Restricted Boltzmann Machines

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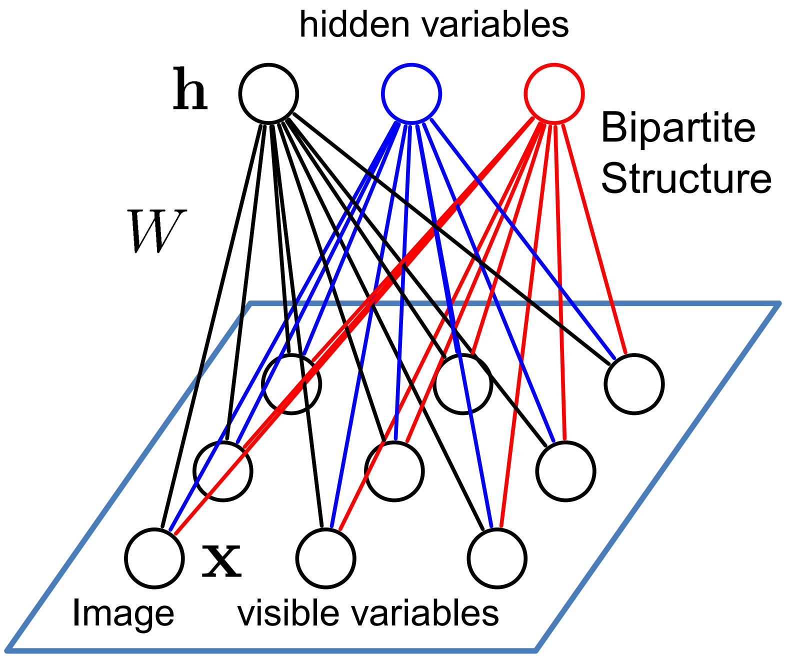
Unsupervised Learning

- Unsupervised learning: we only use the inputs $\mathbf{x}^{(t)}$ for learning
  - automatically extract meaningful features for your data
  - leverage the availability of unlabeled data

- We will consider 3 models for unsupervised learning that will form the basic building blocks for deeper models:
  - Autoencoders
  - Sparse coding models
  - Restricted Boltzmann Machines

Restricted Boltzmann Machines



- Undirected bipartite graphical model
- Stochastic binary visible variables:

$$\mathbf{x} \in \{\mathbf{0}, \mathbf{1}\}^{\mathbf{D}}$$

- Stochastic binary hidden variables:

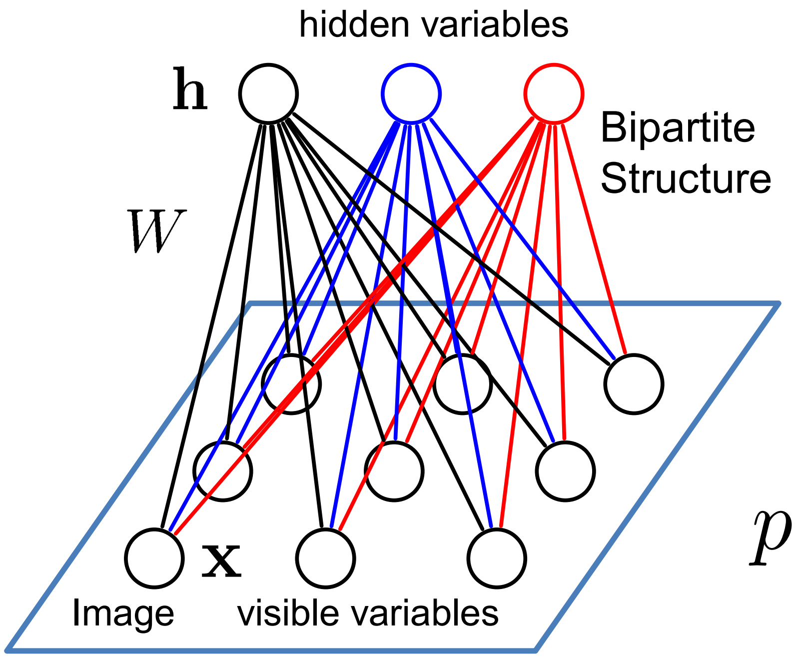
$$\mathbf{h} \in \{0, 1\}^{F}$$

- The energy of the joint configuration:

$$\begin{aligned} E(\mathbf{x}, \mathbf{h}) &= -\mathbf{h}^\top \mathbf{W} \mathbf{x} - \mathbf{c}^\top \mathbf{x} - \mathbf{b}^\top \mathbf{h} \\ &= -\sum_j \sum_k W_{j,k} h_j x_k - \sum_k c_k x_k - \sum_j b_j h_j \end{aligned}$$

Markov random fields, Boltzmann machines, log-linear models.

Restricted Boltzmann Machines



- Probability of the joint configuration is given by the Boltzmann distribution:

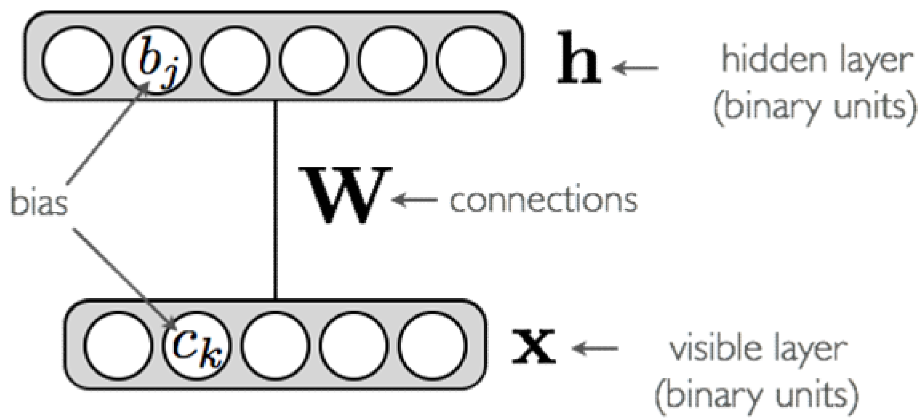
$$p(\mathbf{x}, \mathbf{h}) = \exp(-E(\mathbf{x}, \mathbf{h}))/Z$$

Partition function (intractable)

$$Z = \sum_{\mathbf{x},\mathbf{h}} \exp\big(- E(\mathbf{x}, \mathbf{h})\big)$$

Markov random fields, Boltzmann machines, log-linear models.

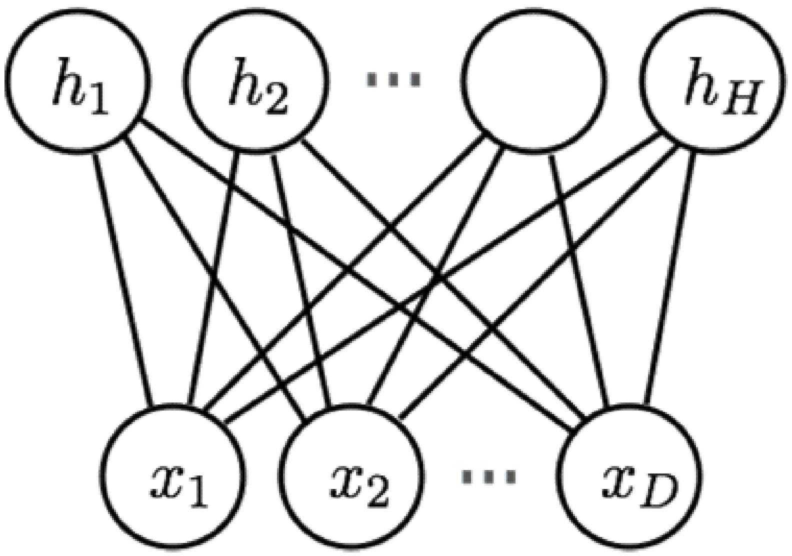
Restricted Boltzmann Machines



$$
\begin{aligned}
p(\mathbf{x}, \mathbf{h}) &= \exp(-E(\mathbf{x}, \mathbf{h}))/Z \\
&= \exp(\mathbf{h}^\top \mathbf{W} \mathbf{x} + \mathbf{c}^\top \mathbf{x} + \mathbf{b}^\top \mathbf{h})/Z \\
&= \underbrace{\exp(\mathbf{h}^\top \mathbf{W} \mathbf{x}) \exp(\mathbf{c}^\top \mathbf{x}) \exp(\mathbf{b}^\top \mathbf{h})}_{\text{Factors}}/Z
\end{aligned}
$$

- The notation based on an energy function is simply an alternative to the representation as the product of factors

Restricted Boltzmann Machines



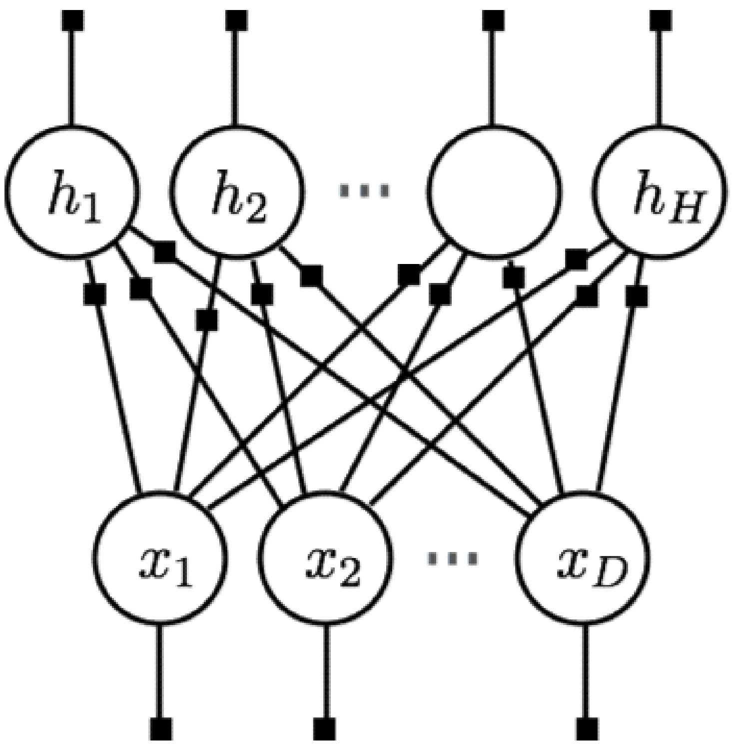
$$p(\mathbf{x},\mathbf{h}) = \frac{1}{Z} \overbrace{\prod_j \prod_k \exp(W_{j,k} h_j x_k)}^{\text{Pair-wise factors}}$$

$$\left.\begin{array}{l} \prod_k \exp(c_k x_k) \\ \prod_j \exp(b_j h_j) \end{array}\right\} \text{Unary factors}$$

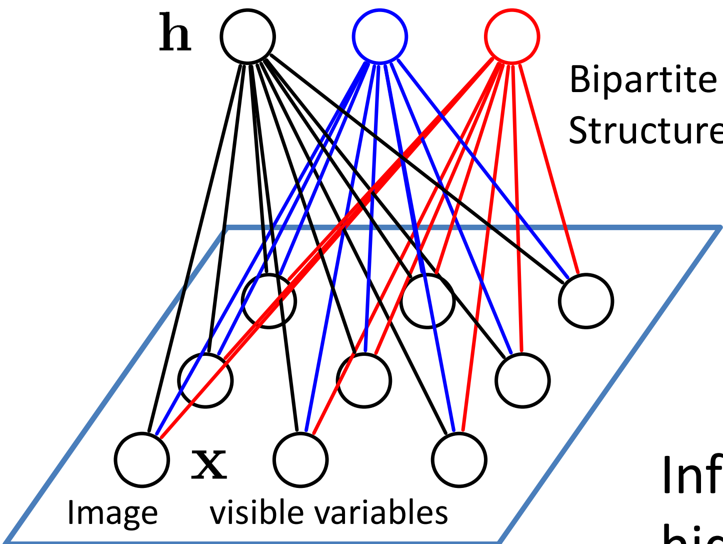
• The scalar visualization is more informative of the structure within the vectors.

Factor Graph View

$$p(\mathbf{x},\mathbf{h}) = \frac{1}{Z}\overbrace{\prod_j \prod_k \exp(W_{j,k}h_j x_k)}^{\text{Pair-wise factors}}\;\left.\begin{array}{l}\displaystyle\prod_k \exp(c_k x_k)\\ \displaystyle\prod_j \exp(b_j h_j)\end{array}\right\}\text{Unary factors}$$



Inference



Restricted: **No interaction between hidden variables**

Inferring the distribution over the hidden variables is easy:

$$p(\mathbf{h}|\mathbf{x}) = \underbrace{\prod_j p(h_j|\mathbf{x})}_{\text{Factorizes: Easy to compute}}$$

Similarly:

$$p(\mathbf{x}|\mathbf{h}) = \prod_k p(x_k|\mathbf{h})$$

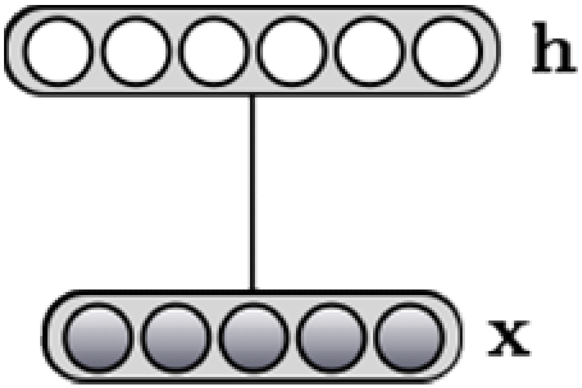
Markov random fields, Boltzmann machines, log-linear models.

In RBM:
1. Every visible node xk connects to all hidden nodes
2. No visible-visible or hidden-hidden edges

Inference

- Conditional Distributions:

A visible unit's neighbors = all hidden units
A hidden unit's neighbors = all visible units

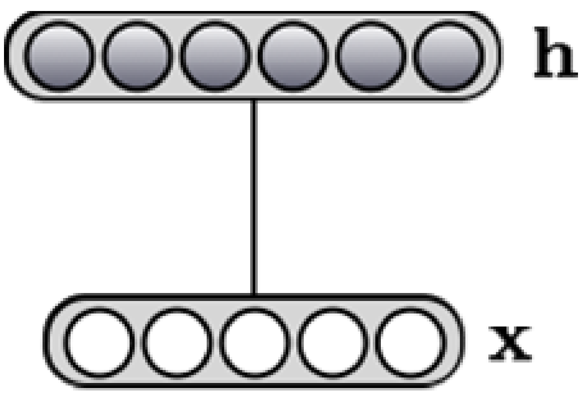


$$p(\mathbf{h}|\mathbf{x}) = \prod_j p(h_j|\mathbf{x})$$

$$p(h_j = 1|\mathbf{x}) = \frac{1}{1 + \exp(-(b_j + \mathbf{W}_{j\cdot}\mathbf{x}))}$$

$$= \mathrm{sigm}(b_j + \mathbf{W}_{j\cdot}\mathbf{x})$$

j^{th} row of W



$$p(\mathbf{x}|\mathbf{h}) = \prod_k p(x_k|\mathbf{h})$$

$$p(x_k = 1|\mathbf{h}) = \frac{1}{1 + \exp(-(c_k + \mathbf{h}^\top \mathbf{W}_{\cdot k}))}$$

$$= \mathrm{sigm}(c_k + \mathbf{h}^\top \mathbf{W}_{\cdot k})$$

k^{th} column of W

Local Markov Property

- In general, we have the following property:

$$
\begin{aligned}
p(z_i | z_1, \ldots, z_V) &= p(z_i | \mathrm{Ne}(z_i)) \\
&= \frac{p(z_i, \mathrm{Ne}(z_i))}{\sum_{z_i'} p(z_i', \mathrm{Ne}(z_i))} \\
&= \frac{\prod_{\substack{f \text{ involving } z_i \\ \text{and any } \mathrm{Ne}(z_i)}} \Psi_f(z_i, \mathrm{Ne}(z_i))}{\sum_{z_i'} \prod_{\substack{f \text{ involving } z_i \\ \text{and any } \mathrm{Ne}(z_i)}} \Psi_f(z_i', \mathrm{Ne}(z_i))}
\end{aligned}
$$

