第41卷第2期
2014年3月

浙 江 大 学 学 报(理学版)
Journal of Zhejiang University(Science Edition)
http://www.journals.zju.edu.cn/sci

Vol. 41 No. 2
Mar. 2014

DOI:10.3785/j.issn.1008-9497.2014.02.001

赋权有限集上的容斥原理及应用

唐善刚

(西华师范大学 数学与信息学院,四川 南充 637009)

摘　要:给出了赋权有限集上具带权表达式的新的容斥原理,并用于推广"夫妻对围坐计数问题",得到了相应的计数公式.

关　键　词:赋权有限集;权和式;容斥原理;广义夫妻对围坐计数问题

中图分类号:O 157.1　　　　**文献标志码**:A　　　　**文章编号**:1008-9497(2014)02-123-04

TANG Shangang (*College of Mathematics and Information*, *China West Normal University*, *Nanchong* 637009, *Sichuan Province*, *China*)

A principle of inclusion-exclusion on a weighted finite set and its applications. Journal of Zhejiang University(Science Edition), 2014,41(2):123-126

Abstract: A principle of inclusion-exclusion with weighted formula is given on a weighted finite set. By using the principle of inclusion-exclusion, ménage enumerating problem is extended and some enumerating theorems are given.

Key Words: weighted finite set; weighted formula; principle of inclusion-exclusion; generalized ménage enumerating problem

容斥原理在计数组合问题中有着广泛的应用.文献[1-4]研究赋权有限集上的容斥原理,得到了$W(r_1, r_2, \cdots, r_m)$[1], $W(A_{r_{k+1},\cdots,r_m(r_1,\cdots,r_k)})$[2], $W(A_{r_1,\cdots,r_k(I_{k+1},\cdots,I_m)})$[3]以及$W(A_{(r_1,\cdots,r_k,I_{k+1},\cdots,I_m)})$[4]的权和公式;文献[5]给出了赋权有限集上的权被重复计算下的$W_{r_{k+1},\cdots,r_m(r_1,\cdots,r_k)}$,但$W_{r_{k+1},\cdots,r_m(r_1,\cdots,r_k)}$的缺陷是不能应用于有限集合有关的计数,文献[6-13]对容斥原理及其应用有较为详细的阐述.本文在文献[1-6]的基础上得出了一个新的容斥原理$W(A_{\leqslant r_1,\cdots,r_k(I_{k+1},\cdots,I_m)})$的权和公式,并用于推广计数组合学中的"夫妻对围坐计数问题",得到了多维"广义夫妻对围坐计数问题"的计数公式.

1　准备知识

有限集A称之为赋权有限集,若对$\forall a\in A$,定义a的权$w(a)$属于某个加群.设$U\subseteq A$,称$W(U)=\sum_{b\in U}w(b)$为集合U的权,特别地当U是空集时,约定$W(U)=0$.设m为正整数,n_j是正整数($j=1,2,\cdots,m$),且$\sum_{j=1}^{m}n_j=n$.令$N_j=\{1,2,\cdots,n_j\}(1\leqslant j\leqslant m)$,设$p_{11},p_{12},\cdots,p_{1n_1},\cdots,p_{m1},p_{m2},\cdots,p_{mn_m}$表示$n$个性质,令$P=\{p_{11},p_{12},\cdots,p_{1n_1},\cdots,p_{m1},p_{m2},\cdots,p_{mn_m}\}$表示性质集.现对$P$作一个$m$部划分,即令$P_j=\{p_{ji}\mid i\in N_j\}(1\leqslant j\leqslant m)$.再设$I_j\subseteq N_j(1\leqslant j\leqslant m)$,$k$是非负整数且$k\leqslant m$,以及$r_1,\cdots,r_k$为非负整数且$r_i\leqslant n_i(i=1,2,\cdots,k)$.设$A_{I_1,\cdots,I_m}$[2-4]表示$A$中具有$P_j$中的性质$p_{jt}$,$t\in I_j(1\leqslant j\leqslant m)$的那些元素组成的集合;$A_{I_1,\cdots,I_k(I_{k+1},\cdots,I_m)}$[3]表示$A$中具有$P_j$中的性质$p_{jt}$,$t\in I_j(1\leqslant j\leqslant k)$,而又恰好具有$P_j$中的性质$p_{jt}$,$t\in I_j(k+1\leqslant j\leqslant m)$的那些元素组成的集合;设$A_{(r_1,\cdots,r_k,I_{k+1},\cdots,I_m)}$[4]表示$A$中恰好具有$P_j$中的性质$p_{jt}$,$t\in I_j(k+1\leqslant j\leqslant m)$,而又恰好具有$P_i$中的$r_i$

收稿日期:2013-04-17.
基金项目:西华师范大学校基金项目(11A032).
作者简介:唐善刚(1978—),男,硕士,副教授,主要从事计数组合与群论研究.

