

$$7 - 13 - 14 - 10 - 19 - 23 - 26 - 27 - 28 - 29 - \underline{\underline{=}} = 44$$

Exercice important

Exam  
Algorithmique  
Sémantique aux 2  
deux 2 aux 2

Question 19 :

Trouver  $m, m$  tels que  $m + m = 76$  &  
 $m = 9m + n$

$$\text{Donc} \quad \begin{cases} m + m = 76 \\ m = 9m + n \quad 0 \leq n < 9 \end{cases}$$

Ainsi  $(9m + n) + m = 76$ , avec  $0 \leq n < 9$

$10m + n = 76$ , avec  $0 \leq n < 9 < 10$

$$10m + n = 7 \times 10 + 6$$

$$m = 7, n = 6 \quad \text{avec } 0 \leq n < 9 < 10$$

$$\text{Or } m = 9m + b \Rightarrow m = 63 + 6 = 69$$

From  $m = 69, m = 7$

Question 20:

$$\text{Sait } a^2 + b^2 = c^2$$

on a :  $(5, 4, 3), (10, 6, 8)$

$$a^2 = b^2 + c^2,$$

or  $a \equiv 1[3]$  &  $b \equiv 1[3]$

$$c^2 = \dots \quad ??$$

Question 21 :

$$\begin{aligned} - \quad & 4^{3m} - 4^m \equiv 0[5] \\ & 4^m \times 4^{2m} - 4^m \equiv 0[5] \\ & 4^m(4^{2m} - 1) \equiv 0[5] \\ & 4^m(4^m \times 4^m - 1) \equiv 0[5] \end{aligned}$$

On se m'a calculer la congruence des  
chiffres mod 5 :

$$4(4 \times 4 - 1) \equiv \dots [5]$$

$$4(16 - 1) \equiv \dots [5]$$

$$4(15) \equiv \dots [5]$$

$$60 \equiv 0[5]$$

Donc  $\forall m \in \mathbb{N}, 4^{3m} - 4^m \equiv 0[5]$

$$\begin{aligned} - \quad & 3^{2m} - 2^m \\ & 3^{2m} - 2^m \\ & = 9^m - 2^m \equiv \dots [7] \end{aligned}$$

On va maintenant calculer la congruence des chiffres mod 7 :

$$9 - 2 \equiv \dots [7]$$

$$7 \equiv 0 [7]$$

Dans  $\forall m \in \mathbb{N}$ ,  $7 \mid 3^{2m} - 2^m$

$$\begin{aligned} & - 4^m + 15m - 1 \\ & 4^m + 15m - 1 \end{aligned}$$

Pareillement, calculons les chiffres et leur congruence modulo 9 :

$$4 + 15 - 1 \equiv \dots [9]$$

$$18 \equiv 0 [9]$$

Dans  $\forall m \in \mathbb{N}$ ,  $9 \mid 4^m + 15m - 1$

$$\begin{aligned} & - 3 \times 5^{2m+1} + 2^{3m+1} \\ & 3 \times 5^{2m} \times 5 + 2^{3m} \times 2 \end{aligned}$$

$$3 \times 5^{2^m} \times 5 + 2^{3^m} \times 2$$

$$3 \times 25^m \times 5 + 8^m \times 2$$

Calculons alors la somme des chiffres modulo 17.

$$3 \times 25 - 75 \times 5 = 375 + 16 = 391$$

$$391 \equiv 0 \pmod{17} \text{ car } 391 = 23 \times 17$$

$$\text{Donc } \forall m \in \mathbb{N}, 17 \mid 3 \times 5^{2m+1} + 2^{3m+1}$$

Question 22 :

$$2391 = 23 \times 100 + 91$$

d'après le lemme d'Euclide qui dit que "si  $a, b \in \mathbb{Z}^2$ ,  $\text{PGCD}(a, b) = \text{PGCD}(b, r)$  avec  $r = b_1 + r$ ".

Or, on a  $a = 2391$ ,  $b = 23$ , et  $r = 91$ .

Or d'après le lemme ci-dessus,  $\text{PGCD}(23, 2391) = \text{PGCD}(91, 23)$ . CQFD

Question 23 :

a)  $\text{PGCD}(1064, 700)$

$$= \left\{ \begin{array}{l} 1064 = 1 \times 700 + \\ \text{PGCD}(700, 364) \end{array} \right\}$$
$$= \left\{ \begin{array}{l} 700 = 1 \times 364 + 336 \\ \text{PGCD}(364, 336) \end{array} \right\} \quad \text{Danc } \text{PGCD}(1064, 700)$$
$$= \left\{ \begin{array}{l} 364 = 1 \times 336 + 28 \\ \text{PGCD}(336, 28) \\ 336 = 12 \times 28 + 0 \checkmark \end{array} \right\} = 28$$

Trouver  $(x, y)$  tel que  $\text{PGCD}(1064, 700) = 1064x + 700y$   
Danc  $28 = 1064x + 700y$

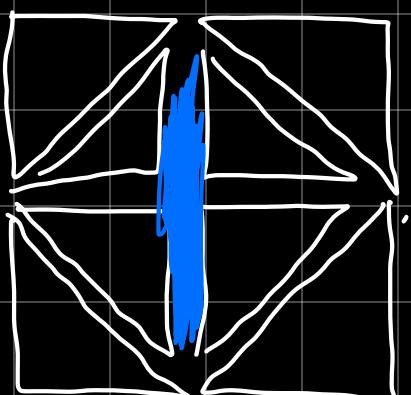
On doit que :

$$28 = 364 - 336$$

$$28 = 364 - (700 - 364)$$

$$28 = 1034 - 700 - (700 - (1034 - 700))$$

$$28 = a - b - (b - (a - b))$$



$$28 = a - b - (l - a + l)$$

$$28 = a - b - (2l - a)$$

$$28 = a - b - 2l + a$$

$$28 = 2a - 3l$$

avec  $a = 1034$  et  $b = 700$

$$\text{Donc } 28 = 2 \times 1034 - 700 \times 3$$

l) PGCD(4567, 91837)

$$91837 = 4567 \times 20 + 497$$

$$4567 = 497 \times 9 + 94$$

$$497 = 94 \times 5 + 27$$

$$94 = 3 \times 27 + 13$$

$$27 = 2 \times 13 + 1$$

$$13 = 1 \times 13 + 0$$

$$1 = 1 \times \boxed{1}$$

Donc le nombre premier entre eux

$$\text{Soit } a = 91837, b = 4567$$

$$l = 27 - 2 \times 13$$

$$1 = (4567 - 5 \times 94) - 2 \times (94 - 3 \times 27)$$

$$1 = (467 - 5 \times 94) - 2 \times (94 - 3 \times (467 - 5 \times 94))$$

$$1 = (467 - 5 \times (4567 - 9 \times 467)) - 2 \times$$

$$( (4567 - 9 \times 467) - 3 \times (467 - 5 \times$$

$$(4567 - 9 \times 467))$$

-

$$467 = a - 20b = c$$

$$1 = c - 5(b - 9c) - 2(b - 9c - 3(c - 5(a - 20b)))$$

$$1 = c - 5(b - 9c) - 2(b - 9c - 3(c - 5b + 45c))$$

$$1 = c - 5b + 45c - 2(b - 9c - 3c + 15b - 135c)$$

$$1 = c - 5b + 45c - 2b + 18c + 6c - 30b + 270c$$

$$1 = -37b + 340c$$

$$1 = -37b + 340(a - 20b)$$

$$1 = -37b + 340a - 6800b$$

$$1 = 340a - 6837b$$

Fawc ...



Question 38: \*\*\*\* (1 of 3 4-star exes!)

$$A = 21a + 2l$$

$$B = 18a + 5b$$

$$a) \text{ Si } 19 | A \Rightarrow 19 | B \wedge \text{ si } 19 | B \Rightarrow 19 | A$$

$$\begin{aligned}47(1+3k_1+2k_2) &= 19K, \\&\Rightarrow 47 + 12k_1 + 8k_2 = 19K \\ \text{or } 47(1+3k_1+2k_2) &= 49K \\&\Rightarrow 47 + 12k_1 + 8k_2 = 49K\end{aligned}$$

## Requirement 1...

$$\begin{aligned}
 & \text{Given } A_1 = A_2 = 5A_3 = 4A_4 = 27 \\
 & A_1 + A_2 + A_3 + A_4 = 81 \\
 & 4A_3 + 4A_3 + A_3 + 2A_3 = 81 \\
 & 10A_3 = 81 \\
 & A_3 = 81 / 10 = 8.1 \\
 & A_3 = 8.1 + 0.9 = 9 \\
 & A_3 = 9 + 0.9 = 9.9 \\
 & A_3 = 9.9 + 0.9 = 10.8 \\
 & A_3 = 10.8 + 0.9 = 11.7 \\
 & A_3 = 11.7 + 0.9 = 12.6 \\
 & A_3 = 12.6 + 0.9 = 13.5 \\
 & A_3 = 13.5 + 0.9 = 14.4 \\
 & A_3 = 14.4 + 0.9 = 15.3 \\
 & A_3 = 15.3 + 0.9 = 16.2 \\
 & A_3 = 16.2 + 0.9 = 17.1 \\
 & A_3 = 17.1 + 0.9 = 18.0 \\
 & A_3 = 18.0 + 0.9 = 18.9 \\
 & A_3 = 18.9 + 0.9 = 19.8 \\
 & A_3 = 19.8 + 0.9 = 20.7 \\
 & A_3 = 20.7 + 0.9 = 21.6 \\
 & A_3 = 21.6 + 0.9 = 22.5 \\
 & A_3 = 22.5 + 0.9 = 23.4 \\
 & A_3 = 23.4 + 0.9 = 24.3 \\
 & A_3 = 24.3 + 0.9 = 25.2 \\
 & A_3 = 25.2 + 0.9 = 26.1 \\
 & A_3 = 26.1 + 0.9 = 27.0 \\
 & A_3 = 27.0 + 0.9 = 27.9 \\
 & A_3 = 27.9 + 0.9 = 28.8 \\
 & A_3 = 28.8 + 0.9 = 29.7 \\
 & A_3 = 29.7 + 0.9 = 30.6 \\
 & A_3 = 30.6 + 0.9 = 31.5 \\
 & A_3 = 31.5 + 0.9 = 32.4 \\
 & A_3 = 32.4 + 0.9 = 33.3 \\
 & A_3 = 33.3 + 0.9 = 34.2 \\
 & A_3 = 34.2 + 0.9 = 35.1 \\
 & A_3 = 35.1 + 0.9 = 36.0 \\
 & A_3 = 36.0 + 0.9 = 36.9 \\
 & A_3 = 36.9 + 0.9 = 37.8 \\
 & A_3 = 37.8 + 0.9 = 38.7 \\
 & A_3 = 38.7 + 0.9 = 39.6 \\
 & A_3 = 39.6 + 0.9 = 40.5 \\
 & A_3 = 40.5 + 0.9 = 41.4 \\
 & A_3 = 41.4 + 0.9 = 42.3 \\
 & A_3 = 42.3 + 0.9 = 43.2 \\
 & A_3 = 43.2 + 0.9 = 44.1 \\
 & A_3 = 44.1 + 0.9 = 45.0 \\
 & A_3 = 45.0 + 0.9 = 45.9 \\
 & A_3 = 45.9 + 0.9 = 46.8 \\
 & A_3 = 46.8 + 0.9 = 47.7 \\
 & A_3 = 47.7 + 0.9 = 48.6 \\
 & A_3 = 48.6 + 0.9 = 49.5 \\
 & A_3 = 49.5 + 0.9 = 50.4 \\
 & A_3 = 50.4 + 0.9 = 51.3 \\
 & A_3 = 51.3 + 0.9 = 52.2 \\
 & A_3 = 52.2 + 0.9 = 53.1 \\
 & A_3 = 53.1 + 0.9 = 54.0 \\
 & A_3 = 54.0 + 0.9 = 54.9 \\
 & A_3 = 54.9 + 0.9 = 55.8 \\
 & A_3 = 55.8 + 0.9 = 56.7 \\
 & A_3 = 56.7 + 0.9 = 57.6 \\
 & A_3 = 57.6 + 0.9 = 58.5 \\
 & A_3 = 58.5 + 0.9 = 59.4 \\
 & A_3 = 59.4 + 0.9 = 60.3 \\
 & A_3 = 60.3 + 0.9 = 61.2 \\
 & A_3 = 61.2 + 0.9 = 62.1 \\
 & A_3 = 62.1 + 0.9 = 63.0 \\
 & A_3 = 63.0 + 0.9 = 63.9 \\
 & A_3 = 63.9 + 0.9 = 64.8 \\
 & A_3 = 64.8 + 0.9 = 65.7 \\
 & A_3 = 65.7 + 0.9 = 66.6 \\
 & A_3 = 66.6 + 0.9 = 67.5 \\
 & A_3 = 67.5 + 0.9 = 68.4 \\
 & A_3 = 68.4 + 0.9 = 69.3 \\
 & A_3 = 69.3 + 0.9 = 70.2 \\
 & A_3 = 70.2 + 0.9 = 71.1 \\
 & A_3 = 71.1 + 0.9 = 72.0 \\
 & A_3 = 72.0 + 0.9 = 72.9 \\
 & A_3 = 72.9 + 0.9 = 73.8 \\
 & A_3 = 73.8 + 0.9 = 74.7 \\
 & A_3 = 74.7 + 0.9 = 75.6 \\
 & A_3 = 75.6 + 0.9 = 76.5 \\
 & A_3 = 76.5 + 0.9 = 77.4 \\
 & A_3 = 77.4 + 0.9 = 78.3 \\
 & A_3 = 78.3 + 0.9 = 79.2 \\
 & A_3 = 79.2 + 0.9 = 80.1 \\
 & A_3 = 80.1 + 0.9 = 81.0
 \end{aligned}$$

# Résumément 2.

Lösungen:  $\text{gau} \in \mathbb{Z}[A]$ ,  $\text{jeweiliges } 19|B$ :  
 $\Rightarrow 19|A + 2B \Leftrightarrow 19|A - 38B$   
 $\Leftrightarrow 19|A + 2B = \frac{1}{19}(A - 38B)$   
 Die Zahl  $m$  erfüllt  $19|m$ ,  $19|B$ ,  $19|A - 38B$   $\Leftrightarrow 19|m_1$   
 $\Leftrightarrow 19|m_1 = 19k_1$ 
  
 Dann ein  $m$  mit  $19|m$  und  $19|B$  ist ein solcher  
 und es gilt  $(B+m) \cdot 19 = 19k_2$   
 $\begin{cases} 19|A + 2B \\ 19|m \\ 19|(B+m) \end{cases}$   
 $\begin{cases} 19|A + 2B \\ 19|m \\ 19|A + 38B \end{cases}$   
 $19|A + 2B \mid 19k_1 + 38k_2 \Leftrightarrow 19|A + 38B \Leftrightarrow 19|m$

# Assignment 3 ..



me right  
more...

A = 110 ± 2 k, 191A et 19XB

$Q_m$  seit 191A  $\Rightarrow$  191B

### Question 26:

$$\begin{cases} \text{PGCD}(m, 527) = 17 \\ \text{PPCM}(m, 527) = 13702 \end{cases}$$

Lien PPCM & PGCD :  $\text{PGCD}(a, b) \times \text{PPCM}(a, b) = a \times b$

Dès lors  $17 \times 13702 = 527m$

$$\frac{17 \times 13702}{527} = m$$

$$\frac{17 \times 13702}{17 \times 31} = m \Leftarrow \text{car } 17 = \text{PGCD}, \text{ donc } 17 \nmid 527$$

$$\frac{13702}{31} = \boxed{m = 442}$$

### Question 27:

$$\begin{cases} a + b = 256 \\ \text{PGCD}(a, b) = 16 \Rightarrow \overbrace{k_1 \cdot 16 + 16k_2}^{\text{PGCD}} = 16(k_1 + k_2) = 256 \end{cases}$$

et  $a = 16k_1$  et  $b = 16k_2$ , donc  $k_1 \wedge k_2$

$$\text{Dn} \quad \left\{ \begin{array}{l} K_1 + K_2 = 16 \\ K_1 \wedge K_2 \end{array} \right.$$

Trix Zahl Legest  
dmawen nappela!

$$\text{Dn a} \quad S = \{(1, 15); (5, 11); (7, 9); (8, 7); (9, 5) \\ \cup (15, 1)\} \quad \text{d} \in (a, b) \in (16\hat{a}, 16\hat{b}) \\ \forall (\hat{a}, \hat{b}) \in S$$

