

Lundi 5 Février 2024



Cours TD #1: Nombres Complexes

$$\mathbb{C} = \{a + ib ; a, b \in \mathbb{R}\}$$

$$a = 0, b = 1, \quad i, i^2 = -1$$

$$x^2 = -1, x = i$$

$$z = a + ib \quad \leftarrow \text{forme algébrique si } a, b \in \mathbb{R}$$

Question 1:

$$1) (2 + 5i) + (i + 3)$$

$$(2 + 5i + i + 3)$$

$$5 + 6i$$

$$2) (3 - 2i) - (-1 - i)$$

$$3 - 2i - (-1 - i)$$

$$3 - 2i + 1 + i$$

$$4 - i$$

$$4) (1 - 4i)(1 + 2i) + 2i + 8$$

$$1 + 2i + 8 - 4i + 2i + 8$$

$$17$$

$$7) (2 + i)^2$$

$$2^2 + i^2 + 2 \times 2 \times i$$

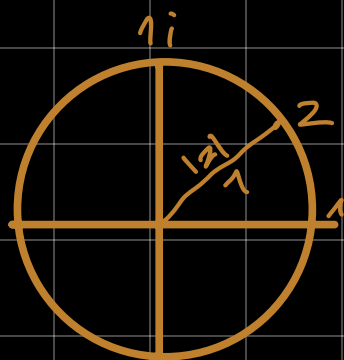
$$4 - 1 + 4i = 3 + 4i$$

Question 3:

$$z = a + ib, \quad \bar{z} = a - ib$$

$$z\bar{z} = a^2 + b^2$$

$$\begin{cases} |z| = 1 \\ |z|^2 = z\bar{z} \\ |z|^2 = a^2 + b^2 \end{cases} \quad \leftarrow \text{important } \triangle$$



$$|z| = 1, \quad z\bar{z} = 1, \quad \frac{1}{z} = \bar{z}$$

$$2) \quad \frac{1}{1-2i} + \frac{1}{1+2i} \Rightarrow \frac{1}{2} + \frac{1}{2} = \frac{1+1}{1+2^2} = \frac{2+\bar{2}}{22^2} = \frac{2}{1+4} = \frac{2}{5}$$

$$\frac{1}{z} + \frac{1}{\bar{z}} = \frac{z+\bar{z}}{z\bar{z}} = \frac{2a}{a^2+b^2}$$

$$3) \quad \frac{2+i}{3-2i} \Rightarrow \frac{2+i(3+2i)}{9+4} = \frac{6-2+3i+4i}{13} = \frac{4+7i}{13} = \frac{4}{13} + \frac{7}{13}i$$

$$4) \left(\frac{1+i}{2-i} \right)^2 = \frac{1+i \times 1-i}{2-i \times 2-i} = \frac{1+i+i-1}{4+1-4i} = \frac{2i}{5}$$

$$5) \frac{2+5i(1+i)}{1-i(1+i)} + \frac{2-5i(1-i)}{(1+i)(1-i)} = \frac{2-5+7i}{2} + \frac{2+5-7i}{2}$$

$$= \frac{-3+7i+7-7i}{2} = \frac{4}{2} = 2$$

↗

$$\frac{2+5i}{1-i} = \overline{\left(\frac{2-5i}{1+i} \right)} \text{ or } \frac{2}{2'} + \overline{\left(\frac{2}{2'} \right)} = 2 \operatorname{Re} \left[\frac{2}{2'} \right]$$

$$\frac{2+5i(1+i)}{(1-i)(1+i)} \Rightarrow \frac{-3+7i}{2} \text{ et } \operatorname{Re} \left[\frac{2+5i}{1-i} \right] = 2 \times -\frac{3}{2}$$

$$\text{Donc } 2 \operatorname{Re} \left[\frac{2+5i}{1-i} \right] = 2 \times -\frac{3}{2} = \boxed{-3}$$

$$6) \left(\frac{1+i}{2-i} \right)^2 + \frac{3+6i}{3-4i} = \frac{2i}{5} + \frac{3+6i}{3-4i} = \frac{2i}{5} + \frac{9-24+12i+18i}{25}$$

$$= \frac{-15+30i}{25} = \frac{-15}{25} + \frac{30}{25}i + \frac{2i}{5} = -\frac{3}{5} + \frac{6}{5} + \frac{2i}{5} = \frac{3}{5} + \frac{2}{5}i$$

Exercises Trigonometry:

$$z = a + ib, \bar{z} = a - ib$$

$$z + \bar{z} = 2a, z - \bar{z} = 2ib$$

⋮

Trivial...

Question 4:

$$r = \sqrt{\frac{3^2}{2} + 0} = \sqrt{\frac{9}{2}}$$

$$1) -3\sqrt{2} + 0i, \quad a = -3\sqrt{2}, \quad b = 0, \quad r = 3\sqrt{2}$$
$$3\sqrt{2} (\cos \pi + i \sin \pi)$$

$$4) 3 + 3i \Rightarrow 3\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$$
$$3\sqrt{2} e^{i\frac{\pi}{4}}$$

$$5) 2 e^{i\frac{\pi}{6}}$$