⚠️ IMPORTANTE ⚠️

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Analisi Complessa

CARATTERIZZAZIONI

$z = a + bi = \cos\alpha + i\sin\alpha = \rho e^{i\alpha}$

$\rho = \sqrt{a^2+b^2}$ $\quad \alpha = \arctan\left(\frac{b}{a}\right) \; (+\pi)$

FORMA ESPONENZIALE

$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$ $\quad \sinh z = \frac{e^{z} - e^{-z}}{2}$

$\cos z = \frac{e^{iz} + e^{-iz}}{2}$ $\quad \cosh z = \frac{e^{z} + e^{-z}}{2}$

ANNULLAMENTO

$\cos z = 0 \quad z = \frac{\pi}{2} + k\pi$

$\cosh z = 0 \quad z = \left(\frac{\pi}{2} + k\pi\right) i$

$\sin z = 0 \quad z = k\pi$

$\sinh z = 0 \quad z = k\pi i$

SINGOLARITÀ

$\frac{\sin z}{z}$ $\quad z = 0$ ELIMINABILE

$\sin \frac{1}{z^N}$ $\quad z = 0$ NON ISOLATA; $\quad z = \frac{1}{(k\pi)^N}$ ESSENZIALI

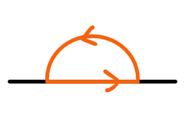
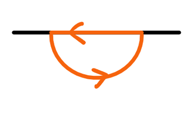
$e^{\frac{1}{z^N}}$ $\quad z = 0$ ESSENZIALE

$\frac{1}{(z-a)^N}$ $\quad z = a$ POLO ORDINE N

$\frac{1}{\sin z}$ $\quad z = k\pi$ POLI SEMPLICI

JORDAN

$\lim_{R \to \infty} \int_{C_R(0)} f(z)\, e^{i\omega z}\, dz = 0$

 $\omega > 0$  $\omega < 0$

POLINDROME

$z = \rho e^{i\alpha} = \rho e^{i(\alpha + 2k\pi)}$ $\quad \infty$

$\sqrt[N]{z} = \sqrt[N]{\rho} \cdot e^{\frac{i(\alpha + 2k\pi)}{N}}$ $\quad k \in [0, N-1]$ $\quad N$

$\log(z) = \log(\rho) + i(\alpha + 2k\pi)$ $\quad \infty$

TEOREMA DEI RESIDUI

$\oint_\gamma f(z) = 2\pi i \sum \mathrm{RES}(f, z_0) \cdot \mathrm{IND}(\gamma, z_0)$

RESIDUI

POLO 1 $\quad \mathrm{RES}(z_0) = \lim_{z \to z_0} f(z)(z - z_0) = \frac{\mathrm{NUM}}{\mathrm{DEN}'}(z_0)$

POLO 2 $\quad \mathrm{RES}(z_0) = \lim_{z \to z_0} D\left(f(z)(z - z_0)^2\right)$

POLO K $\quad \mathrm{RES}(z_0) = \lim_{z \to z_0} \frac{1}{(k-1)!} D^{k-1}\left(f(z)(z - z_0)^k\right)$

ESSENZIALE $\quad \mathrm{RES}(z_0) = C_{-1}$ LAURENT

ELIMINABILE $\quad \mathrm{RES}(z_0) = 0$

PUNTO ALL' INFINITO

$f_\infty(z) = -\frac{1}{z^2} f\left(\frac{1}{z}\right)$

$\mathrm{RES}(f, \infty) = \mathrm{RES}(f_\infty, 0)$

$\sum \mathrm{RES}(z_k) + \mathrm{RES}(\infty) = 0$

SERIE GEOMETRICA

$\sum_0^\infty x^N = \frac{1}{1-x}$ $\quad \sum_1^\infty x^N = \frac{x}{1-x}$

$|z| < b$ $\quad \frac{1}{z-b} = \frac{-1}{b\left(1 - \frac{z}{b}\right)} = -\frac{1}{b} \sum \left(\frac{z}{b}\right)^N$

$|z| > a$ $\quad \frac{1}{z-a} = \frac{-1}{z\left(1 - \frac{a}{z}\right)} = -\frac{1}{z} \sum \left(\frac{a}{z}\right)^N$

