INDEX

PRACTICAL – 1

1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\rightarrow \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$

$\lim_{x \to a} \frac{(a+2x-3x)\ (\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)\ (\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$

$\frac{1}{3} \times \frac{\cancel{4}^{2}\sqrt{a}}{\cancel{2}_{1}\sqrt{3a}}$

$= \frac{2}{3\sqrt{3}}$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\to \lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right]$

$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y \to 0} \frac{\cancel{y}}{\cancel{y}\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$

$= \frac{1}{\sqrt{a+0}(\sqrt{a+0}+\sqrt{a})}$

$= \frac{1}{\sqrt{a}(\sqrt{a}+\sqrt{a})}$

$= \frac{1}{\sqrt{a}(2\sqrt{a})} = \frac{1}{2a}$

3] $\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$

By substituting $x - \frac{\pi}{6} = h$

$x = h + \pi/6$

where $h \to 0$

$$\therefore \lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right) - \sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi - 6\left(h+\frac{\pi}{6}\right)}$$

Using
$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \cdot \sin \pi/6 - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \cdot \sin\frac{\pi}{6}}{\pi - \cancel{6}\left(\frac{6h+\pi}{\cancel{6}}\right)}$$

$\cos\frac{\pi}{6} = \cos 30^\circ$
$= \sqrt{3}/2$
$\sin \pi/6 = \sin 30^\circ$
$= \frac{1}{2}$

$$\lim_{h\to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \cdot \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h\to 0} \frac{\cancel{\cos\frac{\sqrt{3}}{2}h} - \sin\frac{h}{2} - \sin 3\frac{h}{2} - \cancel{\cos\frac{\sqrt{3}}{2}h}}{-6h}$$

$$\lim_{h\to 0} \frac{\cancel{-}\sin\frac{4h}{2}}{\cancel{-}6h}$$

$$\lim_{h\to 0} \frac{\sin \cancel{4}h}{3\cancel{12}h}$$

$$\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h} = \frac{1}{3}\times 1 = \frac{1}{3}.$$

4] $\lim_{x\to\infty}\left[\dfrac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing numerator & denominator both.

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\,(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\ \frac{\overset{4}{\cancel{8}}(\sqrt{x^2+3}+\sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit we get,

$= 4$

5] $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$

at $x = \pi/2$

$$\cancel{f(\pi/2) =} \frac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}} \quad \therefore f(\pi/2) = 0$$

f at $x = \pi/2$ define.

ii) $\lim_{x \to \pi/2^+} f(x) = \lim_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$

By Substituting Method

$x - \pi/2 = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$$ using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \cdot \sin \pi/2}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h \to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2}$$

b) $\lim_{x \to \pi/2^-} f(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ Using $\sin 2x = 2\sin x \cos x$

$\lim_{x \to \pi/2^-} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x \to \pi/2^-} \frac{2\,\cancel{\sin x}\cos x}{\sqrt{2}\,\cancel{\sin x}}$

$\lim_{x \to \pi/2^-} \frac{2\cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

$\therefore$ L.H.L = R.H.L.

$\therefore$ f is not continuous at $x = \pi/2$.

3] ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $6 \le x < 9$

at $x = 3$ & $x = 6$

at $x = 3$

$f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}}$

L·H·L = R·H·L

f is continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{\cancel{27}\,3}{\cancel{9}} = 3$

2] $\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$

$\lim_{x\to 6^+} (x-3) = 6-3 = 3$

$\lim_{x\to 6^-} (x+3) = 3+6 = 9$

$\therefore$ L·H·L $\neq$ R·H·L

function is not continuous.

6] i) $f(x) = \frac{1-\cos 4x}{x^2} \quad x<0$
$= k \quad x=0$ } at $x=0$

$\rightarrow$ f is continuous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} 1 - \frac{\cos 4x}{x^2} = k$$

$$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$\therefore k = 8$$

Tij $f(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$ } at $x = 0$

$= k$ $x = 0$

$\rightarrow$ $f(x) = (\sec^2 x)^{\cot^2 x}$

Using

$$\sec^2 x - \tan^2 x = 1$$

$$\therefore \sec^2 x = 1 + \tan^2 x$$

$$\& \cot^2 x = \frac{1}{\tan^2 x}$$

$$\therefore \lim_{x \to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{1/\tan^2 x}$$

We know that

$$\lim_{x \to 0} (1 + px)^{1/px} = e$$

$$\therefore e$$

$$\therefore k = e$$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$ $\Bigg\}$ at $x = \pi/3$

$= k \quad\quad x = \pi/3$

$x - \pi/3 = h$

$x = h + \pi/3$

where $h \to 0$

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

Using

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\lim_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$

$\lim_{h \to 0} \dfrac{\dfrac{\sqrt{3}\left(1 - \tan \pi/3 \cdot \tan h\right) - \left(\tan \pi/3 + \tan h\right)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

Using

$\tan \pi/3 = \tan 60 = \sqrt{3}$

$\lim_{h \to 0} \dfrac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

$$\lim_{h\to 0} \frac{\frac{(\sqrt{3} - 3\tan h) - (\sqrt{3} + \tan h)}{1-\sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\frac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1-\sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\cancel{-}4\tan h}{\cancel{-}3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)} \qquad \frac{\tan h}{h} = 1$$

$$= \frac{4}{3} \frac{1}{(1-\sqrt{3}(0))}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = 4/3$$

7) i) $f(x) = \frac{1-\cos 3x}{x\tan x}$ $x \neq 0$

$= a$ $x = 0$ at $x = 0$

$$f(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} \frac{2\sin^2 3/2\, x}{x\tan x}$$

$$\lim_{x\to 0} \frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2}\times x^2}{\dfrac{x\cdot\tan x}{x^2}\times x^2}$$

$$= 2\lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1} = 2\times\frac{9}{4} = $$

$\lim_{x\to 0} f(x) = \frac{9}{2}$ $\quad 9 = f(0)$

$\therefore$ f is not continuous at $x = 0$.

Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

7] ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases}$ at $x = 0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} \frac{3\cdot e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$= 3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$

$= 3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is continuous at $x = 0$

8. $f(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x = 0$

∴ given,

f is continuous at $x = 0$

$\lim_{x\to 0} f(x) = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x \cancel{-1} \cancel{+1}}{x^2} = f(0)$

$= \lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \lim_{x\to 0} \frac{(e^{x^2}-1) + (1-\cos x)}{x^2}$

$= \lim_{x\to 0} \frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} \frac{1-\cos x}{x^2}$

$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 on Num^r & Denom^r.

$= 1 + \cancel{2} \times \frac{1}{\cancel{4}\,2} = \frac{3}{2} = f(0)$

$f(x) = \dfrac{\sqrt{3} - \sqrt{1+\sin x}}{\cos^2 x}$, $x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$\lim_{x\to\pi/2} \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x\to\pi/2} \dfrac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \dfrac{1+\sin x}{1-\sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \dfrac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \dfrac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2} + \sqrt{2})}$

$= \dfrac{1}{2(2\sqrt{2})} = \dfrac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}$ //

PRACTICAL - 2

* Derivative :-

Q.1. Show that the following func^n defined from to R are differentiable

i) cot x

→ $f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$\lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$

$= \lim_{x \to 0} \frac{\tan a - \tan x}{1(-a) \tan x \tan a}$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$f'(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(\alpha + h - \alpha) \tan(a+h) \tan a}$

$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$

formula :- $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \ldots$

$$= \lim_{h\to 0} \frac{\tan(\cancel{a} - \cancel{a} - h) - (1 + \tan a \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= -\frac{1}{\cancel{\cos^2 a}} \times \frac{\cancel{\cos^2 a}}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$\therefore f'(a) = -\operatorname{cosec}^2 a$

$\therefore f$ is differentiable $\forall a \in R$

ii) $f(x) = \operatorname{cosec} x$

$$f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f'(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \,(\sin(a+h))}$$

Formula :-

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$$

$$= -\frac{1}{2} \times 2\cos\left(\frac{2a+0}{2}\right)$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f'(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula :- $-2\sin\left(\frac{C+D}{2}\right)\cdot\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{h\times\cos a\,(\cos a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a\cos(a+h)\times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= \frac{-1}{2}\times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a\,(\cos(a+0))}$$

$$= \frac{-1}{2}\times \cancel{-2}\frac{\sin a}{\cos a+\cos a}$$

$$= \tan a \sec a$$

Q. If $f(x) = 4x+1$, $x \le 2$ at $x=2$, then find func^n
$= x^2+5$ $\quad x>0$
is differentiable or not.

$\rightarrow$ L.H.D =

$$f'(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$4(x-2) = 4$

$f'(2^-) = 4$

R.H.D =

$$f'(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 2+2 = 4$$

$f'(2^+) = 4$

$\therefore$ R.H.D = L.H.D

f is differentiable at $x = 2$

Q.3. If $f(x) = 4x+7$, $x < 3$

$= x^2+3x+1$, $x \geq 3$ at $x = 3$ then find f is differentiable or not?

$\rightarrow$ R.H.D :-

$$f(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3-3+1)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{x-3}}$

$= 3 + 6 = 9$

$f'(3^+) = 9$

L.H.D $= f'(3^-)$

$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x-3}$

$= \lim_{x \to 3^-} \frac{4x - 12}{x-3}$

$= \lim_{x \to 3} \frac{4\cancel{(x-3)}}{\cancel{x-3}}$

$f'(3^+) = 4$

R.H.D $\neq$ L.H.D

f is not differentiable at $x = 3$

Q.4. If $f(x) = 8x - 5$, $x \leq 2$

$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$ then find f is differentiable or not

→ $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

R.H.D $=$ $f'(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2 = 8$$

$$f'(2^+) = 8$$

$$L.H.D = f'(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 8$$

$$f'(2^-) = 8$$

$$L.H.D = R.H.D$$

$\therefore f$ is differentiable at $x = 3$

PRACTICAL - 3

Topic :- Application of Derivative

1] Find the intervals in which function is increasing or decreasing.

i) $f(x) = x^3 - 5x - 11$

ii) $f(x) = x^2 - 4x$

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

iv] $f(x) = x^3 - 27x + 5$

v] $f(x) = 69 - 24x - 9x^2 + 2x^3$

2] Find the intervals in which func^n is concave upwards.

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

iii) $y = x^3 - 27x + 5$

iv] $y = 69 - 24x - 9x^2 + 2x^3$

v] $y = 2x^3 + x^2 - 20x + 4$

→ Q.1.

i) $f(x) = x^3 - 5x - 11$

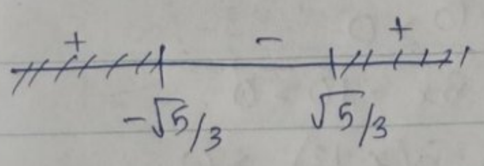
$\therefore f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$



$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3,$

and f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2] $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3] $f'(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 2(3x^2 + x - 10) > 0$

$\therefore 3x^2 + x - 10 > 0$

$\therefore 3x^2 + 6x - 5x - 10 > 0$

$\therefore 3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

$+ \quad -$

-2

$\therefore 6x^2 + 2x - 20$ iff $f'(x) < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$x \in (-2, 5/3)$

4] $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x-3)(x+3) > 0$

+ − +

−3 3

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x-3)(x+3) < 0$

+ − +

−3 3

$\therefore x \in (-3, 3)$

5] $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$\therefore (x^2 - 3x - 4) > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore x(x-4) + 1(x-4) > 0$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

& f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 3x - 4 < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$\therefore x(x-4) + 1(x-4) < 0$

$\therefore (x+1)(x-4) < 0$

$\therefore x \in (-1, 4)$

Q.2.

1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

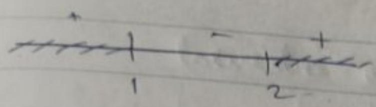
$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$\therefore (x-2)(x-1) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

3] $y = x^3 - 27x + 6$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$x > 0$

$x \in (0, \infty)$

4] $y = 64 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$ $\quad \therefore x > 3/2$ $\quad \therefore x \in (3/2, \infty)$

5] $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x > -1/6$

$\therefore f''(x) > 0$

~~$\therefore$ There exist no interval~~

$x \in (-1/6, \infty)$.

