Template

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November 14, 2019

目录

1 A

1.1 .vimrc

```
set nu ai ci si mouse=a ts=2 sts=2 sw=2
nmap<F2> : vs %<.in <CR>
nmap<F3> : !gedit % <CR>
nmap<F8> : !time ./%< < %<.in <CR>
nmap<F9> : :w <CR> :!g++ % -o %< -O2 -g -std=c++11 -Wall <CR>
nmap<F10> : :w <CR> :make %< <CR>
```

1.2 Head

```
#include<bits/stdc++.h>
using namespace std;
#define fi first
#define se second
#define mp make_pair
#define pb push_back
#define rep(i, a, b) for(int i=(a); i<(b); i++)
#define per(i, a, b) for(int i=(b)-1; i>=(a); i--)
#define sz(a) (int)a.size()
#define de(a) cout << #a << " = " << a << endl
#define dd(a) cout << #a << " = " << a << " "
#define all(a) a.begin(), a.end()
#define pw(x) (1ll<<(x))
#define endl "\n"
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
typedef double db;

int main() {
   std::ios::sync_with_stdio(false);
   std::cin.tie(0);
   // cout << setiosflags(ios::fixed);
   // cout << setprecision(3);
   return 0;
}
```

2 DP

2.1 DigDP

```
ll f[];
ll dfs(int pos, ..., bool lim) {
   if (pos == -1) return ?;
   if (!lim && ~f[...]) return f[...];
   ll res = 0;
   int up = lim ? dig[pos] : 9;
   rep(i, 0, up + 1) {
      if (..) res += dfs(pos - 1, ..., lim & (i == up));
   }
   if (!lim) f[] = res;
   return res;
}
ll solve(ll x) {
   int pos = 0;
   while(x) dig[pos++] = x % 10, x /= 10;
   return dfs(pos - 1, ..., 1);
}
void init() { memset(f, -1, sizeof(f)); }
void solve() {
   init();
   // 可调用 solve(x) 多次
}
```

3 DataStructure

3.1 2DST

```
namespace ST_2D{
   const int N = 1010, M = 11;
   int Log[N], p[M], dep1, dep2;
   short st[M][M][N][N];
   void build(int n, int m, short a[][N]){
      rep(i, 0, M) p[i] = 1 << i;
      rep(i, 2, N) Log[i] = Log[i >> 1] + 1;
      for(dep1 = 0; (1 << dep1) < n; dep1++);
      for(dep2 = 0; (1 << dep2) < m; dep2++);
      rep(i, 1, n + 1) rep(j, 1, m + 1)
         st[0][0][i][j] = a[i][j]; // modify
      rep(i, 1, n + 1) rep(j, 1, dep2 + 1) rep(k, p[j], m + 1)
         st[0][j][i][k] = max(st[0][j-1][i][k], st[0][j-1][i][k - p[j-1]]);
      //attention to range of k
      rep(i, 1, dep1+1) rep(j, p[i], n+1) rep(k, 0, dep2+1) rep(l, p[k], m+1)
         st[i][k][j][l]=max(st[i-1][k][j-p[i-1]][l], st[i-1][k][j][l]);
   }
   int qry(int x1, int y1, int x2, int y2){
      int l1 = Log[x2-x1+1], l2 = Log[y2-y1+1];
      int res1 = max(st[l1][l2][x1+p[l1]-1][y1+p[l2]-1], st[l1][l2][x2][y2]);
      int res2 = max(st[l1][l2][x1+p[l1]-1][y2], st[l1][l2][x2][y1+p[l2]-1]);
      return max(res1, res2);
   }
}
```

3.2 2DSegTree

```
// 修改：将区域内的值修改为区域最大值 + h
// 询问：求区域最大值
const int N=1010;
int n,m,q;
struct seg {
    int ma[N<<2], la[N<<2];
    void upd(int L,int R,int c,int l=0,int r=m,int rt=1) {
```

```
        ma[rt]=max(ma[rt], c);
        if(L<=l&&r<=R) return la[rt]=max(la[rt], c), void();
        int mid=l+r>>1;
        if(L<=mid) upd(L, R, c, l, mid, rt<<1);
        if(R>=mid+1) upd(L, R, c, mid+1, r, rt<<1|1);
    }
    int qry(int L,int R,int l=0,int r=m,int rt=1) {
        int ans=0;
        ans=max(ans, la[rt]);
        if(L<=l&&r<=R) return ans=max(ans, ma[rt]);
        int mid=l+r>>1;
        if(L<=mid) ans=max(ans, qry(L, R, l, mid, rt<<1));
        if(R>=mid+1) ans=max(ans, qry(L, R, mid+1, r, rt<<1|1));
        return ans;
    }
};
struct Seg {
    seg ma[N<<2], la[N<<2];
    void upd(int x1,int x2,int y1,int y2,int c,int l=0,int r=n,int rt=1) {
        ma[rt].upd(y1, y2, c);
        if(x1<=l&&r<=x2) return la[rt].upd(y1, y2, c), void();
        int mid=l+r>>1;
        if(x1<=mid) upd(x1, x2, y1, y2, c, l, mid, rt<<1);
        if(x2>=mid+1) upd(x1, x2, y1, y2, c, mid+1, r, rt<<1|1);
    }
    int qry(int x1,int x2,int y1,int y2,int l=0,int r=n,int rt=1) {
        int ans=0;
        ans=max(ans, la[rt].qry(y1, y2));
        if(x1<=l&&r<=x2) return ans=max(ans, ma[rt].qry(y1, y2));
        int mid=l+r>>1;
        if(x1<=mid) ans=max(ans, qry(x1, x2, y1, y2, l, mid, rt<<1));
        if(x2>=mid+1) ans=max(ans, qry(x1, x2, y1, y2, mid+1, r, rt<<1|1));
        return ans;
    }
}T;
```

3.3 CDQ

```
const int N = 200005;
int p1, p2, pos, n, k, nn, ans[N];
struct node{
   int x, y, z, num, ans;
   bool operator == (const node & b) const{
      return x == b.x && y == b.y && z == b.z;
   }
} a[N], tmp[N];
bool cmp(node a, node b){
   if (a.x != b.x) return a.x < b.x;
   if (a.y != b.y) return a.y < b.y;
   return a.z < b.z;
}
bool cmp2(node a, node b){
   //if (a.y != b.y) return a.y < b.y;
   //return a.z < b.z;
   return a.y < b.y;
}
void CDQ(int l, int r){
   if (l == r) {
      a[l].ans = a[l].num - 1;
      return;
   }
   int mid = l + r >> 1;
   CDQ(l, mid); CDQ(mid+1, r);
   pos = l;
   rep(i, mid+1, r+1) {
      while (pos <= mid && a[pos].y <= a[i].y) {
         fen.add(fen.a1, a[pos].z, a[pos].num);
         pos++;
      }
      a[i].ans += fen.sum(fen.a1, a[i].z);
   }
   rep(i, l, pos) fen.add(fen.a1, a[i].z, -a[i].num);
   p1 = l; p2 = mid+1;
   rep(i, l, r+1){
      if (p1 > mid) {tmp[i] = a[p2]; p2++;}
      else if (p2 > r) {tmp[i] = a[p1]; p1++;}
      else if (a[p1].y <= a[p2].y) {tmp[i] = a[p1]; p1++;}
      else  {tmp[i] = a[p2]; p2++;}
   }
   rep(i, l, r+1) a[i] = tmp[i];
}

int main(){
   cin >> n >> k;
   rep(i, 1, n+1) cin >> a[i].x >> a[i].y >> a[i].z;
   sort(a+1, a+n+1, cmp);
   nn = 0;
   rep(i, 1, n+1) {
      if (i > 1 && a[i] == a[i-1]) { a[nn].num++;
      } else {
         a[++nn] = a[i];
         a[nn].num = 1;
      }
   }
   fen.ini(N);
   CDQ(1, nn);
   rep(i, 1, nn+1) ans[a[i].ans] += a[i].num;
   rep(i, 0, n) cout << ans[i] << endl;
   return 0;
}
```

3.4 CartesianTree

```
// desc : bud a cartesion tree from a[0] .. a[n - 1]
// time : O(N)
// !!!! : return rt, a[n] will be rewrite
int ls[N], rs[N];
int cartesionTree(int a[], int n) {
   a[n] = INT_MAX; vi v(1, n);
   fill_n(ls, n, -1), fill_n(rs, n, -1);
```

```
   rep(i, 0, n) {
      while (a[v.back()] < a[i]) ls[i] = v.back(), v.pop_back();
      v.pb(rs[v.back()] = i);
   }
   return v[1];
}
```

3.5 Fenwick

```
// index : [1, n]
// time : nlogn
// support : segment add, sum
// !!!! : use before init()!
template<class T>
struct Fenwick{
   static const int N =2e5+7;
   int n;T a1[N],a2[N];
   void ini(int _n){ fill_n(a1+1,n=_n,0);fill_n(a2+1,n=_n,0); }
   void add(T *a,int p,T d) { for(; p<=n; p+=p & -p) a[p]+=d; }
   void add(int l,int r,T d) {
      add(a1, l, d), add(a1, r + 1, -d);
      add(a2, l, d * (l - 1)), add(a2, r + 1, -d * r);
   }
   T sum(T *a,int p) { T r=0; for(; p>=1; p-=p & -p) r+=a[p]; return r; }
   T pre(int p) { return !p ? 0 : sum(a1, p) * p - sum(a2, p);}
   T qry(int l,int r) {return pre(r)-pre(l-1); }
};
```

3.6 IntervalMaximumChangeTimes

```
inline int qry(int L, int R, ll &v, int o, bool spe, int l, int r, int rt) {
  if(L > R) return 0;
  if(!spe) {
    if(ma[rt] < v) return 0;
    if(L <= l && r <= R) {
      if(l == r) return v = ma[rt], 1;
      int mid = l + r >> 1;
      down(l, r, mid, rt);
      if(ma[ls | o] < v) return o ? qry(L, R, v, o, 0, l, mid, ls) : qry(L, R, v, o, 0,
      mid + 1, r, rs);
      int ans = cnt[rt][o] - cnt[ls | o][o] + (o == 0 ? qry(L, R, v, o, 0, l, mid, ls) :
       qry(L, R, v, o, 0, mid + 1, r, rs));
      return v = ma[rt], ans;
    }
  }
  int mid = l + r >> 1, ans = 0;
  down(l, r, mid, rt);
  if(o == 0 && L <= mid) ans += qry(L, R, v, o, 0, l, mid, ls);
  if(R > mid) ans += qry(L, R, v, o, 0, mid + 1, r, rs);
  if(o == 1 && L <= mid) ans += qry(L, R, v, o, 0, l, mid, ls);
  return ans;
}
void up(int l, int r, int rt) {
  ma[rt] = max(ma[ls], ma[rs]);
  rep(o, 0, 2) { ll v = 0; cnt[rt][o] = qry(l, r, v, o, 1, l, r, rt); }
}
```

3.7 KDT

```
// init
typedef int T; // modify
namespace KDT {
   const int N = 1e6 + 7, D = 2;
   const T INF = 1e9 + 7;
   const db al = 0.75;

   int rt,L,top,W,sta[N];
   struct P{ T x[D]; bool operator < (const P &c) const { return x[W] < c.x[W]; } }p[N];
   struct Node{ T mi[D],ma[D]; int son[2],sz; P val; }nd[N];

   void init() { rt = L = top = 0; }
   int newnode() { return top ? sta[top--] : ++L; }
   void up(int k) {
      rep(i, 0, D) {
         nd[k].mi[i]=nd[k].ma[i]=nd[k].val.x[i];
         rep(o, 0, 2) if(nd[k].son[o]) {
            int s = nd[k].son[o];
            nd[k].mi[i] = min(nd[k].mi[i], nd[s].mi[i]);
            nd[k].ma[i] = max(nd[k].ma[i], nd[s].ma[i]);
         }
      }
      nd[k].sz = 1; rep(i, 0, 2) nd[k].sz += nd[nd[k].son[i]].sz;
   }
   int build(int l,int r,int w) {
      if(l > r) return 0;
      int mid = l + r >> 1, k = newnode();
      W=w,nth_element(p+l,p+mid,p+r+1),nd[k].val=p[mid];
      nd[k].son[0] = build(l,mid-1, (w + 1) % D);
      nd[k].son[1] = build(mid+1,r, (w + 1) % D);
      up(k); return k;
   }
   void pia(int k,int &cnt) {
      if(nd[k].son[0]) pia(nd[k].son[0],cnt);
      p[++cnt]=nd[k].val,sta[++top]=k;
      if(nd[k].son[1]) pia(nd[k].son[1],cnt);
   }
   void check(int &k,int w) {
      bool o = 0;
      rep(i, 0, 2) if(al * nd[k].sz < nd[nd[k].son[i]].sz) o = 1;
      if(o) { int cnt = 0; pia(k,cnt),k = build(1,cnt,w); }
   }
   void ins(P p,int &k,int w) {
      if(!k) {k=newnode(),nd[k].val=p,nd[k].son[0]=nd[k].son[1]=0,up(k);return;}
      ins(p, nd[k].son[nd[k].val.x[w] < p.x[w]], (w + 1) % D);
      up(k),check(k,w);
   }
   // 抄上面这部分就好了，下面部分是视具体题目定的
   // 最近点（曼哈顿距离）
   // O(nsqrt(n))
```

```
    T dis(P p,int k) {
        T ans = 0;
        rep(i, 0, D) ans += max(0, p.x[i] - nd[k].ma[i]) + max(0, nd[k].mi[i] - p.x[i]);
        // modify
        return ans;
    }
    T dis(P a, P b) {
        T ans = 0; rep(i, 0, D) ans += abs(a.x[i] - b.x[i]);
        return ans;
    }
    void qry(P p, int k, T &ans) {
        ans=min(ans,dis(p,nd[k].val));
        int ls = nd[k].son[0], rs = nd[k].son[1];
        T dl = ls ? dis(p, ls) : INF;
        T dr = rs ? dis(p, rs) : INF;
        if(dl > dr) swap(dl, dr), swap(ls, rs);
        if(dl<ans) qry(p,ls,ans);
        if(dr<ans) qry(p,rs,ans);
    }
    // 矩形区域的最大值（伪代码）
    // O(n ^ (2 - 1 / D))
    void qry(int u, int &ans) {
        if(no_in || ma < ans) return ;
        if(all_in) { ans = max(ans, ma); return ; }
        if(u_in) ans = max(ans, u_val);
        rep(i, 0, 2) if(nd[u].son[i]) qry(nd[u].son[i], ans);
    }
    // 距离点 u 第 k 远
    priority_queue<ll> ans;
    void init() {
        while(!ans.empty()) ans.pop();
        rep(i, 0, k) ans.push(1);
    }
    ll sqr(int x) { return 1ll * x * x; }
    ll Dis(P p, int u) {
        ll ans = 0;
        rep(d, 0, D) ans += max(sqr(nd[u].mi[d] - p.x[d]), sqr(nd[u].ma[d] - p.x[d]));
        return ans;
    }
    void qry(P p, int u) {
        ll dis = 0; rep(d, 0, D) dis += sqr(nd[u].val.x[d] - p.x[d]);
        ans.push(-dis), ans.pop();
        int ls = nd[u].son[0], rs = nd[u].son[1];
        ll dl = ls ? Dis(p, ls) : -1;
        ll dr = rs ? Dis(p, rs) : -1;
        if(dl > dr) swap(dl, dr), swap(ls, rs);
        if(dr > -ans.top()) qry(p, rs);
        if(dl > -ans.top()) qry(p, ls);
    }
}
```

3.8 LCT

```
struct Node { int val, sum, fa, son[2]; bool rev; };
struct LCT {
    static const int N = ::N;
    Node nd[N]; int sta[N];
    // if(no root) return 1
    bool nrt(int x) {
        int fa = nd[x].fa;
        return nd[fa].son[0] == x || nd[fa].son[1] == x;
    }
    void up(int x) {
        if(!x) return ;
        int ls = nd[x].son[0], rs = nd[x].son[1];
        nd[x].sum = nd[ls].sum + nd[rs].sum + nd[x].val;
    }
    void gao(int x) {
        if(!x) return ;
        nd[x].rev ^= 1, swap(nd[x].son[0], nd[x].son[1]);
    }
    void down(int x) { if(nd[x].rev) gao(nd[x].son[0]), gao(nd[x].son[1]), nd[x].rev =
    0; }
    int id(int u) { return nd[nd[u].fa].son[1] == u; }
    void rot(int x) {
        int y = nd[x].fa, z = nd[y].fa;
        int l = id(x), r = (l ^ 1), s = nd[x].son[r];
        if(nrt(y)) nd[z].son[id(y)] = x; nd[x].son[r] = y; nd[y].son[l] = s;
        if(s) nd[s].fa = y; nd[y].fa = x; nd[x].fa = z;
        up(y), up(x);
    }
    void splay(int x) {
        int top = 0;
        for(int i = x; ; i = nd[i].fa) {
            sta[++top] = i;
            if(!nrt(i)) break;
        }
        while(top) down(sta[top--]);
        while(nrt(x)) {
            int y = nd[x].fa;
            if(nrt(y)) (id(x) ^ id(y)) ? rot(x) : rot(y);
            rot(x);
        }
    }
    void access(int x) { for(int y = 0; x ; y = x, x = nd[x].fa) splay(x), nd[x].son[1]
    = y, up(x); }
    // 换根
    void makeroot(int x) { access(x); splay(x); gao(x); }
    // 找根
    int findroot(int x) {
        access(x); splay(x);
        while(nd[x].son[0]) down(x), x = nd[x].son[0];
        splay(x);
        return x;
    }
    // 加边
    void link(int x, int y) {
        makeroot(x);
        if(findroot(y) != x) nd[x].fa = y;
    }
```

```
    // 删边
    void cut(int x, int y) {
        makeroot(x);
        if(findroot(y) == x && nd[y].fa == x && !nd[y].son[0]) nd[y].fa = nd[x].son[1] =
         0, up(x);
    }
    // nd[y]: 路径信息
    void path(int x, int y) { makeroot(x); access(y); splay(y); }
    // 单点修改
    void upd(int x, int c) { splay(x); nd[x].val = c; up(x); }
}T;
```

3.9 LCT_diameter

```
int fir(multiset<int> st) { return sz(st) ? *(st.rbegin()) : 0; }
int sec(multiset<int> st) { return sz(st) > 1 ? *(++st.rbegin()) : 0; }
void Era(multiset<int> &s, int x) { s.erase(s.find(x)); }
struct Node { int fa, son[2], lmx, rmx, mxs, sum; bool rev; multiset<int> chain, path;
};
struct LCT {
   static const int N = 30303;
   Node nd[N]; int sta[N];
   void del(int x, int y) { Era(nd[x].chain, nd[y].lmx), Era(nd[x].path, nd[y].mxs); }
   void add(int x, int y) { nd[x].chain.insert(nd[y].lmx), nd[x].path.insert(nd[y].mxs);
    }
   void init() {
      rep(i, 1, n + m + 1) {
         nd[i].fa = nd[i].son[0] = nd[i].son[1] = nd[i].rev = 0;
         nd[i].lmx = nd[i].rmx = nd[i].mxs = nd[i].sum = 0;
         nd[i].chain.clear(), nd[i].path.clear();
      }
   }
   void up(int x) {
      if(!x) return ;
      int p = x, ls = nd[x].son[0], rs = nd[x].son[1];
      // 以下考虑的都是链 p 与链  p 的所有虚子树
      nd[p].sum = nd[ls].sum + nd[rs].sum + (p > n); // 当前链的长度
      int cha = fir(nd[p].chain); // 从 p 沿虚儿子走的最远距离
      int L = max(cha, nd[ls].rmx) + (p > n); // 从 p 沿父亲走的最远距离
      int R = max(cha, nd[rs].lmx) + (p > n); // 从 p 沿实儿子走的最远距离
      nd[p].lmx = max(nd[ls].lmx, nd[ls].sum + R); // 从链顶出发的最远距离
      nd[p].rmx = max(nd[rs].rmx, nd[rs].sum + L); // 从链底出发的最远距离
      nd[p].mxs = max(nd[ls].mxs, nd[rs].mxs); // mxs[p] 表示当前范围的直径
      Ma(nd[p].mxs, fir(nd[p].path)); // 虚子树的直径
      Ma(nd[p].mxs, nd[ls].rmx + R); // 经过 p 父边的答案
      Ma(nd[p].mxs, nd[rs].lmx + L); // 经过 u 向下实边的答案
      Ma(nd[p].mxs, cha + sec(nd[p].chain) + (p > n)); // 虚子树中到根路径最长的两条拼起来
   }
   void gao(int x) {
      if(!x) return ;
      nd[x].rev ^= 1;
      swap(nd[x].son[0], nd[x].son[1]);
      swap(nd[x].lmx, nd[x].rmx);
   }
   void access(int x) {
      for(int y = 0; x ; y = x, x = nd[x].fa) {
         splay(x);
         int &rs = nd[x].son[1];
         if(y) del(x, y);
         if(rs) add(x, rs);
         rs = y, up(x);
      }
   }
   void link(int x, int y) {
      makeroot(x);
      if(findroot(y) != x) makeroot(y), nd[x].fa = y, add(y, x), up(y);
   }
   int getAns() { access(1); splay(1); return nd[1].mxs; }
}lct;
```

3.10 PerTrie

```
struct Trie {
   static const int N = ::N, M = 60;
   int L, rt[N], ne[N * (M + 1)][2], cnt[N * (M + 1)], ed[N * (M + 1)];
   void init() { L = 0; }
   // 将当前数的信息存在叶子
   inline void upd(int &now, int pre, ll val, int ind, int dep = M - 1) {
      now = ++L;
      ne[now][0] = ne[pre][0];
      ne[now][1] = ne[pre][1];
      cnt[now] = cnt[pre] + 1;
      ed[now] = ed[pre];
      if(dep == -1) return ed[now] = ind, void();
      int c = val >> dep & 1;
      upd(ne[now][c], ne[pre][c], val, ind, dep - 1);
   }
   // 查询区间和 val 异或和最大的数下标
   inline int qry(int L, int R, ll val) {
      if(!(cnt[R] - cnt[L])) return -1;
      per(i, 0, M) {
         int c = val >> i & 1;
         if(cnt[ne[R][c ^ 1]] - cnt[ne[L][c ^ 1]]) c ^= 1;
         R = ne[R][c], L = ne[L][c];
      }
      return ed[R];
   }
}ac;
```

3.11 Rope

```
#include <ext/rope>
using namespace __gnu_cxx;
rope<char> rp;
rp.push_back(ch);              // 在末尾插入字符
rp.insert(cur, 字符数组 );     // 在 cur 处插入字符数组
rp.erase(cur, len);            // 删除 cur 开始的 len 个字符
rp.replace(cur, 字符数组 );    // 删除 cur 处的字符，换成字符数组
rp.copy(cur, len, 字符数组 ); // 复制 cur 处开始的 len 个字符到字符数组
```

```
rp.at(cur);                  // 取第 cur 个字符
rp[cur];                     // 取第 cur 个字符
rp.substr(cur, len);         // 提取从 cur 处开始的 len 个字符
rp[i] = rp[i - 1];           // 可持久化, O(1), 直接拷贝根节点
/*
 * 一）翻转操作
 *  1. 维护一正一反两个 rope
 *  2. 翻转等价于交换两个子串
 * 二）区间循环位移
 *  1. 拆成多个子串，重新安排它们的位置
 * 三）区间 a -> b, b -> c, c -> d ... z -> a
 *  1. 维护 26 个 rope
 */
```

3.12 ST

```
// [0,n)
struct ST{
   static const int N = 101010;
   int a[20][N], lg[N];
   void build(int *v, int n){
      rep(i, 2, n + 1) lg[i] = lg[i >> 1] + 1;
      rep(i, 0, n) a[0][i] = v[i];
      rep(i, 1, lg[n] + 1) rep(j, 0, n - (1 << i) + 1) {
         a[i][j] = max(a[i - 1][j], a[i - 1][j + (1 << i >> 1)]);
      }
   }
   int qry(int l, int r){
      if(l > r) swap(l, r);
      int i = lg[r - l + 1];
      return max(a[i][l] , a[i][r + 1 - (1 << i)]);
   }
};
```

3.13 SegIntervalMax

```
// O(nlogn)
// 区间取 max, 区间求和
struct Seg {
   static const int N = ::N << 2;
   ll sum[N]; int mi[N][2], cnt[N];
   void up(int rt) {
      sum[rt] = sum[ls] + sum[rs];
      rep(i, 0, 2) mi[rt][i] = min(mi[ls][i], mi[rs][i]);
      cnt[rt] = 0;
      rep(i, 0, 2) {
         if(mi[rt][0] == mi[ls | i][0]) cnt[rt] += cnt[ls | i];
         else mi[rt][1] = min(mi[rt][1], mi[ls | i][0]);
      }
   }
   void build(int l, int r, int rt) {
      if(l == r) {
         sum[rt] = mi[rt][0] = a[l]; //modify
         mi[rt][1] = inf; cnt[rt] = 1;
         return ;
      }
      int mid = l + r >> 1;
      build(l, mid, ls); build(mid + 1, r, rs); up(rt);
   }
   void gao(int rt, int c) {
      if(c <= mi[rt][0]) return ;
      sum[rt] += 1ll * cnt[rt] * (c - mi[rt][0]);
      mi[rt][0] = c;
   }
   void down(int rt) { gao(ls, mi[rt][0]); gao(rs, mi[rt][0]); }
   void upd(int L, int R, int c, int l, int r, int rt) {
      if(L > R) return ;
      if(L <= l && r <= R && c < mi[rt][1]) return gao(rt, c), void();
      int mid = l + r >> 1; down(rt);
      if(L <= mid) upd(L, R, c, l, mid, ls);
      if(R > mid) upd(L, R, c, mid + 1, r, rs);
      up(rt);
   }
}seg;
```

3.14 Splay

```
// init
// id starts from 1
// if go to vertex p, must splay(p)
struct Node { int val, fa, son[2], cnt, sz; bool rev; };
struct Splay {
   static const int N = ::N;
   int rt, L; Node nd[N];
   int newnode(int c, int fa = 0, int o = 0) {
      nd[++L].fa = fa;
      nd[fa].son[o] = L;
      nd[L].val = c;
      nd[L].son[0] = nd[L].son[1] = nd[L].rev = 0;
      nd[L].cnt = nd[L].sz = 1;
      return L;
   }
   void init(int n) { rt = L = 0; }
   void gao(int u) {
      if(!u) return ;
      nd[u].rev ^= 1, swap(nd[u].son[0], nd[u].son[1]);
   }
   void down(int u) {
      if(nd[u].rev) gao(nd[u].son[0]), gao(nd[u].son[1]), nd[u].rev = 0;
   }
   void up(int u) {
      if(!u) return ;
      int ls = nd[u].son[0], rs = nd[u].son[1];
      nd[u].sz = nd[ls].sz + nd[rs].sz + nd[u].cnt;
   }
   int id(int u) { return nd[nd[u].fa].son[1] == u; }
   void rot(int x) {
      int y = nd[x].fa, z = nd[y].fa;
      int l = id(x), r = (l ^ 1), s = nd[x].son[r];
```

```
      if(z) nd[z].son[id(y)] = x; nd[x].son[r] = y; nd[y].son[l] = s;
      if(s) nd[s].fa = y; nd[y].fa = x; nd[x].fa = z;
      up(y), up(x);
   }
   void splay(int x, int g = 0) {
      while(nd[x].fa != g) {
         int y = nd[x].fa, z = nd[y].fa;
         if(z != g) (id(x) ^ id(y)) ? rot(x) : rot(y);
         rot(x);
      }
      if(!g) rt = x;
   }
}T;
```

3.15 fhqTreap

```
// init
// id starts from 1
// 不要修改 0 节点的值
struct Node { int val, cnt, sz, ls, rs; ll r; bool rev;};
struct fhqTreap {
   static const int N = ::N;
   int rt, L; Node nd[N];
   void init() { rt = L = 0; srand(time(0)); }
   ll Rand() { return ((rand() * 1ll << 32) ^ (rand() * 1ll << 16) ^ rand()); }
   int newnode(int c) {
      nd[++L].r = Rand();
      nd[L].val = c;
      nd[L].cnt = nd[L].sz = 1;
      nd[L].ls = nd[L].rs = nd[L].rev = 0;
      return L;
   }
   void up(int x) {
      if(!x) return ;
      int ls = nd[x].ls, rs = nd[x].rs;
      nd[x].sz = nd[ls].sz + nd[rs].sz + nd[x].cnt;
   }
   void gao(int x) {
      if(!x) return ;
      nd[x].rev ^= 1, swap(nd[x].ls, nd[x].rs);
   }
   void down(int x) { if(nd[x].rev) gao(nd[x].ls), gao(nd[x].rs), nd[x].rev = 0; }
   // u -> (<= c) (> c)
   void splitc(int u, int c, int &x, int &y) {
      if(u) {
         down(u);
         if(nd[u].val <= c) x = u, splitc(nd[u].rs, c, nd[u].rs, y);
         else y = u, splitc(nd[u].ls, c, x, nd[u].ls);
         up(u);
      } else x = y = 0;
   }
   // u -> (1 ~ k) (k+1 ~ L)
   // !!!: nd[].cnt == 1
   void splitk(int u, int k, int &x, int &y) {
      if(u) {
         down(u);
         int sz = nd[nd[u].ls].sz;
         if(sz < k) x = u, splitk(nd[u].rs, k - sz - 1, nd[u].rs, y);
         else y = u, splitk(nd[u].ls, k, x, nd[u].ls);
         up(u);
      } else x = y = 0;
   }
   int merge(int x, int y) {
      if(x && y) {
         if(nd[x].r < nd[y].r) { down(x), nd[x].rs = merge(nd[x].rs, y), up(x); return x
          ; }
         else { down(y), nd[y].ls = merge(x, nd[y].ls), up(y); return y; }
      } else return x + y;
   }
}T;
```

3.16 lcSegTree

```
// init
struct Node {
    ll k, b;
    Node() : k(0), b(0) {}
    Node(ll k, ll b) : k(k), b(b) {}
    ll getf(int x) const { return k * x + b; }
};
struct Seg {
    static const int N = ::N << 2;
    Node nd[N], mi[N]; // nd: max val; mi: min val;
    void _upd(Node k, int l, int r, int rt) {
        int mid = l + r >> 1;
        if(k.getf(v[mid]) > nd[rt].getf(v[mid])) swap(k, nd[rt]);
        if(l == r) return ;
        if(min(nd[rt].getf(v[l]), nd[rt].getf(v[r])) >= max(k.getf(v[l]), k.getf(v[r])))
         return ;
        if(nd[rt].k > k.k) _upd(k, l, mid, ls);
        else _upd(k, mid + 1, r, rs);
    }
    void _min(Node k, int l, int r, int rt) {
        int mid = l + r >> 1;
        if(k.getf(v[mid]) < mi[rt].getf(v[mid])) swap(k, mi[rt]);
        if(l == r) return ;
        if(max(mi[rt].getf(v[l]), mi[rt].getf(v[r])) <= min(k.getf(v[l]), k.getf(v[r])))
         return ;
        if(mi[rt].k <= k.k) _min(k, l, mid, ls);
        else _min(k, mid + 1, r, rs);
    }
    void upd(int L, int R, Node c, int l, int r, int rt) {
        if(L > R) return ;
        if(L <= l && r <= R) {
            _upd(c, l, r, rt);
            _min(c, l, r, rt);
            return ;
        }
        int mid = l + r >> 1;
        if(L <= mid) upd(L, R, c, l, mid, ls);
```

```
        if(R > mid) upd(L, R, c, mid + 1, r, rs);
    }
    ll qry(int p, int l, int r, int rt) {
        ll ans = max(abs(nd[rt].getf(v[p])), abs(mi[rt].getf(v[p])));
        if(l == r) return ans;
        int mid = l + r >> 1;
        if(p <= mid) ans = max(ans, qry(p, l, mid, ls));
        else ans = max(ans, qry(p, mid + 1, r, rs));
        return ans;
    }
}seg;
```

3.17 perTreap

```
// init
// id starts from 1
// 不要修改 0 节点的值
struct Node { int val, cnt, sz, ls, rs; ll r, sum; bool rev; };
struct fhqTreap {
   static const int N = 3e7;
   int rt[::N], L; Node nd[N];
   void init() { fill_n(rt, L + 1, 0); L = 0; srand(time(0)); }
   ll Rand() { return ((rand() * 1ll << 32) ^ (rand() * 1ll << 16) ^ rand()); }
   int newnode(int c) {
      nd[++L].r = Rand();
      nd[L].val = nd[L].sum = c;
      nd[L].cnt = nd[L].sz = 1;
      nd[L].ls = nd[L].rs = nd[L].rev = 0;
      return L;
   }
   int newcopy(int x) { nd[++L] = nd[x]; return L; }
   void up(int x) {
      if(!x) return ;
      int ls = nd[x].ls, rs = nd[x].rs;
      nd[x].sz = nd[ls].sz + nd[rs].sz + nd[x].cnt;
      nd[x].sum = nd[ls].sum + nd[rs].sum + nd[x].val;
   }
   void gao(int &x) {
      if(!x) return ;
      x = newcopy(x);
      nd[x].rev ^= 1, swap(nd[x].ls, nd[x].rs);
   }
   void down(int x) { if(nd[x].rev) gao(nd[x].ls), gao(nd[x].rs), nd[x].rev = 0; }
   // u -> (<= c) (> c)
   void splitc(int u, int c, int &x, int &y) {
      if(u) {
         u = newcopy(u), down(u);
         if(nd[u].val <= c) x = u, splitc(nd[u].rs, c, nd[u].rs, y);
         else y = u, splitc(nd[u].ls, c, x, nd[u].ls);
         up(u);
      } else x = y = 0;
   }
   // u -> (1 ~ k) (k+1 ~ L)
   // !!!: nd[].cnt == 1
   void splitk(int u, int k, int &x, int &y) {
      if(u) {
         u = newcopy(u), down(u);
         int sz = nd[nd[u].ls].sz;
         if(sz < k) x = u, splitk(nd[u].rs, k - sz - 1, nd[u].rs, y);
         else y = u, splitk(nd[u].ls, k, x, nd[u].ls);
         up(u);
      } else x = y = 0;
   }
   // sometimes do not need to newcopy
   int merge(int x, int y) {
      if(x && y) {
         if(nd[x].r < nd[y].r) { x = newcopy(x), down(x), nd[x].rs = merge(nd[x].rs, y),
          up(x); return x; }
         else { y = newcopy(y), down(y), nd[y].ls = merge(x, nd[y].ls), up(y); return y;
          }
      } else return x + y;
   }
}T;
```

3.18 动态 dp_bst

```
int n, m, a[N], sz[N], wson[N], f[N][2], h[N][2];
int to[N << 1], ne[N << 1], hd[N], _;
inline void ae(int u, int v) { to[++_] = v, ne[_] = hd[u], hd[u] = _; }
void dfs(int u, int fa) {
   sz[u] = 1;
   for(int i = hd[u]; i; i = ne[i]) if(to[i] != fa) {
      int v = to[i];
      dfs(v, u);
      // upd f[u]
      sz[u] += sz[v];
      (sz[v] > sz[wson[u]]) && (wson[u] = v);
   }
   int s = wson[u];
   // h[u] = f[u]
   if(s) {
      // upd h[u]
   }
}
struct BST {
   Mat val[N], sum[N]; bool isr[N];
   int fa[N], son[N][2], rt, sta[N], top, n;
   inline void up(int x) {
      sum[x] = val[x];
      if(son[x][0]) sum[x] = sum[son[x][0]] * sum[x];
      if(son[x][1]) sum[x] = sum[x] * sum[son[x][1]];
   }
   inline int sbuild(int l, int r) {
      if(l > r) return 0;
      int tot = 0, now = 0;
      rep(i, l, r + 1) tot += sz[sta[i]] - sz[wson[sta[i]]];
      rep(i, l, r + 1) {
         now += sz[sta[i]] - sz[wson[sta[i]]];
         if((now << 1) >= tot) {
            int x = sta[i];
```

```
            son[x][0] = sbuild(l, i - 1);
            son[x][1] = sbuild(i + 1, r);
            fa[son[x][0]] = fa[son[x][1]] = x;
            up(x);
            return x;
         }
      }
   }
   int build(int tp, int f) {
      for(int u = tp; u; f = u, u = wson[u]) {
         for(int i = hd[u]; i; i = ne[i]) if(to[i] != f && to[i] != wson[u]) {
            fa[build(to[i], u)] = u;
         }
         // upd val[u]
      }
      top = 0;
      for(int u = tp; u; u = wson[u]) sta[++top] = u;
      return sbuild(1, top);
   }
   void build(int _n) {
      n = _n; rt = build(1, 0);
      rep(i, 1, n + 1) isr[i] = (son[fa[i]][0] != i && son[fa[i]][1] != i);
   }
   inline int upd(int x, int y) {
      // upd h[x], a[x] = y, val[x]
      while(x) {
         if(isr[x] && fa[x]) {
            // get old f[x]
            up(x);
            // get new f[x]
            // u = fa[x], get h[u], val[u]
         } else {
            up(x);
         }
         x = fa[x];
      }
      // get dp[1] by sum[rt]
   }
}bst;
void init() {
   dfs(1, 0);
   bst.build(n);
}
```

3.19 动态 dp_lct

```
int n, m, a[N], f[N][2];
namespace DP {
   struct Node { int fa, son[2]; Mat val, sum; };
   struct LCT {
      inline void up(int x) {
         nd[x].sum = nd[x].val;
         if(ls) nd[x].sum = nd[ls].sum * nd[x].sum;
         if(rs) nd[x].sum = nd[x].sum * nd[rs].sum;
      }
      inline void access(int x) {
         for(int y = 0; x; y = x, x = nd[x].fa) {
            splay(x);
            if(nd[x].son[1]) {
               // upd val[x]
            }
            if(y) {
               // upd val[x]
            }
            nd[x].son[1] = y; up(x);
         }
      }
      inline int upd(int x, int y) {
         access(x); splay(x); nd[x].val.a[1][0] += y - a[x];
         up(x); a[x] = y;
         return max(nd[x].sum.a[0][0], nd[x].sum.a[1][0]);
      }
   }lct;
}
```

3.20 动态 dp_树链剖分

```
int n, a[N]; vi g[N];
namespace DP {
   int sz[N], wson[N], top[N], dep[N], id[N], _, par[N], who[N], leaf[N];
   ll f[N][2], F[N][2];
   struct Mat {
      ll a[3][3];
      inline Mat operator * (const Mat &c) const {
         Mat r; rep(i, 0, 3) rep(j, 0, 3) {
            r.a[i][j] = a[i][0] + c.a[0][j];
            rep(k, 1, 3) r.a[i][j] = max(r.a[i][j], a[i][k] + c.a[k][j]);
         } return r;
      }
      inline void e() { rep(i, 0, 3) rep(j, 0, 3) a[i][j] = (i != j) * (-inf); }
   };
   struct Seg {
      Mat m[N << 2];
      inline void build(int l, int r, int rt) {
         if(l == r) {
            int u = who[l];
            // calc F, f
            // set m[rt]
            return ;
         }
         int mid = l + r >> 1; build(mid + 1, r, rs); build(l, mid, ls);
         m[rt] = m[ls] * m[rs];
      }
      inline void upd(int u, int l, int r, int rt) {
         if(l == r) {
            // set m[rt]
            return ;
         }
         int mid = l + r >> 1;
         (id[u] <= mid) ? upd(u, l, mid, ls) : upd(u, mid + 1, r, rs);
```

```
        m[rt] = m[ls] * m[rs];
    }
    inline void qry(int L, int R, int l, int r, int rt, Mat &ans) {
        if(rt == 1) ans.e();
        if(L <= l && r <= R) return ans = ans * m[rt], void();
        int mid = l + r >> 1;
        if(L <= mid) qry(L, R, l, mid, ls, ans);
        if(R > mid) qry(L, R, mid + 1, r, rs, ans);
    }
}seg;
struct HeavyChain{
    // if(s) top[s] = top[c], dfs2(s, c, g), leaf[c] = leaf[s];
    // else leaf[c] = c;
}hc;
inline pair<ll, ll> qry(int x) {
    Mat tmp; seg.qry(id[x], id[leaf[x]], 1, n, 1, tmp);
    ll f0 = max(tmp.a[0][0], tmp.a[0][1], tmp.a[0][2]);
    ll f1 = max(tmp.a[1][0], tmp.a[1][1], tmp.a[1][2]);
    return mp(f0, f1);
}
void upd(int p, int c) {
    F[p][1] += c - a[p], a[p] = c;
    int v = p;
    while(v) {
        int u = top[v], fa = par[u];
        seg.upd(v, 1, n, 1);
        pair<ll, ll> _f = qry(u);
        if(fa) {
            // upd F[fa]
        }
        f[u][0] = _f.fi, f[u][1] = _f.se, v = fa;
    }
}
void work() {
    hc.Build(g);
    seg.build(1, n, 1);
}
}
```

3.21 常见转化

```
/*
 * 单点修改，区间查询 -> 单点修改，前缀查询 -> 后缀修改，单点查询
 * 树剖路径问题：重链区间修改，轻边暴力维护。轻边深度小的点一定在重链上，深度大的一定是 top 。
 */
```

3.22 标记不下传_区间加区间求和

```
int n; ll a[N];
struct Seg {
    static const int N = ::N << 2;
    ll sum[N], la[N];
    void build(int l = 1, int r = n, int rt = 1) {
        la[rt] = 0;
        if(l == r) return sum[rt] = a[l], void();
        int mid = l + r >> 1;
        build(l, mid, ls); build(mid + 1, r, rs);
        sum[rt] = sum[ls] + sum[rs];
    }
    void upd(int L, int R, ll c, int l = 1, int r = n, int rt = 1) {
        sum[rt] += c * (min(R, r) - max(L, l) + 1);
        if(L <= l && r <= R) return la[rt] += c, void();
        int mid = l + r >> 1;
        if(L <= mid) upd(L, R, c, l, mid, ls);
        if(R > mid) upd(L, R, c, mid + 1, r, rs);
    }
    ll qry(int L, int R, int l = 1, int r = n, int rt = 1) {
        if(L <= l && r <= R) return sum[rt];
        ll ans = la[rt] * (min(R, r) - max(L, l) + 1);
        int mid = l + r >> 1;
        if(L <= mid) ans += qry(L, R, l, mid, ls);
        if(R > mid) ans += qry(L, R, mid + 1, r, rs);
        return ans;
    }
}seg;
```

3.23 线段树优化建图

```
struct SegGraph {
#define ls (rt << 1)
#define rs (ls | 1)
    static const int N = :: N << 2, M = N + Q;
    int id[N][2], p[::N], tim;
    vector<pii> g[M];
    void init() { rep(i, 0, tim+1) g[i].clear(); tim = 0; }
    void liu(int u, int v, int w) { g[u].pb(mp(v, w)); }
    void build(int l, int r, int rt) {
        int *t = id[rt], *fa = id[rt / 2], mid = l + r >> 1;
        t[0] = ++tim, t[1] = ++tim, liu(t[0], t[1], 0);
        if (rt / 2) liu(fa[0], t[0], 0), liu(t[1], fa[1], 0);
        if (l == r) { p[l] = t[0]; return; }
        build(l, mid, ls); build(mid+1, r, rs);
    }
    void link(int l, int r, int rt, int L, int R, int w, int o) {
        int *t = id[rt], mid = l + r >> 1;
        if (L <= l && R >= r) {
            if (o) liu(t[o], tim, w);
            else liu(tim, t[o], w);
            return;
        }
        if (L <= mid) link(l, mid, ls, L, R, w, o);
        if (R > mid)  link(mid+1, r, rs, L, R, w, o);
    }
    // [l1, r1] -> [l2, r2] weight = w
    void link(int l1, int r1, int l2, int r2, int w, int n) {
        ++tim;
        link(1, n, 1, l2, r2, 0, 0);
        link(1, n, 1, l1, r1, w, 1);
    }
```

```
} G;
```

3.24 覆盖大于 k 次的矩形面积

```
// 这里是覆盖次数大于 1 次的
struct Seg {
   static const int N = ::N << 2;
   int la[N], len[2][N];
   void up(int rt, int l, int r) {
      if(la[rt] >= 2) {
         len[0][rt] = r - l + 1;
         len[1][rt] = r - l + 1;
      } else if(la[rt] >= 1) {
         len[0][rt] = r - l + 1;
         len[1][rt] = (l == r) ? 0 : len[0][ls] + len[0][rs];
      } else {
         len[0][rt] = (l == r) ? 0 : len[0][ls] + len[0][rs];
         len[1][rt] = (l == r) ? 0 : len[1][ls] + len[1][rs];
      }
   }
   void upd(int L, int R, int c, int l, int r, int rt) {
      if(L <= l && r <= R) {
         la[rt] += c;
         up(rt, l, r);
         return ;
      }
      int mid = l + r >> 1;
      if(L <= mid) upd(L, R, c, l, mid, ls);
      if(R > mid) upd(L, R, c, mid + 1, r, rs);
      up(rt, l, r);
   }
}seg;
```

3.25 高维偏序

```
// 如果有 O2 比较快, 不然可能比较慢要手写 bitset
namespace PX{
   const int N = :: N, M = sqrt(N) + 5, K = 7;
   int n, k, B, pos[K][N];
   bitset<N> s[K][M];
   vector<pii> V[K];
   struct node { int d[K]; } a[N];
   void init(int _n, int _k) {
      n = _n; k = _k;
      rep(i, 1, n+1) rep(j, 0, k) cin >> a[i].d[j];
      rep(i, 1, n+1) rep(j, 0, k) V[j].pb(mp(a[i].d[j], i));
      rep(j, 0, k) sort(all(V[j]));
      rep(i, 1, n+1) rep(j, 0, k) {
         a[i].d[j] = lower_bound(all(V[j]), mp(a[i].d[j], i)) - V[j].begin();
         pos[j][a[i].d[j]] = i;
      }
      B = sqrt(n);
      rep(j, 0, k) {
         bitset<N> tmp; int id = 1;
         rep(i, 0, n) {
            tmp.set(pos[j][i]);
            if (i == id * B - 1) s[j][id++] = tmp;
         }
      }
   }
   int qry(node a) {
      bitset<N> ans; ans.set();
      rep(j, 0, k) {
         int ed = lower_bound(all(V[j]), mp(V[j][a.d[j]].fi, n+1)) - V[j].begin() - 1;
         bitset<N> tmp;int id = ed / B, st = id ? (id - 1) * B : 0;
         if (id) tmp = s[j][id - 1];
         rep(i, st, ed+1) tmp[pos[j][i]] = 1;
         ans &= tmp;
      }
      return ans.count();
   }
}
```

4 Game

4.1 Nim 积

```
/*
 * 注：高维硬币游戏
 */
namespace Nim {
   int nimPow(int x, int y) {
      if (x < 2) return x && y; int a = 0;
      while (x < (1 << (1 << a)) || x >= (1 << (1 << (a + 1)))) a++;
      int m = 1 << (1 << a), p = x / m, s = y / m, t = y%m;
      int d1 = nimPow(p, s), d2 = nimPow(p, t);
      return (m*(d1^d2)) ^ nimPow(m / 2, d1);
   }
   int Mul(int x, int y) {
      if (x < y) return Mul(y, x);
      if (x < 2) return x && y; int a = 0;
      while (x < (1 << (1 << a)) || x >= (1 << (1 << (a + 1)))) a++;
      int m = 1 << (1 << a), p = x / m, q = x%m, s = y / m, t = y%m;
      int c1 = Mul(p, s), c2 = Mul(p, t) ^ Mul(q, s), c3 = Mul(q, t);
      return (m*(c1^c2)) ^ c3^nimPow(m / 2, c1);
   }
}
```