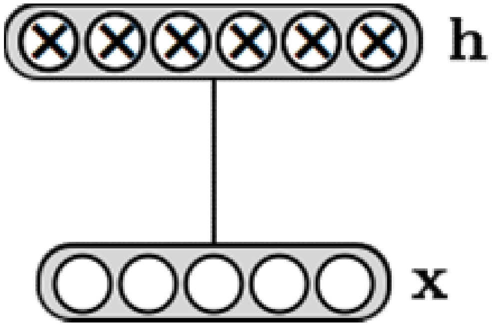
  - z_i is any variable in the Markov network (x_k or h_j in an RBM)
  - $\mathrm{Ne}(z_i)$ are the neighbors of z_i in the Markov network

Free Energy

- What about computing marginal $p(\mathbf{x})$?

$$
\begin{aligned}
p(\mathbf{x}) &= \sum_{\mathbf{h}\in\{0,1\}^H} p(\mathbf{x},\mathbf{h}) = \sum_{\mathbf{h}\in\{0,1\}^H} \exp(-E(\mathbf{x},\mathbf{h}))/Z \\
&= \exp\left(\mathbf{c}^\top\mathbf{x} + \sum_{j=1}^{H}\log(1+\exp(b_j + \mathbf{W}_{j\cdot}\mathbf{x}))\right)/Z \\
&= \exp(-F(\mathbf{x}))/Z
\end{aligned}
$$

Free Energy



Free Energy

- What about computing marginal $p(\mathbf{x})$?

$$
\begin{aligned}
p(\mathbf{x}) &= \sum_{\mathbf{h}\in\{0,1\}^H} \exp(\mathbf{h}^\top \mathbf{W}\mathbf{x} + \mathbf{c}^\top \mathbf{x} + \mathbf{b}^\top \mathbf{h})/Z \\
&= \exp(\mathbf{c}^\top \mathbf{x}) \sum_{h_1\in\{0,1\}} \cdots \sum_{h_H\in\{0,1\}} \exp\left(\sum_j h_j \mathbf{W}_{j\cdot}\mathbf{x} + b_j h_j\right)/Z \\
&= \exp(\mathbf{c}^\top \mathbf{x}) \left(\sum_{h_1\in\{0,1\}} \exp(h_1 \mathbf{W}_{1\cdot}\mathbf{x} + b_1 h_1)\right) \dots \left(\sum_{h_H\in\{0,1\}} \exp(h_H \mathbf{W}_{H\cdot}\mathbf{x} + b_H h_H)\right)/Z \\
&= \exp(\mathbf{c}^\top \mathbf{x})\,(1+\exp(b_1 + \mathbf{W}_{1\cdot}\mathbf{x}))\dots(1+\exp(b_H + \mathbf{W}_{H\cdot}\mathbf{x}))/Z \\
&= \exp(\mathbf{c}^\top \mathbf{x})\exp(\log(1+\exp(b_1 + \mathbf{W}_{1\cdot}\mathbf{x})))\dots\exp(\log(1+\exp(b_H + \mathbf{W}_{H\cdot}\mathbf{x})))/Z \\
&= \exp\left(\mathbf{c}^\top \mathbf{x} + \sum_{j=1}^{H} \log(1+\exp(b_j + \mathbf{W}_{j\cdot}\mathbf{x}))\right)/Z
\end{aligned}
$$

- Also known as Product of Experts model.

Free Energy

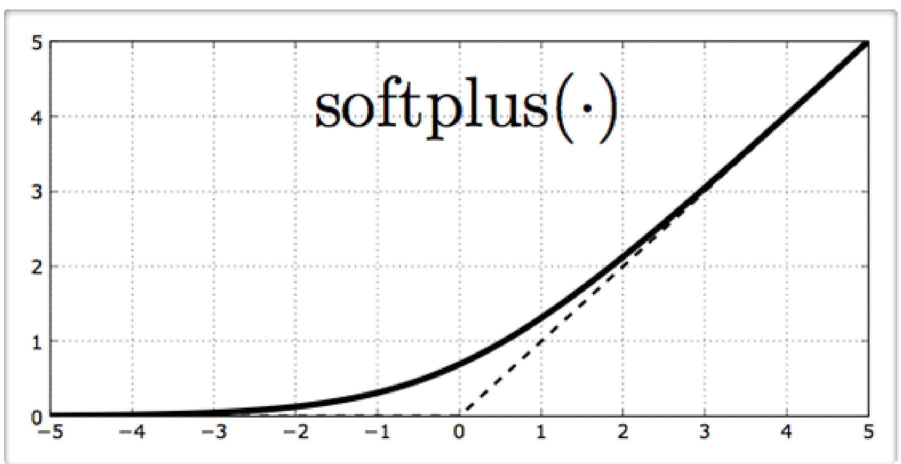
$$p(\mathbf{x}) = \exp\left(\mathbf{c}^\top\mathbf{x} + \sum_{j=1}^{H}\log(1+\exp(b_j + \mathbf{W}_{j\cdot}\mathbf{x}))\right)/Z$$

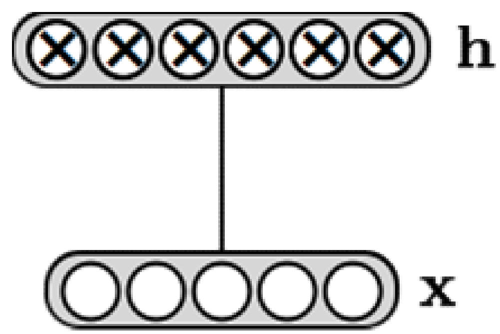
$$= \exp\left(\mathbf{c}^\top\mathbf{x} + \sum_{j=1}^{H}\text{softplus}(b_j + \mathbf{W}_{j\cdot}\mathbf{x})\right)/Z$$

bias the probability of each x_i

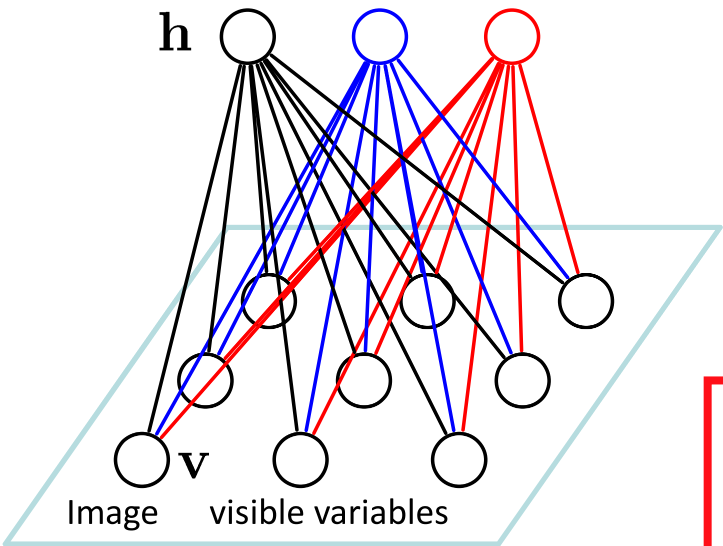
bias of each feature

feature expected in x





Model Learning



- Given a set of *i.i.d.* training examples we want to minimize the average negative log-likelihood (NLL):

$$\frac{1}{T}\sum_t l(f(\mathbf{x}^{(t)})) = \frac{1}{T}\sum_t -\log p(\mathbf{x}^{(t)})$$

Remember:

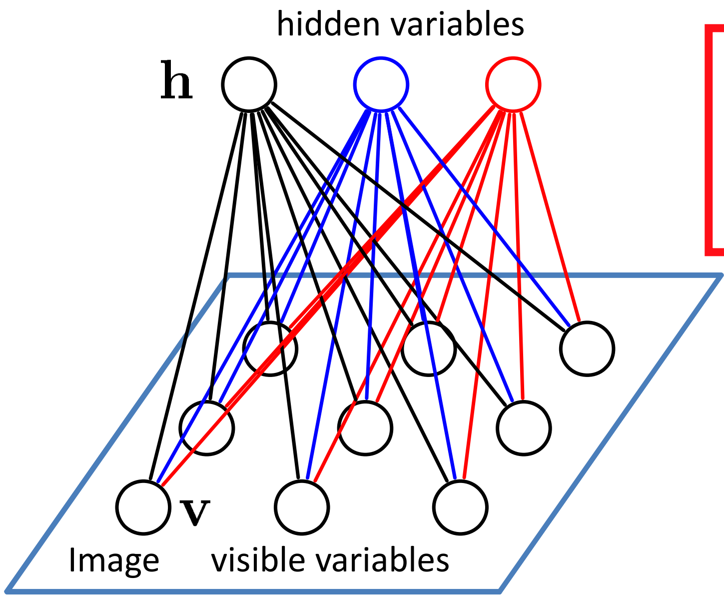
$$p(\mathbf{x},\mathbf{h}) \quad = \quad \exp(-E(\mathbf{x},\mathbf{h}))/Z$$

- Derivative of the negative log-likelihood objective:

$$\frac{\partial -\log p(\mathbf{x}^{(t)})}{\partial \theta} = \underbrace{\mathrm{E}_{\mathbf{h}}\left[\frac{\partial E(\mathbf{x}^{(t)},\mathbf{h})}{\partial \theta}\bigg|\mathbf{x}^{(t)}\right]}_{\text{Positive Phase}} - \underbrace{\mathrm{E}_{\mathbf{x},\mathbf{h}}\left[\frac{\partial E(\mathbf{x},\mathbf{h})}{\partial \theta}\right]}_{\text{Negative Phase}}$$

Negative Phase
Hard to compute

An Important Calculation



Remember:

$$p(\mathbf{x}, \mathbf{h}) \quad = \quad \exp(-E(\mathbf{x}, \mathbf{h}))/Z$$

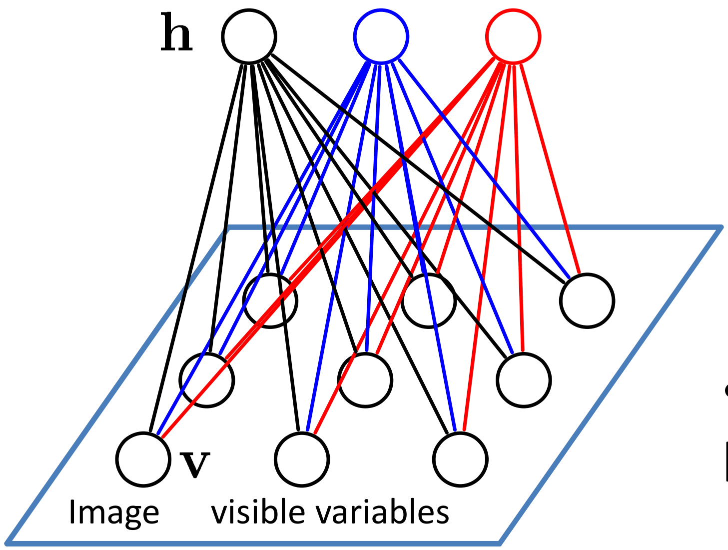
$$Z \; = \Sigma_{x,h} \; \text{exp(-E(x,h))}$$

$$\ln(Z) \; = \ln(\Sigma_{x,h} \; \text{exp(-E(x,h)))}$$

$$\frac{\partial l \quad (Z)}{\partial \theta} = \; -\frac{1}{Z} \; \Sigma_{x,h} \; \text{exp(-E(x,h))} \, \frac{\partial \text{E(x,h)}}{\partial \theta}$$

$$\frac{\partial ln(Z)}{\partial \theta} = \; -\Sigma_{x,h} \; \text{p(x,h)} \; \frac{\partial \text{E(x,h)}}{\partial \theta}$$

Model Learning



$$p(\mathbf{x}, \mathbf{h}) = \exp(-E(\mathbf{x}, \mathbf{h}))/Z$$

$$p(\mathbf{h}|\mathbf{x}) = \prod_j p(h_j|\mathbf{x})$$

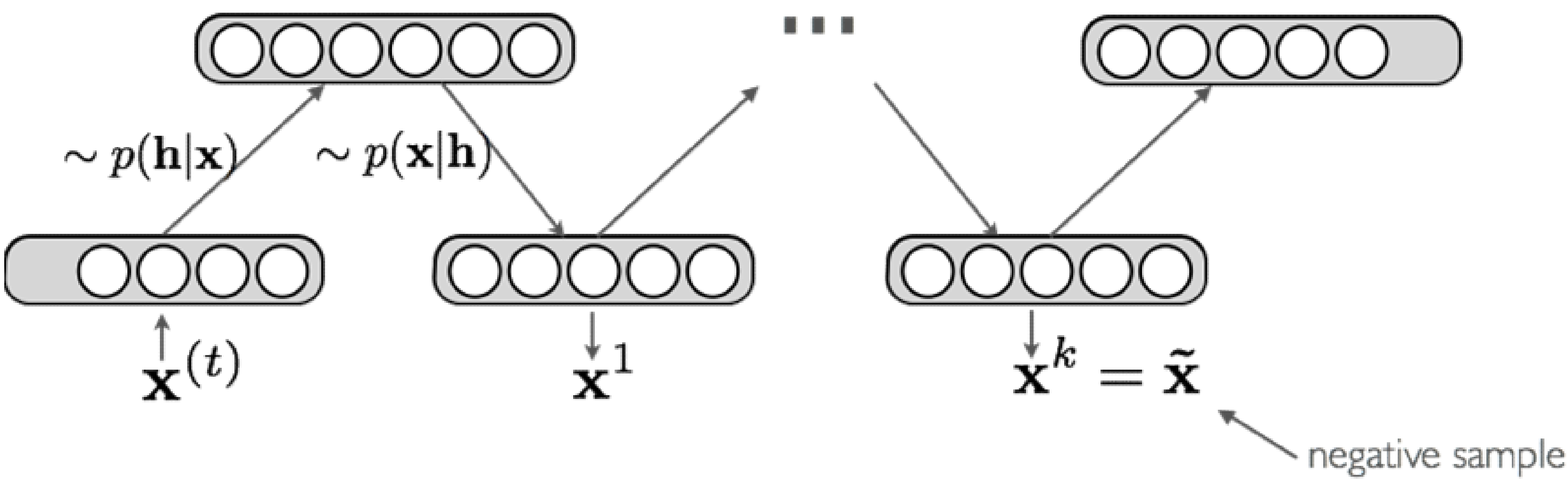
- Derivative of the negative log-likelihood objective:

$$\frac{\partial - \log p(\mathbf{x}^{(t)})}{\partial \theta} = \underbrace{\mathrm{E}_{\mathbf{h}} \left[\frac{\partial E(\mathbf{x}^{(t)}, \mathbf{h})}{\partial \theta} \Big| \mathbf{x}^{(t)} \right]}_{\text{Data-Dependent Expectations w.r.t } P(h|x)} - \underbrace{\mathrm{E}_{\mathbf{x},\mathbf{h}} \left[\frac{\partial E(\mathbf{x}, \mathbf{h})}{\partial \theta} \right]}_{\text{Model: Expectation w.r.t joint } P(x,h)}$$

- Second term: intractable due to exponential number of configurations.