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PRACTICAL - 4

Topic :- Application of derivatives & Newton's Method.

Q.1. Find maximum & minimum values of the following:-

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1 \quad [-1/2, 4]$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

Q.2. Find the root of the following equation by Newton's (Take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5 \quad (\text{take } x_0 = 0)$

ii) $f(x) = x^3 - 4x - 9 \quad \text{in } [2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17 \quad \text{in } [1, 2]$

Q.1. i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now, consider, $f'(x) = 0$

$\therefore \quad 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 32/2$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4 \qquad = 8 > 0$

$f''(2) = 2 + 96/24 \qquad \therefore f$ has minimum value at $x = 2$

$= 2 + 96/16$

$= 2 + 6$

$= 4 + 16/4$
$= 4 + 4$
$= 8$

$\therefore f''(-2) = 2 + 96/(-2)^4$
$= 2 + 96/16$
$= 2 + 6$
$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$.
$\therefore$ Function reaches minimum value at $x = 2$ & $x = -2$

ii) $f(x) = 3 - 5x^3 + 3x^5$
$\rightarrow f'(x) = -15x^2 + 15x^4$
Consider, $f'(x) = 0$
$\therefore -15x^2 + 15x^4 = 0$
$\therefore 15x^4 = 15x^2$
$x^2 = 1$
$\therefore x = \pm 1$
$\therefore f''(x) = -30x + 60x^3$
$f(1) = -30 + 60$
$= 30 > 0$ $\therefore$ f has minimum value at $x = 1$.
$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$
$= 6 - 5$
$= 1$
$\therefore f''(-1) = -30(-1) + 60(-1)^3$
$= 30 - 60$
$= -30 < 0$ $\therefore$ f has maximum value at $x = -1$
$\therefore f(1) = -30 < 0$
$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$
$= 3 + 5 - 3$

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$\therefore$ f has maximum value 5 at $x = -1$ & has the minimum value 1 at $x = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$\therefore f''(0) = -6$

$= -6 < 0$ $\therefore$ f has minimum value at $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has maximum value 1 at $x = 0$ & f has minimum v

-3 at $x = 2$.

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

Consider, $f'(x) = 0$

$\therefore 6(x^2 - x - 2) = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$\therefore (x-2)(x+1) = 0$

$\therefore x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore f$ has minimum value at $x = 2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$\therefore f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

$\therefore f$ has maximum value 8 at $x = -1$ & f has minimum value $= -19$ at $x = 2$.

Q.2.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0) / f'(x_0)$

$\therefore x_1 = 0 + 9.5/55$

$\therefore x_1 = \underline{0.1727}$

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -\underline{0.0829}$

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 5.0895 - 1.0362 - 55$

$= -\underline{55.9467}$

$\therefore x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1727 - 0.0829/55.9467$

$= \underline{0.1712}$

$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= \underline{0.0011}$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 6.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712 + 0.0011/55.9393$

$= \underline{0.1712}$

$\therefore$ The root of equation is $\underline{0.1712}$

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

... initial approximation

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$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1) / f'(x_1)$

$= 2.7392 - 0.596 / 18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= (9.7158 - 10.866 - 9) = -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901 / 17.8943$

$= 2.7015 + 0.0050$

$= 2.7065$

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1,2]$

$f'(x) = 3x^2 - 3.6x - 10$

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$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - 10 + 17$
$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17 = -2.2$

Let $x_0 = 2$ be initial approximation By Newton's Method

$x_n + 1 = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_2) / f'(x_2)$
$= 2 - 2.2/5.2$
$= 2 - 0.4230$
$= \underline{1.577}$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$.
$= 3.9219 - 4.4764 - 15.77 + 17$
$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -\underline{\underline{8.2164}}$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$
$= 1.577 + \cancel{0.6755} / 8.2/64$
$= 1.577 + 0.0822$
$= \underline{\underline{1.6592}}$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.6592 + 0.0204/7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.6618 + \frac{0.0004}{7.6977}$

$= 1.6618$

Practical - 5

Topic :- Integration

Q.1. Solve the following integration

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\, dx$

iii) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

v) $\int t + \sin(2t^4)\, dt$

vi) $\int \sqrt{x}\,(x^2-1)\, dx$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\, dx$

ix) $\int e^{\cos^2 x} \sin 2x\, dx$

x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right) dx$

i) $\int \frac{1}{x^2+2x-3} dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}} dx$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}} dx$

Substituting $x+1=t$

$\therefore dx = \frac{1}{t} dt$

where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}} dt$

using.

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln(|x+\sqrt{x^2+a^2}|)$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln[|x+1+\sqrt{x^2+2x-3}|)+c$

2] $\int (4e^{3x}+1) dx$

$= \int 4e^{3x} dx + \int 1 dx$

$= 4\int e^{3x} dx + \int 1 dx$ # $\int e^{ax} dx = \frac{1}{a} x e^{ax}$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

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3] $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\, dx$ # $\sqrt[n]{a^m} = a^{m/n}$

$= \int 2x^2\, dx - \int 3\sin x\, dx + \int 5x^{1/2}\, dx$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{-2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4] $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$

$= \int \frac{x^3 + 3x + 4}{x^{1/2}}\, dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\, dx$

$= \int x^{5/2}\, dx + \int 3x^{1/2}\, dx + \int \frac{4}{x^{1/2}}\, dx$

$= \frac{x^{5/2} + 1}{5/2 + 1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5] $\int t^7 \times \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3} du$$

$$= \int t^4 \times \sin(2t^4) \times \frac{1}{2\times 4} du$$

$$= \int t^4 \sin(2t^4) \times \frac{1}{8} du = \frac{t^4 \times \sin(2t^4)}{8} du$$