4.2 SurNum

```
int sgn(ll x) { return !x ? 0 : (x > 0 ? 1 : -1); }
struct SurNum {
   ll x, k; int op;
   SurNum() { x = k = op = 0; }
   SurNum(ll x, ll k, ll op = 0) :x(x), k(k), op(op) { }
   SurNum(const SurNum &a) { *this = a; }
   inline SurNum Simplify() {
```

```
    while (x % 2 == 0 && k > 0) x /= 2, k--;
    return *this;
  }
  friend inline int sgn(const SurNum &a) { return sgn(a.x); }
  inline bool Grow(int kk) {
    Simplify();
    if (kk < k) return 0;
    x *= 1ll << kk - k, k = kk;
    return 1;
  }
  friend inline void grow(SurNum &a, SurNum &b) {
    int k = max(a.k, b.k);
    a.Grow(k), b.Grow(k);
  }
  friend inline int compare(SurNum a, SurNum b) {
    if (a.op < b.op) return -1;
    if (a.op > b.op) return 1;
    if (a.op != 0) return 0;
    int opa = sgn(a), opb = sgn(b);
    if (opa < opb) return -1;
    if (opa > opb) return 1;
    grow(a, b);
    return sgn(a.x - b.x);
  }
  friend inline bool operator < (const SurNum &a, const SurNum &b) {
    return compare(a, b) == -1;
  }
  friend inline bool operator > (const SurNum &a, const SurNum &b) {
    return compare(a, b) == 1;
  }
  friend inline bool operator == (const SurNum &a, const SurNum &b) {
    return compare(a, b) == 0;
  }
  friend inline bool operator != (const SurNum &a, const SurNum &b) {
    return compare(a, b) != 0;
  }
  friend inline bool operator <= (const SurNum &a, const SurNum &b) {
    return compare(a, b) <= 0;
  }
  friend inline bool operator >= (const SurNum &a, const SurNum &b) {
    return compare(a, b) >= 0;
  }
  friend inline SurNum operator - (const SurNum &a) {
    return SurNum(-a.x, a.k, -a.op);
  }
  friend inline SurNum operator + (SurNum a, SurNum b) {
    if (a.op == 1 || b.op == 1) return SurNum(0, 0, 1);
    if (a.op == -1 || b.op == -1) return SurNum(0, 0, -1);
    grow(a, b); return SurNum(a.x + b.x, a.k, 0).Simplify();
  }
  friend inline SurNum operator - (SurNum a, SurNum b) {
    return a + (-b);
  }
  friend inline SurNum operator += (SurNum &a, SurNum b) {
    return a = a + b;
  }
  friend inline SurNum operator -= (SurNum &a, SurNum b) {
    return a = a - b;
  }
  friend inline SurNum operator >> (SurNum a, ll k) {
    return a.k += k, a.Simplify();
  }
  friend inline SurNum getMid(SurNum a, SurNum b) {
    return a + b >> 1;
  }
  inline void print() const {
    printf("SurNum:\n");
    if (op == 1) { printf("+inf\n"); return; }
    if (op == -1) { printf("-inf\n"); return; }
    printf("%lld/%lld\n", x, 1 << k);
  }
  inline static SurNum read() {
    ll a1, a2, a3;
    scanf("%lld%lld%lld", &a1, &a2, &a3);
    return SurNum(a1, a2, a3).Simplify();
  }
} _0(0, 0, 0), _inf(0, 0, 1);
struct SurTri {
  SurNum p, x, q;
  SurTri() { p = x = q = _0; }
  SurTri(SurNum p, SurNum x, SurNum q) :p(p), x(x), q(q) {}
  SurTri(const SurTri &a) { *this = a; }
  SurTri goRight() {
    SurNum y;
    if (x.x >= 0 && x.k == 0) y = x, y.x++; else y = x + q >> 1;
    return SurTri(x, y, q);
  }
  SurTri goLeft() {
    SurNum y;
    if (x.x <= 0 && x.k == 0) y = x, y.x--; else y = p + x >> 1;
    return SurTri(p, y, x);
  }
  void print() {
    printf("\n\nSurTri:\n\n");
    p.print(), x.print(), q.print();
    printf("\nend\n\n");
  }
};
struct SurCalculator {
  int getDir(SurTri S, SurNum a, SurNum b) {
    if (a < S.x && S.x < b) return 0;
    if (a <= S.x && b <= S.x) return -1;
    if (a >= S.x && b >= S.x) return 1;
    assert(0);
  }
  SurNum getValue(SurNum a, SurNum b) {
    int op;
    SurTri S(-_inf, _0, _inf);
    while (op = getDir(S, a, b)) S = ((op == 1) ? S.goRight() : S.goLeft());
    return S.x;
```

```
    }
};
```

5 Geo

5.1 1、基础点、向量

```
struct P {
    int quad() const { return sign(y) > 0 || (sign(y) == 0 && sign(x) >= 0); }
    P rot90() { return P(-y, x); }
    P rot(db a) { return P(cos(a) * x - sin(a) * y, cos(a) * y + sin(a) * x); }
    P norm() { return *this / len(); }
};
db rad(P p1, P p2) { return atan2l(det(p1, p2), dot(p1, p2)); } // p1 与 p2 的夹角，有方向
bool cmp(const pii &a, const pii &b) { // 级角排序
    int o = a > pii(0, 0), t = b > pii(0, 0);
    if(o != t) return o < t;
    return det(a, b) > 0;
}
// 【点集中最近点对】
namespace NearestPoints { // sz(A) <= 1e5
    db solve(int l, int r, vector<P> &p) {
        if(l == r) return 1e100;
        int m = l + r >> 1;
        db Xm = p[m].x, lim = min(solve(l, m, p), solve(m + 1, r, p));
        inplace_merge(p.begin() + l, p.begin() + m + 1, p.begin() + r + 1, [&](P a, P b){
        return a.y < b.y;});
        vector<P> V;
        rep(i, l, r + 1) if(fabs(p[i].x - Xm) <= lim) V.pb(p[i]);
        rep(i, 0, sz(V)) rep(j, i + 1, sz(V)) {
            if(fabs(V[j].y - V[i].y) >= lim) break;
            T dis = (V[i] - V[j]).len();
            lim = min(lim, dis);
        }
        return lim;
    }
    db solve(vector<P> A) {
        sort(all(A), [&](P a, P b){return a.x < b.x;});
        return solve(0, sz(A) - 1, A);
    }
}
// 【最小圆覆盖】
C Mincir(P *p,int n){
    random_shuffle(p , p + n);
    P o = p[0];db r = 0;
    rep(i,1,n) {
        if(sgn(abs(o-p[i])-r) <= 0) continue;
        o = p[i] , r = 0;
        rep(j,0,i) {
            if(sgn(abs(o-p[j])-r) <= 0) continue;
            o = (p[i] + p[j]) / 2 , r = abs(o-p[j]);
            rep(k,0,j) {
                if(sgn(abs(o-p[k])-r) <= 0) continue;
                o = outC(p[i],p[j],p[k]) , r = abs(o-p[k]);
            }}}
    return C(o,r);
}
// 【费马点】
// sqrt((a ^ 2 + b ^ 2 + c ^ 2 + 4 * sqrt(3) * area) / 2)
// 如果有重点，大于 2 的直接用模拟退火法
P fermat(vector<P> p) {
  int n = sz(p); assert(n);
  if(n == 1) return p[0];
  if(n == 2) return (p[0] + p[1]) / 2;
  if(n == 3) {
    db a[3];
    rep(i, 0, 3) a[i] = (p[(i + 2) % 3] - p[(i + 1) % 3]).len();
    rep(i, 0, 3) {
      int j = (i + 1) % 3, k = (i + 2) % 3;
      if(sign(a[i] * a[i] - a[j] * a[j] - a[k] * a[k] - a[j] * a[k]) >= 0) return p[i];
    }
    if(det(p[0], p[1], p[2]) < 0) swap(p[1], p[2]);
    P q1 = (p[2] - p[0]).rot(pi / 3) + p[0];
    P q2 = (p[0] - p[1]).rot(pi / 3) + p[1];
    return isLL(L(q1, p[1]), L(q2, p[2]));
  }
  auto Rand = [&] () { return rand() % 10000 / 5000 * pi; };
  P ans(0, 0); rep(i, 0, n) ans = ans + p[i]; ans = ans / n;
  db len = 0; rep(i, 0, n) len += (ans - p[i]).len();
  db t = 10000; // modify
  while(t > eps) {
    db ang = Rand();
    P np(ans.x + t * sin(ang), ans.y + t * cos(ang));
    db k = 0; rep(i, 0, n) k += (np - p[i]).len();
    if(sign(len - k) > 0) ans = np, len = k;
    t *= 0.999;
  }
  return ans;
}
```

5.2 2、线段、直线、曲线

```
// 【点到直线投影（垂足）】
P proj(L l, P p) { return l.a + (l.b - l.a) * (dot(p - l.a, l.b - l.a) / (l.b - l.a).
len2()); }
// 【直线交点】
P isLL(L l1, L l2) {
   db s1 = det(l2.b - l2.a, l1.a - l2.a);
   db s2 = -det(l2.b - l2.a, l1.b - l2.a);
   return (l1.a * s2 + l1.b * s1) / (s1 + s2);
}
P isLL(L l, db a, db b, db c) { // ax + by + c = 0
   db u = a * l.a.x + b * l.a.y + c;
   db v = -(a * l.b.x + b * l.b.y + c);
   return (l.a * v + l.b * u) / (u + v);
}
P isLL(db a0, db b0, db c0, db a1, db b1, db c1) {
   db d = a0 * b1 - a1 * b0;
```

```
   return P(b0 * c1 - b1 * c0, a1 * c0 - a0 * c1) / d;
}
// 【线相交判定】
bool isSSr(const L &a, const L &b){
   db c1 = det(a.t - a.s, b.s - a.s), c2 = det(a.t - a.s, b.t - a.s);
   db c3 = det(b.t - b.s, a.s - b.s), c4 = det(b.t - b.s, a.t - b.s);
   return sign(c1) * sign(c2) < 0 && sign(c3) * sign(c4) < 0;
}
bool isSS(L a,L b){
   db c1 = det(a.t - a.s, b.s - a.s), c2 = det(a.t - a.s, b.t - a.s);
   db c3 = det(b.t - b.s, a.s - b.s), c4 = det(b.t - b.s, a.t - b.s);
   return sign(c1) * sign(c2) <= 0 && sign(c3) * sign(c4) <= 0 &&
      sign(max(a.s.x, a.t.x) - min(b.s.x, b.t.x)) >= 0 &&
      sign(max(b.s.x, b.t.x) - min(a.s.x, a.t.x)) >= 0 &&
      sign(max(a.s.y, a.t.y) - min(b.s.y, b.t.y)) >= 0 &&
      sign(max(b.s.y, b.t.y) - min(a.s.y, a.t.y)) >= 0;
}
bool isLS(P a1, P a2, P b1, P b2) { // 判断直线线段是否相交（端点也算）
   db c1 = det(a2 - a1, b1 - a1), c2 = det(a2 - a1, b2 - a1);
   return sign(c1) * sign(c2) <= 0;
}
// 【点到线距离】
db disToL(L l, P p) {
  return fabs(det(l.a, p, l.b) / (l.b - l.a).len());
}
db disToS(L l, P p) {
  return sign(dot(l.a, p, l.b)) * sign(dot(l.b, p, l.a)) == 1 ? disToL(l, p) : min((p -
  l.a).len(), (p - l.b).len());
}
// 【线到线距离】
db disSS(L a, L b){
   if(isSS(a, b)) return 0;
   return min(min(disToSeg(b, a.s), disToSeg(b, a.t)), min(disToSeg(a, b.s), disToSeg(a,
    b.t)));
}
```

5.3 3、凸包

```
// 【求凸包】
vector<P> convexHull(vector<P> ps) {
  int n = sz(ps); if(n <= 1) return ps;
  sort(all(ps)); vector<P> qs;
  for(int i = 0; i < n; qs.pb(ps[i++])) {
    while(sz(qs) > 1 && sign(det(qs[sz(qs) - 2], qs.back(), ps[i])) <= 0) qs.pop_back();
  }
  for(int i = n - 2, t = sz(qs); i >= 0; qs.pb(ps[i--])) {
    while(sz(qs) > t && sign(det(qs[sz(qs) - 2], qs.back(), ps[i])) <= 0) qs.pop_back();
  }
  qs.pop_back(); return qs;
}
// 【凸包最远点对】
db diameter(vector<P> A) {
   int n = sz(A);
   if(n <= 1) return 0;
   int l = 0, r = 0;
   rep(i, 1, n) (A[i] < A[l]) && (l = i), (A[r] < A[i]) && (r = i);
   db res = (A[l]-A[r]).len();
   int i = l, j = r;
   do (++(det(A[(i + 1) % n]- A[i], A[(j + 1) % n] - A[j]) >= 0 ? j : i)) %= n,
      res = max(res, (A[i] - A[j]).len());
   while(i != l || j != r);
   return res;
}
// 【动态凸包】
// O(nlogn)
// 插入点，询问点在不在凸包内部（包括边界）
namespace DCH {
   map<int, P> h1, h2;
   bool ao(P a, P b, P c) {
      // 包括边界：小等于
      return (b.y - a.y) * 1ll * (c.x - b.x) <= (c.y - b.y) * 1ll * (b.x - a.x);
   }
   bool in(map<int, P> &h, P p) {
      if(!sz(h)) return 0;
      if(p.x < h.begin()->se.x || p.x > h.rbegin()->se.x) return 0;
      auto l = h.lower_bound(p.x);
      if(p.x == l->se.x) return p.y <= l->se.y;
      auto r = l--;
      return ao(l->se, p, r->se);
   }
   void ins(map<int, P> &h, P p) {
      if(in(h, p)) return ;
      h[p.x] = p;
      auto pos = h.find(p.x);
      while(1) {
         auto l = pos; if(l == h.begin()) break; --l;
         auto ll = l; if(ll == h.begin()) break; --ll;
         if(ao(ll->se, l->se, p)) h.erase(l); else break;
      }
      while(1) {
         auto r = pos; r++; if(r == h.end()) break;
         auto rr = r; rr++; if(rr == h.end()) break;
         if(ao(p, r->se, rr->se)) h.erase(r); else break;
      }
   }
   void ins(int x, int y) { ins(h1, P(x, y)); ins(h2, P(x, -y)); }
   bool in(int x, int y) { return in(h1, P(x, y)) && in(h2, P(x, -y)); }
}
// 【凸包交】
namespace ConvecIntersection{ // ?
   const int N = 1005;
   struct Rec {
      P d[10];int dn;// d[dn] = d[0]
      P operator [] (const int&n) {return d[n];}
   }r[N];
   typedef pair<db,int> pdi;
   int n;pdi res[1000005];
   db getLoc(P a,P b,P p){
      if(sgn(b.x - a.x)) return (p.x - a.x) / (b.x - a.x);
      return (p.y - a.y) / (b.y - a.y);
```

```
   }
   db work() {
      db rt=0;
      rep(i,0,n) rep(j,0,r[i].dn){
         int sz=0;
         res[sz++] = pdi(0,0);res[sz++] = pdi(1,0);
         rep(t,0,n) {
            if(t == i) continue;
            rep(g,0,r[t].dn) {
               int du = sgn((r[i][j+1] - r[i][j]) / (r[t][g] - r[i][j]));
               int dv = sgn((r[i][j+1] - r[i][j]) / (r[t][g+1] - r[i][j]));
               if(!du && !dv) {
                  if(sgn((r[i][j+1] - r[i][j]) * (r[t][g+1] - r[t][g])) < 0 || i < t){
                     res[sz++] = pdi(getLoc(r[i][j] , r[i][j+1] , r[t][g]) , 1);
                     res[sz++] = pdi(getLoc(r[i][j] , r[i][j+1] , r[t][g+1]) , -1);
                  }} else {
                     db s1 = (r[i][j] - r[t][g]) / (r[t][g+1] - r[t][g]);
                     db s2 = (r[t][g+1] - r[t][g]) / (r[i][j+1] - r[t][g]);
                     if(du >= 0 && dv < 0) res[sz++] = pdi(s1 / (s1 + s2) , 1);
                     else if(du < 0 && dv >= 0) res[sz++] = pdi(s1 / (s1 + s2) , -1);
                  }}}
         sort(res , res + sz);
         int cnt = 0; --sz;
         rep(t,0,sz) {
            cnt += res[t].se;
            if(cnt == 0 && sgn(res[t].fi - res[t+1].fi)) {
               db a = res[t].fi;
               if(a < 0) a = 0; if(a > 1) break;
               db b = res[t+1].fi;
               if(b < 0) continue; if(b > 1) b = 1;
               rt += ((r[i][j+1] - r[i][j]) * a + r[i][j]) / ((r[i][j+1]-r[i][j]) * b +
                r[i][j]);
            }}}
      return rt / 2;}}
```

5.4 4、三角形

```
// 【心】
P outC(P A, P B, P C) { // 外心
   P b = B - A, c = C - A;
   db dB = b.len2(), dC = c.len2(), d = 2 * det(b, c);
   return A - P(b.y * dC - c.y * dB, c.x * dB - b.x * dC) / d;
}
P baryC(P p[], int n) { // 重心
   P fz(0, 0); db fm = 0;
   rep(i, 1, n - 1) {
      db t = det(p[0], p[i], p[i + 1]);
      fm += t;
      fz = fz + (p[0] + p[i] + p[i + 1]) * t / 3;
   }
   return fz / fm;
}
```

5.5 5、多边形

```
// 【平面图欧拉定理】 V + F - E = 2
// 【简单多边形求面积交】
db polyInter(vector<P> &p, vector<P> &q) {
   int n = sz(p), m = sz(q);
   if(n < 3 || m < 3) return 0;
   // if(area(p) < 0) reverse(all(p));
   // if(area(q) < 0) reverse(all(q));
   db ans = 0;
   rep(i, 1, n - 1) {
      P p1 = p[i], p2 = p[i + 1];
      bool f1 = 0;
      if(det(p[0], p1, p2) < 0) swap(p1, p2), f1 = 1;
      rep(j, 1, m - 1) {
         P q1 = q[j], q2 = q[j + 1];
         bool f2 = 0;
         if(det(q[0], q1, q2) < 0) swap(q1, q2), f2 = 1;
         vector<P> ps({p[0], p1, p2});
         convexCut(ps, L(q[0], q1));
         convexCut(ps, L(q1, q2));
         convexCut(ps, L(q2, q[0]));
         db res = f1 == f2 ? area(ps) : -area(ps);
         ans += res;
      }
   }
   return fabs(ans);
}
```

5.6 6、圆

```
// 【两圆关系】
// 注意相等关系
// 相离4: 外切3: 相交2: 内切1: 内含0:
int relCC(C A, C B) { // 两圆关系
   db dis = (A.o - B.o).len();
   if(sign(dis - (A.r + B.r)) == 1) return 4;
   if(sign(dis - (A.r + B.r)) == 0) return 3;
   if(sign(dis - fabs(A.r - B.r)) == 1) return 2;
   if(sign(dis - fabs(A.r - B.r)) == 0) return 1;
   return 0;
}
// 【点圆切点】
bool tanCP(O c, P p0, P &p1, P &p2) {
   db x = (p0 - c.o).len2(), d = x - c.r * c.r;
   if(d < eps) return 0;
   P p = (p0 - c.o) * (c.r * c.r / x);
   P det = ((p0 - c.o) * (-c.r * sqrt(d) / x)).rot90();
   p1 = c.o + p + det;
   p2 = c.o + p - det;
   return 1;
}
// 【圆圆切点】
vector<P> tanCC(const C &c1, const C &c2) {
   vector<P> res;
   db dis = (c1.o - c2.o).len();
```

```
   if(sign(dis - (c1.r + c2.r)) == 0) {
      res.pb(c1.o + (c2.o - c1.o) * c1.r / (c1.r + c2.r));
   }
   if(sign(dis - fabs(c1.r - c2.r) == 0)) {
      res.pb(c1.o + (c2.o - c1.o) * c1.r / (c1.r - c2.r));
   }
   return res;
}
// 【外公切线】
vector<L> extanCC(C c1, C c2) {
   vector<L> ret;
   if(sign(c1.r - c2.r) == 0) {
      P dir = c2.o - c1.o;
      dir = (dir * (c1.r / dir.len())).rot90();
      ret.pb(L(c1.o + dir, c2.o + dir));
      ret.pb(L(c1.o - dir, c2.o - dir));
   } else {
      P p = (c1.o * -c2.r + c2.o * c1.r) / (c1.r - c2.r);
      P p1, p2, q1, q2;
      if(tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
         if(c1.r < c2.r) swap(p1, p2), swap(q1, q2);
         ret.pb(L(p1, q1));
         ret.pb(L(p2, q2));
      }
   }
   return ret;
}
// 【内公切线】
vector<L> intanCC(C c1, C c2) {
   vector<L> ret;
   P p = (c1.o * c2.r + c2.o * c1.r) / (c1.r + c2.r);
   P p1, p2, q1, q2;
   if(tanCP(c1, p, p1, p2) && tanCP(c2, p, q1, q2)) {
      ret.pb(L(p1, q1));
      ret.pb(L(p2, q2));
   }
   return ret;
}
// 【直线和圆求交】
bool isCL(O a, L l, P &p1, P &p2) {
  db x = dot(l.a - a.o, l.b - l.a);
  db y = (l.b - l.a).len2();
  db d = x * x - y * ((l.a - a.o).len2() - a.r * a.r);
  if(sign(d) < 0) return 0;
  d = max(d, 0.);
  P p = l.a - ((l.b - l.a) * (x / y)), det = (l.b - l.a) * (sqrt(d) / y);
  p1 = p - det, p2 = p + det; // dir : l.a -> l.b
  return 1;
}
// 【圆与三角形交面积】
db areaCT(db r,P s,P t) { // 需要除 2
   P p1, p2;
   bool f = isCL(C(P(0, 0), r), L(s, t), p1, p2);
   if(!f) return r * r * rad(s, t);
   bool b1 = sign(s.len2() - r * r) == 1 , b2 = sign(t.len2() - r * r) == 1;
   if(b1 && b2) {
      if(sign(dot(s - p1, t - p1)) <= 0 && sign(dot(s - p2, t - p2) <= 0))
         return r * r * (rad(s, p1) + rad(p2, t)) + det(p1, p2);
      else return r * r * rad(s, t);
   } else if(b1) return r * r * rad(s, p1) + det(p1, t);
   else if(b2) return r * r * rad(p2, t) + det(s, p2);
   return det(s, t);
}
// 【圆与多边形交面积】
db areaCPoly(C c, vector<P> p) {
   int n = sz(p);
   db ans = 0;
   rep(i, 0, n) {
      P u = p[i], v = p[(i + 1) % n];
      ans += areaCT(c.r, u - c.o, v - c.o);
   }
   return fabs(ans) / 2;
}
// 【圆交】
namespace CircleIntersection{ // ?
   struct E{
      P p;T ang;int delta;
      E(){} E(P p,T ang,int delta):p(p),ang(ang),delta(delta){}
      bool operator < (const E&b) const {return ang<b.ang;}
   };
   bool overlap(C a,C b) {return sgn(a.r-b.r-abs(a.o-b.o))>=0;}
   void solve(C *c,int n,T *ans) {
      memset(ans , 0 , sizeof(T) * (n + 1));
      rep(i,0,n) {
         int cnt=1;
         vector<E> evt;
         rep(j,0,i) if(c[i]==c[j]) cnt++;
         rep(j,0,n) if(j!=i&&!(c[i]==c[j])&&overlap(c[j],c[i])) cnt++;
         rep(j,0,n) if(j!=i){
            vector<P> pts=insCC(c[i],c[j]);
            if(sz(pts)) {
               T a[2];
               rep(j,0,2) a[j]=(pts[j]-c[i].o).arg();
               evt.pb(E(pts[0],a[0],1));
               evt.pb(E(pts[1],a[1],-1));
               cnt += a[0] > a[1];
            }
         }
         if(!sz(evt)) ans[cnt] += pi*c[i].r*c[i].r;
         else{
            sort(all(evt));
            evt.pb(evt.front());
            rep(j,0,sz(evt)-1) {
               cnt+=evt[j].delta;
               ans[cnt] += evt[j].p / evt[j+1].p / 2;
               db ang = evt[j + 1].ang - evt[j].ang;
               if(ang < 0) ang += pi * 2;
               ans[cnt] += ang * c[i].r * c[i].r / 2 - sin(ang) * c[i].r * c[i].r / 2;
            }}}}}
```

5.7 7、3D

```
// 【最小球覆盖】
P3 MinSphere(vector<P3> p) {
  int n = sz(p); assert(n);
  db t = 1; P3 ans(0, 0, 0);
  rep(i, 0, n) ans = ans + p[i]; ans = ans / n;
  while(t > eps) {
    int j = -1; db ret = -1;
    rep(i, 0, n) {
      db tmp = (p[i] - ans).len();
      if(ret < tmp) ret = tmp, j = i;
    }
    ans = ans + (p[j] - ans) * t;
    t *= 0.999;
  }
  return ans;
}
// 【三维向量变换】
struct Mat {
  db a[4][4];
  void set() { rep(i, 0, 4) rep(j, 0, 4) a[i][j] = 0; }
  void e() { rep(i, 0, 4) a[i][i] = 1; }
  Mat operator * (const Mat &c) {
    Mat r; r.set();
    rep(i, 0, 4) rep(j, 0, 4) rep(k, 0, 4) r.a[i][j] += a[i][k] * c.a[k][j];
    return r;
  }
};
Mat kpow(Mat a, int b) {
  Mat r; r.set(); r.e();
  while(b) {
    if(b & 1) r = r * a;
    a = a * a;
    b >>= 1;
  }
  return r;
}
Mat translate(db tx, db ty, db tz) { // 平移，以下矩阵均为左乘
  db p[4][4] = {
    1, 0, 0, tx,
    0, 1, 0, ty,
    0, 0, 1, tz,
    0, 0, 0, 1};
  Mat r; rep(i, 0, 4) rep(j, 0, 4) r.a[i][j] = p[i][j]; return r;
}
Mat scale(db a, db b, db c) { // 缩放
  db p[4][4] = {
    a, 0, 0, 0,
    0, b, 0, 0,
    0, 0, c, 0,
    0, 0, 0, 1};
  Mat r; rep(i, 0, 4) rep(j, 0, 4) r.a[i][j] = p[i][j]; return r;
}
Mat rotate(P3 s, db a) { // 绕 s 为轴旋转 a 度，右手方向
  db l = s.len(), x = s.x / l, y = s.y / l, z = s.z / l, si = sin(a), co = cos(a);
  db p[4][4] = {
    co + (1 - co) * x * x, (1 - co) * x * y - si * z, (1 - co) * x * z + si * y, 0,
    (1 - co) * y * x + si * z, co + (1 - co) * y * y, (1 - co) * y * z - si * x, 0,
    (1 - co) * z * x - si * y, (1 - co) * z * y + si * x, co + (1 - co) * z * z, 0,
    0, 0, 0, 1};
  Mat r; rep(i, 0, 4) rep(j, 0, 4) r.a[i][j] = p[i][j]; return r;
}
```

5.8 HalfPlane_n2

```
// l: a->b 逆时针方向
void convexCut(vector<P> &p, L l) {
   vector<P> q;
   rep(i, 0, sz(p)) {
      P p1 = p[i], p2 = p[(i + 1) % sz(p)];
      int d1 = sign(det(l.a, l.b, p1));
      int d2 = sign(det(l.a, l.b, p2));
      if(d1 >= 0) q.pb(p1);
      if(d1 * d2 < 0) q.pb(isLL(L(p1, p2), l));
   } p = q;
}
// ax + by + c >= 0
void convexCut(vector<P> &p, db a, db b, db c) {
   vector<P> q;
   rep(i, 0, sz(p)) {
      P p1 = p[i], p2 = p[(i + 1) % sz(p)];
      int d1 = sign(a * p1.x + b * p1.y + c);
      int d2 = sign(a * p2.x + b * p2.y + c);
      if(d1 >= 0) q.pb(p1);
      if(d1 * d2 < 0) q.pb(isLL(L(p1, p2), a, b, c));
   } p = q;
}
```

5.9 HalfPlane_nlogn

```
struct P {
   int quad() const { return sign(y) > 0 || (sign(y) == 0 && sign(x) >= 0); }
};
struct L {
   // ax + by + c >= 0, (a != 0 || b != 0)
   L(db a, db b, db c) {
      if(sign(a)==0) {
         this->a=P(0,-c/b);this->b=P(sign(b),-c/b);
      } else if(sign(b)==0) {
         this->a=P(-c/a,0);this->b=P(-c/a,-sign(a));
      } else {
         if(sign(c)!=0) {
            int x=sign(c)*sign(det(P(-c/a,0), P(0,-c/b)));
            if(x==1) this->a=P(-c/a,0),this->b=P(0,-c/b);
            else this->a=P(0,-c/b),this->b=P(-c/a,0);
         } else {
            this->a=P(0,0);this->b=P(sign(b),sign(b)*(-a/b));
         }
```

```
    }
  }
  bool includer(const P &p) const { return sign(det(b - a, p - a)) > 0; }
  bool include(const P &p) const { return sign(det(b - a, p - a)) >= 0; }
  // 向内（右手方向）推
  L push(db len) {
    P det = (b - a).rot90().norm() * len;
    return L(a + det, b + det);
  }
};
bool sameDir(L l0, L l1) {
  P a = l0.a - l0.b, b = l1.a - l1.b;
  return sign(det(a, b)) == 0 && sign(dot(a, b)) == 1;
}
bool operator < (const P &a,  const P &b) {
  if(a.quad() != b.quad()) return a.quad() < b.quad();
  return sign(det(a, b)) > 0;
}
bool operator < (const L &l0, const L &l1) {
  if(sameDir(l0, l1)) return l1.includer(l0.a);
  return (l0.b - l0.a) < (l1.b - l1.a);
}
bool check(L u, L v, L w) { return w.include(isLL(u, v)); }
deque<L> halfPlane(vector<L> l) {
  sort(all(l)); deque<L> q;
  rep(i, 0, sz(l)) {
    if(i && sameDir(l[i], l[i - 1])) continue;
    while(sz(q) > 1 && !check(q[sz(q) - 2], q.back(), l[i])) q.pop_back();
    while(sz(q) > 1 && !check(q[1], q[0], l[i])) q.pop_front();
    q.pb(l[i]);
  }
  while(sz(q) > 2 && !check(q[sz(q) - 2], q.back(), q[0])) q.pop_back();
  while(sz(q) > 2 && !check(q[1], q[0], q.back())) q.pop_front();
  return q;
}
```

5.10 MaxAreaPoly

```
ld solve_poly(vi &S) {
  assert(sz(S) > 0);
  int sum = 0, hi = S[0];
  vi vals;
  rep(i, 1, sz(S)) {
    int cur = S[i];
    if (cur > hi) swap(cur, hi);
    sum += cur;
    vals.pb(cur);
  }
  if (sum <= hi) return 0;
  auto getAngle = [&](ld D) -> ld{
    ld tot = 0;
    for (int l : vals) tot += 2 * asin(ld(l) / ld(D));
    return tot;
  };
  bool isReflex = (getAngle(hi) < PI);
  auto tooSmall = [&](ld D) {
    ld ang = getAngle(D);
    ld hiAng = 2 * asin(ld(hi) / ld(D));
    if (isReflex) return ang < hiAng;
    else return ang + hiAng >= 2 * PI;
  };
  ld mi = hi, ma = hi + 1;
  int numExpand = 0;
  while (tooSmall(ma)) numExpand++, ma += (ma - mi);
  rep(tim, 0, 50 + numExpand) {
    ld md = mi + (ma - mi) / 2;
    if (tooSmall(md)) mi = md;
    else ma = md;
  }
  ld D = mi, area = 0;
  for (int l : vals) area += ld(l) * sqrt(ld(D) * ld(D) - ld(l) * ld(l)) / 4;
  ld hiArea = ld(hi) * sqrt(ld(D) * ld(D) - ld(hi) * ld(hi)) / 4;
  if (isReflex) area -= hiArea;
  else area += hiArea;
  return area;
}
```

5.11 MaxAreaTri

```
// O(n ^ 2)
void maxAreaTri(P *p, int n, P &a, P &b, P &c) {
  int i = 0, j = 1, k = 2;
  a = p[i], b = p[j], c = p[k];
  T res = area(a, b, c), cur = res, tmp;
  do {
    while(1) {
      while(cur <= (tmp = area(p[i], p[j], p[(k + 1) % n]))) (++k) %= n, cur = tmp;
      if(cur <= (tmp = area(p[i], p[(j + 1) % n], p[k]))) (++j) %= n, cur = tmp;
      else break;
    }
    if(cur > res) a = p[i], b = p[j], c = p[k], res = cur;
    (++i) %= n;
    if(i == j) (++j) %= n;
    if(j == k) (++k) %= n;
    cur = area(p[i], p[j], p[k]);
  } while(i);
}
```

5.12 MinAreaTri

```
// 无重点、三点共线
// O(n^2log_2n)
struct P { int x, y, ind, u, v; };
namespace MinAreaTri {
  const int N = 2020;
  const ll inf = 4e18;
  int n, m, pos[N];
  P p[N], l[N * N];
  bool cmp(const P &x, const P &y) { return det(x, y) < 0; }
```

```
   void solve() {
      sort(p + 1, p + 1 + n);
      rep(i, 1, n + 1) p[i].ind = i, pos[i] = i;
      m = 0; rep(i, 1, n + 1) rep(j, i + 1, n + 1) {
         l[++m] = p[i] - p[j];
         if(l[m].x < 0) l[m].x *= -1, l[m].y *= -1;
         else if(l[m].x == 0 && l[m].y < 0) l[m].y *= -1;
         l[m].u = i, l[m].v = j;
      }
      sort(l + 1, l + 1 + m, cmp);
      mi = inf, ma = 0;
      rep(i, 1, m + 1) {
         int u = l[i].u, v = l[i].v;
         int pu = pos[u], pv = pos[v];
         if(pu > pv) swap(u, v), swap(pu, pv);
         if(pu == 1 || pv == n) continue;
         mi = min(mi, area(p[pu - 1], p[pu], p[pv + 1], p[v]));
         ma = max(ma, area(p[1], p[pu], p[n], p[v]));
         swap(p[pu], p[pv]);
         swap(pos[u], pos[v]);
      }
      cout << mi << " " << ma << endl;
   }
}
```

5.13 凹四边形计数

```
const int N = 1010;
int n; P p[N], q[N]; ll s[N];
namespace CNT {
   bool gao(P a) { return a.y > 0 || (a.y == 0 && a.x >= 0); }
   bool cmp(P a, P b) {
      bool o = gao(a), t = gao(b);
      if(o != t) return o > t;
      return det(a, b) > 0;
   }
   void solve(int u, ll &ans) {
      rep(i, 1, n + 1) q[i] = p[i]; swap(q[1], q[u]);
      rep(i, 2, n + 1) q[i] = q[i] - p[u];
      sort(q + 2, q + n + 1, cmp);
      int k = n; while(k >= 2 && q[k].y <= 0) --k;
      int j = k, cnt = 0;
      per(i, k + 1, n + 1) {
         while(j >= 2 && det(q[j], q[i]) > 0) --j, ++cnt;
         s[i] = s[i + 1] + cnt;
      }
      int c = j = k + 1;
      rep(i, 2, k + 1) {
         while(c <= n && det(q[i], q[c]) > 0) ++c;
         while(j <= n && det(q[i], q[j]) >= 0) ++j;
         ans += s[j] + (n - j + 1) * 1ll * (c - k - 1);
      }
   }
   ll solve() {
      ll ans = 0; rep(i, 1, n + 1) solve(i, ans);
      return ans;
   }
}
```

5.14 圆反演

```
P fy(P o, db r, P p) { // 点反演
   db d = (o - p).len();
   db r0 = r * r / d;
   return (p - o) * r0 / d + o;
}
C fy1(P o, db r, C c) { // 不过反演中心的圆反演后还是圆
   P p1, p2; isCL(c, L(o, c.o), p1, p2);
   p1 = fy(o, r, p1);
   p2 = fy(o, r, p2);
   return C((p1 + p2) / 2, (p1 - p2).len() / 2);
}
L fy0(P o, db r, C c) { // 过反演中心的圆反演后是直线
   P p = c.o + c.o - o;
   P p1 = fy(o, r, p);
   P p2 = p1 + (o - p1).rot90();
   return L(p1, p2);
}
C fy(P o, db r, L l) { // 不过反演中心的直线反演后是圆
   P p = fy(o, r, proj(l, o));
   return C((p + o) / 2, (p - o).len() / 2);
}
```

5.15 平面图转对偶图

```
struct Planar {
   static const int N = 101010, M = 101010;
   // ps id starts from 0
   vector<P> ps;
   // cnte id starts from 0
   int cnte, ne[M];
   bool vis[M];
   // u -> (v, cnte)
   vector<pii> g[N];
   pii E[M];
   vector<db> areas;

   void init() {
      rep(i, 0, sz(ps)) g[i].clear();
      fill_n(vis, cnte, false);
      ps.clear(); cnte = 0;
      areas.clear();
   }
   void adde(int u, int v) {
      g[u].pb(mp(v, cnte));
      E[cnte++] = mp(u, v);
      g[v].pb(mp(u, cnte));
      E[cnte++] = mp(v, u);
   }
```

```
   int V;
   bool cmp(const pii &i, const pii &j) {
      P a = ps[i.fi] - ps[V], b = ps[j.fi] - ps[V];
      int o = P(0, 0) < a, t = P(0, 0) < b;
      if(o != t) return o < t;
      return det(a, b) > 0;
   }
   void go(int e) {
      db res = 0;
      while(!vis[e]) {
         res += det(ps[E[e].se], ps[E[e].fi]); vis[e] = 1;
         e = ne[e ^ 1];
      }
      if(res > 0) areas.pb(res / 2);
   }
   void solve(const vector<P> &_ps, const vector<pii> &es) {
      init(); ps = _ps;
      for(auto e : es) adde(e.fi, e.se);
      rep(i, 0, sz(ps)) {
         V = i; sort(all(g[i]), cmp);
         rep(j, 0, sz(g[i])) {
            ne[g[i][j].se] = g[i][(j + 1) % sz(g[i])].se;
         }
      }
      rep(i, 0, cnte) if(!vis[i]) go(i);
   }
};
```

5.16 旋转卡壳

```
// 凸包都是顺时针给出
// 【凸包直径】点 - 点
T diameter(vector<P> ps) {
   n = sz(ps); T ans = 0;
   if(n <= 1) return 0;
   if(n == 2) return (ps[1] - ps[0]).len();
   rep(i, 0, n) {
      P t = ps[i] - ps[(i + 1) % n];
      while(det(t, ps[(p + 1) % n] - ps[p]) > 0) (++p) %= n;
      ans = max(ans, (ps[i] - ps[p]).len());
      ans = max(ans, (ps[(i + 1) % n] - ps[p]).len());
   }
   return ans;
}
// 【凸包宽度】点 - 边
// 【凸包间的最大距离】点 - 点
// 【凸包间的最小距离】
T solve(P p[], int n, P q[], int m) {
   int o = 0, t = 0; T ans = inf;
   rep(i, 1, n) if(p[i].y > p[o].y) o = i;
   rep(i, 1, m) if(q[i].y < q[t].y) t = i;
   rep(i, 0, n) {
      P a = p[(o + 1) % n] - p[o]; db tmp;
      while((tmp = det(a, q[(t + 1) % m] - q[t])) < 0) (++t) %= m;
      if(sign(tmp)) ans = min(ans, disToSeg(L(p[o], p[(o + 1) % n]), q[t]));
      else ans = min(ans, disSS(L(p[o], p[(o + 1) % n]), L(q[t], q[(t + 1) % m])));
      (++o) %= n;
   }
   return ans;
}
T work(P p[], int n, P q[], int m) {
   return min(solve(p, n, q, m), solve(q, m, p, n));
}
// 【凸包最小面积外接矩形】
T solve(vector<P> ps) {
   int n = sz(ps); T ans = 1e18;
   int p = 1, l = 1, r;
   rep(i, 0, n) {
      P t = ps[i] - ps[(i + 1) % n];
      while(det(t, ps[(p + 1) % n] - ps[p]) > 0) (++p) %= n;
      while(dot(t, ps[(l + 1) % n] - ps[l]) < 0) (++l) %= n;
      r = (p + 1) % n;
      while(dot(t, ps[(r + 1) % n] - ps[r]) > 0) (++r) %= n;
      ll et = abs(det(ps[p], ps[i], ps[(i + 1) % n]));
      ll ot = abs(dot(t, ps[l] - ps[r]));
      ans = min(ans, (db)et * ot / t.len2());
   }
   return ans;
}
// 【凸包最小周长外接矩形】
```

6 Graph

6.1 2-sat

```
struct TwoSat {
   static const int N = ::N << 1;
   int dfn[N], low[N], id[N], st[N], _st, _, cc;
   vi g[N];
   int mark[N], n;
   void init(int _n) { per(i, 0, (n = _n << 1)) g[i].clear(); }
   int new_node() { rep(i, 0, 2) g[n++].clear(); return n / 2 - 1; }
   /// optionals begin
   void addedge(int a, int va, int b, int vb) { // va 选了 vb 必选
      a = a << 1 | va; b = b << 1 | vb;
      g[a].pb(b); g[b ^ 1].pb(a ^ 1);
   }
   void add_set(int a, int va) { a = a << 1 | va; g[a ^ 1].pb(a); } // va 必选
   void add_then(int a, int va, int b, int vb) { // va 和 vb 不能同时取
      addedge(a, va, b, vb ^ 1);
   }
   void add_or (int a, int va, int b, int vb) { // va 和 vb 不能同时不取
      addedge(a, va ^ 1, b, vb);
   }
   void add_xor(int a, int va, int b, int vb) { // va 和 vb 同时取或同时不取
      addedge(a, va, b, vb);
      addedge(b, vb, a, va);
   }
   // 需要 sz(vu) 个额外的 dp 变量
```

```
void add_at_most_one(vector<pii> vu) {
   int pre = -1;
   rep(i, 0, sz(vu)) {
      int a = vu[i].fi, va = vu[i].se;
      int dpi = new_node();
      addedge(a, va, dpi, 1);
      if (i) {
         addedge(pre, 1, dpi, 1);
         addedge(pre, 1, a, va ^ 1);
      }
      pre = dpi;
   }
}
// optionals end
void dfs(int c, vi g[]){
   dfn[c] = low[c] = ++cc;
   st[_st++] = c;
   for(auto t : g[c])
      if(!dfn[t]) dfs(t, g), low[c] = min(low[c], low[t]);
      else if(!id[t]) low[c] = min(low[c], dfn[t]);
   if(low[c] == dfn[c]){
      ++_;
      do{id[st[--_st]]=_;}while(st[_st] != c);
   }
}
void find(){
   fill_n(dfn, n, cc=0);
   fill_n(low, n, _st=0);
   fill_n(id, n, _=0);
   rep(i, 0, n) if(!dfn[i]) dfs(i, g);
   rep(i, 0, n) --id[i];
   return;
}
bool solve() { // 构造任意解
   find();
   for (int i = 0; i < n; i += 2) {
      if (id[i] == id[i + 1]) return 0;
      mark[i >> 1] = (id[i] > id[i + 1]);
   }
   return 1;
}
int col[N], ans[N], tot;
bool dfs(int u) {
   if (col[u] == -1) return 0;
   if (col[u] ==  1) return 1;
   ans[tot++] = u;
   col[u] = 1; col[u ^ 1] = -1;
   for (auto v : g[u]) if (!dfs(v)) return 0;
   return 0;
}
bool solve2() { // 构造字典序最小解
   for (int i = 0; i < n; i += 2) if (!col[i]) {
      tot = 0;
      if (!dfs(i)) {
         rep(j, 0, tot) col[ans[j]] = col[ans[j] ^ 1] = 0;
         if (!dfs(i ^ 1)) return 0;
      }
   }
   return 1;
}
}ts;
```

6.2 BCC

```
// key contains the id of edges
// _ starts from 0
namespace BCC{
    const int N = 202020;
    vi key , bcc[N];
    int dfn[N] , low[N] , id[N] , st[N] , _st , _;
    void dfs(int c,int dep,vector<pii> g[]){
        int cc=0;st[_st++]=c;
        dfn[c]=low[c]=dep;
        for(auto e:g[c]){
            int t=e.fi;
            if(!dfn[t]){
                dfs(t,dep+1,g);
                low[c]=min(low[c],low[t]);
                if(low[t]>dfn[c]) key.pb(e.se);
            } else if(dfn[t] != dfn[c] - 1 || cc++)
                low[c] = min(low[c] , dfn[t]);
        }
        if(low[c]==dfn[c]){
            do{id[st[--_st]]=_;}while(st[_st]!=c);
            _++;
        }
    }
    int solve(int n,vector<pii> g[]){
        fill_n(dfn,n,_=0);
        fill_n(low,n,_st=0);
        fill_n(bcc,n,key=vi());
        rep(i,0,n) if(!dfn[i]) dfs(i,1,g);
        rep(i,0,n) for(auto j:g[i]) if(id[i]!=id[j.fi])
            bcc[id[i]].pb(id[j.fi]);
        return _;
    }
};
```

6.3 CircleCount

```
struct circle4 {
   static const int N = 1e5 + 7;
   int n, m, u, v, x, y;
   bool vis[N];
   // cnt3,4 中为包含 i 号点的三 , 四元环数量
   ll f[N][5], du[N], D[N], cnt4[N], cnt3[N], cntl[N], t, ans;
   priority_queue<pii> q;
   vi w[N], gg[N], d2, d1;
   set<int> g[N];
```

```
   void dfs(int u, int d, int fa) {
      if (d == 2) { d2.pb(u); w[u].pb(fa); return; }
      if (d == 1) d1.pb(u), vis[u] = 1;
      for (auto v : g[u]) if (v != fa) dfs(v, d+1, u);
   }
   void solve(int n, vi gg[]) {
      rep(i, 1, n+1) {
         D[i] = du[i] = sz(gg[i]); cnt3[i] = cnt4[i] = 0;
         for (auto v : gg[i]) g[i].insert(v);
      }
      rep(i, 1, n+1) q.push(mp(du[i], i));
      rep(i, 1, n+1) {
         rep(j, 1, 5) f[i][j] = 0; f[i][0] = 1;
      }
      rep(i, 1, 5) rep(j, 1, n+1) for (auto v : gg[j]) f[j][i] += f[v][i-1];
      while (!q.empty()) {
         x = q.top().se; y = q.top().fi; q.pop();
         if (du[x] != y) continue;
         dfs(x, 0, -1);
         for (auto u : d2) {
            ll s = sz(w[u]);
            for (auto v : w[u]) {
               cnt4[v] += s - 1;
               if (vis[u]) cnt3[v]++, t++;
            }
            cnt4[x] += s * (s - 1) / 2;
            cnt4[u] += s * (s - 1) / 2;
            w[u].clear();
         }
         for(auto u : d1) vis[u] = 0; d1.clear(); d2.clear();
         cnt3[x] += t / 2; t = 0;
         for (auto u : g[x]) {
            q.push(mp(--du[u], u));
            g[u].erase(x);
         }
      }
      //以第一次产生重复位置分类计数
      rep(i, 1, n+1) { // 计算边数为 4 的链数
         ans = f[i][4];
         //第一次重复为第 4 步
         ans -= cnt4[i] * 2; ll ans3 = f[i][3] - D[i] * D[i] - 2 * cnt3[i];
         for (auto v : gg[i]) ans3 -= D[v] - 1; ans -= ans3; // 边数为 3 的链数
         for (auto v : gg[i]) ans -= 2 * cnt3[v]; ans += 4 * cnt3[i];
         //第一次重复为第 3 步
         ans -= 2 * cnt3[i] * D[i]; for (auto v : gg[i]) ans -= (D[v] - 1) * D[v];
         //第一次重复为第 2 步
         ans -= D[i] * f[i][2];
         cntl[i] = ans;
      }
   }
} c4;
```

