Lisp in 99 lines of C and how to write one yourself

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"In 1960, John McCarthy published a remarkable paper in which he did for programming something like what Euclid did for geometry. He showed how, given a handful of simple operators and a notation for functions, you can build a whole programming language. He called this language Lisp, for "List Processing," because one of his key ideas was to use a simple data structure called a list for both code and data." – Paul Graham [1]

1 Introduction

McCarthy's paper [2] not only showed how a programming language can be built entirely from lists as code and data, he also showed a function in Lisp that acts like an interpreter for Lisp itself. This function, called eval, takes as an argument a Lisp expression and returns its value. It was a remarkable discovery that Lisp can be written in Lisp itself. Lisp also introduced the concept of functions as first-class objects (closures) with static scoping¹, runtime typing and garbage collection. Features we now take for granted but were radical at that time. To put this into context, other programming languages at that time were Fortran (1957) and Algol (1958). Many new programming languages have appeared since. Most are still "Algol-like", or as some say, "C-like." The Lisp model of computation has regained momentum over the past decade. Contemporary programming languages now include Lisp-like "lambda functions." Lambda functions are syntactic materializations of lambda abstractions from the lambda calculus [3] introduced by Alonzo Church in 1944. It was lambda calculus that inspired McCarthy to write Lisp.

In honor of the contributions made by Church and McCarthy, I wrote this article to show how anyone can write a tiny Lisp interpreter in a few lines of C or any "C-like" programming language. I attempted to preserve the original meaning and flavor of Lisp as much as possible. As a result, the C code in this article is strongly Lisp-like in compact form. Despite being small, these tiny Lisp interpreters in C include 20 built-in Lisp primitives, garbage collection and REPL, which makes them a bit more practical than a toy example. If desired, more Lisp features can be easily added with a few more lines of C as explained in this article with examples that are ready for you to try.

I encourage anyone to explore other Lisp implementations and their code. Many are cool with lots of features. Some are actually incorrect. Sometimes little nuggets surface when digging deeper to achieve perfection. This appears to be the case when writing this article, as it turns out that the list dot operator plays an important and useful role in lambda variable lists and arguments lists. In this case, no special forms are needed to achieve the same in pure Lisp. I hope you will enjoy reading this article as much as I did writing it!

¹Originally dynamic scoping, which has some drawbacks.

Contents

1	Introduction	1
2	Understanding Lisp	4
3	Lisp Expressions as Tagged Structures 3.1 NaN Boxing	7 8 8 9
4	Constructing Lisp Expressions	11
5	Evaluating Lisp Expressions	14
6	Lisp Primitives 6.1 eval 6.2 quote 6.3 cons 6.4 car and cdr 6.5 Arithmetic 6.6 int 6.7 Comparison 6.8 Logic 6.9 cond 6.10 if 6.11 let* 6.12 lambda 6.13 define	16 16 17 17 17 17 17 18 18 18 19 19
7	Reading and Parsing Lisp Expressions	20
8	Printing Lisp Expressions	22
9	Garbage Collection	22
10	The Read-Eval-Print Loop	23
11	Additional Lisp Primitives 11.1 assoc and env 11.2 let and letrec* 11.3 setq 11.4 set-car! and set-cdr! 11.5 macro 11.6 read and print	23 24 24 25 25 26
12	Adding Readline with History	27

Tracing Lisp	28
Adding Error Handling and Exceptions to Lisp 14.1 C setjmp	
Downsizing Lisp to Single Floating Point Precision	31
Conclusions	32
Bibliography	32
Tiny Lisp Interpreter with NaN boxing: 99 Lines of C	33
Tiny Lisp Interpreter with BCD boxing: 99 Lines of C	35
Optimized Lisp Interpreter with NaN boxing	37
Optimized Lisp Interpreter with BCD boxing	39
Example Lisp Functions E.1 Standard Lisp Functions	42 42 44
	Adding Error Handling and Exceptions to Lisp 14.1 C setjmp 14.2 C++ Exceptions Downsizing Lisp to Single Floating Point Precision Conclusions Bibliography Tiny Lisp Interpreter with NaN boxing: 99 Lines of C Tiny Lisp Interpreter with BCD boxing: 99 Lines of C Optimized Lisp Interpreter with NaN boxing Optimized Lisp Interpreter with BCD boxing Example Lisp Functions E.1 Standard Lisp Functions E.2 Math Functions E.3 List Functions

Biography

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2 Understanding Lisp

Lisp programs are composed of anonymous functions written in the form of a ()-delimited list

```
(lambda variables expression)
```

where *variables* is a list of names denoting the function parameters and *expression* is the body of the function. Just like any other ordinary math function, lambdas don't do anything until we apply them to arguments. Application of a lambda to arguments is written as a list

```
(function arguments)
```

The application is performed in two steps. First, we bind the *variables* to the values of the corresponding *arguments*. Then the *expression* is evaluated. The *expression* may reference the function's *variables* by their name. The value of *expression* is "returned" as the result of the application.

Note that "return" is an imperative concept. Lisp has no imperative keywords. The entire Lisp language is built from functions and function applications, all using lists as syntax. Besides lists, Lisp also has symbols (names) for variables, primitives, functions (closures) and numbers. Lisp dialects may also include strings.

Function application is perhaps best illustrated with an example. Consider the function

```
(lambda (x y) (/ (- y x) x))
```

This function returns the value of $\frac{y-x}{x}$ for numeric arguments x and y. The first thing we note is that all arithmetic operations are written in functional form in Lisp. There is no need for any specific rules for operator precedence and associativity in Lisp. To apply our lambda to arguments, say 3 and 9, we write the list

```
((lambda (x y) (/ (- y x) x)) 3 9)
```

Spacing in Lisp is immaterial, so let's add some more spacing

The Lisp interpreter binds x to 3 and y to 9, then evaluates the function body (/ (- y x) x)) to compute 2 as the result of the application. Nice, isn't it?

But our function is not stored anywhere. What if we want to reuse it? After all, programs are composed of functions and those functions should be stored as part of a program to use them². We can save our function by giving it a name using a define

```
(define subdiv (lambda (x y) (/ (-y x) x)))
```

and then apply subdiv to 3 and 9 with

```
(subdiv 3 9)
```

²The beauty of lambda calculus is that this is not an absolute requirement: lambda calculus is Turing-complete without named functions.

which displays 2. A defined name is not required to be alphabetic. Names are syntactically symbolic forms in Lisp. We could have named our lambda -/ for example. Any sequence of characters can be used as a name, as long as it is distinguishable from a number and doesn't use parenthesis, quotes and whitespace characters.

The power and simplicity of Lisp's lambdas is better justified when we take a closer look at closures. A closure combines a function (a lambda) with an environment. An environment defines a set of name-value bindings. An environment is created (or extended) when the variables of a lambda are bound to the argument values in a lambda application. An environment provides a concrete mechanism to create a local scope of variables for the function body. Because lambdas are first-class objects and can therefore be returned as values by lambdas, environments play a crucial role to scope nested lambdas properly through static scoping. Consider for example the make-adder lambda that takes an x to return a new lambda that takes a y and adds them together:

```
(define make-adder (lambda (x) (lambda (y) (+ x y))))
```

Applying make-adder to 5 returns a closure in which the environment includes a binding of x to 5. When this closure is applied to 2 it returns 7 as expected:

```
> (define make-adder (lambda (x) (lambda (y) (+ x y))))
> ((make-adder 5) 2)
7
> (define add5 (make-adder 5))
> (add5 2)
7
```

Note that make-adder returns a closure that combines (lambda (y) (+ x y)) with an environment in which x is bound to 5. The x in the lambda body is not modified. It is Lisp code after all. When the closure is applied, the environment is extended to include a binding of y to 2. With this environment the function body (+ x y) evaluates to 7.

A handful of programming languages both correctly and safely implement the semantics of closures with static scoping. The implementation requires *unlimited extent* of non-local variables in scope to store bindings. Otherwise, non-local variables are "gone" as their values are removed from memory. This requires environments and garbage collection to remove them safely after the work is done. It doesn't suffice that functions can be syntactically nested within other functions.

Lisp also has a collection of built-in *primitives*. These are functions like + and *special forms* like define. A special form is a function that selectively evaluates its arguments rather than all of its arguments as in lambda applications. For example, define does not evaluate its *name* argument. Otherwise the value of the *name* would end up being used by define or an error is produced when *name* is not yet defined, which is more likely.

An overview of Lisp is not complete without a presentation of the basic primitives introduced in McCarthy's paper. We list them here and also include two more primitives if and a let since these are often used in Lisp³. Some of the primitives listed below are special forms, namely quote, cond, if and let:

• (quote x) returns x unevaluated, "as is". Abbreviated 'x.

³The if and let can be defined as macros, but we keep our Lisp interpreter small without macro processing. To add macro processing, see Section 11.5.

```
> (quote a)
a
> 'a
a
> '(a b c)
(a b c)
```

• (cons x y) returns the pair (x . y) where the dot is displayed if y is not the empty list ().

```
> (cons 'a 'b)
(a . b)
> (cons 'a ())
(a)
> (cons 'a (cons 'b (cons 'c ())))
(a b c)
```

• (car x) ("Contents of the Address part of Register") returns the first element of the pair or list x.

```
> (car (cons 'a 'b))
a
> (car (cons 'a (cons 'b (cons 'c ()))))
a
```

• (cdr x) ("Contents of the Decrement part of Register", pronounced "coulder") returns the second element of the pair x. When x is a list, the rest of the list is returned after the first element.

```
> (cdr (cons 'a 'b))
b
> (cdr (cons 'a (cons 'b (cons 'c ()))))
(b c)
```

• (eq? x y) returns the atom #t (representing true) if the values of x and y are identical. Otherwise returns () representing false.

```
> (eq? 2 2)
#t
> (eq? 2 3)
()
> (eq? 'a 'a)
#t
```

• (cond $(x_1 \ y_1)$ $(x_2 \ y_2)$... $(x_n \ y_n)$) evaluates x_i from left to right until x_i is not the empty list (i.e. is true), then returns the corresponding value of y_i .

```
> (cond ((eq? 'a 'b) 1) ((eq? 'b 'b) 2))
2
> (cond (() 1) (#t 2))
2
```

• (if $x \ t \ e$) if x is not the empty list (i.e. is true), then the value of t is returned else the value of e is returned. (if $x \ t \ e$) is a shorthand for (cond ($x \ t$) (#t e)).

```
> (if 'a 1 2)
1
> (if () 1 2)
2
> (if (eq? 'a 'a) 'ok 'fail)
ok
```

• (let $((v_1 \ x_1) \ (v_2 \ x_2) \ \dots \ (v_n \ x_n))$ y) evaluates x_i from left to right and binds each variable v_i to the value of x_i to extend the environment to the body y, then returns the value of y. The same is accomplished with ((lambda $(v_1 \ v_2 \ \dots \ v_n) \ y) \ x_1 \ x_2 \ \dots \ x_n$).

```
> (let ((x 3) (y 9)) (/ (- y x)))
2
```

Lisp implementations include more primitives, notably for arithmetic, logic and runtime type checking. Most Lisp implementations define additional Lisp primitives in Lisp itself.

3 Lisp Expressions as Tagged Structures

Lisp expressions are composed of numbers, atoms (names and symbols), strings (when implemented), primitives, cons pairs and closures. A Lisp expression type can be conveniently defined in C as a tagged union:

```
struct Expr {
  enum { NMBR, ATOM, STRG, PRIM, CONS, CLOS, NIL } tag;
  union {
    double number;
                                 /* NMBR: double precision float number */
                                 /* ATOM: pointer to atom name on the heap */
    const char *atom;
                                 /* STRG: pointer to string on the heap */
    const char *string;
    struct Expr (*fn)(struct Expr, struct Expr);
                                                     /* PRIM: built-in primitive */
    struct Expr *cons;
                                /* CONS: pointer to (car,cdr) pair on the heap */
    struct Expr *closure;
                                 /* CLOS: pointer to closure pairs on the heap */
  } value;
};
```

However, rather than storing this information elaborately in a structure, we can exploit *NaN boxing* to store this information in an IEEE-754 single or double precision float, because all structure members are pointers that are essentially unsigned integer offsets from a base address.

3.1 NaN Boxing

The idea behind NaN boxing is that IEEE 754 floating point NaN ("Not-a-Number") values are not unique. A double precision NaN allows up to 52 bits to be arbitrarily used to stuff any information we want into a double precision NaN:

where

s is the sign bit of the float, s=1 for negative numbers

exponent consists of 11 bits to represent binary exponents -1022 to 2023, when the bits are all 1 the value is NaN or INF

fraction consists of 52 bits with an invisible 1 bit as the leading digit of the mantissa

The tag and other data can be stored in the freely available 52 bit fraction part of a NaN. However, we want to use $quiet\ NaNs$ which means that the first bit of the fraction (the bit before the tag bits) must be 1, leaving 51 bits and the sign bit for both the tag and other data to our disposal. This is plenty of space in a NaN-boxed double precision float to store a tag to identify atoms, strings, primitives, cons pairs, and closures together with their pointers and/or integer indices. In all, 48 bits are available to store an integer, or 49 bits when including the sign bit.

In this article I will also describe a Lisp implementation for the Sharp PC-G850 vintage pocket computer, which poses a bit of a challenge since it does not use IEEE 754 floating point representations. Instead, floating point values are represented internally in BCD (Binary Coded Decimal). This raises the question: can we use similar tricks as NaN boxing with BCD floats? Let's find out.

3.2 BCD Float Boxing

To understand if and how we can exploit BCD floats to store information other than floating point numbers, we will take a closer look at the PC-G850's internal decimal floating point representation. A decimal floating point value is stored in 8 bytes. The first 2 bytes store the exponent in BCD and the sign of the number. The next 5 bytes store the 10 digit BCD mantissa followed by a zero byte:

where

s is the sign bit of the float, s=1 for negative numbers

d is the degree bit, d=1 to display degrees in $D^oM'S.S$ " format in BASIC

u is an unused bit, likely a mantissa carry bit used by the system

10's complement BCD exponent consists of 12 bits for 3 BCD digits to represent exponents -99 (901 BCD) to 99 (099 BCD)

10 digit BCD mantissa consists of 10 BCD digits with the normalized mantissa, the leading BCD digit is nonzero unless all mantissa digits are zero

2 BCD guard digits are always zero after internal rounding to 10 significant digits

To explore opportunities to exploit BCD float $boxing^4$ to store information other than decimal floating point numbers, we can write some C code for testing. Unfortunately, we cannot use the mantissa or its trailing guard digits to store extra information. The mantissa is always normalized to BCD and the guard digits are always reset to zero when passing floats through functions, even when no arithmetic operations are applied to the float. Our target bits to box a tag with data in a float are the three bits in the upper half of the leading byte of the float. These three bits of the float remain unmodified when passing the floating point value through functions as arguments and as return values. A quick test confirms our hypothesis, with some caveats:

```
double func(double x) { return x; }
int main() {
  double x,y,z; char *p = (char*)&x,*q = (char*)&y; int i;
  scanf("%lg",&z);
                                             /* input a value z to check */
  for (i = 0x20; i \le 0x70; i += 0x10) {
                                             /* set x to z and set its tag bits */
    x = z; *p |= i;
                                             /* pass x through func() */
    x = func(x);
    y = z; *q |= i;
                                             /* set y to z and set its tag bits */
                                             /* x and y should be equal */
    if (x != y)
      printf("fail %x\n",i);
 }
}
```

The first caveat is that tag 000 (i==0x00) cannot be used. This tagged value is indistinguishable from a normal float value. Second, tag 001 (i==0x10) cannot be used because all tagged floats appear to fail with an arithmetic error when passed to a function, perhaps because the tagged value corresponds to a non-normalized carry digit in the exponent. Third, all tagged floats with values |x| < 10 are normalized to zero and thus fail this test. This type of failure happens when the two-digit BCD exponent is zero and the third highest order BCD exponent digit nonzero, thus representing a non-normalized two-digit zero BCD exponent.

After confirming our hypothesis by observation, we conclude that we have six possible tag bit patterns 010 to 111 to our disposal, as long as we box integers as tagged floats $|x| \ge 10$. This is not a problem, because we can simply multiply a value by 10 before boxing and divide by 10 after unboxing. When boxing unsigned integers, such as pointers and array indices, it suffices to add 10 before boxing and subtract 10 after unboxing. So we will use this simpler boxing method.

3.3 Types of Lisp Expressions

We define two types⁵ in C that we exclusively use in our Lisp interpreter: a nice and simple type I for unsigned integers and a type L for Lisp expressions defined as floats for NaN or BCD float boxing:

⁴I think the term "BCD float boxing" nails it, but Google seems to think it's sports gear.

⁵PC-G850 C does not support typedef. We use #define instead.

```
#define I unsigned
#define L double
```

We define five tags for Atoms, primitives, constructed pairs, closures and nil (the empty list):

```
/*** with NaN Boxing ***/
I ATOM=0x7ff8,PRIM=0x7ff9,CONS=0x7ffa,CLOS=0x7ffb,NIL=0x7ffc;
/*** with BCD boxing ***/
I ATOM=32,PRIM=48,CONS=64,CLOS=80,NIL=96;
```

To access the tag bits of a tagged float we cast the pointer to the float to uint64_t* or char* using a nice short and sweet T:

```
/*** with NaN Boxing ***/
#define T(x) *(uint64_t*)&x>>48
    /*** with BCD boxing ***/
#define T *(char*)&
```

We will set a tag with T(x) = tag and retrieve a tag with T(x) for Lisp expression L x. Instead of uint64_t to cast the 64 bit double in T(x) we can use unsigned long long instead, which is typically 64 bits⁶. We also need the following two functions to manipulate tagged floats and their ordinal content:

```
/*** with NaN Boxing ***/
L box(I t,I i) { L x; *(uint64_t*)&x = (uint64_t)t<<48|i; return x; }
I ord(L x) { return *(uint64_t*)&x; }
    /*** with BCD boxing ***/
L box(I t,I i) { L x = i+10; T(x) = t; return x; }
I ord(L x) { T(x) &= 15; return (I)x-10; }</pre>
```

The box function returns a float tagged with the specified tag t as ATOM, PRIM, CONS, CLOS or NIL and by boxing unsigned integer i as ordinal content. For BCD boxing we must add 10 to i to avoid the aforementioned caveat when boxing values in BCD floats. The ord function unboxes the unsigned integer (ordinal) of a tagged float. For BCD boxing we first untag the float with T(x) &= 15 then subtract 10 to return the boxed ordinal content of the tagged float.

We should be able to perform arithmetic on floats in our Lisp interpreter. To do so, we could simply assume that the arguments and operands to arithmetic operations are always untagged floats. However, to make sure we aren't applying arithmetic operations on tagged floats by accident, we should define a new function num to check or clear tag bits first, before applying arithmetic operations:

```
/*** with NaN Boxing ***/
L num(L n) { return n; }
   /*** with BCD boxing ***/
L num(L n) { T(n) &= 159; return n; }
```

⁶At least 64 bits, but we want 64 exactly. C is one of the oldest programming languages and had to accommodate systems with strange bit widths like 18 and 36 and so on, hence "at least".

With NaN boxing the number is returned "as is", but we could check if n is a NaN and take some action. For now, we just pass NaNs to perform arithmetic on, which results in a NaN. For BCD boxing we clear two bits of the tag with T(n) &= 159 (0x9f) since negative BCD exponents are represented in BCD 9dd. The high-order digit 9 (binary 1001) should be preserved.

Checking if two values are equal is performed with the equ function. Because equality comparisons == with NaN values always produce false, we just need to compare the 64 bits of the values for equality:

```
/*** with NaN Boxing ***/
I equ(L x,L y) { return *(uint64_t*)&x == *(uint64_t*)&y; }
/*** with BCD boxing ***/
I equ(L x,L y) { return x == y; }
```

Note that BCD boxing does not really require defining an equ function. It just wraps ==. But we include it here since we may have to redefine it depending on the BCD arithmetic performed by a specific machine.

Checking if a Lisp expression is nil (the empty list) only requires checking its tag for NIL:

```
I not(L x) { return T(x) == NIL; }
```

The **not** function comes in handy later when we implement conditionals, since **nil** is considered false in Lisp. Anything else is implicitly true in Lisp.

The C functions we defined here are the only ones specific to NaN or BCD float boxing. The rest of our Lisp interpreter is independent of the tagging method used.

4 Constructing Lisp Expressions

Lisp expressions are composed of atoms (also called symbols in Lisp), primitives, cons pairs, closures and nil. The nil constant represents the empty list () in Lisp. The nil constant is also considered false in Lisp conditionals. Furthermore, we have two pre-defined atoms, namely #t and ERR. The #t atom will be used as an explicit true in Lisp, although any value other than nil is implicitly true in Lisp conditionals. The ERR atom represents an error and is returned to the user when an expression evaluates to an error. These three constants are globally declared since we will often use them in the internals of our Lisp interpreter. They are initialized in the main function as follows:

```
L nil,tru,err;
...
int main() {
    ...
    nil = box(NIL,0); tru = atom("#t"); err = atom("ERR");
    ...
}
```

⁷We initialize globals in main since PC-G850 C does not support initialization of globals with non-constants, i.e., function calls cannot be used as initializers of globals.

where we used the box function defined in Section 3.2. The atom function returns an ATOM-tagged float that is globally unique. The atom function checks if the atom name already exists on the heap and returns the heap index corresponding to the atom name boxed in the ATOM-tagged float. If the atom name is new, then additional heap space is allocated to copy the atom name into the heap as a string. The heap index of the new atom name is boxed in the ATOM-tagged float and returned by the atom function:

```
L atom(const char *s) {
   I i = 0; while (i < hp && strcmp(A+i,s)) i += strlen(A+i)+1;
   if (i == hp && (hp += strlen(strcpy(A+i,s))+1) > sp<<3) abort();
   return box(ATOM,i);
}</pre>
```

where hp is the heap pointer pointing to free bytes available on the heap. When i==hp we execute strlen(strcpy(A+i,s))+1) to copy the string s to the free heap address at A+hp and return the length of the string plus one to increase hp. A is the starting byte address of the heap and sp is the stack pointer pointing to the top of the stack of Lisp values (tagged and untagged floats L):

```
#define A (char*)cell
#define N 1024
L cell[N];
I hp = 0,sp = N;
```

The cell[N] array of 1024 (tagged) floats contains both the heap and the stack. The value of N can be increased to pre-allocate more memory.

The atom function searches the heap at addresses A+i until a matching atom name is found to return box(ATOM,i). If the atom name is new, then space for the atom's string name is allocated and copied into this space with a terminating zero byte. In this way, atoms constructed with box(ATOM,i) are globally unique. The method by which we construct them by looking them up in a pool of names is often referred to as *interning*.

The heap grows upward towards the stack. The stack grows downward, as stacks usually do. The remaining free space is available between the heap and stack. For example, the table below depicts the memory configuration after pushing a pair of cells box(ATOM,0) and nil on the stack and storing two atoms #t and ERR in the heap:



What's actually on the stack here is the Lisp list (#t) containing one element #t in the list. This list is represented by box(CONS,1022) where 1022 is the stack index of the cdr cell of this list pair. The cell above it on the stack contains the car of this list pair. Lisp uses linked lists with the car of a list node (a cons pair) containing the list element and cdr pointing to the next cons pair in the list.

With this memory configuration in mind, constructing cons pairs is easy. We just allocate two cells on the stack, copy the values therein and return box(CONS,sp) since sp points to the cdr cell:

```
L cons(L x,L y) {
  cell[--sp] = x;
  cell[--sp] = y;
  if (hp > sp<<3) abort();
  return box(CONS,sp);
}</pre>
```

If the hp and sp pointers meet we ran out of memory and we should abort or take some other action. Note that hp points to bytes whereas sp points to 8-byte floats. Therefore, the out-of-memory condition is checked in the atom and cons functions by scaling sp by a factor 8 in the conditional if (hp > sp<<3) abort().

Deconstructing a cons pair is trivial. We just need to get the cell of the car or cdr indexed by i in box(CONS,i) by retrieving it with the ord function:

```
L car(L p) { return (T(p) & ^{(CONS^{CLOS})}) == CONS ? cell[ord(p)+1] : err; } L cdr(L p) { return (T(p) & ^{(CONS^{CLOS})}) == CONS ? cell[ord(p)] : err; }
```

where ord(p) is the cell index of the cdr of the cons pair p with the car cell located just above it. However, we do not trust the argument p to be a cons pair. The condition T(p) & ~(CONS^CLOS)) == CONS guards valid car and cdr function calls on cons pairs. The condition is true if p is a cons pair or a closure pair. Closure pairs are just cons pairs tagged as closures. The ~(CONS^CLOS) mask is an efficient way to check for both CONS and CLOS tags in one comparison, because the tag values of CONS and CLOS were carefully chosen to differ by only one bit.

For example, suppose p = box(CONS,sp) representing the list (#t) with the cells on the stack depicted in the memory configuration shown previously. Then car(p) returns box(ATOM,0) representing #t and cdr(p) returns nil, the empty list.

Closures and environment lists are constructed with the cons function applied twice, first to construct the name-value Lisp pair⁸ (v.x), then to place the pair in front of the Lisp environment list. We define a pair function for this purpose:

```
L pair(L v,L x,L e) { return cons(cons(v,x),e); }
```

A closure is a CLOS-tagged pair(v,x,e) representing an instantiation of a Lisp (lambda v x) with either a single atom v as a variable referencing a list of arguments passed to the function, or v is a list of atoms as variables, each referencing the corresponding argument passed to the function. Closures include their static scope as an environment e to reference the bindings of their parent functions, if functions are nested, and to reference the global static scope:

```
L closure(L v,L x,L e) { return box(CLOS,ord(pair(v,x,equ(e,env) ? nil : e))); }
```

⁸The Lisp pair denoted with a dot (v.x) is constructed by the Lisp parser with p = cons(v,x), whereas a Lisp list always ends in nil, such as $(1\ 2)$ constructed by the Lisp parser with cons(1,cons(2,nil)).

