绕任意轴旋转

Rodrigues' Rotation Formula

默认n向量起点是原点

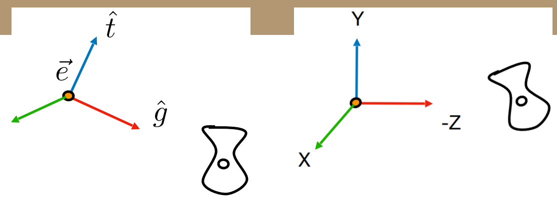
Rotation by angle α around axis $\mathbf{n}$

叉乘矩阵形式

如何按照任意轴（不在原点的轴）进行旋转?
先将点平移到原点，旋转后再做相反的平移

$$\mathbf{R}(\mathbf{n},\alpha) = \cos(\alpha)\,\mathbf{I} + (1-\cos(\alpha))\,\mathbf{n}\mathbf{n}^T + \sin(\alpha)\underbrace{\begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix}}_{\mathbf{N}}$$

转换到View Space



- M_{view} in math?
  - Let's write $M_{view} = R_{view}T_{view}$
  - Translate e to origin

$$T_{view} = \begin{bmatrix} 1 & 0 & 0 & -x_e \\ 0 & 1 & 0 & -y_e \\ 0 & 0 & 1 & -z_e \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  - Rotate g to -Z, t to Y, (g x t) To X
  - Consider its inverse rotation: X to (g x t), Y to t, Z to -g

用于旋转的矩阵，且是正交矩阵，每个向量是View Space下的基向量

WHY?

$$R_{view}^{-1} = \begin{bmatrix} x_{\hat{g}\times\hat{t}} & x_t & x_{-g} & 0 \\ y_{\hat{g}\times\hat{t}} & y_t & y_{-g} & 0 \\ z_{\hat{g}\times\hat{t}} & z_t & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow R_{view} = \begin{bmatrix} x_{\hat{g}\times\hat{t}} & y_{\hat{g}\times\hat{t}} & z_{\hat{g}\times\hat{t}} & 0 \\ x_t & y_t & z_t & 0 \\ x_{-g} & y_{-g} & z_{-g} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