6.4 DAG 剖分

```
namespace DAG {
   const int N = ::N, M = 18, lim = 1e9;
   bool sp[N]; ll dp[N]; // init
   int ne[M][N]; ll cnt[M][N]; vector<ll> pre[N];

   void build(vector<pii> g[]) {
      rep(i, 1, n + 1) {
         ne[0][i] = 0; ll md = -1; int col = 0;
         for(auto j : g[i]) if(dp[j.se] > md) md = dp[j.se], ne[0][i] = j.se, col = j.fi
         ;
         cnt[0][i] = sp[i]; pre[i] = vector<ll>(sz(g[i]));
         rep(j, 0, sz(pre[i])) {
            pii t = g[i][j];
            pre[i][j] = (j == 0 ? sp[i] : pre[i][j - 1]) + dp[t.se];
            if(t.fi < col) cnt[0][i] = min((ll)lim, cnt[0][i] + dp[t.se]);
         }
      }
      rep(i, 1, M) rep(j, 1, n + 1) {
         int t = ne[i - 1][j];
         ne[i][j] = ne[i - 1][t];
         cnt[i][j] = min((ll)lim, cnt[i - 1][j] + cnt[i - 1][t]);
      }
   }
   int qry(int k) {
      int ans = 0, u = 1;
      while(1) {
         per(i, 0, M) if(ne[i][u] && cnt[i][u] < k && k <= cnt[i][u] + dp[ne[i][u]]) {
            ans += pw(i);
            k -= cnt[i][u];
            u = ne[i][u];
         }
         if(k == 1 && sp[u]) break;
         int p = lower_bound(all(pre[u]), k) - pre[u].begin();
         k -= (p == 0 ? sp[u] : pre[u][p - 1]);
         u = g[u][p].se;
         ++ans;
      }
      return ans;
   }
}
```

6.5 DCC

```
// cactus: n multi by 2
// key is cuts
// dcc i->j , i(points) , j(bcc_block)
// st is stack
// _st is top of stack
// _ is number of dcc
// can handle isolate point and not connected graph and muti edge
// can handle self circle ?
namespace DCC{
    const int N = 202020;
    vi key , dcc[N];
    int dfn[N] , low[N] , st[N] , _st , _;
```

```
    void dfs(int c,int dep,const vi g[]){
        int cc=0,out=1<dep;st[_st++]=c;
        dfn[c]=low[c]=dep;
        for(auto t:g[c])
            if(!dfn[t]){
                dfs(t,dep+1,g);
                low[c]=min(low[c],low[t]);
                if(low[t]>=dfn[c]){
                    if(++out==2) key.pb(c);
                    while(st[--_st]!=t) dcc[st[_st]].pb(_);
                    dcc[c].pb(_);dcc[t].pb(_++);
                }
            } else if(dfn[t] != dfn[c] - 1 || cc++)
                low[c] = min(low[c] , dfn[t]);
    }
    int solve(int n,const vi g[]){// n is size of points
        fill_n(dfn,n,_=0);
        fill_n(low,n,_st=0);
        fill_n(dcc,n,key=vi());
        rep(i,0,n) if(!dfn[i]) dfs(i,1,g);
        rep(i,0,n) if(sz(dcc[i]) == 0) dcc[i].pb(_++);
        return _;
    }
}
```

6.6 DLX

```
struct DLX{
#define FOR(i, ne, t) for(int i = ne[t]; i != t; i = ne[i])
   static const int N = 2e4 + 8, D = 4, len = 16;
   int n, m, tim, ansd, row[N], col[N], s[N], ans[N], l[N], r[N], u[N], d[N];
   pair<pii, int> pos[N]; string ss[100];
   void init(int _m) {
      m = _m;
      rep(i, 0, m+1) l[i] = i-1, r[i] = i+1, u[i] = d[i] = i;
      l[0] = m, r[m] = 0, tim = m+1;
      rep(i, 0, m+1) s[i] = 0;
   }
   void add(int R, const vi &tmp){
      int first = tim;
      rep(i, 0, sz(tmp)) {
         int c = tmp[i];
         l[tim] = tim-1, r[tim] = tim+1, u[tim] = u[c], d[tim] = c;
         u[c] = tim; d[u[tim]] = tim;
         row[tim] = R, col[tim] = c;
         tim++, s[c]++;
      }
      if (sz(tmp)) l[first] = tim-1, r[tim-1] = first;
   }
   inline void remove(int c) {
      l[r[c]] = l[c]; r[l[c]] = r[c];
      FOR(i, d, c) FOR(j, r, i) u[d[j]] = u[j], d[u[j]] = d[j], --s[col[j]];
   }
   inline void restore(int c) {
      FOR(i, u, c) FOR(j, l, i) u[d[j]] = j, d[u[j]] = j, ++s[col[j]];
      l[r[c]] = c; r[l[c]] = c;
   }
   bool dance(int dep) {
      if (!r[0]) return ansd = dep, 1;
      int c = r[0];
      FOR(i, r, 0) if (s[c] > s[i]) c = i;
      remove(c);
      FOR(i, d, c) {
         ans[dep] = row[i];
         FOR(j, r, i) remove(col[j]);
         if (dance(dep+1)) return 1;
         FOR(j, l, i) restore(col[j]);
      }
      restore(c); return 0;
   }
   vi tmp;
   void ins(int x, int y, int c) {
      n++; pos[n] = mp(mp(x, y), c);
      int p = ((x - 1) / D * D + (y - 1) / D) * len + c;
      tmp[0] = ((x - 1) * len + y);
      tmp[1] = (len * len * 1 + (x - 1) * len + c);
      tmp[2] = (len * len * 2 + (y - 1) * len + c);
      tmp[3] = (len * len * 3 + p);
      add(n, tmp);
   }
   void work() {
      tmp.resize(4);
      while (cin >> ss[1]) {
         n = 0; init(len * len * 4);
         rep(i, 1, len+1) {
            if (i > 1) cin >> ss[i];
            rep(j, 1, len+1) {
               if (ss[i][j-1] == '-') rep(k, 1, len+1) ins(i, j, k);
               else ins(i, j, ss[i][j-1] - 'A' + 1);
            }
         }
         bool ok = dance(1);
         if (ok) {
            rep(i, 1, ansd) {
               //cout << ans[i] << " \n"[i == ansd - 1];
               int p = ans[i], x = pos[p].fi.fi, y = pos[p].fi.se, c = pos[p].se;
               ss[x][y - 1] = c + 'A' - 1;
            }
            rep(i, 1, len+1) cout << ss[i] << endl;
            cout << endl;
         }
      }
   }
} T;
```

6.7 DMST

```
// id starts from 0
// can handle multi edge, self ring
struct edge {int u, v, d, U, V;bitset<1005> b;};
```

```
struct DMST{
   static const int N = ::N , M = N * N , inf = 2e9;
   edge e[M];int n, m, vis[N], pre[N], id[N], index[N], Pre[N];
   bitset<1005> fang;
   int in[N];
   void ini(int n) {this->n = n, m = 0;}
   void addedge(int u, int v, int d) {e[m] = edge({u,v,d,u,v}); e[m].reset();e[m].b[m] =
    1;m++;}
   int run(int root){
      int ans = 0;
      while(1){
         rep(i, 0, n) in[i] = inf;
         rep(i, 0, m){
            int u = e[i].u ,v = e[i].v;
            if(e[i].d < in[v] && u != v) in[v] = e[i].d, pre[v] = u, index[v] = i;
         }
         rep(i, 0, n) {
            if(i == root) continue;
            if(in[i] == inf) return -1;
            fang ^= e[index[i]].b;
         }
         int cnt = 0;in[root] = 0;
         memset(id, -1, sizeof(*id)*n);
         memset(vis, -1, sizeof(*vis)*n);
         rep(i, 0, n){
            ans += in[i]; int v = i;
            int t = index[i];
            while(vis[v] != i && id[v] == -1 && v!=root) vis[v] = i, v = pre[v];
            if(v != root && id[v] == -1) {
               for(int u=pre[v];u != v;u = pre[u]) id[u] = cnt;
               id[v] = cnt++;
            }
         }
         if(cnt == 0) break;
         rep(i, 0, n) if(id[i] == -1) id[i] = cnt++;
         rep(i, 0, m) {
            int v=e[i].v;
            e[i].u = id[e[i].u]; e[i].v = id[e[i].v];
            if(e[i].u != e[i].v) {e[i].d -= in[v];e[i].b ^= e[index[v]].b;}
         }
         n = cnt; root = id[root];
      }
      return ans;
   }
} dmst;
```

6.8 Dinic

```
// [0,n) init!!
// double need eps
template<class T>
struct Dinic{
    const static int N = 10101 , M = N * 10;
    int s , t , n , h[N] , cur[N] , lv[N] , q[N] , e , ne[M] , to[M];
    T cap[M] , flow;
    void liu(int u,int v,T w){ to[e] = v;ne[e] = h[u];cap[e] = w;h[u] = e++;}
    void link(int u,int v,T w){ liu(u , v , w);liu(v , u , 0);}
    void ini(int _n = N) { fill(h , h + (n=_n) , -1);e = 0;}
    bool bfs(){
        int L = 0 , R = 0;
        fill(lv , lv + n , -1);
        lv[q[R++] = s] = 0;
        while(L < R && !~lv[t]){
            int c = q[L++];
            for(int k = h[c]; ~k ; k = ne[k])
                if(cap[k] > 0 && !~lv[to[k]])
                    lv[q[R++] = to[k]] = lv[c] + 1;
        }
        return ~lv[t];
    }
    T dfs(int c,T mx){
        if(c == t) return mx;
        T ret = 0;
        for(int &k = cur[c]; ~k; k = ne[k]){
            if(lv[to[k]] == lv[c] + 1 && cap[k] > 0){
                T flow = dfs(to[k] , min(mx , cap[k]));
                ret += flow;cap[k] -= flow , cap[k^1] += flow;mx -= flow;
                if(!mx) return ret;
            }
        }
        lv[c] = -1;
        return ret;
    }
    T run(int _s,int _t){
        s = _s , t = _t;
        flow = 0;
        while(bfs()){
            copy(h , h + n , cur);
            flow += dfs(s, ~0U>>1);
        }
        return flow;
    }
};
```

6.9 DominatorTree

```
const int N = 1e5 + 7;
vi revg[N], g[N], buf[N], ord;
int stamp, vis[N], dfn[N], fa[N];
int fs[N], mins[N], dom[N], sem[N], buf2[N];
void dfs(int u) {
   vis[u] = stamp; dfn[u] = sz(ord); ord.pb(u);
   for (auto v : g[u]) if (vis[v] != stamp) fa[v] = u, dfs(v);
}
int find(int u) {
   if (u == fs[u]) return u;
   int v = fs[u];
   fs[u] = find(fs[u]);
   if (~mins[v] && dfn[sem[mins[v]]] < dfn[sem[mins[u]]]) mins[u] = mins[v];
   return fs[u];
```

```
}
void mark(int s) {
   ord.clear(); ++stamp; dfs(s);
   for (auto u : ord) fs[u] = u, mins[u] = buf2[u] = -1;
   per(i, 1, sz(ord)) {
      int u = ord[i], p = fa[u]; sem[u] = p;
      for(auto v : revg[u]) if (vis[v] == stamp) {
         if (dfn[v] > dfn[u]) find(v), v = sem[mins[v]];
         if (dfn[v] < dfn[sem[u]]) sem[u] = v;
      }
      buf[sem[u]].pb(u); mins[u] = u; fs[u] = p;
      per(j, 0, sz(buf[p])) {
         int v = buf[p][j]; find(v);
         if (sem[v] == sem[mins[v]]) dom[v] = sem[v]; else buf2[v] = mins[v];
      }
      buf[p].clear();
   }
   dom[ord[0]] = ord[0];
   for (auto u : ord) if (~buf2[u]) dom[u] = dom[buf2[u]];
}
```

6.10 DualMST

对偶图最小生成树，等于平面图所有边边权和减去平面图最大生成树。

6.11 EulerianPath

```
vi ans; bool vis[N]; int p[N];
vector<pii> g[N];
void dfs(int u) {
   for( ; p[u] < sz(g[u]); ++p[u]) {
      auto v = g[u][p[u]];
      if(!vis[abs(v.se)]) {
         vis[abs(v.se)] = 1;
         dfs(v.fi);
         ans.pb(-v.se);
      }
   }
}
```

6.12 FindCircle

```
// 支持基环树森林和自环重边
const int N = 1e5 + 7;
vector<pair<pii>, int> g[N]; // 点编号边权边编号
int tim, dfn[N], fa[N], d[N], k;
vi cir[N];
int ne[N]; // 有向图的出度
int id[N]; // 点属于的环编号
void dfs(int u, int pre) { // pre 为边编号
   dfn[u] = ++tim;
   rep(i, 0, sz(g[u])) {
      if (g[u][i].se == pre) continue;
      int v = g[u][i].fi.fi, w = g[u][i].fi.se;
      if (dfn[v] && dfn[v] <= dfn[u]) {
         k++;
         int p = u; cir[k].pb(p); id[p] = k;
         if (p != v) {do { p = fa[p]; cir[k].pb(p); id[p] = k;
         } while (p != v);}
         if (sz(cir[k]) > 1 && ne[cir[k][0]] != cir[k][1]) reverse(all(cir[k]));
         continue;
      }
      if (!dfn[v]) {fa[v] = u; d[v] = d[u] + w; dfs(v, g[u][i].se);}
   }
}
```

6.13 Gomory-HuTree

```
Dinic<int> G;
struct GHT{
   static const int N = 1e5 + 100, M = 17; // (1 << M) > n
   int id[N], tmp[N], n, f[N][M], h[N][M], dep[N];
   vector<pii> g[N];
   void ini(int _n) { n = _n; G.ini(n + 5); rep(i, 1, n+1) id[i] = i, g[i].clear(); }
   void link(int u, int v, int w) { G.link(u, v, w); G.link(v, u, w);}
   void solve(int l, int r) {
      if (l == r) return;
      int s = id[l], t = id[l+1];
      for(int i = 0; i < G.e; i += 2) G.cap[i] += G.cap[i+1], G.cap[i+1] = 0;
      int w = G.run(s, t);
      g[s].pb(mp(t, w));
      g[t].pb(mp(s, w));
      int cl = l, cr = 0;
      rep(i, l, r+1) {
         if (G.lv[id[i]] != -1) id[cl++] = id[i];
         else tmp[cr++] = id[i];
      }
      rep(i, 0, cr) id[cl + i] = tmp[i];
      solve(l, cl - 1);
      solve(cl, r);
   }
   void dfs(int u, int fa) {
      dep[u] = dep[fa] + 1;
      for (auto v : g[u]) if (v.fi != fa) {
         f[v.fi][0] = u; h[v.fi][0] = v.se;
         rep(i, 1, M) {
            f[v.fi][i] = f[f[v.fi][i-1]][i-1];
            h[v.fi][i] = min(h[v.fi][i-1], h[f[v.fi][i-1]][i-1]);
         }
         dfs(v.fi, u);
      }
   }
   void build() { solve(1, n); dfs(1, 0); }
   int get(int u, int v) { // 注意 long long
      int res = pw(30);
      if (dep[u] < dep[v]) swap(u, v);
      per(i, 0, M) if (dep[f[u][i]] >= dep[v]) res = min(res, h[u][i]), u = f[u][i];
```

```
      per(i, 0, M) if (f[u][i] != f[v][i]) res = min(res, min(h[u][i], h[v][i])), u = f[
      u][i], v = f[v][i];
      if (u != v) res = min(res, min(h[u][0], h[v][0]));
      return res;
   }
} tr;
```

6.14 KM

```
// init!! , id starts from 0
// n <= m

template<class T>
struct KM {
   static const int N = 505;
   static const T inf = ~0U>>2;
   int n, m, left[N], pre[N], used[N];
   T g[N][N], Lx[N], Ly[N], slack[N];
   void ini(int _n, int _m) {
      n = _n, m = _m;
      rep(i, 0, n) rep(j, 0, m) g[i][j] = -inf;
   }
   void go(int now) {
      rep(i, 0, m+1) used[i] = 0, slack[i] = inf;
      left[m] = now;
      int u, v;
      for(u = m; ~left[u]; u = v) {
         used[u] = 1;
         T d = inf;
         rep(i, 0, m) if(!used[i]) {
            T tmp = Lx[left[u]] + Ly[i] - g[left[u]][i];
            if (tmp < slack[i]) slack[i] = tmp, pre[i] = u;
            if (slack[i] < d) d = slack[v = i];
         }
         rep(i, 0, m+1) if (used[i]) Lx[left[i]] -= d, Ly[i] += d;
         else slack[i] -= d;
      }
      for(; u != m; left[u] = left[pre[u]], u = pre[u]);
   }
   T run() {
      fill_n(Lx, n, 0);
      fill_n(Ly, m, 0);
      fill_n(left, m, -1);
      rep(i, 0, n) go(i);
      T ans = 0;
      rep(i, 0, n) ans += Lx[i];
      rep(i, 0, m) ans += Ly[i];
      return ans;
   }
};
```

6.15 Lindstrom_Gessel_Viennot_Lemma

```
/*
 * 对于一张无边权的 DAG 图，给定 n 个起点和对应的 n 个终点，这 n 条不相交路径的方案数为矩阵
 * e(a1,b1),e(a1,b2)...e(a1,bn)
 * e(a2,b1),e(a2,b2)...e(a2,bn)
 * ....
 * ....
 * e(an,b1),e(an,b2)...e(an,bn)
 * 的行列式。

 * 即 M[i][j]=e(ai,bj)
 * e(a,b) 为 a 到 b 的路径方案数
 */
```

6.16 ManhattanDistance

```
(x, y) -> (x + y, x - y)           Manhattan distance -> Chebyshev distance
(x, y) -> (x + y >> 1, x - y >> 1) Chebyshev distance -> Manhattan distance
```

6.17 ManhattanDistanceMST

```
// 曼哈顿最小距离生成树（可以求最大）
// 这份代码处理的区域是 Y 轴右转 45 度
namespace MMST {
   const int N = 101010, inf = 1e9 + 7;
   vector<pair<int, pii> > E;
   vi V;
   // 最大只要把这里所有 mi 改成 ma 就行了
   pii mi[N];
   void init() { rep(i, 1, sz(V) + 1) mi[i] = mp(inf, inf); }
   void upd(int p, pii c) {
      p = sz(V) + 1 - p;
      for( ; p <= sz(V); p += lb(p)) mi[p] = min(mi[p], c);
   }
   pii qry(int p) {
      p = sz(V) + 1 - p;
      pii ans = mp(inf, inf);
      for( ; p >= 1; p ^= lb(p)) ans = min(ans, mi[p]);
      return ans;
   }
   int F(int x) { return lower_bound(all(V), x) - V.begin() + 1; }
   void _solve(vector<pair<pii, int> > v) {
      V.clear();
      rep(i, 0, sz(v)) v[i].fi.se -= v[i].fi.fi, V.pb(v[i].fi.se);
      sort(all(V));
      V.erase(unique(all(V)), V.end());
      sort(all(v));
      reverse(all(v));
      init();
      for(auto u : v) {
         pii t = qry(F(u.fi.se));
         int s = u.fi.fi * 2 + u.fi.se;
         if(t.se != inf) E.pb(mp(t.fi - s, mp(t.se, u.se)));
         upd(F(u.fi.se), mp(s, u.se));
      }
   }
```

```
   void solve(vector<pair<pii, int> > v) {
      _solve(v);
      rep(i, 0, sz(v)) swap(v[i].fi.fi, v[i].fi.se);
      _solve(v);
      rep(i, 0, sz(v)) v[i].fi.fi *= -1;
      _solve(v);
      rep(i, 0, sz(v)) swap(v[i].fi.fi, v[i].fi.se);
      _solve(v);
   }
}
```

6.18 MaxMatch

```
namespace MaxMatch {
   const int N = 1050;
   int link[N], vis[N], use[N], in[N];
   queue<int> Q;
   int dfs(int u, vi g[]) {
      for(auto v : g[u]) {
         if(!vis[v]) {
            vis[v] = 1;
            if(!link[v] || dfs(link[v], g)) { return link[v] = u, 1; }
         }
      }
      return 0;
   }
   int solve(int n, int m, vi g[]) {
      fill_n(link, m+1, 0);
      int ret = 0;
      rep(i, 1, n+1) {
         fill_n(vis, m+1, 0);
         ret += dfs(i, g);
      }
      return ret;
   }
   void MVC(int n, vi g[]) {
      fill_n(vis, n+1, 0);
      per(i, 1, n+1) link[link[i]] = i;
      rep(i, 1, n+1) if (!link[i]) vis[i] = use[i] = 1, Q.push(i);
      while (!Q.empty()) {
         int u = Q.front(); Q.pop();
         if (use[u] == 1) {
            for (auto v : g[u]) {
               use[v] = 2;
               if (!vis[v]) vis[v] = 1, Q.push(v);
            }
         }else {
            int v = link[u];
            use[v] = 1;
            if (!vis[v]) vis[v] = 1, Q.push(v);
         }
      }
      rep(i, 1, n+1) if (link[i] && !use[link[i]]) use[i] = 2;
      return;
   }
}
```

6.19 Max_clique_BK

```
// g[i][i] should be 0
// g[i] is i's edge
// index [0..N)
// O(3 ^ (n / 3))
typedef unsigned long long T;
struct BK {
   static const int N = 100; T g[N];
   inline int ctz(T s){ return s ? __builtin_ctzll(s) : 64;}
   int n, ans;
   void ini(int _n) {
      //per(i, 0, n = _n) g[i] = (1ull << n) - 1 - (1ull << i); }
      n = _n;rep(i, 0, n) g[i] = 0;
      rep(i, 0, n) rep(j, 0, n) if (a[i][j]) g[i] |= 1ull << j;
   }
   void gao(T cur, T can, T ban) {
      if (!can && !ban) { ans = max(ans, __builtin_popcountll(cur)); return; }
      if (!can) return;
      int piv = ctz(can | ban), ret = 0;
      T z = can & ~g[piv];
      for(int u = ctz(z); u < n; u += ctz(z >> (u + 1)) + 1) {
         gao(cur | (1ull << u), can & g[u], ban & g[u]);
         can ^= 1ull << u, ban |= 1ull << u;
      }
   }
   int run() { gao(ans = 0, (1ull << n) - 1, 0); return ans; }
}  bk;
```

6.20 Max_clique_fastest

```
const int N = 130;
typedef bool BB[N];
struct Maxclique {
   const BB *e; int pk, lv; db Tlimit;
   struct ve {int i, d; ve(int i): i(i),d(0) {}}; //ve : Vertex
   struct sc {int a, b; sc(     ): a(0),b(0) {}}; //sc : StepCount
   typedef vector<ve> ves; ves V;                 //ves: Vertices
   typedef vector<int> cc; cc  Q, QMAX;           //cc : ColorClass
   vector<cc> C;
   vector<sc> S;
   Maxclique(BB *conn, int sz, const db tt = 0.025): pk(0), lv(1), Tlimit(tt) {
      rep(i, 0, sz) V.pb(ve(i)); e = conn;
      C.resize(sz + 1);
      S.resize(sz + 1);
   }
   static bool desc_deg(const ve &a, const ve &b) { return a.d > b.d; }
   void ini_col(ves &v) { per(i, 0, sz(v)) v[i].d = min(i, v[0].d) + 1; }
   void set_deg(ves &v) { rep(i, 0, sz(v)){v[i].d = 0; rep(j, 0, sz(v)) v[i].d += e[v[i
   ].i][v[j].i]; } }
   void deg_sort(ves &R) { set_deg(R); sort(all(R), desc_deg); }
```

```
   bool cut1(int pi , cc  &va) { rep(i, 0, sz(va)) if (e[pi][va[i]]) return true; return
    false; }
   void cut2(ves &va, ves &vb) { rep(i, 0, sz(va) - 1) if (e[va.back().i][va[i].i]) vb.
   pb(va[i].i); }
   void co_sort(ves &R) {
      int j = 0, maxno = 1, min_k = max(sz(QMAX) - sz(Q) + 1, 1);
      rep(i, 1, 3) C[i].clear();
      rep(i, 0, sz(R)) {
         int pi = R[i].i, k = 1;
         while (cut1(pi, C[k])) k++;
         if (k > maxno) C[(maxno = k) + 1].clear(); C[k].pb(pi);
         if (k < min_k) R[j++].i = pi;
      }
      if (j > 0) R[j - 1].d = 0;
      rep(k, min_k, maxno + 1) rep(i, 0, sz(C[k])) R[j].i = C[k][i], R[j++].d = k;
   }
   void exp_dyn(ves &R) {  // expand_dyn
      S[lv].a += S[lv - 1].a - S[lv].b;
      S[lv].b  = S[lv - 1].a;
      for (; sz(R); Q.pop_back(), R.pop_back()) {
         if (sz(Q) + R.back().d <= sz(QMAX)) return;
         Q.pb(R.back().i);
         ves Rp; cut2(R, Rp);
         if (sz(Rp)) {
            if ((db) S[lv].a / ++pk < Tlimit) deg_sort(Rp);
            co_sort(Rp); S[lv++].a++;
            exp_dyn(Rp); --lv;
         } else if (sz(Q) > sz(QMAX)) QMAX = Q;
      }
   }
   void mcqdyn(int *mxc, int &sz) { // mcqdyn(int maxclique, int &siz)
      set_deg(V); sort(all(V), desc_deg);
      ini_col(V); rep(i, 0, sz(V) + 1) S[i].a = S[i].b = 0;
      exp_dyn(V); per(i, 0, sz(QMAX)) mxc[i] = QMAX[i];
      sz = sz(QMAX);
   }
};
```

6.21 MinCostMaxFlow

```
// [0,n) , init!! , inf modify
template<class U,class V>
struct MCMF{
    static const int N = 6000, M = 201010;
    int h[N], ing[N], pre[N], to[M], ne[M], e, s, t, n;
    U cap[M]; V dis[N], cost[M];
    void ini(int _n = N){ fill(h , h + (n=_n) , -1);e = 0;}
    void liu(int u,int v,U c,V w){ to[e] = v;ne[e] = h[u];cap[e] = c;cost[e] = w;h[u] =
    e++;}
    void link(int u,int v,U c,V w){ liu(u,v,c,w);liu(v,u,0,-w); }
    bool spfa(){
        queue<int> Q;
        fill(dis, dis+n, inf);
        Q.push(s), ing[s] = 1, dis[s] = 0;
        while(!Q.empty()){
            int c = Q.front(); Q.pop(); ing[c] = 0;
            for(int k = h[c]; ~k; k = ne[k]) if (cap[k] > 0) {
                int v = to[k];
                if(dis[c] + cost[k] < dis[v]){
                    dis[v] = dis[c] + cost[k];
                    pre[v] = k;
                    if(!ing[v]) Q.push(v) , ing[v] = 1;
                }
            }
        }
        return dis[t] != inf;
    }
    U flow; V mincost;
    pair<U, V> run(int _s,int _t){
        s = _s , t = _t;
        flow = mincost = 0;
        while(spfa()){
            U pl = inf; int p, k;
            for(p = t;p != s;p = to[k^1]) pl = min(pl, cap[k = pre[p]]);
            for(p = t;p != s;p = to[k^1]) cap[k = pre[p]] -= pl, cap[k^1] += pl;
         mincost += pl * dis[t];
            flow += pl;
        }
        return mp(flow, mincost);
    }
};
```

6.22 SCC

```
// _ starts from 0
namespace SCC{
    const int N = 100050;
    int dfn[N],low[N],id[N],st[N],_st,_,cc;
    void dfs(int c,vi g[]){
        dfn[c]=low[c]=++cc;
        st[_st++]=c;
        for(auto t:g[c])
            if(!dfn[t]) dfs(t,g),low[c]=min(low[c],low[t]);
            else if(!id[t]) low[c] =min(low[c],dfn[t]);
        if(low[c]==dfn[c]){
            ++_;
            do{id[st[--_st]]=_;}while(st[_st]!=c);
        }
    }
    vi ng[N];
    int solve(int n,vi g[]){
        fill_n(dfn,n,cc=0);
        fill_n(low,n,_st=0);
        fill_n(id,n,_=0);
        rep(i,0,n) if(!dfn[i]) dfs(i,g);
        rep(i,0,n) --id[i];
        fill_n(ng,_,vi());
        rep(i,0,n) for(auto j:g[i]) if(id[i]!=id[j]) ng[id[i]].pb(id[j]);
        return _;
    }
```

```
}
```

6.23 SteinerTree

```
// 要视图的情况使用 spfa, dijstra, 多源 bfs
const int N = 11, M = 10, inf = 0x3f3f3f3f;
int n, m, k, a[N][N], st[N][N], dp[1 << M][N][N], S, ans;
bool use[N][N], vis[1 << M][N][N];
int dx[] = {1, -1, 0, 0};
int dy[] = {0, 0, 1, -1};
queue<pii> q;
struct node {
   int x, y, msk;
   node(int x = 0, int y = 0, int msk = 0):x(x), y(y), msk(msk){}
} now;
pair<node, node> pre[1 << M][N][N];
void spfa(int msk) {
   while (!q.empty()) {
      pii u = q.front(); q.pop();
      int x = u.fi, y = u.se;
      vis[msk][x][y] = 0;
      rep(i, 0, 4) {
         int nx = x + dx[i], ny = y + dy[i], t = msk | st[nx][ny];
         if (nx > n || nx < 1 || ny > m || ny < 1) continue;
         int &z = dp[t][nx][ny], w = dp[msk][x][y] + a[nx][ny];
         if (z > w) {
            z = w, pre[t][nx][ny] = mp(node(x, y, msk), node(x, y, 0));
            if (t == msk && !vis[msk][nx][ny]) {
               vis[msk][nx][ny] = 1;
               q.push(mp(nx, ny));
            }
         }
      }
   }
}
void dfs(node now) {
   pair<node, node> t = pre[now.msk][now.x][now.y];
   node t1 = t.fi, t2 = t.se;
   use[now.x][now.y] = 1;
   if (!t1.x) return;
   dfs(t1);
   if (t2.msk) dfs(t2);
}
int SteinerTree(int n, int m) {
   memset(dp, 0x3f, sizeof(dp));
   rep(i, 1, n+1) rep(j, 1, m+1) {
      cin >> a[i][j];
      if (!a[i][j]) {
         st[i][j] = pw(k);
         dp[pw(k)][i][j] = 0;
         pre[pw(k++)][i][j] = mp(node(0, 0, 0), node(0, 0, 0));
      }
   }
   S = pw(k) - 1;
   rep(msk, 1, S+1) {
      rep(i, 1, n+1)
         rep(j, 1, m+1) {
            if (st[i][j] && !(st[i][j] & msk)) continue;
            int &z = dp[msk][i][j];
            for (int t = msk & (msk - 1); t > 0; t = (t - 1) & msk) {
               int t1 = t | st[i][j], t2 = msk ^ t | st[i][j];
               int w = dp[t1][i][j] + dp[t2][i][j] - a[i][j];
               if (z > w) z = w, pre[msk][i][j] = mp(node(i, j, t1), node(i, j, t2));
            }
            if (z < inf) q.push(mp(i, j)), vis[msk][i][j] = 1;
         }
      spfa(msk);
   }
   ans = inf;
   rep(i, 1, n+1) rep(j, 1, m+1) if (ans > dp[S][i][j])
      ans = dp[S][i][j], now = node(i, j, S);
   dfs(now);
   return ans == inf ? -1 : ans;
}
```

6.24 StoerWagner_O(n3)

```
struct StoerWagner{
   static const int N = 305, INF = 0x3f3f3f3f;
   int n, g[N][N], val[N];
   bool vis[N], use[N];
   void init(int _n) {
      n = _n;
      fill_n(use + 1, n, 0);
      rep(i, 1, n+1) fill_n(g[i] + 1, n, 0);
   }
   void add_edge(int u, int v, int w) { g[u][v] += w; g[v][u] += w; }
   void merge(int u, int v) {
      rep(i, 1, n+1) {
         g[v][i] += g[u][i];
         g[i][v] += g[i][u];
      }
      use[u] = 1;
   }
   int MinimumCutPhase(int cnt, int &s, int &t) {
      fill_n(val + 1, n, 0);
      fill_n(vis + 1, n, 0);
      t = 1;
      while (--cnt) {
         vis[s = t] = 1;
         rep(i, 1, n+1) if (!vis[i] && !use[i]) val[i] += g[t][i];
         int ma = 0;
         rep(i, 1, n+1) if (!vis[i] && !use[i] && val[i] >= ma) ma = val[i], t = i;
         if (!ma) return 0;
      }
      return val[t];
   }
   int solve() {
      int res = INF;
      for (int i = n, s, t; i > 1; --i) {
```

```
        res = min(res, MinimumCutPhase(i, s, t));
        if (res == 0) break;
        merge(s, t);
      }
      return res;
   }
} SW;
```

6.25 StoerWagner_O(nmlog(m))

```
struct StoerWagner{
   static const int N = 3005, M = 100005 * 2, INF = 0x3f3f3f3f;
   int head[N], val[N], e, n, to[M], ne[M], data[M], fa[N], link[N];
   bool vis[N];
   void init(int _n) {
      n = _n;
      fill_n(head + 1, n, -1);
      fill_n(link + 1, n, -1);
      rep(i, 1, n+1) fa[i] = i;
      e = 0;
   }
   void add_edge(int u, int v, int w) {
      to[e] = v; data[e] = w; ne[e] = head[u]; head[u] = e++;
      to[e] = u; data[e] = w; ne[e] = head[v]; head[v] = e++;
   }
   int findset(int u) { return u == fa[u] ? u : fa[u] = findset(fa[u]); }
   void merge(int u, int v) {
      int p = u;
      while (~link[p]) p = link[p];
      link[p] = v;
      fa[v] = u;
   }
   int MinimumCutPhase(int cnt, int &s, int &t) {
      fill_n(val + 1, n, 0);
      fill_n(vis + 1, n, 0);
      priority_queue<pii> q;
      t = 1;
      while (--cnt) {
         vis[s = t] = 1;
         for (int u = s; ~u; u = link[u]) {
            for (int p = head[u]; ~p; p = ne[p]) {
               int v = findset(to[p]);
               if (!vis[v]) q.push(mp(val[v] += data[p], v));
            }
         }
         while (!q.empty() && (vis[q.top().se] || val[q.top().se] != q.top().fi)) q.pop
         ();
         if (q.empty()) return 0;
         t = q.top().se; q.pop();
      }
      return val[t];
   }
   int solve() {
      int res = INF;
      for (int i = n, s, t; i > 1; --i) {
```

6.26 ZKW

```
// [0,n) , init!! , inf modify
template<class U,class V>
struct ZKW{
    static const int N = 1010 , M = 40404;
    int h[N] , ing[N] , v[N] , to[M] , ne[M] , e , s , t , n;
    U cap[M];V dis[N] , cost[M];
    void ini(int _n = N){ fill(h , h + (n=_n) , -1);e = 0;}
    void liu(int u,int v,U c,V w){ to[e] = v;ne[e] = h[u];cap[e] = c;cost[e] = w;h[u] =
    e++;}
    void link(int u,int v,U c,V w){ liu(u,v,c,w);liu(v,u,0,-w); }
    void spfa(){
        queue<int> Q;
        fill(dis,dis+n,inf);
        ing[t] = true , dis[t] = 0;
        Q.push(t);
        while(!Q.empty()){
            int c = Q.front();Q.pop();ing[c] = false;
            for(int k=h[c];~k;k=ne[k]){
                int v = to[k];
                if(cap[k^1] <= 0) continue;
                if(dis[c] + cost[k^1] < dis[v]){
                    dis[v] = dis[c] + cost[k^1];
                    if(!ing[v]) Q.push(v) , ing[v] = true;
                }
            }
        }
    }
    U flow;V mincost;
    bool modlable(){
        V Min = inf;
        rep(c,0,n) if(v[c]) for(int k=h[c];~k;k=ne[k]){
         int t=to[k];
         if(!v[t] && cap[k] > 0) Min = min(Min , dis[t] + cost[k] - dis[c]);
      }
        if(Min == inf) return false;
        rep(i,0,n) if(v[i]) dis[i] += Min;
        return true;
    }
    U dfs(int c,U mx){
        if(c == t) return flow += mx , mincost += mx * dis[s] , mx;
        v[c] = true;U ret = 0;
        for(int k=h[c];~k;k=ne[k]){
            int t = to[k];
            if(!v[t] && cap[k] > 0 && dis[c] - cost[k] == dis[t]){
                U tmp = dfs(t , min(cap[k] , mx - ret));
```

```
                cap[k] -= tmp , cap[k^1] += tmp;
                ret += tmp;
                if(ret == mx) return ret;
            }
        }
        return ret;
    }
    pair<U,V> run(int _s,int _t){
        s = _s , t = _t;
        spfa();
        flow = mincost = 0;
        do do memset(v,0,sizeof(v[0])*n);
            while(dfs(s,inf));
        while(modlable());
        return make_pair(flow , mincost);
    }
};
```

6.27 k 短路

```
// S -> T 可重复经过点的第 K 短路
// time : O(klogk + mlogn) space : O(nlogn)
const int N = 5050, M = 200005, B = 20;
const db eps = 1e-9, inf = 1e16;
db dis[N], w;
bool vis[N], tree[M];
int n, m, S, T, fa[N], st[N], top, u, v;
struct Graph{
   int h[N], ne[M], to[M], e;
   db w[M];
   inline void add(int u, int v, db val){
      ne[++e] = h[u], h[u] = e, to[e] = v, w[e] = val;
   }
} g, rg;
void Dij(){
   rep(i, 1, n+1) dis[i] = inf;
   priority_queue<pair<db, int> > pq;
   pq.push(mp(dis[T] = 0, T));
   while(!pq.empty()){
      int u = pq.top().se; pq.pop();
      if (vis[u]) continue; vis[u] = 1;
      for (int i = rg.h[u]; i; i = rg.ne[i]) {
         int v = rg.to[i];
         if (dis[v] > dis[u] + rg.w[i] + eps) {
            dis[v] = dis[u] + rg.w[i];
            pq.push(mp(-dis[v], v));
         }
      }
   }
}
void dfs(int u){
   st[++top] = u; vis[u] = 1;
   for (int i = rg.h[u]; i; i = rg.ne[i]) {
      int v = rg.to[i];
      if (!vis[v] && fabs(dis[v] - dis[u] - rg.w[i]) <= eps) {
         fa[v] = u; tree[i] = 1;
         dfs(v);
      }
   }
}
int rt[N];
namespace LT{
   int ls[M*B], rs[M*B], ht[M*B], id[M*B], tot;
   db val[M*B];
   inline int newnode(db _val, int _id, int _dis = 0){
      int p = ++tot;
      val[p] = _val, id[p]=_id, ht[p] = _dis;
      ls[p] = rs[p] = 0;
      return p;
   }
   inline int _copy(int ori){
      int p = ++tot;
      val[p] = val[ori], id[p] = id[ori], ht[p] = ht[ori];
      ls[p] = ls[ori], rs[p] = rs[ori];
      return p;
   }
   inline int merge(int a, int b){
      if(!a || !b) return a|b;
      if(val[a] > val[b]) swap(a, b);
      int now = _copy(a);
      rs[now] = merge(rs[now], b);
      if(ht[ls[now]] < ht[rs[now]]) swap(ls[now],rs[now]);
      ht[now] = ht[rs[now]] + 1;
      return now;
   }
   inline void ins(int &rt, db val, int id){ rt = merge(rt, newnode(val, id)); }
}
void build_heap(){
   rep(j, 1, top+1) {
      int u = st[j];
      rt[u] = rt[fa[u]];
      for (int i = g.h[u]; i ; i = g.ne[i]) {
         int v = g.to[i];
         if (!tree[i] && dis[v] < inf) LT::ins(rt[u], dis[v] - dis[u] + g.w[i], v);
      }
   }
}
typedef pair<db, int> pdi;
db E;
inline int calc_K(){
   int ans = 1; E -= dis[S];
   priority_queue<pdi, vector<pdi>, greater<pdi> > pq;
   if (rt[S]) pq.push(mp(dis[S] + LT::val[rt[S]], rt[S]));
   while(!pq.empty()) {
      pdi t = pq.top(); pq.pop();
      db w = t.fi; int u = t.se, o = LT::id[u];
      E -= w; if (E >= 0) ++ans; else return ans;
      int ls = LT::ls[u], rs = LT::rs[u];
      if (rt[o]) pq.push(mp(w + LT::val[rt[o]], rt[o]));
      if (ls) pq.push(mp(w + LT::val[ls] - LT::val[u], ls));
```

```
      if (rs) pq.push(mp(w + LT::val[rs] - LT::val[u], rs));
   }
   return ans;
}
int main(){
   ios::sync_with_stdio(0);
   cin.tie(0);
   cin >> n >> m >> E;
   S = 1; T = n;
   rep(i, 1, m+1) {
      cin >> u >> v >> w;
      g.add(u, v, w);
      rg.add(v, u, w);
   }
   Dij();
   rep(i, 1, n+1) vis[i] = 0;
   dfs(T);
   build_heap();
   cout << calc_K() << endl;
   return 0;
}
```

6.28 仙人掌最短路

```
/*
 * 建出圆方树，选任意圆点作为根，环的根指的是环上深度最小的点。
 * 圆圆边边权不变，圆方边边权是圆点到它所在环的根的最短距离。
 * 如果询问两点的 lca 是圆点， ans = dep[a] + dep[b] - dep[lca]
 * 如果是方点， ans = dep[a] + dep[b] - dep[A] - dep[B] + dis(A, B)
 */
```

6.29 前向星

```
struct Gra {
   static const int N = ::N << 1;
   int L, hd[::N], ne[N], to[N]; ll val[N];
   inline void init(int n) { L = 0; rep(i, 1, n + 1) hd[i] = -1; }
   inline void _add(int u, int v, ll w) { to[L] = v; val[L] = w; ne[L] = hd[u]; hd[u] =
   L++; }
   inline void add(int u, int v, ll w) { _add(u, v, w); _add(v, u, w); }
};
```

6.30 图绝对中心

```
// id : 0 .. n - 1
// time : O(n ^ 3)
// g[i][i] should be 0

const int N = 1e3 + 7, inf = 1e9 + 7;
int n, m, g[N][N], u, v, w;

pii GraphCenter(int n, int g[][N]) {
   static int rk[N][N], d[N][N];
   rep(i, 0, n) rep(j, 0, n) d[i][j] = g[i][j], rk[i][j] = j;
   rep(k, 0, n) rep(i, 0, n) rep(j, 0, n)
      d[i][j] = min(d[i][j], d[i][k] + d[k][j]);
   rep(i, 0, n) sort(rk[i], rk[i] + n, [&](int a, int b) {return d[i][a] < d[i][b];});
   int ret = inf, s1 = -1, s2 = -1;
   db ds1 = 0, ds2 = 0;
   rep(u, 0, n) {
      if (d[u][rk[u][n - 1]] * 2 < ret) {
         ret = d[u][rk[u][n - 1]] * 2;
         s1 = s2 = u;
         ds1 = ds2 = 0;
      }
      rep(v, 0, n) if (g[u][v] != inf) {
         for (int k = n - 1, i = n - 2; i >= 0; --i) {
            int x = rk[u][i], y = rk[u][k];
            if (d[v][x] > d[v][y]) {
               int tmp = d[u][x] + d[v][y] + g[u][v];
               if (tmp < ret) {
                  ret = tmp, s1 = u, s2 = v;
                  ds1 = 0.5 * tmp - d[u][x];
                  ds2 = g[u][v] - ds1;
               }
               k = i;
            }
         }
      }
   }
   cout << ret / 2.0 << endl;
   return mp(s1, s2);
}
```

6.31 完美消除序列

```
const int N = 1e5 + 7, M = 2e6 + 7;
int ans, use[N], col[N], lab[N], vis[N], a[N], e, m, ne[M], h[N], to[M], u, v, n, ma;
vi g[N];

void ins(int p, int v) { ++e; to[e] = v; ne[e] = h[p]; h[p] = e; }
void del(int p) {
   h[p] = ne[h[p]];
   while (!h[ma]) ma--;
}
int solve(){
   cin >> n >> m;
   rep(i, 0, m) {
      cin >> u >> v;
      g[u].pb(v); g[v].pb(u);
   }
   e = ma = 0;  // 完美消除序列
   rep(i, 1, n+1) ins(0, i);
   per(i, 1, n+1) {
      while (1) {
         u = to[h[ma]]; del(ma);
         if (!vis[u]) break;
      }
```

```
      a[i] = u, vis[u] = 1;
      for (auto v : g[u]) {
         ins(++lab[v], v);
         ma = max(lab[v], ma);
      }
   } ans = 0;
   per(i, 1, n+1) {  // 色数
      for (auto v : g[a[i]]) use[col[v]] = i;
      rep(j, 1, n+1) if (use[j] != i) {
         col[a[i]] = j;
         ans = max(ans, col[a[i]]);
         break;
      }
   }
   return ans;
}
```

6.32 带花树

```
// time : O(n^3)
// id : 0 .. n-1
struct blossom {
   static const int N = 5005;
   vi g[N];
   int u, v, n, match[N], q[N], L, R, pred[N], b[N], s, t, newb;
   bool inq[N], inb[N], inp[N];
   void init(int _n) { n = _n; rep(i, 0, n) g[i].clear(); }
   void link(int u, int v) { g[u].pb(v); g[v].pb(u); }
   void push(int u) { q[R++] = u; inq[u] = 1; }
   int pop() { return q[L++]; }
   int LCA(int u,int v) {
      rep(i, 0, n) inp[i]=0;
      while(1) {
         inp[u = b[u]] = 1;
         if (u == s) break;
         u = pred[match[u]];
      }
      while(1) {
         if (inp[v = b[v]]) break;
         v = pred[match[v]];
      }
      return v;
   }
   void ResetTrace(int u) {
      int v;
      while(b[u] != newb) {
         v = match[u];
         inb[b[u]] = inb[b[v]] = 1;
         u = pred[v];
         if(b[u] != newb) pred[u] = v;
      }
   }
   void Blossom(int u,int v) {
      newb = LCA(u,v);
      rep(i, 0, n) inb[i] = 0;
      ResetTrace(u);
      ResetTrace(v);
      if(b[u] != newb) pred[u] = v;
      if(b[v] != newb) pred[v] = u;
      rep(i, 0, n) if (inb[b[i]]) {
         b[i] = newb;
         if (!inq[i]) push(i);
      }
   }
   bool Find(int u) {
      bool found = 0;
      rep(i, 0, n) pred[i] = -1, b[i] = i, inq[i] = 0;
      s = u, t = -1, L = R = 0;
      push(s);
      while(L < R) {
         int u = pop();
         per(i, 0, sz(g[u])) {
            int v = g[u][i];
            if (b[u] != b[v] && match[u] != v)
               if(v == s || (match[v] >= 0 && pred[match[v]] >= 0))
                  Blossom(u, v);
               else if(pred[v] == -1) {
                  pred[v]=u;
                  if (match[v] >= 0) push(match[v]);
                  else return t = v, 1;
               }
         }
      }
      return found;
   }
   void AugmentPath() {
      int u = t, v, w;
      while(u >= 0) {
         v = pred[u], w = match[v];
         match[v] = u, match[u] = v;
         u = w;
      }
   }
   int solve() {
      int res = 0;
      rep(i, 0, n) match[i] = -1;
      // random_shuffle maybe faster
      rep(i, 0, n) if (match[i] == -1) if (Find(i)) AugmentPath();
      rep(i, 0, n) if (match[i] != -1) res++;
      return res / 2;
   }
} G;
```