The conditional equ(e,env)?nil:e forces the scope of a closure to be nil if e is the global environment env. Later, when we apply the closure, we check if its environment is nil and use the current global environment. This permits recursive calls and calls to forward-defined functions, because the current global environment includes the latest global definitions.

An *environment* in Lisp is implemented as a list of name-value associations, where names are Lisp atoms. Environments are searched with the assoc function given an atom a and an environment e:

```
L assoc(L v,L e) {
  while (T(e) == CONS && !equ(v,car(car(e)))) e = cdr(e);
  return T(e) == CONS ? cdr(car(e)) : err;
}
```

The assoc function returns the Lisp expression associated with the specified atom in the specified environment e or returns err if not found.

Consider for example the Lisp environment e = ((n.3) (p.(1.2)) (x.nil)) where the Lisp expression (1.2) constructs a pair of 1 and 2 instead of a list. This example environment e is represented in memory as follows with boxes for cells and arrows for CONS indices pointing to cells:



Note that a CONS index points to two cells on the stack, the car and cdr cells.

5 Evaluating Lisp Expressions

A Lisp expression is either a number, an atom, a primitive, a cons pair, a closure, or nil. Numbers, primitives, closures and nil are constant and returned by eval as is. Atoms are evaluated by returning their associated value from the environment using the assoc function, see Section 4. The environment includes function parameters and global definitions. Lists are evaluated by applying the first element in the list as a function to the rest of the list as arguments passed to that function:

```
L eval(L x,L e) {
  return T(x) == ATOM ? assoc(x,e) :
      T(x) == CONS ? apply(eval(car(x),e),cdr(x),e) :
      x;
}
```

Note that Lisp expression x evaluates to the value assoc(x,e) of x when x is an atom, or evaluates to the result of a function application apply(eval(car(x),e),cdr(x),e) if x is a list, or evaluates to x itself otherwise. A function application requires evaluating the function eval(car(x),e) first before applying it, because car(x) may be an expression that returns a function such as an atom associated with a Lisp primitive or the closure constructed for a lambda. The apply function applies the primitive or the closure f to the list of arguments f in environment f:

The prim[] array contains all pre-defined primitives. A primitive is stored in a structure with its name as a string and a function pointer to its code. To call the primitive we invoke the function pointer with prim[ord(f)].f(t,e). See also Section 6. If f is a closure then reduce⁹ is called to apply closure f to the list of arguments t:

```
L reduce(L f,L t,L e) {
  return eval(cdr(car(f)),bind(car(car(f)),evlis(t,e),not(cdr(f)) ? env : cdr(f)));
}
```

where f is the closure and t is the list of arguments. The outer eval evaluates the body of closure f retrieved with cdr(car(f)). The evaluation is performed with an extended environment produced with the bind function. This extended environment includes the bindings of the lambda's variables to the evaluated arguments. Remember that a closure was constructed for a lambda to include its static scope or nil as its environment? We retrieve its environment with cdr(f) and check it for nil with not(cdr(f))?env:cdr(f), which gives the current global environment env or the lambda's local static environment cdr(f). This environment is used with bind to bind the lambda variables car(car(f)) with the evaluated arguments evlis(t,e).

Arguments passed to a function or primitive are evaluated with the evlis function:

where evlis recursively traverses the list of expressions t to create a new list with their values. Recursion bottoms out at a non-CONS t, which is evaluated. It is important to handle the dot in argument lists correctly this way, such as (f x.args) by calling eval(t,e) to evaluate args.

The variable-argument bindings for apply are constructed as a list of pairs with the bind function originally called pairlis:

```
L bind(L v,L t,L e) {
  return T(v) == NIL ? e :
        T(v) == CONS ? bind(cdr(v),cdr(t),pair(car(v),car(t),e)) :
        pair(v,t,e);
}
```

where v is a list of variables or a variable (an atom), and t is the list of evaluated arguments. When recursion bottoms out, we either have a NIL or a name. The name is bound to the rest of the list t with pair(v,t,e). The latter happens when a single variable is used like (lambda args args) and when a dot is used in the list of variables like (lambda (x.args) args).

⁹Viz. lambda calculus beta reduction involves a contraction step $(\lambda v.x) y \Rightarrow x[v := y]$ where y may or may not be evaluated first before the contraction. This models strict and lazy evaluation, respectively.

6 Lisp Primitives

The Lisp primitives are defined in an array prim[] of structures containing the name of the primitive as string s and the function pointer f pointing to the implementation in C. The function implementing the primitive takes the list of Lisp arguments as the first parameter and the Lisp environment as its second parameter:

```
struct { const char *s; L (*f)(L,L); } prim[] = {
{"eval",
         f_eval},
{"quote", f_quote},
{"cons",
          f_cons},
{"car",
          f_car},
{"cdr",
          f_cdr},
{"+",
          f_add},
{"-",
          f_sub},
{"*",
          f_mul},
{"/",
          f_div},
{"int",
          f_int},
{"<",
          f_lt},
{"eq?",
          f_eq},
{"or",
          f_or},
{"and",
          f_and},
{"not",
          f_not},
{"cond",
          f_cond},
{"if",
          f_if},
{"let*",
          f_leta},
{"lambda",f_lambda},
{"define",f_define},
{0}};
```

The main program initializes the global environment env with #t to return itself, followed by the Lisp primitives:

```
int main() {
    ...
    env = pair(tru,tru,nil);
    for (i = 0; prim[i].s; ++i) env = pair(atom(prim[i].s),box(PRIM,i),env);
    ...
}
```

Lisp includes so-called *special forms*, which are functions that do not evaluate all arguments passed to them. For example, the if special form evaluates the test. If the test is true, then the then-expression is evaluated and returned. Otherwise the else-expression is evaluated and returned.

6.1 eval

(eval *expr*) evaluates an expression. This primitive is also called "unquote" since *expr* is typically a quoted Lisp expression:

```
L f_eval(L t,L e) { return eval(car(evlis(t,e)),e); }
```

All arguments to eval are evaluated with evlis(t,e) but eval only applies to one argument or the first argument when more than one is specified. Example: (eval (quote (+ 1 2))) gives 3.

6.2 quote

(quote expr) quotes an expression to keep it unevaluated. The Lisp parser also accepts 'expr. Note that expr may be a list which means that the list and all of its elements remain unevaluated.

```
L f_quote(L t,L _) { return car(t); }
```

quote only applies to one argument or the first argument when more than one is specified, hence car(t) is returned. Example: (quote (1 2 3)) and '(1 2 3) give (1 2 3)

6.3 cons

(cons $expr_1$ $expr_2$) constructs a new pair ($expr_1$. $expr_2$). Typically $expr_2$ is a list to construct a list with $expr_1$ at its head.

```
L f_cons(L t,L e) { return t = evlis(t,e),cons(car(t),car(cdr(t))); }
```

Example: (cons 1 ()) gives (1), (cons 1 2) gives the pair (1.2) and (cons 1 (cons 2 ())) gives the list (1 2).

6.4 car and cdr

(car pair) and (cdr pair) give the first and second element of a pair, respectively. This means that (car list) and (cdr list) give the element at the front of the list and the rest of the list, respectively.

```
L f_car(L t,L e) { return car(car(evlis(t,e))); }
L f_cdr(L t,L e) { return cdr(car(evlis(t,e))); }
```

6.5 Arithmetic

The four basic arithmetic operations are variadic functions:

```
L f_add(L t,L e) { L n = car(t = evlis(t,e)); while (!not(t = cdr(t))) n += car(t); return num(n); } L f_sub(L t,L e) { L n = car(t = evlis(t,e)); while (!not(t = cdr(t))) n -= car(t); return num(n); } L f_mul(L t,L e) { L n = car(t = evlis(t,e)); while (!not(t = cdr(t))) n *= car(t); return num(n); } L f_div(L t,L e) { L n = car(t = evlis(t,e)); while (!not(t = cdr(t))) n /= car(t); return num(n); }
```

Note that the arithmetic functions expect at least one argument. They are undefined if no arguments are provided. Example: (+ 1 2 3 4) gives 10, (- 3 2) gives 1, and (- 3) gives 3, not -3. Some other Lisp may give -3, which can be implemented by checking if only one argument is passed to the function. I will leave it to you to change the implementation to support this feature.

6.6 int

(int *expr*) truncates *expr* to an integer.

```
L f_int(L t,L e) { L n = car(evlis(t,e)); return n-1e9 < 0 && n+1e9 > 0 ? (long)n : n; }
```

6.7 Comparison

($< expr_1 expr_2$) and (eq? $expr_1 expr_2$) compare $expr_1$ and $expr_2$ to give #t when true or () when false.

```
L f_lt(L t,L e) { return t = evlis(t,e),car(t) - car(cdr(t)) < 0 ? tru : nil; } L f_eq(L t,L e) { return t = evlis(t,e),equ(car(t),car(cdr(t))) ? tru : nil; }
```

Equality for pairs and lists is only true if the lists are the same objects in memory. Otherwise the pairs or lists are not equal, even when they contain the same elements. Example: (eq? 'a 'a) gives #t and (eq? '(a) '(a)) gives ().

6.8 Logic

(not expr) gives #t when expr is () (the empty list) and () otherwise. (or $expr_1 \ expr_2 \ \dots \ expr_n$) gives #t if any of the $expr_i$ is true (i.e. not ()) and () otherwise if all $expr_1$ are false (i.e. ()). (and $expr_1 \ expr_2 \ \dots \ expr_n$) gives () if any of the $expr_i$ is false (i.e. ()) and #t otherwise if all $expr_1$ are true (i.e. not ()).

```
L f_not(L t,L e) { return not(car(evlis(t,e))) ? tru : nil; } L f_or(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (!not(eval(car(t),e))) return tru; return nil; } L f_and(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (not(eval(car(t),e))) return nil; return tru; }
```

Only the first arguments to or and and are evaluated to determine the result. Example: (and #t ()) gives () and (and 1 2) gives #t since numbers are not ().

6.9 cond

(cond ($test_1 \ expr_1$) ($test_2 \ expr_2$) ... ($test_n \ expr_n$)) evaluates the tests from the first to the last until $test_i$ evaluates to true and then returns $expr_i$.

```
L f_cond(L t,L e) {
  while (T t != NIL && not(eval(car(car(t)),e))) t = cdr(t);
  return eval(car(cdr(car(t))),e);
}
```

Example: (cond ((eq? 'a 'b) 1) ((< 2 1) 2) (#t 3)) gives 3.

6.10 if

(if $test\ expr_1\ expr_2$) evaluates and tests if test is true or false. If true (i.e. not ()) then $expr_1$ is evaluated and returned. Else $expr_2$ is evaluated and returned.

```
L f_if(L t,L e) { return eval(car(cdr(not(eval(car(t),e)) ? cdr(t) : t)),e); }

Example: (if (eq? 'a 'a) 1 2) gives 1.
```

6.11 let*

(let* ($var_1 \ expr_1$) ($var_2 \ expr_2$) ... ($var_n \ expr_n$) expr) defines a set of bindings¹⁰ of variables in the scope of the body expr by evaluating each $expr_i$ sequentially and associating var_i with the result

```
I let(L x) { return T(x) != NIL && !not(cdr(x)); }
L f_leta(L t,L e) {
  while (let(t)) e = pair(car(car(t)),eval(car(cdr(car(t))),e),e),t = cdr(t);
  return eval(car(t),e);
}
```

The loop runs over the pairs in the let*. Each iteration extends the environment e with a pair that binds var_i (i.e. car(car(t))) to the value of $expr_i$ (i.e. eval(car(cdr(car(t))), e)). Example: (let* (a 3) (b (* a a)) (+ a b)) gives 12.

6.12 lambda

(lambda $var \ expr$) and (lambda $(var_1 \ var_1 \ \dots \ var_n) \ expr$) create a closure, i.e. an anonymous function. The first form associates var with the list of arguments passed to the closure when the closure is applied. The second form associates each var_i with the corresponding argument passed to the closure when the closure is applied. Also the list dot may be used in the list of variables (lambda $(var_1 \ var_2) \ expr$) to specify the remaining arguments to be passed as a list in var_2 .

```
L f_lambda(L t,L e) { return closure(car(t),car(cdr(t)),e); }
```

Example: ((lambda (x) (* x x)) 3) gives 9 and ((lambda (x y.args) args) 1 2 3 4) gives (3 4).

