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Task: Determine the relationship between the number of objects (n) and the number of pairs (m) generated to accomplish this (i.e. to reduce the number of components from n to 1).

Relationship Conclusion:

To connect objects until only one component remaining, assume we need to connect m times. After m times connection, there are n – m components remaining. Let n – m equals to 1, then m is:

Connections with number of objects:
$$m = n - 1$$
Pairs with number of objects:

$$numOfPairs = \frac{1}{2}nln(n)$$

The evidence for pairs is followed below:

Evidence to support that conclusion:

Number of connections when n equals to 1 is: 0, average number of pairs is 0
Number of connections when n equals to 2 is: 1, average number of pairs is 1
Number of connections when n equals to 4 is: 3, average number of pairs is 4
Number of connections when n equals to 8 is: 7, average number of pairs is 10
Number of connections when n equals to 16 is: 15, average number of pairs is 27
Number of connections when n equals to 32 is: 31, average number of pairs is 66
Number of connections when n equals to 64 is: 63, average number of pairs is 145
Number of connections when n equals to 128 is: 127, average number of pairs is 355
Number of connections when n equals to 256 is: 255, average number of pairs is 774
Number of connections when n equals to 512 is: 511, average number of pairs is 1726
Number of connections when n equals to 1024 is: 1023, average number of pairs is 3925
Number of connections when n equals to 2048 is: 2047, average number of pairs is 8255
Number of connections when n equals to 4096 is: 4095, average number of pairs is 17710
Number of connections when n equals to 8192 is: 8191, average number of pairs is 39381
Number of connections when n equals to 16384 is: 16383, average number of pairs is 84016
Number of connections when n equals to 32768 is: 32767, average number of pairs is 179755
Number of connections when n equals to 65536 is: 65535, average number of pairs is 385979
Number of connections when n equals to 131072 is: 131071, average number of pairs is 811895

With the code running result, the pairs number is increasing more rapidly than n, but slower than n^2, so I guess the relationship may be a*n*ln(n). Then I used the MATLAB to curve fit data, the result shows that the coefficient a is about 0.5.