6.33 最短路矩阵中第 k 小

```
const int N = 2e5 + 7;
vector<pii> g[N]; // ( 边权 , 终点 ) 需要排序
int n, m, k, u, v, w;
struct data { // 距离起点当前点当前扩展过的边编号
   ll w; int st, last, id;
```

```
   data(ll W, int S, int L, int I) { w = W; st = S; last = L; id = I; }
   bool operator < (const data &c) const { return w > c.w; }
};
// 连通图的话 k <= n * (n - 1)
// 复杂度最坏应该是 O( min(nmlogn, k^2logk) ) 正常应该是 O(klogk + nlogn)
ll solve(int n, vector<pii> g[], int k) {
   priority_queue<data> pq;
   set<pii> vis;
   rep(i, 1, n+1) {
      if (sz(g[i])) pq.push(data(g[i][0].fi, i, i, 0));
      vis.insert(mp(i, i));
   }
   while (!pq.empty()) {
      data u = pq.top(); pq.pop();
      int v = g[u.last][u.id].se;
      if (!vis.count(mp(u.st, v))) {
         vis.insert(mp(u.st, v));
         k--; if (k == 0) return u.w;
         if (sz(g[v])) pq.push(data(u.w + g[v][0].fi, u.st, v, 0));
      }
      if (u.id + 1 < sz(g[u.last]))
         pq.push(data(u.w - g[u.last][u.id].fi + g[u.last][u.id + 1].fi, u.st, u.last, u
         .id + 1));
   }
}
```

6.34 生成树计数与欧拉回路方案数

```
// d[][]:
//     i!=j d[i][j]=0
//     i==j d[i][j]=in_deg(i)
// b[][]:
//     from i to j has b[i][j] directed edges
// a[][] = d[][] - b[][]

// 无向图生成树个数： a[][] 任何一个 n-1 阶主子式的绝对值
// 有向图以 i 为根的生成树个数： a[][] 去掉第 i 行第 i 列的行列式的绝对值

int det(int n) { // det(a[1..n-1][1..n-1])
   int ans=1;
   rep(i, 1, n) {
      rep(j, i+1, n) while(a[j][i]) {
         int t = a[i][i] / a[j][i];
         rep(k, i, n) a[i][k] = sub(a[i][k], mul(a[j][k], t)), swap(a[i][k], a[j][k]);
         ans = P - ans;
      }
      if(a[i][i] == 0) return 0;
      ans = mul(ans, a[i][i]);
   }
   return ans;
}

// 有向图要记得判断每个点的出度入度是否相等
// 无向图需要转换成有向图
// tw(G): 以 w 为根的生成树个数
// ec(G) = tw(G) * pi((deg[v] - 1)!)
// ans = ec(G) * deg[w]; 如果求的不是本质不同的，就还需要这个
// 本质相同：    1231341 1341231
// 本质不同：    1231341 1312341
```

6.35 稳定婚姻匹配

```
int mat1[N], mat[N], pos[N];
vi g1[N], g2[N];
queue<int> q;

void match(int n, vi *g, vi *rank) {
   rep(i, 1, n+1) q.push(i), pos[i] = 0, mat[i] = 0;
   while (!q.empty()) {
      int u = q.front(); q.pop();
      int &p = pos[u], v = g[u][p];
      if (!mat[v]) mat[v] = u;
      else if (rank[v][mat[v]] > rank[v][u]) {
         q.push(mat[v]);
         mat[v] = u;
      }else q.push(u);
      p++;
   }
   rep(i, 1, n+1) mat1[mat[i]] = i;
}
```

7 Math

7.1 BerlekampMassey

```
// O(len^2)
// s_{m} = \sum_{j=0}^{m-1}s_{j}*c_{j} 系数直接适配线性递推
vi BM(vi s) {
   vi C(1, 1), B(1, 1);
   int L = 0, m = 1, b = 1;
   rep(n, 0, sz(s)) {
      ll d = 0;
      rep(i, 0, L+1) (d += 1ll * C[i] * s[n-i]) %= P;
      if(d == 0) ++m;
      else {
         vi T = C;
         ll c = P - d * kpow(b, P - 2) % P;
         while(sz(C) < sz(B) + m) C.pb(0);
         rep(i, 0, sz(B)) C[i + m] = add(C[i + m], mul(c, B[i]));
         if(2 * L <= n) L = n + 1 - L, B = T, b = d, m = 1;
         else ++m;
      }
   }
   reverse(all(C));
   rep(i, 0, sz(C)) C[i] = P - C[i];
   return vi(C.begin(), C.end() - 1);
}
```

7.2 Bernoulli

```
// desc : 0^k + 1^k + 2^k + .. + (n-1)^k
// time-ini : O(n^2)
// time-cal : k + log
namespace Bernoulli {
   const int N = 1000;
   int C[N][N], B[N];
   void ini() {
      rep(i, 0, N) C[i][0] = 1;
      rep(i, 0, N) rep(j, 1, i + 1) C[i][j] = add(C[i - 1][j - 1], C[i - 1][j]);
      B[0] = 1;
      rep(i, 1, N) {
         B[i] = 0;
         rep(j, 0, i) B[i] = add(B[i], MOD - mul(C[i + 1][j], B[j]));
         B[i] = mul(B[i], qpow(C[i + 1][i], MOD - 2)) % MOD;
      }
   }
   int cal(int n, int k) {
      int sum = 0;
      rep(i, 0, k + 1) sum = add(sum, mul(C[k + 1][i], mul(B[i], qpow(n, k + 1 - i))));
      return mul(sum, qpow(k + 1, MOD - 2));
   }
};
```

7.3 CRT

```
const int N = 1e5+7;
ll a[N], mod[N];

struct CRT{
   ll M, R;
   void exgcd(ll a, ll b, ll &x, ll &y){
      if (!b) { x = 1; y = 0; return;}
      exgcd(b, a % b, y, x);
      y -= a / b * x;
   }
   ll Inv(ll a, ll mod){
      ll x = 0, y = 0;
      exgcd(a, mod, x, y);
      x %= mod;
      return x < 0 ? x + mod : x;
   }
   ll solve(int n, ll *a, ll *mod){
      M = mod[1], R = a[1];
      rep(i, 2, n+1) {
         ll g = __gcd(M, mod[i]);
         ll inv = Inv(M / g, mod[i] / g);
         if ((a[i] - R) % g) return -1; // 无解
         R += inv * ((a[i] - R) / g) % (mod[i] / g) * M;
         M = M / g * mod[i];
         R = (R % M + M) % M; // 可能为 0 看是否需要是正整数
      }
      return R;
   }
} crt;
```

7.4 EulerPower

```
// a[l] ^ a[l+1] ^ a[l+2] ... ^ a[r] % mod 注意结果要再模 mod
map<int, int> M;
int phi(int n) {
   if (M.count(n)) return M[n];
   int r = n, nn = n;
   for(int i = 2; i * i <= n; i++) if (n % i == 0){
      r = r / i * (i-1);
      while (n % i == 0) n /= i;
   }
   if (n > 1) r = r / n * (n-1);
   M[nn] = r;
   return r;
}
ll Euler_qpow(ll a, ll b, ll mod) {
   ll res = 1; bool ok = (b > 0 && a >= mod);
   while (b) {
      if (b & 1) {
         res = res * a;
         ok |= (res >= mod);
         res %= mod;
      }
      a = a * a;
      ok |= (b > 1 && a >= mod);
      a %= mod;
      b >>= 1;
   }
   return res + mod * ok;
}
ll work(int l, int r, int mod) {
   if (mod == 1) return 1;
   if (l == r) return a[l];
   return Euler_qpow(a[l], work(l+1, r, phi(mod)), mod);
}
```

7.5 FFT

```
const int M = 1 << 17 << 1;
const db pi = acos(-1);

struct vir{
   db r, i;
   vir(db r = 0.0, db i = 0.0) : r(r), i(i){}
   void print() {printf("%f %f\n", r, i);}
   vir operator +(const vir &c) {return vir(r + c.r, i + c.i);}
   vir operator -(const vir &c) {return vir(r - c.r, i - c.i);}
   vir operator *(const vir &c) {return vir(r * c.r - i * c.i, r * c.i + i * c.r);}
} a[M], b[M], w[2][M];

struct FFT{
```

```
   int N, na, nb, rev[M];
   void fft(vir *a, int f){
      vir x, y;
      rep(i, 0, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
      for (int i = 1; i < N; i <<= 1)
         for (int j = 0, t = N/(i<<1); j < N; j += i<<1)
            for (int k = 0, l = 0; k < i; k++, l += t)
               x = w[f][l] * a[j+k+i], y = a[j+k], a[j+k] = y+x, a[j+k+i] = y-x;
      if (f) rep(i, 0, N) a[i].r /= N;
   }
   void work(){
      int d = __builtin_ctz(N);
      rep(i, 0, N) {
         rev[i] = (rev[i>>1] >> 1) | ((i&1) << (d-1));
         w[1][i] = w[0][i] = vir(cos(2*pi*i/N), sin(2*pi*i/N));
         w[1][i].i = -w[1][i].i;
      }
   }
   void doit(vir *a, vir *b, int na, int nb){ // [0, na)
      for (N = 1; N < na + nb - 1; N <<= 1);
      rep(i, na, N) a[i] = vir(0, 0);
      rep(i, nb, N) b[i] = vir(0, 0);
      work(), fft(a, 0), fft(b, 0);
      rep(i, 0, N) a[i] = a[i] * b[i];
      fft(a, 1);
      //rep(i, 0, N) a[i].print();
   }
} fft;
```

7.6 FFTMOD

```
const db PI = acos(-1);
const int M = 1 << 18 << 1;
int na, nb, a[M], b[M];
struct vir{
   db r, i;
   vir(db r = 0.0, db i = 0.0) : r(r), i(i){}
   inline vir operator +(const vir &c) {return vir(r + c.r, i + c.i);}
   inline vir operator -(const vir &c) {return vir(r - c.r, i - c.i);}
   inline vir operator *(const vir &c) {return vir(r * c.r - i * c.i, r * c.i + i * c.r)
   ;}
   inline vir operator !() const {return vir(r, -i);}
   void print() {printf("%lf %lf\n", r, i);}
};
struct FFTMOD{
   static const int M = 1 << 18 << 1;
   int N, L, MASK;
   vir w[M], A[M], B[M], C[M], D[M];
   void FFT(vir p[], int n) {
      for (int i = 1, j = 0; i < n - 1; ++i) {
         for (int s = n; j ^= s >>= 1, ~j & s;);
         if (i < j) swap(p[i], p[j]);
      }
      for (int d = 0; (1 << d) < n; ++d) {
         int m = 1 << d, m2 = m * 2, rm = n >> (d + 1);
         for (int i = 0; i < n; i += m2) {
            for (int j = 0; j < m; ++j) {
               vir &p1 = p[i + j + m], &p2 = p[i + j];
               vir t = w[rm * j] * p1;
               p1 = p2 - t, p2 = p2 + t;
            }
         }
      }
   }
   void doit(int *a, int *b, int na, int nb){
      for (N = 1; N < na + nb - 1; N <<= 1);
      rep(i, 0, na) a[i] = (a[i] % P + P) % P; rep(i, na, N) a[i] = 0;
      rep(i, 0, nb) b[i] = (b[i] % P + P) % P; rep(i, nb, N) b[i] = 0;
      L = 15;  MASK = (1<<L) - 1;
      rep(i, 0, N) w[i] = vir(cos(2 * i * PI / N), sin(2 * i * PI / N));
      rep(i, 0, N) {
         A[i] = vir(a[i] >> L, a[i] & MASK);
         B[i] = vir(b[i] >> L, b[i] & MASK);
      }
      mul(a);
   }
   void mul(int *a) {
      FFT(A, N), FFT(B, N);
      rep(i, 0, N) {
         int j = (N - i) % N;
         vir da = (A[i] - !A[j]) * vir(0, -0.5),
               db = (A[i] + !A[j]) * vir(0.5, 0),
               dc = (B[i] - !B[j]) * vir(0, -0.5),
               dd = (B[i] + !B[j]) * vir(0.5, 0);
         C[j] = da * dd + da * dc * vir(0, 1);
         D[j] = db * dd + db * dc * vir(0, 1);
      }
      FFT(C, N), FFT(D, N);
      rep(i, 0, N) {
         ll  da = (ll)(C[i].i / N + 0.5) % P,
               db = (ll)(C[i].r / N + 0.5) % P,
               dc = (ll)(D[i].i / N + 0.5) % P,
               dd = (ll)(D[i].r / N + 0.5) % P;
         a[i] = ((dd << (L * 2)) + ((db + dc) << L) + da) % P;
      }
   }
} fft;
```

7.7 FFT_fast

```
const int N = 1 << 21;
const double pi=acos(-1.0);
struct vir{
   double a,b;
   vir(double r=0.0,double i=0.0) {a=r,b=i;}
   vir operator +(const vir &o) const{return vir(a+o.a,b+o.b);}
   vir operator -(const vir &o) const{return vir(a-o.a,b-o.b);}
   vir operator *(const vir &o) const{return vir(a*o.a-b*o.b,b*o.a+a*o.b);}
   vir operator *(const double &o) const{return vir(a*o,b*o);}
   vir operator !() const{return vir(a,-b);}
```

```
} x[N|1], y[N|1], z[N|1], w[N|1];
int K;
void fft(vir x[],int k,int v){
   for(int i=0,j=0; i<k; i++){
      if(i>j)swap(x[i],x[j]);
      for(int l=k>>1; (j^=l)<l; l>>=1);
   }
   w[0] = vir(1, 0);
   for(int i=2; i<=k; i<<=1){
      vir g = vir(cos(2*pi/i), (v ? -1 : 1) * sin(2*pi/i));
      for(int j=(i>>1); j>=0; j-=2) w[j] = w[j>>1];
      for(int j=1; j<i>>1; j+=2) w[j] = w[j-1] * g;
      for(int j=0; j<k; j+=i){
         vir *a = x+j, *b = a+(i>>1);
         for(int l=0; l<i>>1; l++){
            vir o = b[l] * w[l];
            b[l] = a[l] - o;
            a[l] = a[l] + o;
         }
      }
   }
   if (v) for(int i=0; i<k; i++) x[i] = vir(x[i].a/k,x[i].b/k);
}
void doit(int *a, int *b, int na, int nb) {
   for(K = 1; K <= na+nb>>1; K <<= 1);
   rep(i, 0, K) x[i] = y[i] = vir(0, 0);
   for(int i=0; i<=na; i++) (i&1 ? x[i>>1].b : x[i>>1].a) = a[i];
   for(int i=0; i<=nb; i++) (i&1 ? y[i>>1].b : y[i>>1].a) = b[i];
   fft(x, K, 0); fft(y, K, 0);
   rep(i, 0, K){
      int j = K-1 & K-i;
      vir tmp = (i&K>>1) ? vir(1, 0) - w[i^K>>1] : w[i] + vir(1, 0);
      z[i] =(x[i]*y[i]*4 - (x[i] - !x[j])*(y[i] - !y[j])*tmp)*0.25;
   }
   fft(z, K, 1);
   rep(i, 0, na+nb+1) a[i] = i&1 ? z[i>>1].b + 0.1 : z[i>>1].a + 0.1;
}
```

7.8 FWT

```
const int P = 1e9 + 7, inv2 = P + 1 >> 1;  // P is odd prime
void FWT(int *a, int len, int o = 1) {  // o=-1 UFWT
   for (int k = 0; 1 << k < len; ++k) rep(i, 0, len) if (~i >> k & 1) {
      int j = i ^ (1 << k), x, y;
      x = (a[i] + a[j]) % P, y = (a[i] - a[j] + P) % P; // xor
      if (o == -1) x = (ll)x * inv2 % P, y = (ll)y * inv2 % P;
      //x = (a[i] + a[j]) % P, y = a[j]; // and
      //if (o == -1) x = (a[i] - a[j] + P) % P;
      //x = a[i], y = (a[i] + a[j]) % P; // or
      //if (o == -1) y = (a[j] - a[i] + P) % P;
      a[i] = x, a[j] = y;
   }
}
```

7.9 FWT_k 进制_xor 版本

```
const int _p = 998244353;
ll add(ll x, ll y) { x += y; return x%_p; }
ll mul(ll x, ll y) { return x*y%_p; }
ll Pow(ll x, ll k) {
   ll ret = 1;
   for (; k; k >>= 1, x = mul(x, x)) if (k & 1) ret = mul(ret, x);
   return ret;
}

template <int K>
struct Num {
   ll a[K];
   Num(int x = 0) { mem(a, 0), a[0] = x; }
   inline Num& operator = (const Num &t) {
      rep(i, 0, K) a[i] = t.a[i];
      return *this;
   }
   inline Num& operator = (int x) { mem(a, 0), a[0] = x; return *this; }
   inline friend Num operator + (const Num &a, const Num &b) {
      Num c;
      rep(i, 0, K) c.a[i] = add(a.a[i], b.a[i]);
      return c;
   }
   inline friend Num operator - (const Num &a, const Num &b) {
      Num c;
      rep(i, 0, K) c.a[i] = add(a.a[i], -b.a[i]);
      return c;
   }
   inline friend Num operator * (const Num &a, const Num &b) {
      Num c;
      rep(i, 0, K) rep(j, 0, K) (c.a[(i + j) % K] += mul(a.a[i], b.a[j])) %= _p;
      return c;
   }
   inline friend Num operator >> (const Num &a, int k) {
      Num c;
      rep(i, 0, K) c.a[(i + k) % K] = a.a[i];
      return c;
   }
   inline friend Num operator ^ (Num x, ll k) {
      Num ret = 1;
      for (; k; k >>= 1, x = x*x) if (k & 1) ret = ret*x;
      return ret;
   }
   inline ll Value() {
      int cnt = K&-K, L = K / cnt; ll ret = add(a[0], -(L > 1)*a[cnt]);
      if (K & 1 ^ 1) ret -= add(a[K >> 1], -(L > 1)*a[(K >> 1) + cnt]), ret %= _p;
      return ret;
   }
   inline void print(string s = "") {
      printf("\n\n\n%s\n", s.c_str());
      rep(i, 0, K) printf("a[%d] => %d\n", i, a[i]);
   }
};
```

```
template <int M, int N, int K>
struct FT {
   Num<K> tmp[M << 1], a[N], b[N]; int t;
   void FWT(Num<K> a[], int S, int n, int op) {
      if (n == 1) return; int L = n / M;
      rep(i, 0, M) FWT(a, S + L*i, n / M, op);
      rep(i, 0, L) {
         rep(j, 0, M) tmp[j] = 0;
         rep(j, 0, M) rep(k, 0, M) {
            t = op*j*k%M, t < 0 ? t += M : 0;
            tmp[j] = tmp[j] + (a[S + L*k + i] >> t);
         }
         rep(j, 0, M) a[S + L*j + i] = tmp[j];
      }
   }
   void Multiply(ll A[], ll B[], int n, ll C[]) {
      rep(i, 0, n) a[i] = A[i], b[i] = B[i];
      FWT(a, 0, n, 1), FWT(b, 0, n, 1);
      rep(i, 0, n) a[i] = a[i] * b[i];
      FWT(a, 0, n, -1); ll inv = Pow(n, _p - 2);
      rep(i, 0, n) C[i] = mul(a[i].Value(), inv), C[i] < 0 ? C[i] += _p : 0;
   }
   int get(int x, int y) {
      int ret = 0, B = 1;
      for (; x || y; x /= M, y /= M, B *= M) ret += (x%M + y%M) % M*B;
      return ret;
   }
   void Multiply_B(ll A[], ll B[], int n, ll C[]) {
      rep(i, 0, n) rep(j, 0, n) (C[get(i, j)] += mul(A[i], B[j])) %= _p;
      rep(i, 0, n) C[i] < 0 ? C[i] += _p : 0;
   }
};
```

7.10 FWT_k 进制_xor 版本_模域

```
const int _p = (int)1e9 + 9, w0 = 115381398ll;
ll add(ll x, ll y) { x += y; return x%_p; }
ll mul(ll x, ll y) { return x*y%_p; }
ll Pow(ll x, ll k) {
   ll ret = 1;
   for (; k; k >>= 1, x = mul(x, x)) if (k & 1) ret = mul(ret, x);
   return ret;
}
template <int N, int K>
struct FT {
   ll tmp[K << 1], a[N], b[N], w[K]; int t;
   void Init(ll w0) { w[0] = 1; rep(i, 1, K) w[i] = mul(w[i - 1], w0); }
   void FWT(ll a[], int S, int n, int op) {
      if (n == 1) return; int L = n / K;
      rep(i, 0, K) FWT(a, S + L*i, n / K, op);
      rep(i, 0, L) {
         rep(j, 0, K) tmp[j] = 0;
         rep(j, 0, K) rep(k, 0, K) {
            t = op*j*k%K, t < 0 ? t += K : 0;
            tmp[j] = add(tmp[j], mul(a[S + L*k + i], w[t]));
         }
         rep(j, 0, K) a[S + L*j + i] = tmp[j];
      }
   }
   void Multiply(ll A[], ll B[], int n, ll C[]) {
      rep(i, 0, n) a[i] = A[i], b[i] = B[i];
      FWT(a, 0, n, 1), FWT(b, 0, n, 1);
      rep(i, 0, n) a[i] = mul(a[i], b[i]);
      FWT(a, 0, n, -1); ll inv = Pow(n, _p - 2);
      rep(i, 0, n) C[i] = mul(a[i], inv), C[i] < 0 ? C[i] += _p : 0;
   }
   int get(int x, int y) {
      int ret = 0, B = 1;
      for (; x || y; x /= K, y /= K, B *= K) ret += (x%K + y%K) % K*B;
      return ret;
   }
   void Multiply_B(ll A[], ll B[], int n, ll C[]) {
      rep(i, 0, n) rep(j, 0, n) (C[get(i, j)] += mul(A[i], B[j])) %= _p;
      rep(i, 0, n) C[i] < 0 ? C[i] += _p : 0;
   }
};
// w0 表示单位根模域表示，默认是 3 进制的，进制要整除模数 -1
/*
1000000009
3 115381398
4 430477711
6 115381399
7 95932470
8 118835338
9 246325263
12 86475609
14 9196980
18 4138593
21 32705801
24 304035978

998244353
4 86583718
7 14553391
8 372528824
14 467509451
*/
```

7.11 FWT_子集卷积

```
const int P = 1e9 + 7, M = 18;
inline int mul(int x, int y) { return (ll)x*y%P; }
inline int add(int x, int y) { return (x += y) >= P ? x - P : x; }

template <int L>
struct Num {
   array<int, L> a;
   inline int& operator [] (int x) { return a[x]; }
   inline int operator [] (int x) const { return a[x]; }
```

```
    inline void clear() { a.fill(0); }
    inline void operator += (const Num &b) {
        rep(i, 0, L) a[i] = add(a[i], b[i]);
    }
    inline void operator -= (const Num &b) {
        rep(i, 0, L) a[i] = add(a[i], P - b[i]);
    }
    inline friend Num operator * (const Num &a, const Num &b) {
        Num<L> c; c.clear();
        rep(i, 0, L) if (a[i])
            for (int j = 0; i + j < L; ++j) if (b[j])
                c[i + j] = add(c[i + j], mul(a[i], b[j]));
        return c;
    }
};

template <class V>
struct Calculator {
    V aa[1 << M], bb[1 << M];
    void fwt(V a[], int len, int o = 1) {  // o=-1 UFWT
        for (int k = 0; 1 << k < len; ++k)
            rep(i, 0, len) if (~i >> k & 1) {
            int j = i ^ (1 << k);
            (o == 1) ? a[j] += a[i] : a[j] -= a[i];
        }
    }
    void mul(V a[], V b[], int len, V c[]) {
        fwt(a, len), fwt(b, len);
        rep(i, 0, len) c[i] = a[i] * b[i];
        fwt(c, len, -1);
    }
    void pow(V a[], int len, int k, V c[]) {
        fwt(a, len);
        rep(i, 0, len) {
            c[i] = a[i];
            rep(j, 0, k - 1) c[i] = c[i] * a[i];
        }
        fwt(c, len, -1);
    }
    void In(int A[], int len, V a[]) {
        rep(i, 0, len) a[i].clear(), a[i][__builtin_popcount(i)] = A[i];
    }
    void Out(V a[], int len, int A[]) {
        rep(i, 0, len) A[i] = a[i][__builtin_popcount(i)];
    }
    void Mul(int A[], int B[], int len, int C[]) {
        In(A, len, aa), In(B, len, bb), mul(aa, bb, len, aa), Out(aa, len, C);
    }
    void Pow(int A[], int len, int k, int C[]) {
        In(A, len, aa), pow(aa, len, k, bb), Out(bb, len, C);
    }
    void ModP(int a[], int len) {
        rep(i, 0, len) a[i] = add(a[i], P);
    }
};

Calculator<Num<M + 1>> T;

/*
 * 集合幂级数用于计算快速子集卷积
 *
 * In 装箱操作，将普通数组封装成集合幂级数
 * Out 拆箱操作，将集合幂级数转化为普通数组
 * Mul 计算子集卷积
 * Pow 计算多重子集自卷积
 * M 为 bit 数，数组范围 [0,2^M-1] , Num 范围 [0,M]
 * 多组数据， L 可改造用以减少计算量
 */
```

7.12 FWT_染色多项式

```
template<class T>
struct Poly{
    static const int N = 30, P = 1e9 + 7;
    T a1[N], b1[N], c[N];
    T add(T a, T b) {a = (a + b) % P; return a < 0 ? a + P : a;}
    T mul(T a, T b) {a = 1ll * a * b % P; return a < 0 ? a + P : a;}
    T kpow(T a, T b) {T r=1;for(;b;b>>=1,a=mul(a,a)) {if(b&1)r=mul(r,a);}return r;}
    void calc(int n, T *a, T *b) {
        fill_n(c, n+1, 0);
        rep(i, 0, n+1) rep(j, 0, 2) c[i+j] = add(c[i+j], mul(a[i], b[j]));
        memcpy(a, c, sizeof(a[0]) * (n+1));
    }
    void solve(int n, T *x, T *y, T *a){ // a[0]*x^0 ... a[n]*x^n
        fill_n(a, n+1, 0);
        rep(i, 0, n+1) {
            fill_n(a1, n+1, 0); a1[0] = 1;
            rep(j, 0, n+1) if (j != i) a1[0] = mul(a1[0], x[i] - x[j]);
            a1[0] = mul(y[i], kpow(a1[0], P - 2));
            rep(j, 0, n+1) if (j != i) {
                b1[0] = -x[j]; b1[1] = 1;
                calc(n, a1, b1);
            }
            rep(j, 0, n+1) a[j] = add(a[j], a1[j]);
        }
    }
};
const int P = 1e9 + 7, M = 20;  int L;
inline int mul(int x, int y) { return (ll)x * y % P; }
inline int add(int x, int y) { if ((x += y) >= P) x -= P; return x < 0 ? x + P : x; }
struct vec {
    int a[M];
    inline int& operator [] (int x) { return a[x]; }
    inline int operator [] (int x) const { return a[x]; }
    inline void clear() { fill_n(a, L, 0); }
    inline void operator += (const vec &b) { rep(i, 0, L) a[i] = add(a[i], b[i]); }
    inline void operator -= (const vec &b) { rep(i, 0, L) a[i] = add(a[i], -b[i]); }
    inline vec operator *= (const vec &b) {
        vec c; c.clear();
        rep(i, 0, L) if (a[i])
```

```
        for (int j = 0; i + j < L; ++j) if (b[j])
          c[i + j] = add(c[i + j], mul(a[i], b[j]));
      return *this = c;
   }
};
struct Cal {
   vec a[1 << M], b[1 << M];
   void fwt(vec a[], int len, int o = 1) {  // o=-1 UFWT
      for (int k = 0; 1 << k < len; ++k)
         rep(i, 0, len) if (~i >> k & 1) {
         int j = i ^ (1 << k);
         o == 1 ? a[j] += a[i] : a[j] -= a[i];
      }
   }
   void pow(int mask[], int len, int k, int ret[]) {
      L = k + 1;
      rep(i, 0, len) a[i].clear(), a[i][__builtin_popcount(i)] = mask[i];
      fwt(a, len), ret[0] = 0;
      rep(j, 1, k + 1) {
          if (j == 1) rep(i, 0, len) b[i] = a[i];
          else rep(i, 0, len) b[i] *= a[i];
          int &t = ret[j] = 0;
          rep(i, 0, len) if (__builtin_parity((len - 1) ^ i))
            t = add(t, -b[i][k]);else t = add(t, b[i][k]);
        }
   }
};
Cal T;
Poly<int> PP;
const int N = 50;
int a[N], mask[1 << M], col[N], ret[N], n, m, u, v, X[N], Y[N];
void solve(int a[], int n) {
    mask[0] = 1; int L = 1 << n;
    rep(i, 1, L) {
        int t = i & -i, k = __builtin_ctz(t);
        mask[i] = mask[i ^ t] & !(i & a[k]);
    }
    T.pow(mask, L, n, col);
    rep(i, 0, n+1) X[i] = i, Y[i] = col[i];
    PP.solve(n, X, Y, ret);
}
int main() {
   cin >> n >> m;
   rep(i, 0, m) {
      cin >> u >> v;
      a[u] |= pw(v);
      a[v] |= pw(u);
   }
   solve(a, n);
   return 0;
}
/*
Graph: link
5 6
0 2
1 3
0 4
1 4
2 4
3 4

Color Ways
{ 0, 0, 0, 12, 144, 720 }

Chromatic Poly
x (x - 1)^2 (x - 2)^2 = {0, 4, -12, 13, -6, 1}

Graph: link
6 9
0 1
1 2
0 3
1 3
2 3
2 4
3 4
0 5
3 5

Color Ways
{ 0, 0, 0, 6, 192, 1620, 7680 }

Chromatic Poly
{0, -16, 48, -56, 32, -9, 1}
*/
```

7.13 Fib

```
// sum(fib[1..n]) + 1=fib[n + 2]
// gcd(fib[n], fib[m]) = fib[gcd(n, m)]
```

7.14 Fraction

```
template<class T>
struct Fra{
   T a, b;
   Fra() : a(0), b(1) {}
   Fra(string s) {
      stringstream ss(s); char c;
      ss >> a >> c >> b;
      *this = Fra(a, b);
   }
   Fra(T c) : a(c), b(1) {}
   Fra(T _a, T _b) {
      T d = __gcd(_a, _b);
      a = _a / d, b = _b / d;
      if(b < 0) a = -a, b = -b;
   }
   Fra operator + (const Fra &c) const { return Fra(a * c.b + b * c.a, b * c.b); }
```

```
    Fra operator - (const Fra &c) const { return Fra(a * c.b - b * c.a, b * c.b); }
    Fra operator * (const Fra &c) const { return Fra(a * c.a, b * c.b); }
    Fra operator / (const Fra &c) const { return Fra(a * c.b, b * c.a); }
    Fra operator * (const T &c) const { return Fra(a * c, b); }
    Fra operator / (const T &c) const { return Fra(a, b * c); }
    bool operator == (const Fra &c) const { return a == c.a && b == c.b; }
    bool operator != (const Fra &c) const { return !(*this == c); }
    void print() { cout << a << "/" << b; }
};
typedef Fra<ll> fll;
```

7.15 GaussDB

```
namespace GaussDB{
    static const int N = 505;
    db a[N][N], x[N]; //增广矩阵和解集
    int free[N], fnum, k, col, p; // 一组合法自由变元
    const db eps = 1e-14;

    void genx(int var) {
        int pre = var; fnum = 0;
        per(i, 0, k) {
            rep(j, 0, var) if (fabs(a[i][j]) > eps) { p = j; break; }
            rep(j, 0, i) if (fabs(a[j][p]) > eps) {
                db t = a[j][p] / a[i][p];
                    rep(l, p, var+1) a[j][l] -= a[i][l] * t;
            }
            rep(j, p+1, pre) free[fnum++] = j, x[j] = (?); pre = p;
            x[p] = a[i][var];
            rep(j, p+1, var) x[p] -= a[i][j] * x[j];
            x[p] /= a[i][p];
        }
        rep(j, 0, pre) free[fnum++] = j;
    }

    int Gauss(int equ, int var){
        for(k = col = 0; k < equ && col < var; ++k, ++col){
            p = k; rep(i, k+1, equ) if(fabs(a[i][col]) > fabs(a[p][col])) p = i;
            if (p != k) rep(j, col, var+1) swap(a[p][j], a[k][j]);
            if(fabs(a[k][col]) < eps) {k--; continue;}
            rep(i, k+1, equ){
                if (fabs(a[i][col]) < eps) continue;
                db t = a[i][col] / a[k][col];
                rep(j, col, var+1) a[i][j] -= a[k][j] * t;
            }
        }
        rep(i, k, equ) if (fabs(a[i][var]) > eps) return -1;//无解
        if(k < var){
            // genx(var);
            return var - k;//自由变元个数
        }
        per(i, 0, var) {
            db t = a[i][var];
            rep(j, i+1, var) if (fabs(a[i][j]) > eps) t -= x[j] * a[i][j];
            x[i] = t / a[i][i];
        }
        return 0;
    }
}
```

7.16 GaussInt

```
namespace GaussInt{
    static const int N = ::N, P = 1e9 + 7;
    int a[N][N], x[N]; //增广矩阵和解集
    int free[N], fnum, k, col, p; // 一组合法自由变元
    int add(int a, int b) {if ((a += b) >= P) a -= P; return a < 0 ? a + P : a;}
    int mul(int a, int b) {return 1ll * a * b % P;}
    int kpow(int a, int b) {int r=1;for(;b;b>>=1,a=mul(a,a)) {if(b&1)r=mul(r,a);}return r
    ;}

    void genx(int var) {
        int pre = var; fnum = 0;
        per(i, 0, k) {
            rep(j, 0, var) if (a[i][j]) { p = j; break; }
            rep(j, 0, i) if (a[j][p]) {
                int t = a[j][p];
                rep(l, p, var+1) a[j][l] = add(a[j][l], -mul(a[i][l], t));
            }
            rep(j, p+1, pre) free[fnum++] = j, x[j] = (?); pre = p;
            x[p] = a[i][var];
            rep(j, p+1, var) x[p] = add(x[p], -mul(a[i][j], x[j]));
        }
        rep(j, 0, pre) free[fnum++] = j;
    }

    int Gauss(int equ, int var){
        for(k = col = 0; k < equ && col < var; ++k, ++col){
            p = k; rep(i, k, equ) if (a[i][col]) {p = i; break;}
            if (p != k) rep(j, col, var+1) swap(a[p][j], a[k][j]);
            if(!a[k][col]) {k--; continue;}
            int inv = kpow(a[k][col], P - 2);
            rep(i, col, var+1) a[k][i] = mul(a[k][i], inv);
            rep(i, k+1, equ) if (a[i][col]) {
                int t = a[i][col];
                rep(j, col, var+1) a[i][j] = add(a[i][j], -mul(a[k][j], t));
            }
        }
        rep(i, k, equ) if (a[i][var]) return -1;//无解
        if(k < var){
            //genx(var);
            return var - k;//自由变元个数
        }
        per(i, 0, var) {
            int t = a[i][var];
            rep(j, i+1, var) if (a[i][j]) t = add(t, -mul(a[i][j], x[j]));
            x[i] = t;
        }
        return 0;
    }
```

```
}
```

7.17 GaussXor

```
//对 2 取模的 01 方程组
namespace Gauss{
   static const int N = 2e3 + 10;
   //有 equ 个方程， var 个变元。增广矩阵行数为 equ 列数为, [0..var]
   bitset<N> a[N]; //增广矩阵 modif
   int x[N]; //解集
   int p, col, k; // k 为增广矩阵的秩
   int free[N], fnum; //一组合法自由变元（多解枚举自由变元可以使用）
   //返回值为 -1 表示无解，为 0 是唯一解，否则返回自由变元个数
   void genx(int msk, int var) {
      rep(i, 0, fnum) x[free[i]] = (msk >> i) & 1;
      per(i, 0, k) {
         rep(j, 0, var) if(a[i][j]) { p = j; break; }
         x[p] = a[i][var];
         rep(j, p+1, var) x[p] ^= (a[i][j] && x[j]);
      }
   }

   int Gauss(int equ, int var){
      fnum = 0;
      for(k = 0, col = 0; k < equ && col < var; k++, col++){
         p = k; rep(i, k, equ) if (a[i][col]) {p = i; break;}
         if (p != k) swap(a[k], a[p]);
         if (!a[k][col]){
            k--; free[fnum++] = col;//这个是自由变元
            continue;
         }
         rep(i, 0, equ) if (i != k && a[i][col]) a[i] ^= a[k];
      }
      rep(i, col, var) free[fnum++] = i;
      rep(i, k, equ) if (a[i][var]) return -1;
      if(k < var) {
         // genx(0, var);
         return var - k;//自由变元个数
      }
      //唯一解，回代
      per(i, 0, var){
         x[i] = a[i][var];
         rep(j, i+1, var) x[i] ^= (a[i][j] && x[j]);
      }
      return 0;
   }
}
```

7.18 LikeEuclid

```
ll cal(ll a,ll b,ll c,ll n) { // sum_{i=0...n-1}floor((a*i+b)/c)
  if(n == 0) return 0;
  return (b/c)*n+(a/c)*n*(n-1)/2+(a%c?cal(c,(a*n+b)%c,a%c,(a%c*n+b%c)/c):0);
}
```

7.19 LinearBasis

```
// 普通集合线性基
const int M=63;
struct LB{
   ll a[M];
   void Clear() { memset(a,0,sizeof(a)); }
   void ins(ll x){
      for(int i=M-1; ~i && x; --i) if (x>>i&1)
         if (a[i]) x^=a[i]; else { a[i]=x; break; }
   }
};
// 可持久化线性基 ( 序列前缀最右线性基 )
const int M=32;
struct LB{
   ll a[M]; int id[M];
   void Clear() { memset(a,0,sizeof(a)); }
   void Copy(const LB &L) { rep(i,0,M) a[i]=L.a[i],id[i]=L.id[i]; }
   void Ins(LB &L,ll x,int no) {
      Copy(L);
      for (int i=M-1; ~i && x; --i) if (x>>i&1) {
         if (!a[i]) a[i]=x,id[i]=no;
         else if (no>id[i]) swap(a[i],x),swap(id[i],no);
         x^=a[i];
      }
   }
   ll Qry(int l,ll x=0) { per(i,0,M) if (id[i]>=l) x=max(x,x^a[i]); return x; }
} B[N];

// 集合线性基求交与查询
const int M=33;
ll tmp[M];
struct LB{
   ll a[M];
   LB() { mem(a,0); }
   void Clear() { mem(a,0); }
   void Copy(LB &A) { rep(i,0,M) a[i]=A.a[i]; }
   // 向 this 中插入 x ，返回  y 在后来插入元素中的投影
   bool I(ll x,ll &y) {
      y=x;
      for(int i=M-1; ~i && x; --i) if (x>>i&1)
         if (a[i]) x^=a[i],y^=tmp[i]; else { a[i]=x,tmp[i]=y; return 1; }
      return 0;
   }
   bool Q(ll x) {
      for(int i=M-1; ~i && x; --i) if (x>>i&1)
         if (a[i]) x^=a[i]; else return 0;
      return 1;
   }
   friend void Intersect(LB &A,LB &B,LB &C) {
      LB AA; ll y,z; AA.Copy(A),C.Clear(); mem(tmp,0);
      per(i,0,M) if (B.a[i]) if (!AA.I(B.a[i],y)) C.I(y,z);
   }
   // 化为最简型，方便线性空间的  hash
   void build () { per(i,0,M) per(j,0,i) a[i]=min(a[i],a[i]^a[j]); }
```

```
};
```

7.20 LinearRecursion

```
// a_{m} = \sum_{j=0}^{~m1}a_{j}*c_{j} O(m^2lgn)
int linear_recurrence(ll n, int m, vi a, vi c) {
   if (n<m) return (a[n]+P)%P;
   vector<ll> v(m, 0), u(m<<1, 0);
   v[0] = 1;
   for(ll x = 0, W = n ? 1ll<<(63 - __builtin_clzll(n)) : 0; W; W >>= 1, x <<= 1) {
      fill(all(u), 0);
      int b = !!(n & W); if(b) x++;
      if(x < m) u[x] = 1;
      else {
         rep(i, 0, m) rep(j, 0, m) (u[i + b + j] += v[i] * v[j]) %= P;
         per(i, m, 2*m) rep(j, 0, m) (u[i - m + j] += c[j] * u[i]) %= P;
      }
      copy(u.begin(), u.begin() + m, v.begin());
   }
   ll ans = 0;
   rep(i, 0, m) (ans += v[i] * a[i]) %= P;
   return (ans+P)%P;
}
```

7.21 MathFunction

```
const int N = 1e6 + 7;
int n, M, f[N], g[N], h[N], phi[N], u[N], p[N];
// f[n] 为 n 的最小质因子 ; g[n]=f[n]^k; phi[n] 为欧拉函数 ; u[n] 为莫比乌斯函数 ; h[n] 为一般积性函数

void prime(int n) {
   u[1]=phi[1]=1,h[1]=(0); // 1 的时候特判
   rep(i, 2, n+1) {
      if (!f[i]) {
         p[++M]=i;
         f[i] = g[i] = i;
         phi[i] = i - 1;
         u[i] = -1;
         h[i] = (0);
      } // 质数的时候特判
      for (int j = 1, k; j <= M && p[j] <= f[i] && i * p[j] <= n; j++){
         f[k = i * p[j]] = p[j];
         if (p[j] < f[i]) {
            g[k] = p[j];
            phi[k] = phi[i] * phi[p[j]];
            u[k] = u[i] * u[p[j]];
            h[k] = h[i] * h[p[j]];
         }else {
            g[k] = g[i] * p[j];
            phi[k] = phi[i] * p[j];
            u[k] = 0;
            h[k] = h[i / g[i]] * (0);
         } /* 质数次幂特判 */
         // phi[i*p[j]]=phi[i]*(p[j]<f[i]?phi[p[j]]:p[j]);
         // u[i*p[j]]=u[i]*(p[j]<f[i]?u[p[j]]:0);
      }
      /*phi[i*j]=phi[i]*phi[j] (gcd(i,j)=1)
               phi[i]*j (j|i)
        u[i*j]=u[i]*u[j] (gcd(i,j)=1)
               0 (j|i)
      */
   }
}
```

7.22 NTT

```
const int M = 1 << 17 << 1;
int a[M], b[M];

struct NTT{
   static const int G = 3, P = 1004535809; //P = C*2^k + 1
   int N, na, nb, w[2][M], rev[M];
   ll kpow(ll a, int b){
      ll c = 1;
      for (; b; b >>= 1,a = a * a % P) if (b & 1) c = c * a %P;
      return c;
   }
   void FFT(int *a, int f){
      rep(i, 0, N) if (i < rev[i]) swap(a[i], a[rev[i]]);
      for (int i = 1; i < N; i <<= 1)
         for (int j = 0, t = N / (i << 1); j < N; j += i << 1)
            for (int k = 0, l = 0, x, y; k < i; k++, l += t)
               x = (ll) w[f][l] * a[j+k+i] % P, y = a[j+k], a[j+k] = (y+x) % P, a[j+k+i]
                 = (y-x+P) % P;
      if (f) for (int i = 0, x = kpow(N, P-2); i < N; i++) a[i] = (ll)a[i] * x % P;
   }
   void work(){
      int d = __builtin_ctz(N);
      w[0][0] = w[1][0] = 1;
      for (int i = 1, x = kpow(G, (P-1) / N), y = kpow(x, P-2); i < N; i++) {
         rev[i] = (rev[i>>1] >> 1) | ((i&1) << (d-1));
         w[0][i] = (ll)x * w[0][i-1] % P, w[1][i] = (ll) y * w[1][i-1] % P;
      }
   }
   void doit(int *a, int *b, int na, int nb){ // [0, na)
      for (N = 1; N < na + nb - 1; N <<= 1);
      rep(i, na, N) a[i] = 0;
      rep(i, nb, N) b[i] = 0;
      work(), FFT(a,0), FFT(b,0);
      rep(i, 0, N) a[i] = (ll)a[i] * b[i] % P;
      FFT(a, 1);
      //rep(i, 0, N) cout << a[i] << endl;
   }
} ntt;
```

7.23 Polya

```
/*
 * Burnside's lemma
 * 首先列出所有可能的染色方案，然后找出每个置换下保持不变的方案（不动点）数。
 * 等价类数目：所有置换的不动点数的平均值。
 * Polya enumeration theorem
 * 一个循环的颜色需相同
 */
```

7.24 Rho

```
using uint128 = __uint128_t;
using ull = unsigned long long;
using ll = long long;
using uint = unsigned int;
using pli = pair<ull, uint>;

namespace prime {
#define eb emplace_back
   ull mod;
   inline ull mul(ull a, ull b) {
      if (mod < int(2e9)) return a * b % mod;
      ull k = ((long double)a * b / mod + 0.1);
      ull res = a * b - k * mod;
      if ((ll)res < 0) res += mod;
      return res;
   }
   inline ull add(ull a, ull b) {
      if ((a += b) >= mod) a -= mod;
      return a;
   }
   inline ull kpow(ull a, ull b) {
      ull res = 1;
      for (; b; a = mul(a, a), b >>= 1) if (b & 1) res = mul(res, a);
      return res;
   }
   inline ull sqr(ull x) { return x * x; }
   bool composite(ull n, const uint* base, int m) {
      int s = __builtin_ctzll(n - 1);
      ull d = (n - 1) >> s; mod = n;
      for (int i = 0, j; i < m; ++i) {
         ull a = kpow(base[i], d);
         if (a == 1 || a == n - 1) continue;
         for (j = s - 1; j > 0; --j) { a = mul(a, a); if (a == n - 1) break; }
         if (j == 0) return 1;
      }
      return 0;
   }
   bool is_prime(ull n) { // reference: http://miller-rabin.appspot.com
      assert(n < (ull(1) << 63));
      static const uint base[][7] = {
         {2, 3},
         {2, 299417},
         {2, 7, 61},
         {15, 176006322, 4221622697u},
         {2, 2570940, 211991001, 3749873356u},
         {2, 2570940, 880937, 610386380, 4130785767u},
         {2, 325, 9375, 28178, 450775, 9780504, 1795265022}
      };
      if (n <= 1) return 0;
      if (!(n & 1)) return n == 2;
      if (n <= 8) return 1;
      int x = 6, y = 7;
      if (n < 1373653) x = 0, y = 2;
      else if (n < 19471033) x = 1, y = 2;
      else if (n < 4759123141) x = 2, y = 3;
      else if (n < 154639673381) x = y = 3;
      else if (n < 47636622961201) x = y = 4;
      else if (n < 3770579582154547) x = y = 5;
      return !composite(n, base[x], y);
   }
   vector<uint> primes;
   void init(uint n) {
      uint sq = sqrt(n);
      vector<bool> isp(n + 1, 1);
      primes.clear();
      for (uint i = 2; i <= sq; ++i) if (isp[i]) {
         if (i != 2) primes.pb(i);
         for (uint j = i * i; j <= n; j += i) isp[j] =  0;
      }
   }
   ull brent(ull n, ull c) { // n must be composite and odd.
      const ull s = 256;
      ull y = 1; c %= n; mod = n;
      for (ull l = 1; ; l <<= 1) {
         ull x = y;
         for (int i = 0; i < l; ++i) y = add(mul(y, y), c);
         ull p = 1;
         for (int k = 0; k < l; k += s) {
            ull sy = y;
            for (int i = 0; i < min(s, l - k); ++i) {
               y = add(mul(y, y), c);
               p = mul(p, add(y, n - x));
            }
            ull g = __gcd(n, p);
            if (g == 1) continue;
            if (g == n) for (g = 1, y = sy; g == 1; ) y = add(mul(y, y), c), g = __gcd(n
            , add(y, n - x));
            return g;
         }
      }
   }
   vector<pli> factors(ull n) {
      assert(n < (1ull << 63));
      if (n <= 1) return {};
      vector<pli> ret;
      if (!(n & 1)) {
         uint e = __builtin_ctzll(n);
         ret.eb(2, e);
         n >>= e;
```

```
    }
    ull lim = sqr(primes.back());
    for (auto &&p: primes) {
      if (sqr(p) > n) break;
      uint e = 0;
      while (n % p == 0) n /= p, e++;
      if (e) ret.eb(p, e);
    }
    uint s = sz(ret);
    while (n > lim && !is_prime(n)) {
      for (ull c = 1; ; ++c) {
        ull p = brent(n, c);
        if (!is_prime(p)) continue;
        uint e = 1; n /= p;
        while (n % p == 0) n /= p, e++;
        ret.eb(p, e);
        break;
      }
    }
    if (n > 1) ret.eb(n, 1);
    if (sz(ret) - s >= 2) sort(ret.begin() + s, ret.end());
    return ret;
  }
}
```