$$Q(A_{I_{t_1},\cdots,I_{t_m}})=\frac{\left(\sum_{i=1}^{m}t_i\right)!((\lambda+2)!)^{\sum_{i=1}^{m}t_i}\times\left[(\lambda+2)\left(n-\sum_{i=1}^{m}t_i\right)\right]!}{(2\lambda+4)n-(2\lambda+2)\sum_{i=1}^{m}t_i}\times$$

$$\begin{pmatrix}(\lambda+2)n-(\lambda+1)\sum_{i=1}^{m}t_i\\ \sum_{i=1}^{m}t_i\end{pmatrix}.$$

进一步化简上式,即得

$$Q(A_{I_{t_1},\cdots,I_{t_m}})=\frac{1}{2}((\lambda+2)!)^{\sum_{i=1}^{m}t_i}\left((\lambda+2)n-1-(\lambda+1)\sum_{i=1}^{m}t_i\right)!,$$

进而得到

$$Q_{t_1,\cdots,t_k,I_{t_{k+1}},\cdots,I_{t_m}}=\frac{1}{2}((\lambda+2)!)^{\sum_{i=1}^{m}t_i}\left[(\lambda+2)n-1-(\lambda+1)\sum_{i=1}^{m}t_i\right]!\prod_{j=1}^{k}\binom{n_j}{t_j}.$$

于是应用式(3)与(9),有

定理 2

$$Q(A_{\leqslant r_1,\cdots,r_k(I_{r_{k+1}},\cdots,I_{r_m})})=\frac{1}{2}\sum_{\substack{0\leqslant t_i\leqslant n_i\\1\leqslant i\leqslant m}}(-1)^{\sum_{i=1}^{m}(t_i-r_i)}((\lambda+2)!)^{\sum_{i=1}^{m}t_i}\left[(\lambda+2)n-1-(\lambda+1)\sum_{i=1}^{m}t_i\right]!\times\prod_{i=1}^{k}\binom{n_i}{t_i}\binom{t_i-1}{r_i}\prod_{j=k+1}^{m}\binom{n_j-r_j}{t_j-r_j}.\quad(10)$$

$$Q(A_{\leqslant r_1,\cdots,r_m})=\frac{1}{2}\sum_{\substack{0\leqslant t_i\leqslant n_i\\1\leqslant i\leqslant m}}(-1)^{\sum_{i=1}^{m}(t_i-r_i)}((\lambda+2)!)^{\sum_{i=1}^{m}t_i}\left[(\lambda+2)n-1-(\lambda+1)\times\sum_{i=1}^{m}t_i\right]!\prod_{i=1}^{m}\binom{n_i}{t_i}\binom{t_i-1}{r_i}.\quad(11)$$

$$Q(A_{(I_{r_1},\cdots,I_{r_m})})=\frac{1}{2}\sum_{\substack{0\leqslant t_i\leqslant n_i\\1\leqslant i\leqslant m}}(-1)^{\sum_{i=1}^{m}(t_i-r_i)}((\lambda+2)!)^{\sum_{i=1}^{m}t_i}\left[(\lambda+2)n-1-(\lambda+1)\times\sum_{i=1}^{m}t_i\right]!\prod_{j=1}^{m}\binom{n_j-r_j}{t_j-r_j}.\quad(12)$$

推论 2 $m=1$ 时,有

$$Q(A_{\leqslant r})=\frac{1}{2}\sum_{t=0}^{n}(-1)^{t-r}\binom{n}{t}\binom{t-1}{r}((\lambda+2)!)^{t}\times[(\lambda+2)n-1-(\lambda+1)t]!\quad(13)$$

$$Q(A_{(I_r)})=\frac{1}{2}\sum_{t=r}^{n}(-1)^{t-r}\binom{n-r}{t-r}((\lambda+2)!)^{t}\times[(\lambda+2)n-1-(\lambda+1)t]!\quad(14)$$

注 2 在式(10)~(12)中约定,若 $a\geqslant 0>b$, $\binom{a}{b}=0$.

参考文献(References):

[1] **魏万迪. 广容斥原理及其应用[J]. 科学通报**,1980,25(7):296-299.
WEI Wandi. A generalized principle of inclusion-exclusion and its applications[J]. **Chinese Science Bulletin**, 1980,25(7):296-299.

[2] **唐善刚. 关于"容斥原理的拓广及其应用"的注记[J]. 山东大学学报:理学版**,2012,47(10):64-69.
TANG Shangang. A note on "generalized principle of inclusion-exclusion and its application"[J]. **Journal of Shandong University: Natural Science**,2012,47(10):64-69.

[3] **唐善刚. 容斥原理的拓展及其应用(Ⅱ)[J]. 山东大学学报:理学版**,2011,46(12):70-75.
TANG Shangang. A generalization of principle of inclusion-exclusion and its application(Ⅱ)[J]. **Journal of Shandong University: Natural Science**,2011,46(12):70-75.