6.13 define

(define var expr) globally defines var and associates it with the evaluated expr.

```
L f_define(L t,L e) { env = pair(car(t),eval(car(cdr(t)),e),env); return car(t); }
```

Globally defined functions may be (mutually) recursive. Example: after (define pi 3.14) the value of pi is 3.14, after (define square (lambda (x) (* x x))) the application (square 3) gives 9 and after (define factorial (lambda (n) (if (< 1 n) (* n (factorial (- n 1))) 1))) the application (factorial 5) gives 120.

We have not defined an apply primitive often found in Lisp implementations, because apply is not needed. To apply a function to a list of arguments (f. args) suffices, but only if args is a variable associated with a list of arguments¹¹. Otherwise, we shall use (let* (args x) (f. args)) to ensure args is a variable bound to expression x.

¹⁰Other Lisp require pairs (var expr) to be placed in a list such as (let* ((a 3) (b (* a a))) (+ a b)), but I prefer the simpler form shown here. Of course, you can change this implementation in any way you like.

¹¹The reason is that if we place a list after the dot like (f.(g args)), then the arguments passed to f are actually g and args which is not what we want.

7 Reading and Parsing Lisp Expressions

A Lisp tokenizer scans the input for tokens to return to the parser. A token is a single parenthesis, the quote character, or a white space delimited sequence of characters up to 39 characters long. The scan function populates buf [40] with the next token scanned from standard input. The token is stored as a 0-terminated string in buf []:

```
char buf[40],see = ' ';
void look() { see = getchar(); }
I seeing(char c) { return c == ' ' ? see > 0 && see <= c : see == c; }
char get() { char c = see; look(); return c; }
char scan() {
    I i = 0;
    while (seeing(' ')) look();
    if (seeing('(')) || seeing(')') || seeing('\'')) buf[i++] = get();
    else do buf[i++] = get(); while (i < 39 && !seeing('(')) && !seeing(')') && !seeing(' '));
    return buf[i] = 0,*buf;
}</pre>
```

This implementation of scan does not support Lisp comments. I did this intentionally, since it is up to you to write your own implementation with the features that you want.

Lisp comments begin with a semicolon and end at the next line. To support comments, we can change the second line of the scan function by adding a label and adding a line to skip comments, jumping back with a goto¹² to continue scanning:

```
s:while (seeing(' ')) look();
if (seeing(';')) { do look(); while (!seeing('\n')); goto s; }
```

Note that EOF is not checked. A nice trick is to look for an EOF and then reopen standard input to read from the terminal:

```
void look() {
  int c = getchar();
  if (c == EOF) freopen("/dev/tty", "r", stdin), c = ' ';
  see = c;
}
```

By changing look this way, we can now read a collection of Lisp definitions from a file before the interactive session starts, by using the Linux/Unix cat utility:

```
bash$ cat common.lisp list.lisp math.lisp | ./tinylisp
```

This sends the common.lisp, list.lisp and math.lisp files (see Section E) to the interpreter. After EOF of cat, the Lisp interpreter is ready to accept input again, this time from the terminal. Beware that the number of cells N must be increased to import this many definitions!

To parse Lisp expressions we employ a recursive-descent parsing technique. The read function returns a Lisp expression parsed from the input by invoking the scan and parse functions:

¹²Goto's are not bad to use if the code is actually *more readable* with them than without them, despite Edsgar Dijkstra's aversion to goto, there is no harm in using goto's appropriately in my opinion.

```
L read() { return scan(),parse(); }
```

where the parse function parses a list, a quoted expression, or an atomic expression (an atom or a number):

```
L parse() { return *buf == '(' ? list() : *buf == '\'' ? quote() : atomic(); }
```

The list function recursively parses and constructs a list up to the closing parenthesis). In addition, a dot in a list creates a pair:

Note that x = parse() must be called before the parsed value is used in cons(x,list()), because argument evaluation order in C is undefined, i.e. not necessarily left-to-right. You may have noticed by now that I use the C comma operator a lot, including for this specific purpose and to keep the code compact. The C comma operator has a cousin in Lisp (begin $expr_1 expr_2 ... expr_n$) that returns the value of the last expression $expr_n$. So its use is sanctioned by Lisp to sequence expressions e.g. with $side\ effects^{13}$.

Parsing the list (x y z) for example, results in the list construction (cons x (cons y (cons z nil))) and parsing (x y args) results in the construction (cons x (cons y args)). Parsing a quoted expression 'expr produces (quote expr):

```
L quote() { return cons(atom("quote"),cons(read(),nil)); }
```

Parsing an atomic expression produces a number if the token is numeric and an atom otherwise:

```
L atomic() { L n; I i; return sscanf(buf, "%lg%n", &n, &i) > 0 && !buf[i] ? n : atom(buf); }
```

A token must be numeric to convert it to a number. If it is not, then an atom with the specified tokenized name is returned.

The PC-G850 requires this function to be modified as follows:

```
L atomic() {
  L n; I i = strlen(buf);
  return isdigit(buf[*buf == '-']) && sscanf(buf,"%lg%n",&n,&i) && !buf[i] ? n : atom(buf);
}
```

where i must be initialized to the length of the buffer passed to sscanf. Because sscanf on the PC-G850 simply returns 0 for incomplete numeric forms such as a single character -, we check if the token begins with a digit after an optional minus sign.

 $^{^{13}}$ Functions with side effects affect the state of the machine outside of their arguments and locals, such as through assignments to non-local variables and by performing IO operations. Pure functional programming bans them. For C and Lisp we must properly sequence functions with side effects to avoid undefined behavior.

8 Printing Lisp Expressions

Displaying Lisp expression requires a few lines of code. This code should be self-explanatory:

```
void print(L x) {
  if (T(x) == NIL) printf("()");
  else if (T(x) == ATOM) printf("%s",A+ord(x));
  else if (T(x) == PRIM) printf("<%s>",prim[ord(x)].s);
  else if (T(x) == CONS) printlist(x);
  else if (T(x) == CLOS) printf("{%u}",ord(x));
  else printf("%.10lg",x);
}
```

Function **printlist** iterates over the list to display its elements in order. It prints a dot for the last cons pair if the list does not end in **nil**:

```
void printlist(L t) {
   putchar('(');
   while (1) {
      print(car(t));
      if (not(t = cdr(t))) break;
      if (T(t) != CONS) { printf(" . "); print(t); break; }
      putchar(' ');
   }
   putchar(')');
}
```

Note that not(t = cdr(t)) changes t to the next list pair. Then, if t is nil we break from the loop. Otherwise, if the next t is not a cons pair, then we display a dot followed by the value of t. The dot visually separates the pair's values. The dot is also used to construct pairs, see Section 7.

9 Garbage Collection

To keep our Lisp interpreter code small, we should implement a very simple form of garbage collection to delete all temporary cells from the stack. We should preserve all globally-defined names and functions listed in env, To delete all temporary cells and keep env intact, it suffices to restore the stack pointer to the point on the stack where the free space begins, which is right below the global environment env cell on the stack:

```
void gc() { sp = ord(env); }
```

Why does this work? After the last env = pair(name, expr, env) call was made to define name globally, we know for sure that expr is already stored higher up in the stacked cells, i.e. in cells above the last env pair on the stack. These cells are not removed by gc when setting sp to the cell indexed by env.

One possible caveat of this approach is that we cannot support interactive use of the Lisp special forms setq (modifies an association in an environment), set-car! (overwrites the car of

a pair) and set-cdr! (overwrites the cdr of a pair), since these may change previously-defined expressions in the global environment. If the modified global environment references temporary expressions, then gc corrupts the global environment by removing the temporaries. In principle we could support setq, set-car! and set-cdr! if we somehow limit their use to prevent this problem from occurring.

10 The Read-Eval-Print Loop

After the main program initialized the static variables nil, tru, and err (see Section 4) and populated the environment with #t and other primitives (see Section 6), the main program executes the so-called Lisp read-eval-print loop (REPL):

```
int main() {
    ...
    while (1) { printf("\n%u>",sp-hp/8); print(eval(read(),env)); gc(); }
}
```

The prompt in the REPL displays the number of cells freely available, i.e. the space between the heap pointer hp and stack pointer sp. Note that hp points to bytes and sp points to 8-byte floats, so hp is scaled down by a factor 8. Garbage collection is performed in the REPL after the results are displayed.

This completes Lisp in 99 lines¹⁴ of C. See Appendix A and B for the complete listings with NaN and BCD boxing, respectively.

11 Additional Lisp Primitives

The following Lisp primitives are not included in the 99 line C program, since these are not absolutely required to write Lisp programs.

11.1 assoc and env

(assoc var environment) gives the expression associated with var in the specified environment. (env) returns the current environment in which (env) is evaluated.

```
L f_assoc(L t,L e) { return t = evlis(t,e),assoc(car(t),car(cdr(t))); }
L f_env(L _,L e) { return e; }
... prim[] = { ... {"assoc",f_assoc},{"env",f_env} ... };
```

Note that *var* should be quoted when passed to assoc since its arguments are evaluated first. Example: (assoc 'b '((a 1) (b 2) (c 3)) gives 2.

¹⁴That is, a Lisp-like functional style of structured C. Lines are 55 columns wide on average and never wider than 120 columns for convenient editing.

11.2 let and letrec*

The let special form¹⁵ is similar to the let* special form, but evaluates all expressions first before binding the values to the variables.

```
L f_let(L t,L e) {
  L d = e;
  while (let(t)) d = pair(car(car(t)),eval(car(cdr(car(t))),e),d),t = cdr(t);
  return eval(car(t),d);
}
... prim[] = { ... {"let",f_let} ... };
```

The letrec* special form is similar to the let* special form, but allows for local recursion where the name may also appear in the value of a letrec* name-value pair.

```
L f_letreca(L t,L e) {
   while (let(t)) {
      e = pair(car(car(t)),err,e);
      cell[sp+2] = eval(car(cdr(car(t))),e);
      t = cdr(t);
   }
   return eval(car(t),e);
}
... prim[] = { ... {"letrec*",f_letreca} ... };
```

This implementation updates the environment **e** first with a new variable-**err** binding, then sets the **err** cell on the stack to the expression evaluated within the updated scope **e**. Example:

```
> (letrec* (f (lambda (n) (if (< 1 n) (* n (f (- n 1))) 1))) (f 5))
120</pre>
```

11.3 setq

The setq special form sets the value of a variable as a side-effect with (setq var expr):

```
L f_setq(L t,L e) {
  L a = car(t),x = eval(car(cdr(t)),e);
  while (T(e) == CONS && !equ(a, car(car(e)))) e = cdr(e);
  return T(e) == CONS ? cell[ord(car(e))] = x : err;
}
... prim[] = { ... {"setq",f_setq} ... };
```

This function is dangerous, because garbage collection after setq may corrupt the stack if the new value assigned to a global variable is a temporary list (all interactively constructed lists are temporary). On the other hand, atomic values are always safe to assign and setq is safe to use to assign local variables of a lambda and a let.

 $^{^{15}}$ This let syntax differs from other Lisp: like our let* we don't need to put all the var-expr pairs in a list. Change the syntax as you like.

11.4 set-car! and set-cdr!

(set-car! pair expr) sets the value of the car cell of a cons pair to expr as a side-effect, (set-cdr! pair expr) sets the value of the cdr cell of a cons pair to expr as a side-effect.

```
L f_setcar(L t,L e) {
   L p = car(t = evlis(t,e)),x = car(cdr(t));
   if (T(p) == CONS) cell[ord(p)+1] = x;
   return x;
}
L f_setcdr(L t,L e) {
   L p = car(t = evlis(t,e)),x = car(cdr(t));
   if (T(p) == CONS) cell[ord(p)] = x;
   return x;
}
... prim[] = { ... {"set-car!",f_setcar},{"set-cdr!",f_setcdr} ... };
```

Like setq, these functions are dangerous.

11.5 macro

Macros allow Lisp to be syntactically extended. A macro is similar to a lambda, except that its arguments are not evaluated when the macro is applied. Typically a macro constructs Lisp code when applied, thereby *expanding* and evaluating the Lisp code in place.

To add the (macro variables expression) primitive is easy. It only requires a few lines of code to define a new MACR tag, add new the functions macro, f_macro and expand, and make some minor changes to the existing functions car, cdr and apply. First we add a new tag MACR:

```
/*** with NaN Boxing ***/
I ATOM=0x7ff8,PRIM=0x7ff9,CONS=0x7ffa,CLOS=0x7ffb,MACR=0x7ffc,NIL=0x7ffd;
/*** with BCD boxing ***/
I ATOM=32,PRIM=48,CONS=64,CLOS=80,MACR=96,NIL=112;
```

The car and cdr functions should be modified to make them applicable to MACR-tagged floats which are essentially cons pairs containing the list of *variables* of the macro as car cell and *expression* as the cdr cell:

We add a constructor macro and the corresponding Lisp primitive f_macro:

```
L macro(L v,L x) { return box(MACR,ord(cons(v,x))); }
L f_macro(L t,L e) { return macro(car(t),car(cdr(t))); }
... prim[] = { ... {"macro", f_macro} ... };
```

Application of macros is similar to lambdas, but they expand instead. Expansion is performed by a new expand function:

The macro is evaluated in the global environment env. This typically constructs Lisp code that is then evaluated in the current environment e.