Substitute t^4 with $\frac{u}{2}$

$$= \int \frac{u/2 \times \sin(u)}{8} du$$

$$= \int \frac{u \times \sin(u)}{16} du$$

$$= \frac{1}{16} \int u \times \sin(u)\, du$$

$$\int u\, dv = uv - \int v\, du$$

where $u = u$

$dv = \sin u \times du$

$du = 1 du \quad v = -\cos(u)$

$$= \frac{1}{16} (u \times (-\cos(u)) - \int -\cos(u)\, du$$

$$= \frac{1}{16} \times (u \times (-\cos u)) + \int \cos(u)\, du$$

$\int \cos x\, dx = \sin(x)$

$$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$$

Return the substitution $u = 2t^4$

$$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$$

$$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$$

vii) $\int \sqrt{x}\,(x^2-1)\,dx$

$\rightarrow \int \sqrt{x}\, x^2 - \sqrt{x}\, dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\, dx$

$= \int x^{3/2} - x^{1/2}\, dx$

$= \int x^{3/2} dx - \int x^{1/2} dx$

$= I_1 = \frac{x^{3/2+1}}{3/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$

$= I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\, dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\, dx$

put $t = \sin(x)$

$dt = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos(x)}\, dt$

$= \frac{1}{\sin x^{3/2}}\, dt$

$= \frac{1}{t^{2/3}}\, dt$

$I = \int \frac{1}{t^{2/3}}\, dt = \frac{-1}{(2/3-1)\,t^{2/3}-1} = \frac{-1}{(2/3-1)(t^{2/3}-1)}$

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$$\frac{\quad}{-\frac{1}{3}t^{2/3-1}} \qquad \frac{1}{3}t^{-1/3} \qquad \frac{1}{3}$$

$= 3\sqrt[3]{t}$

~~Return~~ substituting $t = \sin(x)$

$= 3\sqrt[3]{\sin(x)} + c$

x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2 6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2 6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt \qquad = \int \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt \qquad \because \int \frac{1}{x}\,dx = \ln|x|$

$= \frac{1}{3} \times \ln|t| + c$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + c$

AK
04/01/2020

PRACTICAL NO.- 6

Topic :- Application of Integration & Numerical Integration.

Q.1. Find the length of following curve.

1] $x = 1 - \sin t$, $y = 1 - \cos t$ $t \in [0, 2\pi]$
2] $y = \sqrt{y - x^2}$, $x \in [-2, 2]$
3] $y = x^{3/2}$, $[0, \pi]$
4] $x = 3\sin t$, $y = 3\cos t$ $t \in [0, 2\pi]$
5] $x = \frac{1}{6} y^3 + \frac{1}{2y}$ $y \in [1, 2]$

Q.2. Using simpson's Rule solve the following :-

① $\int_0^2 e^{x^2} dx$ with $n = 4$

② $\int_0^A x^2 dx$ with $n = 4$

③ $\int_0^{\pi/4} \sqrt{\sin x}\, dx$ with $n = 6$

5

(1) $x = 1 - \sin t \quad y = 1 - \cos t \quad , t \in [0, 2\pi]$

$\rightarrow \quad t = \int_a^l \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} dt$

$\frac{dy}{dt} = \sin t \quad , \quad \frac{dx}{dt} = 1 - \cos t$

$L = \int_0^{2\pi} \sqrt{(\sin t)^2 + (1-\cos t)^2} dt$

$= \int_0^{2\pi} \sqrt{\sin^2 t + 1 + \cos^2 t - 2\cos t} dt$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t} dt$

$= \int_0^{2\pi} \sqrt{1 - \cos t} dt$

$= \sqrt{2} \int_0^{2\pi} \sqrt{2\sin^2\left(\frac{t}{2}\right)} dt$

$= \sqrt{2} \times \sqrt{2}$

$= 2 \times 2 \left[-\cos\left(\frac{t}{2}\right)\right]_0^{2\pi}$

$= 4[-1-1]$

$= 8$

(2) $\sqrt{4 - x^2} \qquad x \in [-2, 2]$

$C = \int_a^l \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$

$\frac{dy}{dx} = \frac{1(-\cancel{2}x)}{\cancel{2}\sqrt{4-x}}$

$\sqrt{4-x}$

$$\therefore L = \int_{-2}^{2} \sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$$

$$= \int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$$

$$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\pi/2 - (-\pi/2)\right]$$

$$= 2\pi$$

(3) $y = x^{3/2} \quad , x \in [0, 4]$

$$L = \int_{u}^{l} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x}$$

$$L = \int_{0}^{4} \sqrt{1+\frac{9}{4}x}\, dx$$

$$= \frac{1}{2}\int_{0}^{4} \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_{0}^{4}$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} \times \frac{1}{9}\left[(4+9x)^{3/2}\right]_{0}^{4}$$

$$= \frac{1}{27}\left[40^{3/2} - 8\right]$$

(4) $x = 3\sin t \quad y = 3\cos t \qquad t \in [0, 2\pi]$

$$L = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx^2}{dt}\right)}\, dt$$

$$\frac{dy}{dt} = -3\sin t \qquad \frac{dx}{dt} = 3\cos t$$

$$L = \int_0^{2\pi} \sqrt{-(3\sin^2 t) + (3\cos^2 t)}\, dt$$

$$L = 3\int_0^{2\pi} \sqrt{\sin^2 t + \cos^2 t}$$

$$= 3\int_0^{2\pi} \sqrt{1}\, dt$$

$$= 3\,[t]_0^{2\pi}$$

$$= 6\pi$$

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y} \qquad y \in [1, 2]$

$$L = \int_u^l \sqrt{1 + \frac{dx}{dy}}\, dy$$

$$\frac{dx}{dy} = \frac{3'y^2}{2} - \frac{1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{1}{2}y^2 - \frac{1}{2y}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^2 - 1}{2y^2}\right)^2}\, dy$$

$$= \int_1^2 \frac{\sqrt{(y^4-1)^2 + 4\times y^4 \times 1}}{4y^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 y\frac{y^4+1}{2y^2}\, dy$$

$$\int_1^2 \frac{y^2}{2}\, dy + \frac{1}{2}\int_1^2 \frac{1}{y^2}\, dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3}\right]_1^2 + \frac{1}{2}\left[\frac{1}{y}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{5}{3} - \frac{1}{3}\right) + \frac{1}{2}\left[\frac{1}{2} - 1\right]$$

$$= \frac{7}{6} - \frac{1}{4}$$

$$= \frac{14-3}{12}$$

$$= \frac{11}{12}$$

Q.2. i) $\int_0^2 e^{x^2} dx$ within $n = 4$

$a = 0, \quad b = 2, \quad n = 4$

$h = \frac{b-a}{n} = \frac{2-0}{4} = \frac{1}{2}$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981

By Simpson's Rule:

$$\int_0^2 e^{x^2} dx = \frac{0.5}{3}\left[(1+54.5981) + 4(1.2840+9.48 \ldots (2.7182)\right]$$

$= 17.3535$

ii) $\int_0^4 x^2 dx$ with $n = 4$

$a = 0, \quad b = 4, \quad n = 4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
y	0	1	4	9	16

By Simpson's rule

$$\int_0^4 \cancel{x^2} dx = \frac{1}{3}\left[(0+16) + 4(1+9) + 2(4)\right]$$

$= \frac{1}{3}(16+40+8)$

$= \frac{1}{3} \times 64$

$= \frac{64}{3}$

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iii) $\int_0^{\pi/3} \sin x \, dx$ with $n = 6$

$a = 0$, $b = \pi/3$, $n = 6$

$h = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

$x = 0 \quad \pi/18 \quad 3\pi/18 \quad 3\pi/18 \quad 4\pi/18 \quad 5\pi/18 \quad 6\pi/18$

$y = 0 \quad 0.9167 \quad 0.9848 \quad 0.7071 \quad 0.8017 \quad 0.8752 \quad 0.9306$

By simpson's rule :-

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{\pi}{54}\left[(0.09306) + 4(0.4167 + 0.7071 + 0.8752) + 2(0.9848 + 0.5017)\right]$$

$$= \pi/54 \times 11.6996$$

$$= 0.6806$$

$$\int_0^{\pi/3} \sqrt{\sin x} = 0.6806$$

PRACTICAL - 7

* Differential equation.

Q.1. Solve the following differential equation.

i) $x\frac{dy}{dx} + y = e^x$

ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

iv) $\frac{x\,dy}{dx} + 3y = \frac{\sin x}{x^2}$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

vi) $\sec^2 x \tan y + \sec^2 y \tan x \cdot dy = 0$

vii) $\frac{dy}{dx} = \sin^2(x - y + 1)$

viii) $\frac{dy}{dx} = \frac{2y + 3y - 1}{6x + 9y + 6}$

i) $x \frac{dy}{dx} + y = e^x$

$\therefore \frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

comparing with $\frac{dy}{dx} + P(x)\, y = Q(x)$

$P(x) = \frac{1}{x}$; $Q(x) = \frac{e^x}{x}$

$I.F = e^{\int \frac{1}{x} dx}$

$= e^{\log x}$

$I.f. = x$

$\therefore y\,(I.F) = \int Q(x)\,(I.f)\,dx$

$y\,(x) = \int \frac{e^x}{x} \cdot x\,dx$

$y\,(x) = \int \frac{e^x}{x} \cdot x\,dx$

$y(x) = \int e^x dx$

$xy = e^x + c$

ii) $e^x \frac{dy}{dx} + 2e^x y = 1$

$e^x \left(\frac{dy}{dx} + 2y\right) = 1$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

Comparing with $\frac{dy}{dx} + P(x)\, y = Q(x)$

$\therefore P(x) = 2 \qquad Q(x) = \frac{1}{e^x}$

$$I.f = e^{\int 2\,dx}$$
$$= e^{2x}$$
$$y(I.f) = \int Q(x)\,(I.F)\,dx$$
$$ye^{2x} = \int \frac{1}{e^x} e^{2x}\,dx$$
$$ye^{2x} = \int e^{-x} \cdot e^{2x}\,dx$$
$$ye^{2x} = \int e^x\,dx$$
$$ye^{2x} = e^x + c$$

iii) $\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$\frac{x\,dy}{dx} + 2y = \frac{\cos x}{x}$$
$$\frac{dy}{dx} + \left(\frac{2}{x}\right)y = \frac{\cos x}{x^2}$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$$\therefore P(x) = \frac{2}{x} \quad ; \quad Q(x) = \frac{\cos x}{x}$$
$$If = e^{\int \frac{2}{x}\,dx}$$
$$= e^{2\log x}$$
$$= e^{\log x^2}$$
$$I.f = x^2$$
$$\therefore y(I.f) = \int Q(x)\,(I.f)\,dx$$
$$y(x^2) = \int \frac{\cos x}{\cancel{x^2}} (\cancel{x^2})\,dx$$

$x^2 y = \ldots$

$$I.f = e^{\int 2\,dx}$$
$$= e^{2x}$$
$$y(I.f) = \int Q(x)\,(I.F)\,dx$$
$$ye^{2x} = \int \frac{1}{e^x} e^{2x}\,dx$$
$$ye^{2x} = \int e^{-x} \cdot e^{2x}\,dx$$
$$ye^{2x} = \int e^{x}\,dx$$
$$ye^{2x} = e^{x} + c$$

iii) $\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$\frac{x\,dy}{dx} + 2y = \frac{\cos x}{x}$$
$$\frac{dy}{dx} + \left(\frac{2}{x}\right) y = \frac{\cos x}{x^2}$$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$$\therefore P(x) = \frac{2}{x} \quad ; \quad Q(x) = \frac{\cos x}{x}$$
$$I.f = e^{\int \frac{2}{x}\,dx}$$
$$= e^{2\log x}$$
$$= e^{\log x^2}$$
$$I.f = x^2$$
$$\therefore y(I.f) = \int Q(x)\,(I.f)\,dx$$
$$y(x^2) = \int \frac{\cos x}{\cancel{x^2}} (\cancel{x^2})\,dx$$

$x^2 y = \ldots$

iv) $\frac{dx}{}$ ... $\frac{}{x^2}$.

$\rightarrow \frac{dy}{dx} + \left(\frac{3}{x}\right)y = \frac{\sin x}{x^3}$.

comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$P(x) = 3x^{-1}$ $\qquad Q(x) = \frac{\sin x}{x^3}$.