Contrastive Divergence

- Key idea behind Contrastive Divergence:
  - Replace the expectation by a point estimate at $\tilde{\mathbf{x}}$
  - Obtain the point $\tilde{\mathbf{x}}$ by Gibbs sampling
  - Start sampling chain at $\mathbf{x}^{(t)}$



Hinton, Neural Computation, 2002

Deriving Learning Rule

- Let us look at derivative of $\frac{\partial E(\mathbf{x}, \mathbf{h})}{\partial \theta}$ for $\theta = W_{jk}$

$$\frac{\partial E(\mathbf{x}, \mathbf{h})}{\partial W_{jk}} = \frac{\partial}{\partial W_{jk}} \left(-\sum_{jk} W_{jk} h_j x_k - \sum_k c_k x_k - \sum_j b_j h_j \right)$$

$$= -\frac{\partial}{\partial W_{jk}} \sum_{jk} W_{jk} h_j x_k$$

$$= -h_j x_k$$

Remember:
$E(\mathbf{x}, \mathbf{h}) \;=\; -\mathbf{h}^\top \mathbf{W} \mathbf{x} - \mathbf{c}^\top \mathbf{x} - \mathbf{b}^\top \mathbf{h}$

- Hence:

$$\nabla_{\mathbf{W}} E(\mathbf{x}, \mathbf{h}) = -\mathbf{h}\, \mathbf{x}^\top$$

Deriving Learning Rule

- Let us now derive $\mathbb{E}_{\mathbf{h}}\left[\frac{\partial E(\mathbf{x},\mathbf{h})}{\partial \theta}\Big|\mathbf{x}\right]$

$$\mathbb{E}_{\mathbf{h}}\left[\frac{\partial E(\mathbf{x},\mathbf{h})}{\partial W_{jk}}\Big|\mathbf{x}\right] = \mathbb{E}_{\mathbf{h}}\left[-h_j x_k \Big|\mathbf{x}\right] = \sum_{h_j \in \{0,1\}} -h_j x_k p(h_j|\mathbf{x})$$

$$= -x_k p(h_j = 1|\mathbf{x})$$

- Hence:

$$\mathrm{E}_{\mathbf{h}}\left[\nabla_{\mathbf{W}} E(\mathbf{x},\mathbf{h})\,|\mathbf{x}\right] = -\mathbf{h}(\mathbf{x})\,\mathbf{x}^\top$$

$$\boxed{\begin{aligned}\mathbf{h}(\mathbf{x}) &\stackrel{\text{def}}{=} \begin{pmatrix} p(h_1=1|\mathbf{x}) \\ \dots \\ p(h_H=1|\mathbf{x}) \end{pmatrix} \\ &= \mathrm{sigm}(\mathbf{b} + \mathbf{W}\mathbf{x})\end{aligned}}$$

Deriving Learning Rule

- Hence:

$$\mathrm{E}_{\mathbf{h}}\left[\nabla_{\mathbf{W}} E(\mathbf{x},\mathbf{h})\,|\mathbf{x}\right] = -\mathbf{h}(\mathbf{x})\;\mathbf{x}^{\top}$$

$$\mathbf{h}(\mathbf{x}) \stackrel{\text{def}}{=} \begin{pmatrix} p(h_1{=}1|\mathbf{x}) \\ \cdots \\ p(h_H{=}1|\mathbf{x}) \end{pmatrix}$$

$$= \mathrm{sigm}(\mathbf{b} + \mathbf{W}\mathbf{x})$$

$$\frac{\partial - \log p(\mathbf{x}^{(t)})}{\partial \theta} = \mathrm{E}_{\mathbf{h}}\left[\frac{\partial E(\mathbf{x}^{(t)},\mathbf{h})}{\partial \theta}\,\middle|\,\mathbf{x}^{(t)}\right] - \mathrm{E}_{\mathbf{x},\mathbf{h}}\left[\frac{\partial E(\mathbf{x},\mathbf{h})}{\partial \theta}\right]$$

$$\sum_{\mathbf{x},\mathbf{h}} -\mathbf{h}(\mathbf{x})\mathbf{x}^{\top}\mathbf{P}(\mathbf{x},\mathbf{h})$$

Easy to
compute exactly

Difficult to compute:
exponentially many
Configurations.

Approximate Learning

- An approximation to the gradient of the log-likelihood objective:

$$\frac{\partial - \log p(\mathbf{x}^{(t)})}{\partial \theta} = \mathrm{E}_{\mathbf{h}}\left[\frac{\partial E(\mathbf{x}^{(t)}, \mathbf{h})}{\partial \theta} \middle| \mathbf{x}^{(t)}\right] - \mathrm{E}_{\mathbf{x},\mathbf{h}}\left[\frac{\partial E(\mathbf{x}, \mathbf{h})}{\partial \theta}\right]$$

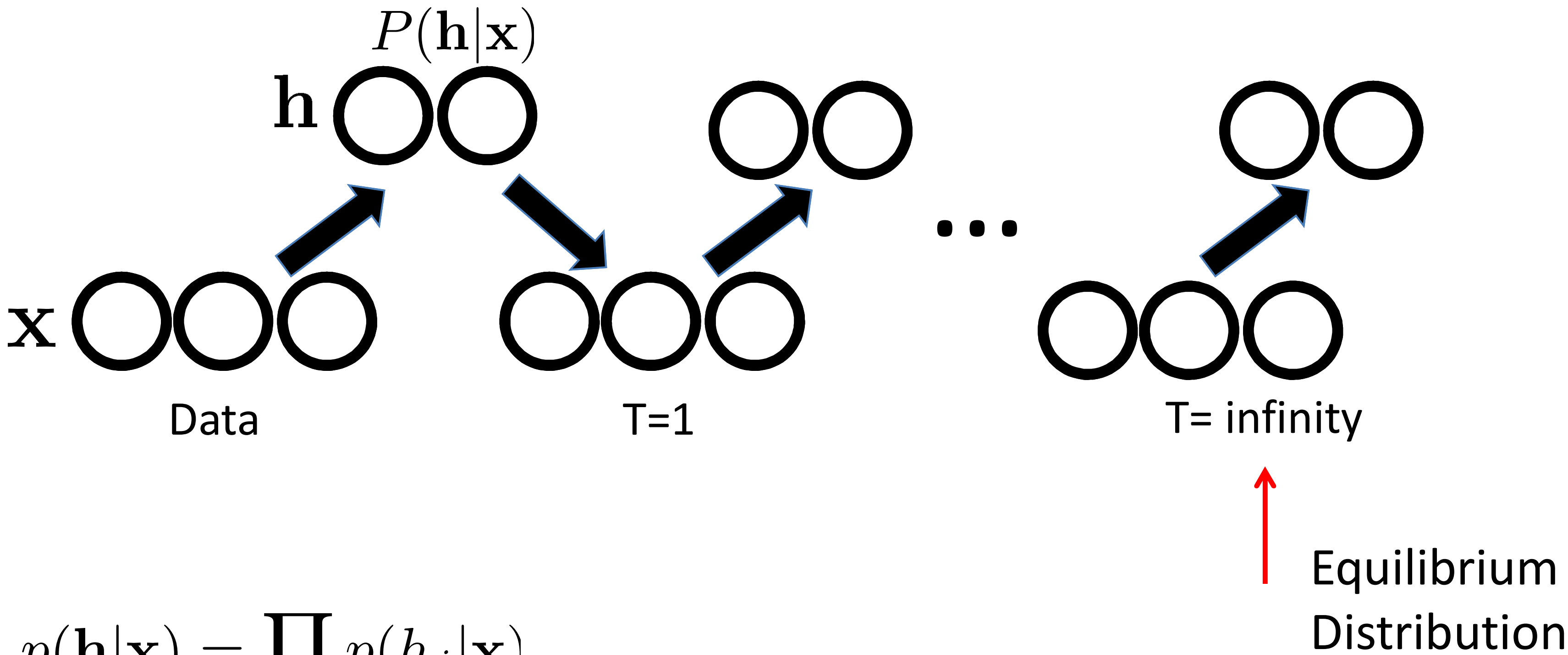
$$\sum_{\mathbf{x},\mathbf{h}} -\mathbf{h}(\mathbf{x})\mathbf{x}^{\top}\mathbf{P}(\mathbf{x}, \mathbf{h})$$

- Replace the average over all possible input configurations by samples.
- Run MCMC chain (Gibbs sampling) starting from the observed examples.

