

Introduction to Computer Graphics

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Lecture 9: Shading 3 (Texture Mapping cont.)



Announcements

- About homework
 - Homework 1 is being graded
 - Homework 2
 - 271 submissions so far
 - Homework 3 will be released soon

Last Lectures

- Shading 1 & 2
 - Blinn-Phong reflectance model
 - Shading models / frequencies
 - Graphics Pipeline
 - Texture mapping

Today

- Shading 3
 - Barycentric coordinates
 - Texture queries
 - Applications of textures
- Shadow mapping

Interpolation Across Triangles: Barycentric Coordinates

(重心坐标)

Interpolation Across Triangles

Why do we want to interpolate?

- Specify values **at vertices**
- Obtain smoothly varying values **across triangles**

What do we want to interpolate?

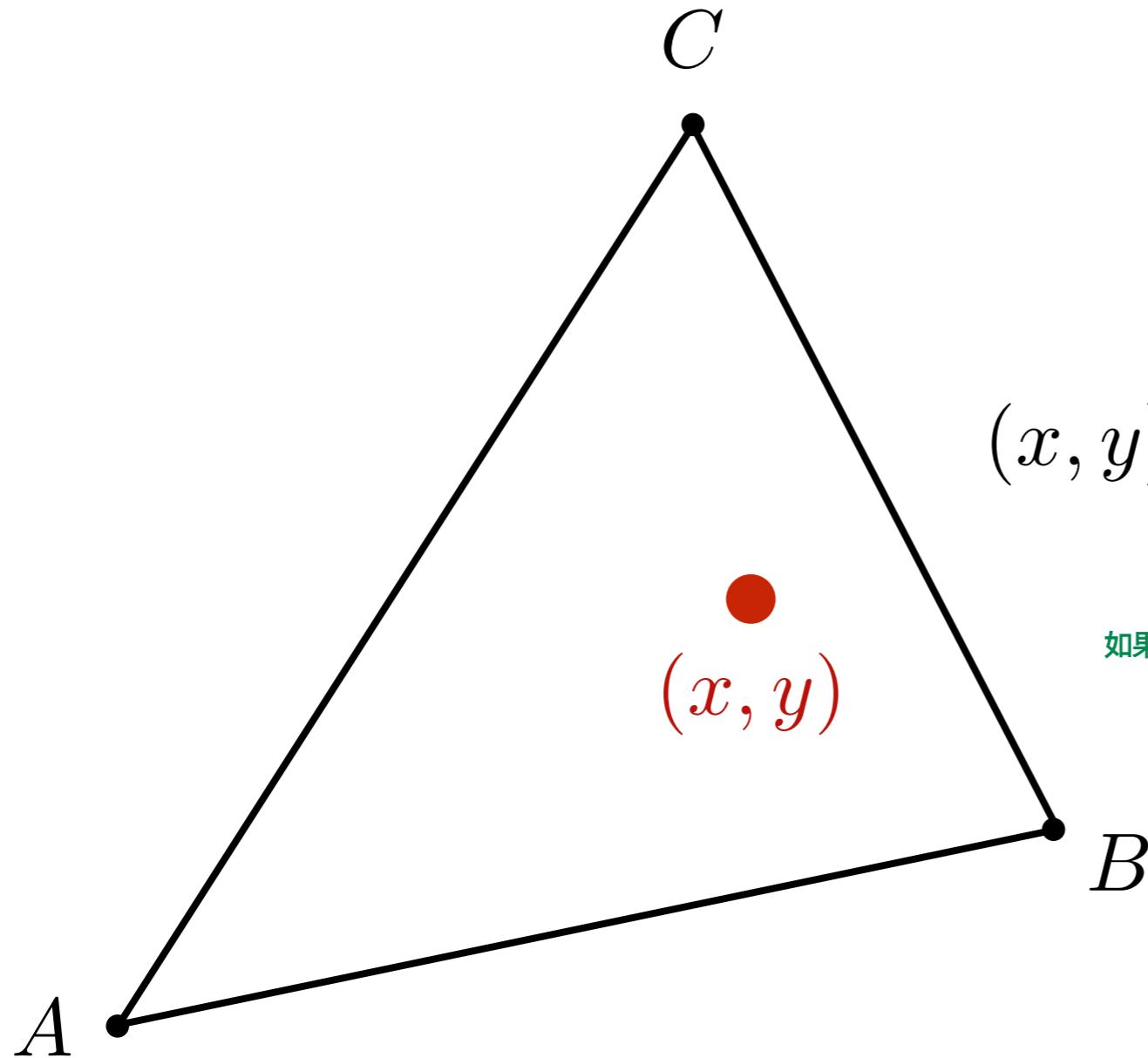
- Texture coordinates, colors, normal vectors, ...

How do we interpolate?

- **Barycentric coordinates**

Barycentric Coordinates

A coordinate system for triangles (α, β, γ)



$$(x, y) = \alpha A + \beta B + \gamma C$$

三个系数之和等于1，为了限制点在三角形内（结论）

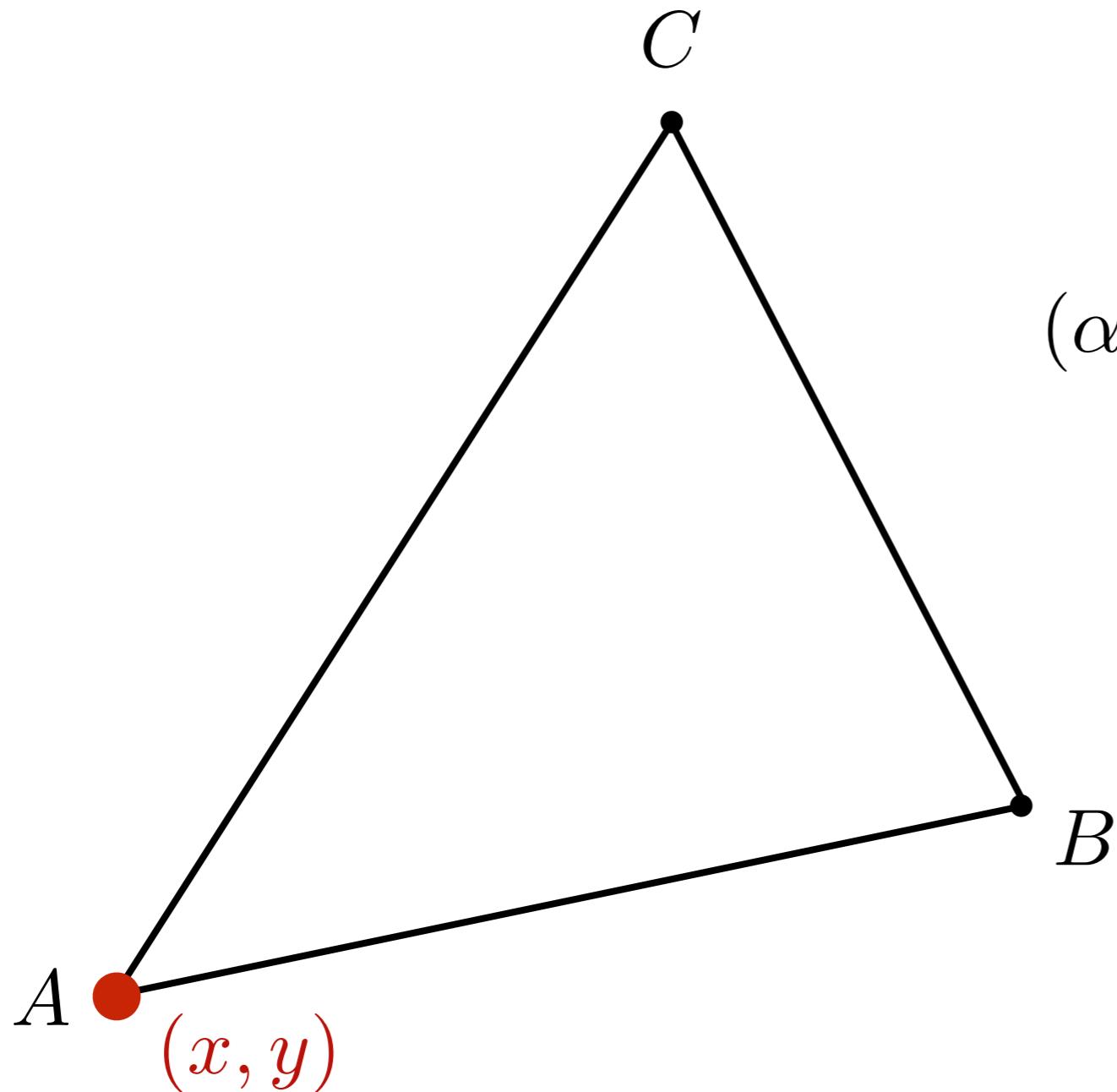
$$\alpha + \beta + \gamma = 1$$

如果要满足这个点在三角形内，必须满足这三个系数大于等于0

**Inside the triangle if
all three coordinates
are non-negative**

Barycentric Coordinates

What's the barycentric coordinate of A?

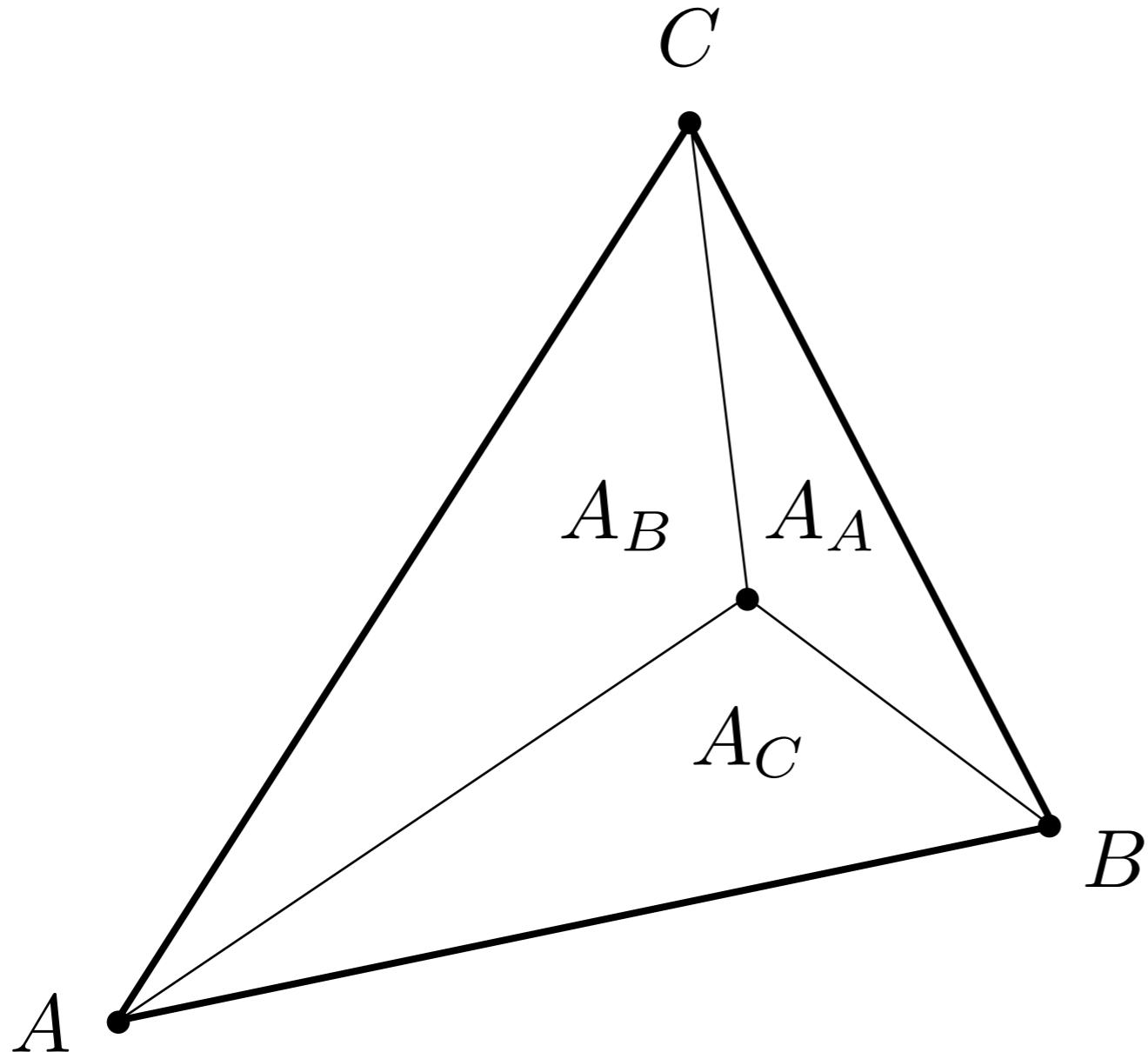


$$(\alpha, \beta, \gamma) = (1, 0, 0)$$

$$\begin{aligned} (x, y) &= \alpha A + \beta B + \gamma C \\ &= A \end{aligned}$$

Barycentric Coordinates

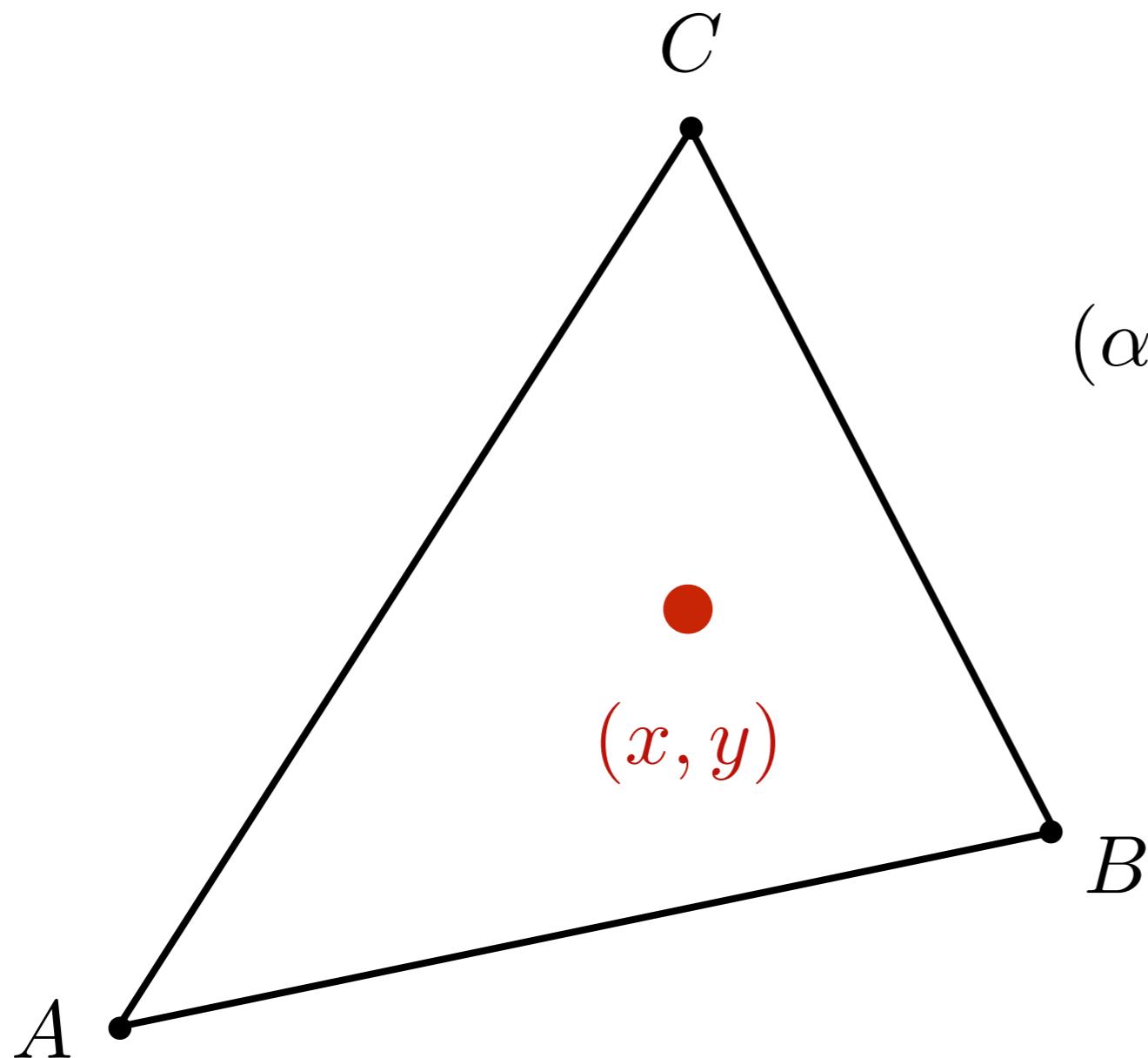
Geometric viewpoint — proportional areas



$$\alpha = \frac{A_A}{A_A + A_B + A_C}$$
$$\beta = \frac{A_B}{A_A + A_B + A_C}$$
$$\gamma = \frac{A_C}{A_A + A_B + A_C}$$

Barycentric Coordinates

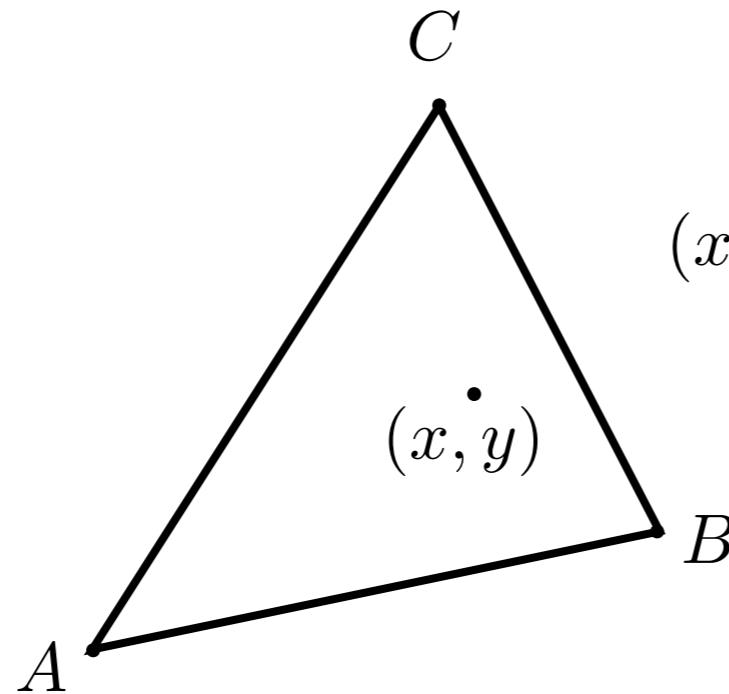
What's the barycentric coordinate of the centroid?



