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On Counting the Number of Non-isomorphic Bipartite Graphs

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Abstract: A combinatoric method which counts the number of non-isomorphic bi-partite graphs with type $[m,n]$ is obtained by classifying their connection matrices.

Key words: bipartite graph; connection matrix; orbit

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1 Introduction

A graph G is called bipartite graph if its vertex set $V = U \cup W = \{u_1, u_2, \cdots, u_m\} \cup \{w_1, w_2, \cdots, w_n\}$, where $U \cap W = \phi$, and edge set E is a collection of pairs $e = \{u, w\}$, where $u \in U$ and $w \in W$. We denote such a bipartite graph by writing $G = (U, E, W)$ and call it an $[m,n]$ type bipartite graph. Without loss of generality, we may assume that $m \leqslant n$.

The combinatoric question determining the number of non-isomorphic graphs with certain character is fundamental and interesting. In this paper, a combinatoric method which counts the number of non-isomorphic bipartite graphs with type $[m,n]$ is given by classifying their connection matrices. Speaking of two graphs to be isomorphic, the general definition comes from [2].

Now we define the *connection matrix* A of an $[m,n]$ type bipartite G with edge set E to be the following $m \times n$ matrix:

$$A = (a_{ij})_{m\times n}; \text{where } a_{ij} = \begin{cases} 1 & \{u_i, w_j\} \in E \\ 0 & \{u_i, w_j\} \notin E \end{cases}.$$

Denote $\boldsymbol{M}$ to be the set of all $m \times n$ matrices over the two elements field $GF(2)$. Let $\sigma = (\sigma_1 \sigma_2) \in S_m \times S_n$; where S_n is the symmetric group of degree n. Then σ may induce an action on $\boldsymbol{M}$ still denoted by σ, as follows:

$A^\sigma := (a_{i^{\sigma_1} j^{\sigma_2}})_{m\times n} \in \boldsymbol{M}$; where $A = (a_{ij})_{m\times n} \in \boldsymbol{M}$.

For any subgroups Γ of $S_m \times S_n$; each orbit of Γ on M is $A^\Gamma := \{A^\sigma \mid \sigma \in \Gamma\}$, where $A \in M$: Obviously, all graphs with connection matrices in A^Γ are isomorphic to each other.

If $m < n$, we easily check that the graphs with type $[m,n]$ are isomorphic if and only if their connection matrices belong to an orbit of $S_m \times S_n$ on $\boldsymbol{M}$. It is means that the above combinatoric question is equivalent to count the number of all orbits of $S_m \times S_n$.

There are the well-known Burnside lemma below:

Burnside Lemma *The number of the orbits of group Γ acting on $\boldsymbol{M}$ is*:

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$$\frac{1}{|\Gamma|}\sum_{\sigma\in\Gamma}|\mathrm{fix}_M(\sigma)|,$$

where $\mathrm{fix}_M(\sigma)=\{A\in\boldsymbol{M}\mid A^\sigma=A\}$ *and* $|\mathrm{fix}_M(\sigma)|$ *is the number of those elements in* $\boldsymbol{M}$ *fixed by* σ.

We denote the set of non-isomorphic bipartite graphs with type $[m,n]$ by $\mathscr{G}_{m,n}$ and let $\Gamma=S_m\times S_n$, then

$$|\mathscr{G}_{m,n}|=\frac{1}{|\Gamma|}\sum_{\sigma\in\Gamma}|\mathrm{fix}_M(\sigma)|, \tag{1}$$

Once $m=n$, the counting problem is little complex. In fact, let $\pi: A\mapsto A^*$, the transpose of A and then denote $\Gamma:=\langle S_n^2,\pi\rangle$, where $S_n^2:=S_n\times S_n$. Clearly, Γ still acts on $\boldsymbol{M}$ and the number of its orbits equals the number of non-isomorphic bipartite graphs with type $[n,n]$:

$$|\mathscr{G}_{n,n}|=\frac{1}{|\Gamma|}\sum_{\alpha\in\Gamma}|\mathrm{fix}_M(\alpha)|. \tag{2}$$

Observing formulas (1) and (2), the key work is how to count $|\mathrm{fix}_M(\sigma)|$ $(\sigma\in S_m\times S_n)$ if $m<n$ and $|\mathrm{fix}_M(\alpha)|$ $(\alpha\in\langle S_n^2,\pi\rangle)$ if $m=n$. This will be done in next section.

