Assignment-3
Latex Report

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- **Exercise 2.101**

1 Find the inverse and QR decomposition of this matrix $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

1.1 Solution

1.1.1 Inverse

Let the given marix be,

A=$\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

To check whether the inverse of a matrix exists or not, if determinant of a matrix is non zero inverse exists, otherwise it dose not exist.
Now,

$$|A| = (2) * (2) - (-1) * (-3)$$

$$|A| = 4 - 3$$

$$|A| = 1$$

Since adjoint of A is gretaer than 0 i.e non zero,therefore inverse of A exists.
Now,
we know,

A$^{-1} = \frac{1}{|A|} Adj(A)$

$$\mathrm{A}^{-1} = \frac{1}{1}\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

Hence,

$$\mathrm{A}^{-1} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}$$

1.1.2 QR Decomposition

IN QR decomposition o a matrix , matrix is decomposed into the upper triangular matrix(R) and an orthogonal matrix(Q).

Since A=QR
where,
Q is an orthogonal matrix(i.e $\mathrm{Q}^T.Q = I$)
and

R is an upper triangle matrix

So,By using Gram-Schmidt method of decomposition let its column vectors be,

$$\mathrm{v}_1 = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \qquad and \qquad v_2 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$$

So,
here

Now, we know A=QR
where,
Q=(q1,q2)
where ,
q1 and q2 are column matrices.
Now to determine q1 and q2
we know

$$\text{q1=} \frac{a}{lengthofa}$$

therefore,

q1 =

$$\frac{1}{\sqrt{2^2+(-1)^2}}\begin{pmatrix}2\\-1\end{pmatrix} \tag{1}$$

Here,
$\frac{1}{\sqrt{2^2+(-1)^2}}$

is the length of $\begin{pmatrix}2\\-1\end{pmatrix}$

q1 =

$$\frac{1}{\sqrt{4+1}}\begin{pmatrix}2\\-1\end{pmatrix} \tag{2}$$

q1 =

$$\frac{1}{\sqrt{5}}\begin{pmatrix}2\\-1\end{pmatrix} \tag{3}$$

q1 =

$$\begin{pmatrix}\frac{2}{\sqrt{5}}\\ \\ \frac{-1}{\sqrt{5}}\end{pmatrix} \tag{4}$$

Now,

q2= $\frac{q2'}{lengthofq2'}$

Now,

$$q2' = v2 - (v2.q1)q1$$

$$q2' = \begin{pmatrix}-3\\2\end{pmatrix} - \left(\begin{pmatrix}-3\\2\end{pmatrix}. \quad \begin{pmatrix}\frac{2}{\sqrt{5}}\\ \frac{-1}{\sqrt{5}}\end{pmatrix}\right)\begin{pmatrix}\frac{2}{\sqrt{5}}\\ \frac{-1}{\sqrt{5}}\end{pmatrix}$$

$$q2' = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \left(\frac{-8}{\sqrt{5}}\right) \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} \end{pmatrix}$$

$$q2' = \begin{pmatrix} -3 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{-16}{5} \\ \\ \frac{8}{5} \end{pmatrix}$$

$$q2' = \begin{pmatrix} \frac{1}{5} \\ \\ \frac{2}{5} \end{pmatrix}$$

Since,
q2= $\dfrac{q2'}{length of q2'}$

Therefore,

$$q2 = \frac{1}{length of q2'} \begin{pmatrix} \dfrac{1}{5} \\ \dfrac{2}{5} \end{pmatrix}$$

Here,

length of q2'= $\sqrt{(2/5)^2 + (1/5)^2}$

length of q2'= $\sqrt{(4/25) + (1/25)}$

length of q2'=

$$\sqrt{(5/25)}$$

length of q2'= $\dfrac{1}{\sqrt{5}}$

Hence,Using

q2= $\frac{q2'}{length of q2'}$

we get,

q2= $\frac{1}{\sqrt{5}}\begin{pmatrix}\frac{1}{5}\\ \\ \frac{2}{5}\end{pmatrix}$

So,

q2= $\begin{pmatrix}\frac{1}{\sqrt{5}}\\ \\ \frac{2}{\sqrt{5}}\end{pmatrix}$

Now,

Q can be obtained by combining

q1

and

q2

Therefore,

Q= $\begin{pmatrix}\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\end{pmatrix}$

Since we know

A=QR

Pre-Multiplying both sides by Q^T,

we get,

$Q^TA = Q^TQR$

But $Q^T Q = I$ **[I means identity matrix]**

So,
$Q^T A = IR$
$Q^T A = R$

Or,

R=$Q^T A$

Hence,
$$R=\begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

[By matrix multiplication]

$$R=\begin{pmatrix} \frac{4}{\sqrt{5}}+\frac{1}{\sqrt{5}} & \frac{-6}{\sqrt{5}}-\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}}-\frac{2}{\sqrt{5}} & \frac{-3}{\sqrt{5}}\frac{4}{\sqrt{5}} \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

$$R=\begin{pmatrix} \frac{5}{\sqrt{5}} & \frac{-8}{\sqrt{5}} \\ 0 & \frac{1}{\sqrt{5}} \end{pmatrix}$$