$$(\alpha, \beta, \gamma) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right)$$

$$(x, y) = \frac{1}{3} A + \frac{1}{3} B + \frac{1}{3} C$$

Barycentric Coordinates: Formulas



$$(x, y) = \alpha A + \beta B + \gamma C$$

$$\alpha + \beta + \gamma = 1$$

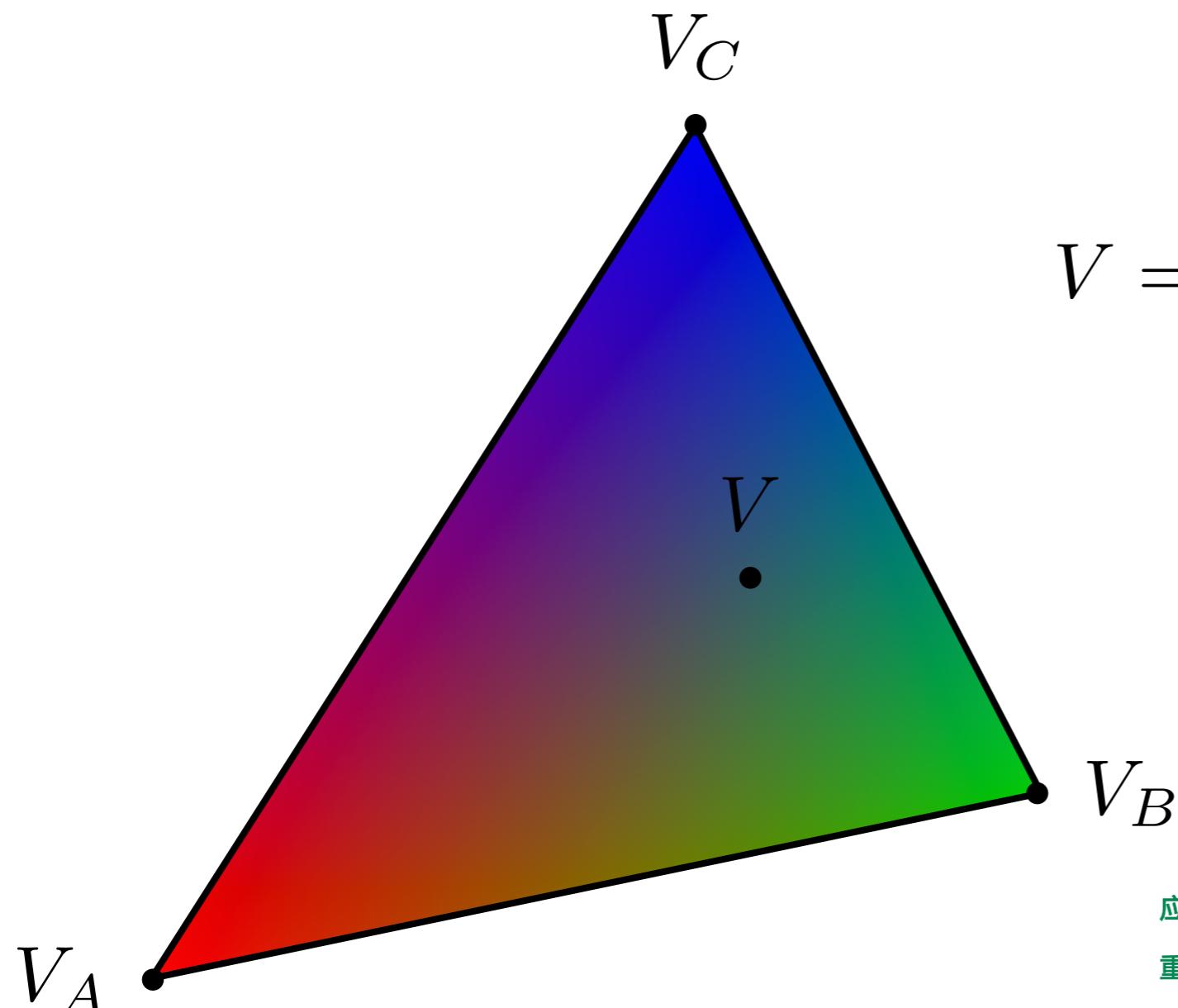
$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

$$\gamma = 1 - \alpha - \beta$$

Using Barycentric Coordinates

Linearly interpolate values at vertices



$$V = \alpha V_A + \beta V_B + \gamma V_C$$

V_A, V_B, V_C can be
positions, texture
coordinates, color,
normal, depth,
material attributes...

应该在三维空间中做插值，而不是投影空间中

重心坐标在投影空间中会发生变化

However, barycentric coordinates are not invariant under projection!

Applying Textures

Simple Texture Mapping: Diffuse Color

for each rasterized screen sample (x, y) :

$(u, v) = \text{evaluate texture coordinate at } (x, y)$

`texcolor = texture.sample(u, v);`

set sample's color to texcolor;



Usually the diffuse albedo K_d
(recall the Blinn-Phong reflectance model)

Usually a pixel's center



Using barycentric
coordinates!

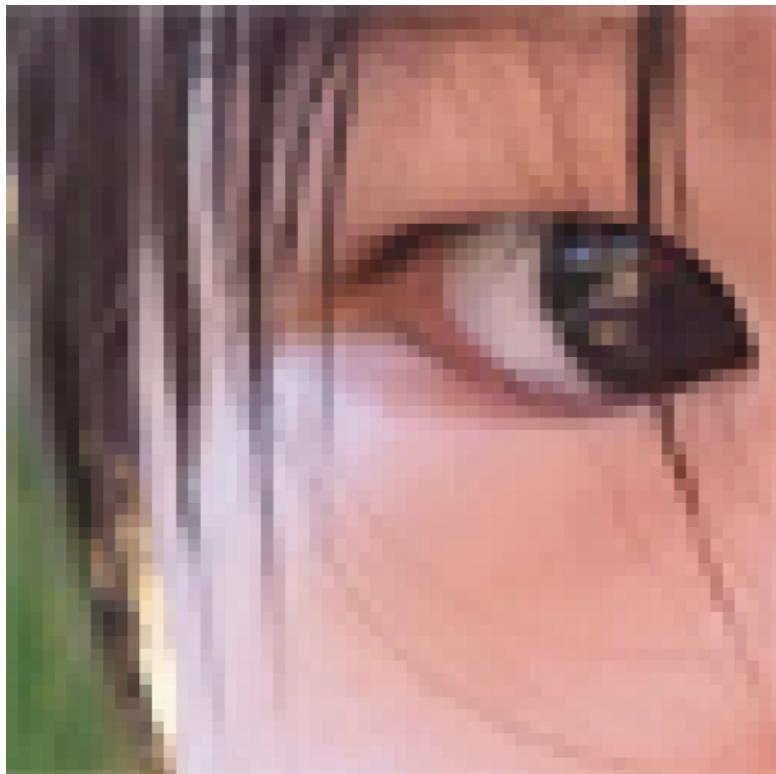
Texture Magnification

(What if the texture is too small?)

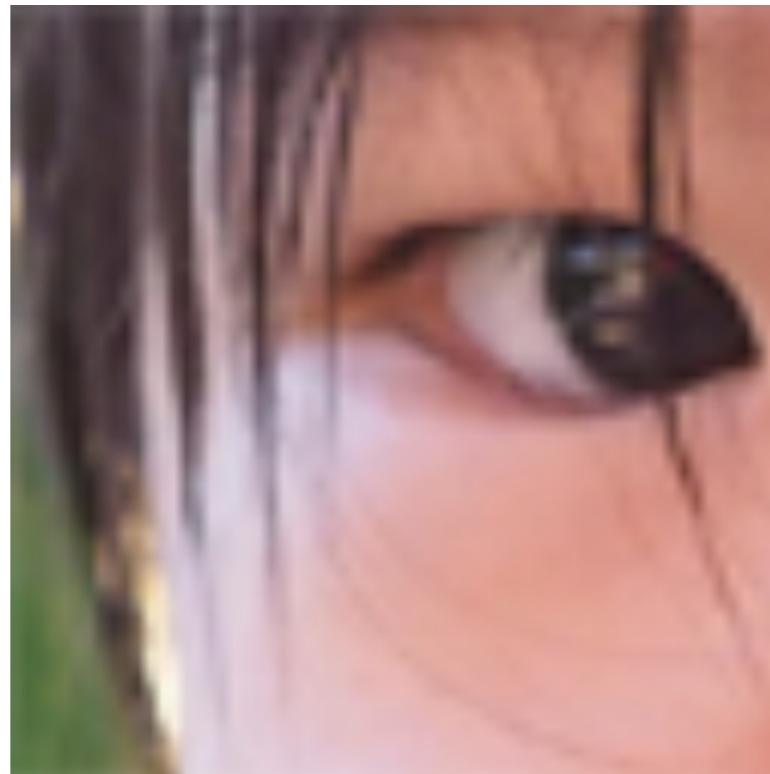
Texture Magnification - Easy Case

Generally don't want this — insufficient texture resolution

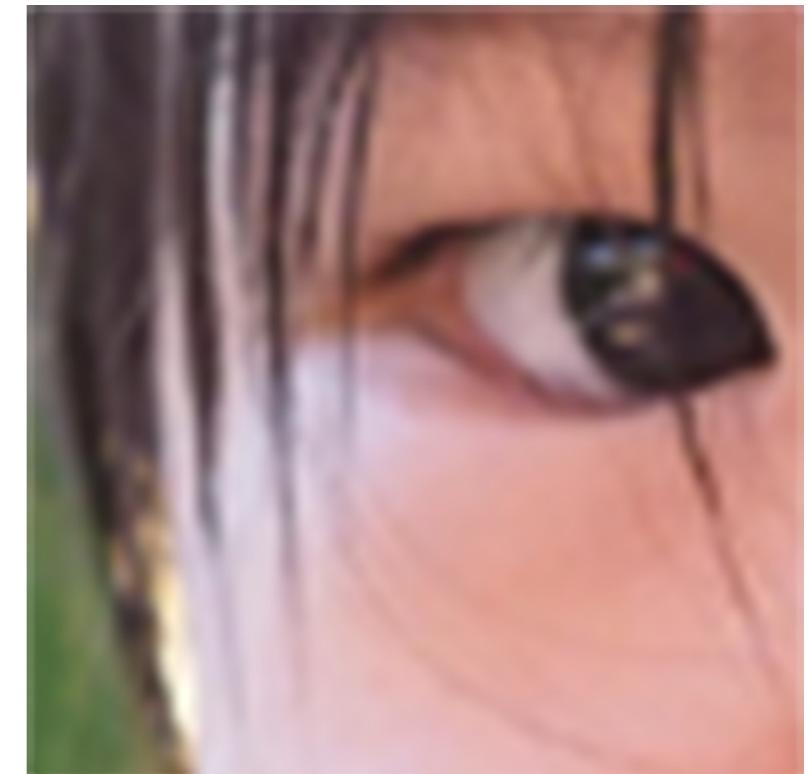
A pixel on a texture — a **texel** (纹理元素、纹素)