2 Preliminaries

We denote $\Omega=\{(i,j)\mid i\in\{1,2,\cdots,m\},j\in\{1,2,\cdots,n\}\}$ for $m,n\in\mathrm{N}$, where N is the set of positive integers. For a permutation subgroup Γ on Ω each orbit of Γ is called a Γ-orbital. If Γ is a cyclic group generated by a permutation σ, then the Γ-orbitals are briefly called σ-orbitals.

Specially, if $\Gamma\leqslant S_m\times S_n$; we may have a natural action of Γ on Ω by $(i,j)^\sigma:=(i^{\sigma_1},j^{\sigma_2})$ for each $\sigma=(\sigma_1,\sigma_2)\in\Gamma$ and $(i;j)\in\Omega$.

Lemma 2.1 *Let* $\sigma=(\sigma_1,\sigma_2)\in\Gamma$ *and denote the number of all* σ*-orbitals in* Ω *by* $c(\sigma)$: *Then* $|\mathrm{fix}_M(\sigma)|=2^{c(\sigma)}$.

The proofs of Lemma 2.1 and the following except Lemma 2.7 are left out due to their simpleness.

Corollary 2.2 *The number of* Γ*-orbits on* $\boldsymbol{M}$ is:

$$\frac{1}{|\Gamma|}\sum_{\sigma\in\Gamma}2^{c(\sigma)}.$$

Thus, the problem of counting $|\mathrm{fix}_M(\sigma)|$ is turned into how to count $c(\sigma)$, the number of the σ-orbitals in Ω.

First, we decompose σ_1,σ_2 respectively into products of several disjoint cycles:

$$\begin{aligned}\sigma_1&=(i_1^{(1)},\cdots,i_{s_1}^{1})(i_1^{(2)},\cdots,i_{s_2}^{(2)})\cdots(i_i^{(u)},\cdots,i_u^{(u)}),\\ \sigma_2&=(j_1^{(1)},\cdots,j_{t_1}^{(1)})(j_1^{(2)},\cdots,j_{t_2}^{(2)})\cdots(j_1^{(w)},\cdots,j_{t_w}^{(w)}).\end{aligned} \tag{3}$$

where $\sum_{k=1}^{u}s_k=m$, $\sum_{l=1}^{w}t_l=n$ and s_k, t_l may equal 1.

For the kth cycle $(i_1^{(k)},\cdots,i_{s_k}^{(k)})$ of σ_1 and the lth cycle $(j_1^{(l)},\cdots,j_{t_l}^{(l)})$ of σ_2, we denote $\Omega_{kl}^{(\sigma)}=\{(i,j)\mid i\in\{i_1^{(k)},\cdots,i_{s_k}^{(k)}\};j\in\{j_1^{(l)},\cdots,j_{t_l}^{(l)}\}\}$.

Obviously,

$$\Omega=\bigcup_{k=1}^{u}\bigcup_{l=1}^{w}\Omega_{kl}^{(\sigma)}. \tag{4}$$

Lemma 2.3 (1) $\Omega_{kl}^{(\sigma)}$ *consists of* (s_k,t_l) σ*-orbitals, where* (s_k,t_l) *is the great common divisor of* s_k *and* t_l; (2) $c(\sigma)=\sum_{k=1}^{u}\sum_{l=1}^{w}(s_k,t_l)$.

Remark If the suffix $r+1$ is greater than s_k or t_l, we always take its value on module s_k or t_l.

Obviously, if h is the least positive integer which satisfies both $i^{\sigma_1^h}=i$ and $j^{\sigma_2^h}=j$ then h equals $[s_k, t_l]$, the least common multiple. Thus, $(i;j)$ belongs to the following orbit:

$$\{(i,j)=(i_1^{(k)},j_1^{(l)}),(i_2^{(k)},j_2^{(l)}),\cdots,(i_h^{(k)},j_h^{(l)})\}\subseteq \Omega_{kl}^{(\sigma)},$$

and its length right equals $h=[s_k, t_l]$ and the length does not relate with the choice of (i,j) yet. Anyway, $\Omega_{kl}^{(\sigma)}$ is a union of several σ-orbitals whose lengths are all equal to h. Since $\Omega_{kl}^{(\sigma)}$ has $s_k\times t_l$ elements, then the number of σ-orbitals there equals:

$$\frac{s_k\times t_l}{[s_k,t_l]}=(s_k,t_l).$$

(2) holds immediately from (1) and formula (4).