### Question 28:

$$\left\{ \begin{array}{l} a \times b = 1734 \\ \text{PGCD}(a, b) = 17 \end{array} \right.$$

$$\text{Sek} \quad K_1, K_2 \in \mathbb{N} \mid 17K_1 \times 17K_2 = 1734 \\ 17(K_1 K_2) = 17 \times 102 \\ K_1 K_2 = 102$$

$$\left\{ \begin{array}{l} K_1 K_2 = 102 \\ \text{PGCD}(K_1, K_2) = 1 \end{array} \right. \quad \begin{array}{l} 1 \times 102 = 102 \\ 2 \times 51 = 102 \\ 3 \times 34 = 102 \\ 6 \times 17 = 102 \end{array}$$

$$\text{Dn } S = \{(1, 102), (2, 51), (3, 34), (6, 17), (102, 1), (51, 2), (34, 3), (17, 6)\} \quad \text{Dn } (a, b) \in (16\hat{a}, 16\hat{b}) \\ \forall (\hat{a}, \hat{b}) \in S$$

### Question 29:

$$\begin{cases} \tilde{m} \wedge \tilde{m} \\ \tilde{m} + \tilde{m} = 60 \end{cases} \quad \boxed{[31, 29]}$$

$$\tilde{m} = 31 \Rightarrow m = 31 \times 47 \quad \tilde{m} = 29 \Rightarrow m = 29 \times 47$$

$$m = 1363$$

$$m = 1457$$

### Question 30:

$$a, b \neq 0 \text{ et } a^2 - b^2 = 2916$$

$$\therefore (a+b)(a-b) = 2916$$

$$\begin{cases} a^2 - b^2 = 2916 \\ \text{PGCD}(a, b) = 18 \end{cases}$$

$\checkmark \quad \text{avec } (\tilde{a}, \tilde{b}) \in \mathbb{N}$

1.  $a = 18\tilde{a}, b = 18\tilde{b}$

\*  $(18\tilde{a} + 18\tilde{b})(18\tilde{a} - 18\tilde{b}) = 2916$

$$(18(\tilde{a} + \tilde{b}))(18(\tilde{a} - \tilde{b})) = 2916$$

$$18 \times 18 (\tilde{a} + \tilde{b})(\tilde{a} - \tilde{b}) = 2916$$

True Lang ...

$$(\tilde{a} + \tilde{l})(\tilde{a} - \tilde{l}) = 9$$

$$\begin{array}{r} 2916 \\ - 150 \\ \hline 2766 \end{array}$$

$$\begin{array}{r} 10 \\ + 180 \\ \hline 190 \end{array}$$

$$\begin{array}{r} 2 \\ + 36 \\ \hline 38 \end{array}$$

$$162 \times 18 = 2916$$

$$9 \times 18 \times 18 = 2916$$

$$\dots$$

$$\textcircled{1} \left\{ \begin{array}{l} \tilde{a} + \tilde{l} = 1 \text{ ou} \\ \tilde{a} - \tilde{l} = 9 \end{array} \right. \quad \textcircled{2} \left\{ \begin{array}{l} \tilde{a} + \tilde{l} = 3 \text{ ou} \\ \tilde{a} - \tilde{l} = 3 \end{array} \right. \quad \textcircled{3} \left\{ \begin{array}{l} \tilde{a} + \tilde{l} = 9 \\ \tilde{a} - \tilde{l} = 1 \end{array} \right.$$

{ On le système  $\textcircled{2}$  implique  $\tilde{l} = 0$ , on re trouve  $\tilde{a} = 3$ ,  $a = 0 \times 18$   
et cela ne fonctionne pas pour l'exercice ??.

{ Et le système  $\textcircled{1}$  implique que  $\tilde{a} = 0$  et  $\tilde{l} = 0$ , on cela revient à dire que  $a = 18 \times 0$  et  $b = 18 \times 0$  et donc  
a et b sont 0 et cela ne fonctionne pas pour l'exercice.  
?????