Nearest

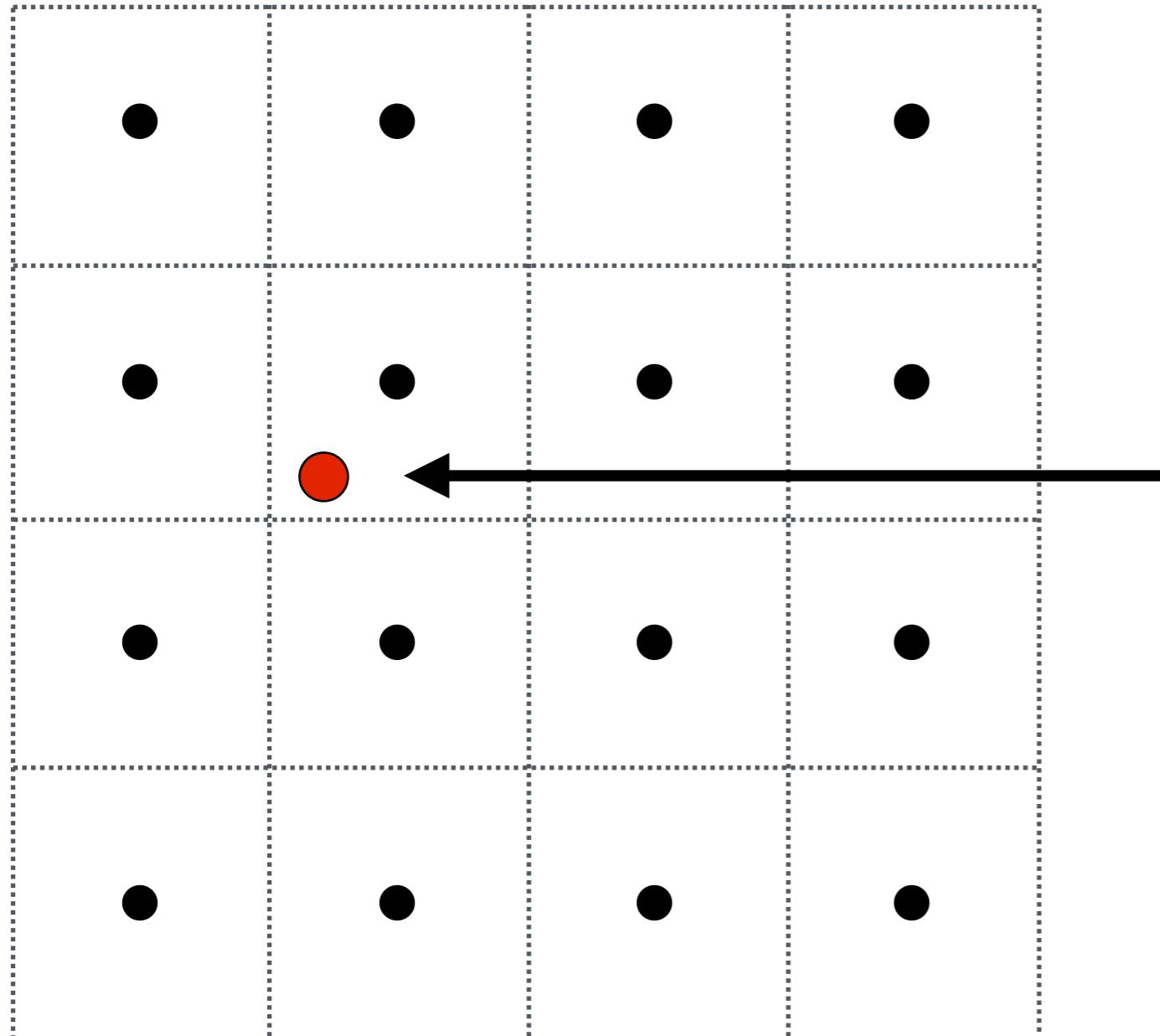


Bilinear



Bicubic

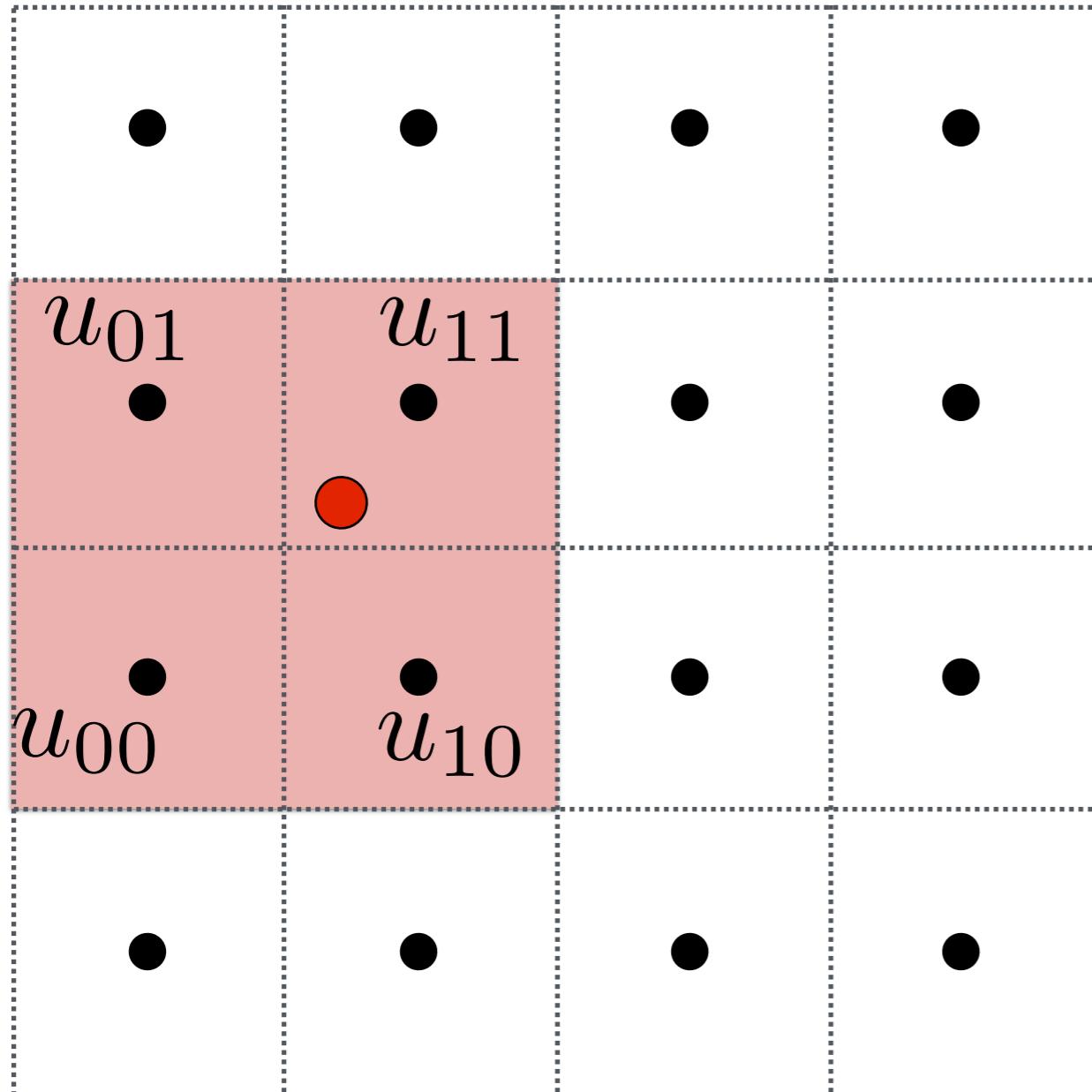
Bilinear Interpolation



Want to sample
texture value $f(x,y)$ at
red point

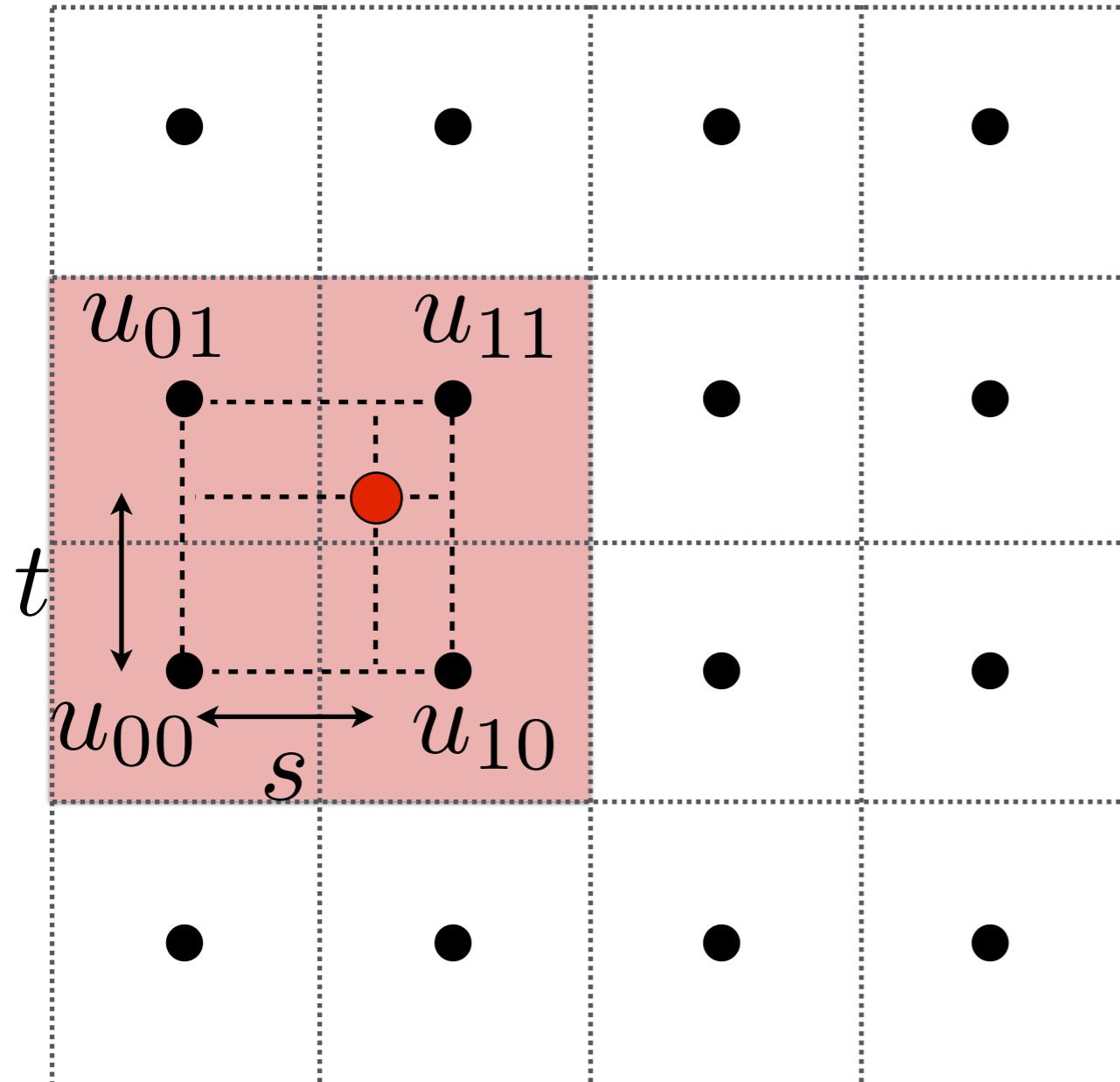
Black points indicate
texture sample
locations

Bilinear Interpolation



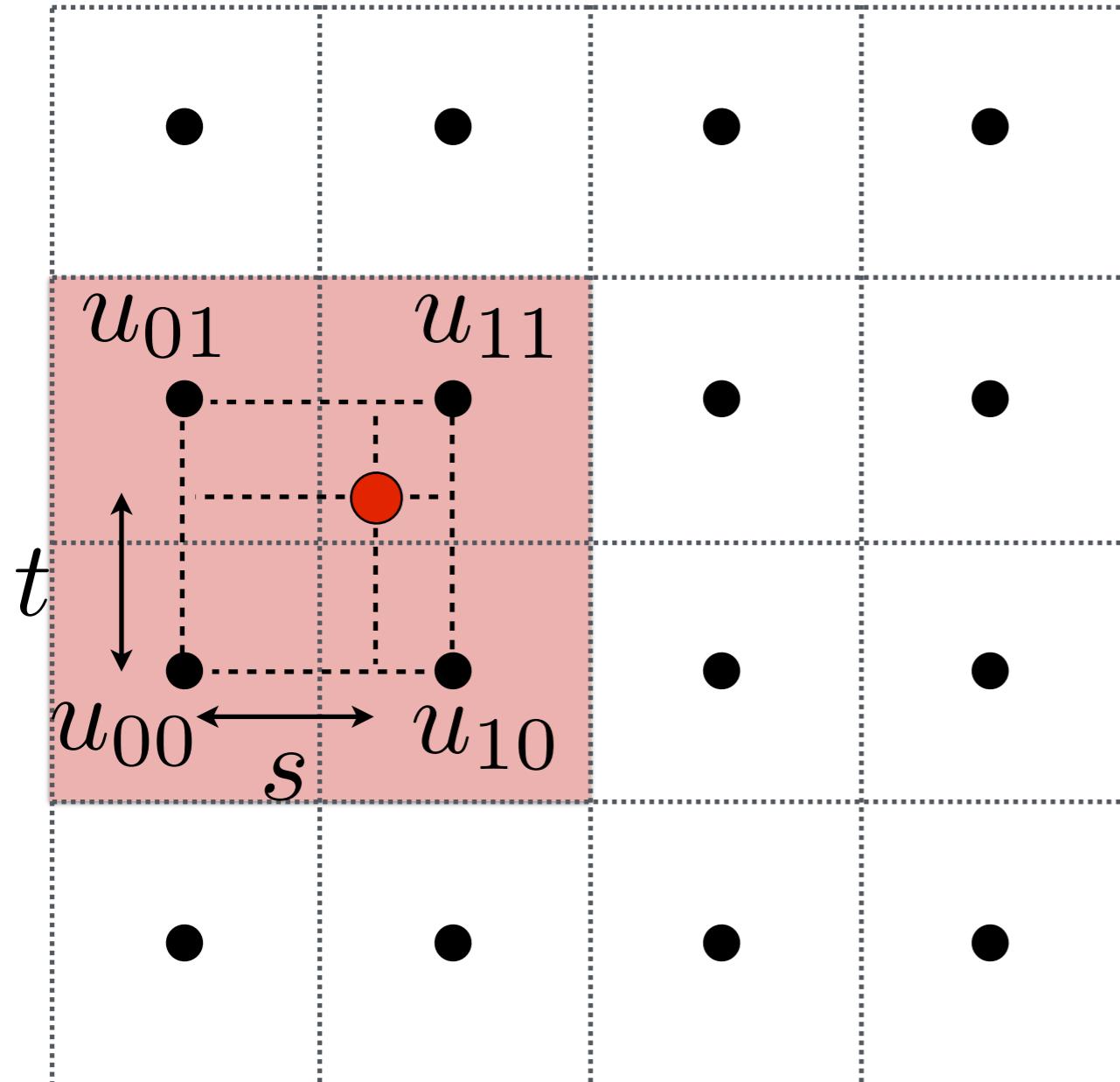
Take 4 nearest sample locations, with texture values as labeled.

Bilinear Interpolation



And fractional offsets,
 (s, t) as shown

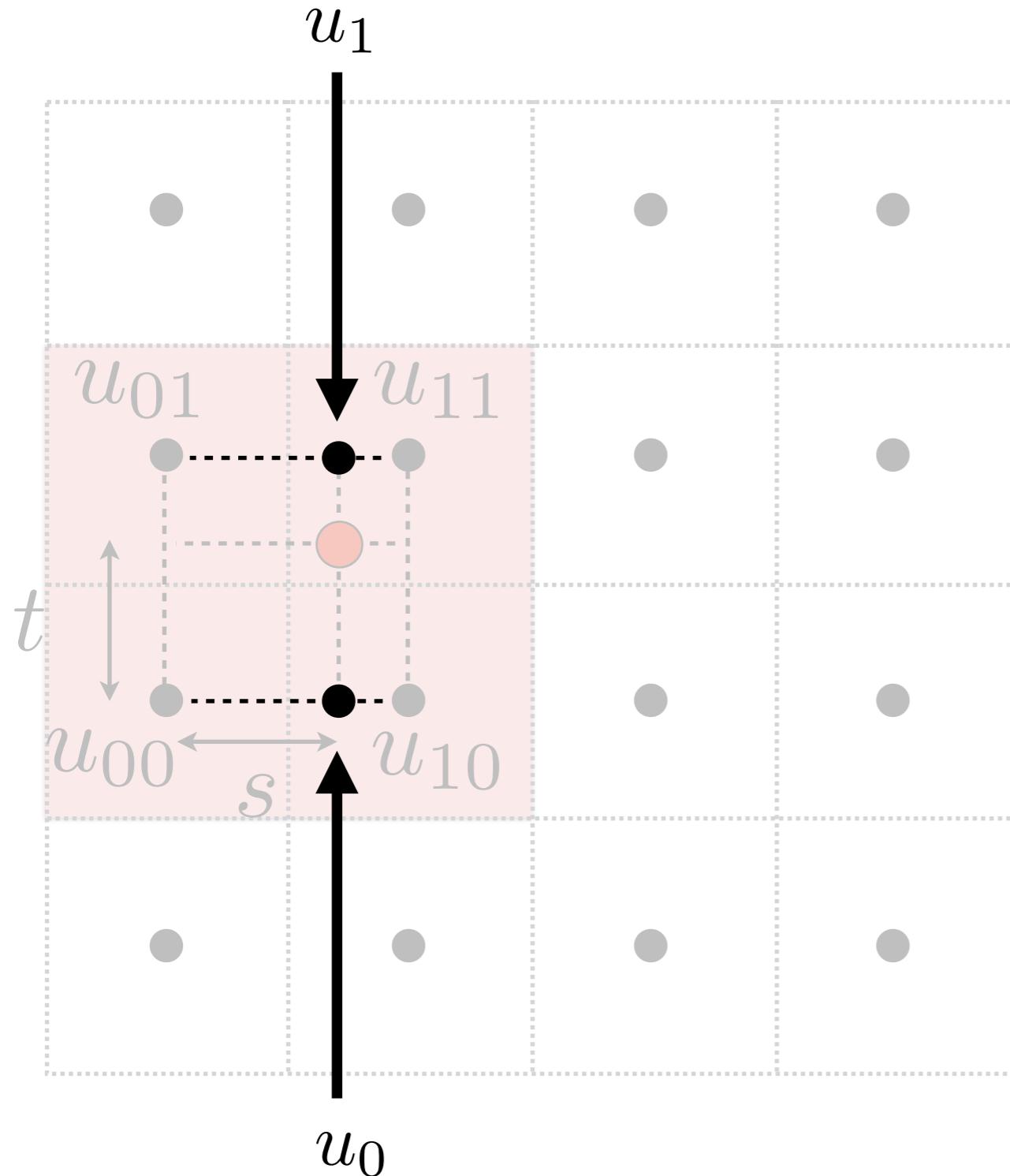
Bilinear Interpolation



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Bilinear Interpolation



Linear interpolation (1D)

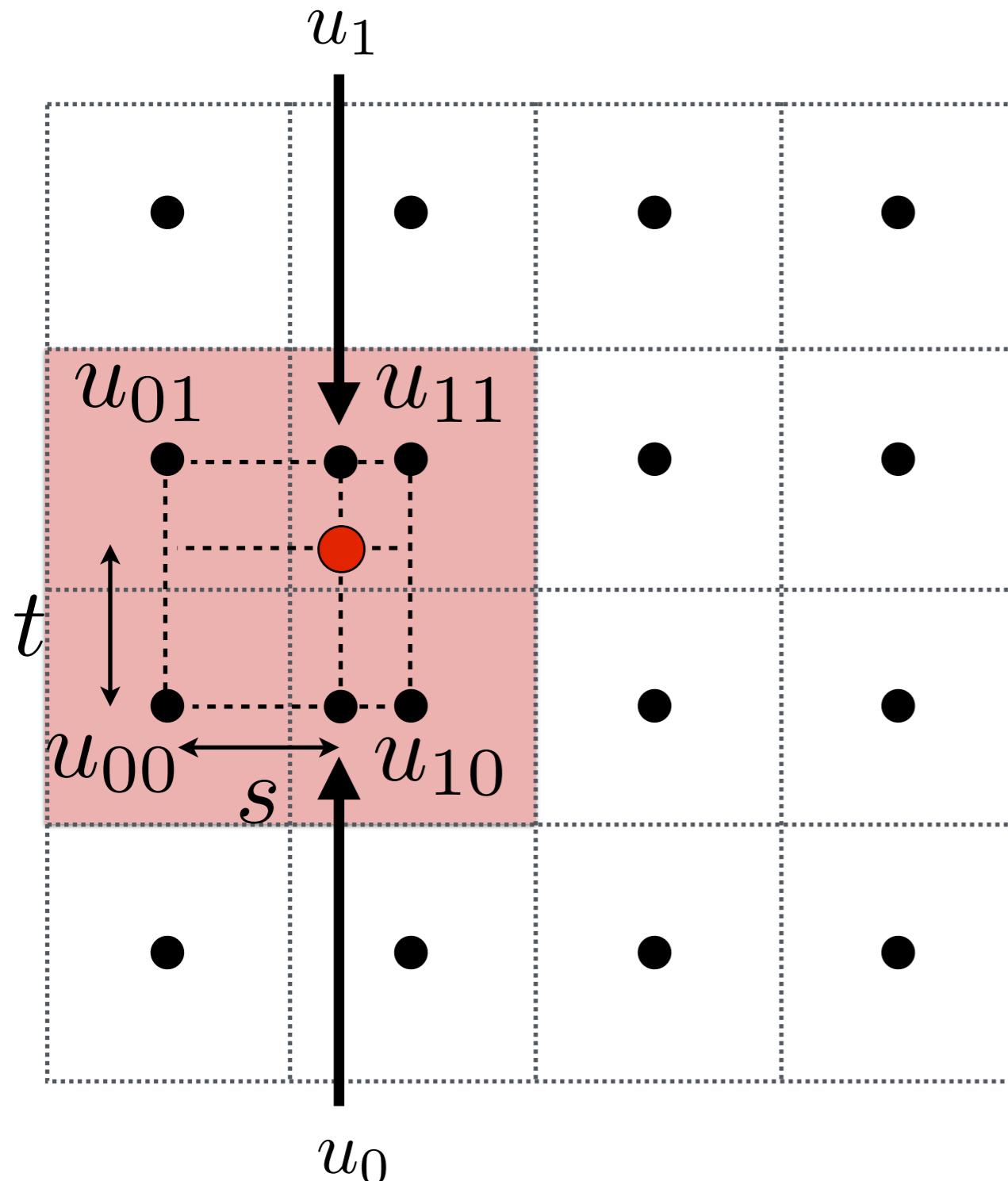
$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Two helper lerps (horizontal)

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

Bilinear Interpolation



Linear interpolation (1D)

$$\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$$

Two helper lerps

$$u_0 = \text{lerp}(s, u_{00}, u_{10})$$

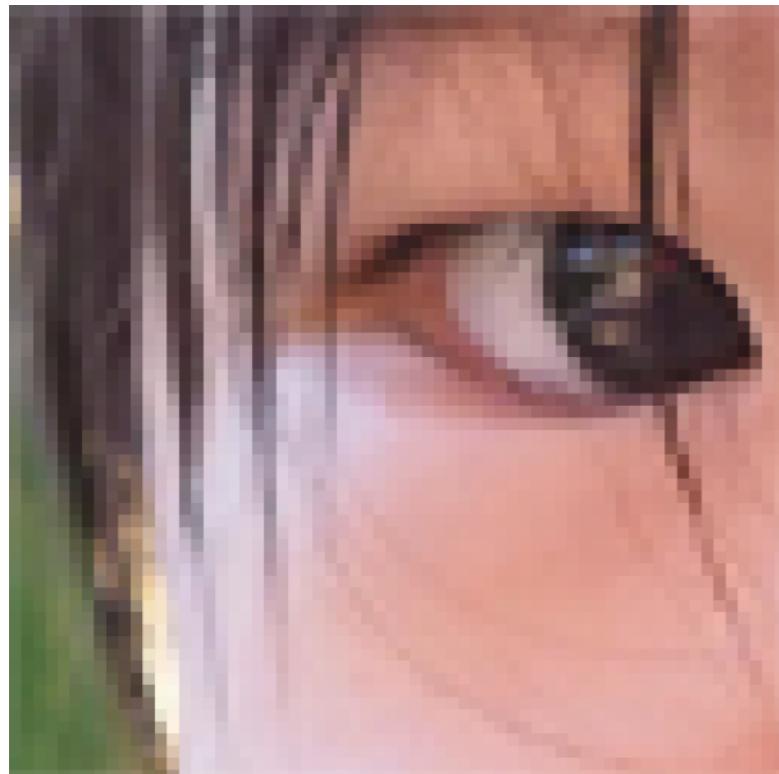
$$u_1 = \text{lerp}(s, u_{01}, u_{11})$$

Final vertical lerp, to get result:

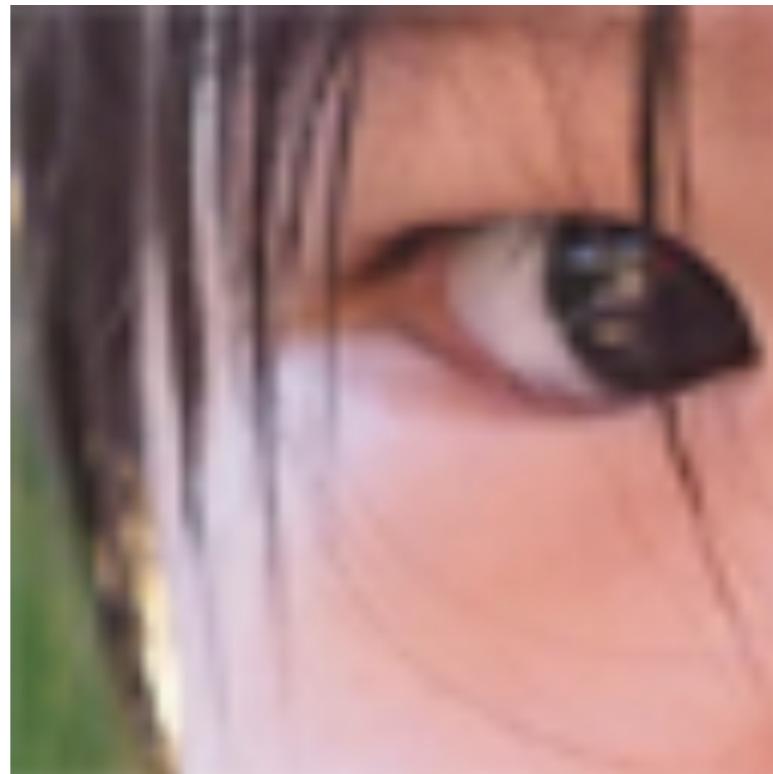
$$f(x, y) = \text{lerp}(t, u_0, u_1)$$

Texture Magnification - Easy Case

Bilinear interpolation usually gives pretty good results at reasonable costs



Nearest



Bilinear

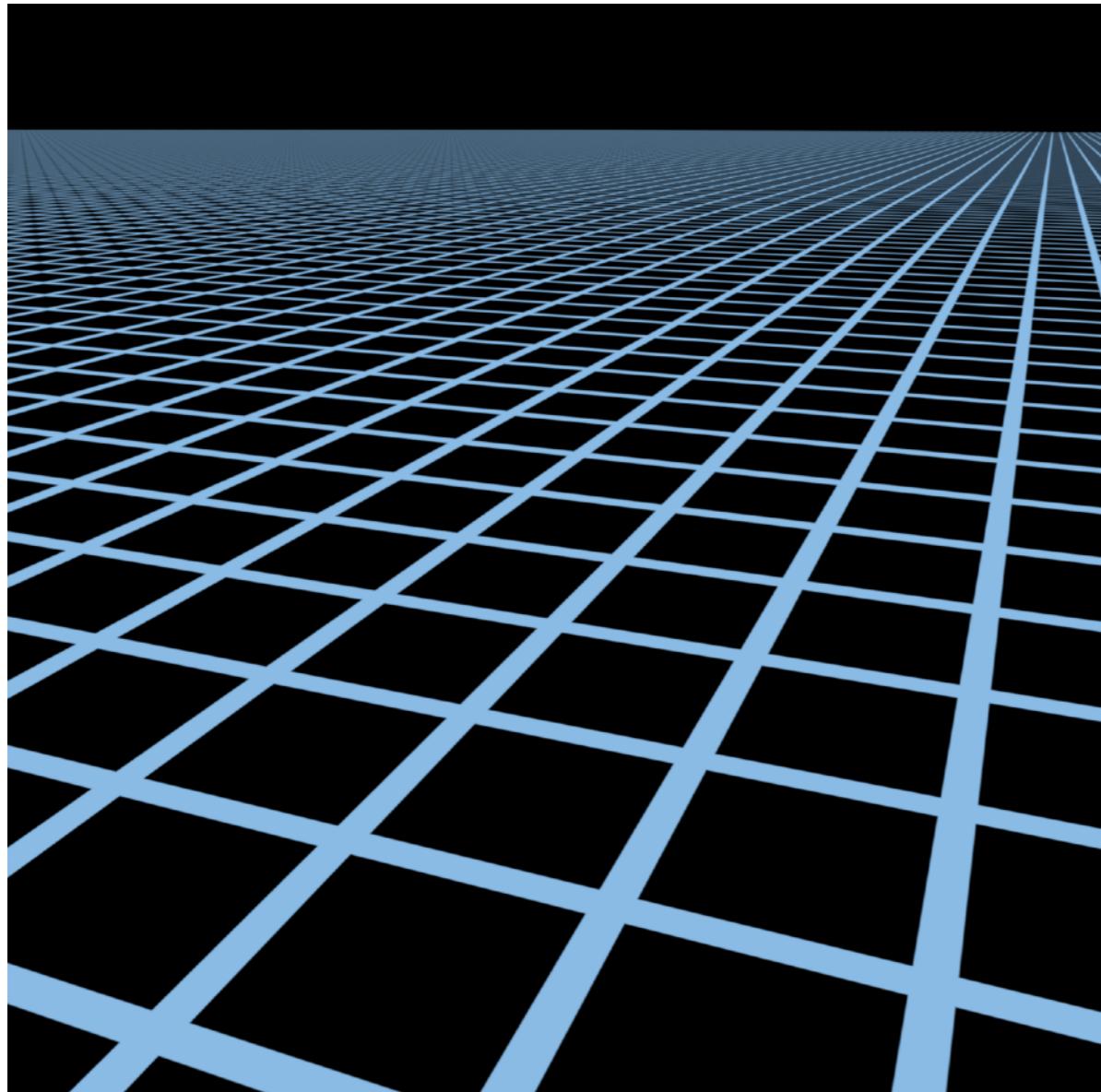


Bicubic

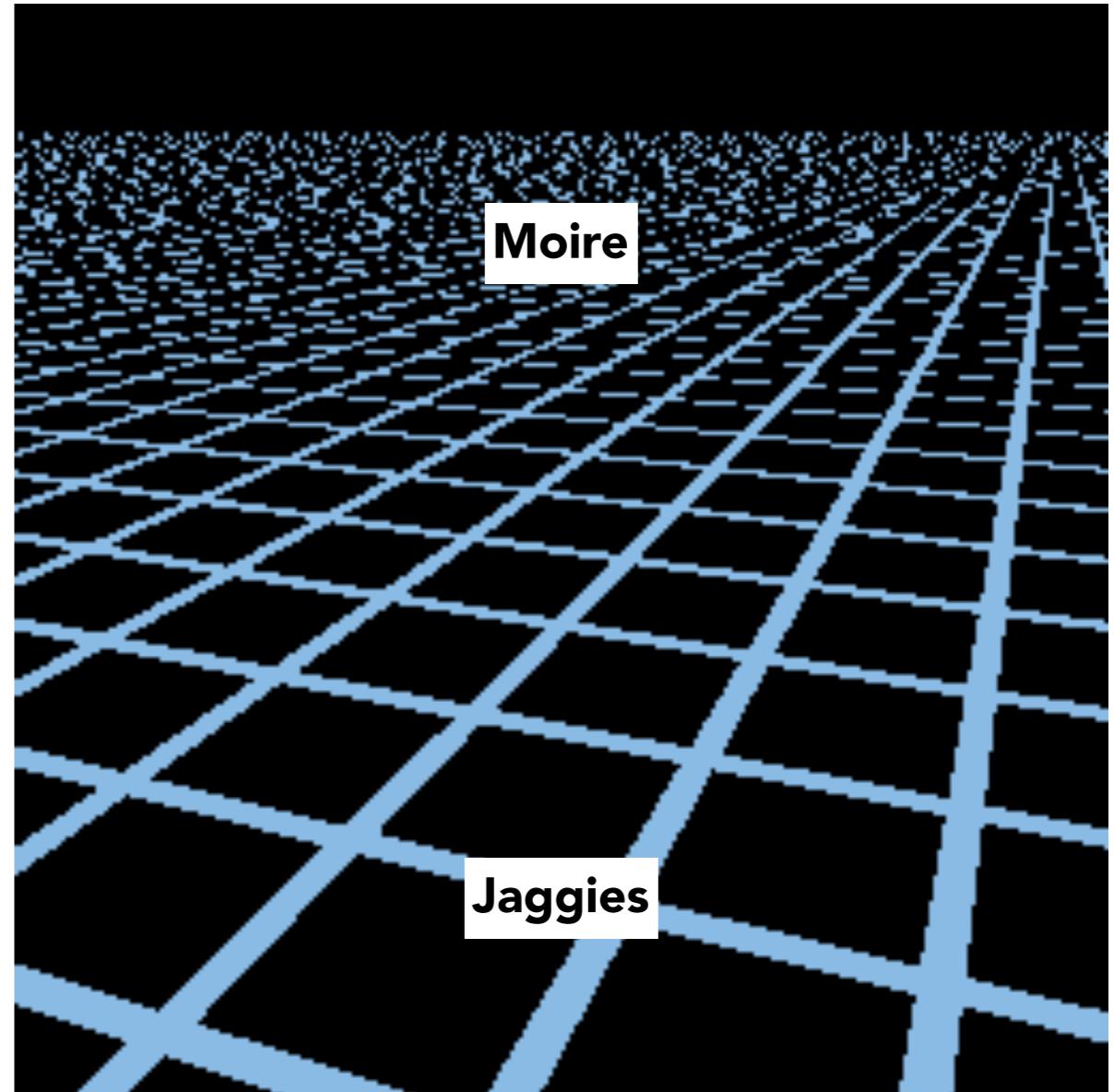
Texture Magnification (**hard case**)

(What if the texture is too large?)

Point Sampling Textures — Problem

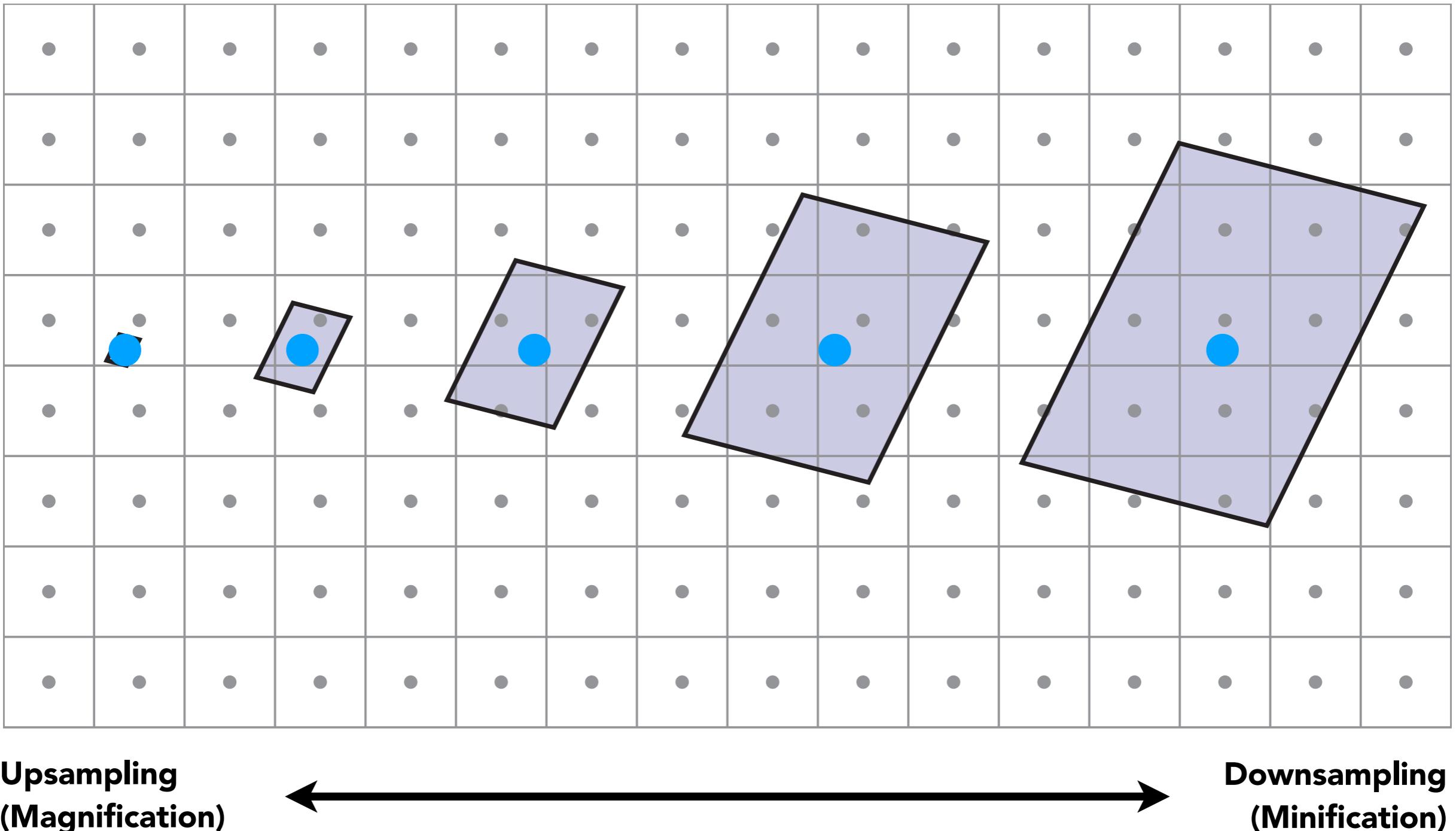


Reference

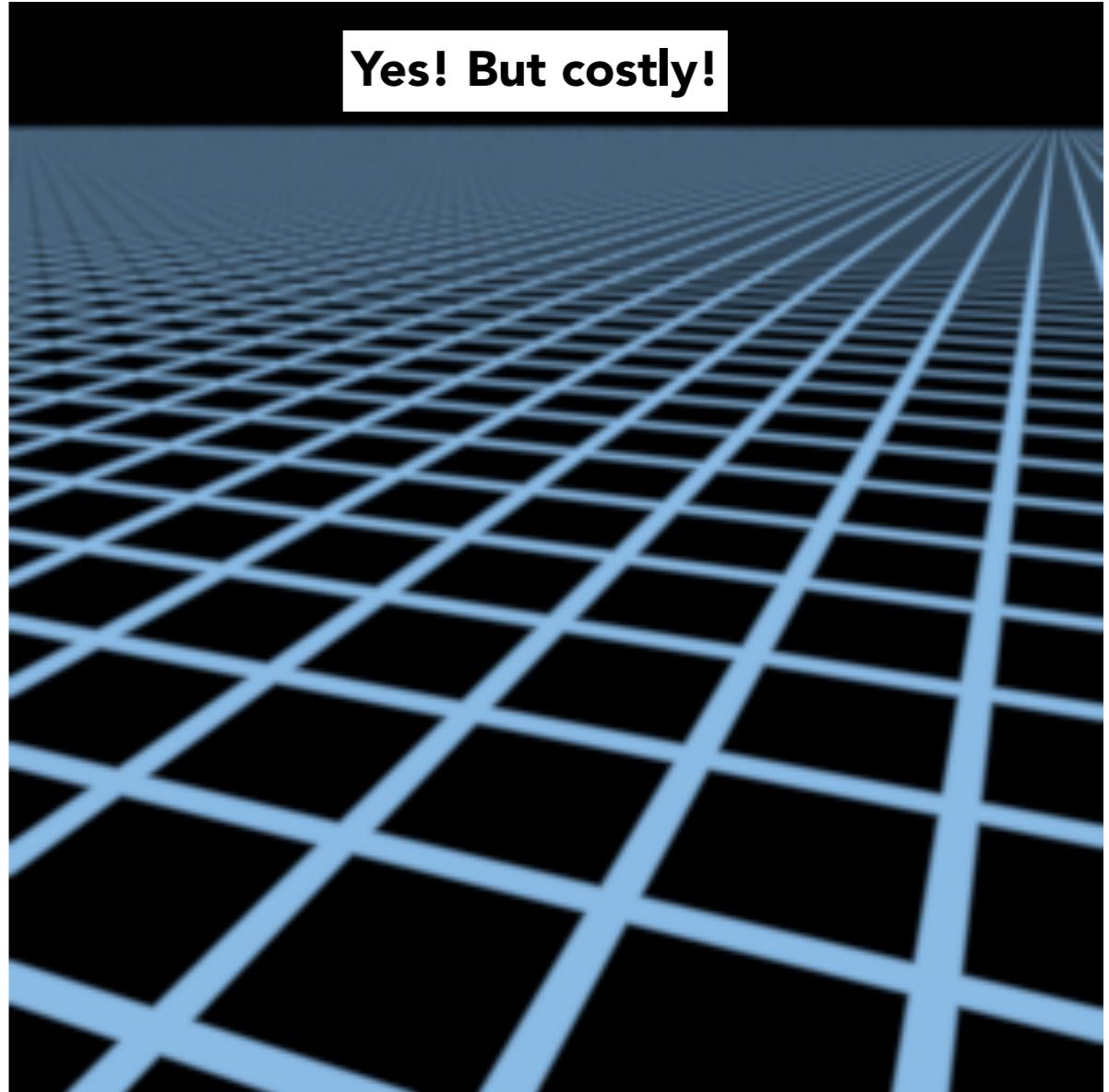
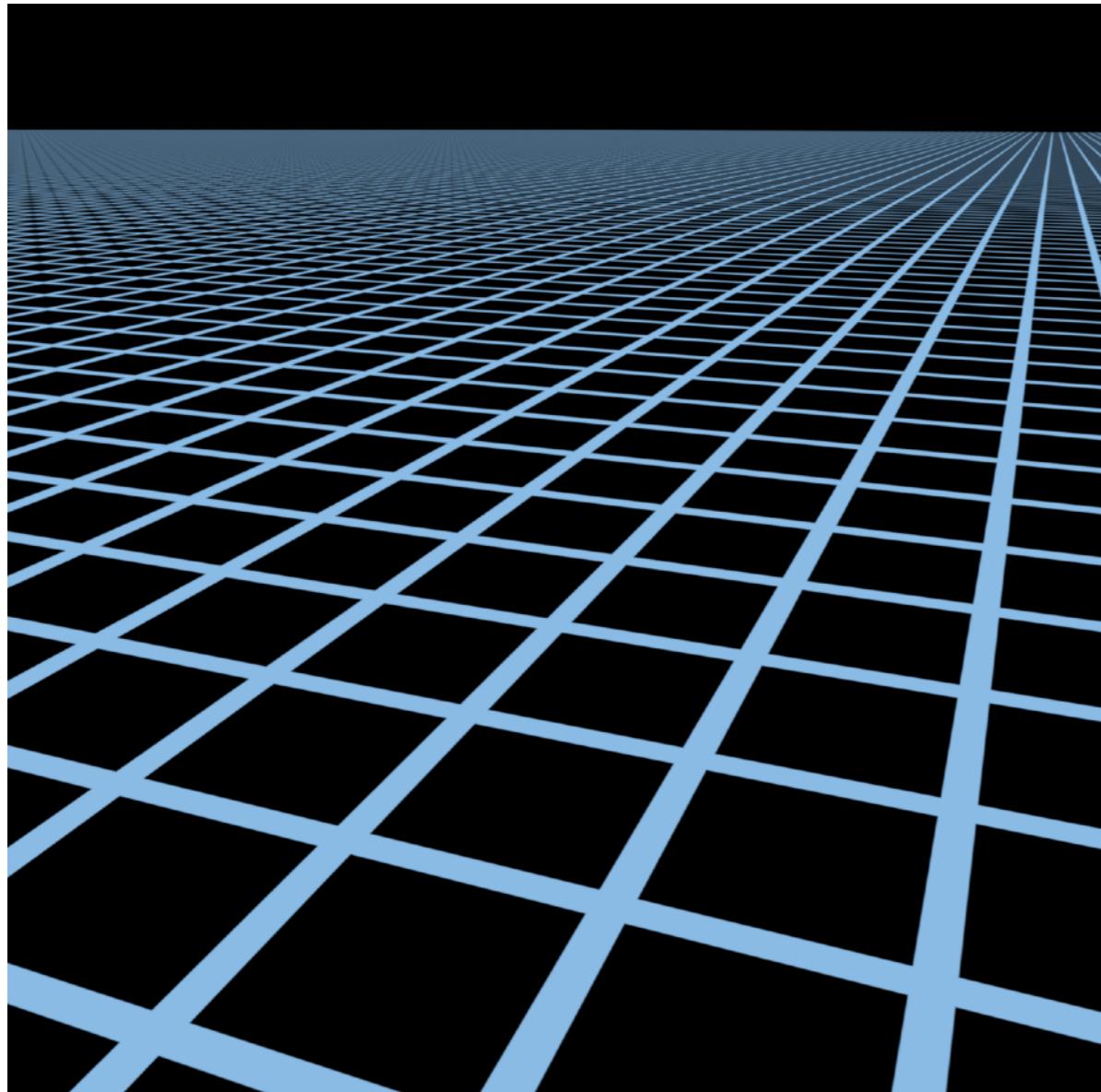


Point sampled

Screen Pixel “Footprint” in Texture



Will Supersampling Do Antialiasing?