RANDOM

$\cos 2x = \cos^2 x - \sin^2 x$ $\quad \cosh 2x = \cosh^2 x + \sinh^2 x$

$\sin 2x = 2 \sin x \cos x$ $\quad \sinh 2x = 2 \sinh x \cosh z$

$\sin^2 x + \cos^2 x = +1$ $\quad \sinh^2 x - \cosh^2 x = +1$

$\cos(iz) = \cosh(z)$

$\sin(iz) = i \sinh(z)$ $\quad i^{-1} = -i$ $\quad |e^{ix}| = 1$

$z \to 0$ $\quad \sin x, \sinh x, \tan x, \arctan x \sim \underline{x}$

$x(\alpha^2 + \beta^2) - 2\alpha\beta \geq 0$ $\quad \underline{x > 1}$

$\gamma : |z - a| < R$ ↻ CENTRATO IN a CON RAGGIO R

$f \times f_{ESS} = f_{ESS}$ $\quad f + f_{ESS} = f_{ESS}$

Trasformata di Fourier

FUNZIONE $\quad u(x)$

TRASFORMATA $\quad \mathcal{F}(u) = \hat{u} = \int_{\mathbb{R}} u(x)\, e^{-i\omega x}$

LA DEFINIZIONE NON E' UNIVOCA

SPAZI

$u \in AC \to u$ INTEGRABILE $\to u \in L$

$u \in L^1 \to \hat{u} \in L^\infty \cap C^0, \; \lim_{\omega \to \infty} \hat{u} = 0$

$u \in L^1 \cap C^0 \to \hat{u} \in L^1$

$u \in L^1 \cap AC \to \hat{u}' \in L^1$

$u x^k \in L^1 \to \hat{u}' \in C^k$

OPERATORE

$\mathcal{F} : L^1 \to L^\infty$

$\mathcal{F} : S \to S$

$\mathcal{F} : L^2 \to L^2$

$\mathcal{F} : S' \to S'$

$u \in S \to u \in L^2$

$\downarrow \qquad \downarrow$

$\hat{u} \in S' \to \hat{u} \in L^2$

PROPRIETÀ

- REGOLARITÀ E CONTINUITÀ
- $\mathcal{F}$(PARI) = REALE PARI
- $\mathcal{F}$(DISPARI) = IMMAGINARIA PURA DISPARI

RANDOM

$\frac{\partial}{\partial x} e^{\pm|x|} = \pm \mathrm{SIGN}(x)\, e^{\pm|x|}$

NOTEVOLI

$\chi_{(a,b)} \to \frac{\sin(b\omega) - \sin(a\omega)}{\omega} + i\, \frac{\cos(b\omega) - \cos(a\omega)}{\omega}$

$e^{-a|x|} \to \frac{2a}{a^2 + \omega^2}$

$\frac{1}{a^2 + \omega^2} \to \frac{\pi}{a} e^{-a|\omega|}$

$u' \to \omega i \hat{u}$

$x u \to i \frac{\partial}{\partial \omega} \hat{u}$

$u(\omega - a) \to e^{-ia\omega}\, \hat{u}$

$\delta(x) \to 1$

$\frac{\sin x}{x} \to \pi \chi_{(-1,+1)}$

$e^{-ax} H(x) \to \frac{1}{a + i\omega}$

$e^{-x^2} \to \sqrt{\pi}\, e^{-\omega^2/4}$

$u \cdot e^{iax} \to \hat{u}(\omega - a)$

$u * v \to \hat{u} \cdot \hat{v}$

EQUAZIONE LINEARE I ORDINE

$y' + ay = f$ $\quad y(t) = e^{-A}\left(c_1 + \int f e^{A} dt\right)$ $\quad A = \int a(t)\, dt$

Lax Milgram

MAGGIORAZIONI

$v \in H_0^1(0, L)$ $\quad \|v\|_{L^2} \leq \frac{L}{\pi} \|v'\|_{L^2}$

$v \in H^1(0, L)$ $\quad |v(x)| \leq \max\left(\sqrt{L}, \frac{1}{\sqrt{L}}\right) \|v\|_{L^2} \|v'\|_{L^2}$

$v \in H^1(0, L)$, $v(0) \vee v(L) = 0$ $\quad \|v\|_{L^2} \leq \frac{2L}{\pi} \|v'\|_{L^2}$

$v \in H^1(0, L)$, $v(c) = 0$ $\quad |v(x)| \leq \sqrt{L} \|v'\|_{L^2}$

1) LINEARITÀ F

$F(\alpha v_1 + \beta v_2) = \alpha F(v_1) + \beta F(v_2)$

2) LIMITATEZZA F

$|F(v)| \leq K \|v\|_V$

3) BILINEARITÀ B

$B(\alpha u_1 + \beta u_2, v) = \alpha B(u_1, v) + \beta B(u_2, v)$

$B(u, \alpha v_1 + \beta v_2) = \alpha B(u, v_1) + \beta B(u, v_2)$

4) LIMITATEZZA B

$|B(w, v)| \leq K \left(\|w'\|_V \cdot \|v'\|_V\right)$

5) COERCITIVITÀ B

$B(w, w) \geq K \|w\|_V^2$