Example: the Lisp delayed evaluation primitives delay and force implemented as a macro:

```
> (define list (lambda args args))
> (define delay (macro (x) (list 'lambda () x)))
> (define force (lambda (f) (f)))
```

The delay macro is used for *lazy evaluation* or *call by need* of arguments, by passing them unevaluated to a function together with their environment as a closure called a *promise*. Hence, (list 'lambda () x) constructs the Lisp code (lambda () x) where x is the unevaluated argument passed to delay. As we saw earlier, the closure of this lambda includes the proper environment in which x should be evaluated, thus respecting its static scope. The force function evaluates a promise:

```
> (force (delay (+ 1 2)))
3
```

More information and examples of delay and force can be found in Lisp textbooks and manuals.

11.6 read and print

Since our Lisp implementation already includes read and print functions, we can add them as primitives as follows:

```
L f_read(L t,L e) { L x; char c = see; see = ' '; x = read(); see = c; return x; }
L f_print(L t,L e) {
  for (t = evlis(t,e); T(t) != NIL; t = cdr(t)) print(car(t));
  return nil;
}
... prim[] = { ... {"read", f_read}, {"print", f_print} ... };
```

Example: (read) gives the Lisp expression typed in (unevaluated) and (print 'hello 123) displays hello123.

With a few more lines of C code, you can add your own Lisp primitives for a more complete IO implementation. You could store open file FILE* streams in an array to support open and close primitives on multiple streams in Lisp. Then index this array by a float number in Lisp to obtain the FILE* for an internal IO operation. Some Lisp implementations include a special *port* type for this, which is basically a FILE*.

12 Adding Readline with History

Command shells such as bash use GNU readline for interactive input. Readline plays nice with interactive input and provides a history mechanism to let us recall previous input. The readline library will be a nice addition to our Lisp. While we are at it, we might as well read a Lisp initialization file init.lisp with one or more Lisp definitions to import before the interactive prompt. First we need to include the usual C headers. There are two for readline:

```
#include <readline/readline.h>
#include <readline/history.h>
```

Besides the buf and see globals, we also need a ptr pointing at the current character in the line string returned by readline. We also add a prompt string ps to display the Lisp prompt and an in pointer to the open file with input, i.e. to read init.lisp when opened successfully:

```
char buf[40],see = ' ',*ptr = "",*line = NULL,ps[20];
FILE *in = NULL;
```

The look function is modified to read a character from in when in is not NULL, or to read a line of input with by readline when we reach the end of the last line read, i.e. when see=='\n':

```
void look() {
  if (in) {
    int c = getc(in);
    see = c;
    if (c != EOF) return;
    fclose(in);
    in = NULL;
  }
  if (see == '\n') {
    if (line) free(line);
    do if (!(ptr = line = readline(ps))) freopen("/dev/tty","r",stdin); while (!line);
    add_history(line);
    strcpy(ps,"?");
 }
  if (!(see = *ptr++)) see = '\n';
To read init.lisp before the prompt, we open init.lisp before running a modified REPL:
int main() {
  . . .
  in = fopen("init.lisp", "r");
 using_history();
  while (1) { putchar('\n'); snprintf(ps,20,"%u>",sp-hp/8); print(eval(read(),env)); gc(); }
}
```

Note that the modified REPL populates the prompt string ps with snprintf to display the number of remaining free cells. The prompt string is set to? in the look function when the interactive input is not yet complete. The libreadline library must be linked with our updated source:

```
bash$ cc -o tinylisp-opt.c -lreadline
```

Now you're all set to enjoy enhanced interactive input!

13 Tracing Lisp

Everything that is happing behind the scenes in our Lisp interpreter is essentially just the traversal of lists as code to return numbers, atoms, closures and lists as data. But what does that actually look like? Well, let's find out by tracing the evaluation steps in our Lisp. When we display every evaluation step performed by eval we see exactly what is happening in detail:

```
tinylisp
930>(define sq (lambda (n) (* n n)))
920: define => <define>
920: lambda => <lambda>
916: (lambda (n) (* n n)) \Rightarrow \{916\}
912: (define sq (lambda (n) (* n n))) \Rightarrow sq
sq
901>(sq 3)
908: sq => \{916\}
908: 3 => 3
908: () => ()
902: * => <*>
902: n \Rightarrow 3
902: n \Rightarrow 3
902: () => ()
898: (* n n) \Rightarrow 9
898: (sq 3) \Rightarrow 9
901>
```

A trace displays the Lisp code and data before eval followed by the Lisp code and data after eval. To implement tracing, we rename eval to step and add a new eval function to display the trace:

```
L step(L x,L e) { ... }  /* this is the old eval() function renamed to step() */
void print(L);  /* function prototype moved up since we need print() */
L eval(L x,L e) {
   L y = step(x,e);
   printf("%u: ",sp); print(x); printf(" => "); print(y);
   while (getchar() >= ' ') continue;
   return y;
}
```

The while-loop in eval waits until the RETURN key is pressed (or any special key or EOF). This always displays the trace of your Lisp. With a few more lines of C, you could make tracing an option enabled with a new (trace *state*) Lisp function that takes *state* 0, 1 or 2 to set a global variable. When this variable is set to 1 or 2, the trace output is displayed in eval and when 2 it also waits for a keypress. With this addition to your Lisp, you can now step through your code on demand. In addition to tracing, you could also dump the stack. For example, when the user types a d for dump followed by RETURN.

14 Adding Error Handling and Exceptions to Lisp

Error handling is practically non-existent in our Lisp at this point. It is an error to take the car or cdr of a non-pair, to use the value of an undefined symbol, or to try to apply a non-function to arguments. These three error conditions produce the ERR value in our Lisp. We could extensively debug our code by tracing (see Section 13) to find all possible bugs and add err? checks to our Lisp functions to catch problems (see Appendix E for a definition of err?). Analyzing your Lisp code for correctness and debugging your Lisp functions is critical, but adding err? calls to Lisp code is cumbersome. Wouldn't it be nice to have a mechanism to catch exceptions?

There are two mechanisms to implement exception handling in our Lisp: in C using setjmp or in C++ with try-catch if we rename and compile our project in C++.

14.1 C setjmp

The setjmp(jmp_buf jb) function saves its calling environment in jb and returns zero. The corresponding longjmp(jmp_buf jb,int n) function restores the calling environment jb saved by the most recent setjmp(jb) and passes n to the return value of this setjmp. What basically happens is that longjmp deletes all active function calls since the last setjmp and immediately returns to setjmp. This mechanism is somewhat similar to C++ exceptions with longjmp acting like throw and setjmp acting like try-catch. As with C++ exceptions, we must be careful to finalize unfinished business before throwing exceptions or add exception handlers to finalize unfinished business, re-throwing the exception when applicable. Unfinished business include open files that must be closed and memory allocations that must freed. Fortunately, we have none of these issues to worry about 16 in our Lisp interpreter.

After including setjmp in our interpreter, we define a global calling environment jmp_buf jb and a new err function to "throw" errors by invoking longjmp(jb,n) for nonzero error codes n:

```
#include <setjmp.h>
...
jmp_buf jb;
L err(I i) { longjmp(jb,i); }
```

Note that the new err function doesn't return a value (you should be able to figure out why it makes no sense to return a value.) After defining a new err function, we change the uses of err in our implementation to throw three different error codes:

```
L car(L p) { return (T(p)&~(CONS^CLOS)) == CONS ? cell[ord(p)+1] : err(1); }
L cdr(L p) { return (T(p)&~(CONS^CLOS)) == CONS ? cell[ord(p)] : err(1); }
...
L assoc(L v,L e) {
  while (T(e) == CONS && !equ(v,car(car(e)))) e = cdr(e);
  return T(e) == CONS ? cdr(car(e)) : err(2);
}
```

¹⁶Unless you've added file open and close to your Lisp interpreter's primitives. In that case, it helps to keep a table of open file descriptors and close them all when an exception occurs.

All other old err values should be removed from the Lisp interpreter's code. In main we invoke setjmp(jb) to initialize jb and also to catch errors when setjmp returns a nonzero value n from err(n):

```
int main() {
    ...
    if ((i = setjmp(jb)) > 0) printf("ERR %u",i);
    while (1) { gc(); printf("\n%u>",sp-hp/8); print(eval(read(),env)); }
}
```

We also moved gc() up. After an error is caught by setjmp we report it and commence the REPL:

```
930>(car 3)

ERR 1

930>(1 2)

ERR 3

930>'(1 2)

(1 2)

930>
```

Another good idea is to replace abort() with err(4) to avoid aborting when we ran out of memory. In addition to our new and practical exception handling mechanism, there are several ways Lisp catch and throw primitives can be defined. The following is a simplified version that allows you to (throw n) a positive error code n that are caught as (ERR. n) pairs returned by (catch expression), when a (throw n) is invoked anywhere and at any time in expression:

```
L f_catch(L t,L e) {
  L x; I i;
  jmp_buf tmp;
  memcpy(tmp,jb,sizeof(jb));
  i = setjmp(jb);
  x = i ? cons(atom("ERR"),i) : eval(car(t),e);
  memcpy(jb,tmp,sizeof(jb));
  return x;
}
L f_throw(L t,L e) { longjmp(jb,(I)num(car(t))); }
... prim[] = { ... {"catch", f_catch},{"throw",f_throw} ... };
```

where throw is a special form that does not evaluate its argument, which must be a constant nonzero integer. With some more C code you could pass along the cell index of a cons pair on the stack instead of error codes. That means you could throw lists as errors with more information. However, care must be taken to distinguish internal error codes from explicit exceptions thrown.

14.2 C++ Exceptions

Compiling our Lisp interpreter in C++ requires almost no change (e.g. renaming not to Not). With C++ we can use try-catch and throw in the REPL:

```
L err(I i) { throw(i); }
...
int main() {
    ...
    while (1) {
        printf("\n%u>",sp-hp/8);
        try { print(eval(read(),env)); }
        catch (I i) { printf("ERR %u",i); }
        gc();
    }
}
```

We also change the car, cdr, assoc and apply functions as shown in the previous section. Lisp catch and throw primitives can be defined with the usual C++ try-catch-throw.

15 Downsizing Lisp to Single Floating Point Precision

Our Lisp supports double precision floating point values and operations on them without compromise. We can halve the memory use of our Lisp by using single precision floating point instead. This is a small compromise to make to reduce memory consumption if memory comes at a premium or if the machine does not support double floating point precision. Despite downsizing our Lisp, it won't be handicapped to run toy Lisp examples only. In fact, it can handle up to $N = 2^{20}/4 = 262144$ cells (1,048,576 bytes) to evaluate Lisp.

A single precision NaN allows up to 22 bits to be arbitrarily used to stuff any information we want into a single precision NaN:

All exponent bits must be set for NaN. This time we use the sign bit s together with our tag since we need 3 bits for tagging. Therefore, NIL has tag 0xfff with the sign bit set as part of the tag. We also want to maximize the use of the 23 bit fraction. Of these 23 bits the most significant bit must be set for quiet NaN. We use 2 bits for the tag and are left with 20 bits to store an integer "ordinal" value to refer to atoms on the heap, to primitives in the prim[] array, and to refer to cons cells and closure cells on the stack.

Only a few minor changes are required to the code to downsize our Lisp accordingly. We first adjust the NaN-boxing tags and 20-bit ordinals and related functions:

```
#define L float
#define T(x) *(uint32_t*)&x>>20
```

```
#define N 1024 /* N should not exceed 262144 = 2^20/4 cells = 1048576 bytes */
I hp=0,sp=N,ATOM=0x7fc,PRIM=0x7fd,CONS=0x7fe,CLOS=0x7ff,NIL=0xfff;
L box(I t,I i) { L x; *(uint32_t*)&x = (uint32_t)t << 20|i; return x; }
I ord(L x) { return *(uint32_t*)&x & 0xfffff; }
L num(L n) { return n; }
I equ(L x,L y) { return *(uint32_t*)&x == *(uint32_t*)&y; }
Furthermore, we tweak the atomic and print functions to correctly parse and print single precision
```

floats, respectively:

```
L atomic() { L n; I i; return sscanf(buf, "%g%n", &n, &i) > 0 && !buf[i] ? n : atom(buf); }
void print(L x) {
  else printf("%g",x);
}
```

And we may also want to change f_int to truncate floats to 32 bit integers:

```
L f_int(L t,L e) { L n = car(evlis(t,e)); return n<1e7 && n>-1e7 ? (int32_t)n: n; }
```

Conclusions 16

This article demonstrated how a fully-functional Lisp interpreter with 20 Lisp primitives, garbage collection and REPL can be written in 99 lines of C or less. The concepts and implementation presented largely follow the original ideas and discoveries made by McCarthy in his 1960 paper. Given the material included in this article, it should not be difficult to expand the Lisp interpreter to support additional features and experiment with alternative syntax and semantics of a hybrid Lisp or a completely new language.