512x supersampling

Antialiasing — Supersampling?

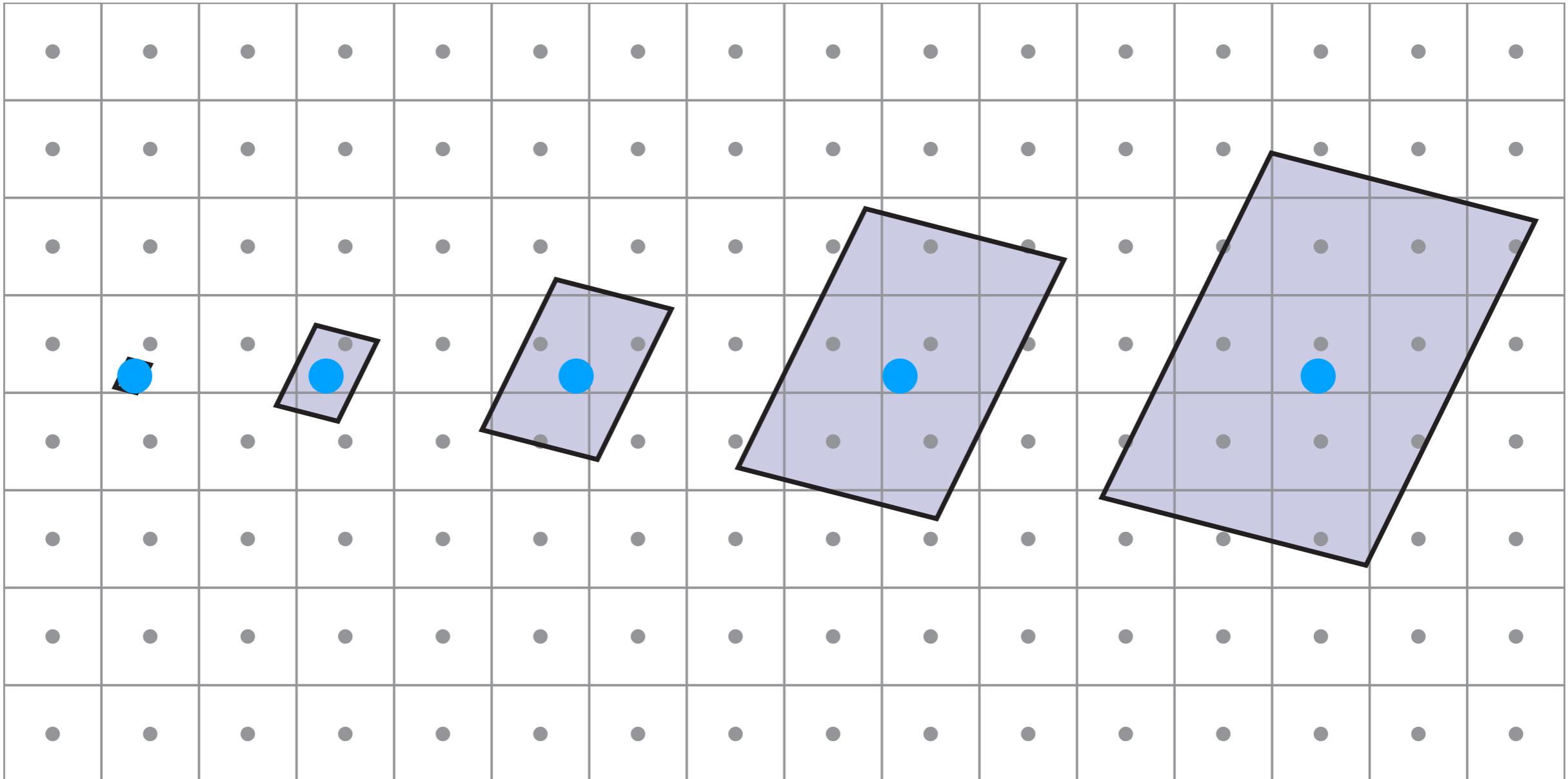
Will supersampling work?

- Yes, high quality, but costly
- When highly minified, many texels in pixel footprint
- Signal frequency too large in a pixel
- Need even higher sampling frequency

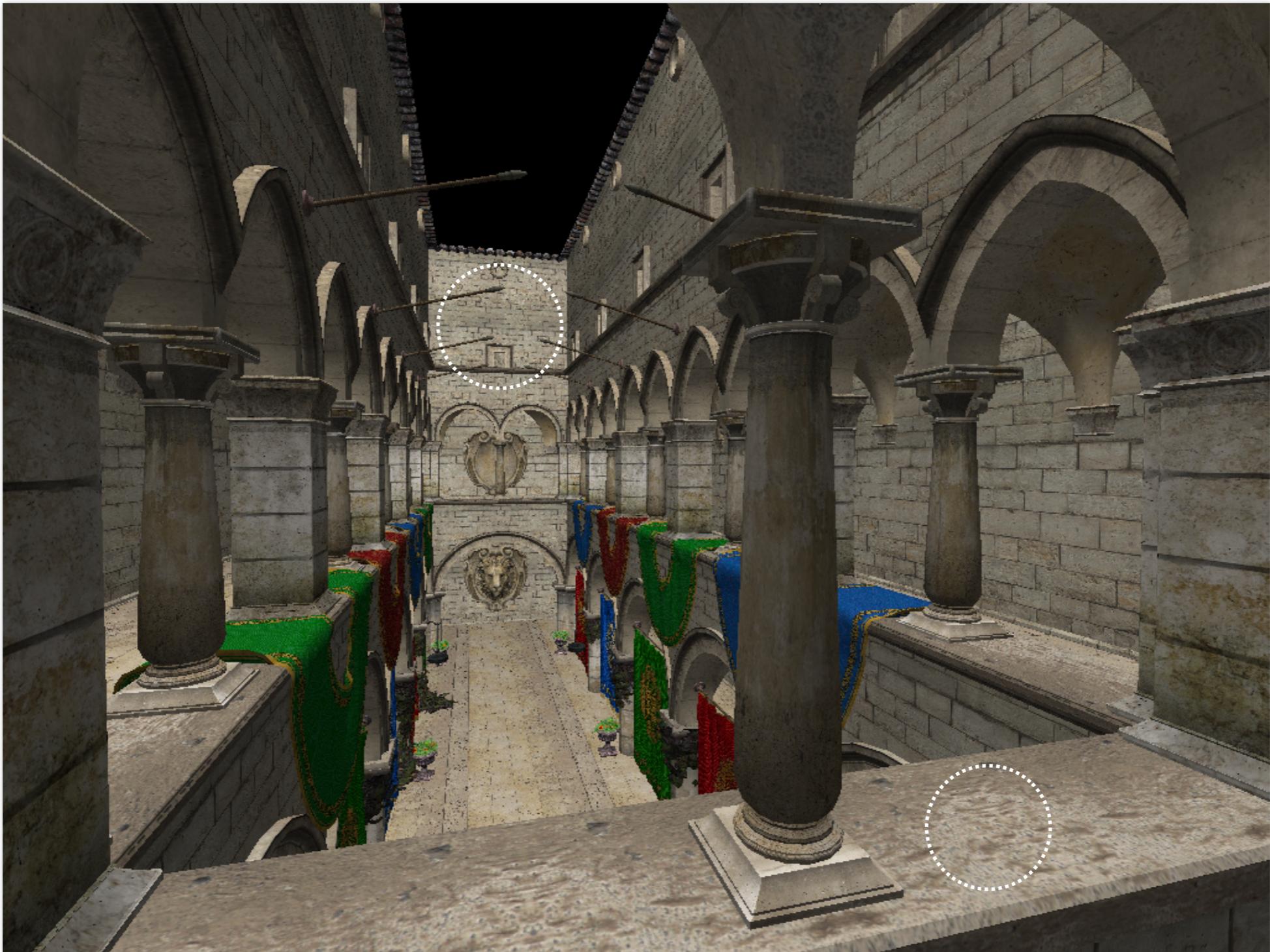
Let's understand this problem in another way

- What if we don't sample?
- Just need to **get the average value within a range!**

Point Query vs. (Avg.) Range Query



Different Pixels -> Different-Sized Footprints



Mipmap

Allowing (fast, approx., square) range queries

Mipmap (L. Williams 83)

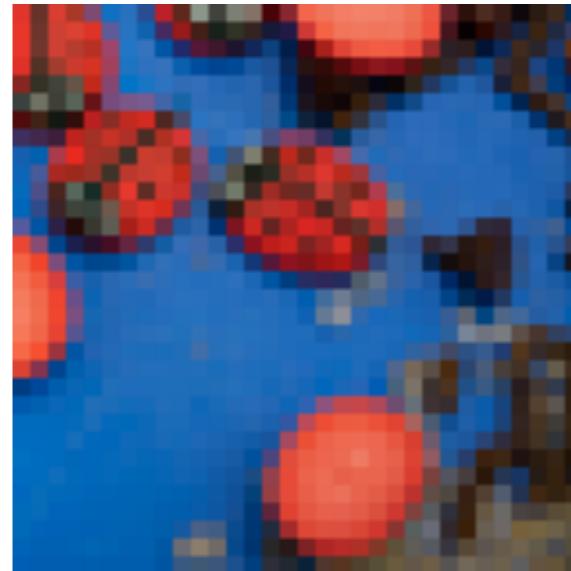
"Mip" comes from the Latin **"multum in parvo"**, meaning a multitude in a small space