- Initialize $x^0 = x$
- Sample h^0 from $P(h \mid x^0)$
- For t=1:T
  - Sample x^t from $P(x \mid h^{t-1})$
  - Sample h^t from $P(h \mid x^t)$

Approximate ML Learning for RBMs

Run Markov chain (alternating Gibbs Sampling):

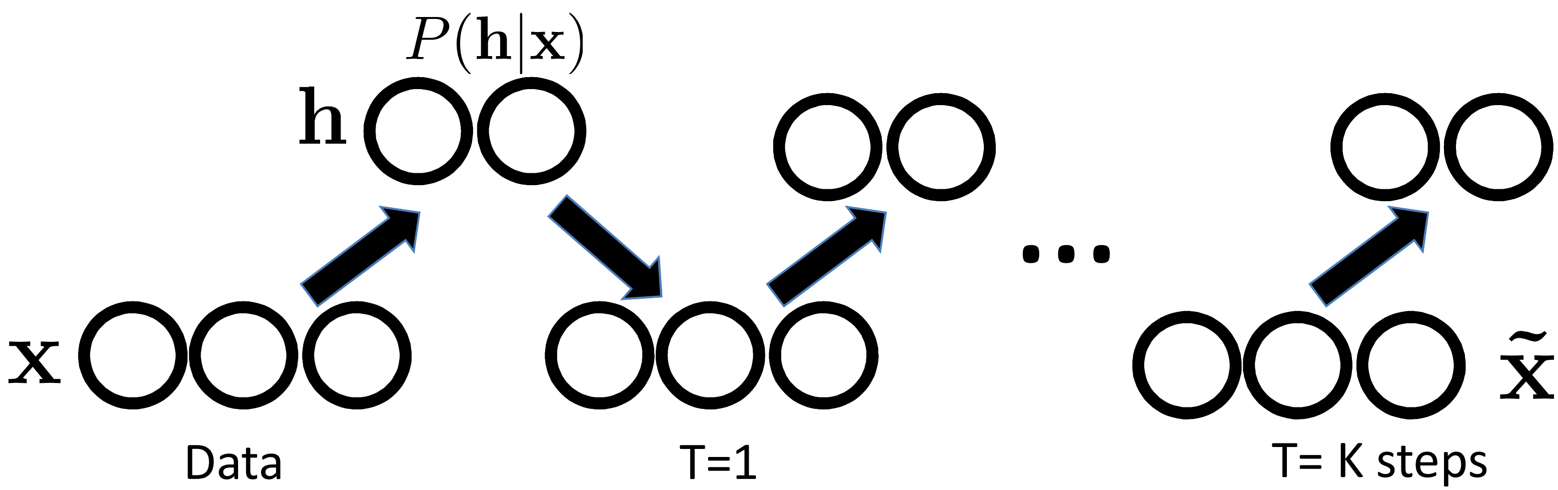


$$p(\mathbf{h}|\mathbf{x}) = \prod_j p(h_j|\mathbf{x})$$

$$p(\mathbf{x}|\mathbf{h}) = \prod_k p(x_k|\mathbf{h})$$

Contrastive Divergence

Run Markov chain (alternating Gibbs Sampling):



- K is typically set to 1.

$$p(\mathbf{h}|\mathbf{x}) = \prod_j p(h_j|\mathbf{x})$$

$$p(\mathbf{x}|\mathbf{h}) = \prod_k p(x_k|\mathbf{h})$$

Deriving Learning Rule

$\mathbf{x}^{(t)}$ $\qquad \tilde{\mathbf{x}}$ $\qquad \theta = \mathbf{W}$

$$
\begin{aligned}
\mathbf{W} \quad & \Longleftarrow \quad \mathbf{W} - \alpha \left(\nabla_{\mathbf{W}} - \log p(\mathbf{x}^{(t)}) \right) \\
& \Longleftarrow \quad \mathbf{W} - \alpha \left(\mathrm{E}_{\mathbf{h}} \left[\nabla_{\mathbf{W}} E(\mathbf{x}^{(t)}, \mathbf{h}) \,\middle|\, \mathbf{x}^{(t)} \right] - \mathrm{E}_{\mathbf{x},\mathbf{h}} \left[\nabla_{\mathbf{W}} E(\mathbf{x}, \mathbf{h}) \right] \right) \\
& \Longleftarrow \quad \mathbf{W} - \alpha \left(\mathrm{E}_{\mathbf{h}} \left[\nabla_{\mathbf{W}} E(\mathbf{x}^{(t)}, \mathbf{h}) \,\middle|\, \mathbf{x}^{(t)} \right] - \mathrm{E}_{\mathbf{h}} \left[\nabla_{\mathbf{W}} E(\tilde{\mathbf{x}}, \mathbf{h}) \,|\tilde{\mathbf{x}} \right] \right) \\
& \Longleftarrow \quad \mathbf{W} + \alpha \left(\mathbf{h}(\mathbf{x}^{(t)}) \, {\mathbf{x}^{(t)}}^{\top} - \mathbf{h}(\tilde{\mathbf{x}}) \, \tilde{\mathbf{x}}^{\top} \right)
\end{aligned}
$$

Learning rate

CD-k Algorithm

- For each training example $\mathbf{x}^{(t)}$
  - Generate a negative sample $\tilde{\mathbf{x}}$ using k steps of Gibbs sampling, starting at the data point $\mathbf{x}^{(t)}$
  - Update model parameters:

$$\mathbf{W} \Longleftarrow \mathbf{W} + \alpha \left(\mathbf{h}(\mathbf{x}^{(t)})\, {\mathbf{x}^{(t)}}^\top - \mathbf{h}(\tilde{\mathbf{x}})\, \tilde{\mathbf{x}}^\top \right)$$

$$\mathbf{b} \Longleftarrow \mathbf{b} + \alpha \left(\mathbf{h}(\mathbf{x}^{(t)}) - \mathbf{h}(\tilde{\mathbf{x}}) \right)$$

$$\mathbf{c} \Longleftarrow \mathbf{c} + \alpha \left(\mathbf{x}^{(t)} - \tilde{\mathbf{x}} \right)$$

  - Go back to 1 until stopping criteria

CD-k Algorithm

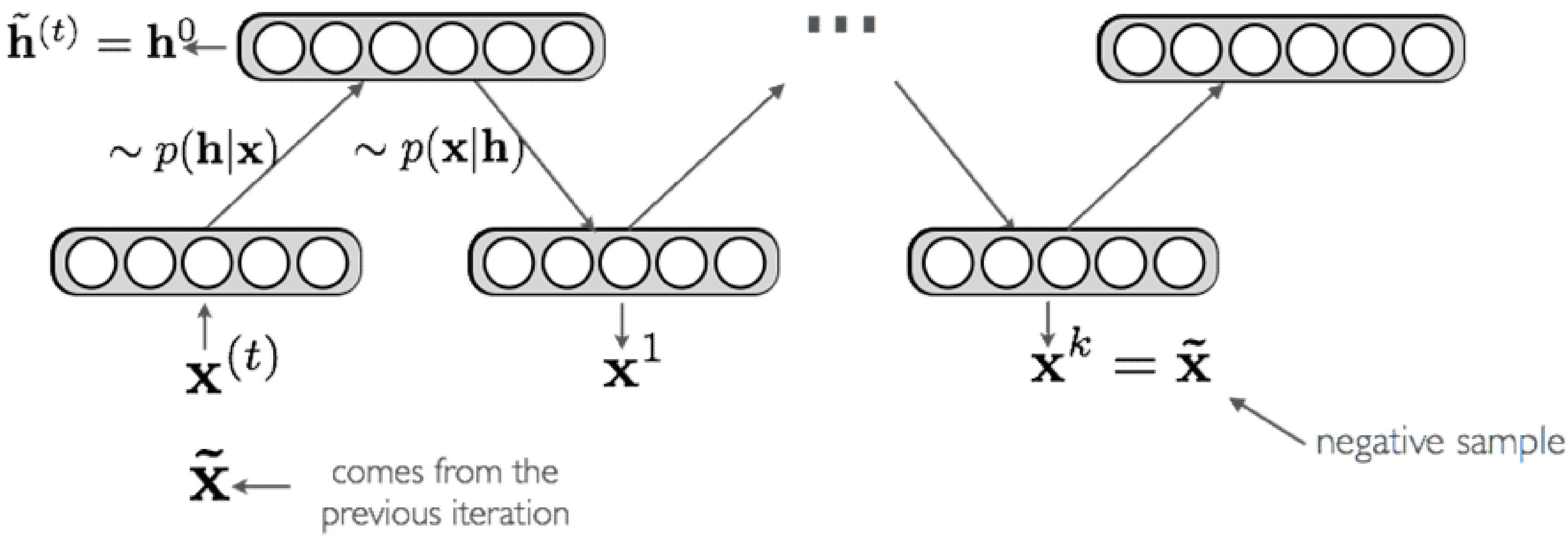
- CD-k: contrastive divergence with k iterations of Gibbs sampling

