

SERGENTI FILIPPO

10743161

2 ∈ ℝ  $|x| \text{ piccolo}$

$$f_2(x) = e^{2x^4 - 3x^6} + \log(1 - x^2 + o(x^4)) - 1 + x^2 - \left(\frac{3}{2} + 2\right)x^4 + \frac{x^6}{2}$$

② SVILUPPA TAYLOR ORDINE 8 IN ORIGINALE,  $f_2^K(u)$

$$\begin{aligned} 2x^4 - 3x^6 &= 1 + (2x^4 - 3x^6) + \frac{1}{2} (2x^4 - 3x^6)^2 + o(x^8) \\ &= 1 + 2x^4 - 3x^6 + \frac{1}{2} (4x^8) + o(x^8) \\ &= \boxed{1 + 2x^4 - 3x^6 + \frac{1}{2} 2x^8 + o(x^8)} \end{aligned}$$

$$\begin{aligned} \log(1 - x^2 + o(x^4)) &= (\cancel{-x^2} + o(x^4)) - \frac{1}{2} (-x^2 + o(x^4))^2 + \frac{1}{3} (-x^2 + o(x^4))^3 \\ &\quad - \frac{1}{4} (-x^2 + o(x^4))^4 \\ &= -x^2 + o(x^4) - \frac{1}{2} (4x^8 - 2x^6) + \frac{1}{3} (-x^6 + 3x^8) - \frac{1}{4} (+x^8) = \\ &= -x^2 + o(x^4) - \frac{x^4}{2} + \cancel{\frac{o^2 x^8}{2}} + \cancel{o(x^6)} \left(-\frac{x^6}{3}\right) + o(x^8) - \frac{x^8}{4} = \\ &= \boxed{-x^2 + \left(\frac{2o - \frac{1}{2}}{2}\right)x^4 + \left(\frac{3o - \frac{1}{3}}{3}\right)x^6 + \left(\frac{-2o^2 + 12o - 1}{4}\right)x^8 + o(x^8)} \end{aligned}$$

$$\begin{aligned} f_2(x) &= \cancel{(2x^4)} \boxed{3x^6} + 2x^8 - \cancel{x^2} + \left(\frac{2o - \frac{1}{2}}{2}\right)x^4 + \left(\frac{3o - \frac{1}{3}}{3}\right)x^6 + \left(\frac{-2o^2 + 12o - 1}{4}\right)x^8 + \\ &\quad + x^2 - 4 - \left(\frac{3}{2} + 2\right)x^4 + \frac{x^6}{3} \end{aligned}$$

~~1 + 2 +  $\frac{2o - 1}{2}$  -  $\frac{3}{2}$  - 2~~  $x^4 + (-3 + \frac{3o - 1}{3})x^6 + \cancel{\left(\frac{-2o^2 + 12o - 1}{4}\right)x^8} + \frac{1}{3}x^6$

$$\begin{aligned} &+ \left(2 + -\frac{2o^2 + 12o - 1}{4}\right)x^8 \end{aligned}$$

$$\begin{aligned} e^x &= 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + o(x^3) \\ \log(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + o(x^4) \end{aligned}$$

$$\begin{aligned}f_2(0) &= 0 \\f_2(1) &= 0 \\f_2(2) &\neq 0 \\f_2(3) &\neq 0 \\f_2(4) &\neq 0\end{aligned}$$

$$\begin{aligned}f_2^1(0) &= 0 \\f_2^2(0) &= 0\end{aligned}$$

$$\begin{aligned}f_2^3(0) &= 0 \\f_2^4(0) &= 2 + \frac{2x-4}{2} - \frac{3x-2}{2} = \frac{4+2x-4-3x+2}{2} = \frac{2-3x}{2} \\f_2^5(0) &= 0 \\f_2^6(0) &= -3 + \frac{2x-1}{3} + \frac{1}{3} = \frac{-9+3x+1}{3} = -\frac{9+3x}{3} = -3+x \\f_2^7(0) &= 0 \\f_2^8(0) &= 2 + \frac{-2x^2+4x-1}{4} = \frac{-2x^2+4x-1+8}{4} = \frac{-2x^2+4x+7}{4}\end{aligned}$$

c)

$$b > 0 \text{ piccolo} \quad \text{TAKE ONE} \quad f_2 \quad [0, b]$$

$$\text{A} \in \mathbb{R} \quad \int_0^b \frac{(\sin x)^2}{f_a(x)} dx \quad \text{È FINITO?}$$

~~DISCONTINUITÀ~~

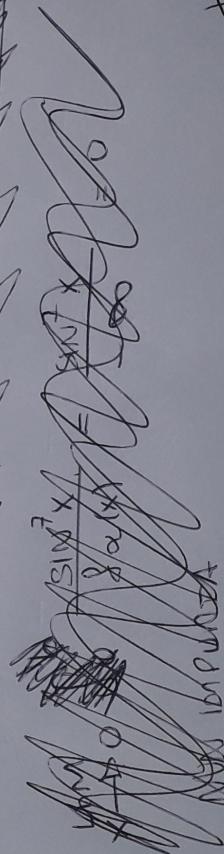
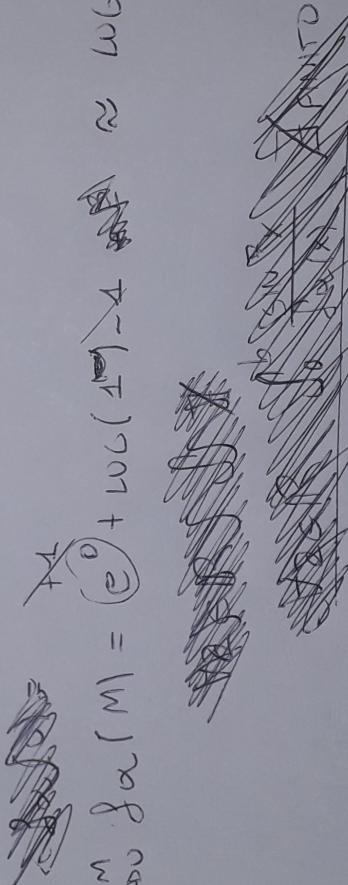
PER STUDIARE L'ESISTENZA DELL'INTEGRALE GENERALIZZATO  
SI STUDIANO I PUNTI DI DISCONTINUITÀ (SINGULARITÀ)  
DELLA SUA DERIVATA

$$\left( \frac{\sin^2 x}{f_a(x)} \right)$$

IN QUESTO CASO  $f_a(x) \neq 0$  ( $\rightarrow$  STUDIARE)  
INOLTRE IL SECONDO NUMERO È COSTANTE POICHÉ

$$-1 \leq \sin^2 x \leq +1$$

~~$$\lim_{x \rightarrow 0} f_a(x) = 0 + \log(-1) \rightarrow -\infty \approx \log(x+1) = -\infty$$~~



~~HORNAL~~

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{f_a(x)} = \lim_{x \rightarrow 0} \frac{x^2}{\log(x+1)} = \lim_{x \rightarrow 0} \frac{x^2}{\frac{1}{x+1}} = \lim_{x \rightarrow 0} \frac{x^3}{1} = 0$$

$$\lim_{x \rightarrow 0} x^2(x+1) = 0$$

CONVERGENTE

INTEGRALE IMPARARIO NON DIPENDE DA  $a \in \mathbb{C}$   
HAZER  $\int_0^b \frac{\sin^2 x}{f_a(x)} dx$  E' FINITO