POLITECNICO
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Electromagnetism Experiments

“The purpose of computing is insight, not numbers”

Richard W. Hamming - The art of doing science & engineering - 1986

Introduction

Theatre is about to begin. We will explore the interactions of *electric & magnetic fields* under different conditions. Electrostatics and magnetostatic phenomena are notorious for being among the most difficult to deal with in the lab. In the following pages, we will show how we have addressed these challenges both experimentally and computationally.

As *John P. A. Ioanidis* showed in (2005) "*Why most published research findings are false*", we live in an era where most published scientific studies are difficult or even impossible to reproduce. Since *reproducibility* of experimental results is an integral part of the scientific method, the inability to replicate works of others has serious implications for many fields of knowledge, not to mention that it slows down technological development in industry and adoption in society. Although this ongoing crisis primarily affects the fields of medicine and social science, we still want to contribute to a *healthier* research environment.

data, methodologies, source code and results are public and free to be examined at the following link.

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Experiment 1

Assumptions and models for the calculation of capacitance

Abstract

In the experiment, we have two circular flat plates of radius R at a fixed initial distance of 1mm charged with a DC power supply. The capacitor can vary its capacitance since one of the plates is fixed, while the other is free to slide along a rail fitted with a metric scale. The aim of this first experiment is to measure the capacitance C and the potential difference ΔV under different assumptions and models.

Theoretical Background

Capacitance

If a potential difference is applied to the plates of a capacitor, the charges separate, and an electric field is generated within the *dielectric* (in this case air). The armature connected to the highest potential is positively charged, while the other is negatively charged. The positive and negative charges on the two plates are equal in absolute value. The ratio between the charge Q and the applied potential ΔV is called capacitance C and it is measured in *farad* (F).

$$globally: \qquad Q = \varepsilon_0 \oint \varepsilon_r \, E \cdot dA \qquad \Delta V = \int_{-}^{+} E \cdot dS \qquad C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 \oint \varepsilon_r \, E \cdot dA}{\int_{-}^{+} E \cdot dS}$$

Hypothesis 1: Capacitor operates in vacuum

The dielectric characteristics of air for electric fields well below $3{,}94 \cdot 10^4 \; V/m$ are very similar to those of *vacuum*. Having calculated the maximum electric field reached inside our capacitor, we find that $2 \cdot 10^3 \; V/m$ amply verifies this assumption. For subsequent calculations, we will neglect the *relative dielectric constant*.

$$\varepsilon_r = 1$$

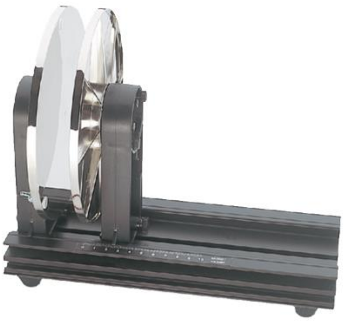
Hypothesis 2: Capacitor is flat

The capacitance of a capacitor with *flat parallel plates* depends entirely on its geometric parameters. The local geometry of a flat capacitor allows important simplifications to be made for all points away from its edge. It is consequently assumed that E is constant throughout the volume between the plates.

The following formulas would apply globally if we were studying an *infinite parallel plate capacitor*.

$$locally: \qquad Q = \varepsilon_0 \, E \, A \qquad \Delta V = E \, d \qquad C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 \, A}{d}$$

Experimental Apparatus



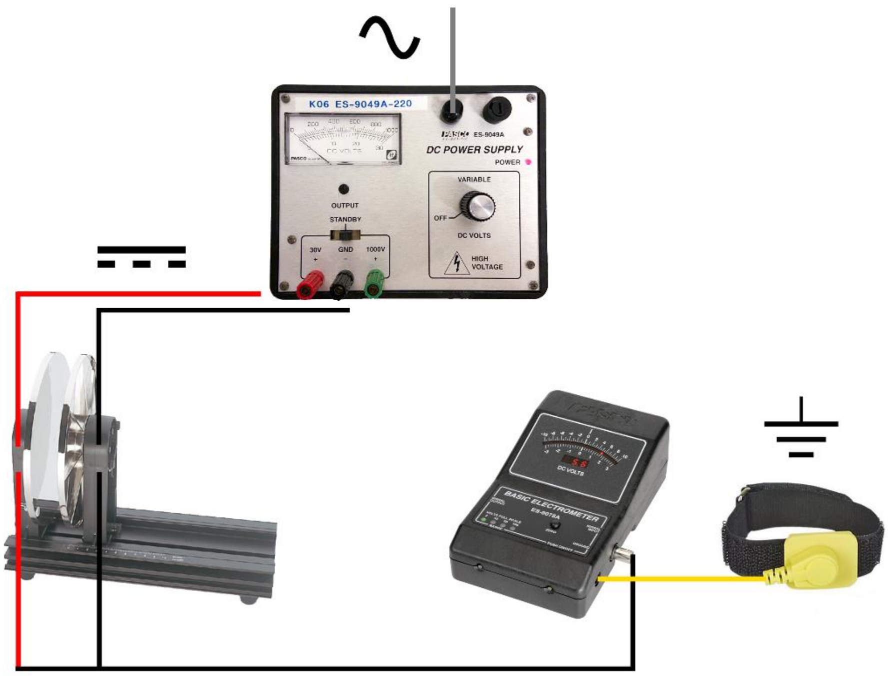
Capacitor: formed by two armatures (disks) of radius 10cm. One of the two disks is free to slide along a rail fitted with a metric scale with 1mm sensitivity.

DC Power Supply: used to charge the plates up. Adjustable voltage.

Voltmeter: used to measure the potential difference between the two armatures. Can be set to three different scale funds to make measurements at different voltages. Analog device.

Anti-Static Wrist Strap Band: important for security reasons

Below, we report how the equipment is connected



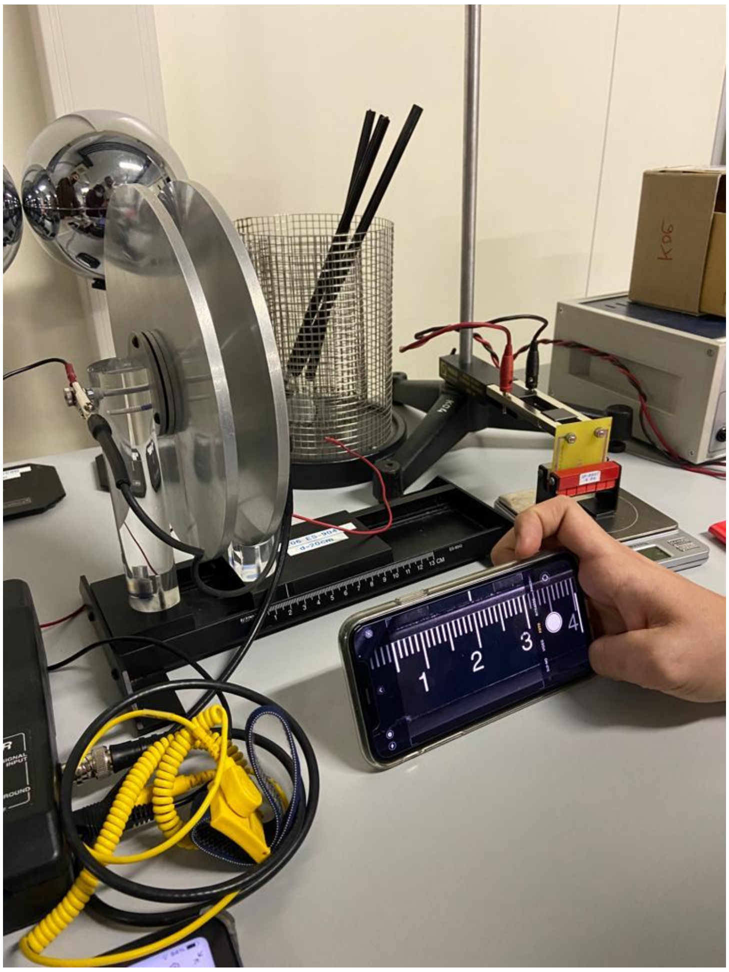
Procedure

To begin, we removed all the objects not related to our experiment to not disperse the charge deposited on the capacitor's plates. This is crucial since we want a *constant charge* throughout the experiment.

After turning on the *voltmeter*, we connected it via terminals at the ends of the two plates. Then, we set the voltage generator to 10 V (*direct current*) connecting it in a similar fashion to the plates. While the connections between *voltmeter* and capacitor remained untouched throughout the experiment, the same cannot be said of the connection between *generator* and moving plate. In fact, the plate and cable made contact only to deposit the charge necessary to report the desired value (10, 15, 20 V) on the voltmeter. During the measurement phase the generator is disconnected to avoid the continuous deposit of new charge to support the voltage. This ensures that it is not the voltage, but the charge deposited on the two armatures that remains constant during the experiment.

Thanks to the voltmeter we measured the potential differences at the various distances to which the two plates are brought through the slide. We bring the plates from the distance of 1mm (initial condition in which all previous operations are carried out) up to 40mm. It is crucial to carry out all measurements in the shortest time possible: In this way we avoid that the charge deposited in the plates is dispersed by contact with the air. Unfortunately, the day we conducted the experiment was particularly humid due to bad weather conditions, a fact that favors the loss of charge since *humid air* is a worse insulator than *dry air*. We repeated the same process for an initial voltage of 15V and 20V at 1mm (default distance).

A cell phone placed parallel to the plane was used to magnify the millimeter scale and avoid *parallax errors*.



This video (not produced by us) shows how to use the instrumentation

Physical Models

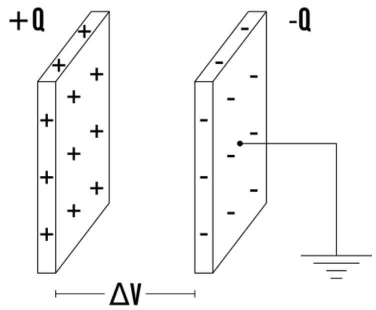
Model 1: Infinite plate capacitor

This precise geometry allows us to extend *hypothesis 2* globally. We assume that the plates of this capacitor are so large and close to each other that the edge effect of the electric field at the ends can be neglected.

$$E_\infty = \frac{Q}{\varepsilon_0 \, A} \quad \forall \, internal \; points$$

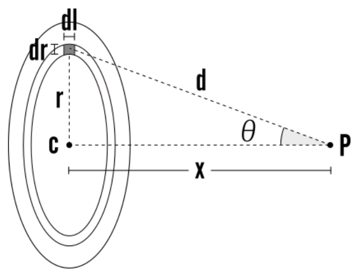
$$V_\infty = E_\infty \, d = \frac{Q}{\varepsilon_0 \, A} \, d_{min}$$

$$C_\infty = \frac{Q}{V_\infty} = \frac{\varepsilon_0 \, A}{d_{min}}$$



Model 2: Circular plate capacitor

In this model we consider the finite geometry of our capacitor and proceed to calculate the electric potential along the axis of a uniformly charged disk.



The electric field generated by charge dQ at point P is given by:

$$dE = \frac{dQ}{4\pi\varepsilon_0 d^2} = \frac{\sigma \, dr \, dl}{4\pi\varepsilon_0 d^2}$$

Each infinitesimal stretch of circular corona generates an electric field identical in modulus to that generated by the diametrically opposite stretch. As a result of symmetry, the two vertical components cancel each other while the two horizontal components are summed for each pair of opposite points.

$$dE_x = \frac{\sigma \, dr \, dl}{4\pi\varepsilon_0 d^2} \cos{(\theta)}$$

We find the electric field generated along the axis by integrating for a distribution of concentric rings:

$$dE_r = \int_L \, dE_x = \frac{\sigma \, dr \; \cos(\theta)}{4\pi\varepsilon_0 d^2} \int_L \, dl = \frac{\sigma \, dr \; \cos(\theta)}{4\pi\varepsilon_0 d^2} \, 2\pi r = \frac{\sigma \, r \, dr}{2\varepsilon_0 d^2} \cos(\theta)$$

From the definition of cosine, it follows that:

$$\cos(\theta) = \frac{x}{d} = \frac{x}{\sqrt{x^2 + r^2}} \qquad since \qquad d = \sqrt{x^2 + r^2}$$

Now it is possible to write the formula of the electric field generated by the corona in P in the following way:

$$dE_r = \frac{\sigma\, r\, dr}{2\varepsilon_0 d^2}\frac{x}{d} = \frac{\sigma\, r\, dr}{2\varepsilon_0}\frac{x}{(x^2 + r^2)^{3/2}}$$

Integrating from 0 to R we get the electric field generated along the axis of a disk. Since there are two disks in a capacitor, we multiply it by two:

$$E_{disk} = 2\int_0^R dE_r = \frac{|\sigma|}{\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right) \qquad (x > 0)$$

By integrating again, it is possible to find the potential difference between the two armatures:

$$V_{disk} = -\int_0^{d_{min}} E_{disk}\, dx = \frac{Q}{\varepsilon_0 A}\left(d_{min} + R - \sqrt{{d_{min}}^2 + R^2}\right)$$

From there, the theoretical capacity of the disk is derived:

$$C_{disk} = \frac{Q}{V_{disk}} = \varepsilon_0 A\left(d_{min} + R - \sqrt{{d_{min}}^2 + R^2}\right)^{-1}$$

Note:

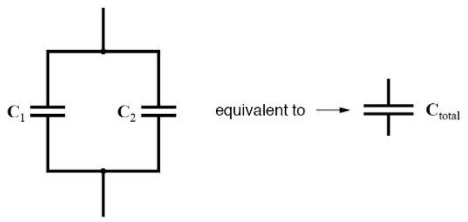
d refers to the generic distance between two points on different plates,
d_{min} refers to the minimum distance, i.e., the segment perpendicular to both plates

Model 3: Parasitic capacity

Noting that the disk voltage does not perfectly match what was obtained from the experimental data. We want to further improve the model by considering even the parasitic capacitance of the voltmeter.

$$C_{disk} + C_{parasitic} = C_{total}$$

$$C_{total} = \frac{Q}{V_{experimental}}$$



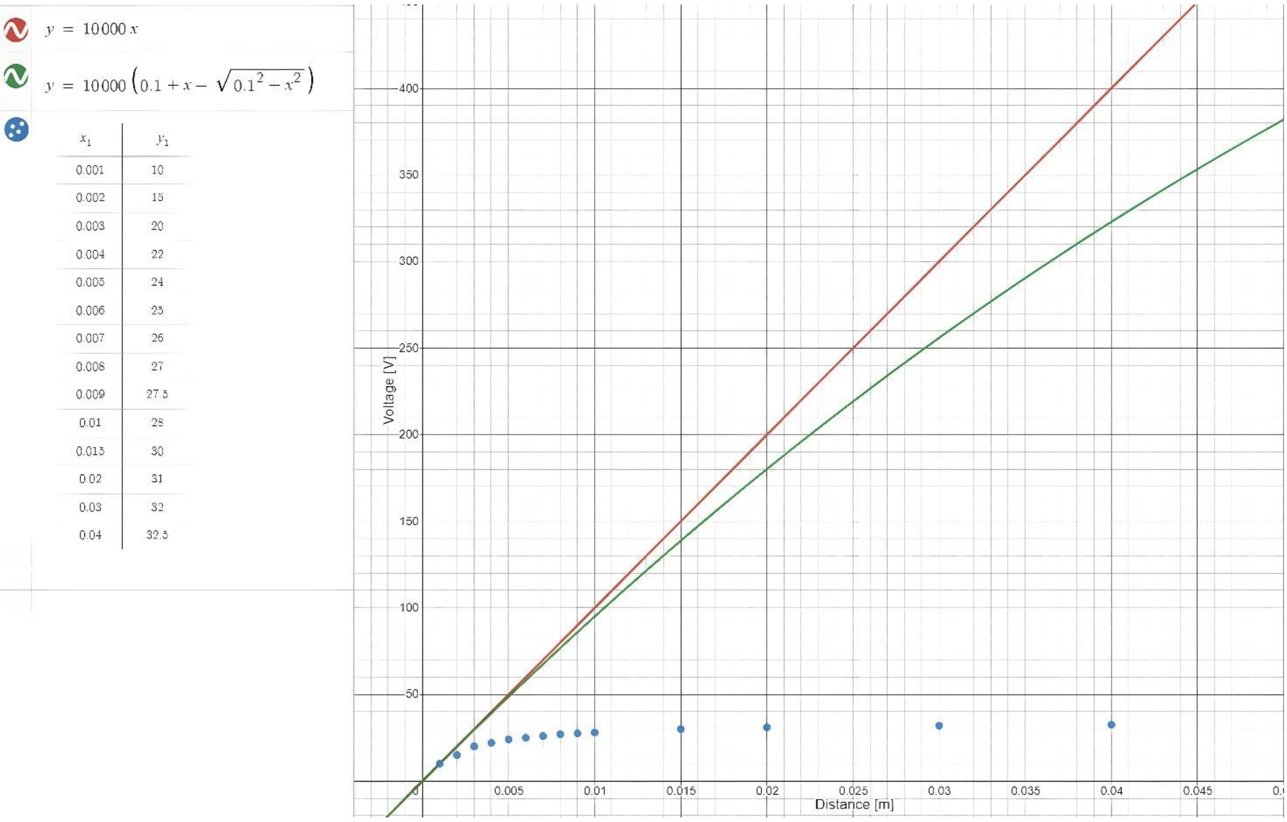
We then proceed to calculate the parasitic capacitance that allows our model to be compatible with the data using the following formula. The capacitances add up since they are in *parallel*.