- In general, the bigger k is, the less biased the estimate of the gradient will be

- In practice, k=1 works well for learning good features and for pre-training

Persistent CD: Stochastic ML Estimator

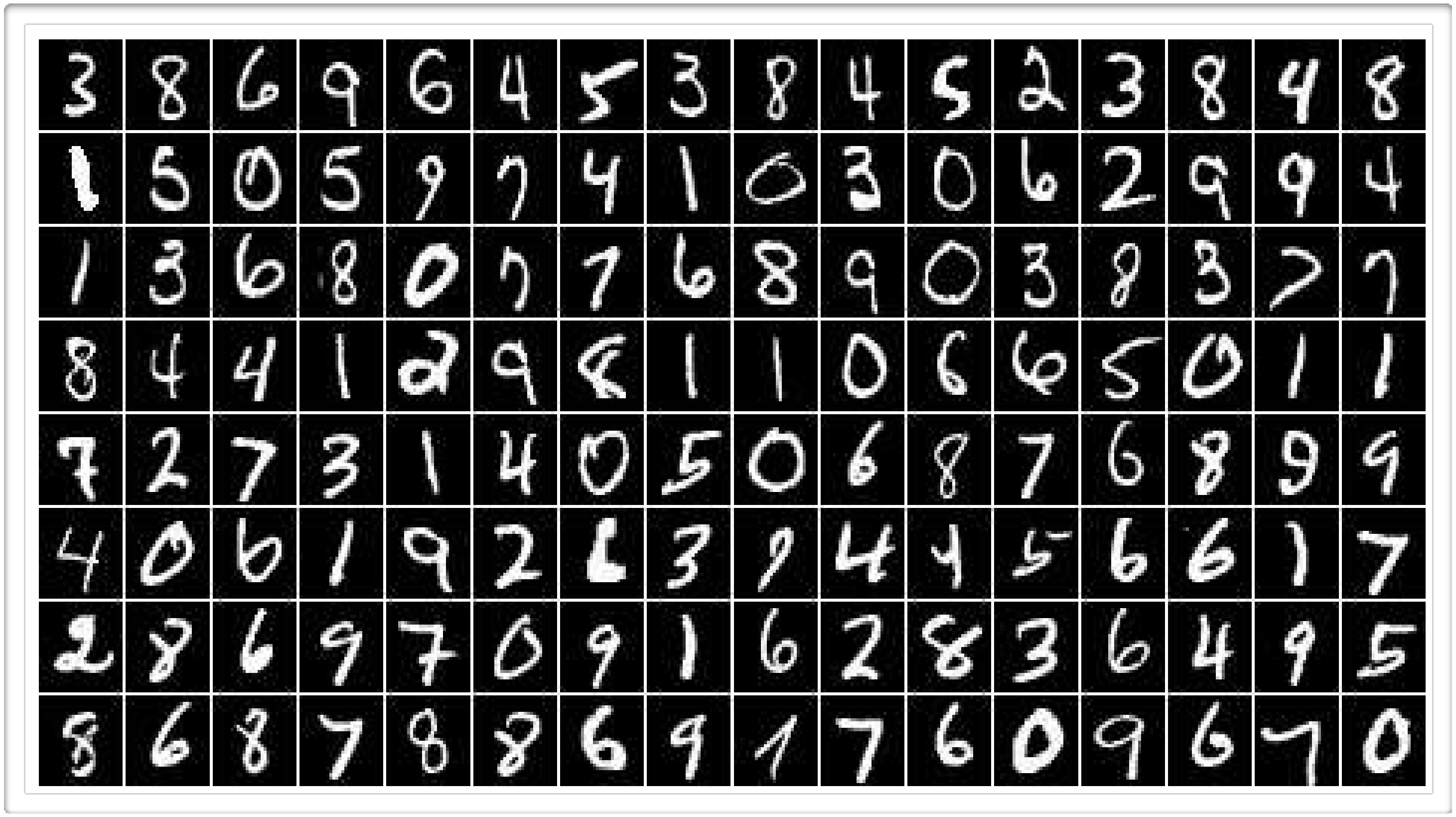
• Idea: instead of initializing the chain to $\mathbf{x}^{(t)}$, initialize the chain to the negative sample of the last iteration



Tieleman, ICML, 2008

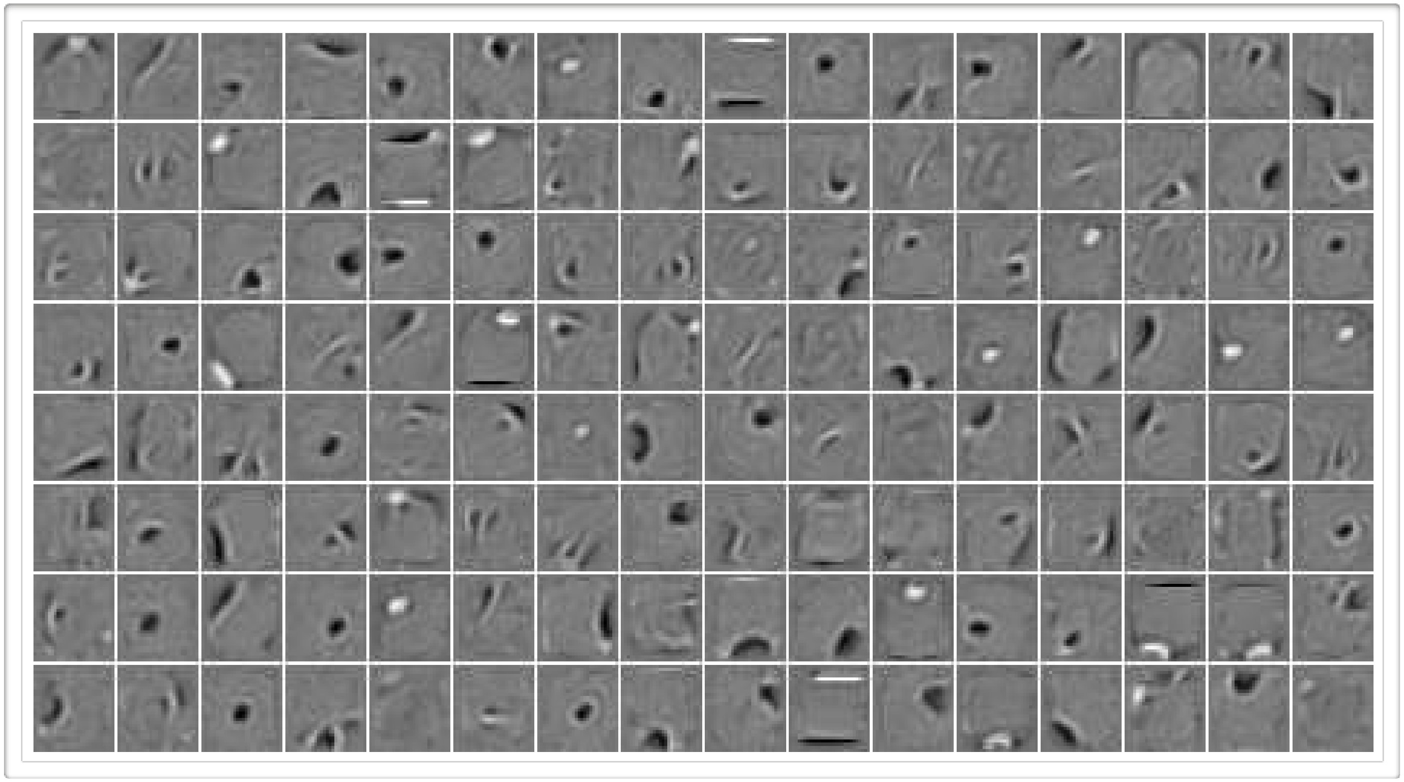
Example: MNIST

- MNIST dataset:



Learned Features

- MNIST dataset:



(Larochelle et al., JMLR 2009)

Gaussian Bernoulli RBMs

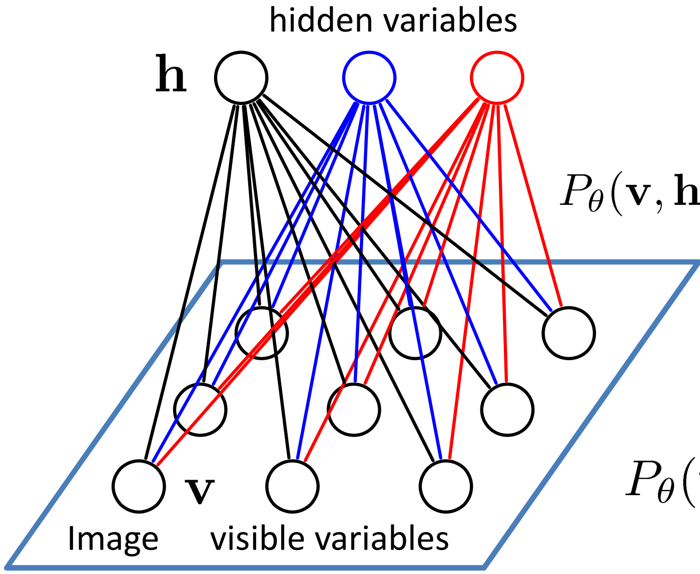
- Let x represent a real-valued (unbounded) input.