Any overlap or resemblance to any other Lisp implementations is coincidental. I wrote the article from scratch based on McCarthy's paper and based on my 20 years of experience teaching programming language courses that include Lisp/Scheme design and programming.

17 Bibliography

References

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- [2] McCarthy, John. "Recursive functions of symbolic expressions and their computation by machine, part I." Communications of the ACM 3.4 (1960): 184-195.
- [3] Church, Alonzo. "The calculi of lambda-conversion." Bull. Amer. Math. Soc 50 (1944): 169-172.

A Tiny Lisp Interpreter with NaN boxing: 99 Lines of C

Lisp in 99 lines of C without comments:

```
#include <stdlib.h>
#include <stdio.h>
#include <string.h>
#define I unsigned
#define L double
#define T(x) *(unsigned long long*)&x>>48
#define A (char*)cell
#define N 1024
I hp=0,sp=N,ATOM=0x7ff8,PRIM=0x7ff9,CONS=0x7ffa,CLOS=0x7ffb,NIL=0x7ffc;
L cell[N], nil, tru, err, env;
L box(I t,I i) { L x; *(unsigned long long*)&x = (unsigned long long)t<<48|i; return x; }
I ord(L x) { return *(unsigned long long*)&x; }
L num(L n) { return n; }
I equ(L x,L y) { return *(unsigned long long*)&x == *(unsigned long long*)&y; }
L atom(const char *s) {
 I i = 0; while (i < hp && strcmp(A+i,s)) i += strlen(A+i)+1;
 if (i == hp && (hp += strlen(strcpy(A+i,s))+1) > sp<<3) abort();
 return box(ATOM,i);
L cons(L x,L y) { cell[--sp] = x; cell[--sp] = y; if (hp > sp<<3) abort(); return box(CONS,sp); }
L \operatorname{car}(L p) \{ \operatorname{return} (T(p)\&^{\sim}(CONS^{\sim}CLOS)) == CONS ? \operatorname{cell}[\operatorname{ord}(p)+1] : \operatorname{err}; \}
L \ cdr(L \ p) \ \{ \ return \ (T(p)\&^{(CONS^CLOS)}) == CONS \ ? \ cell[ord(p)] : err; \ \}
L pair(L v,L x,L e) { return cons(cons(v,x),e); }
L closure(L v,L x,L e) { return box(CLOS,ord(pair(v,x,equ(e,env) ? nil : e))); }
L assoc(L v,L e) { while (T(e) == CONS && !equ(v,car(car(e)))) e = cdr(e); return T(e) == CONS ? cdr(car(e)) : err; }
I not(L x) { return T(x) == NIL; }
I let(L x) { return T(x) != NIL && !not(cdr(x)); }
L eval(L,L),parse();
L evlis(L t,L e) { return T(t) == CONS ? cons(eval(car(t),e),evlis(cdr(t),e)) : eval(t,e); }
L f_eval(L t,L e) { return eval(car(evlis(t,e)),e); }
L f_quote(L t,L _) { return car(t); }
L f_cons(L t,L e) { return t = evlis(t,e),cons(car(t),car(cdr(t))); }
L f_car(L t,L e) { return car(car(evlis(t,e))); }
L f_cdr(L t,L e) { return cdr(car(evlis(t,e))); }
L f_add(L t, L e)  { L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n + car(t); return num(n); }
L f_sub(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n -= car(t); return num(n); \} 
L f_{mul}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n *= car(t); return num(n); \} 
L f_{div}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n /= car(t); return num(n); \} 
L f_int(L t,L e) { L n = car(evlis(t,e)); return n<1e16 && n>-1e16 ? (long long)n : n; }
L f_{t}(L t, L e)  { return t = evlis(t, e), car(t) - car(cdr(t)) < 0 ? tru : nil; }
L f_eq(L t,L e) { return t = evlis(t,e),equ(car(t),car(cdr(t))) ? tru : nil; }
L f_not(L t,L e) { return not(car(evlis(t,e))) ? tru : nil; }
L f_or(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (!not(eval(car(t),e))) return tru; return nil; }
L f_and(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (not(eval(car(t),e))) return nil; return tru; }
L f_{cond}(L t, L e) { while (T(t) != NIL \&\& not(eval(car(car(t)), e))) t = cdr(t); return eval(car(cdr(car(t))), e); }
L f_if(L t,L e) { return eval(car(cdr(not(eval(car(t),e)) ? cdr(t) : t)),e); }
 L f_{\text{leta}}(L t, L e) \{ while (let(t)) e = pair(car(car(t)), eval(car(cdr(car(t))), e), e), t = cdr(t); return eval(car(t), e); \} 
L f_lambda(L t,L e) { return closure(car(t),car(cdr(t)),e); }
L f_define(L t,L e) { env = pair(car(t),eval(car(cdr(t)),e),env); return car(t); }
struct { const char *s; L (*f)(L,L); } prim[] = {
 \label{lem:cons} $$ {\tt "eval",f_eval}, {\tt "quote",f_quote}, {\tt "cons",f_cons}, {\tt "car", f_car}, {\tt "cdr", f_car}, {\tt
                                                                                                                      f_cdr},
                                                                                                                                                       f_add},
                                                                                                                                                                       {"-", f_sub},
{"*", f_mul}, {"/",
                                   f_{div}, {"int", f_{int}, {"<", f_{lt}, {"eq?", f_{eq},
                                                                                                                                       {"or",
                                                                                                                                                                       {"and", f_and},
L bind(L v,L t,L e) { return T(v) == NIL ? e : T(v) == CONS ? bind(cdr(v),cdr(t),pair(car(v),car(t),e)) : pair(v,t,e); }
L reduce(L f,L t,L e) { return eval(cdr(car(f)),bind(car(car(f)),evlis(t,e),not(cdr(f)) ? env : cdr(f))); }
L apply(L f,L t,L e) { return T(f) == PRIM ? prim[ord(f)].f(t,e) : T(f) == CLOS ? reduce(f,t,e) : err; }
L eval(L x,L e) { return T(x) == ATOM ? assoc(x,e) : T(x) == CONS ? apply(eval(car(x),e),cdr(x),e) : x; }
char buf[40],see = ' ';
void look() { see = getchar(); }
I seeing(char c) { return c == ' ' ? see > 0 && see <= c : see == c; }</pre>
```

```
char get() { char c = see; look(); return c; }
char scan() {
I i = 0;
 while (seeing(' ')) look();
 if (seeing('(') || seeing(')') || seeing('\'')) buf[i++] = get();
 else do buf[i++] = get(); while (i < 39 && !seeing('(') && !seeing(')') && !seeing(' '));
return buf[i] = 0,*buf;
L read() { return scan(),parse(); }
 L \ list() \ \{ \ L \ x; \ return \ scan() == ')' \ ? \ nil : \ !strcmp(buf, ".") \ ? \ (x = read(), scan(), x) : \ (x = parse(), cons(x, list())); \ \} 
L quote() { return cons(atom("quote"),cons(read(),nil)); }
 L \ atomic() \ \{ \ L \ n; \ I \ i; \ return \ sscanf(buf,"%lg%n",&n,&i) > 0 \ \&\& \ !buf[i] \ ? \ n \ : \ atom(buf); \ \} 
L parse() { return *buf == '(' ? list() : *buf == '\'' ? quote() : atomic(); }
void print(L);
void printlist(L t) {
 putchar('(');
 while (1) {}
 print(car(t));
  if (not(t = cdr(t))) break;
  if (T(t) != CONS) { printf(" . "); print(t); break; }
 putchar(' ');
putchar(')');
void print(L x) {
if (T(x) == NIL) printf("()");
 else if (T(x) == ATOM) printf("%s",A+ord(x));
 else if (T(x) == PRIM) printf("<%s>",prim[ord(x)].s);
 else if (T(x) == CONS) printlist(x);
 else if (T(x) == CLOS) printf("{{u}",ord(x)};
else printf("%.10lg",x);
void gc() { sp = ord(env); }
int main() {
I i; printf("tinylisp");
nil = box(NIL,0); err = atom("ERR"); tru = atom("#t"); env = pair(tru,tru,nil);
for (i = 0; prim[i].s; ++i) env = pair(atom(prim[i].s),box(PRIM,i),env);
while (1) { printf("\n%u>",sp-hp/8); print(eval(read(),env)); gc(); }
```

B Tiny Lisp Interpreter with BCD boxing: 99 Lines of C

Lisp for the PC-G850 in 99 lines of C without comments:

```
#define I unsigned
#define L double
#define T *(char*)&
#define A (char*)cell
#define N 1024
I hp=0,sp=N,ATOM=32,PRIM=48,CONS=64,CLOS=80,NIL=96;
L cell[N],nil,tru,err,env;
L box(I t,I i) { L x = i+10; T(x) = t; return x; }
I ord(L x) { T(x) \&= 15; return (I)x-10; }
L num(L n) { T(n) &= 159; return n; }
I equ(L x,L y) { return x == y; }
L atom(const char *s) {
 I i = 0; while (i < hp && strcmp(A+i,s)) i += strlen(A+i)+1;
 if (i == hp && (hp += strlen(strcpy(A+i,s))+1) > sp<<3) abort();
 return box(ATOM,i);
L cons(L x,L y) { cell[--sp] = x; cell[--sp] = y; if (hp > sp<<3) abort(); return box(CONS,sp); }
L \operatorname{car}(L p) \{ \operatorname{return} (T(p)\&^{(CONS^{CLOS})}) == \operatorname{CONS} ? \operatorname{cell}[\operatorname{ord}(p)+1] : \operatorname{err}; \}
 L \ cdr(L \ p) \ \{ \ return \ (T(p)\&^{\sim}(CONS^{\sim}CLOS)) \ == \ CONS \ ? \ cell[ord(p)] \ : \ err; \ \}  
L pair(L v,L x,L e) { return cons(cons(v,x),e); }
L closure(L v,L x,L e) { return box(CLOS,ord(pair(v,x,equ(e,env) ? nil : e))); }
L assoc(L v,L e) { while (T(e) == CONS && !equ(v,car(car(e)))) e = cdr(e); return T(e) == CONS ? cdr(car(e)) : err; }
I not(L x) { return T(x) == NIL; }
I let(L x) { return T(x) != NIL && !not(cdr(x)); }
L eval(L,L),parse();
L evlis(L t,L e) { return T(t) == CONS ? cons(eval(car(t),e),evlis(cdr(t),e)) : eval(t,e); }
L f_eval(L t,L e) { return eval(car(evlis(t,e)),e); }
L f_quote(L t,L _) { return car(t); }
L f_cons(L t,L e) { return t = evlis(t,e),cons(car(t),car(cdr(t))); }
L f_car(L t,L e) { return car(car(evlis(t,e))); }
L f cdr(L t.L e) { return cdr(car(evlis(t.e))): }
L f_add(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n += car(t); return num(n); \}
 L f_{sub}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n -= car(t); return num(n); \} 
L f_{mul}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n *= car(t); return num(n); \} 
L f_{div}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n /= car(t); return num(n); \}
L f_int(L t,L e) { L n = car(evlis(t,e)); return n-1e9 < 0 && n+1e9 > 0 ? (long)n : n; }
L f_{t}(L t, L e)  { return t = evlis(t, e), car(t) - car(cdr(t)) < 0 ? tru : nil; }
\label{eq:local_local_local} L \ f_eq(L \ t,L \ e) \ \{ \ return \ t \ = \ evlis(t,e), equ(car(t), car(cdr(t))) \ ? \ tru \ : \ nil; \ \}
L f_not(L t,L e) { return not(car(evlis(t,e))) ? tru : nil; }
L f_or(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (!not(eval(car(t),e))) return tru; return nil; }
L f_and(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (not(eval(car(t),e))) return nil; return tru; }
L f_cond(L t,L e) { while (T(t) != NIL && not(eval(car(car(t)),e))) t = cdr(t); return eval(car(cdr(car(t))),e); }
L f_if(L t,L e) { return eval(car(cdr(not(eval(car(t),e)) ? cdr(t) : t)),e); }
L f_leta(L t,L e) { while (let(t)) e = pair(car(car(t)),eval(car(cdr(car(t))),e),e),t = cdr(t); return eval(car(t),e); }
L f_lambda(L t,L e) { return closure(car(t),car(cdr(t)),e); }
L f_define(L t,L e) { env = pair(car(t),eval(car(cdr(t)),e),env); return car(t); }
struct { const char *s; L (*f)(L,L); } prim[] = {
                                                                                                                                                                                       {"-", f_sub},
 \\ \{"eval", f\_eval\}, \{"quote", f\_quote\}, \{"cons", f\_cons\}, \{"car", f\_car\}, \{"cdr", f\_car\}, \{
                                                                                                                                                    {"+",
                                                                                                                                                                     f add}.
                                                                                                                                 f_cdr},
{"*", f_mul}, {"/", f_div}, {"int", f_int}, {"<", f_lt}, {"eq?", f_eq}, {"or", f_or}, {"and {"not", f_not}, {"cond", f_cond}, {"if", f_if}, {"let*",f_leta},{"lambda",f_lambda},{"define",f_define},{0}};
                                                                                                                                                                                       {"and",f_and},
L bind(L v,L t,L e) { return T(v) == NIL ? e : T(v) == CONS ? bind(cdr(v),cdr(t),pair(car(v),car(t),e)) : pair(v,t,e); }
L reduce(L f,L t,L e) { return eval(cdr(car(f)),bind(car(car(f)),evlis(t,e),not(cdr(f)) ? env : cdr(f))); }
L apply(L f,L t,L e) { return T(f) == PRIM ? prim[ord(f)].f(t,e) : T(f) == CLOS ? reduce(f,t,e) : err; }
L eval(L x,L e) { return T(x) == ATOM ? assoc(x,e) : T(x) == CONS ? apply(eval(car(x),e),cdr(x),e) : x; }
char buf[40],see = ' ';
void look() { see = getchar(); }
I seeing(char c) { return c == ' ' ? see > 0 && see <= c : see == c; }
char get() { char c = see; look(); return c; }
char scan() {
 I i = 0:
```

```
while (seeing(' ')) look();
 if (seeing('(') || seeing(')') || seeing('\'')) buf[i++] = get();
 else do buf[i++] = get(); while (i < 39 && !seeing('(') && !seeing(')') && !seeing(' '));
return buf[i] = 0,*buf;
}
L read() { return scan(),parse(); }
 L \ list() \ \{ \ L \ x; \ return \ scan() == ')' \ ? \ nil : \ !strcmp(buf, ".") \ ? \ (x = read(), scan(), x) : \ (x = parse(), cons(x, list())); \ \} 
L quote() { return cons(atom("quote"),cons(read(),nil)); }
L atomic() {
  L n; I i = strlen(buf);
 return isdigit(buf[*buf == '-']) && sscanf(buf,"%lg%n",&n,&i) && !buf[i] ? n : atom(buf);
L parse() { return *buf == '(' ? list() : *buf == '\'' ? quote() : atomic(); }
void print(L);
void printlist(L t) {
 putchar('(');
 while (1) {}
 print(car(t));
  if (not(t = cdr(t))) break;
  if (T(t) != CONS) { printf(" . "); print(t); break; }
 putchar(' ');
putchar(')');
void print(L x) {
if (T(x) == NIL) printf("()");
 else if (T(x) == ATOM) printf("%s",A+ord(x));
 else if (T(x) == PRIM) printf("<%s>",prim[ord(x)].s);
 else if (T(x) == CONS) printlist(x);
 else if (T(x) == CLOS) printf("{%u}",ord(x));
else printf("%.10lg",x);
void gc() { sp = ord(env); }
int main() {
I i; printf("lisp850");
nil = box(NIL,0); err = atom("ERR"); tru = atom("#t"); env = pair(tru,tru,nil);
for (i = 0; prim[i].s; ++i) env = pair(atom(prim[i].s),box(PRIM,i),env);
while (1) { printf("\n%u>",sp-hp/8); print(eval(read(),env)); gc(); }
```