$$C_{parasitic} = C_{total} - C_{disk} = Q \cdot \left(\frac{1}{V_{experimental}} - \frac{1}{V_{disk}}\right)$$

Results

Interactive data and graphs are available by clicking on the following links:

[10 volts] [15 volts] [20 volts]



We report only the graph related to the measurements with initial ΔV of 10V because all three graphs are similar. As we can see the first two models are unsatisfactory for distances greater than 5mm, indeed towards 10mm we can see that the electric potential tends to *saturate* (reach a constant value).

	distance [m]	voltage inf [V]	voltage disk [V]	voltage exp [V]	capacitance inf [F]	capacitance disk [F]	capacitance total [F]	capacitance par [F]	net charge [C]
0	0.001	10.0	9.950001	10.0	2.781625e-10	2.795603e-10	2.781625e-10	0.000000e+00	2.781625e-09
1	0.002	20.0	19.800020	15.0	1.390813e-10	1.404860e-10	1.854417e-10	4.495570e-11	2.781625e-09
2	0.003	30.0	29.550101	20.0	9.272084e-11	9.413251e-11	1.390813e-10	4.494875e-11	2.781625e-09
3	0.004	40.0	39.200320	22.0	6.954063e-11	7.095925e-11	1.264375e-10	5.547826e-11	2.781625e-09
4	0.005	50.0	48.750780	24.0	5.563250e-11	5.705806e-11	1.159010e-10	5.884298e-11	2.781625e-09
5	0.006	60.0	58.201617	25.0	4.636042e-11	4.779292e-11	1.112650e-10	6.347209e-11	2.781625e-09
6	0.007	70.0	67.552994	26.0	3.973750e-11	4.117693e-11	1.069856e-10	6.580865e-11	2.781625e-09
7	0.008	80.0	76.805104	27.0	3.477031e-11	3.621667e-11	1.030232e-10	6.680648e-11	2.781625e-09
8	0.009	90.0	85.958168	27.5	3.090695e-11	3.236022e-11	1.011500e-10	6.878979e-11	2.781625e-09
9	0.010	100.0	95.012438	28.0	2.781625e-11	2.927643e-11	9.934375e-11	7.006732e-11	2.781625e-09
10	0.015	150.0	138.812579	30.0	1.854417e-11	2.003871e-11	9.272084e-11	7.268213e-11	2.781625e-09
11	0.020	200.0	180.196097	31.0	1.390813e-11	1.543666e-11	8.972984e-11	7.429319e-11	2.781625e-09
12	0.030	300.0	255.969349	32.0	9.272084e-12	1.086702e-11	8.692579e-11	7.605876e-11	2.781625e-09
13	0.040	400.0	322.967039	32.5	6.954063e-12	8.612721e-12	8.558847e-11	7.697574e-11	2.781625e-09

We believe that every researcher should provide maximum *transparency* on how they analyze their data; therefore, we report the following code in Python used to perform our analysis.

As we were advised by the professors, if a negative parasitic capacitance was encountered in the calculation, it will be considered null (row 0). A negative capacitance has no physical meaning.

By averaging the measurements of the parasitic capacitance, we conclude that it has a mean value of:

$$C_{parasitic} = 5.994141716657975 \cdot 10^{-11}\, F$$

$$\sigma\left(C_{parasitic}\right) = 1.9376964418567223 \cdot 10^{-11}$$

```python
# Apparatus
r = 0.1
area = pi * r**2

def analysis (d, v_exp):

    # Charge homogeneously distributed on the capacitor's plates (constant)
    # Measured in coulomb [C]
    q =  (epsilon_0 * area * v_exp[0]) / d[0]

    # Voltage under the hypothesis of an infinite parallel plane capacitor
    # Measured in volt [V]
    v_inf = q * d / (epsilon_0 * area)

    # Capacitance under the hypothesis of an infinite parallel plane capacitor
    # Measured in farad [F]
    c_inf = q / v_inf

    # Voltage along the axis of a flat disk
    # Measured in volt [V]
    v_disk = q * (d + r - np.sqrt(r**2 + np.square(d))) / (epsilon_0 * area)

    # Capacitance under the hypothesis of an finite circular parallel plate capacitor (disk)
    # Measured in farad [F]
    c_disk = q / v_disk

    # Total capacitance of our apparatus (includes parasitic)
    # Measured in farad [F]
    c_tot = q / v_exp

    # Parasitic capacitance from the voltmeter
    # Measured in farad [F]
    c_par = c_tot - c_disk

    # Parasitic capacitance must be positive
    c_par[c_par < 0] = 0

    # Display parasitic capacitance
    print(f"Parasitic capacitance has a mean value of {np.mean(c_par)} farad with a standard error of {np.std(c_par)}")

    # Create dataframe
    results = pd.DataFrame({
        "distance [m]": d,
        "voltage inf [V]": v_inf,
        "voltage disk [V]": v_disk,
        "voltage exp [V]": v_exp,
        "capacitance inf [F]": c_inf,
        "capacitance disk [F]": c_disk,
        "capacitance total [F]": c_tot,
        "capacitance par [F]": c_par,
        "net charge [C]": q,
    })

    display(results)
```

More information about how the code was written is available by clicking here.

Statistical Analysis

Linear Regression

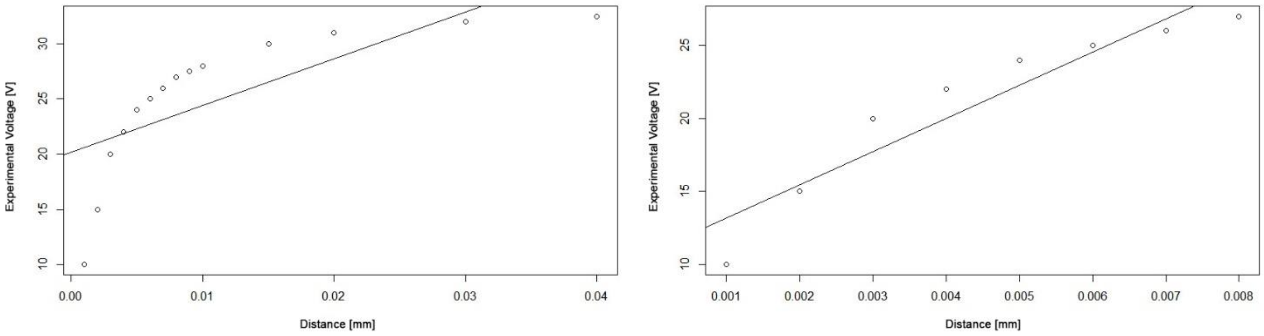
We report only the analysis for measurements with initial ΔV of 10V since all three cases are similar. However, the complete analysis of the other cases can be found at the following link.

The theoretical models initially proposed should approximate the experimental voltage by excess. We study the linear relationship that most closely approximates these models while attempting to optimize the significance of the predictors.