When $m=n$, we refer to Lemma 2.1 to have $|\mathrm{fix}_M(\alpha)|=2^{c(\alpha)}$, where $\alpha\in\langle S_n^2,\pi\rangle$ and $c(\alpha)$ is still defined as the number of α-orbitals in Ω.

Lemma 2.4　(1) *For any* $\sigma=(\sigma_1,\sigma_2)\in S_n^2$, *we have* $\pi^{-1}\sigma\pi=\sigma^*:=(\sigma_2,\sigma_1)$;

(2) $|\langle S_n^2,\pi\rangle|=2|S_n^2|$, *that is*, $\langle S_n^2,\pi\rangle=S_n^2\times\langle\pi\rangle=S_n^2+S_n^2\pi$.

Applying the above lemma to formula (2), we have

$$|\mathscr{G}_{n,n}|=\frac{1}{2|S_n^2|}\sum_{\sigma\in S_n^2}(2^{c(\sigma)}+2^{c(\sigma\pi)}).\tag{5}$$

so, the further work is how to count $c(\sigma\pi)$ and $\sum_{\sigma\in S_n^2}2^{c(\sigma\pi)}$.

Lemma 2.5　*Let* $\sigma_1,\sigma_2\in S_n$. *We denote* $\alpha:=(\sigma_1,\sigma_2)\pi, H:=\langle\alpha\rangle$ *and* $\sigma=(\sigma_1\sigma_2,\sigma_2\sigma_1), K:=\langle\sigma\rangle$. *Then* $H=K+\alpha K$.

Obviously, $\forall(i;j)\in\Omega$, $(i,j)^K$ is a σ-orbital and $(i,j)^H$ is an α-orbital.

Lemma 2.6　(1) $\forall(i,j)\in\Omega$; $(i,j)^H=(i,j)^K\cup(i,j)^{\alpha K}$;

(2) $(i,j)^\alpha\in(i,j)^K\Leftrightarrow(i,j)^H=(i,j)^K$.

Remark　The lemma 2.6 illustrates that an α-orbital which contains (i,j) equals either a σ-orbital which contains (i,j) or a union of the two σ-orbitals, one of which contains (i,j) and the other contains $(i,j)^\alpha$. It depends on whether $(i,j)^\alpha$ belongs to that σ-orbital which contains (i,j).

We now count $c(\alpha)$, the number of α-orbitals in Ω as follows.

Lemma 2.7　*Let* $\sigma_1\sigma_2$ *be decomposed into a product of several disjoint cycles below*:

$$\sigma_1\sigma_2=(i_1^{(1)},\cdots,i_{t_1}^{(1)})(i_1^{(2)},\cdots,i_{t_2}^{(2)})\cdots(i_1^{(u)},\cdots,i_{t_u}^{(u)}),\tag{6}$$

where $\sum_{k=1}^{u}t_k=n$ *and* t_k *may equal* 1. *Then*

$$c(\alpha)=\sum_{k=1}^{u}\left[\frac{t_k+1}{2}\right]+\sum_{k<l}(t_k,t_l),$$

where $[\frac{t}{2}]$ *denotes the integer part of* $\frac{t}{2}$.

Proof　Suppose $\sigma_1: i_t^{(k)}\to j_t^{(k)}$ $(t=1,2,\cdots,t_k; k=1,2,\cdots,u)$ By formula (6) and $\sigma_2\sigma_1=\sigma_1^{-1}(\sigma_1\sigma_2)\sigma_1$, we are easy to check

$$\sigma_2\sigma_1=(j_1^{(1)},\cdots,j_{t_1}^{(1)})(j_1^{(2)},\cdots,j_{t_2}^{(2)})\cdots(j_1^{(u)},\cdots,j_{t_u}^{(u)})\tag{7}$$

and

$$(j_t^{(k)})^{\sigma_2}=(i_t^{(k)})^{\sigma_1\sigma_2}=i_{t+1}^{(k)}(t=1,2,\cdots,t_k; k=1,2,\cdots,u).\tag{8}$$

According to formulas (4), (6), (7) and $\sigma=(\sigma_1\sigma_2,\sigma_2\sigma_1)$, and from Lemma 2.3(1), Ω-contains u^2 subsets as $\Omega_{kl}^{(\sigma)}(k,l=1,2,\cdots,u)$ and each $\Omega_{kl}^{(\sigma)}$ has (t_k,t_l) $\sigma-$orbitals.