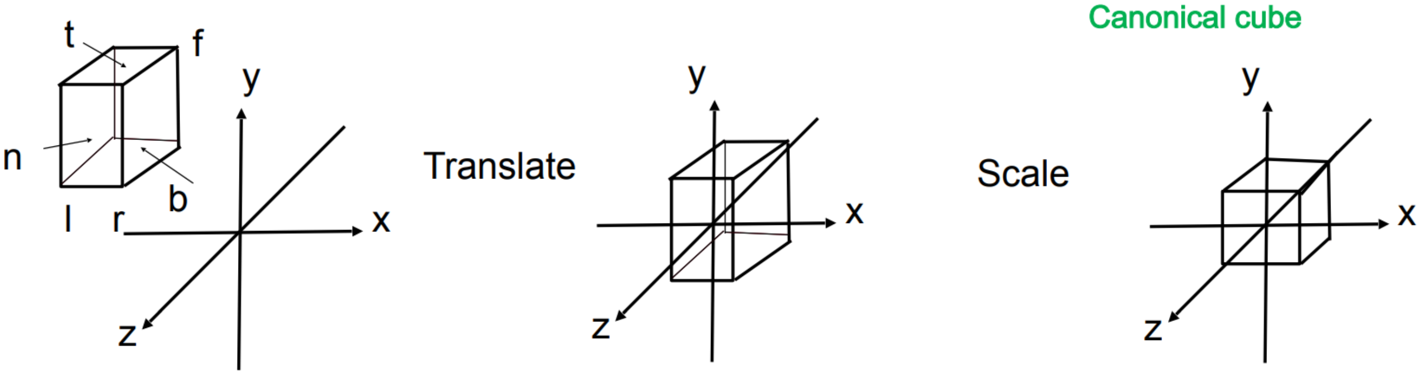
与Shader入门精要中的构建父或子空间变换矩阵的方法对比

先将相机移动到原点，然后旋转g到-Z，t到Y，g×t到X
但将世界坐标轴旋转到视角坐标轴更方便，然后求逆得到从世界坐标旋转到视角坐标的矩阵

正交投影

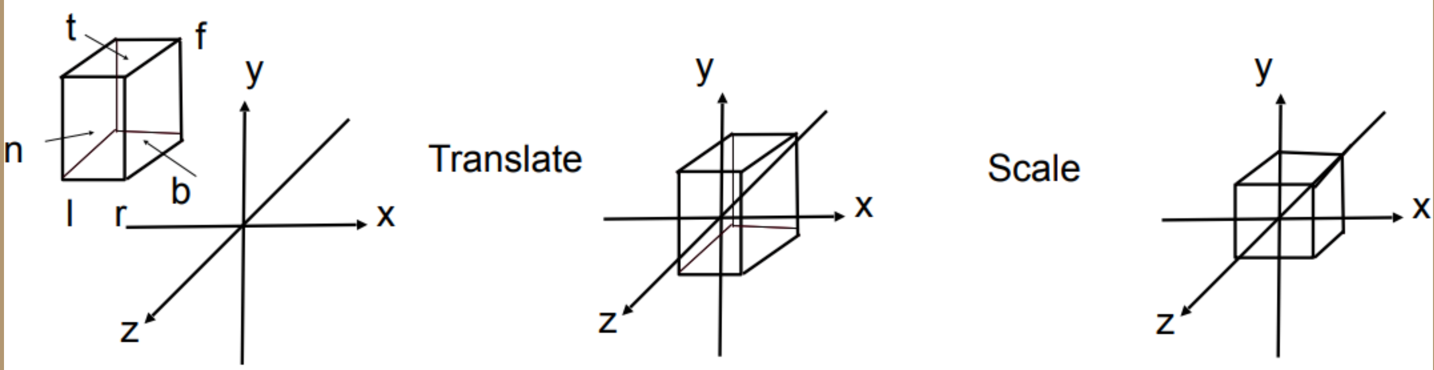
想象去掉Z轴

- We want to map a cuboid [l, r] x [b, t] x [**f, n**] to the “canonical (正则、规范、标准)” cube $[-1, 1]^3$



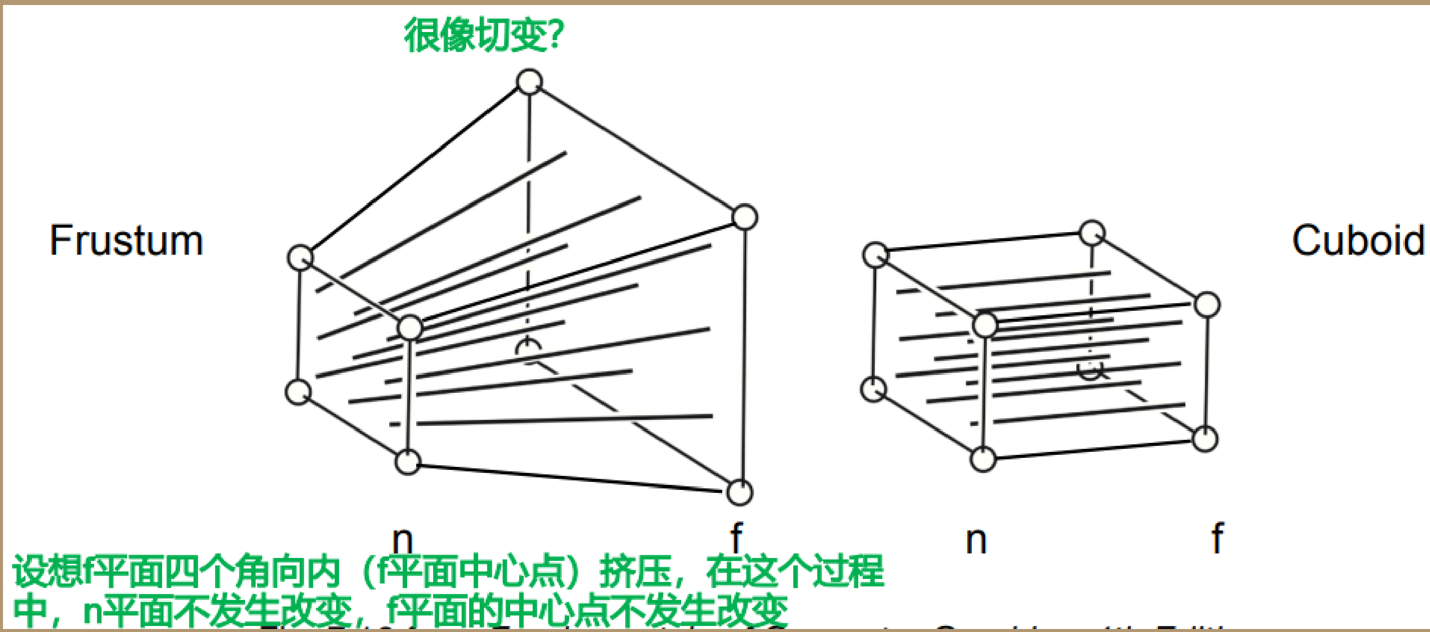
- **Transformation matrix?**
  - Translate (**center** to origin) first, then scale (length/width/height to **2**)