7.25 Simplex

```
const db EPS = 1e-8, DINF = 1e15;
struct Simplex {
  static const int M = 550;
  int n, m, B[M], N[M];
  db v, ans[M], b[M], c[M], A[M][M]; // 全幺模矩阵可以改整数
  /* n - variables, m - equations
   * maxf(x)=cx
   * s.t.Ax<=b,x>=0
   */
  void init(int _n, int _m) {
    n = _n, m = _m, v = 0;
    rep(i, 1, n + 1) N[i] = i;
    rep(i, 1, m + 1) B[i] = i + n;
  }
  inline int sgn(db x) { return (x > EPS) - (x < -EPS); }
  void pivot(int l, int e) {
    db tmp = A[l][e];
    b[l] /= tmp, A[l][e] = 1 / tmp;
    rep(i, 1, n + 1) if (i != e) A[l][i] /= tmp;
    rep(i, 1, m + 1) if (i != l && sgn(A[i][e])) {
      b[i] -= A[i][e] * b[l];
      rep(j, 1, n + 1) A[i][j] -= (j!=e) * A[i][e] * A[l][j]; // 可以链式优化
      A[i][e] = - A[i][e] / tmp;
    }
    rep(i, 1, n + 1) c[i] -= (i!=e) * c[e] * A[l][i];
    v += b[l] * c[e]; c[e] *= -A[l][e]; swap(B[l], N[e]);
  }
  bool ini(){ // 随机化初始解
    while(1){
      int l = -1, e = -1;
      rep(i, 1, m+1) if (sgn(b[i]) < 0 && (l == -1 || (rand() & 1))) l = i;
      if(l == -1) break;
      rep(j, 1, n+1) if (sgn(A[l][j]) < 0 && (e == -1 || (rand() & 1))) e = j;
      if(e == -1) return 0;
      pivot(l, e);
    }
    return 1;
  }
  db run() {
    //if (!ini()) return -DINF; // 无解  b < 0 need ini()
    rep(i, 1, n+1) ans[i] = 0;
    while (1) {
      int r, l, e = -1;
      db delt = -DINF;
      rep(j, 1, n + 1) if (sgn(c[j]) > 0) { // 找非基变量
        db tmp = DINF;
        rep(i, 1, m + 1) if (sgn(A[i][j]) > 0 && b[i] / A[i][j] < tmp) // 找基变量
          r = i, tmp = b[i] / A[i][j];
        if (tmp == DINF) return DINF; // 无界
        if (delt < tmp * c[j]) l = r, e = j, delt = tmp * c[j];
        break;
        // 贪心取最大如果矩阵为全幺模或  0 很多可以加上 break 因为转轴代价可能较小
      }
      if (e == -1) break; // 找到最优解
      pivot(l, e);
    }
    rep(i, 1, m+1) if (B[i] <= n) ans[B[i]] = b[i];
    return v;
  }
} sp;
```

7.26 Simpson

```
namespace Simpson {
  const db eps = 1e-10; // 精度感觉一般要多设 1e-3 左右
  inline db F(db x) { F(x) = (?) }
  inline db simpson(db fa, db fb, db fc, db a, db c) {
    return (fa + 4 * fb + fc) * (c - a) / 6;
  }
  db asr(db a, db b, db c, db esp, db A, db fa, db fb, db fc) {
    db ab = (a + b) / 2, bc = (b + c) / 2;
    db fab = F(ab), fbc = F(bc);
    db L = simpson(fa, fab, fb, a, b), R = simpson(fb, fbc, fc, b, c);
    if (fabs(L + R - A) <= 15 * esp) return L + R + (L + R - A) / 15.0;
    return asr(a, ab, b, esp / 2, L, fa, fab, fb) + asr(b, bc, c, esp / 2, R, fb, fbc,
     fc);
  }
  // f(a, c)
  db asr(db a, db c, db eps) {
    db b = (a + c) / 2;
    db fa = F(a), fb = F(b), fc = F(c);
    return asr(a, b, c, eps, simpson(fa, fb, fc, a, c), fa, fb, fc);
  }
```

```
}
```

7.27 SternBrocotTree

```
namespace SBT {
   typedef long double db;
   typedef int U;
   typedef pair<U, U> pii;
   const U INF = 1e9 + 7;
   typedef __int128 T;
   typedef pair<T, T> V; // V = [double|long double|fraction]
   inline int cmp(const V &a, const V &b) {
      T x = a.fi * b.se - a.se * b.fi;
      return (x > 0) - (x < 0);
   }
   inline bool in(const V &a, const V &b, const V &c) {
      return 0 <= cmp(c, a) && cmp(c, b) < 0;
   }
   pii operator+(const pii &a, const pii &b) { return mp(a.fi + b.fi, a.se + b.se); }
   pii operator*(const pii &a, U x) { return mp(a.fi * x, a.se * x); }
   bool search(V v, U MAXB, pii &lo, pii &hi, int f) {
      V x;
      U l = 0, r = f > 0 ? (hi.se ? (MAXB - lo.se) / hi.se : INF) :
         (lo.se ? (MAXB - hi.se) / lo.se : INF);
      while (l + 1 < r) {
         U z = (l + r) >> 1;
         x = f > 0 ? lo + hi * z : lo * z + hi;
         f * cmp(x, v) <= 0 ? l = z : r = z;
      }
      x = f > 0 ? lo + hi * r : lo * r + hi;
      r = f * cmp(x, v) <= 0 ? r : l;
      f > 0 ? lo = lo + hi * r : hi = lo * r + hi;
      return r > 0;
   }
   pii solve(V v, U MAXB) { // find ROUND_HALF_UP(a / b) = v, b <= MAXB
      V L = mp(v.fi * 10 - 5, v.se * 10);
      V R = mp(v.fi * 10 + 5, v.se * 10);
      pii lo(0, 1), hi(1, 0);
      while (true) {
         bool ok = 0;
         //V m = mp(lo.fi + hi.fi, lo.se + hi.se);
         //if (in(L, R, m)) return mp(m.fi, m.se);
         ok |= search(v, MAXB, lo, hi, 1);
         ok |= search(v, MAXB, lo, hi, -1);
         if (!ok) break;
      }
      db t1 = (db) lo.fi / lo.se;
      db t2 = (db) hi.fi / hi.se;
      db t3 = (db) v.fi / v.se;
      if (t2 - t3 <= t3 - t1) return hi;else return lo;
      //if (in(L, R, lo)) return lo;
      //if (in(L, R, hi)) return hi;
      return mp(-1, -1);
   }
};
```

7.28 min_25

```
struct Min_25{
   // F(i) 要拆成多个完全积性函数的和
   // 或者 F(i) 的质数位置前缀和能通过埃氏筛法 dp 求出，如求模 4 余 1 的质数个数
   static const int N = 1e6 + 7;
   int Sqr, m, p[N], id1[N], id2[N], tot, cntp;
   ll g[N], sp[N], h[N], n, w[N];
   bool isp[N];
   // f(p) = p ^ k
   ll f(int p) { return 1;}

   // 要求的积性函数 F(p ^ e)
   ll F(int p, int e) { return e == 1 ? -1 : 0;}

   // 假设都是质数的完全积性函数前缀和去掉 f(1)
   ll calc(ll n) { return n - 1;}

   void prime(int n){
      cntp = 0;isp[1] = 1;
      rep(i, 2, n+1) {
         if(!isp[i]) p[++cntp] = i;
         for(int j = 1; j <= cntp && i * p[j] <= n; j++){
               isp[i * p[j]] = 1;
               if(i % p[j] == 0)break;
         }
      }
      rep(i, 1, cntp+1) sp[i] = sp[i - 1] + f(p[i]);
      p[++cntp] = INT_MAX;
   }

   // S[x][y] 表示 [2, x] 中最小质因子大于等于 p[y] 的 F(i) 的和

   ll S(ll x, int y){
      if(x <= 1 || p[y] > x) return 0;
      int k = (x <= Sqr ? id1[x] : id2[n/x]);
      ll ret = -(g[k] - sp[y-1]);// 质数的答案
      for(int i = y; i <= tot && 1ll * p[i] * p[i] <= x; i++){
         ll t1 = p[i], t2 = 1ll * p[i] * p[i];
         for(int e = 1; t2 <= x; e++, t1 = t2, t2 *= p[i]) {
            if (F(p[i], e)) ret += S(x / t1, i + 1) * F(p[i], e);
            ret += F(p[i], e + 1);// 合数的答案
         }
      }
      return  ret;
   }

   // g[i] 表示 [2, w[i]] 中质数位置 f(i) 的和

   ll solve(ll _n) {
      n = _n;if (n == 0) return 0;
      m = 0;Sqr = sqrt(n);
      tot = upper_bound(p + 1, p + cntp + 1, Sqr) - (p + 1);
```

```
      for(ll i = 1, j; i <= n; i = j + 1){
         j = n / (n / i);
         w[++m] = n / i;
         g[m] = calc(w[m]);
         w[m] <= Sqr ? id1[w[m]] = m : id2[j] = m;
      }
      rep(j, 1, tot + 1)
         for(int i = 1; i <= m && 1ll * p[j] * p[j] <= w[i]; i++){
            ll t = w[i] / p[j];
            int k = t <= Sqr ? id1[t] : id2[n / t];
            g[i] -= f(p[j]) * (g[k] - sp[j - 1]);
         }
      return S(n,1) + 1;
   }
} _U;
```

7.29 ploynomial

```
template<class T>
struct polynomial{
   static const int N = 101010;
   static const int P = 998244353;
   T a1[N], b1[N], c[N], a[N], pre[N], suf[N], ifac[N], fac[N];
   T add(T a, T b) {a = (a + b) % P; return a < 0 ? a + P : a;}
   T mul(T a, T b) {a = 1ll * a * b % P; return a < 0 ? a + P : a;}
   T kpow(T a, T b) {T r=1;for(;b;b>>=1,a=mul(a,a)) {if(b&1)r=mul(r,a);}return r;}
   void calc(int n, T *a, T *b) {
      fill_n(c, n+1, 0);
      rep(i, 0, n+1) rep(j, 0, 2) c[i+j] = add(c[i+j], mul(a[i], b[j]));
      memcpy(a, c, sizeof(a[0]) * (n+1));
   }
   void solve(int n, T *x, T *y){ // a[0]*x^0 ... a[n]*x^n
      fill_n(a, n+1, 0);
      rep(i, 0, n+1) {
         fill_n(a1, n+1, 0); a1[0] = 1;
         rep(j, 0, n+1) if (j != i) a1[0] = mul(a1[0], x[i] - x[j]);
         a1[0] = mul(y[i], kpow(a1[0], P - 2));
         rep(j, 0, n+1) if (j != i) {
            b1[0] = -x[j]; b1[1] = 1;
            calc(n, a1, b1);
         }
         rep(j, 0, n+1) a[j] = add(a[j], a1[j]);
      }
   }
   T get(int n, int k, T *x, T *y) { // f(k)
      T res = 0;
      rep(i, 0, n+1) {
         T s1 = y[i], s2 = 1;
         rep(j, 0, n+1) if (j != i) s1 = mul(s1, k - x[j]);
         rep(j, 0, n+1) if (j != i) s2 = mul(s2, x[i] - x[j]);
         res = add(res, mul(s1, kpow(s2, P - 2)));
      }
      return res;
   }
   T get(int n, int k, T *y) { // x is [1..n]
      fac[0] = 1;rep(i, 1, n+1) fac[i] = mul(fac[i-1], i);
      ifac[n] = kpow(fac[n], P - 2);
      per(i, 0, n) ifac[i] = mul(ifac[i+1], i+1);
      pre[0] = suf[n+1] = 1;
      rep(i, 1, n+1) pre[i] = mul(pre[i-1], k - i);
      per(i, 1, n+1) suf[i] = mul(suf[i+1], k - i);
      T ans=0;
      rep(i, 1, n+1){
         T s1 = mul(pre[i-1], suf[i+1]);
         T s2 = mul(ifac[i-1], ifac[n-i]);
         T fg = (n-i)&1 ? -1 : 1;
         ans = add(ans, mul(fg*s1, mul(s2, y[i])));
      }
      return ans;
   }
};
```

7.30 polysum

```
struct polysum {
   static const int D = 101000, P = 998244353;
   ll a[D], fac[D], ifac[D], p1[D], p2[D], h[D][2], C[D];
   ll add(ll a, ll b) {a = (a + b) % P; return a < 0 ? a + P : a;}
   ll mul(ll a, ll b) {a = 1ll * a * b % P; return a < 0 ? a + P : a;}
   ll kpow(ll a, ll b) {ll r=1;for(;b;b>>=1,a=mul(a,a)) {if(b&1)r=mul(r,a);}return r;}
   void init(int M) {
      fac[0] = 1; rep(i, 1, M+5) fac[i] = mul(fac[i-1], i);
      ifac[M+4] = kpow(fac[M+4], P - 2);
      per(i, 0, M+4) ifac[i] = mul(ifac[i+1], i+1);
   }
   ll calcn(int d, ll *a, ll n) { // a[0].. a[d]  a[n]
      if (n <= d) return a[n];
      p1[0] = p2[0] = 1;
      rep(i, 0, d+1) p1[i+1] = mul(p1[i], (n - i) % P);
      rep(i, 0, d+1) p2[i+1] = mul(p2[i], (n - d + i) % P);
      ll ans=0;
      rep(i, 0, d+1) {
         ll s1 = mul(p1[i], p2[d - i]);
         ll s2 = mul(ifac[i], ifac[d - i]);
         ll t = mul(mul(s1, s2), a[i]);
         ans = (d-i)&1 ? add(ans, -t) : add(ans, t);
      }
      return ans;
   }
   ll Polysum(ll n, ll *a, ll m) { // a[0].. a[m] \sum_{i=0}^{n-1} a[i]
      a[m+1] = calcn(m, a, m+1);
      rep(i, 1, m+2) a[i] = add(a[i-1], a[i]);
      return calcn(m+1, a, n-1);
   }
   ll qpolysum(ll R, ll n, ll *a, ll m) { // a[0].. a[m] \sum_{i=0}^{n-1} a[i]*R^i
      if (R == 1) return Polysum(n, a, m);
      a[m+1] = calcn(m, a, m+1);
      ll r = kpow(R, P - 2), p3 = 0, p4 = 0, c, ans;
      h[0][0] = 0; h[0][1] = 1;
      rep(i, 1, m+2) {
```

```
        h[i][0] = mul(h[i-1][0] + a[i-1], r);
        h[i][1] = mul(h[i-1][1], r);
      }
      rep(i, 0, m+2) {
        ll t = mul(ifac[i], ifac[m+1-i]);
        p3 = i & 1 ? add(p3, -mul(h[i][0], t)) : add(p3, mul(h[i][0], t));
        p4 = i & 1 ? add(p4, -mul(h[i][1], t)) : add(p4, mul(h[i][1], t));
      }
      c = mul(kpow(p4, P - 2), -p3);
      rep(i, 0, m+2) h[i][0] = add(h[i][0], h[i][1] * c);
      rep(i, 0 ,m+2) C[i] = h[i][0];
      ans = add(mul(calcn(m, C, n), kpow(R, n)), -c);
      return ans;
   }
};
```

7.31 prime

```
// time : O(n)
// low[] : optional
const int N = 1e6 + 6;
int low[N],cntp,p[N];
bool isp[N];
void getprime() {
   fill_n(isp + 2, N - 2, 1);
   rep(i, 2, N) {
      if (isp[i]) p[cntp++]=i;
      for (int j=0;j<cntp&&p[j]*i<N;j++){
         //low[p[j] * i] = p[j];
         isp[p[j] * i] = 0;
         if (i % p[j] == 0) break;
      }
   }
}

// 优化版欧拉筛法 bitset 需要 O2
const int N = 3e7 + 6, M = 2e6 + 6;
// int low[N],
bitset<N / 3 + 1> isp;
int cntp,p[M];
void getprime(int N) {
   cntp = 2;p[0] = 2;p[1] = 3;
   for (int i = 5, k = 1; i <= N; (k & 1) ? i+=2 : i+=4 , k++){
      if (!isp[k]) {
         p[cntp++]=i;
         // low[i] = i;
      }
      for (int j = 2; j < cntp && p[j] * i <= N; j++) {
         // low[p[j] * i] = p[j];
         isp[p[j] * i / 3] = 1;
         if (i % p[j] == 0) break;
      }
   }
}
```

```
// 优化埃氏筛法空间最小可以不存质数
const int N = 3e8 + 6, M = 2e7 + 6;
int cntp,p[M];
bitset<N / 3 + 1> bit;

void getprime(int n){
   int i, j;
   cntp = 2; p[0] = 2; p[1] = 3;
   for(i = 5, j = 1; i * i <= n; (j & 1) ? i += 2 : i += 4 , j++) {
      if(bit[j] == 0) {
         p[cntp++]=i;
         for(int j = i * i; j <= n ; j += i)
            if(j % 2 != 0 && j % 3 != 0) bit[j / 3] = 1;
      }
   }
   for( ; i <= n; (j&1) ? i+=2 : i+=4 , j++) if(bit[j] == 0) p[cntp++]=i;
}
```

7.32 三进制向量

```
struct vec3{
   static const int N = ::N;
   bitset<N> a[3];
   vec3() { a[0].set(); }
   inline void ini() { a[0].set(); a[1].reset(); a[2].reset(); }
   inline void set(int p, int x) { x = (x % 3 + 3) % 3; rep(i, 0, 3) if (i == x) a[i].
   set(p); else a[i].reset(p); }
   inline int operator [] (int x) { rep(i, 0, 3) if (a[i][x]) return i; }
   int getval() { return (a[1].count() + a[2].count() * 2) % 3; }
   inline vec3 operator * (const vec3 &c) const {
      vec3 r;
      r.a[0] = a[0] | c.a[0];
      r.a[1] = (a[1] & c.a[1]) | (a[2] & c.a[2]);
      r.a[2].set(); r.a[2] ^= r.a[0] ^ r.a[1];
      return r;
   }
   inline vec3 operator + (const vec3 &c) const {
      vec3 r;
      r.a[1] = (a[0] & c.a[1]) | (a[1] & c.a[0]) | (a[2] & c.a[2]);
      r.a[2] = (a[0] & c.a[2]) | (a[2] & c.a[0]) | (a[1] & c.a[1]);
      r.a[0] ^= r.a[1] ^ r.a[2];
      return r;
   }
   inline vec3 operator - (const vec3 &c) const {
      return (*this) + (c * -1);
   }
   inline vec3 operator * (int x) const {
      vec3 r = *this;
      x = (x % 3 + 3) % 3;
      if (x == 0) { r.ini(); }
      if (x == 2) { swap(r.a[1], r.a[2]); }
      return r;
   }
   inline vec3 operator + (int x) const{
      vec3 r = *this;
```

```
      x = (x % 3 + 3) % 3;
      if (x == 1) { swap(r.a[0], r.a[2]); swap(r.a[1], r.a[2]); }
      if (x == 2) { swap(r.a[0], r.a[2]); swap(r.a[0], r.a[1]); }
      return r;
   }
};
```

7.33 划分数

```
const int N = 1e6 + 5, P = 998244353;
int n, f[N], fv[N];
inline int add(int a, int b) { if((a += b) >= P) a -= P; return a < 0 ? a + P : a;}
void init(int n) {
   f[0] = f[1] = 1;
   int m = sqrt(n) + 1;
   rep(i, 1, m+1) fv[i] = i * (3 * i - 1) / 2;
   rep(i, 2, n+1) {
      for(int j = 1; fv[j] <= i; j++) {
         f[i] = add(f[i], j & 1 ? f[i - fv[j]] : -f[i - fv[j]]);
         if (fv[j] + j <= i)
            f[i] = add(f[i], j & 1 ? f[i - fv[j] - j] : -f[i - fv[j] - j]);
      }
   }
}
```

7.34 原根

```
ll Pow(ll x, ll k, ll p) {
   ll ret = 1;
   for (; k; k >>= 1, x = x*x%p) if (k & 1) ret = ret*x%p;
   return ret;
}
struct Euler {
   vector<ll> c;
   inline bool check_g(ll g, ll p) {
      rep(i, 0, sz(c)) if (Pow(g, c[i], p) == 1) return 0;
      return 1;
   }
   inline ll getRoot(ll p) {
      c.clear();
      ll tmp = p - 1, g;
      for (ll k = 2; k*k <= tmp; ++k) if (tmp % k == 0) {
         c.pb(k);
         while (tmp % k == 0) tmp /= k;
      }
      if (tmp != 1) c.pb(tmp);
      rep(i, 0, sz(c)) c[i] = (p - 1) / c[i];
      for (g = 1; !check_g(g, p); ++g);
      return g;
   }
};
```

7.35 原根__合数

```
ll kpow(ll x, ll k, ll p) {
   ll ret = 1;
   for (; k; k >>= 1, x = x*x%p) if (k & 1) ret = ret*x%p;
   return ret;
}
struct Euler {
   vector<ll> P, A; ll phi, g;
   inline bool check_g(ll g, ll p) {
      rep(i, 0, sz(P)) if (kpow(g, P[i], p) == 1) return 0;
      return 1;
   }
   inline void factor(ll m) {
      P.clear(), A.clear();
      for (ll k = 2; k*k <= m; ++k) if (m%k == 0) {
         int cnt = 0;
         while (m%k == 0) m /= k, cnt++;
         P.pb(k), A.pb(cnt);
      }
      if (m > 1) P.pb(m), A.pb(1);
   }
   inline bool check(ll m) {
      //if (m==1 || m==2 || m==4) return 1;
      factor(m);
      if (sz(P) > 2 || sz(P) == 1 && P[0] == 2) return 0;
      if (sz(P) == 1) return 1;
      if (P[0] != 2 || P[0] == 2 && A[0] > 1) return 0;
      return 1;
   }
   inline ll getRoot(ll p) {
      if (p == 1 || p == 2 || p == 4) return phi = p + 1 >> 1, p - 1;
      if (!check(p)) return -1;
      phi = p;
      for (auto t : P) phi = phi / t*(t - 1);
      factor(phi);
      for (auto &t : P) t = phi / t;
      for (g = 1; __gcd(g, p) != 1 || !check_g(g, p); ++g);
      return g;
   }
   inline vector<ll> getAllRoot(ll p) {
      vector<ll> ret; ll g = getRoot(p);
      if (g == -1) return ret;
      rep(i, 0, phi) if (__gcd((ll)i, phi) == 1) ret.pb(kpow(g, i, p));
      sort(all(ret)); return ret;
   }
};
```

7.36 带权拟阵交

```
const int N = 85, INF = pw(30);
int c[N], k[N], col[N], u[N], v[N], w[N], sum, ans, T, n, m, tot, tot2;
struct GM {
   vi g[N]; bool vis[N], exist[N];
   void dfs(int u) {
      vis[u] = 1;
```

```
      for(auto v : g[u]) if (!vis[v]) dfs(v);
   }
   bool test(vi &vec) {
      rep(i, 1, n+2) g[i].clear(), vis[i] = 0;
      memset(exist, 1, sizeof(exist));
      for(auto x : vec) exist[x] = 0;
      rep(i, 1, tot+1) if (exist[i]) g[u[i]].pb(v[i]), g[v[i]].pb(u[i]);
      dfs(1);
      rep(i, 1, n+2) if(!vis[i]) return 0;
      return 1;
   }
};
struct CM {
   int cnt[125];
   bool test(vi &vec) {
      memset(cnt, 0, sizeof(cnt));
      for(auto x : vec) cnt[col[x]]++;
      rep(i, 1, m+1) if(cnt[i] > c[i] - k[i]) return 0;
      return 1;
   }
};
template <class MT1, class MT2>
struct MI {
   int n, S, T, pre[N], d[N], cost[N]; bool inq[N], has[N]; vi g[N]; queue<int> q;
   MI(int n) : n(n) {}
   void clear() {
      rep(i, 1, n+3) {
         inq[i] = pre[i] = 0;
         d[i] = -INF;
         g[i].clear();
      }
      while(!q.empty()) q.pop();
   }
   vi getcur() {
      vi ret;
      rep(i, 1, n+1) if(has[i]) ret.pb(i);
      return ret;
   }
   pair<vi, ll> run() {
      ll ans = 0; MT1 mt1; MT2 mt2;
      memset(has, 0, sizeof(has));
      S = n + 1, T = n + 2, cost[S] = cost[T] = 0;
      while (1) {
         clear();
         rep(i, 1, n+1) {
            if(!has[i]) {
               cost[i] = w[i];
               has[i] ^= 1;
               vi tmp = getcur();
               if (mt1.test(tmp)) g[S].pb(i); // X1
               if (mt2.test(tmp)) g[i].pb(T); // X2
               has[i] ^= 1;
            } else cost[i] = -w[i];
         }
         rep(i, 1, n+1) if (has[i]) {
            has[i] ^= 1;
            vi tmp = getcur();
            rep(j, 1, n+1) if (!has[j] && i != j) {
               tmp.pb(j);
               if (mt1.test(tmp)) g[i].pb(j);
               if (mt2.test(tmp)) g[j].pb(i);
               tmp.pop_back();
            }
            has[i] ^= 1;
         }
         d[S] = 0; q.push(S); inq[S] = 1;
         while(!q.empty()) {
            int u = q.front(); q.pop(); inq[u] = 0;
            for(auto v : g[u])
               if(d[v] < d[u] + cost[v]) {
                  d[v] = d[u] + cost[v];
                  pre[v] = u;
                  if (!inq[v]) q.push(v), inq[v] = 1;
               }
         }
         if (!pre[T]) return mp(getcur(), ans);
         ans += d[T];
         int la = pre[T];
         while (la != S) has[la] ^= 1, la = pre[la];
      }
   }
};
//hdu 6636 Milk Candy
int main() {
   cin >> T;
   rep(cas, 0, T) {
      tot = ans = sum = 0;
      cin >> n >> m;
      rep(i, 1, m+1) {
         cin >> c[i] >> k[i];
         sum += c[i] - k[i];
         rep(j, 0, c[i]) {
            int l, r, cost;
            cin >> l >> r >> cost;
            col[++tot] = i;
            u[tot] = l, v[tot] = r + 1;
            w[tot] = cost;
            ans += cost;
         }
      }
      MI<GM, CM> mi(tot);
      auto res = mi.run();
      GM gm;
      if (sz(res.fi) != sum || !gm.test(res.fi)) cout << -1 << endl;
      else cout << ans - res.se << endl;
   }
   return 0;
}
```

7.37 拟阵交

```
const int N = 5005;
int col[N], n, m, tot, tot2, k; ll val[N], x;
struct LM { // 线性拟阵
    ll base[63];
    LM() { memset(base, 0, sizeof(base)); }
    void add(ll x) {
        per(j, 0, 63) if ((x >> j) & 1){
            if(!base[j]) {
                base[j] = x;
                break;
            } else x ^= base[j];
         if (!x) break;
      }
    }
    bool test(ll x) {
        per(j, 0, 63) if ((x >> j) & 1){
            if(!base[j]) return 1; else x ^= base[j];
         if (!x) break;
      }
        return 0;
    }
};
struct CM { // 高维均匀拟阵
    int cnt[125];
    CM() { memset(cnt,0,sizeof(cnt)); }
    void add(int x) { cnt[x]++; }
    bool test(int x) { return cnt[x] == 0; }
};
template <class MT1, class MT2>
struct MI {
    int n, pre[N], id[N]; bool vis[N], sink[N], has[N]; queue<int> q;
    MI(int n) : n(n) {}
    void clear() {
      rep(i, 1, n+1) vis[i] = sink[i] = pre[i] = 0;
        while (!q.empty()) q.pop();
    }
    vi getcur() {
        vi ret;
        rep(i, 1, n+1) if (has[i]) ret.pb(i), id[i] = sz(ret) - 1;
        return ret;
    }
    void push(int v, int p) { vis[v] = 1, pre[v] = p, q.push(v); }
    vi run() {
        MT1 mt1;  MT2 mt2;
        memset(has, 0, sizeof(has));
        while(1) {
            vi cur = getcur(); clear();
            MT1 mt1; MT2 mt2;
            for(auto x : cur) mt1.add(val[x]), mt2.add(col[x]);
            rep(i, 1, n+1) if (!has[i]) {
                if(mt1.test(val[i])) push(i, 0); // X1;
                if(mt2.test(col[i])) sink[i] = 1; // X2;
            }
            bool ok = 0;
            rep(i, 1, n+1) if (sink[i] && vis[i]) { has[i] ^= 1; ok = 1; break;}
            if (ok) continue;

            vector<MT1> vmt1(sz(cur)); vector<MT2> vmt2(sz(cur));
            rep(i, 0, sz(cur))
               rep(j, 0, sz(cur)) if (i != j) {
                    vmt1[i].add(val[cur[j]]);
                    vmt2[i].add(col[cur[j]]);
                }

            int t = -1;
            while(!q.empty()) {
                int u = q.front(); q.pop();
                if (sink[u]) { t = u; break;}
                rep(v, 1, n+1) if (!vis[v] && has[u] != has[v]){
                    if(has[u]) {
                        if(vmt1[id[u]].test(val[v])) push(v, u);
                    }
                    else {
                        if(vmt2[id[v]].test(col[u])) push(v, u);
                    }
                }
            }
            if (t == -1) return cur;
            while (t) has[t] ^= 1, t = pre[t];
        }
    }
};

//Pick Your Own Nim
//In real cases, Linear Matroid Need Optimization to Pass
int main() {
    cin >> n;
    rep(i, 0, n) cin >> x, val[++tot] = x, col[tot] = ++tot2;
    cin >> m;
    rep(i, 0, m) {
        cin >> k; tot2++;
        rep(j, 0, k) cin >> x, val[++tot] = x, col[tot] = tot2;
    }
    MI<LM, CM> matint(tot);
    vi res = matint.run();
    if (sz(res) < n + m)  cout << -1 << endl;
    else for(auto x : res) if (col[x] > n) cout << val[x] << endl;
    return 0;
}
```

7.38 离散对数

```
ll kpow(ll a, ll b, ll P) {
   ll r = 1;
   for (; b; b >>= 1, a = a * a % P) if (b & 1) r = r * a % P;
   return r;
}
```

```
void ex_gcd(int a, int b, int &x, int &y) {
   b ? (ex_gcd(b, a % b, y, x), y -= a / b * x) : (x = 1, y = 0);
}

inline int Inv(int a, int P) {
   int x, y; ex_gcd(a, P, x, y);
   return x < 0 ? x + P : x;
}

struct BSGS {
   unordered_map<ll, int> M;
   ll bsgs(ll x, ll z, ll P) {
      if (x % P == 0) return -1;
      ll res = z % P, sa, t = 1, sq = sqrt(P); M.clear();
      rep(i, 0, sq + 1) { if (M.count(t)) break; M[t] = i, t = t * x % P; }
      t = P / sq, sa = Inv(kpow(x, sq, P), P);
      rep(i, 0, t + 1) if (M.count(res))
         return i * sq + M[res]; else res = res * sa % P;
      return -1;
   }
   ll ex_bsgs(ll x, ll z, ll P) { //x^y==z(mod P)
      ll t = 1 % P, w = 1, ans, c = 0; z %= P;
      rep(i, 0, 51) { if (t == z) return i; t = t * x % P; }
      for (t = __gcd(x, P); t != 1; t = __gcd(x, P)) {
         if (z % t) return -1;
         z /= t, P /= t, w = w * x / t % P, c++;
         if (z == w) return c;
      }
      z = z * Inv(w, P) % P, ans = bsgs(x, z, P);
      return ans + (ans != -1) * c;
   }
};
```

7.39 高次同余_合数

```
ll kpow(ll a, ll b, ll P) {
   ll r = 1; assert(b >= 0);
   for (; b; b >>= 1, a = a * a % P) if (b & 1) r = r * a % P;
   return r;
}
void ex_gcd(ll a, ll b, ll &x, ll &y) {
   b ? (ex_gcd(b, a % b, y, x), y -= a / b * x) : (x = 1, y = 0);
}
inline ll Inv(ll a, ll P) {
   ll x, y; ex_gcd(a, P, x, y);
   return x < 0 ? x + P : x;
}
struct BSGS {
   map<ll, int> M;
   ll bsgs(ll x, ll z, ll P) {
      if (x % P == 0) return -1;
      ll res = z % P, sa, t = 1, sq = sqrt(P); M.clear();
      rep(i, 0, sq + 1) { if (M.count(t)) break; M[t] = i, t = t * x % P; }
      t = P / sq, sa = Inv(kpow(x, sq, P), P);
      rep(i, 0, t + 1) if (M.count(res))
         return i * sq + M[res]; else res = res * sa % P;
      return -1;
   }
   ll ex_bsgs(ll x, ll z, ll P) { //x^y==z(mod P)
      ll t = 1 % P, w = 1, ans, c = 0; z %= P;
      rep(i, 0, 51) { if (t == z) return i; t = t * x % P; }
      for (t = __gcd(x, P); t != 1; t = __gcd(x, P)) {
         if (z % t) return -1;
         z /= t, P /= t, w = w * x / t % P, c++;
         if (z == w) return c;
      }
      z = z * Inv(w, P) % P, ans = bsgs(x, z, P);
      return ans + (ans != -1) * c;
   }
};
struct CRT {
   ll M, R; static const int N = 55;
   ll a[N], mod[N];
   void exgcd(ll a, ll b, ll &x, ll &y) {
      if (b == 0) { x = 1; y = 0; return; }
      exgcd(b, a % b, y, x);
      y -= a / b * x;
   }
   ll Inv(ll a, ll mod) {
      ll x = 0, y = 0;
      exgcd(a, mod, x, y);
      x %= mod;
      return x < 0 ? x + mod : x;
   }
   ll solve(ll n) {
      M = mod[1], R = a[1];
      rep(i, 2, n + 1) {
         ll g = __gcd(M, mod[i]);
         ll inv = Inv(M / g, mod[i] / g);
         if ((a[i] - R) % g) return -1; // 无解
         R += inv * ((a[i] - R) / g) % (mod[i] / g) * M;
         M = M / g * mod[i];
         R = (R % M + M) % M; // 可能为 0 看是否需要是正整数
      }
      return R;
   }
} crt;

typedef vector<ll> vll;
typedef pair<ll, ll> pll;
struct Euler {
   vll P, A, _P, _A; ll phi, g, phi_phi; BSGS T;
   inline bool check_g(ll g, ll p) {
      rep(i, 0, sz(P)) if (kpow(g, P[i], p) == 1) return 0;
      return 1;
   }
   inline void factor(ll m, vll &P, vll &A) {
      P.clear(), A.clear();
      for (ll k = 2; k*k <= m; ++k) if (m%k == 0) {
         int cnt = 0;
```

```
      while (m%k == 0) m /= k, cnt++;
      P.pb(k), A.pb(cnt);
   }
   if (m > 1) P.pb(m), A.pb(1);
}
inline void norm(ll &x, ll p) { x = (x%p + p) % p; }
inline ll get_phi(ll p) {
   ll phi = p;
   //    for (auto t:P) phi=phi/t*(t-1);
   rep(i, 0, sz(P)) phi = phi / P[i] * (P[i] - 1);
   return phi;
}
inline bool check(ll m) {
   //if (m==1 || m==2 || m==4) return 1;
   factor(m, P, A);
   if (sz(P) > 2 || sz(P) == 1 && P[0] == 2) return 0;
   if (sz(P) == 1) return 1;
   if (P[0] != 2 || P[0] == 2 && A[0] > 1) return 0;
   return 1;
}
inline ll getRoot(ll p) {
   if (p == 1 || p == 2 || p == 4) return phi = p + 1 >> 1, phi_phi = 1, p - 1;
   if (!check(p)) return -1;
   phi = get_phi(p);
   factor(phi, P, A), phi_phi = get_phi(phi);
   //       for (auto &t:P) t=phi/t;
   rep(i, 0, sz(P)) P[i] = phi / P[i];
   for (g = 1; __gcd(g, p) != 1 || !check_g(g, p); ++g);
   return g;
}
// solve equation: ax=b(%p), gcd(a,p)!=1
pll solve(ll a, ll b, ll p) {
   norm(a, p); norm(b, p); ll g = __gcd(a, p);
   if (b%g) return mp(-1, g);
   a /= g, b /= g, p /= g;
   return mp(kpow(a, phi_phi - 1, p)*b%p, g);//note that phi_phi
}
ll get_pow(ll p, int k) {
   ll ret = 1; assert(k >= 0);
   rep(i, 0, k) ret = ret*p;
   return ret;
}
// solve equation: x^a=b(%pp^k), pp is a prime
pll solve_high(ll a, ll b, ll pp, int k) {
   assert(pp > 1), assert(k > 0);
   ll p = get_pow(pp, k); norm(b, p); ll t1, t2, t3;
   if (!a) return b == 1 ? mp(0, p) : mp(-1, 0ll);
   if (!b) return mp(!a, get_pow(pp, k - (k - 1) / a - 1));
   ll g = getRoot(p);
   if (g == -1) return mp(-1, 0);
   int cnt = 0; while (b%pp == 0) b /= pp, cnt++;
   if (cnt%a) return mp(-1, 0); bool ok = 0;
   if (cnt) t1 = get_pow(pp, cnt), t2 = get_pow(pp, cnt / a),
      t3 = t1 / t2, ok = 1, p /= t1;
   ll _b = T.ex_bsgs(g, b, p);
   if (_b == -1) return mp(-1, 0);
   ll _p = p / pp*(pp - 1);
   pair<ll, ll> t = solve(a, _b, _p);
   if (t.fi == -1) return mp(-1, 0);
   ll _g = t.se, x = t.fi, ans = kpow(g, x, p), d = kpow(g, _p / _g, p), ret = _g;
   if (ok) ans *= t2, ret *= t3;
   return mp(ans, ret);
  }
  // solve equation: x^a=b(%p), p could not be a prime, but p must have a primitive
  root, that is 8 cannot divide p
  pll solve_high(ll a, ll b, ll p) {
   assert(p > 0); norm(b, p);
   if (p == 1) return mp(0, 1);
   factor(p, _P, _A); int tot = sz(_P); ll ret = 1, ans; pll tmp[32];
   rep(i, 0, tot) {
      tmp[i + 1] = solve_high(a, b, _P[i], _A[i]),
      crt.a[i + 1] = tmp[i + 1].fi,
      crt.mod[i + 1] = get_pow(_P[i], _A[i]),
      ret *= tmp[i + 1].se;
   }
   if (!ret) return mp(-1, 0);
   ans = crt.solve(tot);
   return mp(ans, ret);
  }
};
// 注: 返回 pair( 最小非负解 , [0,p) 中解的个数 )
```

7.40 高次同余_质数

```
ll kpow(ll a, ll b, ll P) {
   ll r = 1; assert(b >= 0);
   for (; b; b >>= 1, a = a * a % P) if (b & 1) r = r * a % P;
   return r;
}
void ex_gcd(ll a, ll b, ll &x, ll &y) {
   b ? (ex_gcd(b, a % b, y, x), y -= a / b * x) : (x = 1, y = 0);
}
inline ll Inv(ll a, ll P) {
   ll x, y; ex_gcd(a, P, x, y);
   return x < 0 ? x + P : x;
}
struct BSGS {
   map<ll, int> M;
   ll bsgs(ll x, ll z, ll P) {
      if (x % P == 0) return -1;
      ll res = z % P, sa, t = 1, sq = sqrt(P); M.clear();
      rep(i, 0, sq + 1) { if (M.count(t)) break; M[t] = i, t = t * x % P; }
      t = P / sq, sa = Inv(kpow(x, sq, P), P);
      rep(i, 0, t + 1) if (M.count(res))
         return i * sq + M[res]; else res = res * sa % P;
      return -1;
   }
   ll ex_bsgs(ll x, ll z, ll P) { //x^y==z(mod P)
      ll t = 1 % P, w = 1, ans, c = 0; z %= P;
      rep(i, 0, 51) { if (t == z) return i; t = t * x % P; }
```

```
      for (t = __gcd(x, P); t != 1; t = __gcd(x, P)) {
         if (z % t) return -1;
         z /= t, P /= t, w = w * x / t % P, c++;
         if (z == w) return c;
      }
      z = z * Inv(w, P) % P, ans = bsgs(x, z, P);
      return ans + (ans != -1) * c;
   }
};

typedef vector<ll> vll;
typedef pair<ll, ll> pll;
struct Euler {
   vll P, A; ll phi, g, phi_phi; BSGS T;
   inline bool check_g(ll g, ll p) {
      rep(i, 0, sz(P)) if (kpow(g, P[i], p) == 1) return 0;
      return 1;
   }
   inline void factor(ll m) {
      P.clear(), A.clear();
      for (ll k = 2; k*k <= m; ++k) if (m%k == 0) {
         int cnt = 0;
         while (m%k == 0) m /= k, cnt++;
         P.pb(k), A.pb(cnt);
      }
      if (m > 1) P.pb(m), A.pb(1);
   }
   inline void norm(ll &x, ll p) { x = (x%p + p) % p; }
   inline ll get_phi(ll p) {
      ll phi = p;
      for (auto t : P) phi = phi / t*(t - 1);
      //    rep(i,0,sz(P)) phi=phi/P[i]*(P[i]-1);
      return phi;
   }
   inline bool check(ll m) {
      //if (m==1 || m==2 || m==4) return 1;
      factor(m);
      if (sz(P) > 2 || sz(P) == 1 && P[0] == 2) return 0;
      if (sz(P) == 1) return 1;
      if (P[0] != 2 || P[0] == 2 && A[0] > 1) return 0;
      return 1;
   }
   inline ll getRoot(ll p) {
      if (p == 1 || p == 2 || p == 4) return phi = p + 1 >> 1, phi_phi = 1, p - 1;
      if (!check(p)) return -1;
      phi = p - 1;
      factor(phi), phi_phi = get_phi(phi);
      for (auto &t : P) t = phi / t;
      //    rep(i,0,sz(P)) P[i]=phi/P[i];
      for (g = 1; __gcd(g, p) != 1 || !check_g(g, p); ++g);
      return g;
   }
   // solve equation: ax=b(%p), gcd(a,p)!=1
   pll solve(ll a, ll b, ll p) {
      norm(a, p); norm(b, p); ll g = __gcd(a, p);
      if (b%g) return mp(-1, g);
      a /= g, b /= g, p /= g;
      return mp(kpow(a, phi_phi - 1, p)*b%p, g);//note that phi_phi
   }
   // solve equation: x^a=b(%p), p must be a prime
   vll solve_high(ll a, ll b, ll p) {
      vll ret; norm(b, p); assert(p > 0);
      if (!a == b) ret.pb(0);
      if (!b) return ret;
      ll g = getRoot(p);
      if (g == -1) return ret;
      ll _b = T.bsgs(g, b, p);
      if (_b == -1) return ret;
      ll _p = p - 1;
      pll t = solve(a, _b, _p);
      if (t.fi == -1) return ret;
      ll _g = t.se, x = t.fi, ans = kpow(g, x, p), d = kpow(g, _p / _g, p);
      ret.pb(ans);
      rep(i, 1, _g) ans = ans*d%p, ret.pb(ans);
      sort(all(ret));
      return ret;
   }
};
// 注 : 返回所有 [0,p) 中的非负整数解
```

8 Others

8.1 BitOperation

```
// 枚举子集
for(int i = x; i; (--i) & x) {
    //
}
// 统计子集的答案
rep(i, 0, n) {
    rep(j, 0, 1 << n) if(j >> i & 1) {
        upd(s[j], s[j ^ (1 << i)]);
    }
}
// 统计超集的答案
rep(i, 0, n) {
    for(int j = (1 << n) - 1; ~j; --j) if(!(j >> i & 1)) {
        upd(s[j], s[j | (1 << i)]);
    }
}
//
int __builtin_ffs (unsigned int x)
int __builtin_ffsl (unsigned long)
int __builtin_ffsll (unsigned long long)
Returns one plus the index of the least significant 1-bit of x, or if x is zero, returns
 zero.
//
int __builtin_clz (unsigned int x)
```

```
Returns the number of leading 0-bits in x, starting at the most significant bit position
. If x is 0, the result is undefined.
//
int __builtin_ctz (unsigned int x)
Returns the number of trailing 0-bits in x, starting at the least significant bit
position. If x is 0, the result is undefined.
//
int __builtin_popcount (unsigned int x)
Returns the number of 1-bits in x.
//
int __builtin_parity (unsigned int x)
Returns the parity of x, i.e. the number of 1-bits in x modulo 2.
```

8.2 Bitset

```
// Base
   b.any();   // has 1 ?
   b.none();  // all 0 ?
   b.count(); // cnt of 1

   b.set();   // all to 1
   b.reset(); // all to 0
   b.flip();  // all = 0 <-> 1

   b.set(p);  // b[p] = 1
   b.test(p); // b[p] is 1
   b.reset(p);// b[p] = 0
   b.flip(p); // b[p] = 0 <-> 1

// Black tech
   // __builtin_ctz in bitst
   b._Find_first();

   // travel all 1
   for (int i = b._Find_first(); i < sz(b); i = b._Find_next(i));
```

8.3 ExpressionParse

```
// 二元运算左结合
vector<char> rpn, ch, sta;
// 定义运算符优先级
int pri(char ch) {
   if(ch == '(') return 0;
   // ...
   return -1;
}
char solve(string s) {
   // 中缀转后缀
   rpn.clear(); ch.clear();
   rep(i, 0, sz(s)) {
      char c = s[i];
      if(c == '(') { ch.pb(c);
      } else if(c == ')') {
         while(ch.back() != '(') rpn.pb(ch.back()), ch.pop_back();
         ch.pop_back();
      } else if(pri(c) > 0) {
         while(sz(ch) && pri(ch.back()) >= pri(c)) rpn.pb(ch.back()), ch.pop_back();
         ch.pb(c);
      } else { rpn.pb(c); }
   }
   reverse(all(ch)); rpn.insert(rpn.end(), all(ch));
   // 后缀表达式计算
   sta.clear();
   rep(i, 0, sz(rpn)) {
      char u = rpn[i];
      if(pri(u) > 0) {
         char b = sta.back(); sta.pop_back();
         sta[sz(sta) - 1] = calc(u, sta.back(), b);
      } else { sta.pb(u); }
   }
   return sta[0];
}
```

