}{p_k}$ 位信息。

对于一个树的所有内节点 a 所取的 p(a)H(a)之和等于叶节点概率分布的熵，其中 p(a)是达到节点 a 的概率，H(a)是 a 的熵。对下图中的树，有 $H(A)+(p_1+p_2+p_3+p_4)H(B)+p_2H(C)+(p_3+p_4)H(D)+(p_6+p_7+p_8+p_9)H(E)=p_1\lg\left(\frac{1}{p_1}\right)+p_2\lg\left(\frac{1}{p_2}\right)+p_3\lg\left(\frac{1}{p_3}\right)+\ldots+p_9\lg\left(\frac{1}{p_9}\right)$



设 X 和 Y 是具有有限范围的随机变量，设$p_i=P(X=x_i)$，$q_j=P(Y=y_j)$，$r_{ij}=P(X=x_i,Y=y_j)$，$H(X)=H(p_1,\ldots p_m)$和 $H(Y)=H(q_1,\ldots q_n)$是单个变量的熵，$H(XY)=H(r_{11},\ldots r_{mn})$是它们联合分布的熵。证明 $H(X)\le H(XY)\le H(X)+H(Y)$
提示：如果 f 是任何凹函数，有 $Ef(x)\le f(Ex)$（需要自行证明）
3. Can we maintain the black-heights of nodes in a red-black tree as attributes in the nodes of the tree without affecting the asymptotic performance of any of the red black tree operations? Show how, or argue why not. How about maintaining the depths of nodes?

4. Write the programs to implement the operations: RB-INSERT and RB-DELETE.

5. Please write the pseudo-code of B-TREE-DELETE.

6. Intuitively, it is easier to find an element that is nearby an element you've already seen. In a dynamic-set data structure, ***a finger search from x to y*** is the following query: given the node in the data structure that stores the element x, and given another element y , find the node in the data structure that stores y. Skip lists support fast finger searches in the following sense.

Give an algorithm for finger searching from x to y in a skip list. Your algorithm should run in O(lg(1+|rank(x)-rank(y)|)) time with high probability, where rank(x) denotes the current rank of element x in the sorted order of the dynamic set.

When we say "with high probability" we mean high probability with respect to m = 1+|rank(x)-rank(y)|. That is, your algorithm should run in O(lgm) time with probability $1 - 1/m^{\alpha}$ for any $\alpha \geq 1$. .

Assume that the finger-search operation is given the node in the bottommost list of the skip list that stores the element x.

7 Suppose we have a potential function ϕ such that $\phi(D_i) \geq \phi(D_0)$ for all i, but $\phi(D_0) \neq 0$. Show that there exists a potential function ϕ' such that $\phi'(D_0) = 0$, $\phi'(D_i) \geq 0$ for all $i \geq 0$, and the amortized costs using ϕ' are the same as the amortized costs using ϕ.

8 编程题：

在一个圆形操场四周摆放着 N 堆石子，现要将石子合并成一堆，规定每次只能选相邻的 2 堆合并成新的一堆，并将新的一堆的石子数，记为该次合并的代价。设计一个算法，计算出将 N 堆石子合并成 1 堆的最小代价。

输入格式：

数据的第一行是一个整数 N，表示有 N 堆石子；第二行有 N 个数，分别表示每堆石子的个数。

输出格式：

输出一个数，表示最小代价。

输入输出样例：

Input：

3

1 2 3

Output：

9