

Commonly Used Taylor Series

SERIES	WHEN IS VALID/TRUE
$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \dots$ $= \sum_{n=0}^{\infty} x^n$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">NOTE THIS IS THE GEOMETRIC SERIES. JUST THINK OF x AS r</div> $x \in (-1, 1)$
$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ $= \sum_{n=0}^{\infty} \frac{x^n}{n!}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">SO: $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$ $e^{(17x)} = \sum_{n=0}^{\infty} \frac{(17x)^n}{n!} = \sum_{n=0}^{\infty} \frac{17^n x^n}{n!}$ </div> $x \in \mathbb{R}$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$ $= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">NOTE $y = \cos x$ IS AN <u>EVEN</u> FUNCTION (I.E., $\cos(-x) = +\cos(x)$) AND THE TAYLOR SERIS OF $y = \cos x$ HAS ONLY <u>EVEN</u> POWERS.</div> $x \in \mathbb{R}$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots$ $= \sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^{2n-1}}{(2n-1)!} \stackrel{\text{or}}{=} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">NOTE $y = \sin x$ IS AN <u>ODD</u> FUNCTION (I.E., $\sin(-x) = -\sin(x)$) AND THE TAYLOR SERIS OF $y = \sin x$ HAS ONLY <u>ODD</u> POWERS.</div> $x \in \mathbb{R}$
$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots$ $= \sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^n}{n} \stackrel{\text{or}}{=} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">QUESTION: IS $y = \ln(1+x)$ EVEN, ODD, OR NEITHER?</div> $x \in (-1, 1]$
$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots$ $= \sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^{2n-1}}{2n-1} \stackrel{\text{or}}{=} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">QUESTION: IS $y = \arctan(x)$ EVEN, ODD, OR NEITHER?</div> $x \in [-1, 1]$