8.4 FastIO

```
// read untill EOF (xint)
struct FastIO {
    static const int S = 1310720;
    int wpos;
    char wbuf[S];
    bool ed;
    FastIO() : wpos(0), ed(0) { }
    inline int xchar() {
        static char buf[S];
        static int len = 0, pos = 0;
        if (pos == len) pos = 0, len = fread(buf, 1, S, stdin);
        if (pos == len) return -1;
        return buf[pos++];
    }
    inline int xint() {
        int c = xchar(), x = 0, s = 1;
        while (c <= 32) {
            if(!~c) return ed = 1;
            c = xchar();
        }
        if (c == '-') s = -1, c = xchar();
        for (; '0' <= c && c <= '9'; c = xchar()) x = x * 10 + c - '0';
        return x * s;
    }
    inline int xuint() {
        int c = xchar(), x = 0;
        while (c <= 32) c = xchar();
        for (; '0' <= c && c <= '9'; c = xchar()) x = x * 10 + c - '0';
        return x;
    }
    inline void xstring(char *s) {
        int c = xchar();
        while (c <= 32) c = xchar();
        for (; c >  32; c = xchar()) *s++ = c;
```

```
        *s = 0;
    }
    inline void wchar(int x) {
        if (wpos == S) fwrite(wbuf, 1, S, stdout), wpos = 0;
        wbuf[wpos++] = x;
    }
    inline void wint(int x) {
        if (x < 0) wchar('-'), x = -x;
        char s[24];
        int n = 0;
        while (x || !n) s[n++] = '0' + x % 10, x /= 10;
        while (n--) wchar(s[n]);
    }
    inline void wstring(const char *s) { while (*s) wchar(*s++); }
    ~FastIO() { if (wpos) fwrite(wbuf, 1, wpos, stdout), wpos = 0; }
} io;
```

8.5 FastMod

```
template<class T1, class T2>
struct FastD {
   const static int wb = sizeof(T1) * 8;
   int len; T1 m, x;
   FastD() = default;
   FastD(T1 n): m(n) {
      if (n == 1) x = 1, len = 0;
      else {
         if (wb == 32) len = 31 - __builtin_clz(n - 1) + wb;
         else len = 63 - __builtin_clzll(n - 1) + wb;
         x = ((T2(1) << len) + n - 1) / n;
      }
   }
   friend T1 operator / (const T1 &n, const FastD &d) { return T2(n) * d.x >> d.len; }
   friend T1 operator % (const T1 &n, const FastD &d) { return n - n / d * d.m; }
};

template<class T> // 只能用于奇数
struct ExactD {
   T t, i;
   ExactD() = default;
   ExactD(const T &n): t(T(-1) / n), i(mul_inv(n)) {}
   constexpr static T mul_inv(T n, int e = 6, T x = 1) {
      return !e ? x : mul_inv(n, e - 1, x * (2 - x * n));
   }
   friend T operator / (const T &n, const ExactD &d) { return n * d.i; }
   bool divide(const T &n) const { return n * i <= t; }
};

using FastDiv32 = FastD<uint32, uint64>;
using FastDiv64 = FastD<uint64, uint128>;
using ExactDiv32 = ExactD<uint32>;
using ExactDiv64 = ExactD<uint64>;
```

8.6 Java

```
import java.io.*;
import java.util.*;
import java.math.*;
public class code {
   public static void main(String[] args) {
      Scanner cin=new Scanner(System.in);
      BigInteger a=cin.nextBigInteger();
      BigInteger b=cin.nextBigInteger();
      System.out.println(a.add(b));
      Integer a,b;
      a=cin.nextInt();
      b=cin.nextInt();
      List<String> mylist1 = new ArrayList<>();
      List<String> mylist2 = new LinkedList<>();
      List<String> mylist3 = new Vector<>();
      Vector<String> vec = new Vector<>();
      Queue<String> que = new LinkedList<>();
      Stack<String> sta = new Stack<>();
      Set<String> myset = new HashSet<>();
      Set<String> myset2 = new TreeSet<>();
      Map<String, Integer> mymap = new HashMap<>();
      Map<String, Integer> mymap2 = new TreeMap<>();
   }
}
```

8.7 Rand

```
mt19937 gen(998244353);
typedef uniform_int_distribution<ll> RR;
ll rnd(ll l, ll r) { RR dis(l, r); return dis(gen); }
typedef uniform_real_distribution<db> RR;
db rnd(db l, db r) { RR dis(l, r); return dis(gen); }
```

8.8 RomanNumerals

```
const int rom[30] = {
   3000,2000,1000,900,800,700,600,500,400,300,200,100,
   90,80,70,60,50,40,30,20,10,
   9,8,7,6,5,4,3,2,1
};
string smb[30]={
   "MMM","MM","M",
   "CM","DCCC","DCC","DC","D","CD","CCC","CC","C",
   "XC","LXXX","LXX","LX","L","XL","XXX","XX","X",
   "IX","VIII","VII","VI","V","IV","III","II","I"
};
string toRoman(ll d) {
   string r;
   rep(i, 0, 30) if (d >= rom[i]) d -= rom[i], r += smb[i];
   return r;
}
```

8.9 Strtok

```
char s[111]; gets(s); vector<string> a;
for(char* p=strtok(s," .,()");p;p=strtok(NULL," .,()")) a.pb(p);
```

8.10 Time

```
clock_t st = clock(); CLOCKS_PER_SEC;
```

8.11 duipai

```
#!/bin/bash
while true; do
    ./gen > gen.in
    ./sol <gen.in >sol.out
    ./dp <gen.in >dp.out
    if diff sol.out dp.out; then
        printf "AC\n"
    else
        printf "Wa\n"
        exit 0
    fi
done

// sh duipai.sh
```

8.12 回溯时还原标记

```
pair<int*, int> sta[N * 5]; int top;

void add(int &a) { sta[++top] = mp(&a, a); }
void dfs(int u) {
   int ttop = top;

   add(var); modify var;

   // .. dfs

   while(top > ttop) *sta[top].fi = sta[top].se, --top;
}
```

9 String

9.1 ACAutomaton

```
/*
 * [0,L) , N-1 is virtual , 0 is rt
 * init!!
 * addition: end[] end[c] |= end[fail[c]]
 */
struct Trie{
    static const int N = 101010 , M = 26;
    int ne[N][M] , fail[N] , fa[N] , rt , L;
    void ini(){ fill_n(ne[fail[0] = N-1],M,0);L = 0;rt = newnode();}
    int newnode(){ fill_n(ne[L],M,0); return L++; }
    void add(char *s){
        int p = rt;
        for(int i=0;s[i];++i){
            int c = s[i] - 'a';// modify
            if(!ne[p][c]) ne[p][c] = newnode() , fa[L-1] = p;
            p = ne[p][c];
        }
    }
    void Build(){
        vi v;v.pb(rt);
        rep(i,0,sz(v)){
            int c = v[i];
            rep(i,0,M) ne[c][i] ?
                v.pb(ne[c][i]) , fail[ne[c][i]] = ne[fail[c]][i] :
                ne[c][i] = ne[fail[c]][i];
        }
    }
};
```

9.2 DoublingArray

```
namespace Doubling{
   static const int N = 101010;
   // sa[0~n]: 排名第的后缀是以i sa[i] 开头
   // h[1~n]:S[sa[i-1]] 与 S[sa[i]] 的最长公共前缀长度为 h[i]
   int t[N] , wa[N] , wb[N] , sa[N] , h[N];
   void sort(int *x,int *y,int n,int m){
      rep(i,0,m) t[i] = 0;
      rep(i,0,n) t[x[y[i]]]++;
      rep(i,1,m) t[i] += t[i-1];
      per(i,0,n) sa[--t[x[y[i]]]] = y[i];
   }
   bool cmp(int *x,int a,int b,int d){ return x[a] == x[b] && x[a+d] == x[b+d]; }
   void da(int *s,int n,int m){
      int *x=wa,*y=wb;
      rep(i,0,n) x[i] = s[i] , y[i] = i;
      sort(x , y , n , m);
      for(int j=1,p=1;p<n;m=p,j<<=1){
         p = 0;rep(i,n-j,n) y[p++] = i;
         rep(i,0,n) if(sa[i] >= j) y[p++] = sa[i] - j;
         sort(x , y , n , m);
         swap(x , y);p = 1;x[sa[0]] = 0;
         rep(i,1,n) x[sa[i]] = cmp(y,sa[i],sa[i-1],j)?p-1:p++;
      }
   }
   void cal_h(int *s,int n,int *rk){
      int j,k=0;
      for(int i=1;i<=n;++i) rk[sa[i]] = i;
      for(int i=0;i<n;h[rk[i++]] = k) for(k&&--k,j=sa[rk[i]-1];s[i+k]==s[j+k];++k);
   }
}
// rank[0~n-1]: 以 i 开头的后缀排名 rank[i]
```

```
struct DA{ // [0,n] , in[n] = 0 , n load
   static const int N = 101010;
   int p[18][N] , rk[N] , in[N] , Log[N] , n;
   void Build(){
      Doubling::da(in,n+1,300);
      Doubling::cal_h(in,n,rk);
      Log[0] = -1;for(int i=1;i<=n;++i) Log[i] = Log[i-1] + (i==(i&(-i)));
      for(int i=1;i<=n;++i) p[0][i] = Doubling::h[i];
      for(int j=1;1<<j<=n;++j){
         int lim = n+1-(1<<j);
         for(int i=1;i<=lim;++i) p[j][i] = min(p[j-1][i] , p[j-1][i+(1<<j>>1)]);
      }
   }
   // 某两个后缀的最长公共前缀
   int lcp(int a,int b){
      a = rk[a] , b = rk[b];
      if(a > b) swap(a , b);++a;
      int t = Log[b-a+1];
      return min(p[t][a] , p[t][b-(1<<t)+1]);
   }
};
```

9.3 Exkmp

```
/*
 * s 串的每个后缀与 t 串的最长公共前缀
 * t:  a  b  a
 * nt: 0  0  1
 * s:  a  b  a  c  a  b  a
 * ns: 3  0  1  0  3  0  1
 */
void exkmp(char *s,int *z,char *t,int *p){
    int lens = strlen(s);
    int lent = strlen(t);
    p[0]=0;
    for(int i=0,x=0,y=0;i<lens;++i){
        z[i] = i <= y ? min(y-i,p[i-x]) : 0;
        while(i + z[i] < lens && z[i] < lent && s[i + z[i]] == t[z[i]]) ++z[i];
        if(y <= i + z[i]) x = i, y = i + z[i];
    }
}

void Exkmp(){
    scanf("%s%s",s,t);
    exkmp(t+1,nt+1,t,nt);
    exkmp(s,ns,t,nt);
}
```

9.4 Kmp

```
/*
t:  a  b  a
nt:-1 -1  0
s:  a  b  a  c  a  b  a
ns: 0  1  2 -1  0  1  2
*/
void kmp(char *s,int *ns,char *t,int *nt){
    int lens = strlen(s);
    int lent = strlen(t);
    nt[0] = -1;
    for(int i=0,j=-1;i<lens;++i){
        while(j >= 0 && s[i] != t[j + 1]) j = nt[j];
        if(s[i] == t[j + 1]) ++j;
        ns[i] = j;
        if(j + 1 == lent) j = nt[j];
    }
}
void KMP(){
    scanf("%s%s",s,t);
    kmp(t+1,nt+1,t,nt);
    kmp(s,ns,t,nt);
}
```

9.5 LyndonWord

```
// O(n) 分解为字典序非严格降的 lyndon word 分解唯一
vector<int> duval(char s[]){
  vector<int> ret;
  int n = strlen(s) + 1; // zero used here
  int start = 0, mid = 1, cur = 0;
  ret.push_back(0);
  for (int i = 0; i < n; ++i){
    if (s[i] == s[cur]){
      if (++cur == mid) cur = start;
    } else if (s[i] > s[cur]){
      mid = i + 1;
      cur = start;
    } else if (s[i] < s[cur]){
      int temp = mid - start;
      while (start + temp <= i){
        start += temp;
        ret.push_back(start);
      }
      i = cur = start;
      mid = start + 1;
    }
  }
  return ret;
}
/*
cbaabc
0 1 2 6
*/

// 生成字符集为 m , 长度不超过 n 的所有 lyndon word , 字符集从 a 开始
void lyndon_generate(int n, int m) {
  char z = 'a' + m - 1, s[1000];
  s[0] = 'a' - 1;
  for (int i = 1, x = 1; ; ++i) {
```

```
    s[x - 1]++; s[x] = 0;
    puts(s);
    if (strlen(s)==1 && s[0]=='a'+m-1) return;
    for (int j = x; j < n; ++j) s[j] = s[j - x];
    for (x = n; s[x - 1] == z; --x);
  }
}
/*
3 2
a
aab
ab
abb
b
*/
```

9.6 Manacher

```
/*
 * length of pa is two size of str
 * i: [0, n)     pa[i<<1] : odd string 整个回文长度为 2*pa[i<<1]-1
 * i: [0, n - 1) pa[i<<1|1] : even string 整个回文长度为 2*pa[i<<1]
 * N>2*n
*/
void Manacher(char *s,int n,int *pa){
    pa[0] = 1;
    for(int i=1,j=0;i<(n<<1)-1;++i){
        int p = i >> 1 , q = i - p , r = ((j + 1)>>1) + pa[j] - 1;
        pa[i] = r < q ? 0 : min(r - q + 1 , pa[(j<<1) - i]);
        while(0 <= p - pa[i] && q + pa[i] < n && s[p - pa[i]] == s[q + pa[i]]) pa[i]++;
        if(q + pa[i] - 1 > r) j = i;
    }
}
```

9.7 PAM

```
// [0,p) , 0(even) and 1(odd) is virtual , init!!
struct PAM {
   static const int N = ::N, M = 26;
   int s[N], len[N], ne[N][M], fail[N], cnt[N], dep[N], id[N], no[N], last, n, p;
   inline int newnode(int l) { fill_n(ne[p], M, 0); cnt[p] = dep[p] = 0; len[p] = l;
   return p++; }
   inline void init() { newnode(p = 0), newnode(s[0] = -1), fail[last = n = 0] = 1; }
   inline int getfail(int x) {
      while(s[n - len[x] - 1] != s[n]) x = fail[x];
      return x;
   }
   inline void add(int c) {
      s[++n] = c;
      int cur = getfail(last);
      if(!ne[cur][c]) {
         int now = newnode(len[cur] + 2);
         fail[now] = ne[getfail(fail[cur])][c];
         ne[cur][c] = now;
         dep[now] = dep[fail[now]] + 1;
      }
      last = ne[cur][c], cnt[last]++;
      id[n] = last, no[last] = n;
   }
   inline void build() { per(i, 0, p) cnt[fail[i]] += cnt[i]; }
};
```

9.8 PAM_multi

```
struct PAM {
   const int K = 11, N = ::N * K, M = 26;
   int s[N], len[N], ne[N][M], fail[N], cnt[N][K], dep[N], id[N], no[N], last, n, p, cs;
   inline int newnode(int l) {
      fill_n(ne[p], M, 0);
      fill_n(cnt[p], K, 0);
      dep[p] = 0; len[p] = l;
      return p++;
   }
   inline void init() { newnode(p = 0), newnode(s[0] = -1), fail[last = n = 0] = 1; cs =
    0; }
   inline int getfail(int x) {
      while(s[n - len[x] - 1] != s[n]) x = fail[x];
      return x;
   }
   inline void add(int c) {
      if(c < 0) { s[++n] = c; last = 1; return; }
      s[++n] = c;
      int cur = getfail(last);
      if(!ne[cur][c]) {
         int now = newnode(len[cur] + 2);
         fail[now] = ne[getfail(fail[cur])][c];
         ne[cur][c] = now;
         dep[now] = dep[fail[now]] + 1;
      }
      last = ne[cur][c], cnt[last][cs]++;
      id[n] = last, no[last] = n;
   }
   inline void ins() {
      for(all string) {
         ++cs; add(- cs - 1);
         for(all char) add(char);
      }
   }
   inline void build() { per(i, 0, p) rep(j, 0, cs) cnt[fail[i]][j] += cnt[i][j]; }
};
```

9.9 SAIS

```
/*
 * Ensure that str[n] is the unique lexicographically smallest character in str.
 * time complexity: O(n)
 */
namespace SA {
```

```
   const static int N = 100000 + 10;
   int sa[N], rk[N], ht[N], s[N << 1], t[N << 1], p[N], cnt[N], cur[N];
#define pushS(x) sa[cur[s[x]]--] = x
#define pushL(x) sa[cur[s[x]]++] = x
#define inducedSort(v) std::fill_n(sa, n, -1); std::fill_n(cnt, m, 0);      \
   for (int i = 0; i < n; i++) cnt[s[i]]++;                                  \
   for (int i = 1; i < m; i++) cnt[i] += cnt[i-1];                           \
   for (int i = 0; i < m; i++) cur[i] = cnt[i]-1;                            \
   for (int i = n1-1; ~i; i--) pushS(v[i]);                                  \
   for (int i = 1; i < m; i++) cur[i] = cnt[i-1];                            \
   for (int i = 0; i < n; i++) if (sa[i] > 0 &&  t[sa[i]-1]) pushL(sa[i]-1); \
   for (int i = 0; i < m; i++) cur[i] = cnt[i]-1;                            \
   for (int i = n-1;  ~i; i--) if (sa[i] > 0 && !t[sa[i]-1]) pushS(sa[i]-1)
   void sais(int n, int m, int *s, int *t, int *p) {
      int n1 = t[n - 1] = 0, ch = rk[0] = -1, *s1 = s + n;
      for (int i = n - 2; ~i; i--) t[i] = s[i] == s[i + 1] ? t[i + 1] : s[i] > s[i + 1];
      for (int i = 1; i < n; i++) rk[i] = t[i - 1] && !t[i] ? (p[n1] = i, n1++) : -1;
      inducedSort(p);
      for (int i = 0, x, y; i < n; i++) if (~(x = rk[sa[i]])) {
         if (ch < 1 || p[x + 1] - p[x] != p[y + 1] - p[y]) ch++;
         else for (int j = p[x], k = p[y]; j <= p[x + 1]; j++, k++)
            if ((s[j] << 1 | t[j]) != (s[k] << 1 | t[k])) { ch++; break; }
         s1[y = x] = ch;
      }
      if (ch + 1 < n1) sais(n1, ch + 1, s1, t + n, p + n1);
      else for (int i = 0; i < n1; i++) sa[s1[i]] = i;
      for (int i = 0; i < n1; i++) s1[i] = p[sa[i]];
      inducedSort(s1);
   }
   template<typename T>
   int mapCharToInt(int n, const T *str) {
      int m = *max_element(str, str + n);
      std::fill_n(rk, m + 1, 0);
      for (int i = 0; i < n; i++) rk[str[i]] = 1;
      for (int i = 0; i < m; i++) rk[i + 1] += rk[i];
      for (int i = 0; i < n; i++) s[i] = rk[str[i]] - 1;
      return rk[m];
   }
   template<typename T>
   void suffixArray(int n, const T *str) {
      int m = mapCharToInt(++n, str);
      sais(n, m, s, t, p);
      for (int i = 0; i < n; i++) rk[sa[i]] = i;
      for (int i = 0, h = ht[0] = 0; i < n - 1; i++) {
         int j = sa[rk[i] - 1];
         while (i + h < n && j + h < n && s[i + h] == s[j + h]) h++;
         if (ht[rk[i]] = h) h--;
      }
   }
};
```

9.10 SAM

```
/*
 * [0,L] , 0 is virtual , 1 is rt , init!!
 * [l[par[s]] + 1, l[s]]
 */
struct SAM {
   static const int N = ::N << 1, M = 26;
   int par[N], l[N], ne[N][M], rt, last, L;
   void add(int c) {
      int p = last;
      /* ex
      if(ne[p][c] && l[ne[p][c]] == l[p] + 1) { last = ne[p][c]; return ; }
      */
      int np = ++L;
      fill(ne[np], ne[np] + M, 0);
      l[np] = l[p] + 1;
      last = np;
      while(p && !ne[p][c]) ne[p][c] = np, p = par[p];
      if(!p) par[np] = rt;
      else{
         int q = ne[p][c];
         if(l[q] == l[p] + 1) par[np] = q;
         else {
            int nq = ++L;
            l[nq] = l[p] + 1;
            copy(ne[q], ne[q] + M, ne[nq]);
            par[nq] = par[q];
            par[q] = par[np] = nq;
            while(p && ne[p][c] == q) ne[p][c] = nq, p = par[p];
         }
      }
   }
   void ini() {
      rt = last = L = 1;
      fill(ne[rt], ne[rt] + M, 0);
      l[0] = -1;
   }
};
// BucketSort
rep(i, 1, L + 1) ++cnt[l[i]];
rep(i, 1, L + 1) cnt[i] += cnt[i - 1];
rep(i, 1, L + 1) cur[cnt[l[i]]--] = i;
```

9.11 SA_trie

```
// trie 树点带字母，每个点到根的字符串排序， O(nlogn)
// C 为字符集大小，从 a 开始， M 为倍增深度
// 调用 Init 之后，取 sa[]
const int N = 5e5, M = 21, C = 26;
int n, fa[N]; char s[N];
struct SA {
   int R[N], RF[N], tmp[N], pos[N], tax[N], tp[N], sa[N], siz, n, pa[N][M];
   int h(int c) { return c - 'a' + 1; }
   void Qsort(int *sa, int *R, int *tp, int siz) {
      rep(i, 0, siz + 1) tax[i] = 0;
      rep(i, 1, n + 1) tax[R[tp[i]]]++;
      rep(i, 1, siz + 1) tax[i] += tax[i - 1];
      per(i, 1, n + 1) sa[tax[R[tp[i]]]--] = tp[i];
```

```
   }
   // s[] 表示字母点权，下标从 1 开始
   // fa[] 表示树上父节点编号，根为 1
   void Init(int _n, int fa[], char s[]) {
      n = _n, pa[1][0] = 0; rep(i, 2, n + 1) pa[i][0] = fa[i];
      rep(i, 2, n + 1) rep(j, 1, M) pa[i][j] = pa[pa[i][j - 1]][j - 1];
      rep(i, 1, n + 1) R[i] = h(s[i]), tp[i] = i;
      Qsort(sa, R, tp, C); rep(i, 1, n + 1) pos[sa[i]] = i;
      for (int w = 1, p = 0; w < n; w <<= 1, p++) {
         rep(i, 1, n + 1) RF[i] = pos[pa[i][p]];
         Qsort(tp, RF, sa, n);
         Qsort(sa, R, tp, R[sa[n]]);
         rep(i, 1, n + 1) tmp[i] = R[i]; R[sa[1]] = 1;
         rep(i, 2, n + 1) R[sa[i]] =
            (tmp[sa[i]] == tmp[sa[i - 1]] && tmp[pa[sa[i]][p]] == tmp[pa[sa[i - 1]][p]])
             ?
               R[sa[i - 1]] : R[sa[i - 1]] + 1;
         rep(i, 1, n + 1) pos[sa[i]] = i;
      }
   }
} T;
```

9.12 StrHash

```
const int P = 1e9 + 7, N = 101010;
struct Str {
   int B[N], h[N], ba;
   Str(int ba) : ba(ba) { B[0] = 1; rep(i, 1, N) B[i] = mul(B[i - 1], ba); }
   int upd(int a, int b) {
      if((a += b) >= P) a -= P;
      return a < 0 ? a + P : a;
   }
   int mul(int a, int b) { return 1ll * a * b % P; }
   void init(vi &s) {
      h[0] = s[0] + 1; rep(i, 1, sz(s)) h[i] = upd(mul(h[i - 1], ba), s[i] + 1);
   }
   int sub(int l, int r) {
      if(!l) return h[r];
      return upd(h[r], -mul(h[l - 1], B[r - l + 1]));
   }
}ha1(233), ha2(241);
```

9.13 StrHash_双哈希

```
const int P = 1e9 + 7;
inline int upd(int a, int b) {
   if((a += b) >= P) a -= P;
   return a < 0 ? a + P : a;
}
inline int mul(int a, int b) {return 1ll * a * b % P; }
struct Int{
   int a, b;
   Int(int a = 0, int b = 0) : a(a), b(b) {}
   inline Int operator + (const Int &c) const { return Int(upd(a, c.a), upd(b, c.b)); }
   inline Int operator - (const Int &c) const { return Int(upd(a, -c.a), upd(b, -c.b));
   }
   inline Int operator * (const Int &c) const { return Int(mul(a, c.a), mul(b, c.b)); }
   inline bool operator == (const Int &c) const {return a == c.a && b == c.b;}
} _0 = Int(), _1 = Int(1, 1), B[N];
void init(int n){
   B[0] = _1; B[1] = Int(233, 241);
   rep(i, 2, n+1) B[i] = B[i-1] * B[1];
}
struct Str{
   Int a; int len;
   Str(Int a = _0, int len = 0) : a(a), len(len) {}
   Str(int x) {a = Int(x, x); len = 1;}
   inline Str operator + (const Str &c) const { return Str(a * B[c.len] + c.a, len + c.
   len); }
   // 减去一个前缀
   inline Str operator - (const Str &c) const { return Str(a - c.a * B[len - c.len], len
    - c.len); }
   inline bool operator == (const Str &c) const { return a == c.a && len == c.len;}
} ha[N], hb[N];
void init(vi &s, Str *ha) {
   rep(i, 0, sz(s)) ha[i] = i > 0 ? ha[i-1] + Str(s[i] + 1) : Str(s[0] + 1);
}
Str sub(Str *ha, int l, int r) {
   if (l > r) return Str();
   return l > 0 ? ha[r] - ha[l-1] : ha[r];
}
```

9.14 序列自动机

```
/*
 * 0 is root
 * a is char size
 * n is string lenth
 * include empty string
 */
// 构建
for(LL i=n;i>=1;--i){
   for(LL j=1;j<=a;++j) nxt[i-1][j]=nxt[i][j];
   nxt[i-1][s[i]]=i;
}
// 求两串的公共子序列个数
LL Dfs(LL x,LL y){
   if(f[x][y]) return f[x][y];
   for(LL i=1;i<=a;++i) if(nxt1[x][i]&&nxt2[y][i])
      f[x][y]+=Dfs(nxt1[x][i],nxt2[y][i]);
   return ++f[x][y];
}
// 求回文子序列个数
LL Dfs(LL x,LL y){
   if(f[x][y]) return f[x][y];
   for(LL i=1;i<=a;++i) if(nxt1[x][i]&&nxt2[y][i]){
      if(nxt1[x][i]+nxt2[y][i]>n+1) continue;
      if(nxt1[x][i]+nxt2[y][i]<n+1) f[x][y]++;
      f[x][y]=(f[x][y]+Dfs(nxt1[x][i],nxt2[y][i]))%mod;
```

```
    }
    return ++f[x][y];
}
// 求一个 A , B 的最长公共子序列 S ，使得 C 是 S 的子序列
for(LL i=1;i<=a;++i) nxt[n][i]=n;
for(LL i=0;i<n;++i){
    for(LL j=1;j<=a;++j) nxt[i][j]=i;
    nxt[i][c[i+1]]=i+1;
}
```

9.15 最小表示法

```
// 下标从 0 开始
// s[] 开两倍长度
int MINR(char s[],int L){
    rep(i,0,L) s[L+i]=s[i]; s[2*L]=0;
    int i=0, j=1;
    while(i<L && j<L){
        int k=0;
        while(s[i+k]==s[j+k] && k<L)++k;
        if(k==L)return min(i,j);
        // 最大改成 <
        if(s[i+k]>s[j+k])i=max(i+k+1,j+1);
        else j=max(j+k+1,i+1);
    }
    return min(i,j);
}
```

10 Tree

10.1 DsuOnTree

```
// id starts with 1
namespace QuerySubtree{
    static const int N = ::N;
    int sz[N] , wson[N] , par[N];
    void dfs(int c,int fa,vi g[]){
        sz[c]=1;par[c]=fa;int &s=wson[c]=0;
        for(auto t:g[c]) if(t!=fa) dfs(t,c,g),sz[c]+=sz[t],(sz[t]>=sz[s])&&(s=t);
    }
    void solve(int c,int fa,bool iswson,vi g[]){
        for(auto t : g[c]) if(t != wson[c] && t != fa) solve(t , c , false , g);
        if(wson[c]) solve(wson[c] , c , true , g);
        for(auto t : g[c]) if(t != wson[c] && t != fa) {
            // 将该子树的信息加入
        }
        // 将当前节点的信息加入
        // 查询
        if(!iswson) {
            // 如果当前子树是轻儿子，删除这棵子树的信息
        }
    }
    void solve(vi g[]){
        dfs(1,0,g);
        solve(1,0,false,g); // 如果输入是单组数据，改成 true 可以优化常数
    }
}
```

10.2 HeavyChain

```
// id starts with 1
struct HeavyChain{
    static const int N = ::N;
    int sz[N], wson[N], top[N], dep[N], id[N], _, par[N], who[N];
    void dfs(int c, int fa, vi g[]){
        sz[c] = 1;
        par[c] = fa;
        dep[c] = dep[fa] + 1;
        int &s = wson[c] = top[c] = 0;
        for(auto t : g[c]) if(t != fa) {
            dfs(t, c, g);
            sz[c] += sz[t];
            if(sz[t] >= sz[s]) s = t;
        }
    }
    void dfs2(int c, int fa, vi g[]){
        id[c] = ++_;
        who[_] = c;
        int s = wson[c];
        if(!top[c]) top[c] = c;
        if(s) top[s] = top[c], dfs2(s, c, g);
        for(auto t : g[c]) if(t != fa && t != s) dfs2(t, c, g);
    }
    void Query(int a, int b){
        int fa = top[a], fb = top[b];
        while(fa != fb){
            if(dep[fa] < dep[fb]) swap(a, b), swap(fa, fb);
            // Cal id[fa] .. id[a]
            a = par[fa]; fa = top[a];
        }
        if(dep[a] < dep[b]) swap(a, b);
        // Cal id[b] .. id[a]
        // b is lca
    }
    void Build(vi g[]){
        dfs(1, 0, g);
        _=0;
        dfs2(1, 0, g);
    }
}hc;
```

10.3 LCARMQ

```
// N is 2 size of tree , id of nodes start from 1
struct LCARMQ{
    static const int N = 101010 << 1;
    int a[20][N] , lft[N] , dep[N] , lg[N] , L;
```

```
    int rmin(int x,int y){return dep[x] < dep[y] ? x : y;}
    void add(int x){ a[0][L++] = x;}
    void dfs(int c,int fa,const vi g[]){
        lft[c]=L;add(c);
        for(auto t : g[c]) if(t!=fa) dep[t]=dep[c]+1,dfs(t,c,g),add(c);
    }
    void Build(const vi g[]){
        L = 0;dfs(1,0,g);dep[0] = -1;
        rep(i,2,L) lg[i]=lg[i>>1]+1;
        rep(i,1,20){
            int lim = L+1-(1<<i);
            rep(j,0,lim) a[i][j] = rmin(a[i-1][j] , a[i-1][j+(1<<i>>1)]);
        }
    }
    int lca(int x,int y){
        x = lft[x] , y = lft[y];
        if(x > y) swap(x , y);
        int i = lg[y-x+1];
        return rmin(a[i][x] , a[i][y+1-(1<<i)]);
    }
};
```

10.4 LongChain

```
struct LongChain{
   static const int N = ::N;
   int wson[N] , top[N] , dep[N] , lg[N];
   int jump[N][20] , id[N] , who[N] , rwho[N] , _;
   void dfs(int c,int fa,vi g[]){
      dep[c]=1;int &s=wson[c]=top[c]=0;
      jump[c][0]=fa;rep(i,1,20) jump[c][i]=jump[jump[c][i-1]][i-1];
      for(auto t:g[c]) if(t!=fa)
         dfs(t,c,g),dep[c]=max(dep[t]+1,dep[c]),(dep[t]>=dep[s])&&(s=t);
   }
   void dfs2(int c,int fa,int rc,vi g[]){
      if(!top[c]) top[c]=c,rc=c;
      who[id[c]=++_]=c;rwho[_]=rc;
      int s=wson[c];
      if(s) top[s]=top[c],dfs2(s,c,jump[rc][0],g);
      for(auto t:g[c]) if(t!=fa&&t!=s) dfs2(t,c,t,g);
   }
   void Build(vi g[]){
      dfs(1,0,g);_=0;dfs2(1,0,1,g);
      rep(i,2,N) lg[i]=lg[i>>1]+1;
   }
   void solve(int c, int fa, vi g[]) {
      for(auto t : g[c]) if(t != fa) solve(t, c, g);
      if(wson[c]) {
         // upd c by wson[c], O(1) or O(log(n))
      } else {
         // c is leaf
      }
      for(auto t : g[c]) if(t != fa && t != wson[c]) {
         // brute force upd c by t
      }
      // 注意统计以 c 为起点的链的答案，注意深度的限制（两棵子树都要注意）
   }
   // kth_par should exist
   int kth_par(int x,int k){
      if(k==0) return x;
      int j0=1<<lg[k];
      int p0=jump[x][lg[k]];
      int j1=k-j0;
      int del=id[p0]-id[top[p0]];
      if(del>=j1) return who[id[p0]-j1];
      else return rwho[id[top[p0]]+j1-del];
   }
}hc;
```

10.5 MoOnTree_Path

```
// 不带修改莫队
// 带修改莫队：块大小 N^(2/3) 按照 l 所在块， r 所在块， time 排序
namespace MoOnTree {
   const int N = ::N, SZ = sqrt(N), M = 17;
   int cd; // starts from 1
   int dep[N], pre[N][M], st[N], ed[N], dfn[N << 1], B[N << 1], cnt[N];
   struct Node {
      int l, r, id, lca;
      Node(int id, int l, int r, int lca = 0) : id(id), l(l), r(r), lca(lca) {}
      bool operator < (const Node &c) const {
         if(B[l] != B[c.l]) return B[l] < B[c.l];
         return (r < c.r) ^ (B[l] & 1);
      }
   };
   vector<Node> nds;
   void dfs(int u, int fa, vi g[]) {
      dep[u] = dep[fa] + 1;
      pre[u][0] = fa;
      for(int i = 1; i < M && pre[u][i - 1]; ++i) pre[u][i] = pre[pre[u][i - 1]][i - 1];
      dfn[++cd] = u, st[u] = cd;
      for(auto v : g[u]) if(v != fa) dfs(v, u, g);
      dfn[++cd] = u, ed[u] = cd;
   }
   int lca(int x, int y) {
      if(dep[x] > dep[y]) swap(x, y);
      per(i, 0, M) if(dep[pre[y][i]] >= dep[x]) y = pre[y][i];
      per(i, 0, M) if(pre[x][i] != pre[y][i]) x = pre[x][i], y = pre[y][i];
      if(x == y) return x;
      return pre[x][0];
   }
   void adde(int u, int v, int id) {
      if(st[u] > st[v]) swap(u, v);
      int f = lca(u, v);
      if(f == u) { nds.pb(Node(id, st[u], st[v]));
      } else {
         int l = ed[u], r = st[v];
         if(l > r) swap(l, r);
         nds.pb(Node(id, l, r, f));
      }
```

```
   }
   // p is index in tree
   void add(int p) { }
   void sub(int p) { }
   void upd(int p, int c) {
      p = dfn[p];
      cnt[p] += c;
      (cnt[p] == 1) ? add(p) : sub(p);
   }
   void solve(vi g[]) {
      rep(i, 0, N << 1) B[i] = i / SZ;
      dfs(1, cd = 0, g);
      // adde(u, v)
      sort(all(nds));
      int l = 1, r = 0;
      for(auto &nd : nds) {
         while(r < nd.r) upd(++r, 1);
         while(l > nd.l) upd(--l, 1);
         while(r > nd.r) upd(r--, -1);
         while(l < nd.l) upd(l++, -1);
         if(nd.lca) upd(st[nd.lca], 1);
         // save ans
         if(nd.lca) upd(st[nd.lca], -1);
      }
   }
}
```

10.6 VTree

```
// sort !
namespace Vtree{
   const int N = ::N << 1;
   int tp[N], _, del[N], cntd, l[N], cntl;
   void solve(vi&v,LCARMQ&R){
      _ = cntd = 0; del[++cntd] = tp[_++] = v[0];
      rep(i, 1, sz(v)){
         int lca = R.lca(tp[_-1] , v[i]);
         cntl = 0; while(_ > 0 && R.dep[lca] < R.dep[tp[_-1]]) l[++cntl] = tp[--_];
         if(_ == 0 || lca != tp[_-1]) del[++cntd] = tp[_++] = lca;
         l[++cntl] = tp[_-1]; del[++cntd] = tp[_++] = v[i];
         rep(i, 2, cntl + 1) {
            int u = l[i], v = l[i - 1];
            // g[u].pb(v);
         }
      }
      per(i, 0, _ - 1) {
         int u = tp[i], v = tp[i + 1];
         // g[u].pb(v);
      }
      rep(i, 1, cntd + 1) {
         // del
      }
   }
}
```

10.7 点分树

```
// id starts from 1
namespace Centriod {
   const int N = ::N;
   bool vis[N]; int sz[N], par[N]; vi G[N];
   void dfssz(int c,int fa,int Sz,int &rt){
      sz[c] = 1;
      for(auto t : g[c]) if(!vis[t]&&t!=fa) dfssz(t,c,Sz,rt) , sz[c]+=sz[t];
      if(!rt && sz[c]*2>Sz) rt=c;
   }
   int dfs(int c){
      int rt=0;dfssz(c,0,0,rt);dfssz(c,0,sz[c],rt=0);
      vis[rt] = true;
      for(auto v : g[rt]) if(!vis[v]) {
         int t = dfs(v);
         G[rt].pb(t);
         par[t] = rt;
      }
      return rt;
   }
   void init() {
      fill_n(G + 1, n, vi());
      fill_n(par + 1, n, 0);
   }
};
```

10.8 点分治

```
// id starts from 1
namespace Centriod {
   const int N = ::N;
   bool vis[N]; int sz[N];
   void dfssz(int c,int fa,int Sz,int &rt){
      sz[c] = 1;
      for(auto t : g[c]) if(!vis[t]&&t!=fa) dfssz(t,c,Sz,rt) , sz[c]+=sz[t];
      if(!rt && sz[c]*2>Sz) rt=c;
   }
   void dfs(int c){
      int rt=0;dfssz(c,0,0,rt);dfssz(c,0,sz[c],rt=0);
      vis[rt] = true;
      /*
       * 注意计算以 rt 为起点的路径、只包含 rt 的路径
       * 注意 v != vis[rt]
       */
      for(auto t : g[rt]) if(!vis[t]) dfs(t);
   }
};
```

10.9 边分树

```
// init
namespace ET {
   const int N = ::N << 1;
```

```
   Gra g, T; int L, n, sz[N]; bool vis[N << 1];
   void dfssz(int u, int fa, int Sz, int &rt) {
      sz[u] = 1;
      for(int i = g.hd[u]; ~i; i = g.ne[i]) if(!vis[i] && g.to[i] != fa) {
         int v = g.to[i];
         dfssz(v, u, Sz, rt);
         sz[u] += sz[v];
         if(rt == -1 || max(sz[g.to[rt]], Sz - sz[g.to[rt]]) > max(sz[v], Sz - sz[v]))
         rt = i;
      }
   }
   void init(int n) { fill_n(vis, n << 1, 0); }
   int dfs(int u) {
      int I = 0; dfssz(u, 0, 0, I);
      if(sz[u] == n) { T.init(n); }
      if(sz[u] == 1) return u;
      dfssz(u, 0, sz[u], I = -1);
      vis[I] = vis[I ^ 1] = true;
      int _ = (I >> 1) + 1 + n, st = g.fr[I >> 1 << 1], ed = g.to[I >> 1 << 1];
      T._add(_, dfs(st));
      T._add(_, dfs(ed));
      return _;
   }
   void rebuild(int u, int fa, const Gra &G) {
      if(u == 1) L = n = ::n, g.init(n << 1);
      bool F = 0; int pre = u;
      for(int i = G.hd[u]; ~i; i = G.ne[i]) if(G.to[i] != fa) {
         if(F) {
            if(~G.ne[i]) {
               g.add(pre, ++n, 0);
               g.add(n, G.to[i], G.val[i]);
               pre = n;
            } else {
               g.add(pre, G.to[i], G.val[i]);
            }
         } else {
            g.add(u, G.to[i], G.val[i]);
            F = 1;
         }
         rebuild(G.to[i], u, G);
      }
   }
}
```

11 ZProblems

11.1 K 小子序列

```
#define M 1001000
int n, a[M]; long long k;
struct Segtree{
   Segtree *ls, *rs;
   long long sum, pos;
   Segtree(Segtree *_, Segtree *__, long long ___, long long ____):
      ls(_), rs(__), sum(___), pos(____){}
   friend Segtree* Insert(Segtree *p, int l, int r, long long x, long long val, long
   long pos){
      int mid = l + r >> 1;
      if(l == r)return new Segtree(0x0, 0x0, val, pos);
      if(x <= mid){
            Segtree *temp = Insert(p->ls, l, mid, x, val, pos);
            return new Segtree(temp, p->rs, min(temp->sum + p->rs->sum, k+1), 0);
      }
      else{
            Segtree *temp = Insert(p->rs, mid+1, r, x, val, pos);
            return new Segtree(p->ls, temp, min(temp->sum + p->ls->sum, k+1), 0);
      }
   }
   friend int Find(Segtree *p, int l, int r){
      int mid = l + r >> 1;
      if(l == r)return p->pos;
      if(k <= p->ls->sum) return Find(p->ls, l, mid);
      else{
         k -= p->ls->sum;
         return Find(p->rs, mid+1, r);
      }
   }
};
// 求字典序第 k 小的子序列
Segtree *tree[M];
int st[M], top;
int main(){
   cin >> n >> k;
   int mx = 0;
   for(int i = 1; i <= n; i ++) scanf("%d", &a[i]), mx = max(mx, a[i]);
   tree[n+1] = new Segtree(0x0, 0x0, 0, 0);
   tree[n+1]->ls = tree[n+1]->rs = tree[n+1];
   for(int i = n+1; i >= 0; i --)
       tree[i-1] = Insert(tree[i], 0, mx, a[i], i == n+1 ? 1 : tree[i]->sum, i);
   if(k > tree[0]->sum) return puts("-1"), 0;
   int now = 0;
   while(true){
      now = Find(tree[now], 0, mx);
      if(now == n+1)break;
      st[++ top] = a[now];
   }
   printf("%d\n", top);
   for(int i = 1; i <= top; i ++) printf("%d ", st[i]);
   return 0;
}
```