  - add a quadratic term to the energy function

$$E(\mathbf{x}, \mathbf{h}) = -\mathbf{h}^\top \mathbf{W}\mathbf{x} - \mathbf{c}^\top \mathbf{x} - \mathbf{b}^\top \mathbf{h} + \frac{1}{2}\mathbf{x}^\top \mathbf{x}$$

  - In this case $p(\mathbf{x}|\mathbf{h})$ becomes a Gaussian distribution with mean $\boldsymbol{\mu} = \mathbf{c} + \mathbf{W}^\top \mathbf{h}$ and identity covariance matrix
  - recommend to normalize the training set by:
    - subtracting the mean off each input
    - dividing each input by the training set standard deviation
  - should use a smaller learning rate than in the regular RBM

Gaussian Bernoulli RBMs

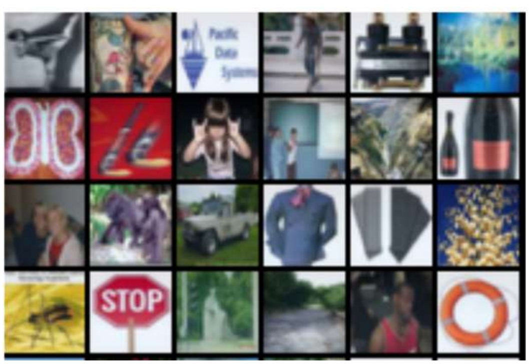


$$P_\theta(\mathbf{v},\mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\sum_{i=1}^{D}\sum_{j=1}^{F} \overbrace{W_{ij}h_j\frac{v_i}{\sigma_i}}^{\textbf{Pair-wise}} + \sum_{i=1}^{D} \overbrace{\frac{(v_i-b_i)^2}{2\sigma_i^2}}^{\textbf{Unary}} + \sum_{j=1}^{F} \overbrace{a_j h_j}^{} \right)$$

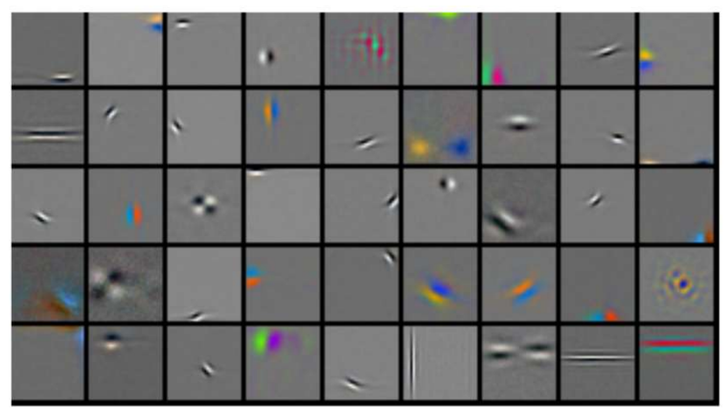
$$\theta = \{W, a, b\}$$

$$P_\theta(\mathbf{v}|\mathbf{h}) = \prod_{i=1}^{D} P_\theta(v_i|\mathbf{h}) = \prod_{i=1}^{D} \mathcal{N}\left(b_i + \sum_{j=1}^{F} W_{ij}h_j, \sigma_i^2 \right)$$

4 million **unlabelled** images



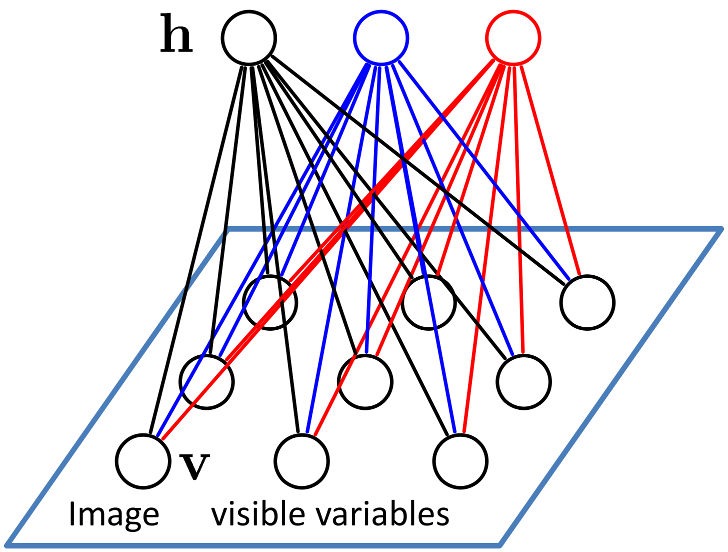
Learned features (out of 10,000)



(Notation: vector x is replaced with v).

Gaussian Bernoulli RBMs

Gaussian-Bernoulli RBM:



Interpretation: Mixture of exponentially growing number of Gaussians

$$P_\theta(\mathbf{v}) = \sum_{\mathbf{h}} P_\theta(\mathbf{v}|\mathbf{h}) P_\theta(\mathbf{h}),$$

where

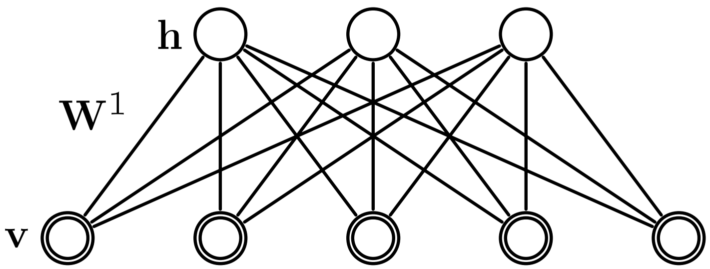
$$P_\theta(\mathbf{h}) = \int_{\mathbf{v}} P_\theta(\mathbf{v}, \mathbf{h}) d\mathbf{v}$$ is an implicit prior, and

$$P(v_i = x|\mathbf{h}) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(x - b_i - \sigma_i \sum_j W_{ij} h_j)^2}{2\sigma_i^2}\right)$$ Gaussian

Example: Collaborative Filtering

$$P_\theta(\mathbf{v}, \mathbf{h}) = \frac{1}{\mathcal{Z}(\theta)} \exp\left(\sum_{ijk} W_{ij}^k v_i^k h_j + \sum_{ik} b_i^k v_i^k + \sum_j a_j h_j \right)$$

Binary hidden: user preferences



Multinomial visible: user ratings

Netflix dataset:

480,189 users

17,770 movies

Over 100 million ratings

Learned features: ``genre''

Fahrenheit 9/11	Independence Day
Bowling for Columbine	The Day After Tomorrow
The People vs. Larry Flynt	Con Air
Canadian Bacon	Men in Black II
La Dolce Vita	Men in Black
Friday the 13th	Scary Movie
The Texas Chainsaw Massacre	Naked Gun
Children of the Corn	Hot Shots!
Child's Play	American Pie
The Return of Michael Myers	Police Academy

State-of-the-art performance on the Netflix dataset.

(Salakhutdinov, Mnih, Hinton, ICML 2007)