$I.f = e^{\int \frac{3}{x} dx}$

$= e^{3\log x}$

$= e^{\log x^3}$

$= x^3$

$\therefore y\,(I.f) = \int Q(x)\,(I f)\,dx$

$y\,(x^3) = \int \frac{\sin x}{\cancel{x^3}} \,(\cancel{x^3})\, dx$

$x^3 y = -\cos x + c$

1] $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\rightarrow e^{2x}\left(\frac{dy}{dx} + 2y\right) = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

Comparing with $\frac{dy}{dx} + P(x)y = Q(x)$

$\therefore P(x) = 2 \qquad Q(x) = \frac{2x}{e^{2x}}$

$\therefore I.f. = e^{\int 2\, dx}$

$\therefore y(I.f) = \int Q(x) \quad (I.f)\,dx$

$y(e^{2x}) = \int \frac{2x}{e^{2x}} \cdot (e^{2x})\,dx$

$ye^{2x} = \cancel{2}\frac{x^2}{\cancel{2}} + c$

$ye^{2x} = x^2 + c$

vi) $\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\sec^2 x \tan y \, dx = -\sec^2 y \tan x \, dy$.

$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 y}{\tan y} dy$

$\therefore \int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

$= \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x| + \log|\tan y| \pm c$

$\log|\tan x . \tan y| = c$

$\Rightarrow \tan x . \tan y = e^c$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

Put $x-y+1 = v$

$1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = 1 - \frac{dv}{dx}$

$\therefore 1 - \frac{dv}{dx} = \sin^2 v$

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$$1-\sin^2 v = \frac{dv}{dx}$$

$$1x = \frac{dv}{1-\sin^2 v}$$

$$\int dx = \int \sec^2 \cdot v \, dv$$

$$x = \tan v + c$$

But $v = x + y - 1$

$$\therefore x = \tan(x+y-1) + c$$

viii] $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

$$\frac{dy}{dx} = \frac{2x+3y-1}{3(2x+3y+2)}$$

Put $2x + 3y = V$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{v-1}{3(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$\frac{dv}{dx} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = \frac{3v+3}{v+2}$$

$$\frac{v+2\,dv}{3(v+1)} = dx$$

$$\frac{1}{3}\int \frac{(v+1+1)}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\int 1 + \frac{1}{v+1}\,dv = \int dx$$

$$\frac{1}{3}\left(v + \log(v+1)\right) = x + c$$

Put $v = 2x+3y$

$\therefore 2x+3y+\log|2x+3y+1| = 3x+c$

$\therefore 3y - x - \log|2x+3y+1| + c$

11/01/2020

PRACTICAL - 8

* Using Euler's Method find the following :-

1) $\frac{dy}{dx} = y + e^x - 2$, $y(0) = 2$, $h = 0.5$ Find $y(2)$

$f(x) = y + e^x - 2$, $x_0 = 0$

$y(0) = 2$, $h = 0.5$

$y(0.2) = ?$

n	x_n	$y(0)$	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.14787	3.5743
2	1	3.5743	4.2926	5.7205
3	1.5	5.7205	8.2021	9.8215

$y(2) = \underline{\underline{9.8215}}$

② $\frac{dy}{dx} = 1+y^2$, $y(0) = 0$, $h = 0.2$ find $y(1)$

→ $x_0 = 0$, $y_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0.	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$, $y(0) = 1$, $h = 0.2$ Find $y(1)$

$y(0) = 1$, $x_0 = 0$, $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2106
3	0.6	1.2106	0.7040	1.3513
4	0.8	1.3513	0.7694	1.5051
5	1	1.5051		

$y(1) = 1.5051$

(4) $\frac{dy}{dx} = 3x^2+1$, $y(1) = 2$. find $y(2)$

For $h = 0.5$ & $h = 0.25$

→ $h = 0.5$ & $h = 0.25$ $y_0 = 2$ $x_0 = 1$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	2	4
1	1.5	1.5	4	7.875
2	2	2	7.875	

$y(2) = 7.875$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.8875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6482	299.9966
4	2	299.9966		

$y(1) = 299.9966$

1) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1) = 1$ find $y(1.2)$ with $h = 0.2$

$y(0) = 1$ $x(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1) = 3.6$

11/01/2020

PRACTICAL – 9

Aim :- Limits & partial order derivatives

i) $\lim_{(x,y) \to (-4,7)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

$\lim_{(x,y) \to (-4,7)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4, -1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-4) + (-4)^2 - 1}{-4(7) + 5}$

$= \frac{-61}{9}$

ii) $\lim_{(x,y) \to (-2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

$\lim_{(x,y) \to (-2,0)} \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

At $(-2, 0)$ Denominator $\neq 0$

$\therefore$ By applying limit

$= \frac{(0+1)(4 + 0 - 8)}{2}$

$= \frac{-4}{2}$

$= -2$

iii) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

At $(1,1,1)$ Denominator $\neq 0$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)}{x^2 y}$

on applying limit

$= \frac{1+1(1)}{1^2}$

$= \underline{\underline{1}}$

Q.2 i) $f(x,y) = xye^{x^2} + y^2$

→ $\therefore f(x) = \frac{\delta f}{\delta x}(x,y)$

$= \frac{\delta(xye^{x^2}+y^2)}{\delta x}$

$= ye^{x^2+y^2}(2x)$

$\therefore f_x = 2xye^{x^2+y^2}$

$f_y = \frac{\delta f(x,y)}{\delta y}$

$$= \frac{\delta (xye^{x^2+y^2})}{\delta x}$$

$$= xe^{x^2+y^2}(2y)$$

$$\therefore f_y = 2yxe^{x^2+y^2}$$

ii) $f(x,y) = e^x \cos y$

$\Rightarrow f_x = \frac{\partial x}{\partial x} f(x,y)$

$$= \frac{\partial (e^x \cos y)}{\delta x}$$

$$f_x = e^x \cos y$$

$$f_y = \frac{\delta f(x,y)}{\delta y}$$

$$= \frac{\delta (e^x \cos y)}{\delta y}$$

$$f_y = -e^x \sin y$$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f_x = \frac{\delta f(x,y)}{\delta x}$$