C Optimized Lisp Interpreter with NaN boxing

```
#include <stdlib.h>
#include <stdio.h>
#include <string.h>
#define I unsigned
#define L double
#define T(x) *(unsigned long long*)&x>>48
#define A (char*)cell
#define N 1024
I hp=0,sp=N,ATOM=0x7ff8,PRIM=0x7ff9,CONS=0x7ffa,CLOS=0x7ffb,NIL=0x7ffc;
L cell[N],nil,tru,err,env;
L box(I t,I i) { L x; *(unsigned long long*)&x = (unsigned long long)t<<48|i; return x; }
I ord(L x) { return *(unsigned long long*)&x; }
L num(L n) { return n; }
I equ(L x,L y) { return *(unsigned long long*)&x == *(unsigned long long*)&y; }
L atom(const char *s) {
I i = 0; while (i < hp && strcmp(A+i,s)) i += strlen(A+i)+1;
 if (i == hp && (hp += strlen(strcpy(A+i,s))+1) > sp<<3) abort();
return box(ATOM,i);
L cons(L x,L y) { cell[--sp] = x; cell[--sp] = y; if (hp > sp<<3) abort(); return box(CONS,sp); }
L \operatorname{car}(L p) \{ \operatorname{return} (T(p)\&^{(CONS^{CLOS})}) == \operatorname{CONS} ? \operatorname{cell}[\operatorname{ord}(p)+1] : \operatorname{err}; \}
L \ cdr(L \ p) \ \{ \ return \ (T(p)\&^(CONS^CLOS)) == CONS \ ? \ cell[ord(p)] : err; \ \}
L pair(L v,L x,L e) { return cons(cons(v,x),e); }
L closure(L v,L x,L e) { return box(CLOS,ord(pair(v,x,equ(e,env) ? nil : e))); }
L assoc(L v,L e) { while (T(e) == CONS && !equ(v,car(car(e)))) e = cdr(e); return T(e) == CONS ? cdr(car(e)) : err; }
I not(L x) { return T(x) == NIL; }
I let(L x) { return T(x) != NIL && !not(cdr(x)); }
L eval(L,L),parse();
L evlis(L t,L e) {
L s = nil,*p = &s;
 while (T(t) == CONS) *p = cons(eval(car(t),e),nil),p = cell+sp,t = cdr(t);
 if (T(t) != NIL) *p = eval(t,e);
}
L f_eval(L t,L e) { return eval(car(evlis(t,e)),e); }
L f_quote(L t,L _) { return car(t); }
L f_cons(L t,L e) { return t = evlis(t,e),cons(car(t),car(cdr(t))); }
L f_car(L t,L e) { return car(car(evlis(t,e))); }
L f_cdr(L t,L e) { return cdr(car(evlis(t,e))); }
L f_{add}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n += car(t); return num(n); \}
L f_sub(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n -= car(t); return num(n); \}
 L \ f_{\tt mul}(L \ t, L \ e) \ \{ \ L \ n = car(t = evlis(t,e)); \ while \ (!not(t = cdr(t))) \ n \ *= car(t); \ return \ num(n); \ \} 
L f_{div}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n /= car(t); return num(n); \}
L f_{int}(L t, L e) \{ L n = car(evlis(t, e)); return n<1e16 && n>-1e16 ? (long long)n : n; \}
L f_{t}(L t, L e)  { return t = evlis(t, e), car(t) - car(cdr(t)) < 0 ? tru : nil; }
L f_eq(L t,L e) { return t = evlis(t,e),equ(car(t),car(cdr(t))) ? tru : nil; }
L f_not(L t,L e) { return not(car(evlis(t,e))) ? tru : nil; }
L f_or(L t,L e) { for (; T(t) != NIL; t = cdr(t)) if (!not(eval(car(t),e))) return tru; return nil; }
L f_{and}(L t, L e)  { for (; T(t) != NIL; t = cdr(t)) if (not(eval(car(t), e))) return nil; return tru; }
L f_{cond}(L t, L e) { while (T(t) != NIL \&\& not(eval(car(car(t)), e))) t = cdr(t); return eval(car(cdr(car(t))), e); }
L f_if(L t,L e) { return eval(car(cdr(not(eval(car(t),e)) ? cdr(t) : t)),e); }
L f_leta(L t,L e) { while (let(t)) e = pair(car(car(t)),eval(car(cdr(car(t))),e),e),t = cdr(t); return eval(car(t),e); }
L f_lambda(L t,L e) { return closure(car(t),car(cdr(t)),e); }
L f_define(L t,L e) { env = pair(car(t),eval(car(cdr(t)),e),env); return car(t); }
struct { const char *s; L (*f)(L,L); } prim[] = {
{"eval",f_eval},{"quote",f_quote},{"cons",f_cons},{"car", f_car}, {"cdr",
                                                                                                              {"-", f_sub},
                                                                                         {"+",
                                                                              f_cdr},
                                                                                                   f_add},
{"or",
                                                                                                              {"and",f_and},
                                                                                                   f or .
{"not", f_not}, {"cond", f_cond}, {"if", f_if}, {"let*",f_leta},{"lambda",f_lambda},{"define",f_define},{0}};
L eval(L x,L e) {
 L f, v, d;
 while (1) {
 if (T(x) == ATOM) return assoc(x,e);
  if (T(x) != CONS) return x;
  f = eval(car(x),e),x = cdr(x);
```

```
if (T(f) == PRIM) return prim[ord(f)].f(x,e);
  if (T(f) != CLOS) return err;
  v = car(car(f)), d = cdr(f);
  if (T(d) == NIL) d = env;
  while (T(v) == CONS) && T(x) == CONS) d = pair(car(v), eval(car(x), e), d), v = cdr(v), x = cdr(x);
  if (T(v) == CONS) x = eval(x,e);
  while (T(v) == CONS) d = pair(car(v), car(x), d), v = cdr(v), x = cdr(x);
  if (T(x) == CONS) x = evlis(x,e);
 else if (T(x) != NIL) x = eval(x,e);
 if (T(v) != NIL) d = pair(v,x,d);
 x = cdr(car(f)), e = d;
}
char buf [40], see = ' ';
void look() { see = getchar(); }
I seeing(char c) { return c == ' ' ? see > 0 && see <= c : see == c; }</pre>
char get() { char c = see; look(); return c; }
char scan() {
I i = 0;
 while (seeing(' ')) look();
 if (seeing('(') || seeing(')') || seeing('\'')) buf[i++] = get();
 else do buf[i++] = get(); while (i < 39 && !seeing('(') && !seeing(')') && !seeing(' '));
return buf[i] = 0,*buf;
L read() { return scan(),parse(); }
L list() {
 L t = nil,*p = &t;
 while (1) {
 if (scan() == ')') return t;
 if (*buf == '.' && !buf[1]) return *p = read(),scan(),t;
  *p = cons(parse(),nil),p = cell+sp;
}
L parse() {
Ln; Ii;
 if (*buf == '(') return list();
 if (*buf == '\'') return cons(atom("quote"),cons(read(),nil));
return sscanf(buf, "%lg%n", &n, &i) > 0 && !buf[i] ? n : atom(buf);
}
void print(L);
void printlist(L t) {
 putchar('(');
 while (1) {
 print(car(t));
 if (not(t = cdr(t))) break;
 if (T(t) != CONS) { printf(" . "); print(t); break; }
 putchar(' ');
putchar(')');
}
void print(L x) {
if (T(x) == NIL) printf("()");
 else if (T(x) == ATOM) printf("%s",A+ord(x));
 else if (T(x) == PRIM) printf("<%s>",prim[ord(x)].s);
 else if (T(x) == CONS) printlist(x);
 else if (T(x) == CLOS) printf("{{u}}",ord(x));
 else printf("%.10lg",x);
}
void gc() { sp = ord(env); }
int main() {
I i; printf("tinylisp");
nil = box(NIL,0); err = atom("ERR"); tru = atom("#t"); env = pair(tru,tru,nil);
for (i = 0; prim[i].s; ++i) env = pair(atom(prim[i].s),box(PRIM,i),env);
 while (1) { printf("\n\u>",sp-hp/8); print(eval(read(),env)); gc(); }
```

D Optimized Lisp Interpreter with BCD boxing

```
#define I unsigned
#define L double
#define T *(char*)&
#define A (char*)cell
#define N 1024
I hp=0,sp=N,ATOM=32,PRIM=48,CONS=64,CLOS=80,NIL=96;
L cell[N], nil, tru, err, env;
L box(I t,I i) { L x = i+10; T x = t; return x; }
I ord(L x) { T x &= 15; return (I)x-10; }
L num(L n) { T n &= 159; return n; }
L atom(const char *s) {
 I i = 0; while (i < hp && strcmp(A+i,s)) i += strlen(A+i)+1;
 if (i == hp && (hp += strlen(strcpy(A+i,s))+1) > sp<<3) abort();
 return box(ATOM,i);
}
L cons(L x,L y) \{ cell[--sp] = x; cell[--sp] = y; if (hp > sp<<3) abort(); return box(CONS,sp); \}
L car(L p) \{ return (T p&224) == CONS ? cell[T p &= 15,(I)p-9] : err; \}
L cdr(L p) { return (T p&224) == CONS ? cell[T p &= 15,(I)p-10] : err; }
L pair(L v,L x,L e) { return cons(cons(v,x),e); }
L assoc(L v,L e) { while (T e == CONS && v != car(car(e))) e = cdr(e); return T e == CONS ? cdr(car(e)) : err; }
I not(L x) { return T x == NIL; }
I let(L x) { return T(x) != NIL && !not(cdr(x)); }
L eval(L,L),parse();
L evlis(L t,L e) {
L s = nil,*p = &s;
 while (T t == CONS) *p = cons(eval(car(t),e),nil),t = cdr(t),p = cell+sp;
 if (T t != NIL) *p = eval(t,e);
return s;
}
L f_eval(L t,L e) { return eval(car(evlis(t,e)),e); }
L f_quote(L t,L _) { return car(t); }
L f_cons(L t,L e) { return t = evlis(t,e),cons(car(t),car(cdr(t))); }
L f_car(L t,L e) { return car(car(evlis(t,e))); }
L f_cdr(L t,L e) { return cdr(car(evlis(t,e))); }
L f_add(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n += car(t); return num(n); \}
L f_sub(L t,L e) { L n = car(t = evlis(t,e)); while (!not(t = cdr(t))) n -= car(t); return num(n); }
L f_{mul}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n *= car(t); return num(n); \} 
L f_{div}(L t, L e) \{ L n = car(t = evlis(t, e)); while (!not(t = cdr(t))) n /= car(t); return num(n); \}
L f_{int}(L t, L e)  { return t = car(evlis(t,e)),t-1e9 < 0 && t+1e9 > 0 ? (long)t : t; }
L f_{t}(L t, L e)  { return t = evlis(t, e), car(t) - car(cdr(t)) < 0 ? tru : nil; }
L f_{eq}(L t, L e)  { return t = evlis(t, e), car(t) == car(cdr(t)) ? tru : nil; }
L f_not(L t,L e) { return not(car(evlis(t,e))) ? tru : nil; }
L f_or(L t,L e) { for (; T t == CONS; t = cdr(t)) if (!not(eval(car(t),e))) return tru; return nil; }
L f_{and}(L t, L e)  { for (; T t == CONS; t = cdr(t)) if (not(eval(car(t), e))) return nil; return tru; }
L f_cond(L t,L e) { while (T t == CONS && not(eval(car(car(t)),e))) t = cdr(t); return eval(car(cdr(car(t))),e); }
L f_if(L t,L e) { return eval(car(cdr(not(eval(car(t),e)) ? cdr(t) : t)),e); }
L f_{\text{leta}}(L t, L e) { while (\text{let}(t)) e = \text{pair}(\text{car}(\text{car}(t)), \text{eval}(\text{car}(\text{cdr}(\text{car}(t))), e), e), t = cdr(t); return eval(car(t), e); }
L f_lambda(L t,L e) { return box(CLOS,ord(pair(car(t),car(cdr(t)),e == env ? nil : e))); }
L f_define(L t,L e) { env = pair(car(t),eval(car(cdr(t)),e),env); return car(t); }
struct { const char *s; L (*f)(L,L); } prim[] = {
{"eval",f_eval},{"quote",f_quote},{"cons",f_cons},{"car", f_car}, {"cdr",
                                                                                         <"+".
                                                                                                              {"-", f_sub},
                                                                                                   f_add},
                                                                              f_cdr},
{"*", f_mul}, {"/", f_div}, {"int", f_int}, {"<", f_lt}, {"eq?", f_eq},
                                                                                         {"or",
                                                                                                   f_or},
{"not", f_not}, {"cond", f_cond}, {"if", f_if}, {"let*",f_leta},{"lambda",f_lambda},{"define",f_define},{0}};
L eval(L x.L e) {
 L f, v, d;
 while (1) {
  if (T x == ATOM) return assoc(x,e);
  if (T x != CONS) return x;
  f = eval(car(x),e),x = cdr(x);
  if (T f == PRIM) return prim[T f &= 15,(I)f-10].f(x,e);
  if (T f != CLOS) return err;
  v = car(car(f)), d = cdr(f);
  if (T(d) == NIL) d = env;
```

```
while (T(v) == CONS \&\& T(x) == CONS) d = pair(car(v),eval(car(x),e),d), v = cdr(v), x = cdr(x);
  if (T(v) == CONS) x = eval(x,e);
  while (T(v) == CONS) d = pair(car(v), car(x), d), v = cdr(v), x = cdr(x);
  if (T(x) == CONS) x = evlis(x,e);
  else if (T(x) != NIL) x = eval(x,e);
 if (T(v) != NIL) d = pair(v,x,d);
 x = cdr(car(f)), e = d;
}
}
char buf[40],see = ' ';
void look() { see = getchar(); }
I seeing(char c) { return c == ' ' ? see > 0 && see <= c : see == c; }
char get() { char c = see; look(); return c; }
char scan() {
I i = 0;
 while (seeing(' ')) look();
 if (seeing('(') || seeing(')') || seeing('\'')) buf[i++] = get();
 else do buf[i++] = get(); while (i < 39 && !seeing('(') && !seeing(')') && !seeing(' '));
return buf[i] = 0,*buf;
}
L read() { return scan(),parse(); }
L list() {
L t = nil,*p = &t;
 while (1) {
 if (scan() == ')') return t;
 if (*buf == '.' && !buf[1]) return *p = read(),scan(),t;
  *p = cons(parse(),nil),p = cell+sp;
}
L parse() {
Ln; Ii;
 if (*buf == '(') return list();
if (*buf == '\'') return cons(atom("quote"),cons(read(),nil));
 i = strlen(buf);
return isdigit(buf[*buf == '-']) && sscanf(buf, "%1g%n", &n, &i) > 0 && !buf[i] ? n : atom(buf);
void print(L);
void printlist(L t) {
putchar('(');
 while (1) {
 print(car(t));
 if (not(t = cdr(t))) break;
 if (T t != CONS) { printf(" . "); print(t); break; }
 putchar(' ');
putchar(')');
}
void print(L x) {
 if (T x == NIL) printf("()");
 else if (T x == ATOM) printf("%s",A+ord(x));
 else if (T x == PRIM) printf("<s",prim[ord(x)].s);
 else if (T x == CONS) printlist(x);
 else if (T x == CLOS) printf("\{u\}",ord(x));
 else printf("%.10lg",x);
void gc() { sp = ord(env); }
int main() {
I i; printf("lisp850");
nil = box(NIL,0); err = atom("ERR"); tru = atom("#t"); env = pair(tru,tru,nil);
for (i = 0; prim[i].s; ++i) env = pair(atom(prim[i].s),box(PRIM,i),env);
 while (1) { printf("\n^u", sp-hp/8); print(eval(read(),env)); gc(); }
```

E Example Lisp Functions

E.1 Standard Lisp Functions

The following functions should be self-explanatory, for details see further below:

```
(define null? not)
(define err? (lambda (x) (eq? x 'ERR)))
(define number? (lambda (x) (eq? (* 0 x) 0)))
(define pair? (lambda (x) (not (err? (cdr x)))))
(define symbol?
    (lambda (x)
        (and
            (not (err? x))
            (not (number? x))
            (not (pair? x)))))
(define atom?
    (lambda (x)
        (or
            (not x)
            (symbol? x))))
(define list?
    (lambda (x)
        (if (not x)
            #t.
            (if (pair? x)
                (list? (cdr x))
                ()))))
(define equal?
    (lambda (x y)
        (or
            (eq? x y)
            (and
                (pair? x)
                (pair? y)
                (equal? (car x) (car y))
                (equal? (cdr x) (cdr y))))))
(define negate (lambda (n) (- 0 n)))
(define > (lambda (x y) (< y x)))
(define \leftarrow (lambda (x y) (not (< y x))))
(define >= (lambda (x y) (not (< x y))))
(define = (lambda (x y) (eq? (- x y) 0)))
(define list (lambda args args))
(define cadr (lambda (x) (car (cdr x))))
(define caddr (lambda (x) (car (cdr (cdr x)))))
(define begin (lambda (x . args) (if args (begin . args) x)))
```

Explanation:

- equal? tests equality of two values recursively (eq? tests exact equality only)
- symbol? tests if the value is an atom excluding the empty list.
- atom? tests if the value is an atom including the empty list, but beware that some Lisp implementation also return #t for numbers.

- list returns a list of the values of the arguments. For example, (list 1 2 (+ 1 2)) gives (1 2 3).
- begin returns the last value of its last argument. For example, (begin 1 2 (+ 1 2)) gives 3. This function is often used as a code block in Lisp, to evaluate a sequence of expressions, which only makes sense if the expressions have side effects, such as setq to change the value of a variable.

E.2 Math Functions

The following functions should be self-explanatory:

```
(define abs
    (lambda (n)
        (if (< n 0)
            (-0n)
           n)))
(define frac (lambda (n) (- n (int n))))
(define truncate int)
(define floor
    (lambda (n)
        (int
            (if (< n 0)
                (-n1)
                n))))
(define ceiling (lambda (n) (- 0 (floor (- 0 n)))))
(define round (lambda (n) (+ (floor n) 0.5)))
(define mod (lambda (n m) (- n (* m (int (/ n m))))))
(define gcd
    (lambda (n m)
        (if (eq? m 0)
            (gcd m (mod n m)))))
(define lcm (lambda (n m) (/ (* n m) (gcd n m))))
(define even? (lambda (n) (eq? (mod n 2) 0)))
(define odd? (lambda (n) (eq? (mod n 2) 1)))
```

E.3 List Functions

The following functions should be self-explanatory, for details see further below:

```
(rev1 (cons (car t) r) (cdr t))
           r)))
(define reverse (lambda (t) (rev1 () t)))
(define member
    (lambda (x t)
        (if t
            (if (equal? x (car t))
                (member x (cdr t)))
            t)))
(define foldr
   (lambda (f x t)
        (if t
            (f (car t) (foldr f x (cdr t)))
           x)))
(define foldl
   (lambda (f x t)
        (if t
            (foldl f (f (car t) x) (cdr t))
           x)))
(define min
    (lambda args
        (foldl
            (lambda (x y)
                (if (< x y)
                    X
                    y))
            9.99999999e99
            args)))
(define max
   (lambda args
        (foldl (lambda (x y)
            (if (< x y)
                у
                x))
        -9.99999999e99
        args)))
(define filter
   (lambda (f t)
        (if t
            (if (f (car t))
                (cons (car t) (filter f (cdr t)))
                (filter f (cdr t)))
            ())))
(define all?
    (lambda (f t)
        (if t
            (if (f (car t))
                (all? f (cdr t))
                ())
            #t)))
(define any?
    (lambda (f t)
        (if t
            (if (f (car t))
                #t
```

```
(any? f (cdr t)))
            ())))
(define map1
    (lambda (f t)
        (if t
            (cons (f (car t)) (map1 f (cdr t)))
            ())))
(define map
    (lambda (f . args)
        (if (any? null? args)
            (let*
                (mapcar (map1 car args))
                (mapcdr (map1 cdr args))
                (cons (f . mapcar) (map f . mapcdr))))))
(define zip (lambda args (map list . args)))
(define range
    (lambda (n m . args)
        (if args
            (if (< 0 (* (car args) (- m n)))
                (cons n (range (+ n (car args)) m (car args)))
            (if (< n m)
                (cons n (range (+ n 1) m))
                ()))))
```

Explanation:

- member checks list membership and returns the rest of the list where x was found.
- foldr and foldl return the value of right- and left-folded lists t using an operator f and initial value: x (foldl $\oplus x_0$ ' $(x_1 \ x_2 \ \dots \ x_n)$) = $(\cdots ((x_0 \oplus x_1) \oplus x_2) \oplus \cdots x_n)$ and x (foldr $\oplus x_0$ ' $(x_1 \ x_2 \ \dots \ x_n)$) = $(x_1 \oplus (x_2 \oplus (\cdots (x_n \oplus x_0)))$).
- filter returns a list with elements x from the list t for which (f x) is true.
- all? returns #t if all elements x of the list t satisfy (f x) is true, and returns () otherwise.
- any? returns #t if any one of the elements x of the list t satisfy (f x) is true, and returns () otherwise.
- map applies f to the elements of a list or to n lists for n-ary function f.
- zip takes n lists of length m to return a list of length m with lists of length n.
- range generates a list of successive values n up to but not including m with an optional step value.

E.4 Higher-Order Functions

The following functions take one or more functions as arguments to construct new functions are explained further below:

```
(define curry (lambda (f x) (lambda args (f x . args))))
(define compose (lambda (f g) (lambda args (f (g . args)))))
(define Y (lambda (f) (lambda args ((f (Y f)) . args))))
```

Explanation:

- curry takes a function f and an argument x and returns a function that applies f to x and the given arguments.
- compose takes two function f and g and returns a function that applies g to the arguments followed by the application of f to the result.
- Y is the Y combinator that takes a function f to return a function that applies f to (Y f) that is a copy of itself, and in turn returns a self-applying (recursive) function. The Y combinator can be used for recursion without naming the function. For example the factorial of 5 is 120:

```
> ((Y (lambda (f) (lambda (k) (if (< 1 k) (* k (f (- k 1))) 1)))) 5) 120
```

E.5 Revealing Closures

The following function reveals the arguments and body of the closure of a lambda:

```
(define reveal (lambda (f) (cons 'lambda (cons (car (car f)) (cons (cdr (car f)) ())))))
```

Explanation:

• reveal takes a closure and "unparses" it as a lambda. For example (reveal reveal) shows (lambda (f) (cons (quote lambda) (cons (car (car f)) (cons (cdr (car f)) ())))).