Exos Entraînement

Question 2 :

1) $z_1 = 2 + 3i$, $z_2 = 1 - i$

$\bar{z}_1 = 2 - 3i$, $\bar{z}_2 = 1 + i$

$$z_1 + \bar{z}_2 = 2 + 3i + 1 + i = 3 + 4i$$

$$\bar{z}_1 z_2 = (1-i)(2-3i) = 2 - 3i - 2i - 3 = -1 - 5i$$

$$|z_1| = \sqrt{2^2+3^2}$$

$$|z_1| = \sqrt{13}$$

Exercice 5 :

$$z = r(\cos(\theta) + i\sin(\theta))$$

$$\bar{z} = r(\cos(\theta) - i\sin(\theta)) \text{ ou } r(\cos(\theta) + i\sin(-\theta))$$

$$-z = -r(\cos(\theta) + i\sin(\theta)) \text{ ou } r(-\cos(\theta) - i\sin(\theta))$$

$$\frac{1}{z} = \frac{1}{a+ib} = \frac{1(a-ib)}{(a+ib)(a-ib)} = \frac{a-ib}{a^2+b^2}$$

Donc $z_\theta^{-1} = \dfrac{r(\cos(\theta) - i\sin(\theta))}{r^2}$

Exercice 6 :

1) $\forall \alpha \in \mathbb{R},\ e^{i\alpha} + e^{i-\alpha} = 2\cos(\alpha)$

On sait que $\begin{cases} e^{i\alpha} = \cos(\alpha) + i\sin(\alpha) \\ e^{i-\alpha} = \cos(-\alpha) + i\sin(-\alpha) \end{cases}$

Or $\begin{cases} \cos(-\alpha) = \cos(\alpha) \\ \sin(-\alpha) = -\sin(\alpha) \end{cases}$ Donc $e^{i-\alpha} = \cos(\alpha) - \sin(\alpha)$

Donc
$$\begin{aligned} e^{i\alpha} + e^{i-\alpha} &= \cos(\alpha) + \sin(\alpha) + (\cos(\alpha) - \sin(\alpha)) \\ &= \cos(\alpha) + \sin(\alpha) + \cos(\alpha) - \sin(\alpha) \\ &= 2\cos(\alpha) \end{aligned}$$

2) $e^{i\theta} + e^{i2\theta} = \cos(\theta) + i\sin(\theta) + \cos(2\theta) + \sin(2\theta)$