n	$numOfPairs$	$\frac{1}{2}nln(n)$
1	0	0

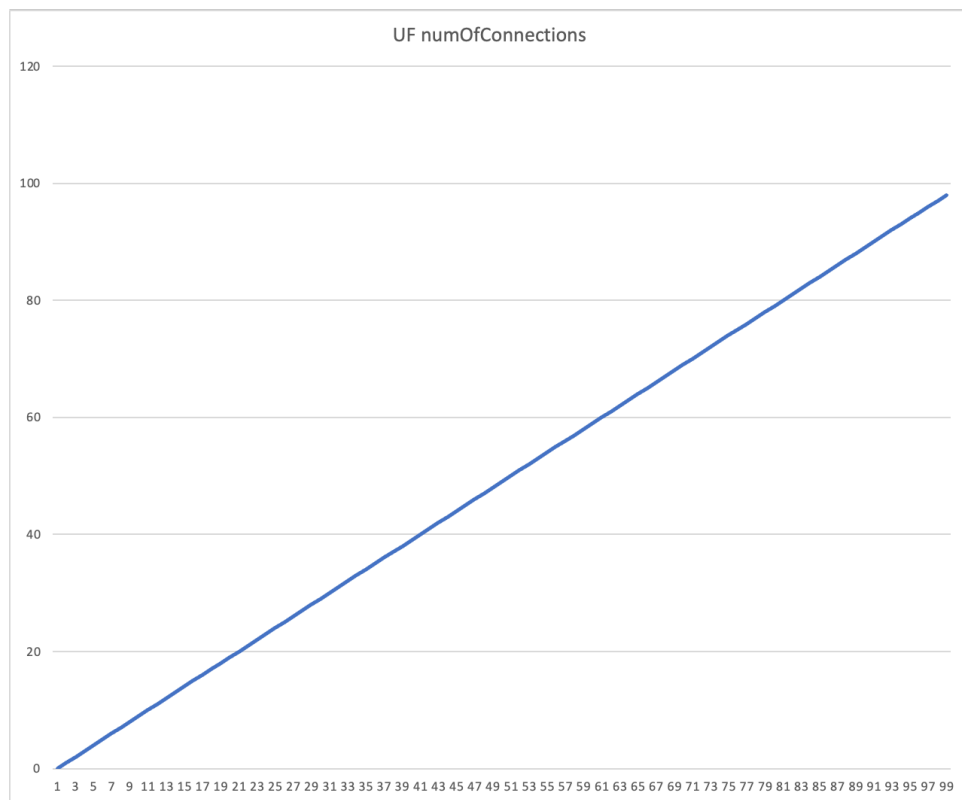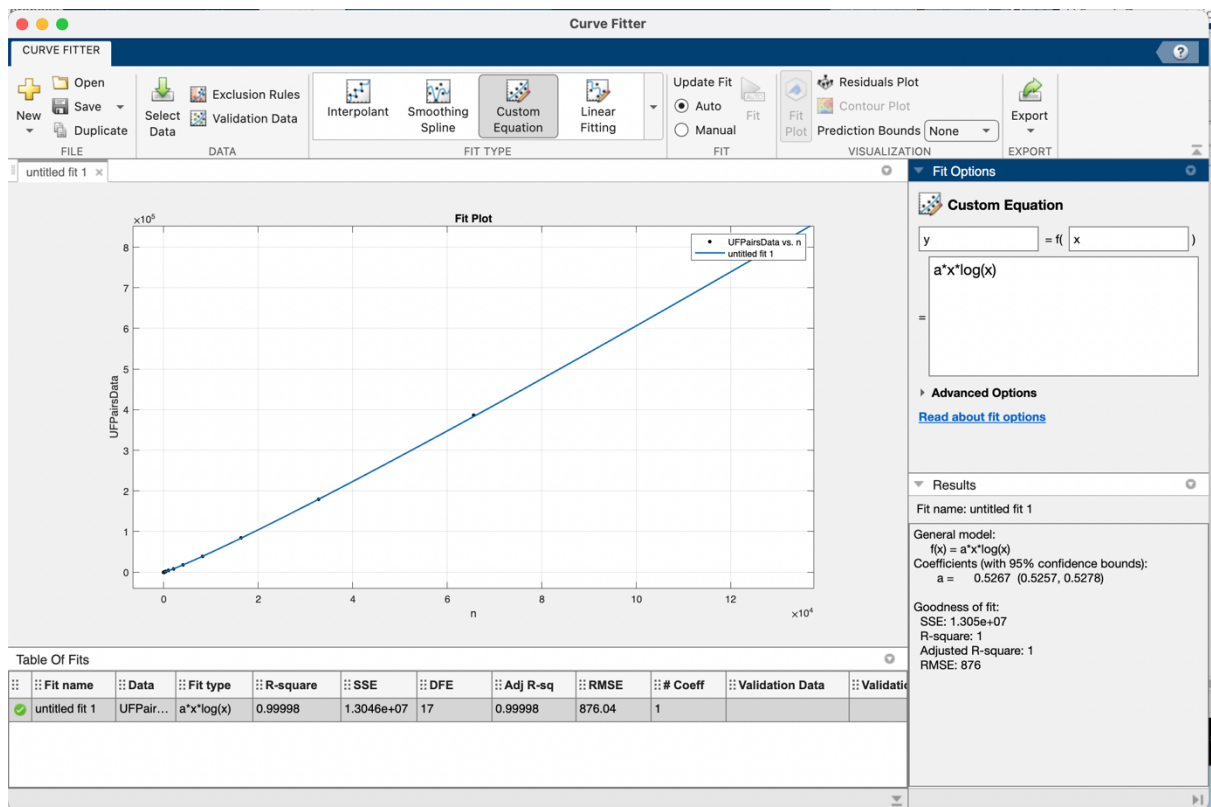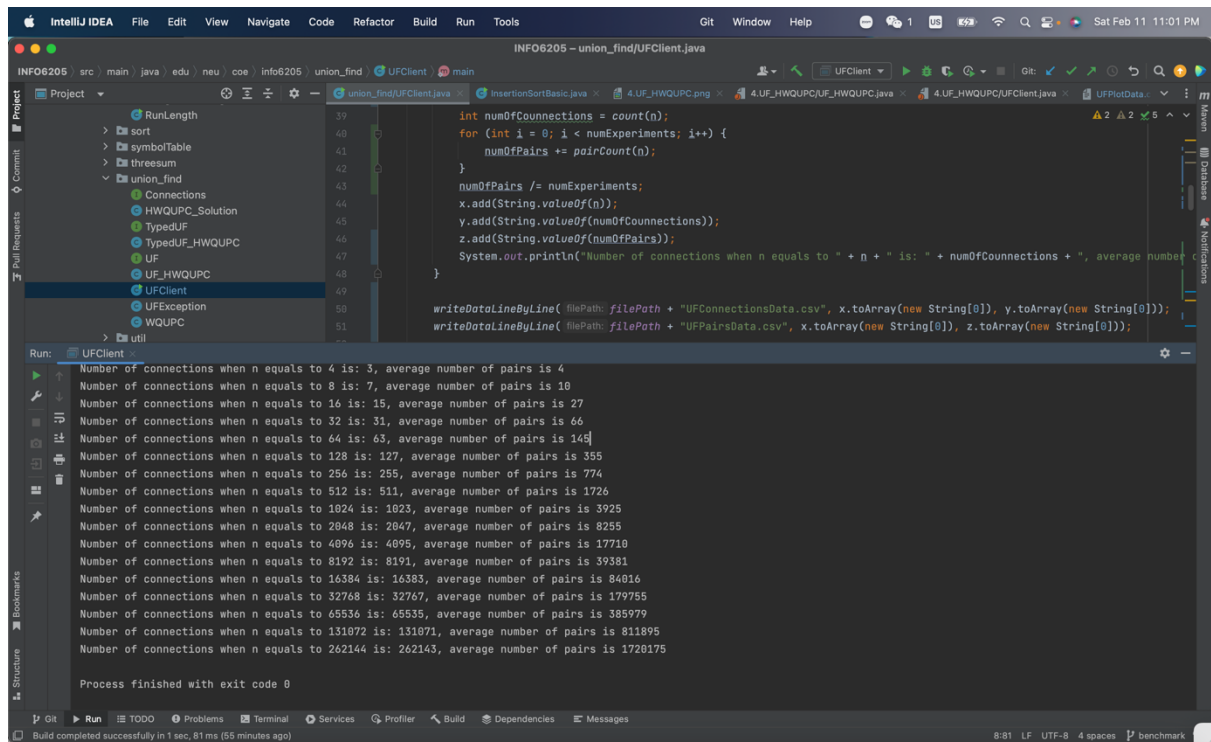|		
2	1	0.6931
4	4	2.7725
8	11	8.3177
16	26	22.1807
32	66	55.4517
64	145	133.08
128	355	310.52
256	774	709.78
512	1726	1597.01
1024	3925	3548
2048	8255	7807
4096	17710	17034
8192	39381	36908
16384	84016	79495
32768	179755	170347
65536	385979	363408
131072	811895	772243
262144	1720175	1635332

Graphical Representation:

Figure 1

Figure 2 curve fit of pairs

Figure 3

Unit Test Screenshots: