In how many ways can m balls be distributed into n boxes?				
Boxes distinguishable?	Balls distinguishable?	Minimum number of balls allowed in a box	Maximum number of balls allowed in a box	Number of distributions
✓	✓	0	∞	n^m
✓	✓	0	1	$\dfrac{n!}{(n-m)!}$
✓	✓	1	∞	$n!\,S(m,n)$
✓	✗	0	∞	$\dbinom{n+m-1}{m}$
✓	✗	0	1	$\dbinom{n}{m}$
✓	✗	1	∞	$\dbinom{m-1}{m-n}$
✗	✓	0	∞	$\displaystyle\sum_{i=1}^{n} S(m,i)$
✗	✓	1	∞	$S(m,n)$
✗	✗	0	∞	$\displaystyle\sum_{i=1}^{n} p(m,i)$
✗	✗	1	∞	$p(m,n)$

Stirling number of the second kind:

$$S(m,n) = \frac{1}{n!}\sum_{i=0}^{n}(-1)^i\binom{n}{i}(n-i)^m$$

is the number of ways of partitioning a set with m elements into n nonempty sets.

Catalan number:

$$b_n = \binom{2n}{n} - \binom{2n}{n-1} = \frac{1}{n+1}\binom{2n}{n} = \frac{(2n)!}{n!(n+1)!}$$

is the number of length $2n$ binary strings in any initial segment of which the number of 0's never exceeds the number of 1's.

Derangement number:

$$d_n = n!\sum_{i=0}^{n}\frac{(-1)^i}{i!} = n!\sum_{i=2}^{n}\frac{(-1)^i}{i!}$$

is the number of permutations of n objects that leave no object in its original place.

Inclusion-Exclusion Formulas:

$$E_m = \sum_{i=0}^{n-m}(-i)^i\binom{m+i}{i}S_{m+i}$$

is the number of elements that lie in exactly m of the sets $A_1, A_2, \ldots, A_n$;

$$L_m = \sum_{i=0}^{n-m}(-i)^i\binom{m-1+i}{i}S_{m+i}$$

is the number of elements that lie in at least m of the sets $A_1, A_2, \ldots, A_n$ where $S_k = \sum_{1\le i_1<i_2<\cdots<i_k\le n} |A_{i_1}\cap A_{i_2}\cap\cdots\cap A_{i_k}|$.