SISTEMI DEL PRIMO ORDINE

19-4-2021

ESERCIZIO 1. Risolvere il seguente sistema del primo ordine

$$\begin{cases} x' = -5y \\ y' = -x - 4y \end{cases}$$

SOL.

$$y'' = -x' - 4y' \longrightarrow y'' = 5y - 4y' \longrightarrow y'' + 4y' - 5y = 0$$

$$p(\lambda) = 0 \iff \lambda^2 + 4\lambda - 5 = 0 \iff \lambda_{1,2} = -2 \pm \sqrt{4+5} = -2 \pm 3 \begin{cases} -5 \\ 1 \end{cases}$$

$$y(t) = c_1 e^{t} + c_2 e^{-5t}$$

$$x = -y' - 4y$$

$$y'(t) = c_1 e^{t} - 5c_2 e^{-5t}$$

$$x(t) = -\left(c_1 e^{t} - 5c_2 e^{-5t}\right) - 4\left(c_1 e^{t} + c_2 e^{-5t}\right)$$

$$x(t) = -c_1 e^{t} - 4c_1 e^{t} + 5c_2 e^{-5t} - 4c_2 e^{-5t}$$

$$x(t) = -5c_1 e^{t} + c_2 e^{-5t}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} -5c_1 e^{t} + c_2 e^{-5t} \\ c_1 e^{t} + c_2 e^{-5t} \end{bmatrix} =$$

$$= \begin{bmatrix} -5 \\ 1 \end{bmatrix} c_1 e^{t} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} c_2 e^{-5t}.$$

SOLUZIONE ALTERNATIVA:

$$\begin{cases} x' = -5y \\ y' = -x - 4y \end{cases} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & -5 \\ -1 & -4 \end{bmatrix}$$

$$p_A(\lambda) = \det[A - \lambda I] = \det \begin{bmatrix} -\lambda & -5 \\ -1 & -4-\lambda \end{bmatrix} =$$

$$= -\lambda(-4-\lambda) - 5 =$$

$$= 4\lambda + \lambda^2 - 5 = \lambda^2 + 4\lambda - 5$$

$$p_A(\lambda) = 0 \iff \lambda_{1,2} = \begin{cases} 1 \\ -5 \end{cases}$$ (AUTOVALORI DI A).

$$V_1 = \ker[A - I] = \ker \begin{bmatrix} -1 & -5 \\ \cancel{-1} & \cancel{-5} \end{bmatrix}$$

$$-a - 5b = 0$$

$$\begin{cases} a = -5b \\ b \in \mathbb{R} \end{cases}$$

$$\begin{cases} a = -5 \\ b = 1 \end{cases}$$

$$V_1 = \mathrm{Span}\left(\begin{bmatrix} -5 \\ 1 \end{bmatrix} \right)$$

$$V_{-5} = \ker[A + 5I] =$$

$$= \ker \begin{bmatrix} 5 & -5 \\ \cancel{-1} & \cancel{1} \end{bmatrix}$$

$5a - 5b = 0 \rightarrow a = b$

$$\begin{cases} a = b \\ b \in \mathbb{R} \end{cases}$$

Una sol. è, per es., $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow V_{-5} = \mathrm{Span}\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$

$$\begin{bmatrix} x \\ y \end{bmatrix} = c_1 e^{1t} \begin{bmatrix} -5 \\ 1 \end{bmatrix} + c_2 e^{-5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

oss.

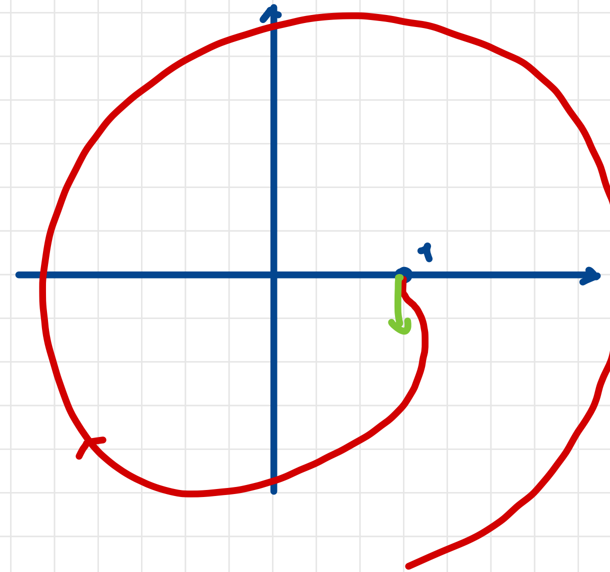
$$\begin{cases} x(t) = -5c_1 e^{t} + c_2 e^{-5t} \\ y(t) = c_1 e^{t} + c_2 e^{-5t} \end{cases}$$