11.2 TT_全功能_Claris

```
#define N 200010
const int inf = ~0U >> 1;
struct tag {
   int a, b; //ax+b
   tag() { a = 1, b = 0; }
   tag(int x, int y) { a = x, b = y; }
```

```
   inline bool ex() { return a != 1 || b; }
   inline tag operator+(const tag&x) { return tag(a*x.a, b*x.a + x.b); }
};
inline int atag(int x, tag y) { return x*y.a + y.b; }
struct data {
   int sum, minv, maxv, size;
   data() { sum = size = 0, minv = inf, maxv = -inf; }
   data(int x) { sum = minv = maxv = x, size = 1; }
   data(int a, int b, int c, int d) { sum = a, minv = b, maxv = c, size = d; }
   inline data operator+(const data&x) { return data(sum + x.sum, min(minv, x.minv), max
   (maxv, x.maxv), size + x.size); }
};
inline data operator+(const data&a, const tag&b) { return a.size ? data(a.sum*b.a + a.
size*b.b, atag(a.minv, b), atag(a.maxv, b), a.size) : a; }
//son: :0—1 重链儿子， 2—3 : AAA 树儿子
int f[N], son[N][4], a[N], tot, rt, rub, ru[N], val[N]; bool rev[N], in[N];
data csum[N], tsum[N], asum[N];
tag ctag[N], ttag[N];
inline bool isroot(int x, int t) {
   if (t) return !f[x] || !in[f[x]] || !in[x];
   return !f[x] || (son[f[x]][0] != x&&son[f[x]][1] != x) || in[f[x]] || in[x];
}
inline void rev1(int x) {
   if (!x) return;
   swap(son[x][0], son[x][1]); rev[x] ^= 1;
}
inline void tagchain(int x, tag p) {
   if (!x) return;
   csum[x] = csum[x] + p;
   asum[x] = csum[x] + tsum[x];
   val[x] = atag(val[x], p);
   ctag[x] = ctag[x] + p;
}
inline void tagtree(int x, tag p, bool t) {
   if (!x) return;
   tsum[x] = tsum[x] + p;
   ttag[x] = ttag[x] + p;
   if (!in[x] && t)tagchain(x, p); else asum[x] = csum[x] + tsum[x];
}
inline void pb(int x) {
   if (!x)return;
   if (rev[x])rev1(son[x][0]), rev1(son[x][1]), rev[x] = 0;
   if (!in[x] && ctag[x].ex())tagchain(son[x][0], ctag[x]), tagchain(son[x][1], ctag[x])
   , ctag[x] = tag();
   if (ttag[x].ex()) {
      tagtree(son[x][0], ttag[x], 0), tagtree(son[x][1], ttag[x], 0);
      tagtree(son[x][2], ttag[x], 1), tagtree(son[x][3], ttag[x], 1);
      ttag[x] = tag();
   }
}
inline void up(int x) {
   tsum[x] = data();
   for (int i = 0; i < 2; i++)if (son[x][i])tsum[x] = tsum[x] + tsum[son[x][i]];
   for (int i = 2; i < 4; i++)if (son[x][i])tsum[x] = tsum[x] + asum[son[x][i]];
   if (in[x]) {
      csum[x] = data();
      asum[x] = tsum[x];
   }
   else {
      csum[x] = data(val[x]);
      for (int i = 0; i < 2; i++)if (son[x][i])csum[x] = csum[x] + csum[son[x][i]];
      asum[x] = csum[x] + tsum[x];
   }
}
inline int child(int x, int t) { pb(son[x][t]); return son[x][t]; }
inline void rotate(int x, int t) {
   int y = f[x], w = (son[y][t + 1] == x) + t;
   son[y][w] = son[x][w ^ 1];
   if (son[x][w ^ 1])f[son[x][w ^ 1]] = y;
   if (f[y])for (int z = f[y], i = 0; i < 4; i++)if (son[z][i] == y)son[z][i] = x;
   f[x] = f[y]; f[y] = x; son[x][w ^ 1] = y; up(y);
}
inline void splay(int x, int t = 0) {
   int s = 1, i = x, y; a[1] = i;
   while (!isroot(i, t))a[++s] = i = f[i];
   while (s)pb(a[s--]);
   while (!isroot(x, t)) {
      y = f[x];
      if (!isroot(y, t)) { if ((son[f[y]][t] == y) ^ (son[y][t] == x))rotate(x, t); else
       rotate(y, t); }
      rotate(x, t);
   }
   up(x);
}
inline int newnode() {
   int x = rub ? ru[rub--] : ++tot;
   son[x][2] = son[x][3] = 0; in[x] = 1;
   return x;
}
inline void setson(int x, int t, int y) { son[x][t] = y; f[y] = x; }
inline int pos(int x) { for (int i = 0; i < 4; i++)if (son[f[x]][i] == x)return i;
return 4; }
inline void add(int x, int y) { // 从 x 连出一条虚边到 y
   if (!y)return;
   pb(x);
   for (int i = 2; i < 4; i++)if (!son[x][i]) {
      setson(x, i, y);
      return;
   }
   while (son[x][2] && in[son[x][2]])x = child(x, 2);
   int z = newnode();
   setson(z, 2, son[x][2]);
   setson(z, 3, y);
   setson(x, 2, z);
   splay(z, 2);
}
inline void del(int x) { // 将 x 与其虚边上的父亲断开
   if (!x)return;
   splay(x);
   if (!f[x])return;
```

```
   int y = f[x];
   if (in[y]) {
      int s = 1, i = y, z = f[y]; a[1] = i;
      while (!isroot(i, 2))a[++s] = i = f[i];
      while (s)pb(a[s--]);
      if (z) {
         setson(z, pos(y), child(y, pos(x) ^ 1));
         splay(z, 2);
      }
      ru[++rub] = y;
   }
   else {
      son[y][pos(x)] = 0;
      splay(y);
   }
   f[x] = 0;
}
inline int fa(int x) { // x 通过虚边的父亲
   splay(x);
   if (!f[x])return 0;
   if (!in[f[x]])return f[x];
   int t = f[x];
   splay(t, 2);
   return f[t];
}
inline int access(int x) {
   int y = 0;
   for (; x; y = x, x = fa(x)) {
      splay(x);
      del(y);
      add(x, son[x][1]);
      setson(x, 1, y);
      up(x);
   }
   return y;
}
inline int lca(int x, int y) {
   access(x);
   return access(y);
}
inline int root(int x) {
   access(x);
   splay(x);
   while (son[x][0])x = son[x][0];
   return x;
}
inline void makeroot(int x) {
   access(x);
   splay(x);
   rev1(x);
}
inline void link(int x, int y) {
   makeroot(x);
   add(y, x);
   access(x);
}
inline void cut(int x) {
   access(x);
   splay(x);
   f[son[x][0]] = 0;
   son[x][0] = 0;
   up(x);
}
inline void changechain(int x, int y, tag p) {
   makeroot(x);
   access(y);
   splay(y);
   tagchain(y, p);
}
inline data askchain(int x, int y) {
   makeroot(x);
   access(y);
   splay(y);
   return csum[y];
}
inline void changetree(int x, tag p) {
   access(x);
   splay(x);
   val[x] = atag(val[x], p);
   for (int i = 2; i < 4; i++) if (son[x][i])tagtree(son[x][i], p, 1);
   up(x);
   splay(x);
}
inline data asktree(int x) {
   access(x);
   splay(x);
   data t = data(val[x]);
   for (int i = 2; i < 4; i++) if (son[x][i])t = t + asum[son[x][i]];
   return t;
}
int n, m, x, y, z, k, i, ed[N][2];
int main() {
   read(n); read(m);
   tot = n;
   for (i = 1; i < n; i++) read(ed[i][0]), read(ed[i][1]);
   for (i = 1; i <= n; i++) read(val[i]), up(i);
   for (i = 1; i < n; i++) link(ed[i][0], ed[i][1]);
   read(rt);
   makeroot(rt);
   while (m--) {
      read(k);
      if (k == 1) { // 换根
         read(rt);
         makeroot(rt);
      }
      if (k == 9) { // x 的父亲变成 y
         read(x), read(y);
         if (lca(x, y) == x)continue;
         cut(x);
         link(y, x);
```

```
        makeroot(rt);
      }
      if (k == 0) { // 子树赋值
        read(x), read(y);
        changetree(x, tag(0, y));
      }
      if (k == 5) { // 子树加
        read(x), read(y);
        changetree(x, tag(1, y));
      }
      if (k == 3) { // 子树最小值
        read(x);
        printf("%d\n", asktree(x).minv);
      }
      if (k == 4) { // 子树最大值
        read(x);
        printf("%d\n", asktree(x).maxv);
      }
      if (k == 11) { // 子树和
        read(x);
        printf("%d\n", asktree(x).sum);
      }
      if (k == 2) { // 链赋值
        read(x), read(y), read(z);
        changechain(x, y, tag(0, z));
        makeroot(rt);
      }
      if (k == 6) { // 链加
        read(x), read(y), read(z);
        changechain(x, y, tag(1, z));
        makeroot(rt);
      }
      if (k == 7) { // 链最小值
        read(x), read(y);
        printf("%d\n", askchain(x, y).minv);
        makeroot(rt);
      }
      if (k == 8) { // 链最大值
        read(x), read(y);
        printf("%d\n", askchain(x, y).maxv);
        makeroot(rt);
      }
      if (k == 10) { // 链和
        read(x), read(y);
        printf("%d\n", askchain(x, y).sum);
        makeroot(rt);
      }
    }
    return 0;
}
```

11.3 basic

```
#pragma once
#include <cassert>
#include <algorithm>
using ll = long long;
using ull = unsigned long long;
using uill = __uill_t;
// return a % b
inline ull mod128_64_small(uill a, ull b) {
  ull q, r;
  __asm__ (
    "divq\t%4"
    : "=a"(q), "=d"(r)
    : "0"(ull(a)), "1"(ull(a >> 64)), "rm"(b)
  );
  return r;
}
// mod should be not greater than 2^62 (about 4e18)
// return a * b % mod when mod is less than 2^31
inline ull mul_mod(ull a, ull b, ull mod) {
  assert(0 <= a && a < mod);
  assert(0 <= b && b < mod);
  if (mod < int(1e9)) return a * b % mod;
  ull k = (ull)((long double)a * b / mod);
  ull res = a * b - k * mod;
  if ((ll)res < 0) res += mod;
  return res;
}
inline ll add_mod(ll x, ll y, ll mod) { return (x + y) % mod; }
inline ll sub_mod(ll x, ll y, ll mod) { return (x - y + mod) % mod; }
inline ull mul_add_mod(ull a, ull b, ull c, ull mod) {
  return mod128_64_small(uill(a) * b + c, mod);
}
ll pow_mod(ll a, ll n, ll m) {
  ll res = 1;
  for (a %= m; n; n >>= 1) {
    if (n & 1) res = mul_mod(res, a, m);
    a = mul_mod(a, a, m);
  }
  return res;
}
template<typename T>
T gcd(T a, T b) { return !b ? a : gcd(b, a % b); }
// ax + by = gcd(a, b), |x| + |y| is minimum
void exgcd(ll a, ll b, ll &g, ll &x, ll &y) {
  if (!b) x = 1, y = 0, g = a;
  else {
    exgcd(b, a % b, g, y, x);
    y -= x * (a / b);
  }
}
// return x, where ax = 1 (mod mod)
ll mod_inv(ll a, ll mod) {
  if (gcd(a, mod) != 1) return -1;
  ll b = mod, s = 1, t = 0;
  while (b) {
    ll q = a / b;
    std::swap(a -= q * b, b);
```

```
    std::swap(s -= q * t, t);
  }
  return s < 0 ? s + mod : s;
}
ull crt2(ull r1, ull mod1, ull r2, ull mod2) {
  ull inv = mod_inv(mod1, mod2);
  return mul_mod(r2 + mod2 - r1,  inv, mod2) * mod1 + r1;
}
//ax + by = c,x >= 0, x is minimum
//xx = x + t * b, yy = y - t * a
bool linear_equation(ll a, ll b, ll c, ll &x, ll &y) {
  ll g;
  exgcd(a,b,g,x,y);
  if (c % g) return false;
  b /= g, a /= g, c /= g;
  x = (x % b * (c % b) % b + b) % b;
  y = (c - a * x) / b;
  return true;
}
// 求的欧拉函数值，简易版n
ll euler_phi(ll n) {
  ll ret = n;
  for (ll i = 2; i * i <= n; ++i) if (n % i == 0) {
    ret = ret / i * (i - 1);
    while (n % i == 0) n /= i;
  }
  if (n > 1) ret = ret / n * (n - 1);
  return ret;
}
```

11.4 factors_pe

```
typedef unsigned long long ull;
typedef __uint128_t dw;
typedef vector<ull> vl;
ll mod_inv(ll a, ll mod) {
   if (__gcd(a, mod) != 1) return -1;
   ll b = mod, s = 1, t = 0;
   while (b) {
      ll q = a / b;
      swap(a -= q * b, b);
      swap(s -= q * t, t);
   }
   return s < 0 ? s + mod : s;
}
// n! / p^{v_p(n!)} mod p^e, assume p^e < 2^63 - 1, pe < 10^6
// (n!)_p = \stirlingfirst{p}{1}^u f_{p,e}(u) \sum_{k=0}^{e-1} (up)^k \stirlingfirst{v
+1}{k+1} \bmod p^e
// f_{p,e} = \prod_{i=0}^{x-1}(1 + \sum_{k=1}^{e-1}\frac{\stirlingfirst{p}{k+1}}{\
stirlingfirst{p}{1}} (ip)^k)
ull fact_pe(ull n, ull p, ull e) {
   vl pows(e + 1, 1);
   ull pe = 1, min_pe = min(p, e);
   rep(i, 1, e+1) pows[i] = (pe *= p);
   ull period = pe / p * 2, deg = e * 2 - 1;
   if (p == 2 && e >= 3) period >>= 1;

   // first kind stirling number: O(p * min(p, e))
   vl s1(p * min_pe); s1[0] = 1;
   rep(i, 1, p) {
      int o = i * min_pe;
      s1[o] = (dw)s1[o - min_pe] * i % pe;
      rep(j, 1, min_pe)
         s1[o + j] = (s1[o + j - min_pe - 1] + (dw)s1[o + j - min_pe] * i) % pe;
   }

   // product of {up + 1, ..., up + v} mod p^e
   auto fact_range = [&] (ull u, ull v) {
      ull coef = (dw)u % pe * p % pe, prod = 1, ret = 0;
      for (ull k = 0; k < min_pe; ++k) {
         ret = (ret + (dw)prod * s1[v * min_pe + k]) % pe;
         prod = (dw)prod * coef % pe;
      }
      return ret;
   };

   // f_{p,e}(0..2e-2): O(e * min(p, e) + e log(p))
   ull fac = fact_range(0, p - 1), ifac = mod_inv(fac, pe);
   vl f_pe(deg, 1);
   for (ull i = 1; i < deg; ++i) {
      f_pe[i] = (dw)f_pe[i - 1] * fact_range(i - 1, p - 1) % pe * ifac % pe;
   }

   // coprime factorials: O(e + e log(p))
   vl cifac(deg, 1), cfac_vs(deg);
   ull prod = 1;
   for (ull i = 1; i < deg; ++i) {
      ull j = i, v = 0;
      for (; j % p == 0; j /= p, ++v);
      cfac_vs[i] = cfac_vs[i - 1] + v;
      cifac[i - 1] = j;
      prod = (dw)prod * j % pe;
   }
   cifac[deg - 1] = mod_inv(prod, pe);
   for (int i = deg - 2; i >= 0; --i) {
      cifac[i] = (dw)cifac[i + 1] * cifac[i] % pe;
   }

   // find the value of f_{p, e}(x): O(e log x)
   auto evaluate = [&](ull x) {
      if (x < deg) return f_pe[x];
      vl vs(deg), inv(deg);
      ull v = 0, prod = 1;
      for (ull i = 0; i < deg; ++i) {
         ull m = x - i;
         for (; m % p == 0; m /= p, ++vs[i]);
         v += vs[i];
         inv[i] = prod;
         prod = (dw)prod * m % pe;
      }
```

```
      ull iprod = mod_inv(prod, pe);
      for (int i = deg - 1; i >= 0; --i) {
         inv[i] = (dw)iprod * inv[i] % pe;
         iprod = (dw)iprod * ((x - i) / pows[vs[i]]) % pe;
      }
      ull ret = 0;
      for (ull i = 0; i < deg; ++i) {
         ull j = deg - 1 - i, ex = v - vs[i] - cfac_vs[i] - cfac_vs[j];
         if (ex >= e) continue;
         ull add = (dw)cifac[j] * cifac[i] % pe;
         if (j & 1) add = pe - add;
         add = (dw)pows[ex] * prod % pe * inv[i] % pe * add % pe * f_pe[i] % pe;
         ret = (ret + add) % pe;
      }
      return ret;
   };

   // ((up+v)!)_p mod p^e: O(min(p, e))
   auto fact_p = [&](ull u, ull v) {
      return (dw)fact_range(u, v) * evaluate(u) % pe;
   };

   ull ret = 1, ex = 0;
   while (n > 0) {
      ull q = n / p, v = n % p;
      ull u = q % period;
      ret = (dw)ret * fact_p(u, v) % pe;
      ex += u, n = q;
   }
   for (ex %= period; ex; ex >>= 1) {
      if (ex & 1) ret = (dw)ret * fac % pe;
      fac = (dw)fac * fac % pe;
   }
   return ret;
}
```

11.5 min_26

```
/*
 * f() 函数中 (31-37 行 ) 填函数在质数幂次处的表达式
 * pow_sum() 函数中 (38-43 行 ) 填幂和函数（如果需要更高次的话可以在这里添加）
 * 202-205 行按要求填写
 * f_p[][0/1/2/3/...] 分别代表质数个数 / 质数和 / 质数平方和 / 质数三次方和 /... 根据自己需要添加
 * 例：如果该函数在质数处表达式为 f(p) = p^2+3*p+1 ，则表明需要质数个数 / 质数和 / 质数平方和，即 f_p[][0],f_p[][1],f_p[][2]
 */
const int maxn = 2000000+100;
ll poww(ll a,ll b){
   ll res = 1;
   ll base = a;
   while(b){
      if(b&1){
         res *= base;
         //res %= mod;
      }
      base *= base;
      //base %= mod;
      b>>=1;
   }
   return res;
}
inline ll f(ll p,int e){
   if(p==1||e==0) return 1;
   ///return f(p^e)
   ll res = poww(p,e);
   return res*res+3*res+1;
}
ll pow_sum(ll n,int k){
   ///return sum(i^k),i from 1 to n.
   if(k==0) return n;
   if(k==1) return n*(n+1)/2;
   if(k==2) return n*(n+1)*(2*n+1)/6;
}
ll f_p[maxn][3];///F_prime(id(n/i))
ll n;
int n_2; ///(int)sqrt(n)
int n_3; ///(int)pow(n,1.0/3.0)
int n_6; ///(int)pow(n,1.0/6.0)
ll val_id[maxn]; ///give the id, return the id-th number like 'n/i' ,(val_id[1] = 1)
int val_id_num; ///how many numbers like 'n/i'
int val_id_num_3; ///how many numbers like 'n/i' below n/n_3;
int p[200000+100];
bool isp[maxn];
int p_sz_2; ///pi(n_2)
int p_sz_3; ///pi(n_3)
int p_sz_6; ///pi(n_6)
void init(){
   n_2 = (int)sqrt(n);
   n_3 = (int)pow(n,1.0/3.0);
   n_6 = (int)pow(n,1.0/6.0);
   val_id_num = 0;
   for(ll i=1;i<=n;){
      val_id[++val_id_num] = i;
      if(i==n) break;
      i = n/(n/(i+1));
   }
   memset(isp,1,sizeof isp);
   isp[1] = 0;
   for(int i=2;i<=n_2;i++){
      if(isp[i]){
         p[++p_sz_2] = i;
         if(i<=n_3) p_sz_3++;
         if(i<=n_6) p_sz_6++;
      }
      for(int j=1;j<=p_sz_2&&p[j]*i<=n_2;j++){
         isp[i*p[j]] = 0;
         if(i%p[j]==0) break;
      }
   }
```

```
}
inline int get_id(ll k){ ///give a number like 'n/i', return the id of it
   if(k>n_2) return val_id_num-n/k+1;
   else return k;
}
ll c[maxn];
int lowbit(int n){return n & (-n);}
void add(int x,ll d){
   while(x<maxn){
      c[x]+=d;
      x+=lowbit(x);
   }
}
ll sum(int x){
   ll ans=0;
   while(x){
      ans+=c[x];
      x-=lowbit(x);
   }
   return ans;
}
struct node{
   int k_max;
   ll val;
   ll f_val;
};
void update_bfs(int k,int type){
   queue<node> q;
   while(!q.empty()) q.pop();
   int e = 1;
   for(ll i=p[k];i<n/n_3;i*=p[k]){
      node st;
      st.k_max = k;
      st.val = i;
      if(type==-1)st.f_val = f(p[k],e);
      else st.f_val = poww(i,type);
      q.push(st);
      e++;
   }
   while(!q.empty()){
      node hd = q.front();
      q.pop();
      if((hd.val!=p[hd.k_max]&&type>=0)||type==-1) {//if(type==-1)cout << "****" << hd.
      val << "****" << hd.f_val << endl;
         ll w = n/hd.val;
         w = n/w;//cout << hd.val << "[" << w<<" , " << val_id[val_id_num] << "]" <<
         endl;
         if(type==-1){
            add(get_id(w),hd.f_val);
            add(val_id_num+1,-1ll*hd.f_val);
         }
         else{
            add(get_id(w),-1ll*hd.f_val);
            add(val_id_num+1,hd.f_val);
         }
      }
      for(int i=hd.k_max+1;hd.val*p[i]<n/n_3&&i<=p_sz_2;i++){
         ll res = p[i];
         for(int e=1;;e++){
            if(hd.val*res<n/n_3){
               node nxt;
               nxt.k_max = i;
               nxt.val = hd.val*res;
               if(type==-1) nxt.f_val = hd.f_val*f(p[i],e);
               else nxt.f_val = hd.f_val*poww(res,type);
               q.push(nxt);
            }
            else break;
            res *= p[i];
         }
      }
   }
}
void get_f_p(ll n,int times){
   for(int i=1;i<=val_id_num;i++){
      for(int j=0;j<=times;j++){
         f_p[i][j] = pow_sum(val_id[i],j)-1;
      }
   }
   int now;
   //for(now=1;now<=p_sz_2;now++){
   for(now=1;p[now]<=n_6;now++){
      for(int j=val_id_num;j>=1;j--){
         ll w = val_id[j]/p[now];
         if(w<p[now]) break;
         ll val=1;
         for(int k = 0;k<=times;k++){
            f_p[j][k] = f_p[j][k] - val*(f_p[get_id(w)][k]-f_p[p[now-1]][k]);
            val *= p[now];
         }
      }
   }
   int nnow = now, val = 1;
   for(int tt = 0;tt<=times;tt++){
      now = nnow;
      memset(c,0,sizeof c);
      add(1,f_p[1][tt]);
      for(int i=2;val_id[i]<n/n_3;i++) add(i,f_p[i][tt] - f_p[i-1][tt]);
      for(;p[now]<=n_3;now++){
         for(int j=val_id_num;j>=1;j--){
            ll w = val_id[j]/p[now];
            if(val_id[j]<n/n_3) break;
            if(w<p[now]) break;
            if(w<n/n_3) f_p[j][tt] = f_p[j][tt] - (sum(get_id(w)) - sum(p[now-1]))*poww(
            p[now],tt);
            else f_p[j][tt] = f_p[j][tt] - (f_p[get_id(w)][tt]-sum(p[now-1]))*poww(p[now
            ],tt);
         }
         update_bfs(now,tt);
      }
```

```
      for(int i=1;i<=val_id_num&&val_id[i]<n/n_3;i++) f_p[i][tt] = sum(i);
      for(;now<=p_sz_2;now++){
         for(int j=val_id_num;j>=1;j--){
            ll w = val_id[j]/p[now];
            if(val_id[j]<n/n_3) break;
            if(w<p[now]) break;
            f_p[j][tt] -= (f_p[get_id(w)][tt]-f_p[p[now-1]][tt])*poww(p[now],tt);
         }
      }
   }
   for(int i=1;i<=val_id_num;i++){
      ///if f(p) = p^2+3p+1,then write:f_p[i][0] = f_p[i][2] + 3*f_p[i][1] + f_p[i][0];
      f_p[i][0] = f_p[i][2] + 3*f_p[i][1] + f_p[i][0];
   }
}
ll F[2000000+100];
void get_f_3(ll n){ ///V(F_{pi(n^(1/3))+1},n)
   ll q = p[p_sz_3+1];
   for(int now=1;now<=val_id_num;now++){
      if(val_id[now]<q){
         F[now] = 1;
      } else if(val_id[now]<q*q){
         F[now] = 1+(f_p[now][0]-f_p[q-1][0]);
      } else{
         F[now] = 1+(f_p[now][0]-f_p[q-1][0]);
         for(int pp=p_sz_3+1;p[pp]<=(int)(sqrt(val_id[now]))&&pp<=p_sz_2;pp++){
            F[now] += f(p[pp],2) + (f(p[pp],1))*(f_p[get_id(val_id[now]/p[pp])][0]-f_p[
            get_id(p[pp])][0]);
         }
      }
   }
}
void get_f_6(ll n){ ///V(F_{pi(n^(1/6))+1},n)
   memset(c,0,sizeof c);
   add(1,F[1]);
   for(int i=2;val_id[i]<n/n_3;i++) add(i,F[i] - F[i-1]);
   for(int k=p_sz_3;k>p_sz_6;k--){
      int now = val_id_num;
      for(;val_id[now]>=n/n_3;now--){
         int e = 1;
         ll _p = p[k];
         while(val_id[now]/_p){
            if(val_id[now]/_p>=n/n_3){
               F[now] += F[get_id(val_id[now]/_p)]*f(p[k],e);
            } else{
               F[now] += sum(get_id(val_id[now]/_p))*f(p[k],e);
            }
            _p *= p[k];
            e++;
         }
      }
      if(k==1) break;
      //cout << "******" << p[k] << "******" << n/n_3 << endl;
      update_bfs(k,-1);///bfs to update [lpf(i)==P{k-1}]f(i)
   }
   for(int i=1;i<=val_id_num&&val_id[i]<n/n_3;i++) F[i] = sum(i);
}
void get_f(ll n){
   for(int k=p_sz_6;k>=1;k--){
      for(int now = val_id_num;now>=1;now--){
         int e = 1;
         ll _p = p[k];
         while(val_id[now]/_p){
            F[now] += F[get_id(val_id[now]/_p)]*f(p[k],e);
            _p *= p[k];
            e++;
         }
      }
   }
}
int main(){//n = 1000000000; //1e10:455052511,0.83s/0.58s 1e12:37607912018 9.224s/5.105s
   cin >> n;
   init();
   get_f_p(n,2);
   get_f_3(n);
   get_f_6(n);
   get_f(n);
   for(int i=1;i<=val_id_num;i++) cout << val_id[i] << " : " << F[i] << endl;
}
```

11.6 三元闭包边计数

```
/*
 * 三元闭包边计数
 * 染色 : 能用 A , B , C 三种颜色染色，满足对所有边 u->v 有 v 的颜色是 u 的下一种颜色
 * 对每一个弱连通子图：
 * 1. 如果能染色且没用够三种颜色，不能增加边
 * 2. 如果能染色且用了三种颜色，把点按颜色分为三类，三类点中相邻两类都有边
 * 3. 如果不能染色，所有点之间都有边
 */
const int P = 1e9 + 7, N = 1e5 + 8;
int add(int a, int b) {if((a += b) >= P) a -= P; return a < 0 ? a + P : a;}
int mul(int a, int b) {return 1ll * a * b % P;}
int kpow(int a, int b) {int r=1;for(;b;b>>=1,a=mul(a,a)) {if(b&1)r=mul(r,a);}return r;}
bool ok; int vis[N], n, m, u, v; ll use[N], ans; vi g[N], gg[N], tmp;
int inc(int x) {
   x++;
   if (x == 4) x = 1;
   return x;
}
int dec(int x) {
   x--;
   if (x == 0) x = 3;
   return x;
}
void dfs(int u) {
   tmp.pb(u);
   for (auto v : g[u]) {
      if (!vis[v]) {
         vis[v] = inc(vis[u]);
```

```
         dfs(v);
      }else {
         if (vis[v] != inc(vis[u])) ok = 0;
      }
   }
   for (auto v : gg[u]) {
      if (!vis[v]) {
         vis[v] = dec(vis[u]);
         dfs(v);
      }else {
         if (vis[v] != dec(vis[u])) ok = 0;
      }
   }
}
int main() {
   cin >> n >> m;
   rep(i, 1, m+1) {
      cin >> u >> v;
      g[u].pb(v);
      gg[v].pb(u);
   }
   rep(i, 1, n+1) if (!vis[i]) {
      ok = 1; tmp.clear(); vis[i] = 1;
      dfs(i);
      if (!ok) { ans += 1ll * sz(tmp) * sz(tmp);
      }else {
         rep(i, 1, 4) use[i] = 0;
         for (auto u : tmp) use[vis[u]]++;
         rep(i, 1, 4) ok &= use[i] > 0;
         if (ok) ans += use[1] * use[2] + use[2] * use[3] + use[3] * use[1];
         else {
            int t = 0;
            for (auto u : tmp) ans += sz(g[u]);
         }
      }
   }
   cout << ans << endl;
   return 0;
}
```

11.7 六元环计数

```
// ans = 非六元简单环计数
// 六元环 = ( 只用走六步的所有方案 - ans) / 6
// time : O(n^3 / 64)
const int N = 1e3 + 7;
bitset<N> a[N]; int n; string s; ll ans, b[9], f[N], g[N], d[N];
ll c[9] = {24, 48, 2, 12, 6, 12, 36, 12, 24};
int main(){
   rep(i, 1, N) f[i] = i * (i-1) / 2, g[i] = i * (i-1) * (i-2) / 6;
   while (cin >> n){
      rep(i, 0, n) a[i].reset();
      rep(i, 0, n) {
         cin >> s;
         rep(j, 0, n) if (s[j] == '1') a[i].set(j);
         d[i] = a[i].count();
      }
      rep(i, 0, n) rep(j, i+1, n) {
         int x = (a[i] & a[j]).count();
         if (a[i][j]) b[0] += x;
         b[1]+=f[x];
         if (a[i][j]) b[2]++;
         if (a[i][j]) b[4] += (d[i]-1)*(d[j]-1);
         if (a[i][j]) b[6] += f[x];
         b[7] += f[x] * (d[i] + d[j] - 4);
      }
      rep(i, 0, n) {
         b[3] += f[d[i]];
         b[5] += g[d[i]];
         ll res = 0;
         rep(j, 0, n) if (a[i][j]) res += (a[i] & a[j]).count();
         res /= 2; b[8] += res * (res - 1) / 2;
      }
      b[0]/=3; b[1]/=2; b[4]-=3*b[0]; b[7]-=2*b[6]; b[8]-=2*b[6]; ans=0;
      rep(i, 0, 9) ans += b[i] * c[i], b[i] = 0;
      cout << ans << endl;
   }
   return 0;
}
```

11.8 动态 k 大

```
// zoj 2112 动态区间 k 大
const int N = 50505, M = 10101;
int n, m, a[N], rt[N<<1];
vi V, add, sub;
inline int rk(int x) { return lower_bound(all(V), x) - V.begin(); }
struct Q {
   bool op;
   int a, b, k;
}q[M];
struct Seg {
   static const int N = 2500005;//(::N + 32 * ::M) * 16;
   int cntn, cnt[N], ls[N], rs[N];
   void init() { fill_n(rt+1, n, cntn = 0); }
   void upd(int pre, int &now, int p, int c, int l, int r) {
      now = ++cntn;
      cnt[now] = cnt[pre] + c;
      ls[now] = ls[pre];
      rs[now] = rs[pre];
      if(l == r) return ;
      int mid = l+r>>1;
      if(p<=mid) upd(ls[pre], ls[now], p, c, l, mid);
      else upd(rs[pre], rs[now], p, c, mid+1, r);
   }
   int qry(int L, int R, int k, int l, int r) {
      if(l == r) return l;
      int mid = l+r>>1;
      int lc = 0;
      for(auto i : add) lc += cnt[ls[i]];
```

```
      for(auto i : sub) lc -= cnt[ls[i]];
      if(lc>=k) {
         rep(i, 0, sz(add)) add[i] = ls[add[i]];
         rep(i, 0, sz(sub)) sub[i] = ls[sub[i]];
         return qry(L, R, k, l, mid);
      } else {
         rep(i, 0, sz(add)) add[i] = rs[add[i]];
         rep(i, 0, sz(sub)) sub[i] = rs[sub[i]];
         return qry(L, R, k-lc, mid+1, r);
      }
   }
}seg;
struct Fenwick {
#define lb(x) ((x)&(-x))
   void init() { fill_n(rt+1+n, n, 0); }
   void upd(int x, int p, int c) {
      for(; x<=n; x+=lb(x)) seg.upd(rt[x+n], rt[x+n], p, c, 0, sz(V)-1);
   }
   int qry(int l, int r, int k) {
      add.clear();sub.clear();
      add.pb(rt[r]);sub.pb(rt[l-1]);
      int x = r;
      for(; x; x^=lb(x)) add.pb(rt[n+x]);
      x = l-1;
      for(; x; x^=lb(x)) sub.pb(rt[n+x]);
      return seg.qry(l, r, k, 0, sz(V)-1);
   }
}fw;
int main() {
   int T; cin >> T;
   while(T--) {
      cin >> n >> m;
      V.clear(); seg.init(); fw.init();
      ///read
      rep(i, 1, n+1) cin >> a[i], V.pb(a[i]);
      rep(i, 1, m+1) {
         string s;
         cin >> s >> q[i].a >> q[i].b;
         q[i].op = (s[0]=='Q');
         if(s[0]=='Q') cin >> q[i].k;
         else V.pb(q[i].b);
      }
      ///solve
      sort(all(V));
      V.erase(unique(all(V)), V.end());
      rep(i, 1, n+1) seg.upd(rt[i-1], rt[i], rk(a[i]), 1, 0, sz(V)-1);
      rep(i, 1, m+1) {
         if(q[i].op) { cout << V[fw.qry(q[i].a, q[i].b, q[i].k)] << endl;
         } else {
            int p = q[i].a, c = q[i].b;
            fw.upd(p, rk(a[p]), -1);
            fw.upd(p, rk(a[p] = c), 1);
         } } }
   return 0; }
```

11.9 动态树上路径 k 大

```
int n, m, L, dfn, val[N], rt[N], cnt[M], ls[M], rs[M], st[N], ed[N], fw[N], par[N];
pair<int, pii> Q[N]; vi V, res[2], g[N]; LCARMQ R;
int F(int x) { return lower_bound(all(V), x) - V.begin() + 1; }
// seg
void upd(int &now, int pre, int p, int c, int l, int r) {
  now = ++L;
  cnt[now] = cnt[pre] + c;
  ls[now] = ls[pre];
  rs[now] = rs[pre];
  if(l == r) return ;
  int mid = l + r >> 1;
  if(p <= mid) upd(ls[now], ls[pre], p, c, l, mid);
  else upd(rs[now], rs[pre], p, c, mid + 1, r);
}
int qry(int k, int l, int r) {
  if(l == r) return l;
  int mid = l + r >> 1;
  int cntr = 0;
  rep(i, 0, sz(res[0])) cntr += cnt[rs[res[0][i]]];
  rep(i, 0, sz(res[1])) cntr -= cnt[rs[res[1][i]]];
  if(cntr >= k) {
    rep(i, 0, sz(res[0])) res[0][i] = rs[res[0][i]];
    rep(i, 0, sz(res[1])) res[1][i] = rs[res[1][i]];
    return qry(k, mid + 1, r);
  } else {
    rep(i, 0, sz(res[0])) res[0][i] = ls[res[0][i]];
    rep(i, 0, sz(res[1])) res[1][i] = ls[res[1][i]];
    return qry(k - cntr, l, mid);
  }
}
// build 主席树
void dfs(int u, int fa) {
  st[u] = ++dfn;
  par[u] = fa;
  upd(rt[u], rt[fa], F(val[u]), 1, 1, sz(V));
  rep(i, 0, sz(g[u])) if(g[u][i] != fa) dfs(g[u][i], u);
  ed[u] = dfn;
}
// fenwick
void upd(int p, int o, int c) { for( ; p <= n; p += lb(p)) upd(fw[p], fw[p], c, o, 1, sz
(V)); }
void upd(int u, int o) {
  res[o].pb(rt[u]);
  int p = st[u]; for( ; p; p ^= lb(p)) res[o].pb(fw[p]);
}
void solve() {
  R.Build(g);
  dfs(1, 0);
  rep(i, 1, m + 1) {
    if(!Q[i].fi) {
      int p = Q[i].se.fi, c = Q[i].se.se;
      upd(st[p], -1, F(val[p]));
      upd(ed[p] + 1, 1, F(val[p]));
```

```
      val[p] = c;
      upd(st[p], 1, F(val[p]));
      upd(ed[p] + 1, -1, F(val[p]));
    } else {
      rep(o, 0, 2) res[o].clear();
      int a = Q[i].se.fi, b = Q[i].se.se, k = Q[i].fi;
      int c = R.lca(a, b), d = par[c];
      upd(a, 0); upd(b, 0);
      upd(c, 1); upd(d, 1);
      cout << V[qry(k, 1, sz(V)) - 1] << endl;
    } } }
```

11.10 勾股数计数

```
const int N = 34000;
int mu[N], p[N], tot = 0, Sqr, n;
bool vis[N];
int w1[N * 2], w2[N * 2], id1[N * 2], id2[N * 2], t1;// 注意 longlong
void init(){
    mu[1] = 1;
    rep(i, 2, N) {
        if (!vis[i]) p[++tot] = i,mu[i]=-1;
        for(int j = 1; j <= tot && p[j] * i < N; j++){
            int u = p[j] * i;
            vis[u] = 1;
            if(i % p[j] == 0) { mu[u] = 0; break; }
            mu[u] = -mu[i];
        }
    }
}
int id(int x) { return x <= Sqr ? id1[x] : id2[n / x];}
ll h1(int n){
    ll ans = 0;
    for(int l = 1, r; l<=n; l = r + 1){
        r = n / (n / l);
        ans += 1ll * (n / l) * (w1[id(r)] - w1[id(l-1)]);
    }
    return ans;
}
ll h2(int n){
    ll ans = 0;
    for(int l = 1, r; l <= n; l = r + 1){
        r = n / (n / l);
        ans += 1ll * (n / l) * (w2[id(r)] - w2[id(l-1)]);
    }
    return ans;
}
ll solve(int _n) {
   n = _n;
   Sqr = sqrt(n); t1 = 0;
    for(int l = 1, r; l <= n; l = r + 1){
        r = n / (n / l);
        int cnt1 = 0, cnt2 = 0;
        for(int i = 1; i * i < r; i++) cnt1 += sqrt(r - i * i);
        for(int i = 1; i * i < r; i += 2) cnt2 += (sqrt(r - i * i) + 1) / 2;
        w1[++t1] = cnt1, w2[t1] = cnt2;
        r <= Sqr ? id1[r] = t1 : id2[n / r] = t1;
    }
    ll ans = 0;
    for(int k = 1; k * k <= n; k++){
        if (mu[k] == 0) continue;
        ans += h1(n/k/k) * mu[k];
        if (k & 1) ans -= h2(n/k/k) * mu[k];
    }
    return ans / 2;
}
int T, nn;
int main() {
   init();
   cin >> T;
   rep(cas, 0, T) {
      cin >> nn;
      cout << solve(nn) << endl;
   }
   return 0;
}
```

11.11 区间 border 查询

```
const int N = 2e5 + 100, maxlog = 20;
struct Seq{
    /** l + k*d <=r **/
    int l,r,d;
   Seq(int l = 0, int r = 0, int d = 0) : l(l), r(r), d(d) {}
    Seq(const vi & pos){
        if (pos.empty()){
            l = r = d = 0;
        }else if (pos.size() == 1){
            l = pos.front();
            r = pos.front();
            d = 1;
        }else{
            l = pos.front();
            r = pos.back();
            d = pos[1] - pos[0];
        }
    }
    bool has(int x){ return d and x >= l and x <= r and x % d == l % d; }
    int count(){
        if (d == 0)return 0;
        return (r - l) / d + 1;
    }
    vi to_list(){
        vi list(0);
        if (d == 0)return list;
        for (int i=l;i<=r;i+=d)list.pb(i);
        return list;
    }
};
Seq operator -(int X, Seq S){return Seq(X - S.r,X - S.l, S.d);}
```

```
Seq operator -(Seq S, int X){return Seq(S.l - X, S.r - X, S.d);}
Seq operator &(Seq S1, Seq S2){
    int cnt1 = S1.count(), cnt2 = S2.count();
    if (cnt1 == 0 || cnt2 == 0) return Seq(0,0,0);
    if (cnt1 > cnt2) swap(S1,S2), swap(cnt1,cnt2);
    if (cnt1 < 3){
        vi pos(0);
        for (int x : S1.to_list()){
            if (S2.has(x)) pos.pb(x);
        }
        return Seq(pos);
    }else{
        if (S1.d == S2.d){
            int l = max(S1.l,S2.l), r = min(S1.r,S2.r);
            if (r >= l && S1.l % S1.d == S2.l % S1.d)return Seq(l,r,S1.d);
            else return Seq(0,0,0);
        }else assert(0);
    }
}
struct Dictionary_of_Basic_Factories{
    /** 1-base **/
    vector<vi> pos[maxlog];
    int name[N][maxlog], cntA[N],cntB[N],tsa[N],A[N],B[N], sa[N],rk[N];
    void init(char *ch, int n){
        ch[0] = ch[n+1] = -1;
        for (int i=1;i<=n;i++)cntA[ch[i]]++;
        for (int i=1;i<N;i++)cntA[i] += cntA[i-1];
        for (int i=n;i>=1;i--)sa[cntA[ch[i]]--] = i;
        rk[sa[1]] = 1;
        for (int i=2;i<=n;i++){
            rk[sa[i]] = rk[sa[i-1]];
            if (ch[sa[i]] != ch[sa[i-1]])rk[sa[i]] ++;
        }
        pos[0].resize(rk[sa[n]] + 1,vi(0));
        for (int i=1;i<=n;i++){
            name[i][0] = rk[i];
            pos[0][rk[i]].pb(i);
        }
        for (int step = 1,l=1;l <= n;l<<=1,step ++){
            for (int i=0;i<=n;i++)cntA[i] = cntB[i] = 0;
            for (int i=1;i<=n;i++){
                cntA[A[i] = rk[i]] ++;
                cntB[B[i]=(i+l<=n)?rk[i+l]:0]++;
            }
            for (int i=1;i<=n;i++)cntB[i] += cntB[i-1];
            for (int i=n;i>=1;i--)tsa[cntB[B[i]]--] = i;
            for (int i=1;i<=n;i++)cntA[i] += cntA[i-1];
            for (int i=n;i>=1;i--)sa[cntA[A[tsa[i]]]--] = tsa[i];
            rk[sa[1]] = 1;
            for (int i=2;i<=n;i++){
                rk[sa[i]] = rk[sa[i-1]];
                if (A[sa[i]] != A[sa[i-1]] || B[sa[i]] != B[sa[i-1]])rk[sa[i]] ++;
            }
            pos[step].resize(rk[sa[n]] + 1,vi (0));
            for (int i = 1;i <=n;i++){
                name[i][step] = rk[i];
                pos[step][rk[i]].pb(i);
            }
        }
    }
    // get sequence [2^step,2^(step+1))
    Seq get_seq(vi & list, int l, int r){
        vi pos(0);
        int idx = lower_bound(list.begin(),list.end(),l) - list.begin();
        while (idx < list.size() && pos.size() < 3 && list[idx] <= r){
            pos.pb(list[idx]);idx ++;
        }
        if (pos.size() < 3)return Seq(pos);
        else{
            int last = upper_bound(list.begin(),list.end(),r) - list.begin() - 1;
            int L = pos.front(), d = pos[1] - pos[0], R = list[last];
            return Seq(L,R,d);
        }
    }
    Seq get_border(int l,int r,int step){
        int len = r - l + 1;
        int baby = 1 << step, giant = min(len-1,(baby * 2-1));
        int namel = name[l][step], namer = name[r - baby + 1][step];
        Seq seql = get_seq(pos[step][namel],r - giant + 1,r - baby + 1),
            seqr = get_seq(pos[step][namer],l,l + giant - baby);
        seql = (r + 1) - seql; seqr = seqr - (l -baby);
        return  seql & seqr;
    }
    /** return O(logn) border series of S[l,r].
     * Attention: can contain empty sequence (0,0,0)
     * if [2^i,2^(i+1)) border does not exist.*/
    vector<Seq> get_border_series(int l,int r){
        vector<Seq> ret(0);
        for (int step = 0;(1<<step) < r - l + 1;step++) ret.pb(get_border(l,r,step));
        return ret;
    }
    int get_biggest_border(int l,int r){
        int len = r - l + 1;
        for (int k = maxlog - 1; k >=0;k--){
            if ((1<<k) >= len)continue;
            Seq seq = get_border(l,r,k);
            if (seq.r)return seq.r;
        }
        return 0;
    }
}dbf;
char s[N]; int n,q;
int main(){
    scanf("%d%d",&n, &q);
    scanf("%s",s + 1);
    dbf.init(s, n);
    while (q--){
        int l,r;
        scanf("%d%d",&l,&r);
        printf("%d\n",dbf.get_biggest_border(l,r));
```

```
    }
    return 0;
}
```

11.12 区间本质不同回文子串计数

```
namespace Space {
   const int N = 600005;
   char s[N]; int m, n, l, r; ll ans = 0, ret[N]; vector<pii> Q[N];
   struct SegTree {
      int a[N << 2];
      int query(int x, int l, int r, int fl, int fr) {
         if (l == fl && r == fr) return a[x];
         int t = (l + r) >> 1;
         if (fr <= t) return query(x << 1, l, t, fl, fr);
         else if (fl > t) return query(x << 1 | 1, t + 1, r, fl, fr);
         else return max(query(x << 1, l, t, fl, t), query(x << 1 | 1, t + 1, r, t + 1,
         fr));
      }
      void updata(int x, int l, int r, int pos, int y) {
         if (l == r) { a[x] = y; return; }
         int t = (l + r) >> 1;
         if (pos <= t)  updata(x << 1, l, t, pos, y);
         else updata(x << 1 | 1, t + 1, r, pos, y);
         a[x] = max(a[x << 1], a[x << 1 | 1]);
      }
   }Seg;
   struct BIT {
      int d[N];
      void updata(int x, int y) { for (int i = x; i < N; i += i&-i)d[i] += y; }
      int sum(int x) {
         int res = 0; for (int i = x; i; i -= i&-i)res += d[i];
         return res;
      }
   }BT;
   struct PAM {
      int next[N][26], fail[N], len[N], s[N], id[N], last, n, p;
      int in[N], out[N], d[N], up[N], dfn = 0;
      vector<int> G[N];    // dfs 序
                           // r 为结尾的回文串的长度一定可以分成 logn 段等差数列
      inline int newnode(int l) {
         for (int i = 0; i < 26; ++i) next[p][i] = 0;
         len[p] = l;
         return p++;
      }
      inline void init() {
         p = 0; newnode(0); newnode(-1);
         last = n = 0; s[n] = -1; fail[0] = 1;
      }
      inline int get_fail(int x) {
         while (s[n - len[x] - 1] != s[n]) x = fail[x];
         return x;
      }
      inline void add(int c, int cc) {
         c -= 'a';
         s[++n] = c;
         int cur = get_fail(last);
         if (!next[cur][c]) {
            int now = newnode(len[cur] + 2);
            fail[now] = next[get_fail(fail[cur])][c];
            next[cur][c] = now;
            d[now] = len[now] - len[fail[now]];
            up[now] = (d[fail[now]] == d[now] ? up[fail[now]] : now);
         }
         last = next[cur][c];
         id[cc] = last;
      }
      void build() { for (int i = 0; i < p; i++)if (i != 1) G[fail[i]].pb(i); }
      void dfs(int x = 1) {
         in[x] = ++dfn;
         for (auto i : G[x]) dfs(i);
         out[x] = dfn;
      }
      void solve() {
         for (int i = 1; i <= n; i++) {
            for (int j = id[i]; j; j = fail[up[j]]) {
               int l = max(1, Seg.query(1, 1, dfn, in[j], out[j]) - len[j] + 2);
               int r = i - len[up[j]] + 2;
               BT.updata(l, 1); BT.updata(r, -1);
            }
            Seg.updata(1, 1, dfn, in[id[i]], i);
            for (auto j : Q[i]) ret[j.se] = BT.sum(j.fi);
         }
      }
   } A;
   void work(int n, int m) {
      scanf("%s", s + 1);
      A.init();
      for (int i = 1; i <= n; i++) A.add(s[i], i);
      A.build();
      A.dfs();
      for (int i = 1; i <= m; i++) scanf("%d%d", &l, &r), Q[r].pb(mp(l, i));
      A.solve();
   }
}
int n, m, l, r;
int main() {
   scanf("%d%d", &n, &m);
   Space::work(n, m);
   rep(i, 1, m + 1) printf("%lld\n", Space::ret[i]);
   return 0;
}
/*注：离线算法
, O(nlogn^2) , 下标从 1 开始

12 5
abcddcbaabcd
1 2
1 4
1 8
```

```
4 8
1 12

2
4
8
5
12
*/
```