For $\Omega_{kl}^{(\sigma)}(k,l=1,2,\cdots,u)$, we may get the α-orbitals from some σ-orbitals by Lemma 2.6 in two cases:

(1) If $k=l$, we claim that $\Omega_{kk}^{(\sigma)}$ has $[\frac{t_k+1}{2}]$ α-orbitals.

In fact, $\Omega_{kk}^{(\sigma)}$ has t_k σ-orbitals.

Let a σ-orbital $(i,j)^K\subseteq\Omega_{kk}^{(\sigma)}$ and $i=i_1^{(k)}, j=j_t^{(k)}$ without loss of generality, then $i^{\sigma_1}=j_1^{(k)}$ by the assumption of σ_1 and $j^{\sigma_2}=i_{t+1}^{(k)}$ by formula (8). Furthermore,

$$(i,j)^\alpha=(i^{\sigma_1},j^{\sigma_2})^\pi=(i_{t+1}^{(k)},j_1^{(k)})\in\Omega_{kk}^{(\sigma)}. \tag{9}$$

It follows that if a σ-orbitals $(i,j)^K$ belongs to $\Omega_{kk}^{(\sigma)}$, then so does another σ-orbitals $(i,j)^{\alpha K}$. Hence, by Lemma 2.6(1), the α- orbital $(i,j)^H=(i,j)^K\cup(i,j)^{\alpha K}$ also belongs to $\Omega_{kk}^{(\sigma)}$.

Still by lemma 2.6(2), $(i,j)^H=(i,j)^K\Leftrightarrow(i,j)^\alpha\in(i,j)^k\Leftrightarrow(i,j)^\alpha=(i,j)^{\sigma^r}$ for $r\in\mathrm{N}$.

Since $(i,j)^\sigma=(i^{\sigma_1\sigma_2},j^{\sigma_2\sigma_1})=(i_2^{(k)},j_{t+1}^{(k)})$, then

$$(i,j)^{\sigma^r}=(i_{1+r}^{(k)},j_{t+r}^{(k)}). \tag{10}$$

Comparing formula (9) with formula (10), we have $i_{t+1}^{(k)}=i_{1+r}^{(k)}$ and $j_1^{(k)}=j_{t+r}^{(k)}$, that is, $t\equiv r(\bmod t_k)$ and $t+r\equiv 1(\bmod t_k)\Rightarrow 2t\equiv 1(\bmod t_k)\Rightarrow t_k$ is an odd positive integer.

Thus, if t_k is even, then each α-orbital $(i,j)^H$ in $\Omega_{kk}^{(\sigma)}$ does not equal any σ-orbital and it only consists of two different σ-orbitals $(i,j)^K$ and $(i,j)^{\alpha K}$. By Lemma 2.3(1), $\Omega_{kk}^{(\sigma)}$ has $\frac{t_k}{2}=[\frac{t_k+1}{2}]$ α-orbitals.

If t_k is odd, by $2t\equiv 1(\bmod t_k)$ and $t\leqslant t_k\Rightarrow t$ exists uniquely. It infers that $\Omega_{kk}^{(\sigma)}$ has only a σ-orbital $(i_1^{(k)},j_{\frac{t_k}{2}}^{(k)})^K$ to equal an α-orbital and others to unite else α-orbitals per pair. Thus, $\Omega_{kk}^{(\sigma)}$ has $1+\frac{t_k-1}{2}=[\frac{t_k+1}{2}]$ α-orbitals.

Anyway, the claim holds.

(2) If $k\neq l$, we claim that $\Omega_{kl}^{(\sigma)}\cup\Omega_{lk}^{(\sigma)}$ has (t_k,t_l) α-orbitals.

In fact, the union has $2(t_k,t_l)$ σ-orbitals.

Let a σ-orbital $(i,j)^K\subseteq\Omega_{kl}^{(\sigma)}$ and $i=i_s^{(k)}$, $j={}_t^{(l)}$, then $i^{\sigma_1}=j_s^{(k)}$, $j^{\sigma_2}=i_{t+1}^{(l)}\Rightarrow(i,j)^\alpha=(i^{\sigma_1},j^{\sigma_2})^\pi=(i_{t+1}^{(l)},j_s^{(k)})\in\Omega_{lk}^{(\sigma)}\Rightarrow(i,j)^{\alpha K}\subseteq\Omega_{lk}^{(\sigma)}$.