$t = 0$ $\begin{cases} x(0) = -5c_1 + c_2 \\ y(0) = c_1 + c_2 \end{cases}$

passaggio per $(1; 0)$

$$\begin{cases} -5c_1 + c_2 = 1 \\ c_1 + c_2 = 0 \end{cases} \rightarrow \begin{cases} c_1 = -\frac{1}{6} \\ c_2 = \frac{1}{6} \end{cases}$$

$$\begin{cases} x(t) = +\frac{5}{6}e^{t} + \frac{1}{6}e^{-5t} \\ y(t) = -\frac{1}{6}e^{t} + \frac{1}{6}e^{-5t} \end{cases}$$



ESERCIZIO 2.

Determinare l'integrale generale del sistema:

$$\begin{cases} x' = 2x - 5y \\ y' = 5x - 6y \end{cases}$$

Sol.

$$\begin{cases} 5y = 2x - x' \longrightarrow \underline{y = \frac{2}{5}x - \frac{1}{5}x'} \longrightarrow y' = \frac{2}{5}x' - \frac{1}{5}x'' \\ \frac{2}{5}x' - \frac{1}{5}x'' = 5x - \frac{12}{5}x + \frac{6}{5}x' \end{cases}$$

$$2x' - x'' = 25x - 12x + 6x'$$

$$x'' + 4x' + 13x = 0$$

$$p(\lambda) = \lambda^2 + 4\lambda + 13 \qquad p(\lambda) = 0 \iff \lambda_{1,2} = -2 \pm 3i$$

$$x(t) = e^{-2t}\left(c_1 \cos 3t + c_2 \operatorname{sen} 3t\right)$$

$$y(t) = \frac{2}{5}x(t) - \frac{1}{5}x'(t)$$

$$x'(t) = -2e^{-2t}\left(c_1 \cos 3t + c_2 \operatorname{sen} 3t\right) +$$
$$+ e^{-2t}\left(-3c_1 \operatorname{sen} 3t + 3c_2 \cos 3t\right) =$$

$$= e^{-2t}\left(-2c_1\cos 3t - 2c_2 \operatorname{sen} 3t - 3c_1 \operatorname{sen} 3t + 3c_2 \cos 3t\right) =$$

$$= e^{-2t}\left(\left(-2c_1+3c_2\right)\cos 3t + \left(-3c_1-2c_2\right)\operatorname{sen} 3t\right)$$

$$y(t) = \frac{2}{5}e^{-2t}\left(c_1\cos 3t + c_2 \operatorname{sen} 3t\right) - \frac{1}{5}e^{-2t}\left(\left(-2c_1+3c_2\right)\cos 3t + \left(-3c_1-2c_2\right)\operatorname{sen} 3t\right).$$

In definitiva:

$$\begin{cases} x(t) = e^{-2t}\left(c_1\cos 3t + c_2 \operatorname{sen} 3t\right) \\ y(t) = \frac{1}{5}e^{-2t}\left(\left(3c_1+4c_2\right)\operatorname{sen} 3t + \left(4c_1-3c_2\right)\cos 3t\right) \end{cases}$$