Level 0 = 128x128



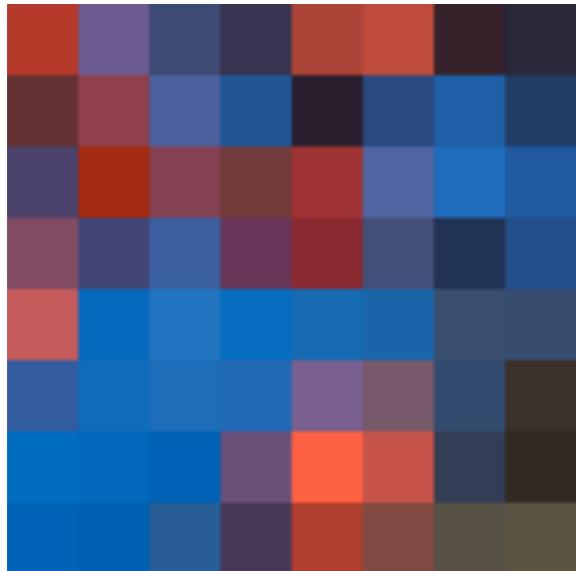
Level 1 = 64x64



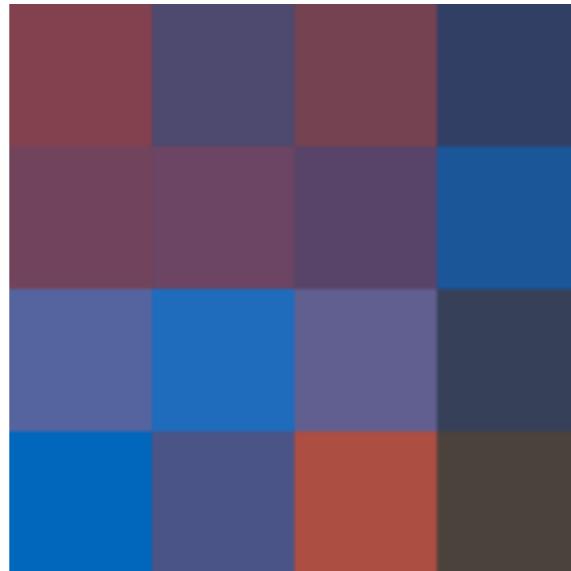
Level 2 = 32x32



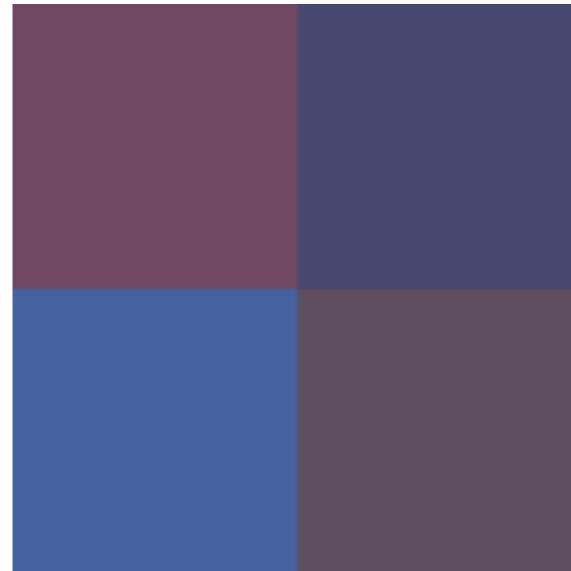
Level 3 = 16x16



Level 4 = 8x8



Level 5 = 4x4

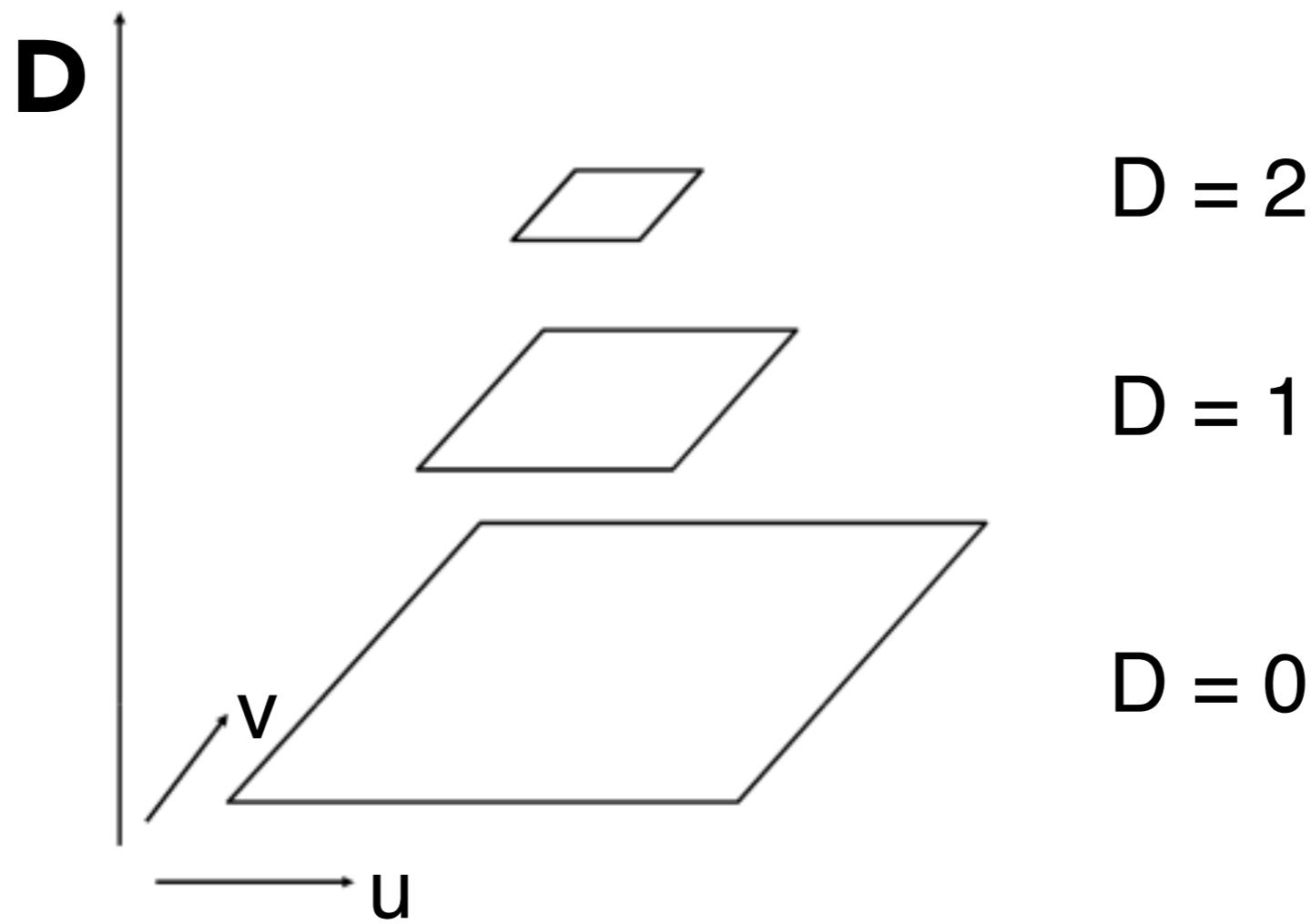


Level 6 = 2x2



Level 7 = 1x1

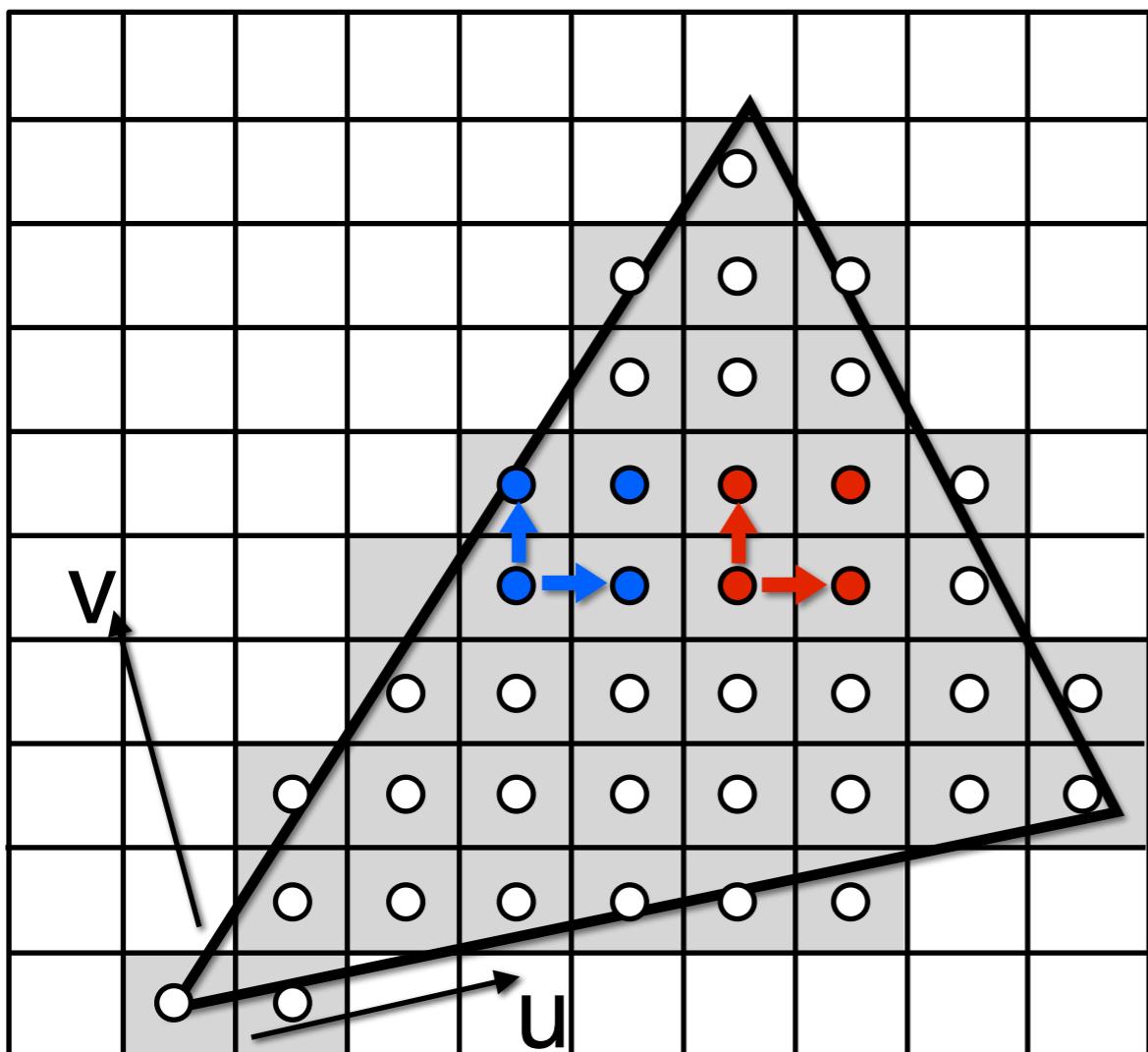
Mipmap (L. Williams 83)



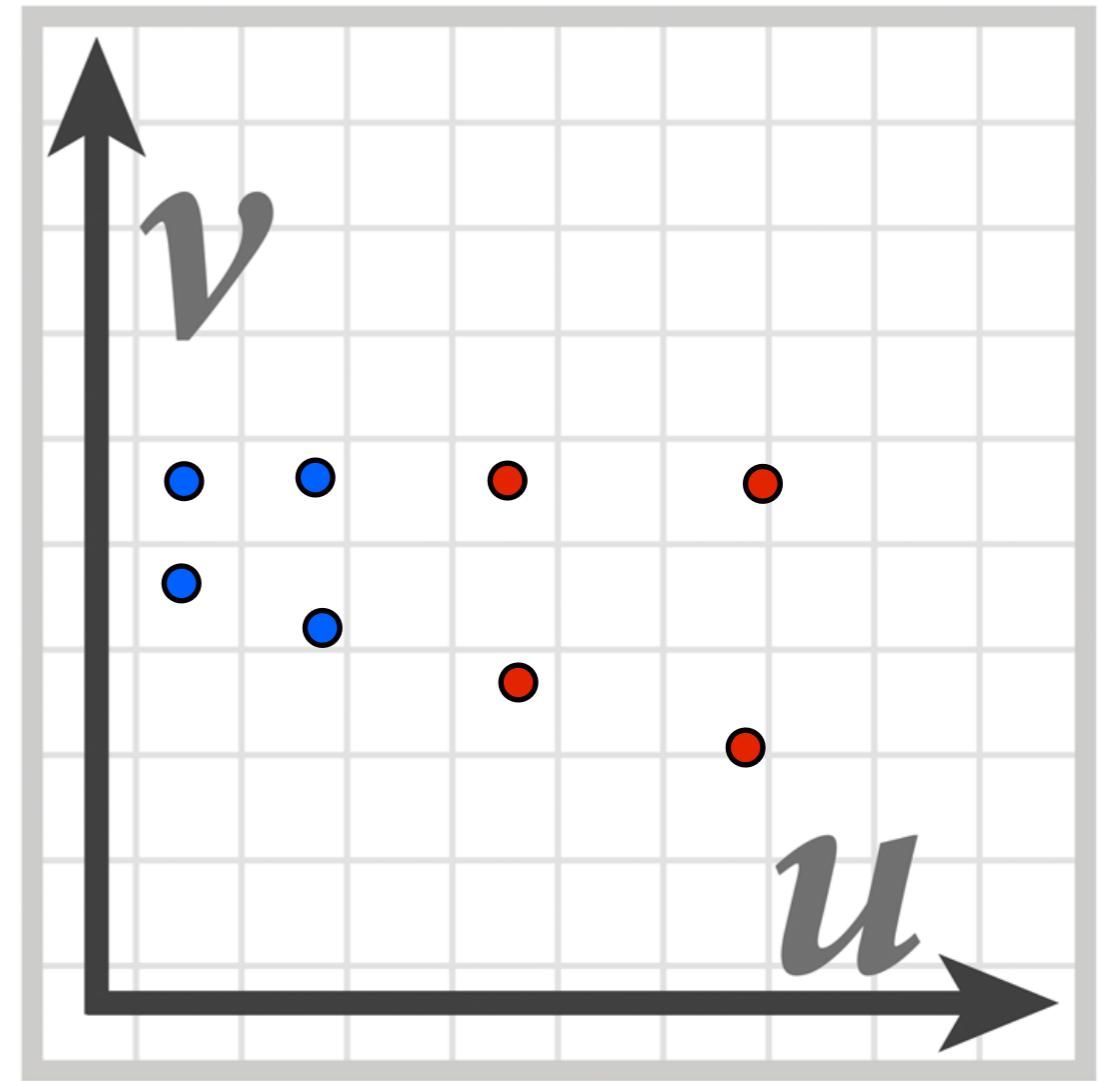
“Mip hierarchy”
level = D

What is the storage overhead of a mipmap?

Computing Mipmap Level D



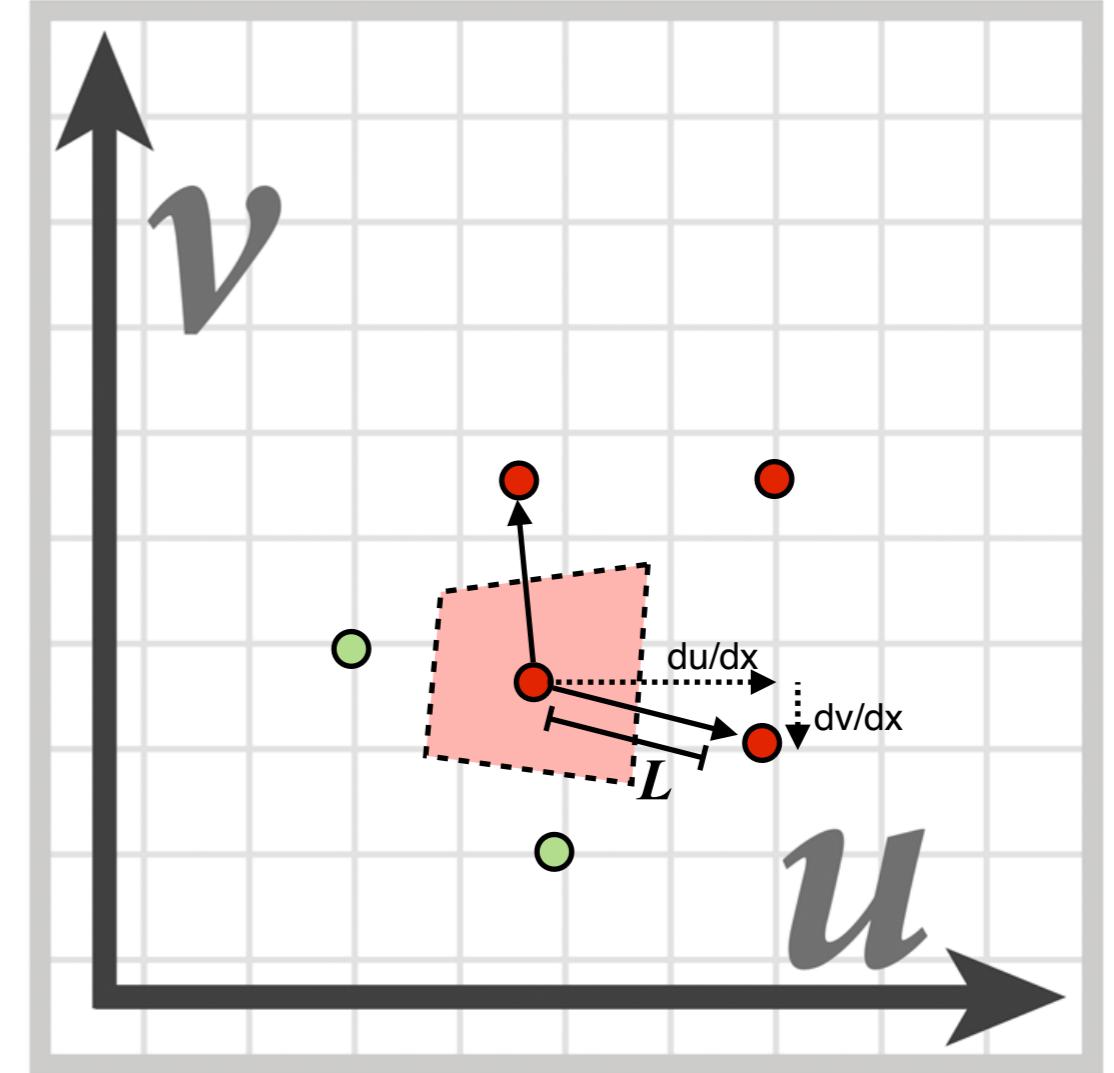
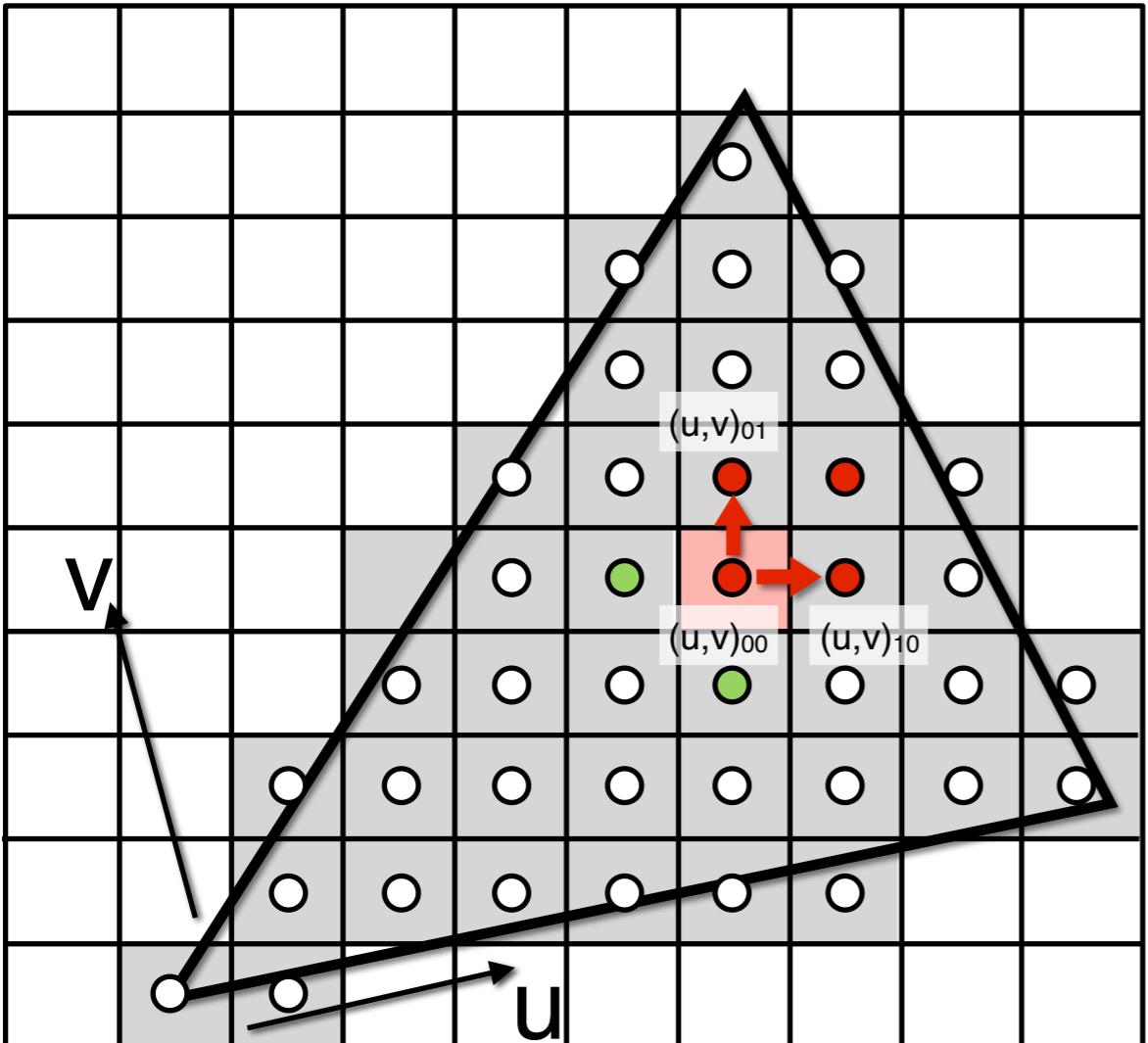
Screen space (x,y)



Texture space (u,v)