It illustrates that if $(i,j)^K$ is a σ-orbital in $\Omega_{kl}^{(\sigma)}$, then $(i,j)^{\alpha K}$ is another α-orbital in $\Omega_{lk}^{(\sigma)}$ and both σ- orbitals $(i,j)^K$ and $(i,j)^{\alpha K}$ unite an α-orbital $(i,j)^H$. Thus, the claim holds.

To sum up (1) and (2), we finally have

$$c(\alpha)=\sum_{k=1}^{u}[\frac{t_k+1}{2}]+\sum_{k<l}^{u}(t_k,t_l). \qquad \square$$

The lemma above shows that the value of $c(\alpha)=c((\sigma_1,\sigma_2)\pi)$ is only related with the type of $\sigma_1\sigma_2$. So, we definite a function $\lambda:S_n\rightarrow\mathrm{N}$ as follows:

$$\lambda(\sigma_0):=\sum_{k=1}^{u}[\frac{t_k+1}{2}]+\sum_{k<l}^{u}(t_k,t_l),$$

where $\sigma_0\in S_n$ and

$$\sigma_0=(i_1^{(1)},\cdots,i_{t_1}^{(1)})(i_1^{(2)},\cdots,i_{t_2}^{(2)})\cdots(i_1^{(u)},\cdots,i_{t_u}^{(u)}). \tag{11}$$

Lemma 2.8

$$\sum_{\sigma\in S_n^2}2^{c(\sigma\pi)}=n!\sum_{\sigma_0\in S_n}2^{\lambda(\sigma_0)}.$$

Corollary 2.9 *The number of orbits of* $\langle S_n^2,\pi\rangle$ *acting on* Ω*-is*:

$$\frac{1}{2|S_n^2|}\sum_{\sigma\in S_n^2}2^{c(\sigma)}+\frac{1}{2|S_n|}\sum_{\sigma_0\in S_n}2^{\lambda(\sigma_0)}.$$

So $c((\sigma_1,\sigma_2))$ and $\lambda(\sigma_0)$ only depends on the lengths of disjoint cycles of which σ_1,σ_2 and σ_0 are decomposed in S_m or S_n.

To express σ_0 conveniently and depending on its decomposition as formula (11) we denote the lengths shortly by $t_1 t_2 \cdots t_u$ and called it the type of σ_0 in S_n. We do so to σ_1, σ_2, and other permutations.

The remained problems is to count how many permutations whose types equal to the certain one and what kinds of type in S_n.

Lemma 2.10[1] *Let $\sigma_0 \in S_n$ with type $t_1^{r_1} t_2^{r_2} \cdots t_v^{r_v}$ $(r_1 t_1 + r_2 t_2 + \cdots + r_v t_v = n; t_1 > t_2 > \cdots > t_v)$ and denote $l_n(\sigma_0)$ to be the number of those permutations which have the same type as σ_0, then*

$$l_n(\sigma_0) = \frac{n!}{r_1! r_2! \cdots r_v! t_1^{r_1} t_2^{r_2} \cdots t_v^{r_v}}.$$

Lemma 2.11[1] *The number of all different types in S_n is equal to the number of nonnegative integer solutions $x_1, \cdots, x_n$ of equation*

$$\begin{cases} 1x_1 + 2x_2 + \cdots + nx_n = n \\ x_1 \geqslant 0, x_2 \geqslant 0, \cdots, x_n \geqslant 0 \end{cases}.$$

And the $n^{x_n} \cdots 2^{x_2} 1^{x_1}$ with the nonnegative integer solutions $x_1, \cdots, x_n$ of equation are total types of the permutations in S_n after omitting the term i^{x_i} whose exponent $x^i = 0$.

3 Main Results

According to Corollary 2.2 and Corollary 2.9, we now obtain two main results of this paper as follows.

Theorem 3.1 *Let $m < n$, and $C = C_1 \times C_2$, where C_1 and C_2 denote the sets of all permutation's types in S_m and S_n respectively, $l(\sigma) = l_m(\sigma_1) \cdot l_n(\sigma_2)$, where $\sigma = (\sigma_1, \sigma_2) \in S_m \times S_n$: Then the number of non-isomorphic bipartite graphs with type $[m, n]$ equals:*

$$|\mathcal{G}_{m,n}| = \frac{1}{m!n!} \sum_{\sigma \in C} l(\sigma) 2^{c(\sigma)}.$$

