# Lundi 29 Janvier 2024 :

## Resolution d'equation diophantienne

1) Si $PGCD(a, b) \nmid c$ → Pas de solution

2) $a x_0 + b y_0 = PGCD(a, b)$

3) $c = PGCD(a, b) \times c'$

$$a x_0 c' + b y_0 c' = c$$

Or $x = x_0 + K \times \frac{b}{pgcd(a, b)}$

$y = y_0 - K \times \frac{a}{pgcd(a, b)}$

Donc $a x + b y = c$

## Question 44 :

2) $5x + 13y = 6$

On peut calculer que $x, y$ peuvent valoir :

$x = -30, \ y = 12$

$\vdots$

3) $504x + 1188y = 144$

1) $PGCD(1188, 504)$

$\Leftrightarrow PGCD(504, 180)$

$\Leftrightarrow PGCD(180, 144)$

$\Leftrightarrow PGCD(144, 36)$

$\Leftrightarrow PGCD = 36$

Donc $36 | 144$, donc des solutions existent.

2) $36 = 180 - 144$

$36 = 180 - (504 - 2 \times 180)$

$36 = (1188 - 2 \times 504) - (504 - 2(1188 - 2 \times b)$

$36 = (a - 2b) - (b - 2(a - 2b))$

$36 = a - 2b - b + 2(a - 2b)$

$36 = a - 2b - b + 2a - 4b$

$36 = 3a - 7b$

Donc $3 \times 1188 - 7 \times 504 = 36$

On $144 = 4 \times 36$

Donc $36 \times 4 = 3 \times 1188 \times 4 - 7 \times 504 \times 4$

$\Leftrightarrow 144 = 12 \times 1188 - 28 \times 504$

3) On a $x_0 = 12$, $y_0 = -28$, Trouvons toutes les solutions

$$X = -28 + K \frac{504}{PGCD(a,b}$$

$$Y = 12 - K \frac{1188}{PGCD(a,b)}$$

$$\begin{cases} x = -28 + \frac{1188}{36} K \\ y = 12 - \frac{504}{36} K \end{cases}$$

$$\boxed{\begin{cases} x = -28 + 33K \\ y = 12 - 14K \end{cases}}$$

## Question 26 :

$$\begin{cases} PGCD(n, 527) = 17 \\ PPCM(n, 527) = 1370 \end{cases}$$

Lien PPCM & PGCD : $PGCD(a, b) \times PPCM(a, b) = a \times b$

Donc $17 \times 13702 = 527\, n$

$13702 = 31\, n$

$n = \dfrac{13702}{31}$

$n = 442$ CQFD

## Question 27 :

Preuve $\boxed{a' \wedge b' = 1}$

↳ $a = \hat{a}\,\delta$ avec $PGCD(\hat{a}, \tilde{b}) = 1$

$b = \tilde{b}\,\delta$

Si $\mathrm{pgcd}(\tilde{a}, \hat{b}) \neq 1 \Rightarrow PGCD(\hat{a}, \hat{b}) > 1$

Donc $\left\{ \begin{array}{l} \hat{a} = K\,\tilde{\hat{a}} \\ \hat{b} = K\,\tilde{\hat{b}} \end{array} \right\} K > 1$

$a = \hat{a}\,\delta = \delta K \tilde{\hat{a}}$ Or si $K > 1$, donc $\delta K > \delta$, donc $a = \tilde{a} \times$ quelque chose de plus grand que $\delta$, alors que $\delta = \mathrm{pgcd}(a, b)$ !

$$\begin{cases} a + b = 256 \\ PGCD(a, b) = 16 \end{cases}$$

↳ $a = 16\tilde{a}$, $b = 16\tilde{b}$ & $PGCD(\hat{a}, \tilde{b}) = 1$

⋮

⚠ à Terminé

# Les Congruences :

**Définition :** $a \equiv b\ [n]$ ssi $\exists K \mid a-b = Kn$

## Question 15 :

$55 \equiv 6\ [7]$

$2048 \equiv 2\ [3]$

$406 \equiv 406\ [1056]$

## Question 16 :

2) $\left.\begin{array}{l} a \equiv b\ [n] \\ b \equiv c\ [n] \end{array}\right\}$ donc $a \equiv c\ [n]$

↳ preuve : $\exists k \in \mathbb{Z} \mid a-b = kn$ ①

$\exists k' \in \mathbb{Z} \mid b-c = k'n$ ②

Or si on fait ① + ② :

$a - b + b - c = lm + km$

$a - c = (l+k)m$

donc $m \mid (a-c)$ donc $a \equiv c\,[m]$

e) Si $a \equiv b\,[m] \Rightarrow m \times a \equiv m \times b\,[m \times m]$

$\hookrightarrow$ preuve : $\exists k \in \mathbb{Z} \mid a - b = km$

Si on fait $m \times$ "$(a-b) = km$"

$\Rightarrow m(a-b) = m(km)$

$\Rightarrow ma - mb = mmk$

$\Leftrightarrow ma \equiv mb\,[mm]$

## Question 18 :

a) $X \equiv 6\,[7]$, $Y \equiv 2\,[7]$

$$\begin{cases} X + Y \equiv 8\,[7] \equiv 1\,[7] \\ X - Y \equiv 4\,[7] \\ Y - X \equiv -4\,[7] \equiv 3\,[7] \\ X \times Y \equiv 12\,[7] \equiv 5\,[7] \end{cases}$$

b) $46 \times 23 \equiv \dots [7]$

$46 \equiv 4 [7]$ & $23 \equiv 2 [7]$

Donc $46 \times 23 \equiv 4 \times 2 [7]$

$= 46 \times 23 \equiv 8 [7$

$= 46 \times 23 \equiv 1 [7]$ CQFD

c) $13^8 \equiv ? [7]$

$13^1 \equiv 6 [7] \equiv (-1) [7]$

$13^2 \equiv (-1)^2 [7] \equiv 1 [7]$

$13^{2^4} = 13^8 \equiv 1^4 \equiv 1 [7]$

d) $2^{123} \equiv \dots [29]$

| $n$ | $2^n \equiv [29]$ |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

| $n$ | $2^n \equiv [29]$ |
|---|---|
| 5 | 3 |
| 6 | 6 |
| 7 | 12 |
| 8 | 24 |
| ⋮ | |

$2^{14} \equiv -1 [29]$

$2^{28} \equiv 1 [29]$

Or $123 = 4 \times 28 + 11$

$$2^{123} \equiv 2^{4 \times 28 + 11} \equiv 1 \times 2^{11}$$

Or $2^{11} \equiv 18 [29]$

Donc $2^{123} \equiv 18 [29]$