Donc  $\textcircled{3} \Rightarrow \begin{cases} \tilde{a} = 5 \\ \tilde{l} = 4 \end{cases} \Rightarrow \begin{cases} 5 + 4 = 9 \\ 5 - 4 = 1 \end{cases}$  ✓

Donc  $a = 5 \times 18$  et  $b = 4 \times 18$

$$a = 90 \quad \text{et} \quad b = 72$$

On vérifie que  $a^2 - b^2 = 2916$

$$a^2 = 90^2$$

$$l^2 = 72^2$$

$$a^2 = 8100$$

$$l^2 = 5040 + 144 = 5184$$

$$a^2 - l^2 = 8100 - 5184 = 2916$$

Dans ce cas  $a = 90$  et  $b = 72$  avec  $\begin{cases} a^2 - l^2 = 2916 \\ \text{PGCD}(a, b) = 18 \end{cases}$

Type de correction du professeur :

CED.

- \* Expliquer le détail de la démonstration de ce
  - \* prouver mathématiquement - >>> prouver grammaticalement
  - \* Parfois, détaillé complique au lieu de simplifier
- 
- \*\* Si j'aurais introduit  $\tilde{a}$  et  $\tilde{l}$  dans  $N^{2k}$ , j'aurais pu en finir de prouver pourquoi c'est évident que  $\tilde{a}$  et  $\tilde{l}$  valent 0.
- \*\* Le "L" est une implication, donc une vérification qu'il faut prouver, alors que j'ai fait à la fin dans le "Vérification que", mais je peut m'éviter le besoin de vérifier en ôtant le "L".

- 1) calculer  $7234 \bmod 7$
- 2) calculer  $2020 \bmod 6$
- 3) calculer  $7234^{2020} \bmod 7$

$$7234 \equiv 3 \pmod{7}$$

$$2020 \equiv 5 \pmod{6}$$

$$7234^{2020} \equiv 3^{2020} \pmod{7}$$

$$\text{" " } \equiv 3^{6k+4}$$

$$3^4 \pmod{7} \quad 87 \pmod{7} = 4 \pmod{7}$$

### Question 44 \*

Déterminer si les équations suivantes ont des solutions dans  $\mathbb{Z}$ . Dans le cas affirmatif, les expliciter.

(a)  $3x + 2y = 4$

(b)  $5x + 13y = 6$

(c)  $504x + 1188y = 144$

$$5x + 13y = 6$$

$$\text{P6CD}(5, 13) = 1$$

$$13 = 2 \times 5 + 3$$

$$5 = 1 \times 3 + 2$$

$$3 = 1 \times 2 + 1$$

$$1 = 3 - 2$$

$$1 = 3 - (5 \times 1 - 3)$$

$$1 = 3 - 5 + 3$$

$$1 = 2 \times 3 - 7 \times 5$$

$$1 = 2 \times (13 - 2 \times 5) - 7 \times 3$$

$$1 = 2 \times 13 - 4 \times 5 - 1 \times 3$$

$$1 = 2 \times 13 - 5 \times 5$$

$$a = 2, b = -5$$

$$13 \times 2 + -5 \times 5 = 1$$

$$12 \times 12 - 30 \times 5 = 6$$

$$a = 13, b = -30$$

Dans on a  $x_0 = 12, y_0 = -30$

Dans  $S = \{(13 - 12k, -30 + 5k), k \in \mathbb{Z}\}$

$$45 \in 5x[-20]$$

$$4 \in x[14]$$

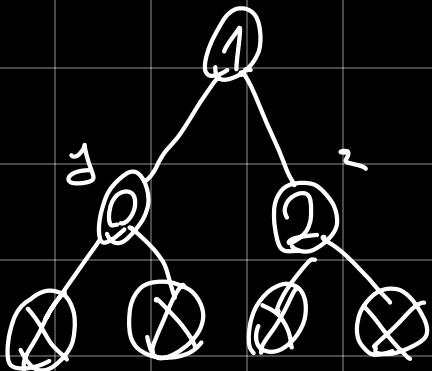
$$9 \in 5[14]$$

$E \times 38 \sqrt{2}:$

$$11(1 + 5K) - 2(5 - 11K) = 1$$

$$15 \times 2 - 5 \times 7 = 1$$

$$18(2 + 5K) - 5(7 - 16K) = 19$$



let  $T = (y, 1, 2)$