Theorem 3.2 *Let $C = C_0^2$, where C_0 denote the sets of all permutation's types in S_n, $l(\sigma) = l_n(\sigma_1) \cdot l_n(\sigma_2)$, where $\sigma = (\sigma_1, \sigma_2) \in S_n^2$, then the number of non-isomorphic bipartite graphs with type $[n, n]$ equals:*

$$|\mathcal{G}_{n,n}| = \frac{1}{2(n!)^2} \sum_{\sigma \in C} l(\sigma) 2^{c(\sigma)} + \frac{1}{2n!} \sum_{\sigma_0 \in C_0} l_n(\sigma_0) 2^{\lambda(\sigma_0)}.$$

3.1 The Bipartite Graphs with Type $[m, n]$

As an application of Theorem 3.1, we may count the number of non-isomorphic bipartite graphs with type $[3, 4]$.

Example 3.3 $|\mathcal{G}_{3,4}| = 87$.

First, the permutations in S_3 and S_4 have the types as: $3, 21, 1^3$ and $4, 31, 2^2, 21^2, 1^4$.

Second, by Lemma 2.10 and Lemma 2.3(2), we obtain two tables about $l(\sigma)$ and $c(\sigma)$ as follows (table 1 and table 2).

table 1

$l(\sigma)$	4	31	2^2	21^2	1^4
3	12	16	6	12	2
21	18	24	9	18	3
1^3	6	8	3	6	1

table 2

$c(\sigma)$	4	31	2^2	21^2	1^4
3	1	4	2	3	4
21	3	4	6	7	8
1^3	3	6	6	9	12

Third, by Theorem 3.1, we have

$$|\mathcal{G}_{3,4}| = \frac{1}{144}(12\cdot 2^1 + 16\cdot 2^4 + 6\cdot 2^2 + 12\cdot 3^3 + 2\cdot 2^4 + 18\cdot 2^3 + 24\cdot 2^4 + 9\cdot 2^6 + 18\cdot 2^7 + 3\cdot 2^8 + 6\cdot 2^3 + 8\cdot 2^6 + 3\cdot 2^6 + 6\cdot 2^9 + 1\cdot 2^{12}) = 87.$$

3.2 The Bipartite Graphs with Type $[n,n]$

As a application of Theorem 3.2, we count the number of non-isomorphic bipartite graphs with type [4,4].

Example 3.4 $|\mathcal{G}_{4,4}| = 808$.

First, the permutations in S_4 have the types as: $4, 31, 2^2, 21^2, 1^4$.

Second, by Lemma 2.10 and Lemma 2.7, we obtain three tables about $l(\sigma), c(\sigma), l_4(\sigma_0)$ and $\lambda(\sigma_0)$ as follows(table 3, table 4 and table 5):

table 3

$l(\sigma)$	4	31	2^2	21^2	1^4
4	36	48	18	36	6
31	48	64	24	48	8
2^2	18	24	9	18	3
21^2	36	48	18	36	6
1^4	6	8	3	6	1

table 4

$c(\sigma)$	4	31	2^2	21^2	1^4
4	4	2	4	4	4
31	2	6	4	6	8
2^2	4	4	8	8	8
21^2	4	6	8	10	12
1^4	4	8	8	12	16

table 5

C_0	4	31	2^2	21^2	1^4
$l_4(\sigma_0)$	6	8	3	6	1
$\lambda(\sigma_0)$	2	4	4	6	10

Third, by Theorem 3.2, we have

$$|\mathcal{G}_{4,4}| = \frac{1}{1152}[36\cdot 2^4 + 64\cdot 2^6 + 9\cdot 2^8 + 36\cdot 2^{10} + 1\cdot 2^{16} + 2(48\cdot 2^2 + 18\cdot 2^4 + 36\cdot 2^4 + 6\cdot 2^4 + 24\cdot 2^4 + 48\cdot 2^6 + 8\cdot 2^8 + 18\cdot 2^8 + 3\cdot 2^8 + 6\cdot 2^{12})] + \frac{1}{48}(6\cdot 2^2 + 8\cdot 2^4 + 3\cdot 2^4 + 6\cdot 2^6 + 1\cdot 2^{10}) = 774\cdot 5 + 33\cdot 5 = 808.$$

References:

[1] Richard A Brualdi. Introductory Combinatorics(Third Edition)[M]. Prentice Hall.
[2] Reinhard Diestel. Graph Theory[M]. Springer.

非同构二部图的计数

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摘　要:该文研究的是$[m,n]$型二部图的计数问题,利用这类图连接矩阵的分类导出了一种组合计数方法.

关键词:二部图;连接矩阵;轨道

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