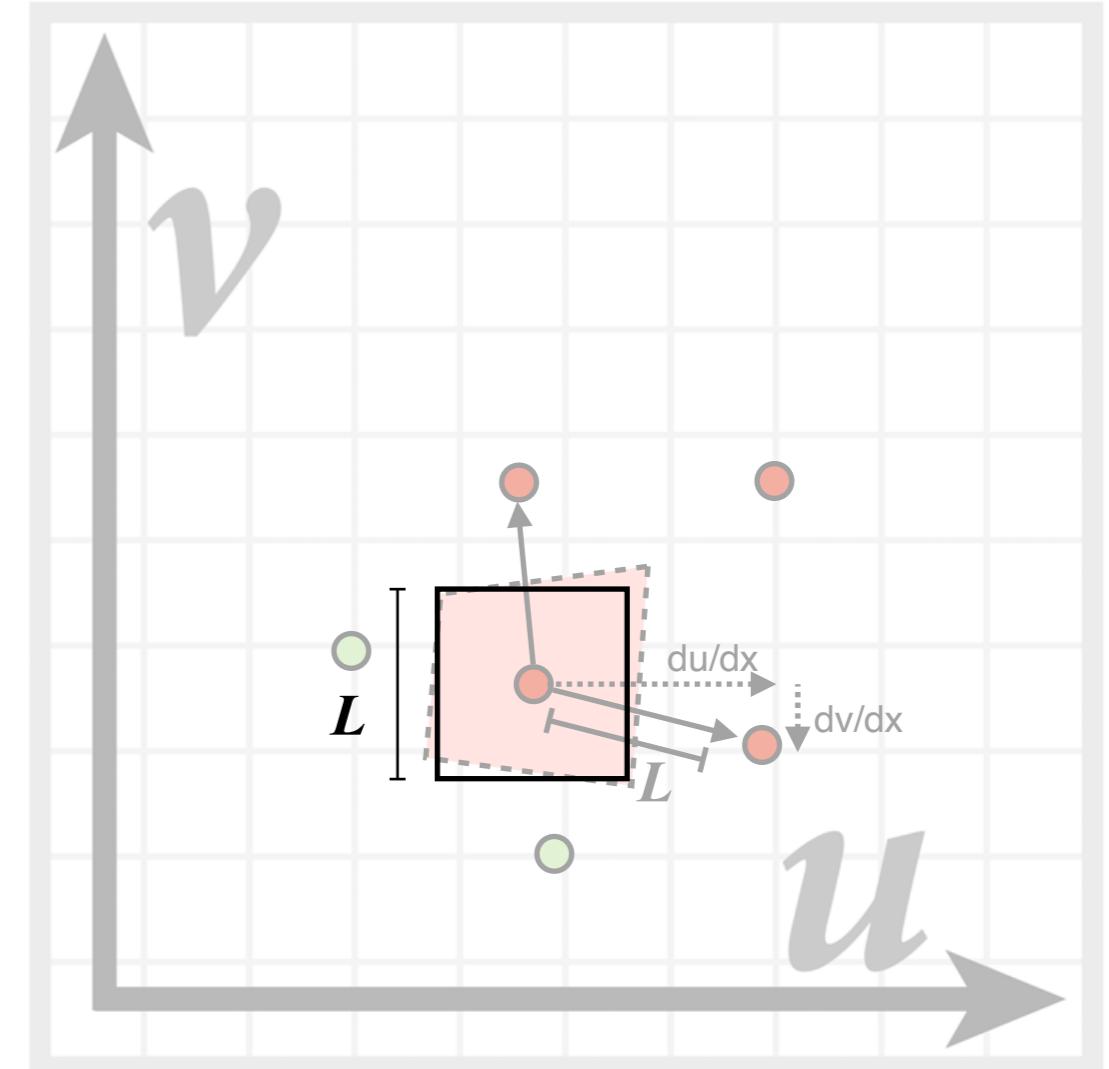
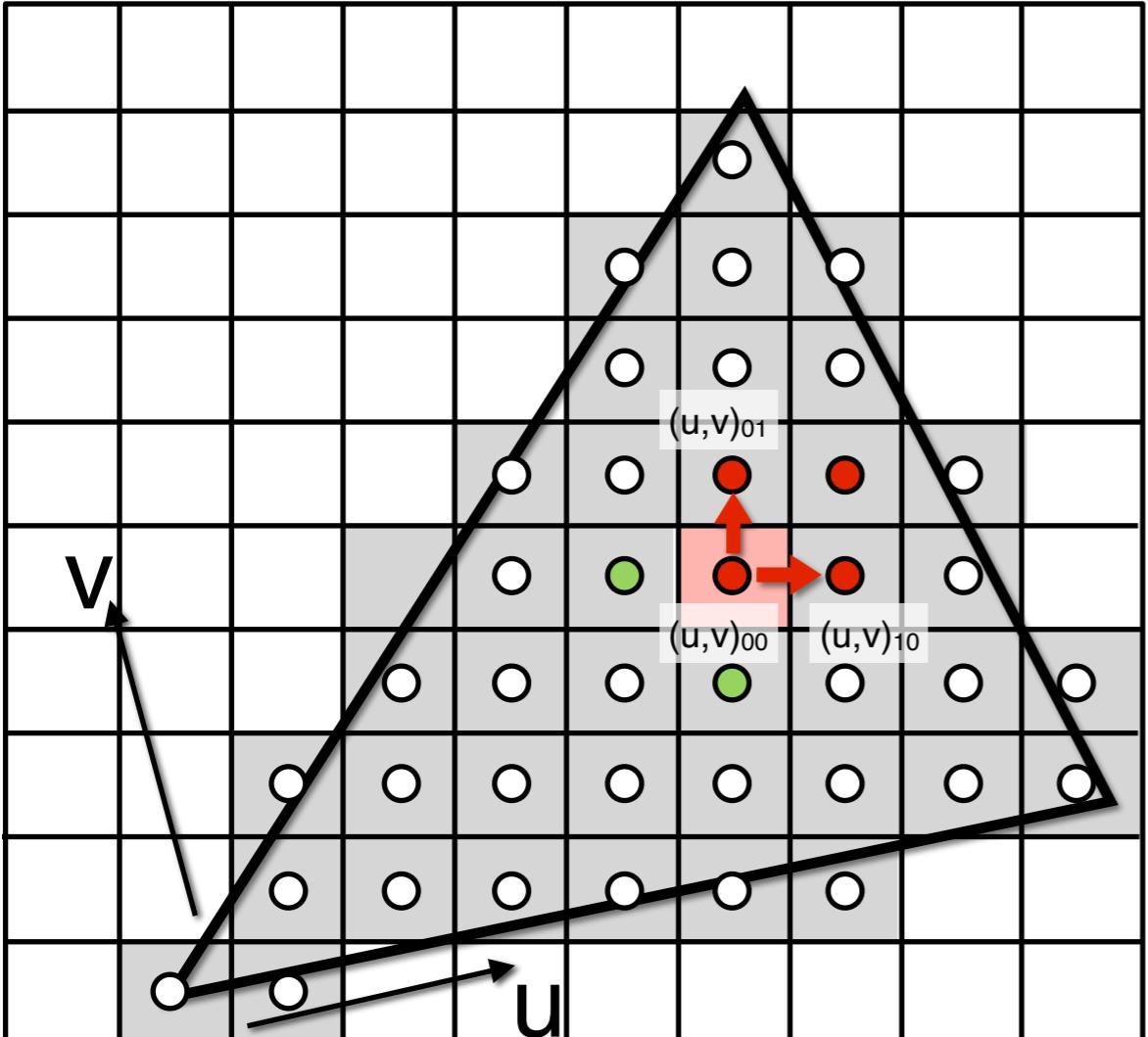
Estimate texture footprint using texture coordinates of neighboring screen samples

Computing Mipmap Level D



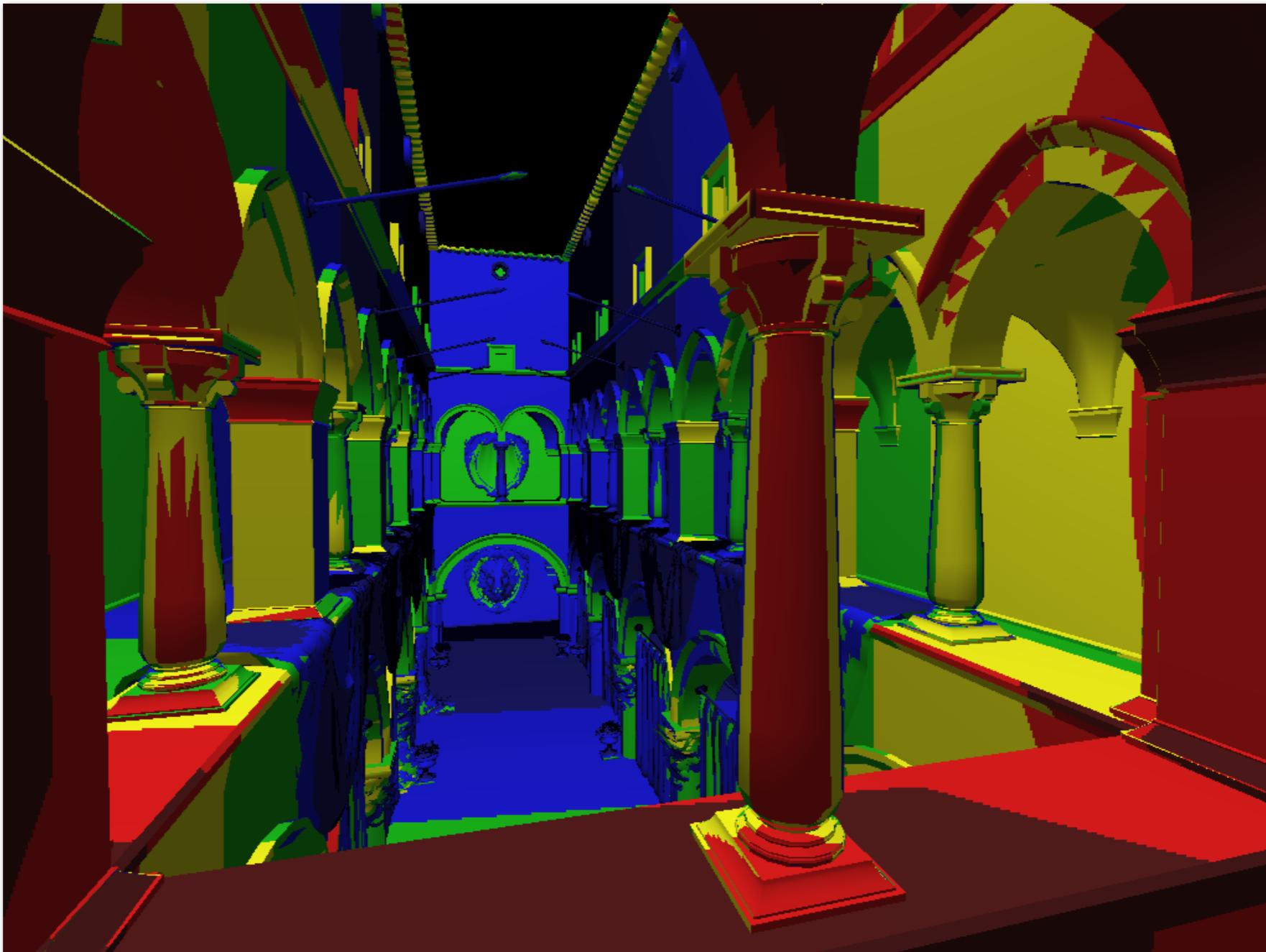
$$D = \log_2 L \quad L = \max \left(\sqrt{\left(\frac{du}{dx} \right)^2 + \left(\frac{dv}{dx} \right)^2}, \sqrt{\left(\frac{du}{dy} \right)^2 + \left(\frac{dv}{dy} \right)^2} \right)$$

Computing Mipmap Level D



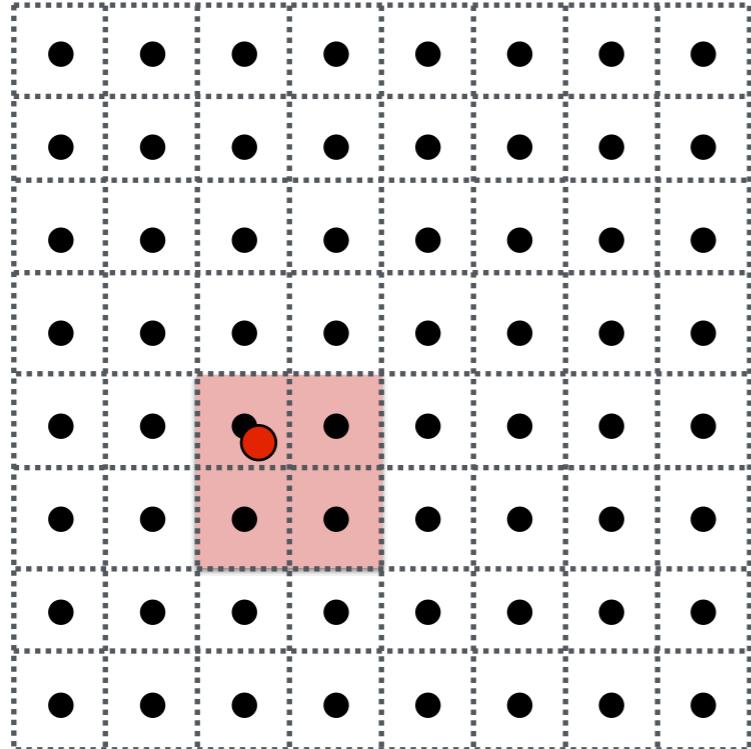
$$D = \log_2 L \quad L = \max \left(\sqrt{\left(\frac{du}{dx} \right)^2 + \left(\frac{dv}{dx} \right)^2}, \sqrt{\left(\frac{du}{dy} \right)^2 + \left(\frac{dv}{dy} \right)^2} \right)$$

Visualization of Mipmap Level

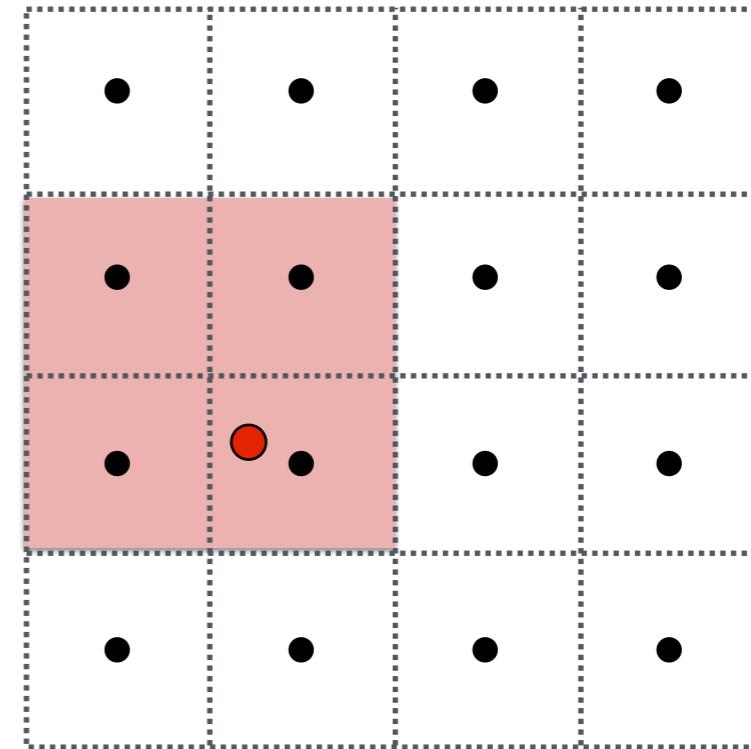


D rounded to nearest integer level

Trilinear Interpolation



Mipmap Level D



Mipmap Level D+1

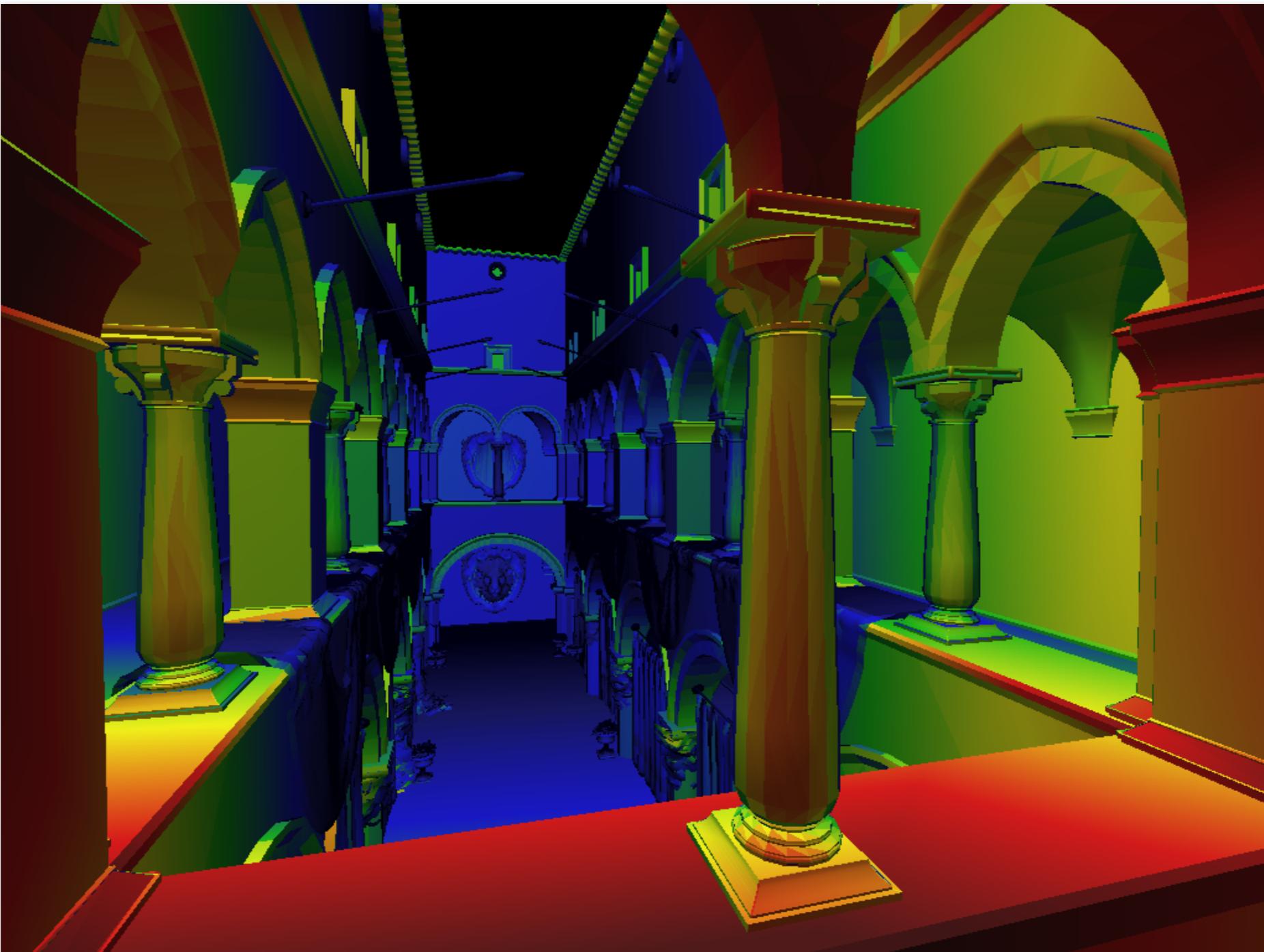
Bilinear result



Bilinear result

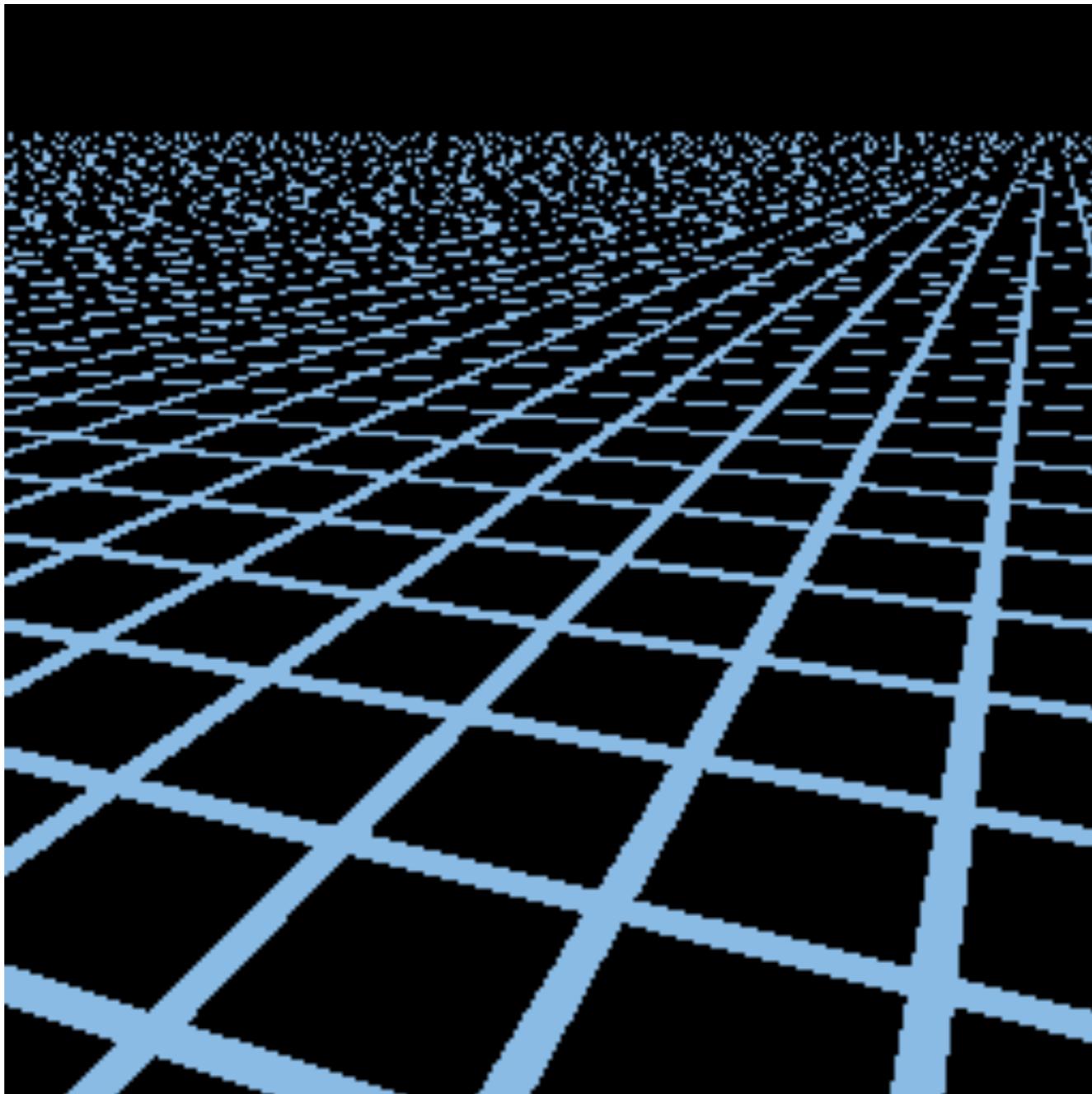
Linear interpolation based on continuous D value