$$M_{ortho} = \begin{bmatrix} \frac{2}{r-l} & 0 & 0 & 0 \\ 0 & \frac{2}{t-b} & 0 & 0 \\ 0 & 0 & \frac{2}{n-f} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -\frac{r+l}{2} \\ 0 & 1 & 0 & -\frac{t+b}{2} \\ 0 & 0 & 1 & -\frac{n+f}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

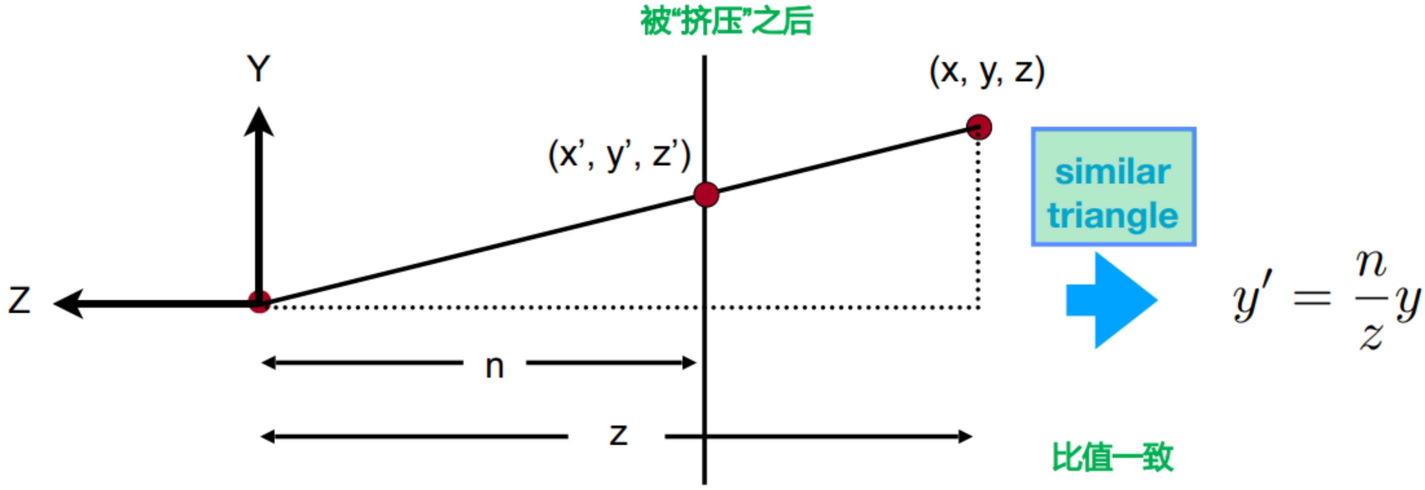


缩放时，两倍的长度分之一，返回值在[0,2]之间，以方便在[-1,1]之间存储

透视投影



- In order to find a transformation
  - Recall the key idea: Find the relationship between transformed points (x', y', z') and the original points (x, y, z)



$$y' = \frac{n}{z}y$$

$$M_{persp\to ortho} = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ (0 & 0 & A & B) \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$A = n + f$$
$$B = -nf$$

How to do perspective projection

- First "squish" the frustum into a cuboid (n -> n, f -> f) ($M_{persp\to ortho}$)
- Do orthographic projection (M_{ortho}, already known!)

$$\mathbf{M}_{\text{per}} = \begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$