[4] **唐善刚. 容斥原理的拓展及其应用[J]. 山东大学学报:理学版**,2010,45(12):12-15.
TANG Shangang. A generalization of principle of inclusion-exclusion and its application[J]. **Journal of Shandong University: Natural Science**,2010,45(12):12-15.

[5] **万宏辉. 容斥原理的拓广及其应用[J]. 科学通报**,1984,29(16):972-975.
WAN Honghui. A generalized principle of inclusion-exclusion and its applications[J]. **Chinese Science Bulletin**,1984,29(16):972-975.

[6] **唐善刚. 广义容斥原理及其应用[J]. 山东大学学报:理学版**,2009,44(1):83-90.
TANG Shangang. Generalized principle of inclusionexclusion and its application[J]. **Journal of Shandong University: Natural Science**,2009,44(1):83-90.

[7] **唐善刚. 广义容斥原理的应用[J]. 佳木斯大学学报:自然科学版**,2010,28(4):573-575.
TANG Shangang. Application of generalized principle of inclusion-exclusion[J]. **Journal of Jiamusi University: Natural Science**,2010,28(4):573-575.

[8] BENDER E A, GOLDMAN J R. Mobius inversion in combinatorial analysis[J]. **Amer Math Monthly**, 1975(82):789-802.

(下转第 137 页)

Acknowladgement　The author thanks the referees for valuable comments.

References：

[1] DONG Y. Index theory, nontrivial solutions and asymptotically linear second order Hamiltonian systems[J]. **J Differ Equations**, 2005, 214: 233-255.

[2] EKELAND I. **Convexity Methods in Hamiltonian Mechanics**[M]. Berlin: Springer-Verlag, 1990.

[3] CLARKE F H, EKELAND I. Nonlinear oscillations and boundary value problems for Hamiltonian systems [J]. **Arch Rational Mech Anal**, 1982, 78: 315-333.

[4] DONG Y. On equivalent conditions for the solvability of equation $(p(t)x')'+f(t,x)=h(t)$ satisfying linear boundary conditions with f restricted by linear growth conditions[J]. **J Math Anal Appl**, 2000, 245: 204-220.

[5] DONG Y. On the solvability of asymptotically positively homogeneous equations with S-L boundary value conditions [J]. **Nonlinear Analysis**, 2000, 42: 1351-1363.

[6] MAWHIN J, WILLEM M. **Critical Point Theory and Hamiltonian Systems**[M]. Berlin: Springer, 1989.

[7] EKELAND I, GHOUSSOUB N, TEHRANI H. Multiple solutions for a classical problem in the calculus of variations [J]. **J Differential Equations**, 1996, 131: 229-243.

[8] LONG Y. The minimal period problem for classical Hamiltonian systems with even potentials[J]. **Ann Inst H Poincare Anal non lineaire**, 1993, 10: 605-626.

[9] LONG Y. The minimal period problem of periodic solutions for autonomous superquadratic second order Hamiltonian systems [J]. **J Differential Equations**, 1994, 111: 147-174.

[10] LONG Y. Index theory for symplectic paths with applications[C]//**Progress in Math**. Basel: Birkhäuser, 2002.

[11] CHANG K C. **Critical Point Theory and Its Application**[M]. Shanghai: Shanghai Sci Tech Press, 1986.

（上接第 126 页）

[9] **柯召，魏万迪. 组合论：上册**[M]. **北京：科学出版社**，1981.
KE Zhao, WEI Wandi. **Combinatorial Theory** [M]. Beijing: Science Press, 1981.

[10] **林翠琴. 组合学与图论**[M]. **北京：清华大学出版社**，2009.
LIN Cuiqin. **Combinatorial Mathematics and Graph Theory** [M]. Beijing: Tsinghua University Press, 2009.

[11] **李乔. 组合数学基础**[M]. **北京：高等教育出版社**，1993.
LI Qiao. **Combinatorial Mathematics Basic**[M]. Beijing: Higher Education Press, 1993.

[12] **曹汝成. 组合数学**[M]. **广州：华南理工大学出版社**，2000.
CAO Rucheng. **Combinatorial Mathematics** [M]. Guangzhou: South China University of Technology Press, 2000.

[13] **陈景润. 组合数学简介**[M]. **天津：天津科学技术出版社**，1998.
CHEN Jingrun. **Combinatorial Mathematics Synopsis** [M]. Tianjin: Tianjin Science and Technology Press, 1998.

[14] **李磊. 关于几个组合计数公式的推广**[J]. **工程数学学报**，1996，13(4)：95-98.
LI Lei. Generalization of some combinatorial formulas [J]. **Journal of Engineering Mathematics**, 1996, 13(4): 95-98.