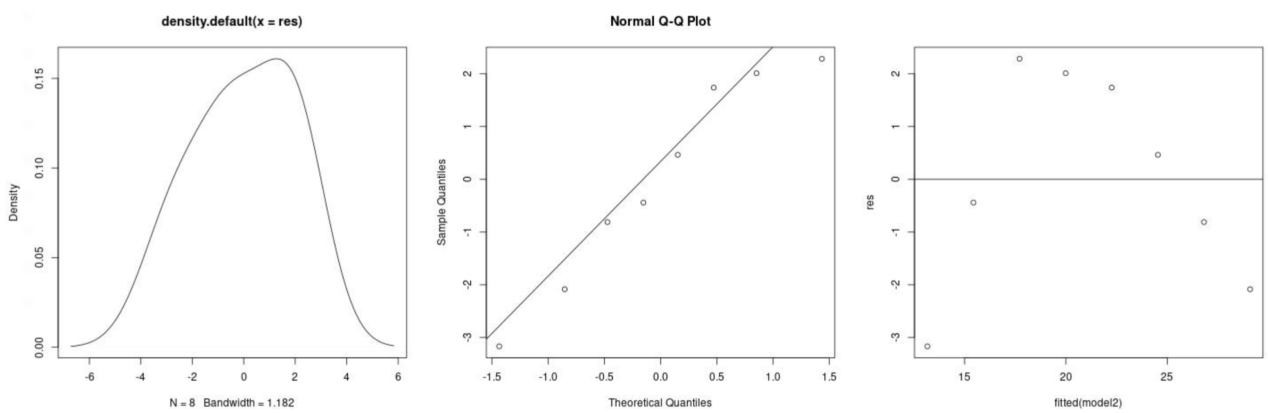
Linear regression using all data first. Refined model later using only the first eight.

```
Call:
lm(formula = v_exp ~ d)

Residuals:
    Min      1Q  Median      3Q     Max
-10.575  -1.288   2.016   3.311   3.606

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    20.15       1.74  11.579 7.19e-08 ***
d             424.34     109.91   3.861  0.00226 **

Residual standard error: 4.507 on 12 degrees of freedom
Multiple R-squared:  0.554, Adjusted R-squared:  0.5168
F-statistic: 14.91 on 1 and 12 DF,  p-value: 0.002265
```

```
Call:
lm(formula = v_exp[0:8] ~ d[0:8])

Residuals:
    Min      1Q  Median      3Q     Max
-3.1667 -1.1280  0.0119  1.8065  2.2857

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   10.893      1.675   6.503 0.000630 ***
d[0:8]      2273.810    331.700   6.855 0.000474 ***

Residual standard error: 2.15 on 6 degrees of freedom
Multiple R-squared:  0.8868, Adjusted R-squared:  0.8679
F-statistic: 46.99 on 1 and 6 DF,  p-value: 0.0004743
```



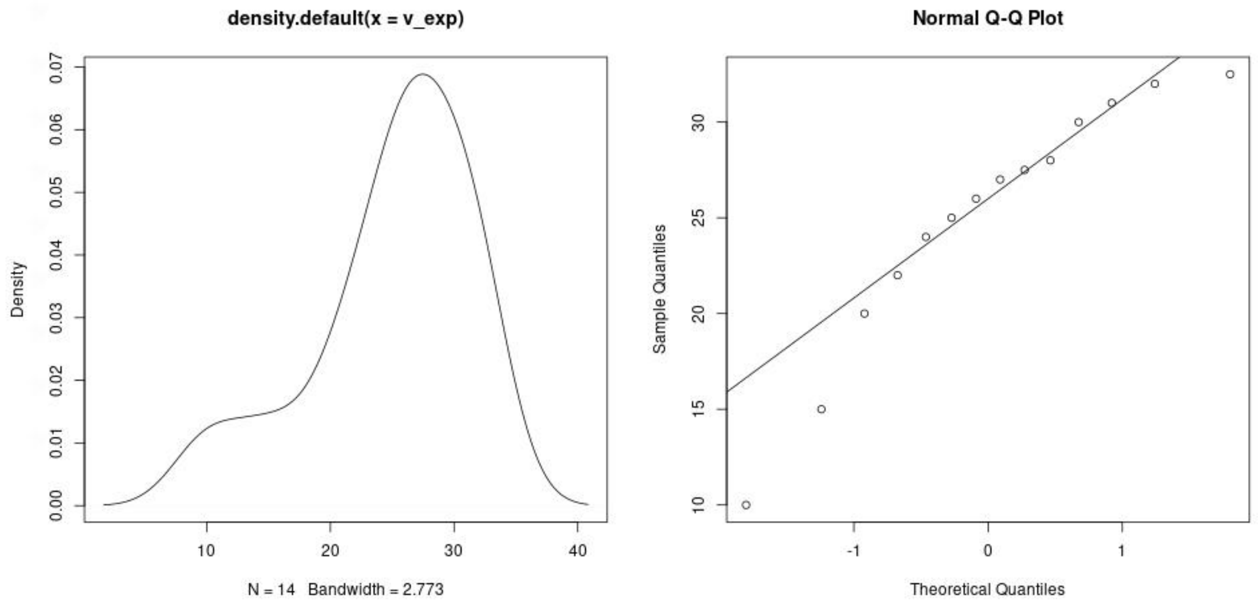
Density and *QQplot* for the residuals. *SW* test p-value: 0.5627 - gaussianity not rejected.
From the third plot we note that the residuals are homoscedastic and have a *parabolic bias*.



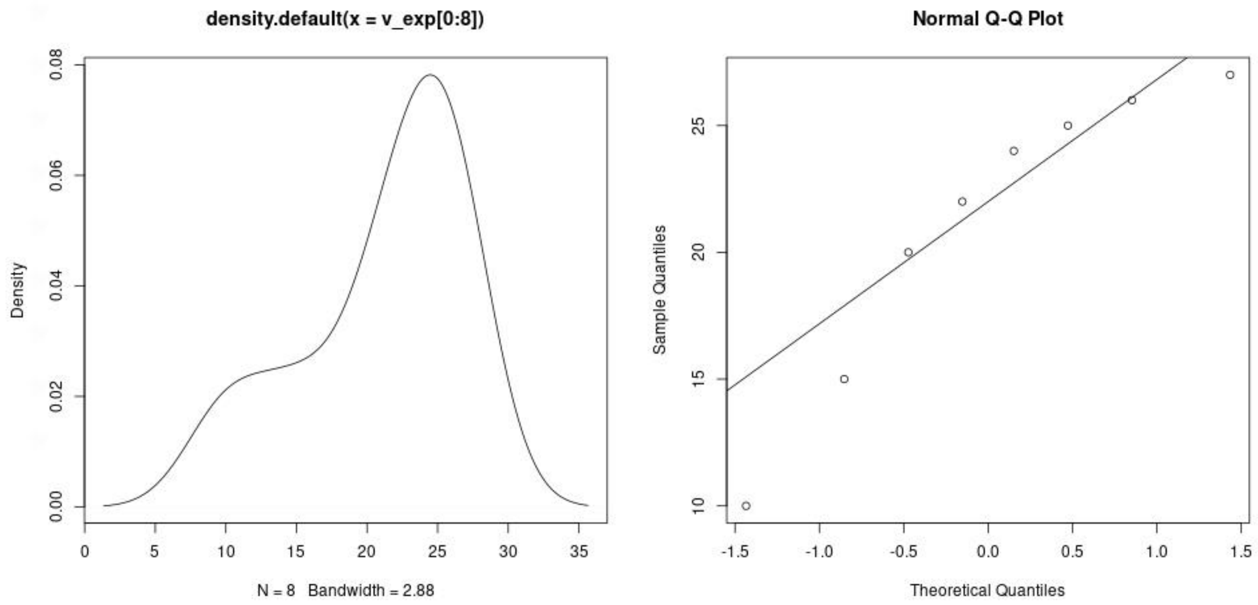
Normality Analysis

Studying the QQ-plots we notice that the measurements obtained follow a *normal trend*; a following Shapiro-Wilk test with a p-value of *0.159 > 0.05* leads to accept the Gaussianity of the samples. It is important to remember that the Shapiro-Wilk test places the Gaussianity in the *null hypothesis* H_0; this means that it is not possible to strongly assert the Gaussianity of the samples. However, we also do not have strong evidence to reject this hypothesis either. Thus, we have a conclusion that statisticians call "weak". There are no tests that strongly assert the Gaussianity of a random sample.

Density and *QQplot* of all experimental data



Density and *QQplot* for the first eight experimental data (used in the regression)
SW test p-value: 0.2287 - gaussianity not rejected



Again, we feel it is our duty to show the R code used for our analysis.

```r
# Data
d = c(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 15, 20, 30, 40) / 1000
v_exp= c(10, 15, 20, 22, 24, 25, 26, 27, 27.5, 28, 30, 31, 32, 32.5)

# Linear regression (all data)
model = lm(v_exp ~ d)
summary(model)

# Linear regression (only first 8)
model2 = lm(v_exp[0:8] ~ d[0:8])
summary(model2)

# Plots with regression line
plot(d, v_exp)
abline(model)
plot(d[0:8], v_exp[0:8])
abline(model2)

# The second model is better so we proceed studying its residuals
res = resid(model2)

# Residual VS Fitted plot
plot(fitted(model2), res)
abline(0,0)

# Shapiro-Wilk
shapiro.test(v_exp)
shapiro.test(v_exp[0:8])
shapiro.test(res)

# QQ-plot
qqnorm(v_exp)
qqline(v_exp)
qqnorm(v_exp[0:8])
qqline(v_exp[0:8])
qqnorm(res)
qqline(res)

# Density
plot(density(v_exp))
plot(density(v_exp[0:8]))
plot(density(res))
```

Experiment 2

Experimental verification of Laplace's law

Abstract

The objective of these experiments is to verify Laplace's law by studying the mutual dependence between the variables involved (current I, length of the circuit L and angle ϑ between current I and magnetic field B). The theoretical models are compatible with the experimental data, this allows us to study the intensity of the field generated by the magnets with great accuracy.