$$= \frac{\delta f(x^3 y^3 - 3x^2y + y^3 + 1)}{\delta x}$$

$$= 3x^2y^2 - 6xy$$

$$f_y = \frac{\delta f(x,y)}{\delta y}$$

$$= \frac{\delta f(x^3y^2 - 3x^2y + y^3 + 1)}{\delta y}$$

$$= 2x^3y - 3x^2 + 3y^2$$

Q.S. if $f(x,y) = \frac{2x}{1+y^2}$

$$\Rightarrow f_x = \frac{\delta f(x,y)}{\delta x}$$

$$= \frac{\delta f (2x)(1+y^2)}{\delta x}$$

$$= \frac{1+y^2 \frac{\delta(2x)}{\delta x} - 2x \frac{\delta(1+y^2)}{\delta x}}{(1+y^2)^2}$$

$$= \frac{2+2y^2}{(1+y^2)^2}$$

$$= 2/1+y^2$$

At $f(0,0)$

$$= \frac{2}{1+0} = 2$$

$$f_y = \frac{\delta f\,(2x)(1+y^2)}{\delta y}$$

$$= \frac{1+y^2 \frac{\delta\,(2x)}{\delta x} - 2x \frac{\delta}{\delta x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2} = \underline{\underline{0}}$$

4. i) $f(x,y) = \cancel{\dfrac{y^2 xy}{x^2}}$

$$f_x = \frac{x^2 \frac{\delta f(y^2 - xy)}{\delta x} - (y^2 - xy)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\delta}{\delta x}\left(\frac{-x^2y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\delta/\delta x\,(-x^2y - 2xy^2 + 2x^2y)\right) - (-x^2y - 2xy + 2x^2y)\,\delta/\delta x\,(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2}{x^6} \quad \text{------ (i)}$$

$$f_{yy} = \frac{\delta}{\delta y}\left(2y - \frac{x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad \text{------ (ii)}$$

$$f_{yx} = \frac{\delta}{\delta x}\left(2y - \frac{x}{x^2}\right)$$

$$= \frac{x^2\frac{\delta}{\delta x}(xy - x) - (2y - x)\frac{\delta}{\delta x}(x^2}{(x^2)^2}$$

$$= \frac{-x - 4xy - 2x^2}{x^4} \text{ ------ (iv)}$$

From (iii) & (iv)

$$f_{xy} = f_{yx}$$

(2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$\Rightarrow f_x = \frac{\delta}{\delta x}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x+1}$$

$$f_y = \frac{\delta}{\delta y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \frac{(x^2+1\frac{\delta(2x)}{\delta x} - 2x\left(\frac{\delta}{\delta x}(x^2+1)\right)}{(x^2+1)^2}$$

$$= 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \text{ ...... (i)}$$

$$f_{yy} = \frac{\delta}{\delta y}(6x^2y)$$

$$= 6x^2 \text{ ------- (ii)}$$

$$f_{xy} = \frac{\delta}{\delta y}\left(8x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy + 0$$

$$= 12xy \text{ ------- (iii)}$$

$$f_{yx} = \frac{\delta}{\delta x}(6x^2y)$$

$$= 12xy \quad \text{- - - - (ii)}$$

From (iii) & (iv)

$$f_{xy} = f_{yx}$$

(3) $f(x,y) = \sin(xy) + e^{x+y}$

$\rightarrow$ $f_x = y\cos(xy) + e^{x+y}(1)$

$$= y\cos(xy) + e^{x+y}$$

$$f_y = x\cos(xy) + e^{x+y}(1)$$

$$= x\cos(xy) + e^{x+y}$$

$$f_x = \frac{\delta}{\delta x}(y\cos(xy) + e^{x+y})$$

$$= -y\sin(xy)(y) + e^{1+x}$$

$$= -y^2\sin(xy) + e^{x+y} \quad \text{- - - - - - (i)}$$

$$f_{yy} = \frac{\delta}{\delta y}(x\cos(xy) + e^{x+y})$$

$$= \cancel{-x\sin(xy)(x)} + e^{x+y}(1)$$

$$= -x^2\sin(xy) + e^{x+y} \quad \text{- - - - (ii)}$$

$$f_{xy} = \frac{\delta}{\delta y}(y\cos(xy) + e^{x+y})$$

$$= -y^2\sin(xy) + \cos(xy) + e^{x+y} \quad \text{- - - - (iii)}$$

$$f_{yx} = \frac{\delta}{\delta x}(x\cos(xy) + e^{x+y})$$

$$= -x^2\sin(xy) + \cos(xy) + e^{x+y} \quad \text{------ (iv)}$$

From (iii) & (iv)

$$f_{xy} \neq f_{yx}$$

Q.5. i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$\rightarrow f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$$

$$= \frac{x}{\sqrt{x^2+y^2}}$$

$$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$$

$$= \frac{y}{\sqrt{x^2+y^2}}$$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$$l(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

(2) $f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$\rightarrow$ $f(\pi/2, 0) = 1 - \pi/2, 0 - 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

f_x at $(\pi/2, 0)$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \sin \pi/2$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-b)$$

$$= 1 - \pi/2 - x + \pi/2 + y$$

$$= 1 - x + y$$

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(3) $f(x,y) = \log x + \log y$ at $(1,1)$

$\rightarrow f(1,1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x}$

f_x at $(1,1) = \underline{\underline{1}}$

$f_y = 1/y$

f_y at $(1,1) = \underline{\underline{1}}$

$l(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= \underline{\underline{x + y - 2}}$

25/01/2020

PRACTICAL - 10

1. Find the directional derivative of the following function at given point & in the direction of given vector.

$f(x,y) = x + 2y - 3 \qquad a = (1,-1) \qquad u = 3i - j$

→ Here, $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$.

Unit vector along u is $\frac{u}{|\bar{u}|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)_{-3}$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)_{-3}$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right)_{-3}$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}}$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h\to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$$

$$D_u f(a) = \frac{1}{\sqrt{10}}$$

ii) $f(x) = y^2 - 4x + 1$ $a = (3,4)$ $u = i + 5j$ then $u = i + 5j$ is not a unit vector.