11.13 大阶乘取模

```
#define R register
#define fd(i, a, b) for (R int i = (a), I = (b)-1; i > I; --i)
#define go(u) for (int i = head[u], v = e[i].v; i; i = e[i].nx, v = e[i].v)
const int N = (1 << 17) + 5;
int P;
inline int add(R int x, R int y) { return 0ll + x + y >= P ? 0ll + x + y - P : x + y; }
inline int dec(R int x, R int y) { return x - y < 0 ? x - y + P : x - y; }
inline int mul(R int x, R int y) { return 1ll * x * y - 1ll * x * y / P * P; }
int ksm(R int x, R int y) {
   R int res = 1;
   for (; y; y >>= 1, x = mul(x, x)) (y & 1) ? res = mul(res, x) : 0;
   return res;
}
const double Pi = acos(-1.0);
struct cp {
   double x, y;
   inline cp(R double xx = 0, R double yy = 0) : x(xx), y(yy) {}
   inline cp operator+(const cp &b) const { return cp(x + b.x, y + b.y); }
   inline cp operator-(const cp &b) const { return cp(x - b.x, y - b.y); }
   inline cp operator*(const cp &b) const { return cp(x * b.x - y * b.y, x * b.y + y * b
   .x); }
   inline cp operator*(const double &b) const { return cp(x * b, y * b); }
   inline cp operator~() const { return cp(x, -y); }
} w[2][N];
int r[21][N], ifac[N], lg[N], inv[N]; double iv[21];
void Pre() {
   iv[0] = 1;
   rep(d, 1, 17) {
      rep(i, 0, (1 << d) - 1) r[d][i] = (r[d][i >> 1] >> 1) | ((i & 1) << (d - 1));
      lg[1 << d] = d, iv[d] = iv[d - 1] * 0.5;
   }
   inv[0] = inv[1] = ifac[0] = ifac[1] = 1;
   rep(i, 2, 131072) inv[i] = mul(P - P / i, inv[P % i]), ifac[i] = mul(ifac[i - 1], inv
   [i]);
   for (R int i = 1, d = 0; i < 131072; i <<= 1, ++d) rep(k, 0, i - 1) {
      w[1][i + k] = cp(cos(Pi * k * iv[d]), sin(Pi * k * iv[d]));
      w[0][i + k] = cp(cos(Pi * k * iv[d]), -sin(Pi * k * iv[d]));
   }
}
int lim, d;
void FFT(cp *A, int ty) {
   rep(i, 0, lim - 1) if (i < r[d][i]) swap(A[i], A[r[d][i]]);
   cp t;
   for (R int mid = 1; mid < lim; mid <<= 1) for (R int j = 0; j < lim; j += (mid << 1))
    {
      rep(k, 0, mid - 1) {
         A[j + k + mid] = A[j + k] - (t = w[ty][mid + k] * A[j + k + mid]);
         A[j + k] = A[j + k] + t;
      }
   }
   if (!ty) rep(i, 0, lim - 1) A[i] = A[i] * iv[d];
}
void MTT(int *a, int *b, int len, int *c) {
   static cp f[N], g[N], p[N], q[N];
   lim = len, d = lg[lim];
   rep(i, 0, len - 1) f[i] = cp(a[i] >> 16, a[i] & 65535), g[i] = cp(b[i] >> 16, b[i] &
   65535);
   rep(i, len, lim - 1) f[i] = g[i] = cp(0, 0);
   FFT(f, 1), FFT(g, 1);
   rep(i, 0, lim - 1) {
      cp t, f0, f1, g0, g1;
      t = ~f[i ? lim - i : 0], f0 = (f[i] - t) * cp(0, -0.5), f1 = (f[i] + t) * 0.5;
      t = ~g[i ? lim - i : 0], g0 = (g[i] - t) * cp(0, -0.5), g1 = (g[i] + t) * 0.5;
      p[i] = f1 * g1, q[i] = f1 * g0 + f0 * g1 + f0 * g0 * cp(0, 1);
   }
   FFT(p, 0), FFT(q, 0);
   rep(i, 0, lim - 1)
      c[i] = ((((ll)(p[i].x + 0.5) % P << 16) % P << 16) + ((ll)(q[i].x + 0.5) << 16) +
      ((ll)(q[i].y + 0.5))) % P;
}
void calc(int *a, int *b, int n, int k) {
   static int f[N], g[N], h[N], sum[N], isum[N];
   int len = 1;
   while (len <= n + n) len <<= 1;
   rep(i, 0, n) f[i] = mul(a[i], mul(ifac[i], ifac[n - i]));
   for (R int i = n - 1; i >= 0; i -= 2) f[i] = P - f[i];
   int t = dec(k, n);
   rep(i, 0, n + n) g[i] = add(i, t);
   sum[0] = g[0];
   rep(i, 1, n + n) sum[i] = mul(sum[i - 1], g[i]);
   isum[n + n] = ksm(sum[n + n], P - 2);
   fd(i, n + n, 1) isum[i - 1] = mul(isum[i], g[i]);
   rep(i, 1, n + n) g[i] = mul(isum[i], sum[i - 1]);
   g[0] = isum[0];
   rep(i, n + 1, len - 1) f[i] = 0;
   rep(i, n + n + 1, len - 1) g[i] = 0;

   MTT(f, g, len, h);
   int res = 1, p1 = k - n, p2 = k;
   rep(i, p1, p2) res = 1ll * res * i % P;
   res = add(res, 0);

   rep(i, 0, n) g[i] = (0ll + P + p1 + i) % P;
   sum[0] = g[0];
   rep(i, 1, n) sum[i] = mul(sum[i - 1], g[i]);
   isum[n] = ksm(sum[n], P - 2);
   fd(i, n, 1) isum[i - 1] = mul(isum[i], g[i]);
   rep(i, 1, n) g[i] = mul(isum[i], sum[i - 1]);
```

```
   g[0] = isum[0];

   for (R int i = 0; i <= n; p2 = add(p2, 1), ++i)
      b[i] = mul(h[i + n], res), res = mul(res, mul(g[i], p2 + 1));
}
int solve(int bl) {
   static int a[N], b[N], c[N];
   int s = 0;
   for (int p = bl; p; p >>= 1) ++s;
   a[0] = 1, --s;
   int qwq = ksm(bl, P - 2);
   for (int p = 0; s >= 0; --s) {
      if (p) {
         calc(a, b, p, p + 1);
         rep(i, 0, p) a[p + i + 1] = b[i];
         a[p << 1 | 1] = 0;
         calc(a, b, p << 1, mul(p, qwq));
         p <<= 1;
         rep(i, 0, p) a[i] = mul(a[i], b[i]);
      }
      if (bl >> s & 1) {
         rep(i, 0, p) a[i] = mul(a[i], (1ll * bl * i + p + 1) % P);
         p |= 1, a[p] = 1;
         rep(i, 1, p) a[p] = mul(a[p], (1ll * bl * p + i) % P);
      }
   }
   int res = 1;
   rep(i, 0, bl - 1) res = mul(res, a[i]);
   return res;
}
int GetFac(int n) {
   int s = sqrt(n), res = solve(s);
   rep(i, s * s + 1, n) res = mul(res, i);
   return res;
}
int Fac(int n) {
   if (n >= P) return 0;
   if (n > P - 1 - n) {
      int res = ksm(GetFac(P - 1 - n), P - 2);
      return n & 1 ? res : P - res;
   }
   return GetFac(n);
}
int n;
int main() {
   int T;
   scanf("%d", &T);
   while (T--) {
      scanf("%d%d", &n, &P), Pre();
      printf("%d\n", Fac(n));
   }
   return 0;
}
```

11.14 带花树最大权匹配

```
// from vfleaking
// 求的是在权最大情况下的匹配权值大优先应该要是正数
// id :  1 .. n
// time : 应该也是 O(n^3)
#define INF INT_MAX
#define N 600
#define M N*2+1
struct E{
   int u,v,w;
   E(){}
   E(int u,int v,int w):u(u),v(v),w(w){}
};
int n,n_x, lab[M], match[M],slack[M],st[M],pa[M], flower_from[M][N+1],S[M],vis[M];
E g[M][M]; vi flower[M]; queue<int> q;
inline int e_delta(const E &e){ // does not work inside blossoms
   return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
}
inline void update_slack(int u,int x){
   if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]][x])) slack[x]=u;
}
inline void set_slack(int x){
   slack[x]=0;
   for(int u=1;u<=n;++u) if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0) update_slack(u,x);
}
void q_push(int x){
   if(x<=n) q.push(x);
   else for(size_t i=0;i<flower[x].size();i++)q_push(flower[x][i]);
}
inline void set_st(int x,int b){
   st[x]=b;
   if(x>n) for(size_t i=0;i<flower[x].size();++i) set_st(flower[x][i],b);
}
inline int get_pr(int b,int xr){
   int pr=find(flower[b].begin(),flower[b].end(),xr)-flower[b].begin();
   if(pr%2==1){ // 检查他在前一层图是奇点还是偶点
      reverse(flower[b].begin()+1,flower[b].end());
      return (int)flower[b].size()-pr;
   }else return pr;
}
inline void set_match(int u,int v){
   match[u]=g[u][v].v;
   if(u>n){
      E e=g[u][v];
      int xr=flower_from[u][e.u],pr=get_pr(u,xr);
      for(int i=0;i<pr;++i)set_match(flower[u][i],flower[u][i^1]);
      set_match(xr,v);
      rotate(flower[u].begin(),flower[u].begin()+pr,flower[u].end());
   }
}
inline void augment(int u,int v){
   for(;;){
      int xnv=st[match[u]];
      set_match(u,v);
```

```
      if(!xnv)return;
      set_match(xnv,st[pa[xnv]]);
      u=st[pa[xnv]],v=xnv;
   }
}
inline int get_lca(int u,int v){
   static int t=0;
   for(++t;u||v;swap(u,v)){
      if(u==0)continue;
      if(vis[u]==t)return u;
      vis[u]=t; // 这种方法可以不用清空 v 数组
      u=st[match[u]];
      if(u)u=st[pa[u]];
   }
   return 0;
}
inline void add_blossom(int u,int lca,int v){
   int b=n+1;
   while(b<=n_x&&st[b])++b;
   if(b>n_x)++n_x;
   lab[b]=0,S[b]=0;
   match[b]=match[lca];
   flower[b].clear();
   flower[b].pb(lca);
   for(int x=u,y;x!=lca;x=st[pa[y]]) flower[b].pb(x), flower[b].pb(y=st[match[x]]),
   q_push(y);
   reverse(flower[b].begin()+1,flower[b].end());
   for(int x=v,y;x!=lca;x=st[pa[y]]) flower[b].pb(x), flower[b].pb(y=st[match[x]]),
   q_push(y);
   set_st(b,b);
   for(int x=1;x<=n_x;++x)g[b][x].w=g[x][b].w=0;
   for(int x=1;x<=n;++x)flower_from[b][x]=0;
   for(size_t i=0;i<flower[b].size();++i){
      int xs=flower[b][i];
      for(int x=1;x<=n_x;++x) if(g[b][x].w==0||e_delta(g[xs][x])<e_delta(g[b][x]))
         g[b][x]=g[xs][x], g[x][b]=g[x][xs];
      for(int x=1;x<=n;++x) if(flower_from[xs][x]) flower_from[b][x]=xs;
   }
   set_slack(b);
}
inline void expand_blossom(int b){ // S[b] == 1
   for(size_t i=0;i<flower[b].size();++i) set_st(flower[b][i],flower[b][i]);
   int xr=flower_from[b][g[b][pa[b]].u],pr=get_pr(b,xr);
   for(int i=0;i<pr;i+=2){
      int xs=flower[b][i],xns=flower[b][i+1];
      pa[xs]=g[xns][xs].u;
      S[xs]=1,S[xns]=0;
      slack[xs]=0,set_slack(xns);
      q_push(xns);
   }
   S[xr]=1,pa[xr]=pa[b];
   for(size_t i=pr+1;i<flower[b].size();++i){
      int xs=flower[b][i];
      S[xs]=-1,set_slack(xs);
   }
   st[b]=0;
}
inline bool on_found_E(const E &e){
   int u=st[e.u],v=st[e.v];
   if(S[v]==-1){
      pa[v]=e.u,S[v]=1;
      int nu=st[match[v]];
      slack[v]=slack[nu]=0;
      S[nu]=0,q_push(nu);
   }else if(S[v]==0){
      int lca=get_lca(u,v);
      if(!lca)return augment(u,v),augment(v,u),true;
      else add_blossom(u,lca,v);
   }
   return false;
}
inline bool matching(){
   memset(S+1,-1,sizeof(int)*n_x);
   memset(slack+1,0,sizeof(int)*n_x);
   q=queue<int>();
   for(int x=1;x<=n_x;++x) if(st[x]==x&&!match[x]) pa[x]=0,S[x]=0,q_push(x);
   if(q.empty())return false;
   for(;;){
      while(q.size()){
         int u=q.front();q.pop();
         if(S[st[u]]==1)continue;
         for(int v=1;v<=n;++v) if(g[u][v].w>0&&st[u]!=st[v]){
            if(e_delta(g[u][v])==0){
               if(on_found_E(g[u][v]))return true;
            } else update_slack(u,st[v]);
         }
      }
      int d=INF;
      for(int b=n+1;b<=n_x;++b) if(st[b]==b&&S[b]==1) d=min(d,lab[b]/2);
      for(int x=1;x<=n_x;++x) if(st[x]==x&&slack[x]){
         if(S[x]==-1)d=min(d,e_delta(g[slack[x]][x]));
         else if(S[x]==0)d=min(d,e_delta(g[slack[x]][x])/2);
      }
      for(int u=1;u<=n;++u){
         if(S[st[u]]==0){
            if(lab[u]<=d)return 0;
            lab[u]-=d;
         }else if(S[st[u]]==1)lab[u]+=d;
      }
      for(int b=n+1;b<=n_x;++b) if(st[b]==b){
         if(S[st[b]]==0)lab[b]+=d*2;
         else if(S[st[b]]==1)lab[b]-=d*2;
      }
      q=queue<int>();
      for(int x=1;x<=n_x;++x) if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x
      ]][x])==0)
         if(on_found_E(g[slack[x]][x]))return true;
      for(int b=n+1;b<=n_x;++b) if(st[b]==b&&S[b]==1&&lab[b]==0) expand_blossom(b);
   }
   return false;
```

```
}
inline pair<long long,int> weight_blossom(){
   memset(match+1,0,sizeof(int)*n);
   n_x=n;
   int n_matches=0;
   long long tot_weight=0;
   for(int u=0;u<=n;++u)st[u]=u,flower[u].clear();
   int w_max=0;
   for(int u=1;u<=n;++u) for(int v=1;v<=n;++v){
      flower_from[u][v]=(u==v?u:0);
      w_max=max(w_max,g[u][v].w);
   }
   for(int u=1;u<=n;++u)lab[u]=w_max;
   while(matching())++n_matches;
   for(int u=1;u<=n;++u) if(match[u]&&match[u]<u) tot_weight+=g[u][match[u]].w;
   return mp(tot_weight,n_matches);
}
inline void init_weight_graph(){
   for(int u=1;u<=n;++u) for(int v=1;v<=n;++v) g[u][v]=E(u,v,0);
}
int main(){
   int m; scanf("%d%d",&n,&m);
   init_weight_graph();
   for(int i=0;i<m;++i){
      int u,v,w; scanf("%d%d%d",&u,&v,&w);
      g[u][v].w=g[v][u].w=w;
   }
   printf("%lld\n",weight_blossom().first);
   for(int u=1;u<=n;++u)printf("%d ",match[u]);puts("");
   return 0;
}
```

11.15 强连通子图计数

```
// n 点 m 边强连通子图计数
// 首先要会 n 点 m 边 DAG 计数，然后把枚举出度为零的点集容斥改为枚举缩点后出度为零的点集容斥
const int N = 1 << 15;
int n, cnt[N], S, g[N], f[N], u, v, a[N], m, p[300];
int get(int x) {
   int res = 0;
   while (x) {
      res++;
      x = (x - 1) & x;
   }
   return res;
}
int cal(int x, int y) {
   int res = 0, t;
   while (x) {
      t = x & (-x);
      res += cnt[a[t] & y];
      x ^= t;
   }
   return res;
}
int main() {
   cin >> n >> m;
   rep(i, 1, m+1) {
      cin >> u >> v;
      a[pw(u-1)] |= pw(v-1);
   }
   S = pw(n) - 1;
   rep(i, 1, S+1) cnt[i] = get(i);
   p[0] = 1;
   rep(i, 1, m+1) p[i] = mul(p[i-1], 2);
   rep(i, 1, S+1) {
      u = i & (-i);
      for (int x = (i - 1) & i; x > 0; x = (x - 1) & i) {
         if (x & u) g[i] = sub(g[i], mul(f[x], g[i ^ x]));
      }
      f[i] = p[cal(i, i)];
      for (int x = i; x > 0; x = (x - 1) & i) {
         f[i] = sub(f[i], mul(g[x], p[cal(i ^ x, i)]));
      }
      g[i] = add(g[i], f[i]);
   }
   cout << f[S];
   return 0;
}
```

11.16 快速 Rho

```
using uint128 = __uint128_t;
using ull = unsigned long long;
using ll = long long;
using uint = unsigned int;
using pli = pair<ull, uint>;
namespace prime {
   inline ull sqr(ull x) { return x * x; }
   inline uint isqrt(ull x) { return sqrtl(x); }
   inline uint ctz(ull x) { return __builtin_ctzll(x); }
   template <class T>
   T gcd(T a, T b) {  while (b) { T t = a % b; a = b; b = t; } return a; }

   template <class T, class dT, class sT>
   struct Mod {
      static T mod, inv, r2;
      static const int wb = sizeof(T) * 8;
      T x;
      Mod(): x(0) {}
      Mod(T _x): x(init(_x)) {}
      bool operator == (const Mod& rhs) const { return x == rhs.x; }
      bool operator != (const Mod& rhs) const { return x != rhs.x; }
      Mod operator += (const Mod& rhs) { if ((x += rhs.x) >= mod) x -= mod; return *this
      ; }
      Mod operator -= (const Mod& rhs) { if (sT(x -= rhs.x) < 0) x += mod; return *this;
       }
      Mod operator *= (const Mod& rhs) { x = reduce(dT(x) * rhs.x); return *this; }
      Mod operator + (const Mod &rhs) const { return Mod(*this) += rhs; }
      Mod operator - (const Mod &rhs) const { return Mod(*this) -= rhs; }
```

```
    Mod operator * (const Mod &rhs) const { return Mod(*this) *= rhs; }
    Mod operator - () const { return Mod() - *this; }
    Mod pow(ull e) const {
        Mod r(1); for (Mod a = *this; e; e >>= 1, a *= a) if (e & 1) r *= a;
        return r;
    }
    T get() const { return reduce(x); }
    static T modulus() { return mod; }
    static T init(T w) { return reduce(dT(w) * r2); }
    static void set_mod(T m) { mod = m, inv = mul_inv(mod), r2 = -dT(mod) % mod; }
    static T reduce(dT x) {
        T y = T(x >> wb) - T((dT(T(x) * inv) * mod) >> wb);
        return sT(y) < 0 ? y + mod : y;
    }
    static T mul_inv(T n, int e = 6, T x = 1) {
        return !e ? x : mul_inv(n, e - 1, x * (2 - x * n));
    }
};

using Mod64 = Mod<ull, uint128, ll>;
using Mod32 = Mod<uint, ull, int>;
template <> ull Mod64::mod = 0;
template <> ull Mod64::inv = 0;
template <> ull Mod64::r2 = 0;
template <> uint Mod32::mod = 0;
template <> uint Mod32::inv = 0;
template <> uint Mod32::r2 = 0;

template <class T, class mod>
bool composite(T n, const uint* bases, int m) {
    mod :: set_mod(n);
    int s = __builtin_ctzll(n - 1);
    T d = (n - 1) >> s;
    mod one(1), fone(n - 1);
    for (int i = 0, j; i < m; ++i) {
        mod a = mod(bases[i]).pow(d);
        if (a == one || a == fone) continue;
        for (j = s - 1; j > 0; --j) { if ((a *= a) == fone) break; }
        if (j == 0) return 1;
    }
    return 0;
}
bool is_prime(ull n) { // reference: http://miller-rabin.appspot.com
    assert(n < (ull(1) << 63));
    static const uint bases[][7] = {
        {2, 3},
        {2, 299417},
        {2, 7, 61},
        {15, 176006322, 4221622697u},
        {2, 2570940, 211991001, 3749873356u},
        {2, 2570940, 880937, 610386380, 4130785767u},
        {2, 325, 9375, 28178, 450775, 9780504, 1795265022}
    };
    if (n <= 1) return 0;
    if (!(n & 1)) return n == 2;
    if (n <= 8) return 1;
    int x = 6, y = 7;
    if (n < 1373653) x = 0, y = 2;
    else if (n < 19471033) x = 1, y = 2;
    else if (n < 4759123141) x = 2, y = 3;
    else if (n < 154639673381) x = y = 3;
    else if (n < 47636622961201) x = y = 4;
    else if (n < 3770579582154547) x = y = 5;
    if (n < (1u << 31)) return !composite<uint, Mod32>(n, bases[x], y);
    return !composite<ull, Mod64>(n, bases[x], y);
}
struct ExactDiv {
    ExactDiv() {}
    ExactDiv(ull n) : n(n), i(Mod64::mul_inv(n)), t(ull(-1) / n) {}
    friend ull operator / (ull n, ExactDiv d) { return n * d.i; };
    bool divide(ull n) { return n / *this <= t; }
    ull n, i, t;
};
vector<ExactDiv> primes;
void init(uint n) {
    uint sqrt_n = sqrt(n);
    vector<bool> is_prime(n + 1, 1);
    primes.clear();
    for (uint i = 2; i <= sqrt_n; ++i) if (is_prime[i]) {
        if (i != 2) primes.pb(ExactDiv(i));
        for (uint j = i * i; j <= n; j += i) is_prime[j] =  0;
    }
}

template <class T, class mod>
T brent(T n, T c) { // n must be composite and odd.
    const ull s = 256;
    mod::set_mod(n);
    const mod one = mod(1), mc = mod(c);
    mod y = one;
    for (ull l = 1; ; l <<= 1) {
        auto x = y;
        for (int i = 0; i < (int)l; ++i) y = y * y + mc;
        mod p = one;
        for (int k = 0; k < (int)l; k += s) {
            auto sy = y;
            for (int i = 0; i < (int)min(s, l - k); ++i) {
                y = y * y + mc;
                p *= y - x;
            }
            T g = gcd(n, p.x);
            if (g == 1) continue;
            if (g == n) for (g = 1, y = sy; g == 1; ) y = y * y + mc, g = gcd(n, (y - x)
            .x);
            return g;
        }
    }
}
ull brent(ull n, ull c) {
    if (n < (1u << 31)) return brent<uint, Mod32>(n, c);
```

```
      return brent<ull, Mod64>(n, c);
   }
   vector<pli> factors(ull n) {
      assert(n < (1ull << 63));
      if (n <= 1) return {};
      vector<pli> ret;
      ull v2 = sqrtl(n), v3 = cbrtl(n), v = v2, b = 2;
      if (v2 * v2 == n || v3 * v3 * v3 == n) {
         if (v2 * v2 != n) v = v3, b = 3;
         ret = factors(v);
         for (auto &&e: ret) e.se *= b;
         return ret;
      }
      if (!(n & 1)) {
         uint e = ctz(n);
         ret.emplace_back(2, e);
         n >>= e;
      }
      ull lim = sqr(primes.back().n);
      for (auto &&p: primes) {
         if (sqr(p.n) > n) break;
         if (p.divide(n)) {
            uint e = 1; n = n / p;
            while (p.divide(n)) n = n / p, e++;
            ret.emplace_back(p.n, e);
         }
      }
      uint s = ret.size();
      while (n > lim && !is_prime(n)) {
         for (ull c = 1; ; ++c) {
            ull p = brent(n, c);
            if (!is_prime(p)) continue;
            uint e = 1; n /= p;
            while (n % p == 0) n /= p, e += 1;
            ret.emplace_back(p, e);
            break;
         }
      }
      if (n > 1) ret.emplace_back(n, 1);
      if (ret.size() - s >= 2) sort(ret.begin() + s, ret.end());
      return ret;
   }
}
```

11.17 扩展类欧几里得

```
//\sum_{x = 0} ^ {n} x ^ {k_1} {\left \lfloor \frac{ax + b}{c} \right \rfloor} ^ {k_2}
namespace _lo {
   static const int K = 10, P = 998244353; // e1+e2 <= K
   int inv[K + 2], C[K + 2][K + 2], B[K + 1]; // Ŭ▫▫▫▫²®
   int cof[K + 1][K + 1], tmp[K + 1][K + 1], k;
   inline void add(int &a, int b) { if ((a += b) >= P) a -= P; }
   inline int mul(int a, int b) { return 1ll * a * b % P; }
   int reduce(int &a, int b) {
      int res = a / b;
      if ((a %= b) < 0) a += b, --res;
      return res;
   }
   void init() {
      inv[1] = 1; rep(i, 2, K+2) inv[i] = mul(P - P / i, inv[P % i]);
      rep(i, 0, K+2) {
         C[i][0] = 1;
         rep(j, 1, i+1) C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % P;
      }
      rep(i, 0, K+1) {
         int sum = 0;
         rep(j, 0, i) add(sum, mul(C[i + 1][j], B[j]));
         B[i] = (1 + mul(P - sum, inv[i + 1])) % P;
      }
   }
   inline void calc(int x, bool o) {
      rep(i, 0, k) memcpy(tmp[i]+1, cof[i]+1, 4 * (k - i));
      for (int i = 1, u = x; i <= k; ++i, u = mul(u, x))
         rep(s, i, k+1) {
            int v = mul(C[s][i], u);
            rep(r, 0, k-s+1) add(cof[r + o * i][s - i], mul(v, tmp[r][s]));
         }
   }
   int run(int n, int a, int b, int c, int k1, int k2) {
      k = k1 + k2;
      int res = 0, sign = 1;
      if (k1 == 0) { res = 1; rep(i, 0, k2) res = mul(res, b / c); }
      rep(i, 0, k) memset(cof[i]+1, 0, 4 * (k - i));
      cof[k1][k2] = 1;
      while (1) {
         int cur = 0, x = reduce(a, c), y = (reduce(b, c) + P) % P;
         calc(x, 1); calc(y, 0);
         int l = ((ll)a * n + b) / c;
         rep(r, 0, k+1) {
            int sum = 0;
            for (int i = 1, u = n; i <= r + 1; ++i, u = mul(u, n))
               add(sum, mul(u, mul(C[r + 1][i], B[r + 1 - i])));
            sum = mul(sum, inv[r + 1]);
            for (int s = 0, u = sum; s <= k - r; ++s, u = mul(u, l)) add(cur, mul(u, cof
            [r][s]));
            cof[r][0] = 0;
         }
         add(res, mul(sign, cur));
         if (!a) break;
         rep(i, 0, k) memset(tmp[i], 0, 4 * (k - i));
         rep(s, 1, k+1) rep(i, 1, s+1) {
            int u = mul(C[s][i], i & 1 ? 1 : P - 1);
            rep(r, 0, k-s+1) add(tmp[r][s - i], mul(u, cof[r][s]));
         }
         rep(i, 0, k) memset(cof[i]+1, 0, 4 * (k - i));
         rep(r, 0, k) rep(i, 0, r+1) {
            int u = mul(C[r+1][i], mul(B[i], inv[r+1]));
            rep(s, 0, k-r) add(cof[s][r + 1 - i], mul(u, tmp[r][s]));
         }
         n = l; swap(a, c);
```

```
        b = -b - 1; sign = P - sign;
      }
      return res;
    }
}

// f = \sum\limits_{i=0}^{n}\lfloor \frac{ai+b}{c} \rfloor
// g = \sum\limits_{i=0}^{n}{\lfloor \frac{ai+b}{c} \rfloor}^2
// h = \sum\limits_{i=0}^{n}i\lfloor \frac{ai+b}{c} \rfloor

#define R register
const int P=998244353,inv2=499122177,inv6=166374059;
inline int add(R int x,R int y){return x+y>=P?x+y-P:x+y;}
inline int dec(R int x,R int y){return x-y<0?x-y+P:x-y;}
inline int mul(R int x,R int y){return 1ll*x*y-1ll*x*y/P*P;}
inline int pow(R int x){return 1ll*x*x%P;}
inline int s(R int x){return 1ll*x*(x+1)%P*inv2%P;}
inline int ss(R int x){return 1ll*x*(x+1)%P*((x<<1)+1)%P*inv6%P;}
int ksm(R int x,R int y){
    R int res=1;
    for(;y;y>>=1,x=mul(x,x))if(y&1)res=mul(res,x);
    return res;
}
struct node{int f,g,h;}res;
void get(int a,int b,int c,int n){
    int x=a/c,y=b/c;
    if(!a){
        res.f=1ll*y*(n+1)%P;
        res.g=1ll*pow(y)*(n+1)%P;
        res.h=1ll*y*s(n)%P;
        return;
    }
    if(a>=c||b>=c){
        get(a%c,b%c,c,n);
        res.g=add(res.g,add(1ll*(x<<1)*res.h%P,add(1ll*(y<<1)*res.f%P,add(1ll*ss(n)*pow(
        x)%P,add(1ll*n*(n+1)%P*x%P*y%P,1ll*(n+1)*pow(y)%P)))));
        res.h=add(res.h,add(1ll*ss(n)*x%P,1ll*s(n)*y%P));
        res.f=add(res.f,add(1ll*s(n)*x%P,1ll*(n+1)*y%P));
        return;
    }
    int M=(1ll*a*n+b)/c;
    get(c,c-b-1,a,M-1);
    int h=res.h,g=res.g,f=res.f;
    res.f=dec(1ll*n*M%P,res.f);
    res.g=dec(dec(dec(1ll*n*M%P*(M+1)%P,res.f),mul(h,2)),mul(f,2));
    res.h=1ll*inv2*dec(dec(1ll*M*n%P*(n+1)%P,g),f)%P;
    return;
}
```

11.18 插头 dp_两通路

```
/*
 * 给定 9 * 9 的棋盘，格子四联通，有两类格子， 1 不能走 0 可以走。要求从 S1 走到 T1 ，从 S2 走
 到 T2 ，且路径不相交，求最短路径。
 */
int n, m, a[22][22], t[22];
struct HM {
  static const int INF = pw(18) - 1, N = 8e5;
  int hd[INF + 1], ne[N], s[N], L, f[N];
  void init() {
    memset(hd, -1, sizeof(hd));
    L = 0;
  }
  void upd(int u, int v) {
    int p = u & INF;
    for(int i = hd[p]; ~i; i = ne[i]) if(u == s[i]) {
      f[i] = min(f[i], v);
      return ;
    }
    s[L] = u; f[L] = v; ne[L] = hd[p]; hd[p] = L++;
  }
}hm[2];
int solve() {
  int o = 0; hm[o].init(); hm[o].upd(0, 0);
  rep(i, 1, n + 1) {
    rep(k, 0, hm[o].L) {
      if(hm[o].s[k] >> t[m]) hm[o].f[k] = -1;
      else hm[o].s[k] <<= 3;
    }
    rep(j, 1, m + 1) {
      o ^= 1; hm[o].init();
      rep(k, 0, hm[o ^ 1].L) {
   int x = hm[o ^ 1].s[k], y = hm[o ^ 1].f[k] + 1;
   if(x >> t[m + 1] || y <= 0) continue;
   int p = x >> t[j - 1] & 7, q = x >> t[j] & 7;
   int tx = x ^ (p << t[j - 1]) ^ (q << t[j]);
   if(a[i][j] == 1) {
     if(!p && !q) hm[o].upd(x, y - 1);
   } else if(a[i][j] == 0) {
     if(!p && !q) {
       hm[o].upd(x | (1 << t[j - 1]) | (2 << t[j]), y);
       hm[o].upd(x, y - 1);
     } else if(!p || !q) {
       int k = max(p, q);
       hm[o].upd(x, y);
       hm[o].upd(x ^ (k << t[j - 1]) ^ (k << t[j]), y);
     } else if(p == 1 && q == 2) {
     } else if(p == 2 && q == 1 || p == q && p >= 3) {
       hm[o].upd(tx, y);
     } else if(min(p, q) <= 2) {
       int mi = min(p, q), now = 1, nx = tx;
       for(int _ = mi == 1 ? j + 1 : j - 2; ; mi == 1 ? ++_ : --_) {
         int c = x >> t[_] & 7;
         if(c == mi) ++now;
         if(c == (mi ^ 3)) --now;
         if(!now) {
      hm[o].upd(nx ^ ((3 ^ p ^ q) << t[_]), y);
      break;
         }
       }
```

```
    }
   } else {
    int kk = a[i][j] + 1;
    if(!p && !q) {
      hm[o].upd(x | (kk << t[j - 1]), y);
      hm[o].upd(x | (kk << t[j]), y);
    } else if(!p || !q) {
      int ma = max(p, q);
      if(ma == kk) {
        hm[o].upd(tx, y);
      } else if(ma <= 2) {
        int now = 1, nx = tx;
        for(int _ = ma == 1 ? j + 1 : j - 2; ; ma == 1 ? ++_ : --_) {
           int c = x >> t[_] & 7;
           if(c == ma) ++now;
           if(c == (ma ^ 3)) --now;
           if(!now) {
             hm[o].upd(nx ^ ((3 ^ ma ^ kk) << t[_]), y);
             break;
     }
         }
       }
    }
   }
     }
    }
  }
  int ans = 0;
  rep(i, 0, hm[o].L) if(!hm[o].s[i]) ans = hm[o].f[i] - 2;
  return ans;
}
int main() {
  rep(i, 0, 22) t[i] = i + i + i;
  while(cin >> n >> m) {
    if(!n && !m) break;
    rep(i, 1, n + 1) rep(j, 1, m + 1) cin >> a[i][j];
    cout << solve() << endl;
  }
  return 0;
}
```

11.19 插头 dp_回路

```
/*
 * 给定 12 * 12 的棋盘，格子四联通，有三类格子， X 不能走 0 必走 * 可走可不走。画一条回路，求方案数。
 */
int n, m, edx, edy, t[22], ok[22][22];
char s[22][22];
struct HM {
  static const int INF = pw(18) - 1, N = 8e5;
  int hd[INF + 1], ne[N], s[N], L; ll f[N];
  void init() {
    memset(hd, -1, sizeof(hd));
    L = 0;
  }
  void upd(int u, ll v) {
    int p = u & INF;
    for(int i = hd[p]; ~i; i = ne[i]) if(u == s[i]) {
      f[i] += v;
      return ;
    }
    s[L] = u; f[L] = v; ne[L] = hd[p]; hd[p] = L++;
  }
}hm[2];
void gao() {
  per(i, 1, n + 1) per(j, 1, m + 1) {
    ok[i][j] = 1;
    if(s[i][j] == '0') return ;
  }
}
ll solve() {
  cin >> n >> m;
  rep(i, 1, n + 1) cin >> (s[i] + 1);
  rep(i, 1, n + 1) rep(j, 1, m + 1) ok[i][j] = 0;
  gao();
  ll ans = 0;
  int o = 0; hm[o].init(); hm[o].upd(0, 1);
  rep(i, 1, n + 1) {
    rep(j, 0, hm[o].L) hm[o].s[j] <<= t[1];
    rep(j, 1, m + 1) {
      o ^= 1; hm[o].init();
      rep(k, 0, hm[o ^ 1].L) {
   int x = hm[o ^ 1].s[k];
   if(x >> t[m + 1]) continue;
   int p = x >> t[j - 1] & 3, q = x >> t[j] & 3;
   int tx = x ^ (p << t[j - 1]) ^ (q << t[j]);
   ll y = hm[o ^ 1].f[k];
   if(s[i][j] == 'X') {
     if(!p && !q) hm[o].upd(x, y);
   } else {
     if(!p && !q) {
       hm[o].upd(x | (1 << t[j - 1]) | (2 << t[j]), y);
       if(s[i][j] == '*') hm[o].upd(x, y);
     } else if(!p || !q) {
       hm[o].upd(x, y);
       hm[o].upd(x ^ ((p | q) << t[j - 1]) ^ ((p | q) << t[j]), y);
     } else if(p == q) {
       int now = 0, nx = tx;
       for(int _ = p == 1 ? j + 1 : j - 2; ; p == 1 ? ++_ : --_) {
         int c = x >> t[_] & 3;
         if(!c) continue;
         c == p ? ++now : --now;
         if(now == -1) {
            hm[o].upd(nx ^ (3 << t[_]), y);
     break;
         }
       }
     } else if(p == 1 && q == 2) {
       if(ok[i][j] && !tx) ans += y;
```

```
    } else if(p == 2 && q == 1) {
      hm[o].upd(tx, y);
    }
  }
    }
  }
  return ans;
}
int main() {
  rep(i, 0, 22) t[i] = i + i;
  int T; cin >> T;
  rep(i, 1, T + 1) cout << "Case " << i << ": " << solve() << endl;
  return 0;
}
```

11.20 插头 dp_矩阵加速通路

```
/*
 * 给定 7 * 1e9 的棋盘，格子四联通，每个格子必走。求左上走到左下的方案数。
 */
const int P = 7777777, N = 150;
int n, m, id[20202], t[22], k, dp[2][N]; vi sta;
inline int add(int a, int b) {
  if((a += b) >= P) a -= P;
  return a;
}
inline int mul(int a, int b) { return 1ll * a * b % P; }
struct HM {
  static const int N = pw(16);
  int hd[N], s[N], L, f[N];
  inline void init() {
    rep(i, 0, L) hd[s[i]] = -1;
    L = 0;
  }
  inline void upd(int u, int v) {
    if(~hd[u]) {
      int i = hd[u];
      f[i] = add(f[i], v);
    } else {
      s[L] = u, f[L] = v; hd[u] = L++;
    }
  }
}hm[2];
struct Mat {
  ll a[N][N];
  Mat() { rep(i, 0, k) rep(j, 0, k) a[i][j] = 0; }
  void reset() { rep(i, 0, k) rep(j, 0, k) a[i][j] = 0; }
  void set() { rep(i, 0, k) a[i][i] = 1; }
  inline Mat operator * (const Mat &c) const {
    Mat r;
    rep(i, 0, k) rep(j, 0, k) rep(t, 0, k) r.a[i][j] += a[i][t] * c.a[t][j];
    rep(i, 0, k) rep(j, 0, k) r.a[i][j] %= P;
    return r;
  }
}A;
Mat kpow(Mat a, int b) {
  Mat r; r.set();
  while(b) {
    if(b & 1) r = r * a;
    a = a * a;
    b >>= 1;
  }
  return r;
}
void gao(int s) {
  int o = 0; hm[o].init(); hm[o].upd(sta[s] << t[1], 1);
  rep(j, 1, n + 1) {
    o ^= 1; hm[o].init();
    rep(k, 0, hm[o ^ 1].L) {
      int x = hm[o ^ 1].s[k], y = hm[o ^ 1].f[k];
      if(x >> t[n + 1]) continue;
      int p = x >> t[j - 1] & 3, q = x >> t[j] & 3;
      int tx = x ^ p << t[j - 1] ^ q << t[j];
      if(!p && !q) {
   hm[o].upd(x | (1 << t[j - 1]) | (2 << t[j]), y);
      } else if(!p || !q) {
   hm[o].upd(x, y);
   hm[o].upd(x ^ ((p | q) << t[j - 1]) ^ ((p | q) << t[j]), y);
      } else if(p == q) {
   int now = 0, nx = tx;
   for(int _ = p == 1 ? j + 1 : j - 2; ; p == 1 ? ++_ : --_) {
     int c = x >> t[_] & 3;
     if(!c) continue;
     c == p ? ++now : --now;
     if(now == -1) {
       hm[o].upd(nx ^ (3 << t[_]), y);
       break;
     }
   }
      } else if(p == 2 && q == 1) {
   hm[o].upd(tx, y);
      }
    }
  }
  rep(_, 0, hm[o].L) if(!(hm[o].s[_] >> t[n])) {
    int i = id[hm[o].s[_]];
    A.a[i][s] = add(A.a[i][s], hm[o].f[_]);
  }
}
bool check(int x) {
  int now = 0;
  rep(i, 0, n) {
    int c = x >> t[i] & 3;
    if(c == 3) return 0;
    if(c == 1) ++now;
    if(c == 2) --now;
    if(now < 0) return 0;
  }
  return now == 0;
```

```
}
int solve() {
  if((n & 1) && !(m & 1)) return 0;
  sta.clear();
  rep(i, 0, pw(n << 1)) if(check(i)) {
    id[i] = sz(sta);
    sta.pb(i);
  }
  k = sz(sta); A.reset();
  rep(j, 0, k) gao(j);
  A = kpow(A, m);
  int i = id[1 ^ 2 << t[n - 1]];
  return A.a[i][0];
}
int main() {
  rep(i, 0, 22) t[i] = i + i;
  memset(hm[0].hd, -1, sizeof(hm[0].hd));
  memset(hm[1].hd, -1, sizeof(hm[0].hd));
  while(cin >> n >> m) {
    int ans = solve();
    if(ans) cout << ans << endl;
    else cout << "Impossible" << endl;
  }
  return 0;
}
```

11.21 插头 dp_通路

```
/*
 * 给定 7 * 7 的棋盘，格子四联通，格子有收益或不能走。求通路的最大收益。
 */
int n, m, t[22], a[22][22];
struct HM {
  static const int INF = pw(18) - 1, N = 8e5;
  int hd[INF + 1], ne[N], s[N], L, f[N];
  void init() {
    memset(hd, -1, sizeof(hd));
    L = 0;
  }
  void upd(int u, int v) {
    int p = u & INF;
    for(int i = hd[p]; ~i; i = ne[i]) if(u == s[i]) {
      f[i] = max(f[i], v);
      return ;
    }
    s[L] = u; f[L] = v; ne[L] = hd[p]; hd[p] = L++;
  }
}hm[2];
int solve() {
  int ans = -INT_MAX;
  cin >> n >> m;
  rep(i, 1, n + 1) rep(j, 1, m + 1) {
    cin >> a[i][j];
    ans = max(ans, a[i][j]);
  }
  int o = 0; hm[o].init(); hm[o].upd(0, 0);
  rep(i, 1, n + 1) {
    rep(j, 0, hm[o].L) hm[o].s[j] <<= t[1];
    rep(j, 1, m + 1) {
      o ^= 1; hm[o].init();
      rep(k, 0, hm[o ^ 1].L) {
   int x = hm[o ^ 1].s[k], y = hm[o ^ 1].f[k] + a[i][j];
   if(x >> t[m + 1]) continue;
   int p = x >> t[j - 1] & 3, q = x >> t[j] & 3;
   int tx = x ^ (p << t[j - 1]) ^ (q << t[j]);
   if(a[i][j] == 0) {
     if(!p && !q) hm[o].upd(x, y - a[i][j]);
   } else {
     if(!p && !q) {
       hm[o].upd(x | (1 << t[j - 1]) | (2 << t[j]), y);
       hm[o].upd(x | (3 << t[j - 1]), y);
       hm[o].upd(x | (3 << t[j]), y);
       hm[o].upd(x, y - a[i][j]);
     } else if(!p || !q) {
       int k = max(p, q);
       hm[o].upd(x, y);
       hm[o].upd(x ^ (k << t[j - 1]) ^ (k << t[j]), y);
       if(k == 3) {
         if(!tx) ans = max(ans, y);
       } else {
         int now = 1, nx = tx;
         for(int _ = k == 1 ? j + 1 : j - 2; ; k == 1 ? ++_ : --_) {
            int c = x >> t[_] & 3;
            if(c == k) ++now;
            if(c == (k ^ 3)) --now;
            if(!now) {
              hm[o].upd(nx ^ (k << t[_]), y);
              break;
    }
         }
       }
     } else if(p == 1 && q == 2) {
     } else if(p == 2 && q == 1) {
       hm[o].upd(tx, y);
     } else if(min(p, q) <= 2) {
       int k = min(p, q), now = 1, nx = tx;
       for(int _ = k == 1 ? j + 1 : j - 2; ; k == 1 ? ++_ : --_) {
         int c = x >> t[_] & 3;
         if(c == k) ++now;
         if(c == (k ^ 3)) --now;
         if(!now) {
    hm[o].upd(nx ^ ((3 ^ p ^ q) << t[_]), y);
    break;
         }
       }
     } else {
       if(!tx) ans = max(ans, y);
     }
   }
    }
  }
```

```
    }
  }
  return ans;
}
int main() {
  rep(i, 0, 22) t[i] = i + i;
  int T; cin >> T;
  while(T--) cout << solve() << endl;
  return 0;
}
```

11.22 相邻差值小于等于 4 的排列数量

```
// 求相邻差值小于等于 k(k<=4) 的排列数量
// 一种枚举排列的方法
#pragma GCC optimize("Ofast")
#pragma GCC target("sse2")
#pragma GCC optimize("unroll-loops")
const int N = 10;
int n, k, a[N], b[N], ok, p, ans, c[200], ans2, S, m; bool o; map<vi, int> f[2][4];
int main() {
   while (cin >> n >> k) {
      if (n == 1) {
         cout << 1 << endl;
         continue;
      }
      rep(i, 0, 2) rep(j, 0, 4) f[i][j].clear();
      f[0][0][vi()] = 1; o = 0;
      rep(i, 1, n+1) {
         o ^= 1;
         rep(t, 0, 4) f[o][t].clear();
         rep(t, 0, 4) {
            for (auto v : f[!o][t]) {
               m = sz(v.fi); vi a = v.fi;
               rep(l, 0, m) {
                  if (l == 0 && (t & 1)) continue;
                  if (l == m - 1 && (t & 2)) continue;
                  a[l]++;
               }
               rep(l, 0, m) { // 放左或右
                  if (l == 0 && (t & 1)) continue;
                  if (l == m - 1 && (t & 2)) continue;
                  vi tmp = a; tmp[l] = 0; p = 1;
                  rep(i, 0, sz(tmp)) if (tmp[i]+1 > k) p = 0;
                  if (!p) continue;
                  (f[o][t][tmp] += v.se) %= P;
                  if (l == 0) (f[o][t | 1][tmp] += v.se) %= P;
                  if (l == m - 1) (f[o][t | 2][tmp] += v.se) %= P;
               }
               for (int l = 0; l <= m; l += 2) { // 新开区间
                  if (l == 0 && (t & 1)) continue;
                  if (l == m && (t & 2)) continue;
                  vi tmp;
                  rep(j, 0, l) tmp.pb(a[j]); tmp.pb(0); tmp.pb(0);
                  rep(j, l, m) tmp.pb(a[j]);
                  p = 1;
                  rep(i, 0, sz(tmp)) if (tmp[i]+1 > k) p = 0;
                  if (!p) continue;
                  (f[o][t][tmp] += v.se) %= P;
                  if (l == 0) (f[o][t | 1][tmp] += v.se) %= P;
                  if (l == m) (f[o][t | 2][tmp] += v.se) %= P;
               }
               for (int l = 1; l <= m - 2; l += 2) { // 合并区间
                  vi tmp;
                  rep(j, 0, l) tmp.pb(a[j]);
                  rep(j, l+2, m) tmp.pb(a[j]);
                  p = 1;
                  rep(i, 0, sz(tmp)) if (tmp[i]+1 > k) p = 0;
                  if (!p) continue;
                  (f[o][t][tmp] += v.se) %= P;
               }
            }
         }
      }
      ans = 0;
      for (auto v : f[o][3]) if (sz(v.fi) == 2) ans = add(ans, v.se);
      cout << ans << endl;
   }
   return 0;
}
```

11.23 边双联通子图计数

```
// 求边双联通子图个数
const int N = 1 << 10;
int n, S, e[N][N], way[N][N], a[100], t, x, y, dc[N], c[N], c2[N], c1[N], pw2[500], T, m
, u, v;
// c 联通子图个数
// dc 不联通子图个数
// c1 单联通子图个数
// c2 边双联通子图个数
void solve1() {
   S = pw(n) - 1;
   rep(i, 0, S+1) rep(j, 0, S+1) {
      e[i][j] = 0;
      rep(k, 0, n) if (pw(k) & i) e[i][j] += __builtin_popcount(a[k] & j);
   }
   rep(i, 1, S+1) {
      dc[i] = 0;
      for (int msk = (i - 1) & i; msk >= 0; msk = (msk - 1) & i) {
         if (msk & lb(i)) dc[i] = add(dc[i], mul(c[msk], pw2[e[i ^ msk][i ^ msk] / 2]));
         if (msk == 0) break;
      }
      c[i] = sub(pw2[e[i][i] / 2], dc[i]);
   }
}
void solve2() {
   rep(j, 1, S+1) way[0][j] = 1;
   rep(i, 1, S+1) rep(j, 1, S+1) {
      if ((j & i) || lb(j) > lb(i)) continue;
```

```
      way[i][j] = 0;
      for (int msk = i; msk >= 0; msk = (msk - 1) & i) {
         if (msk & lb(i)) way[i][j] = add(way[i][j], mul(mul(way[i ^ msk][j], c[msk]), e
         [msk][j]));
         if (msk == 0) break;
      }
   }
   rep(i, 1, S+1) {
      c1[i] = 0;
      for (int msk = (i - 1) & i; msk >= 0; msk = (msk - 1) & i) {
         if (msk & lb(i)) c1[i] = add(c1[i], mul(way[i ^ msk][msk], c2[msk]));
         if (msk == 0) break;
      }
      c2[i] = sub(c[i], c1[i]);
   }
}
int main() {
   cin >> T;
   pw2[0] = 1;
   rep(i, 1, 300) pw2[i] = pw2[i-1] * 2 % P;
   rep(cas, 0, T) {
      cin >> n >> m;
      rep(i, 0, n) a[i] = (pw(n) - 1) ^ pw(i);
      rep(i, 0, m) {
         cin >> u >> v;
         u--;v--;
         a[u] ^= pw(v);
         a[v] ^= pw(u);
      }
      solve1();
      solve2();
      cout << c2[S] << endl;
   }
   return 0;
}
```