Theoretical Background

Lorentz force:

It is the combination of electric and magnetic force on a point charge due to electromagnetic fields. A particle of charge q moving with a velocity v experiences a force of:

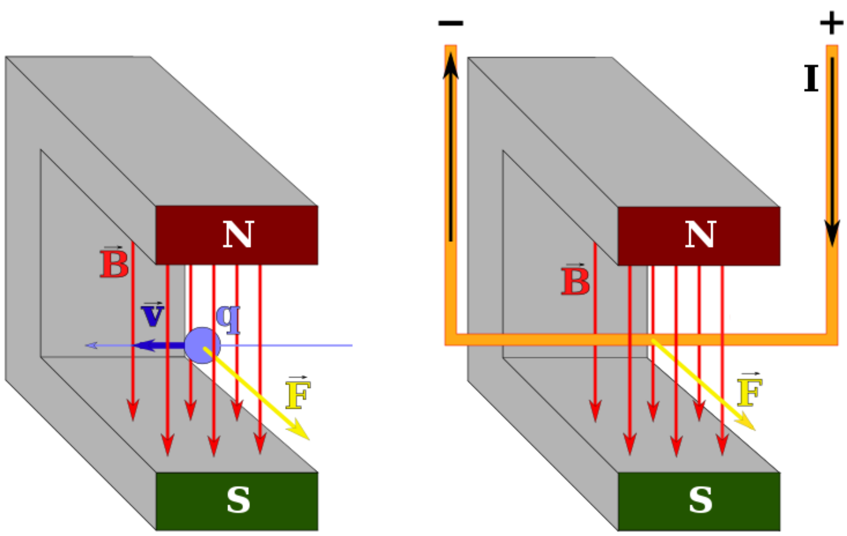
$$\vec{F}_{Lorentz} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$$

Laplace force:

When a wire carrying an electric current is placed in a magnetic field, each of the moving charges, which comprise the current, experiences the Lorentz force and together they can create a macroscopic force on the wire (sometimes called the *Laplace force*). By combining the Lorentz force law above with the definition of electric current, the following equation results, in the case of a straight stationary wire:

$$\vec{F}_{Laplace} = I\,\vec{L} \times \vec{B} \qquad\qquad |F| = I\,L\,B\,\sin(\vartheta)$$

The direction of the force follows directly from the fact that force F, magnetic field B and current I form a *right-handed triad* (the *right-hand rule* can be applied)



Experimental Apparatus

Set of circuits: important for the second part of the experiment. Circuits range from 10 to 80 mm in length.

Horseshoe magnet: convenient because it generates a uniform magnetic field within its cavity.

Precision Scale: digital device. Sensitivity: 10^{-5} Kg (centigram). Tare functionality included.

DC power supply: digital device. Used to pass direct current within the circuit.

Rectangular coil: mounted on a support that allows a rotation up to 180°. 11 windings.

Procedure

1) Variable current:

At this stage of the experiment the length of the circuit is fixed at 4 cm while the current is varied from 1 to 4 amperes (A). The circuit is placed inside the magnet cavity perpendicular to the work surface using the conductive support. It is important to keep the circuit parallel to the magnet walls to prevent the angle from affecting the measurements. The field is considered *uniform* within the cavity, so it is not necessary to place the circuit perfectly in the center, however we still recommend it.

The magnet is located on the precision scale because with this configuration we can indirectly measure the vertical force acting on it in correspondence of the different current values set on the generator. This is possible because of *Newton's third law of motion* (For every action, there is an equal and opposite reaction)

Magnet and current are oriented in such a way as to produce a downward force, since our precision scale was more accurate for positive changes in mass and not vice versa.

The total force acting along the vertical direction is the vector sum of the weight force and the (reaction of the) Laplace force; fortunately, the balance can measure relative variations with respect to an indicated mass, this allows to know with simplicity the variation of apparent mass due to the action of the force.

The Laplace force is obtained by multiplying the variation of mass in kilograms by the acceleration of gravity.

$$F_{Laplace} = \Delta m \cdot g = I\,L\,B$$

$$we\ assume \qquad \sin(\vartheta) = 1 \qquad \rightarrow \qquad \vartheta = \pi/2$$

2) Variable Length:

In this phase of the experiment the current is fixed at 3 amperes (A) while the length of the circuit was varies. Under these different conditions we repeat the same procedure as previously explained.

The length L varies since we mount different circuits included in the kit. Lengths vary from 10 to 80 mm.

For safety reasons, it is imperative to turn off the power generator before changing the circuit. In any case the current of 3 A is high enough to have visible results but not enough to create dangerous situations.

$$F_{Laplace} = \Delta m \cdot g = I\,L\,B \qquad\qquad (\,\vartheta = \pi/2\,)$$

3) Variable Angle:

In the last phase we use a special tool that allows us to vary the angle between current and magnetic field precisely without affecting the structure of the system (which is similar to the previous cases). The circuit length and current intensity are both constant at 1 cm and 3 A, respectively. The loop has N=11 windings.

$$F_{Laplace} = \Delta m \cdot g = N\,I\,L\,B\,\sin(\vartheta)$$

Results

Interactive data and graphs are available by clicking on the following links:

current length angle

1) Variable current:

	current [A]	length [m]	angle [deg]	angle [rad]	Δmass [Kg]	force [N]	magnetic field [T]
0	1.0	0.04	90	1.570796	0.00038	0.003727	0.093163
1	1.5	0.04	90	1.570796	0.00057	0.005590	0.093163
2	2.0	0.04	90	1.570796	0.00079	0.007747	0.096841
3	2.5	0.04	90	1.570796	0.00098	0.009611	0.096105
4	3.0	0.04	90	1.570796	0.00118	0.011572	0.096432
5	3.5	0.04	90	1.570796	0.00137	0.013435	0.095965
6	4.0	0.04	90	1.570796	0.00153	0.015004	0.093776

$$B_1 = \frac{F_{Laplace}}{I\ L} = \quad 0.09506363038690477\ \ T$$

$$\sigma\ (B_1) = \ 0.0015028110582847372$$

2) Variable Length:

	current [A]	length [m]	angle [deg]	angle [rad]	Δmass [Kg]	force [N]	magnetic field [T]
0	3	0.01	90	1.570796	0.00031	0.003040	0.101335
1	3	0.02	90	1.570796	0.00059	0.005786	0.096432
2	3	0.03	90	1.570796	0.00087	0.008532	0.094798
3	3	0.04	90	1.570796	0.00116	0.011376	0.094798
4	3	0.06	90	1.570796	0.00169	0.016573	0.092074
5	3	0.08	90	1.570796	0.00220	0.021575	0.089894

$$B_1 = \frac{F_{Laplace}}{I\ L} = \ 0.09488841898148148\, T$$

$$\sigma\ (B_1) = \ 0.0035783486084375207$$

3) Variable Angle:

	current [A]	length [m]	angle [deg]	angle [rad]	Δmass [Kg]	force [N]	magnetic field [T]
0	3	0.11	-90	-1.570796	-0.00077	-0.007551	0.022882
1	3	0.11	-80	-1.396263	-0.00075	-0.007355	0.022632
2	3	0.11	-70	-1.221730	-0.00072	-0.007061	0.022769
3	3	0.11	-60	-1.047198	-0.00066	-0.006472	0.022647
4	3	0.11	-50	-0.872665	-0.00058	-0.005688	0.022500
5	3	0.11	-40	-0.698132	-0.00048	-0.004707	0.022191
6	3	0.11	-30	-0.523599	-0.00036	-0.003530	0.021396
7	3	0.11	-20	-0.349066	-0.00024	-0.002354	0.020853
8	3	0.11	-10	-0.174533	-0.00010	-0.000981	0.017113
9	3	0.11	0	0.000000	0.00000	0.000000	NaN
10	3	0.11	10	0.174533	0.00014	0.001373	0.023959
11	3	0.11	20	0.349066	0.00026	0.002550	0.022591
12	3	0.11	30	0.523599	0.00038	0.003727	0.022585
13	3	0.11	40	0.698132	0.00049	0.004805	0.022654
14	3	0.11	50	0.872665	0.00059	0.005786	0.022888
15	3	0.11	60	1.047198	0.00066	0.006472	0.022647
16	3	0.11	70	1.221730	0.00072	0.007061	0.022769
17	3	0.11	80	1.396263	0.00077	0.007551	0.023235
18	3	0.11	90	1.570796	0.00078	0.007649	0.023179

$$B_2 = \frac{F_{Laplace}}{N\ I\ L \sin(\vartheta)} = \ 0.02230510145458343\ \ T$$

$$\sigma\ (B_2) = \ 0.0014147961692990447$$

The magnet used in the first two experiments (B_1) is different from the one used in the third (B_2). Considered with their errors, the two values of B_1 are compatible and converge towards *0.095 T*.

The magnetic field in the third table in line 9 (highlighted in yellow) is "*NaN*" or in other words *undefined* since we are trying to divide by zero. This is because when the angle approaches zero, so does its sine.

For the correct calculation of the magnetic field, it is necessary to convert the angles in *radians*.

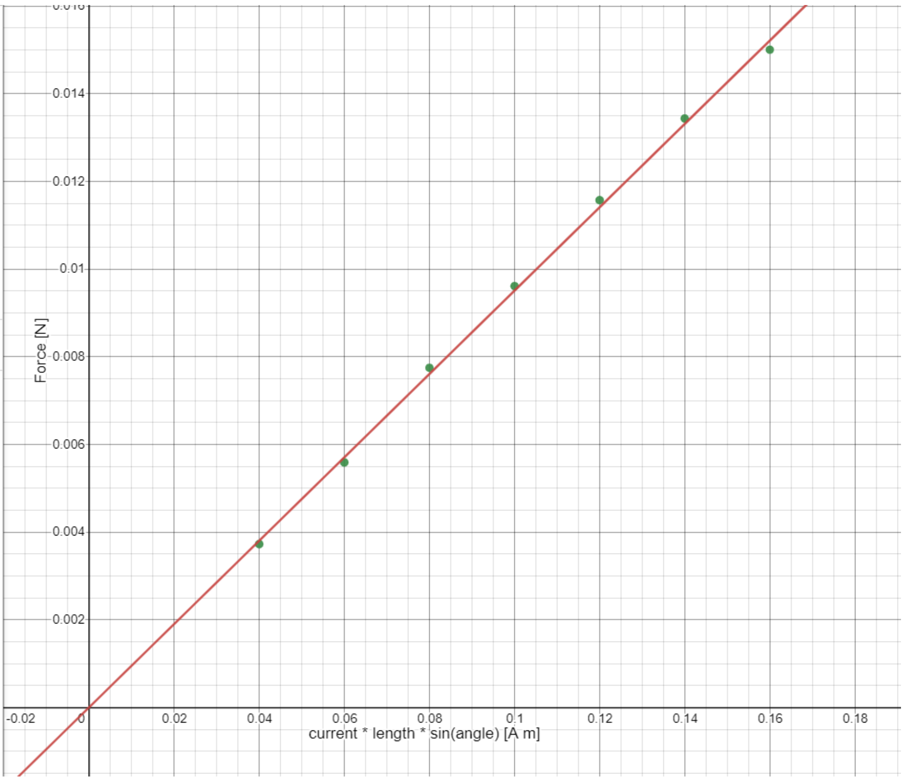
The average value of the magnetic field can also be found as the first order coefficient in a linear regression that we will discuss later. (The intercept has very high p-value, so we can exclude it from the model)

As in the previous cases here is the code for our analysis. More information here.

```python
def analysis(mass, current, length, angle):

    # Laplace force
    # Measured in newton [N]
    force = mass * g

    # Angle in radians
    angle_rad = angle * pi / 180

    # Magnetic field
    # Measured in tesla [T]
    field = force / ( current * length * np.sin(angle_rad) )

    # If the angle is zero, its sine is zero as well
    # Since it is not possible to divide by zero the script gives us back "NaN" (not a number)
    # We proceed to clean the data in order to make a meaningful analysis
    clean_field = [x for x in field if isnan(x) == False]

    # Display magnetic field
    print(f"Magnetic field has a mean value of {np.mean(clean_field)} tesla with a standard error of {np.std(clean_field)}")

    # Create dataframe
    results = pd.DataFrame({
    'current [A]': current,
    'length [m]': length,
    'angle [deg]': angle,
    'angle [rad]': angle_rad,
    'Δmass [Kg]': mass,
    'force [N]': force,
    'magnetic field [T]': field,
    })

    display(results)
```

x_1	y_1
$1 \cdot 0.04$	0.003727
$1.5 \cdot 0.04$	0.005590
$2 \cdot 0.04$	0.007747
$2.5 \cdot 0.04$	0.009611
$3 \cdot 0.04$	0.011572
$3.5 \cdot 0.04$	0.013435
$4 \cdot 0.04$	0.015004

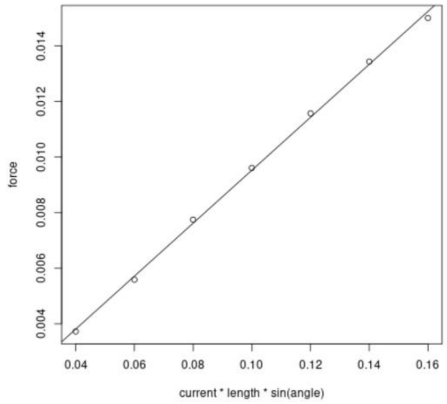
$$y = 0.0950636303869047 7 \cdot x$$

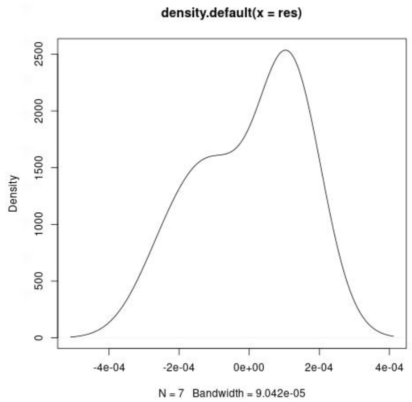


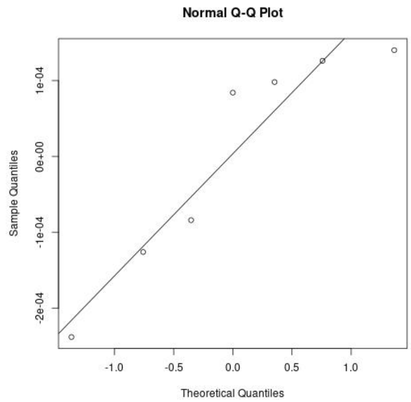
Statistical Analysis

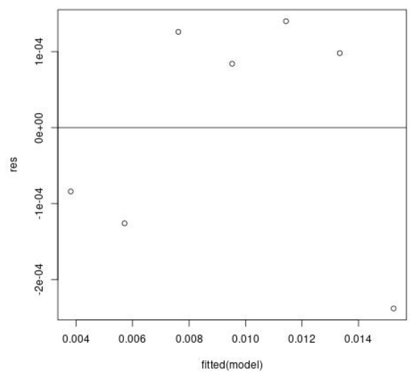
1) Variable Current:

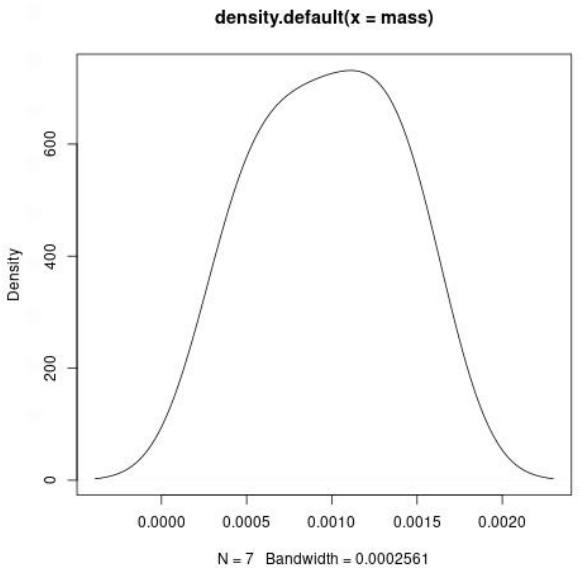
```
Call:
lm(formula = force ~ I(current * length * sin(angle)))

Residuals:
         1          2          3          4          5          6          7
-8.406e-05 -1.261e-04  1.261e-04  8.406e-05  1.401e-04  9.807e-05 -2.382e-04

Coefficients:
                              Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)                 -5.245e-18   1.653e-04     0.00         1
current * length * sin(angle) 9.526e-02   1.535e-03    62.08  2.05e-08 ***

Residual standard error: 0.0001624 on 5 degrees of freedom
Multiple R-squared:  0.9987,    Adjusted R-squared:  0.9984
F-statistic:  3853 on 1 and 5 DF,  p-value: 2.054e-08
```

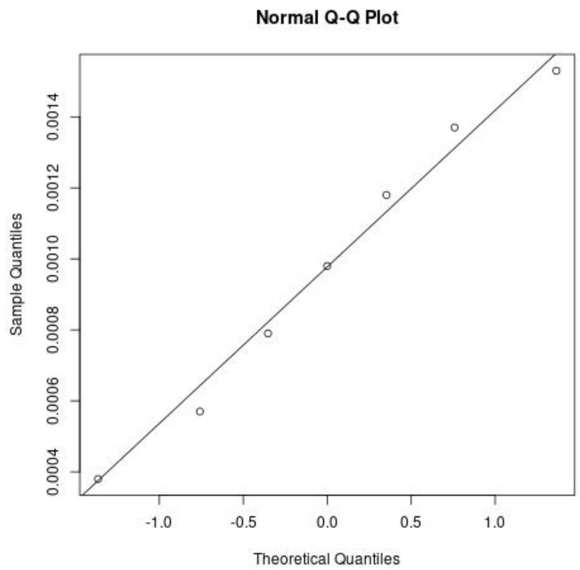








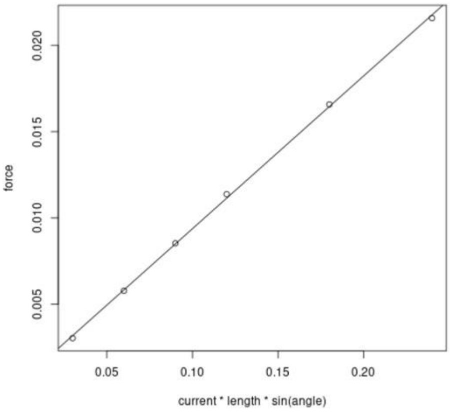


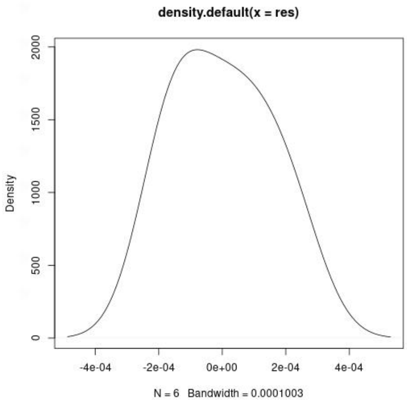


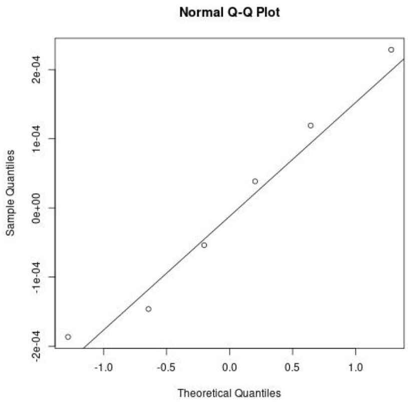
Gaussianity is not rejected for both samples and residuals; further confirmation of this comes from the *Shapiro-Wilk* test: *0.9099 >> 0.05* for mass, *0.1685 > 0.05* for residuals. From the *"res VS fitted"* plot, we can see some hint of *homoscedasticity*: the residuals have finite variance and a cloud-like arrangement. As the low p-value of the *F-test* tells us, the overall model is highly significant with satisfactory R^2 e R^2_{adj}. Since the p-value of the intercept is maximum (1) this makes us sure not to consider it in the model.

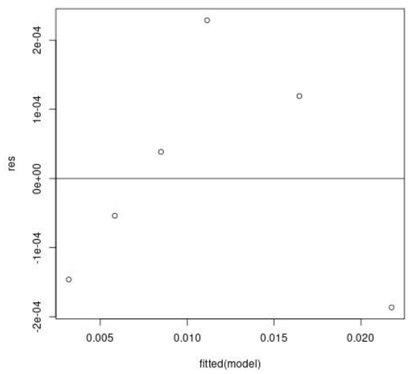
2) Variable Length:

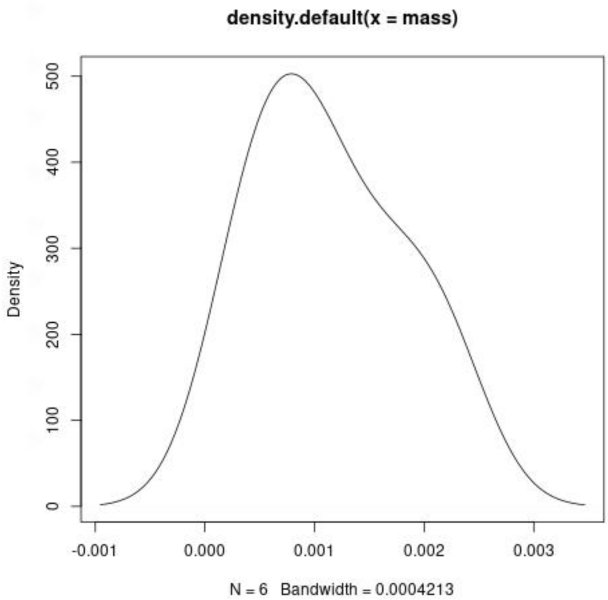
```
Call:
lm(formula = force ~ I(current * length * sin(angle)))

Residuals:
         1          2          3          4          5          6
-1.461e-04 -5.384e-05  3.846e-05  2.288e-04  1.192e-04 -1.865e-04

Coefficients:
                               Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)                   0.0005326  0.0001423    3.743    0.0201 *
current * length * sin(angle)  0.0884521  0.0010190   86.803  1.06e-07 ***

Residual standard error: 0.0001783 on 4 degrees of freedom
Multiple R-squared:  0.9995,     Adjusted R-squared:  0.9993
F-statistic:  7535 on 1 and 4 DF,  p-value: 1.056e-07
```

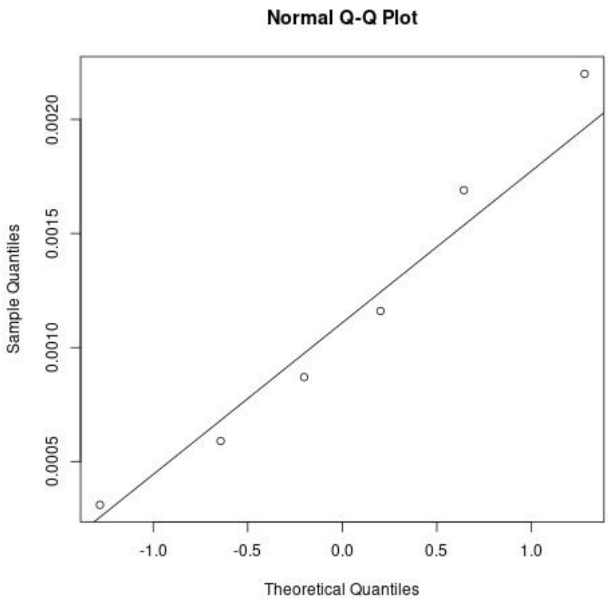








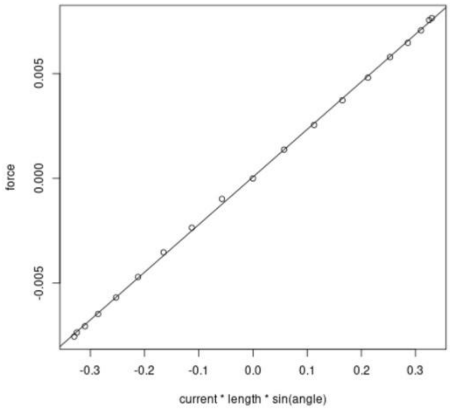


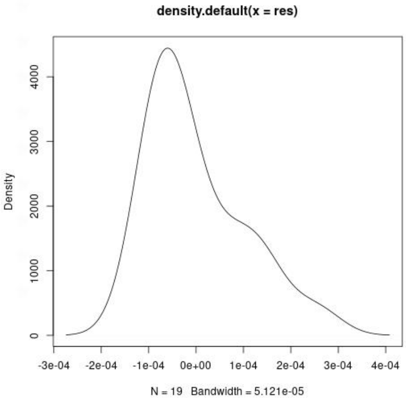


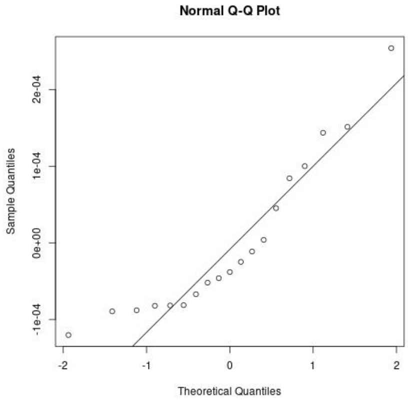
Gaussianity not rejected for both samples and residuals; further confirmation of this comes from the *Shapiro-Wilk* test: *0.8505 >> 0.05* for mass, *0.8359 >> 0.05* for residuals. From the "*res VS fitted*" plot, we can see there is a certain *homoscedasticity* with *non-linear bias* (maybe parabolic). In any case the residuals are perfectly kept within the ±2 margins of the normalized plot. It is not possible to conclude more due to the lack of data. As the low p-value of the *F-test* tells us, the overall model is highly significant with satisfactory R^2 e R^2_{adj}. Intercept less significant than the other predictor ($0.0201 >> 10^{-7}$) this allows not to consider it in the model.

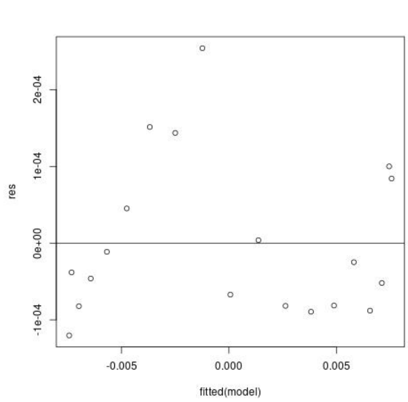
3) Variable Angle:

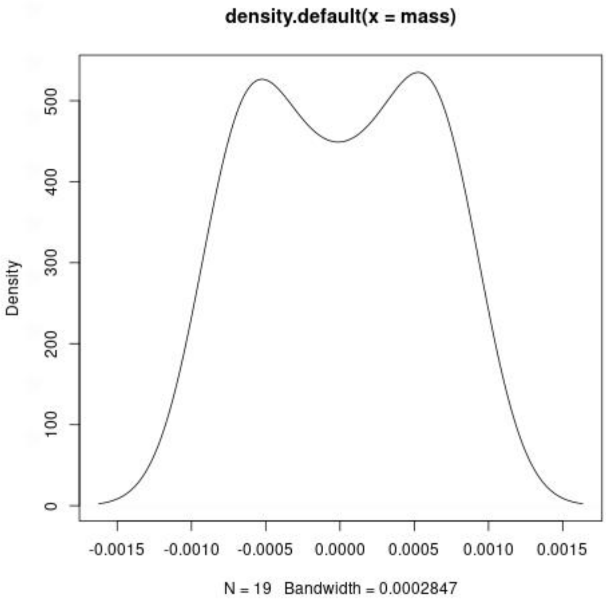
```
Call:
lm(formula = force ~ I(current * length * sin(angle)))

Residuals:
       Min         1Q     Median         3Q        Max
-1.204e-04 -8.158e-05 -3.814e-05  6.474e-05  2.542e-04

Coefficients:
                                 Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)                     6.710e-05   2.421e-05    2.772    0.0131 *
current * length * sin(angle)   2.272e-02   1.011e-04  224.725    <2e-16 ***

Residual standard error: 0.0001055 on 17 degrees of freedom
Multiple R-squared:  0.9997,     Adjusted R-squared:  0.9996
F-statistic: 5.05e+04 on 1 and 17 DF,  p-value: < 2.2e-16
```

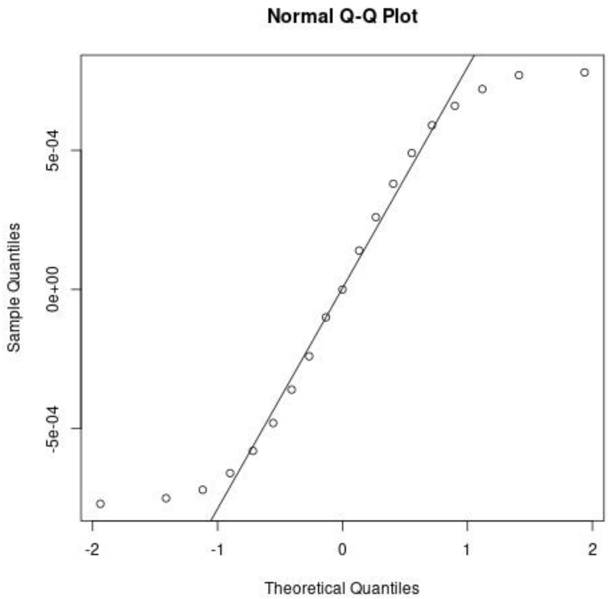




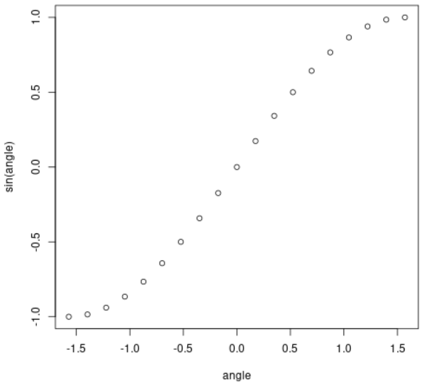








Non-rejectable Gaussianity for samples only. From the Shapiro-Wilk test we get *0.05728 > 0.05* for mass, *0.02078 < 0.05* for residuals. There is no certainty about the *homo* or *heteroscedasticity* of the residuals, all we can say from the plot *"res vs fitted"* is that the residuals do not have a linear trend, they have finite variance but the pattern they describe does not seem to have a random trend. As the low p-value of the *F-test* tells us, the overall model is highly significant with satisfactory R^2 e R^2_{adj}. Intercept less significant than the other predictor (0.0201 >> 10-16) this allows not to consider it in the model.



As per practice we leave the code used for the statistical analysis. More info here.

```r
# Near Earth's surface the gravitational acceleration is approximately g
g = 9.80665

# Data
current = 3
length = 0.11
angle = c(-90, -80, -70, -60, -50, -40, -30, -20, -10, 0, 10, 20, 30, 40, 50, 60, 70, 80, 90) * pi/180

# Laplace Force
mass = c(-77, -75, -72, -66, -58, -48, -36, -24, -10, 0, 14, 26, 38, 49, 59, 66, 72, 77, 78) / 100000
force = mass * g

# Linear regression
model = lm(force ~ I(current * length * sin(angle)))
summary(model)

# Plots with regression line
plot(current * length * sin(angle), force)
abline(model)

# Residuals
res = resid(model)

# Residual VS Fitted plot
plot(fitted(model), res)
abline(0,0)

# Shapiro-Wilk
shapiro.test(res)
shapiro.test(mass)

# QQ-plot
qqnorm(res)
qqline(res)
qqnorm(mass)
qqline(mass)

# Density
plot(density(res))
plot(density(mass))

# Sin plot
plot(angle, sin(angle))
```

Conclusions

In general, this experience turned out to be very instructive, not only from a notional point of view but above all because it allowed us to appreciate the differences between theoretical models and experimental data, pushing us to find reasons behind these inconsistencies. Physics is fascinating even when it doesn't work!

Acknowledgements

Software:

R: programming language

Python: programming language

NumPy: python library

Pandas: python library

Seaborn: python library

SciPy: python library

Jupiter Notebook: IDE for python

Adobe Photoshop: photo editing software

Web Services:

Carbon.now: easy sharing of source code via images

Rdrr.io: online IDE for R (R studio alternative)

Imgonline.com.ua: online tool for quick image editing

Github.com: opensource repositories. All the material of this research can be found here

Desmos.com: online graphic calculator. Useful for sharing interactive graphs

Images:

Pasco Scientific: experimental apparatus

Wikipedia: illustrations

YouMath: illustrations

Literature:

Ioannidis (2005) "Why Most Published Research Findings Are False"