IN ALTERNATIVA:

$$\begin{cases} x' = 2x - 5y \\ y' = 5x - 6y \end{cases}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ 5 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$p_A(\lambda) = \det \begin{bmatrix} 2-\lambda & -5 \\ 5 & -6-\lambda \end{bmatrix} = (2-\lambda)(-6-\lambda) + 25 =$$
$$= -12 + 4\lambda + \lambda^2 + 25 =$$
$$= \lambda^2 + 4\lambda + 13$$

$$p_A(\lambda) = 0 \iff \lambda_{1,2} = -2 \pm 3i$$

$$V_{-2+3i} : \quad \operatorname{Ker}\left[A - (-2+3i) I\right] = \operatorname{Ker} \begin{bmatrix} 4-3i & -5 \\ \cancel{5} & \cancel{-4-3i} \end{bmatrix}$$

$$\begin{cases} (4-3i)a - 5b = 0 \\ b \in \mathbb{C} \end{cases} \longrightarrow \begin{cases} a = \dfrac{5}{4-3i} b \\ b \in \mathbb{C} \end{cases}$$

Una sol è, per esempio:

$$\begin{cases} a = 5 \\ b = 4-3i \end{cases} \qquad \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 5+0i \\ 4-3i \end{bmatrix}$$

$$V_{-2+3i} = \text{Span}\left(\begin{bmatrix} 5+0i \\ 4-3i \end{bmatrix}\right) = \text{Span}\left(\begin{bmatrix} 5 \\ 4 \end{bmatrix} + i\begin{bmatrix} 0 \\ -3 \end{bmatrix}\right)$$

L'integrale generale del sistema è:

$$\underline{u}(t) = c_1 \underline{u}_1(t) + c_2 \underline{u}_2(t) \qquad c_1, c_2 \in \mathbb{R}.$$

dove
$$\underline{u}_1(t) = e^{-2t}\left(\cos 3t \begin{bmatrix} 5 \\ 4 \end{bmatrix} - \sin 3t \begin{bmatrix} 0 \\ -3 \end{bmatrix}\right)$$
$$\underline{u}_2(t) = e^{-2t}\left(\sin 3t \begin{bmatrix} 5 \\ 4 \end{bmatrix} + \cos 3t \begin{bmatrix} 0 \\ -3 \end{bmatrix}\right)$$

oss. L'origine è asintoticamente stabile.

ESERCIZIO 3. Determinare l'integrale generale del sistema

$$\begin{cases} x' = -x + 4y \\ y' = -x + 3y \end{cases}$$

Sol.

$$\begin{cases} -y'' + 3y' = y' - 3y + 4y \longrightarrow \boxed{y'' - 2y' + y = 0} \\ x = -y' + 3y \longrightarrow x' = -y'' + 3y' \end{cases}$$

$$p(\lambda) = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2 \qquad \lambda = 1$$

$$y(t) = c_1 e^t + c_2 t e^t$$

$$y'(t) = c_1 e^t + c_2 e^t + c_2 t e^t$$

$$x(t) = -y'(t) + 3y(t)$$

$$x(t) = -c_1 e^t - c_2 e^t - c_2 t e^t + 3c_1 e^t + 3c_2 t e^t$$

$$x(t) = 2c_1 e^t + c_2 e^t (2t-1)$$

$$\begin{cases} x(t) = 2c_1 e^t + c_2 e^t (2t-1) \\ y(t) = c_1 e^t + c_2 t e^t \end{cases}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^t \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^t \begin{bmatrix} 2t-1 \\ t \end{bmatrix} =$$

$$= c_1 e^t \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 e^t \left(t \begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right)$$

In modo alternativo:

$$\begin{cases} x' = -x + 4y \\ y' = -x + 3y \end{cases} \qquad A = \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$$

$$p_A(\lambda) = \det\begin{bmatrix} -1-\lambda & 4 \\ -1 & 3-\lambda \end{bmatrix} = (-1-\lambda)(3-\lambda)+4 =$$

$$= -3-2\lambda+\lambda^2+4 =$$

$$= \lambda^2-2\lambda+1 = (\lambda-1)^2$$

$\lambda = 1$ autovalore di m.a. $= 2$.

$$V_1 = \operatorname{Ker}[A-I] = \operatorname{Ker}\begin{bmatrix} -2 & 4 \\ \cancel{-1} & \cancel{2} \end{bmatrix} \quad \begin{cases} -\cancel{2}a + \overset{2}{\cancel{4}}b = 0 \\ b \in \mathbb{R} \end{cases}$$

$$\begin{cases} a = 2b \\ b \in \mathbb{R} \end{cases} \longrightarrow \begin{cases} a = 2 \\ b = 1 \end{cases}$$

$$V_1 = \operatorname{Span}\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) \qquad \dim V_1 = 1 = \text{m.g.} \neq \text{m.a.}$$

$\underline{w}$

A non è diagonalizzabile.

Cerco un vettore $\underline{v} = \begin{bmatrix} a \\ b \end{bmatrix}$ tale che:

$$[A - \lambda'' I]\, \underline{v} = \underline{w}$$

$$\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} \cancel{-2} & \cancel{4} & \cancel{2} \\ -1 & 2 & 1 \end{array}\right] \qquad -a + 2b = 1 \qquad \begin{cases} a = 2b - 1 \\ b \in \mathbb{R} \end{cases}$$

$$\begin{cases} a = -1 \\ b = 0 \end{cases} \implies v = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

L'integrale generale è:

$$\underline{u}(t) = c_1 \underline{u}_1(t) + c_2 \underline{u}_2(t) \qquad c_1, c_2 \in \mathbb{R}$$

con $u_1(t) = e^{\lambda = 1 \cdot t} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$u_2(t) = e^t\left(t\begin{bmatrix}2\\1\end{bmatrix}+\begin{bmatrix}-1\\0\end{bmatrix}\right).$$

L'int. generale è:

$$\underline{u}(t) = c_1 e^t\begin{bmatrix}2\\1\end{bmatrix} + c_2 e^t\left(t\begin{bmatrix}2\\1\end{bmatrix}+\begin{bmatrix}-1\\0\end{bmatrix}\right).$$

SISTEMI LINEARI DEL I ORDINE 2x2

$$\begin{cases} x' = ax + by \\ y' = cx + dy \end{cases} \qquad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} x \\ y \end{bmatrix} \quad \underline{u}' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\Updownarrow$$

$$\underline{u}' = A\,\underline{u}$$

L'integrale generale del sistema è

$$\underline{u}(t) = c_1 \underline{u}_1(t) + c_2 \underline{u}_2(t)$$

$c_1, c_2 \in \mathbb{R}$ e :

1) Se A ha due autovettori reali e linearmente indipendenti $\underline{w}_1$ e $\underline{w}_2$ relativi agli autovalori λ_1 e λ_2

$$\underline{u}_1(t) = e^{\lambda_1 t} \underline{w}_1$$

$$\underline{u}_2(t) = e^{\lambda_2 t} \underline{w}_2$$

2) Se A ha un autovalore complesso $\lambda = \alpha + i\beta$ con $\beta \neq 0$ e $\underline{w} = \underline{A} + i \underline{B}$ è un autovettore corrispondente, allora

$$\underline{u}_1(t) = e^{\alpha t}\left(A\cos\beta t - B\,\text{sen}\,\beta t\right)$$

$$\underline{u}_2(t) = e^{\alpha t}\left(A\,\text{sen}\,\beta t + B\cos\beta t\right)$$

3) Se A ha un unico autovalore reale λ con molteplicità geometrica 1 e $\underline{w}$ è un autovettore corrispondente a λ allora esiste un vettore $\underline{v}$ t.c.

$$[A - \lambda I]\,\underline{v} = \underline{w} \qquad \text{e}$$

$$\underline{u}_1(t) = e^{\lambda t}\,\underline{w}$$

$$\underline{u}_2(t) = e^{\lambda t}\left(t\,\underline{w} + \underline{v}\right).$$

OSS. Per ogni matrice quadrata 2x2 si è in uno dei casi precedenti. Se A ha un unico autovalore con m.g. = 2 si scelgono come autovettori i vettori della base canonica.

ESERCIZIO 4. Determinare l'integrale generale del sistema NON OMOGENEO seguente:

$$\begin{cases} x' = 2x + 3y \\ y' = 2x + y + 2e^{-2t} \end{cases}$$

SOL.

$$\begin{cases} y = \frac{1}{3}(x' - 2x) \longrightarrow y' = \frac{1}{3}(x'' - 2x') \\ \cancel{\frac{1}{3}}(x'' - 2x') = \overset{6}{\cancel{2}}x + \cancel{\frac{1}{3}}(x' - 2x) + \overset{6}{\cancel{2}}e^{-2t} \end{cases}$$

$$x'' - 2x' - 6x - x' + 2x = 6e^{-2t}$$

$$x'' - 3x' - 4x = 6e^{-2t}$$

INT. GEN. OMOGENEA:

$$p(\lambda) = \lambda^2 - 3\lambda - 4$$

$$p(\lambda) = 0 \iff \lambda_{1,2} = \frac{3 \pm \sqrt{9+16}}{2} = \begin{cases} 4 \\ -1 \end{cases}$$

$$x_0(t) = c_1 e^{-t} + c_2 e^{4t}$$

SOL. PARTICOLARE:

$$x_p(t) = A e^{-2t}$$

$$x_p'(t) = -2A e^{-2t}$$

$$x_p''(t) = 4A e^{-2t}$$

$$x_p'' - 3x_p' - 4x_p = 6e^{-2t}$$

$$\cancel{4Ae^{-2t}} + 6Ae^{-2t} - \cancel{4Ae^{-2t}} = 6e^{-2t} \quad \rightarrow 6A = 6$$

$$\rightarrow A = 1$$

$$x_p(t) = e^{-2t}$$

L'int. generale è: $x(t) = x_0(t) + x_p(t)$

$$x(t) = c_1 e^{-t} + c_2 e^{4t} + e^{-2t}$$

$$x'(t) = -c_1 e^{-t} + 4c_2 e^{4t} - 2e^{-2t}$$

$$y(t) = \frac{1}{3}\left(-c_1 e^{-t} + 4c_2 e^{4t} - 2e^{-2t} - 2c_1 e^{-t} - 2c_2 e^{4t} - 2e^{-2t}\right)$$

$$= \frac{1}{3}\left(-3c_1 e^{-t} + 2c_2 e^{4t} - 4e^{-2t}\right) =$$

$$= -c_1 e^{-t} + \frac{2}{3}c_2 e^{4t} - \frac{4}{3}e^{-2t}$$

L'int. generale del sistema è:

$$\begin{cases} x(t) = c_1 e^{-t} + c_2 e^{4t} + e^{-2t} \\ y(t) = -c_1 e^{-t} + \frac{2}{3} c_2 e^{4t} - \frac{4}{3} e^{-2t} \end{cases}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{4t} \begin{bmatrix} 1 \\ 2/3 \end{bmatrix} + e^{-2t} \begin{bmatrix} 1 \\ -4/3 \end{bmatrix}.$$

ESERCIZIO 5. Siano assegnati $\underline{z}(t)$ e A nel modo seguente:

$$z(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$$

1) Scrivere l'int. generale di $z' = Az$.
2) Risolvere il Pb. d. C. $z(0) = \begin{bmatrix} 7 \\ 0 \end{bmatrix}$
3) Tra tutte le soluzioni infinitesime per $t \to +\infty$ determinare quella per cui

$x(0) = 10.$

SOL.

1) $A = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$

$p_A(\lambda) = \det \begin{bmatrix} 1-\lambda & 5 \\ 2 & 4-\lambda \end{bmatrix} = (1-\lambda)(4-\lambda) - 10 =$

$= 4 - 5\lambda + \lambda^2 - 10 = \lambda^2 - 5\lambda - 6$

$p_A(\lambda) = 0 \iff \lambda_{1,2} = \begin{cases} -1 \\ 6 \end{cases}.$

$V_{-1} = \ker \begin{bmatrix} 2 & 5 \\ 2 & 5 \end{bmatrix}$ $\quad 2a + 5b = 0 \quad 2a = -5b$

$\begin{cases} a = 5 \\ b = -2 \end{cases}$ $\quad V_{-1} = \mathrm{Span}\left(\begin{bmatrix} 5 \\ -2 \end{bmatrix}\right)$

$V_6 = \ker \begin{bmatrix} -5 & 5 \\ 2 & -2 \end{bmatrix}$ $\quad -5a + 5b = 0 \rightarrow a = b$

$$V_6 = \mathrm{Span}\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$$

$$\underline{z}(t) = c_1 e^{-t}\begin{bmatrix}5\\-2\end{bmatrix} + c_2 e^{6t}\begin{bmatrix}1\\1\end{bmatrix} \qquad c_1, c_2 \in \mathbb{R}$$

$$\begin{cases} x(t) = 5c_1 e^{-t} + c_2 e^{6t} \\ y(t) = -2c_1 e^{-t} + c_2 e^{6t} \end{cases}$$

2) $$\underline{z}(0) = \begin{bmatrix}7\\0\end{bmatrix} \longrightarrow \begin{cases} x(0) = 7 \\ y(0) = 0 \end{cases} \qquad \begin{cases} 5c_1 + c_2 = 7 \\ -2c_1 + c_2 = 0 \end{cases}$$

$$\longrightarrow \begin{cases} 5c_1 + c_2 = 7 \longrightarrow c_2 = 2 \\ 7c_1 = 7 \rightarrow c_1 = 1 \end{cases}$$

Sol. Pb. di C. : $$\underline{\bar{z}}(t) = e^{-t}\begin{bmatrix}5\\-2\end{bmatrix} + 2e^{6t}\begin{bmatrix}1\\1\end{bmatrix}.$$

3) Tutte le sol. infinitesime sono quelle per cui $c_2 = 0$

$$\underline{z}(t) = c_1 e^{-t} \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$x(0) = 10 \longrightarrow 5 c_1 \cancel{e^0} = 10 \longrightarrow c_1 = 2.$$

$$\boxed{\underline{\tilde{z}}(t) = 2 e^{-t} \begin{bmatrix} 5 \\ -2 \end{bmatrix}}.$$

RIPASSO.

ESERCIZIO 6. Si consideri la curva piana di eq. parametriche:

$$\underline{r}(t) = \operatorname{sen} t \; \underline{i} + \operatorname{sen} 2t \; \underline{j} \qquad t \in [0; 2\pi].$$

1) Dimostrare che la curva è chiusa, regolare ma non semplice.

2) Verificare che il sostegno γ della curva soddisfa l'eq. $4x^4 - 4x^2 + y^2 = 0$.
Disegnare γ e indicarne il verso di percorrenza.

3) Trovare al variare di $c \in \mathbb{R}$ gli eventuali punti singolari della curva definita implicitamente dall'eq. $4x^4 - 4x^2 + y^2 = c$.

SOL.

$$\underline{r}(t) = \begin{bmatrix} \operatorname{sen} t \\ \operatorname{sen} 2t \end{bmatrix} \qquad t \in [0; 2\pi]$$

1) $\underline{r}(0) = \underline{r}(2\pi) = (0; 0) \longrightarrow r$ è chiusa.

$$\underline{r}'(t) = \begin{bmatrix} \cos t \\ 2\cos 2t \end{bmatrix} \qquad \| \underline{r}'(t) \| = \sqrt{\cos^2 t + 4\cos^2 2t} \neq 0$$

$\Rightarrow$ regolare.

$\underline{r}(0) = \underline{r}(\pi) = (0;0)$ si interseca in $(0,0)$

$\Rightarrow$ non è semplice.

2)

$$4x^4 - 4x^2 + y^2 = 0$$

$$4\operatorname{sen}^4 t - 4\operatorname{sen}^2 t + \operatorname{sen}^2 2t =$$

$$= 4\operatorname{sen}^4 t - 4\operatorname{sen}^2 t + 4\operatorname{sen}^2 t\cos^2 t =$$

$$= 4\operatorname{sen}^2 t\left(\operatorname{sen}^2 t - 1 + \cos^2 t\right) = 0$$

$$4x^4 - 4x^2 + y^2 = 0 \longrightarrow y^2 = 4x^2 - 4x^4$$

$$y^2 = 4x^2(1 - x^2)$$

$$y = \pm\sqrt{4x^2(1-x^2)}$$

$$y = \pm 2|x|\sqrt{1-x^2}$$

- $y = 2|x|\sqrt{1-x^2}$ $\qquad \mathbb{D} = [-1,1]$

$$y = \begin{cases} 2x\sqrt{1-x^2} & \text{se } 0 \le x \le 1 \\ -2x\sqrt{1-x^2} & \text{se } -1 \le x < 0 \end{cases}$$