Also, from above we got

$$Q=\begin{pmatrix} \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \end{pmatrix}$$

- **Exercise 2.102**

2 Find the inverse and QR decomposition of this matrix $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

2.1 Solution

2.1.1 Inverse

Let the given matrix be,

A=$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

we know,
A$^{-1} = \frac{1}{|A|} Adj(A)$

To check whether the inverse of a matrix exists or not, if determinant of a matrix is non zero inverse exists, otherwise it does not exist.
Now,

$$|A| = (2) * (2) - (4) * (1)$$

$$|A| = 4 - 4$$

$$|A| = 0$$

Therefore,inverse of the given matrix A does not exist.

2.1.2 QR decomposition

Since A=QR
where,
Q is an orthogonal matrix(i.e $Q^T.Q = I$)
and

R is an upper triangle matrix
So, By using Gram-Schmidt method of decomposition let its column vectors be,
$v_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$
and
$v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Therefore,

So,
here

Now, we know A=QR
where,
Q=(q1,q2)
where ,
q1 and q2 are column matrices.
Now to determine q1 and q2
we know

q1= $\frac{a}{length of a}$

therefore,

q1 =

$$\frac{1}{\sqrt{2^2+4^2}}\begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{5}$$

Here,

$$\frac{1}{\sqrt{2^2+4^2}}$$

is the length of $\begin{pmatrix}2\\4\end{pmatrix}$

q1 =

$$\frac{1}{\sqrt{20}}\begin{pmatrix}2\\4\end{pmatrix} \tag{6}$$

q1 =

$$\begin{pmatrix}\frac{2}{\sqrt{20}}\\ \\ \frac{4}{\sqrt{20}}\end{pmatrix} \tag{7}$$

Now,

q2= $\frac{q2'}{length of q2'}$

Now,

$$q2' = v2 - (v2.q1)q1$$

$$q2' = \begin{pmatrix}1\\2\end{pmatrix} - \left(\begin{pmatrix}1\\2\end{pmatrix} \cdot \quad \begin{pmatrix}\frac{2}{\sqrt{20}}\\ \frac{4}{\sqrt{20}}\end{pmatrix}\right)\begin{pmatrix}\frac{2}{\sqrt{20}}\\ \frac{4}{\sqrt{20}}\end{pmatrix}$$

$$q2' = \begin{pmatrix}1\\2\end{pmatrix} - \left(\frac{10}{\sqrt{20}}\right)\begin{pmatrix}\frac{2}{\sqrt{20}}\\ \frac{4}{\sqrt{20}}\end{pmatrix}$$

$$q2' = \begin{pmatrix}1\\2\end{pmatrix} - \begin{pmatrix}\frac{20}{20}\\ \\ \frac{40}{20}\end{pmatrix}$$

$$q2' = \begin{pmatrix}0\\0\end{pmatrix}$$

Since,

q2= $\frac{q2'}{length of q2'}$

Therefore,

$$q2 = \frac{1}{length of q2'} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Here,

length of q2'= $\sqrt{(0)^2 + (0)^2}$

$length of q2' = 0$

Hence,Using

q2= $\frac{q2'}{length of q2'}$

we get,

q2=0.$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

So,

q2=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Now,
Q can be obtained by combining
q1
and

q2
Therefore,

$$Q=\begin{pmatrix} \frac{2}{\sqrt{20}} & 0 \\ \\ \frac{4}{\sqrt{20}} & 0 \end{pmatrix}$$

Since we know

A=QR

Pre-Multiplying both sides by Q^T,

we get,

$Q^T A = Q^T Q R$

But $Q^T Q = I$ **[I means identity matrix]**

So,
$Q^T A = IR$
$Q^T A = R$

Or,

$R=Q^T A$

Hence,
$$R=\begin{pmatrix} \frac{2}{\sqrt{20}} & \frac{4}{\sqrt{20}} \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$$

R= $\begin{pmatrix} \frac{20}{\sqrt{20}} & \frac{10}{\sqrt{20}} \\ 0 & 0 \end{pmatrix}$

Therefore,
A=QR

$$\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{20}} & 0 \\ \frac{4}{\sqrt{20}} & 0 \end{pmatrix} \begin{pmatrix} \frac{20}{\sqrt{20}} & \frac{10}{\sqrt{20}} \\ 0 & 0 \end{pmatrix}$$

The End