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

Unit vector along u is $\frac{u}{|\bar{u}|} = \frac{1}{\sqrt{26}}(1,5)$

$$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= f = \left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$f_{(x,y)}(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$\frac{h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)}{h}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

iii) $2x + 3y \quad a = (1,2) \ , \ u = (3i + 4j)$

Here $u = 3i + 4j$ is not a unit vector

$$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

Unit vector along u is $\frac{V}{|u|} = \frac{1}{5}(3,4)$

$$= \left(\frac{3}{5}, \frac{4}{5}\right)$$

$$f(a) = f(1,2) = 2(1) + 3(2) = 8$$

$$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$$

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$$

$$f(a+hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$$= \frac{18h}{5} + 8$$

$$D_u f(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + \cancel{8} - 8}{h}$$

$$= 18$$

Q.2. Find gradient ... following func^n at given point :-

i) $f(x,y) = x^y + y^x \quad a = (1,1)$

$f_x = yx^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y,\ x^y \log x + xy^{x-1})$

$\nabla f(1,1) = (1+0, 1+0)$

$= \underline{\underline{(1,1)}}$

ii) $f(x,y) = (\tan^{-1}x)\cdot y^2 \quad , a = (1,-1)$

$f_x = \frac{1}{1+x^2}\cdot y^2$

$f_y = 2y \tan^{-1}x$

$\therefore \nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2},\ 2y\tan^{-1}x\right)$

$\nabla f(1,-1) = \left(\frac{1}{2},\ \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2},\ \frac{\pi}{\cancel{4}_2}\cancel{(-2)}\right)$

$= \cancel{\left(\frac{1}{2},\ -\pi/2\right)}$

iii) $f(x,y,z) = xyz - e^{x+y+z} \quad , a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\therefore f(x,y,z) = f_x, f_y, f_z$
$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = \left((-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0}\right)$

$= (0 - e^0, 0 - e^0, -1 - e^0)$
$= (-1, -1, -2)$

3. Find the equation of tangent & normal to each of the following using curves at given points.

$\rightarrow$ i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f(x) = \cos y \, 2x + e^{xy} y$
$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1,0) = \therefore x_0 = 1, y_0 = 0$

eq^n of tangent
$f_x(x - x_0) + f_y(y - y_0) = 0$
$f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 0$
$= 1(2) + 0$
$= 2$
$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$
$= 0 + 1 \cdot 1$
$= 1$

$2(x-1) + 1(y-0) = 0$
$2x - 2 + y = 0$
$2x + y - 2 = 0$ $\therefore$ It is the required eq^n

85

eq^n of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

i) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$\rightarrow f(x) = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$fy = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2, y_0 = -2$

$fx(x_0, y_0) = 2(2) - 2 = 2$

$fy(x_0, y_0) = 2(-2) + 3 = -1$

eq^n of tangent

$fx(x - x_0) + fy(y - y_0) = 0$

$2(x-2) + (-1)(y+2) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ $\rightarrow$ It is required eq^n of tangent

eq^n of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore \boxed{d = 6}$

Q.4. Find the eq^n of tangent & normal line to each of the following surface :-

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$fx = 2x - 0 + 0 + z$

$fx = 2x + z$

$fy = 0 - 2z + 3 + 0$

$= 2z + 3$

$fz = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$ $\therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$fx(x_0, y_0, z_0) = 2(2) + 0 = 4$

$fy(x_0, y_0, z_0) = 2(0) + 3 = 3$

$fz(x_0, y_0, z_0) = -2(1) + 2 = 0$

eq^n of tangent

$fx(x - x_0) + fy(y - y_0) + fz(z - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$4x + 3y - 11 = 0 \longrightarrow$ This is required eq^n of tangent

Eq^n of normal at $(4,3,-11)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

ii) $3xyz - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + z + 4 = 0$ at $(1,-1,2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1,-1,2)$ $\therefore x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0 \longrightarrow$ This is required eq^n of tangent

Eq^n of normal at $(-7, 5, -2)$

$$\frac{x-x_0}{f_x} = \frac{y-y_0}{f_y} = \frac{z-z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q. 5. Find the local maxima & minima for the following function.

$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$fx = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6$

$fy = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x - 4$

$fx = 0$

$6x - 3y + 6 = 0$
$3(2x - y + 2) = 0$
$2x - y + 2 = 0$
$\therefore 2x - y = -2 \text{ ---- (i)}$

$fy = 0$

$2y - 3x - 4 = 0$
$2y - 3x = 4 \text{ ..... (ii)}$

Multiply eq (i) with 2

$\therefore 4x - \cancel{2y} = -4$
$\cancel{2y} - 3x = 4$
$x = 0$

Substitute value of x in eqⁿ (i)

$2(0) - y = -2$
$\cancel{-}y = \cancel{-}2$
$\therefore y = 2$

$\therefore$ Critical points are $(0, 2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$3 > 0$

$\therefore$ f has maximum at (0,2)

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ ...... (i)

$f_y = 0$

$3x^2 - 2y = 0$ ...... (ii)

Multiply eqn (i) with 3

(ii) with 4

$\cancel{12x^2} + 9y = 0$

$-\cancel{12x^2} - 8y = 0$

Substitute value of y in eq^n (1)

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical points is (0,0)

$r = fxx = 24x^2 + 6x$

$t = fyy = 0 - 2 = -2$

$s = fxy = 6x - 0 \quad 6x = 6(0) = 0$

r at (0,0)

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - s^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

f(x,y) at (0,0)

$2(0)^4 + 3(0)^2(0) - 0$

$= 0$ to 0

$= 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$\therefore fx = 2x + 2$

$fy = -2y + 8$

$fx = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2} \quad \therefore x = -1$

$fy = 0 \quad -2y + 8 = 0$

$y = \frac{-8}{-2} \quad 4$

$\therefore y = 4$

$\therefore$ Critical points is (-1,4)

01/02/2022

$r = fxx = 2$

$t = fyy = -2$

$s = fxy = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

f(x,y) at (-1,4)

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70 = \underline{\underline{33}}$