$[0;1] \rightarrow y \geq 0$

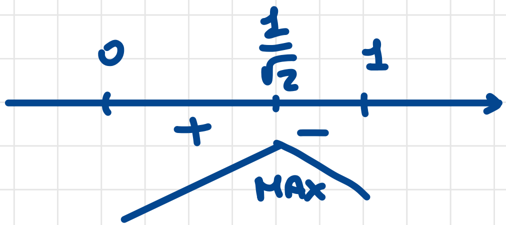
$x=0 \rightarrow y=0, \quad x=1 \Rightarrow y=0$

$$y' = 2\sqrt{1-x^2} + 2x \cdot \frac{1\cdot(-\cancel{2}x)}{\cancel{2}\sqrt{1-x^2}} = \frac{2-2x^2-2x^2}{\sqrt{1-x^2}} = \frac{2(1-2x^2)}{\sqrt{1-x^2}}$$

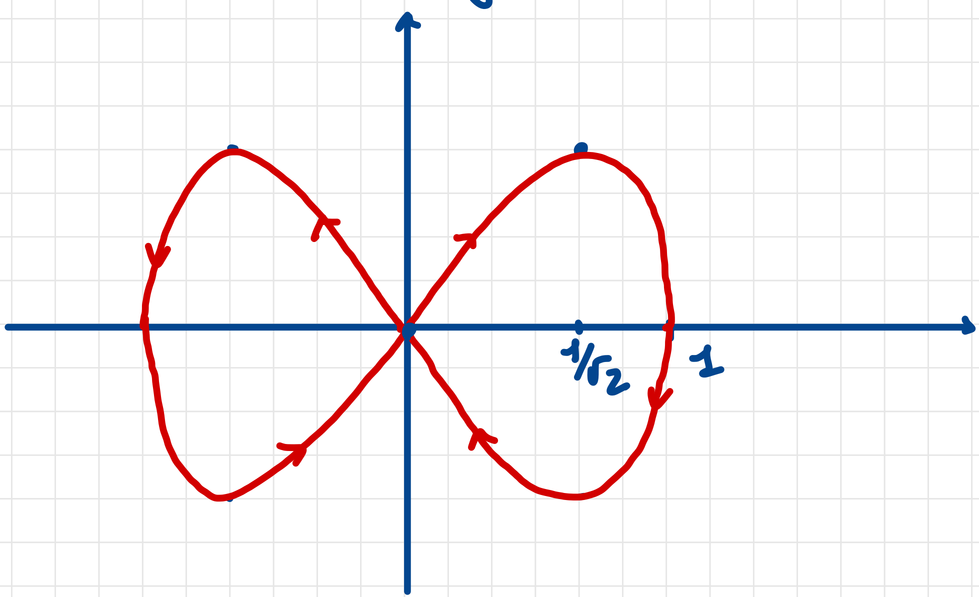
$$y' = 0 \iff 1-2x^2=0 \iff x = \cancel{\pm}\frac{1}{\sqrt{2}}$$

$$y' > 0 \iff 1 - 2x^2 > 0 \iff -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \iff$$

$$\iff 0 < x < \frac{1}{\sqrt{2}}$$



$$\lim_{x \to 1^-} y'(x) = \infty \Rightarrow \text{tg verticale.}$$



3) $4x^4 - 4x^2 + y^2 = c$

$$\begin{cases} 16x^3 - 8x = 0 \rightarrow 8x(2x^2-1) = 0 \begin{cases} x = 0 \\ x = \pm \frac{1}{\sqrt{2}} \end{cases} \\ 2y = 0 \rightarrow y = 0 \\ 4x^4 - 4x^2 + y^2 = c \end{cases}$$

$(0,0)$ è un punto singolare x $c=0$

$\left(\pm \frac{1}{\sqrt{2}}; 0\right)$ sono punti singolari x $c = -1$

Ci sono punti singolari solo per $c = 0 \vee c = -1$.