Visualization of Mipmap Level



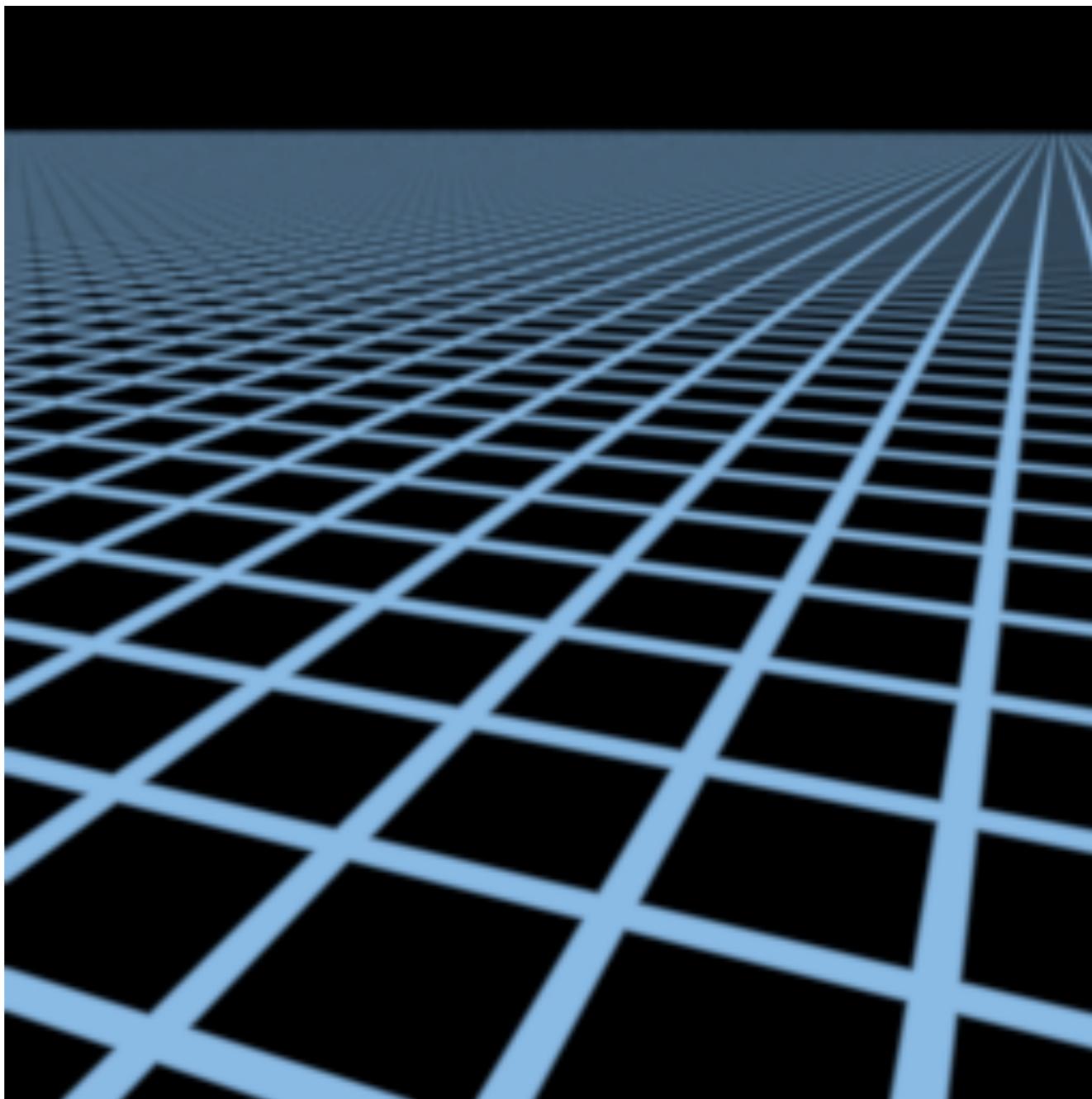
Trilinear filtering: visualization of continuous D

Mipmap Limitations



Point sampling

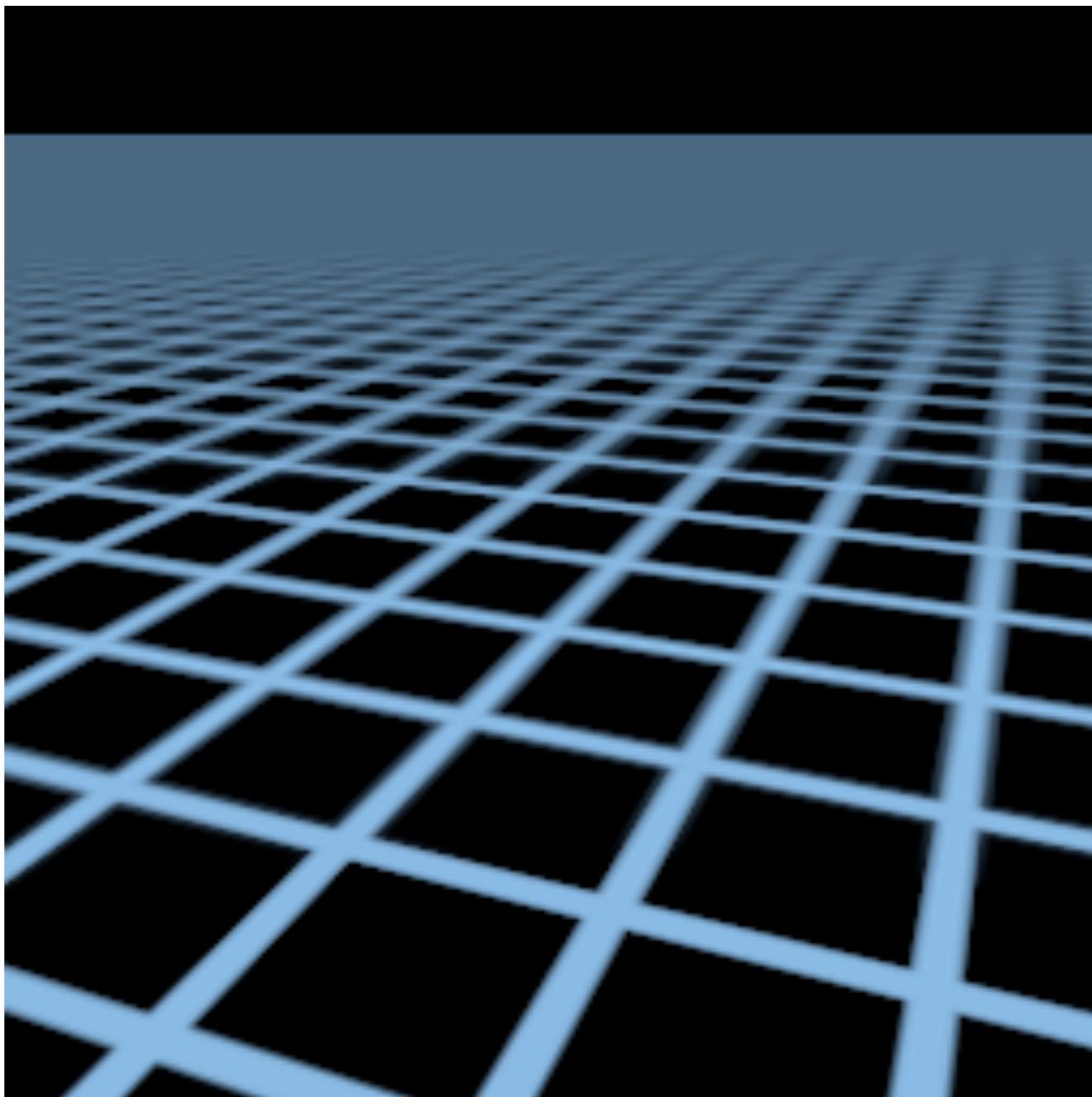
Mipmap Limitations



Supersampling 512x (assume this is correct)

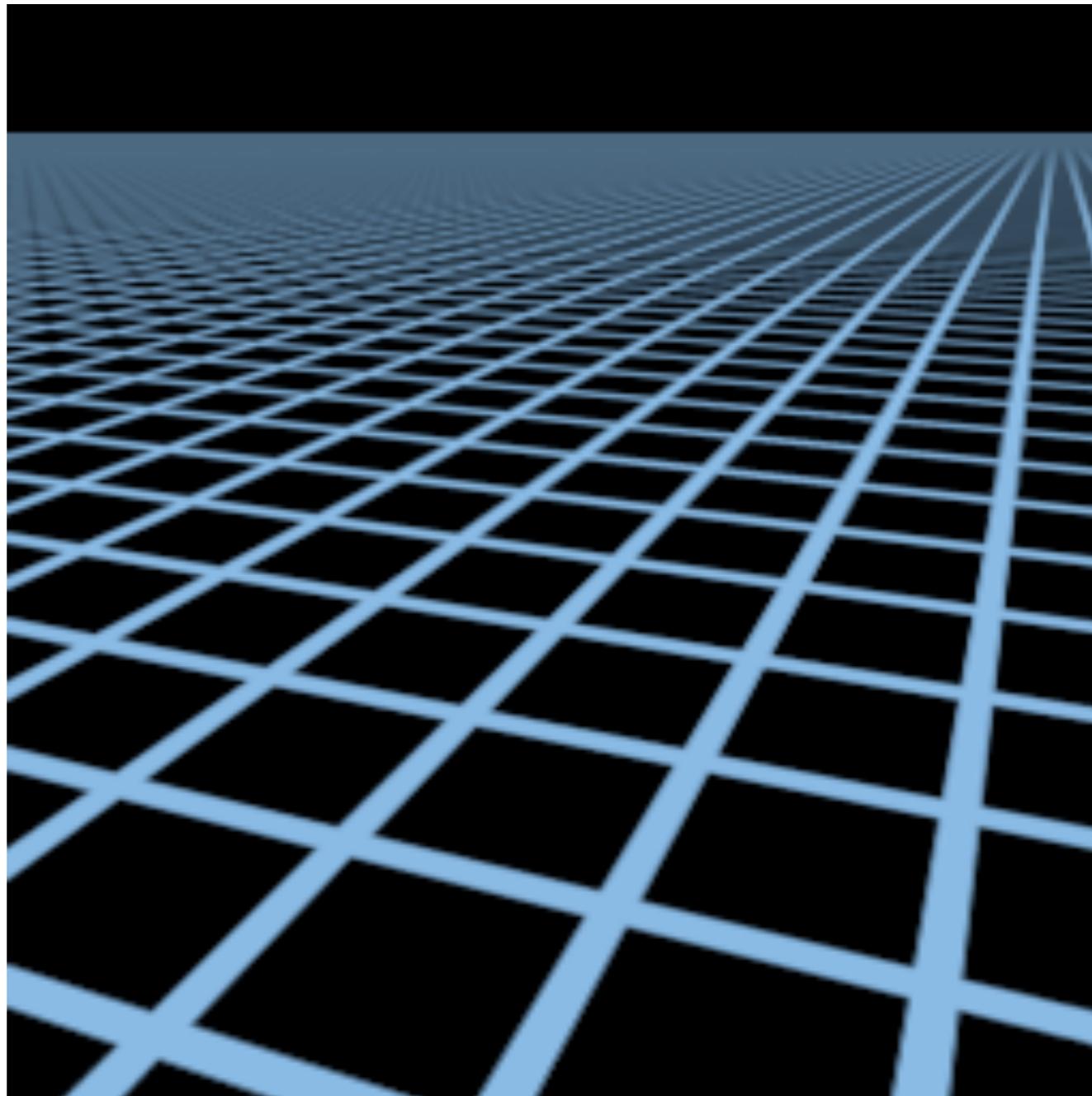
Mipmap Limitations

Overblur
Why?



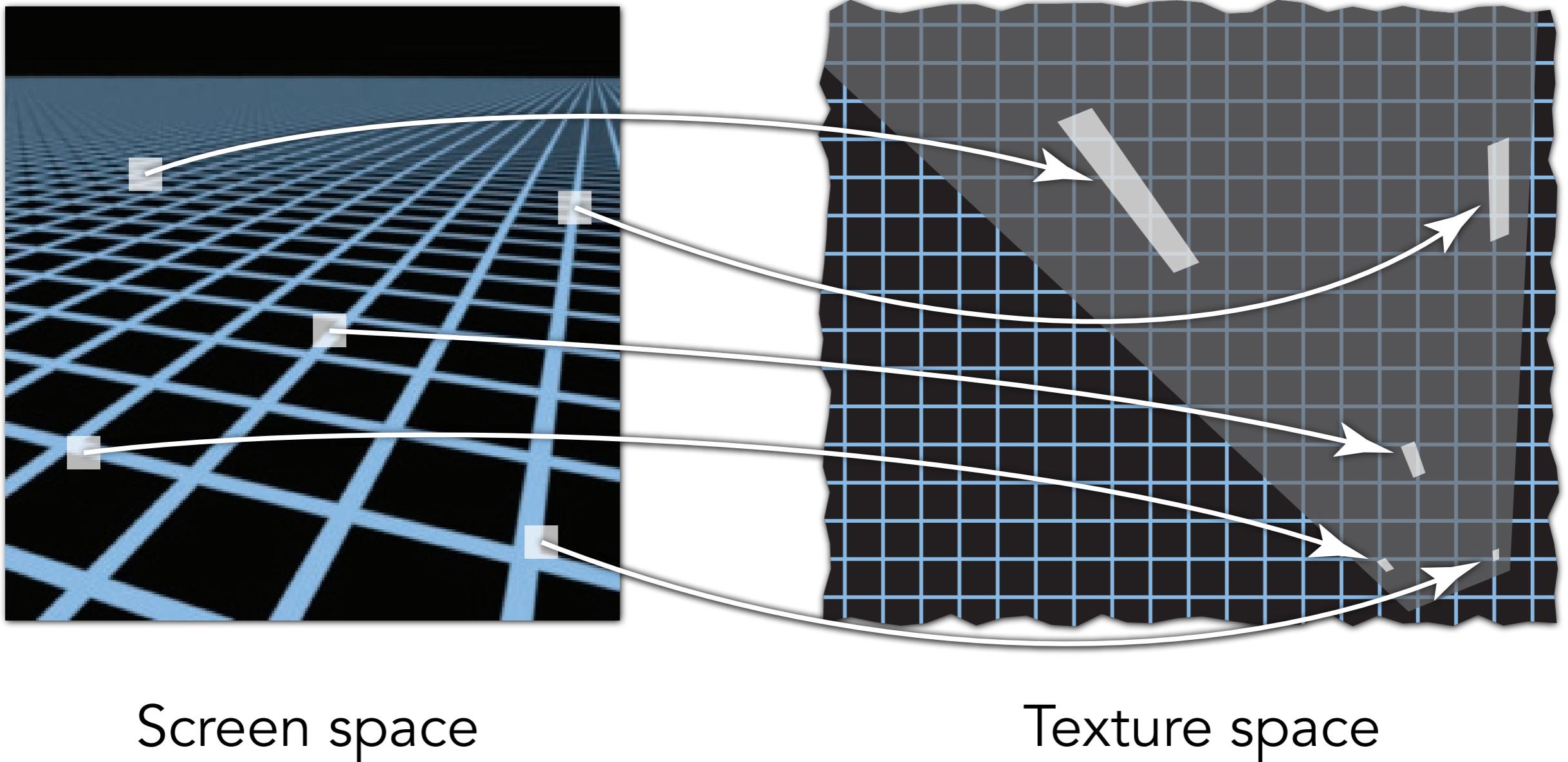
Mipmap trilinear sampling

Anisotropic Filtering



Better than Mipmap!

Irregular Pixel Footprint in Texture



Anisotropic Filtering

Ripmaps and summed area tables

- Can look up **axis-aligned rectangular zones**
- Diagonal footprints still a problem



Wikipedia

Anisotropic Filtering

Ripmaps and summed area tables

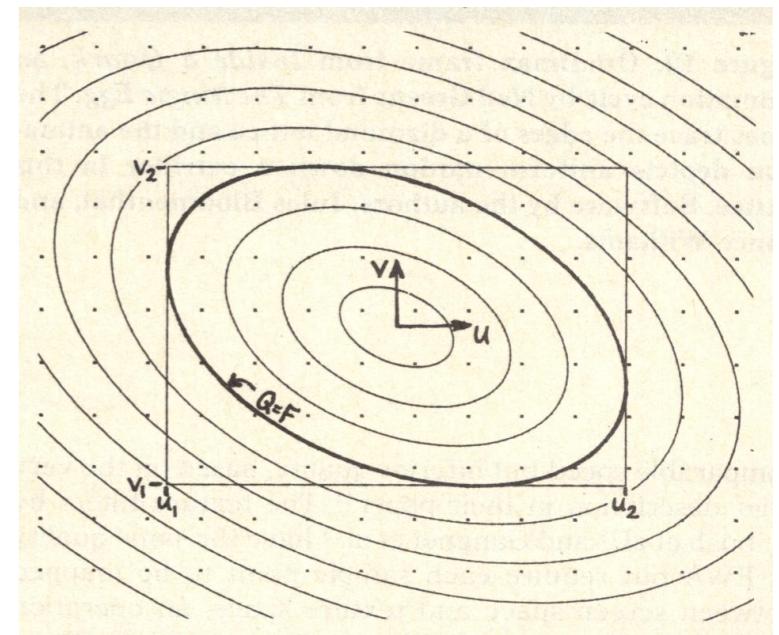
- Can look up axis-aligned rectangular zones
- Diagonal footprints still a problem



Wikipedia

EWA filtering

- Use multiple lookups
- Weighted average
- Mipmap hierarchy still helps
- Can handle irregular footprints



Greene & Heckbert '86

Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)