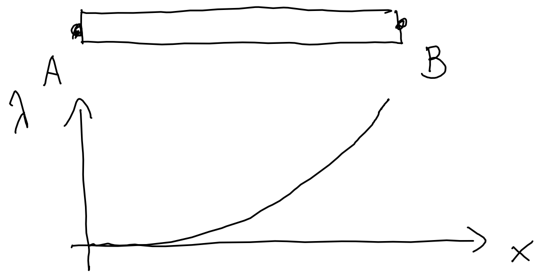
ESERCIZIO 1

$m = 200\,g$ $L = 1\,m$ $S = 3\,cm^2$

$\lambda(x) = Hx^2$



a) calcolo del centro di massa e H

$$X_{cm} = \frac{\int_0^L \lambda x\,dx}{m} = \int_0^L \frac{Hx^3}{m}\,dx = \frac{HL^4}{4m}$$

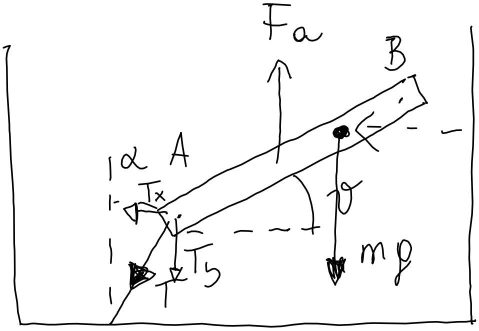
$$m = \int_0^L \lambda\,dx = \int_0^L Hx^2\,dx = HL^3/3$$

$$\hookrightarrow \quad H = \frac{3m}{L^3} = \frac{3 \cdot 0.2\,kg}{(1m)^3} = 0.6\,\frac{kg}{m^3}$$

$$X_{cm} = \frac{HL^4}{4m} = \frac{3\cancel{m}}{\cancel{L^3}}\frac{L^4}{4\cancel{m}} = \frac{3}{4}L = 75\,cm$$

(b)

b)



nel centro di massa

Eq Newton lungo $x \Rightarrow T_x = 0$

$$\Rightarrow \alpha = \operatorname{atan}\left(\frac{T_x}{T_y}\right) = 0$$

Eq Newton lungo y

$$F_a - mg - T = 0 \qquad (T = T_y)$$

$$T = F_a - mg = \rho V g - mg = g(\rho V - m)$$

dato che $T > 0$, $\rho > \frac{m}{V}$ $\boxed{\text{densità media del corpo}}$

c) 2ª eq cardinale

$$F_a \frac{L}{2} \cos\vartheta - F_g \frac{3}{4} L \cos\vartheta = 0 \qquad \underline{\text{EQUILIBRIO}}$$

$$\cos\vartheta \, \frac{L}{2} \left(\rho V - \frac{3}{2} m\right) g = 0$$

SOLUZIONI : $\vartheta = 0$, $\vartheta = \pi$ sono sempre punti di equilibrio. $\forall$ valore di ρ.

$\forall \vartheta$, $\rho V = \frac{3}{2} m$ EQUILIBRIO

Se $\rho = \frac{3m}{V} = \frac{3m}{2SL}$, allora si ha equilibrio statico $\forall \theta$.

$$\rho = \frac{3m}{2SL} = \frac{3 \cdot 0.2\,kg}{2 \cdot 3 \cdot 10^{-4}\,m^2 \cdot 1\,m} = 10^3 \frac{kg}{m^3}$$

In tal caso:

$$T = g(\rho V - m) = \frac{3}{2} mg - mg = \frac{1}{2} mg = 0.1\,kg \cdot 9.81 \frac{m}{s^2} \simeq 1\,N$$

ESERCIZIO 2

$r = 20\,cm = 0.2\,m$

$n = 4\,mol$ biatomico

$p = 1\,atm$ $T_1 = 20°C$

a) $pV = nRT = pSh_0 = p\pi r^2 h_0$

$$h_0 = \frac{nRT_1}{\pi r^2 p} = \frac{4\,\cancel{mol} \cdot 8.314 \frac{\cancel{J}\,Nm}{\cancel{mol}\,\cancel{K}} \cdot 293.15\,\cancel{K}}{\pi (0.2)^2\,\cancel{m^2} \cdot 101325\,\cancel{Pa}\,\frac{N}{m^2}}$$

$= 0.766\,m = 76.6\,cm$

b) trasf. rev. isobara

$T_2 = 0°C = 273.15\,K$

$$h_2 = \frac{nRT_2}{\pi r^2 p} = \frac{h_0}{T_1} \cdot T_2 = 76.6\,cm \cdot \frac{273.15}{293.15} =$$

$= 0.713\,m = 71.3\,cm$ $\Delta h = -5.3\,cm$

$\Delta S_{univ} = 0$ (trasf. rev.)

$\Delta S_{sorgenti} + \Delta S_{gas} = 0$

$\delta Q_{isobara} = nc_p\,dT$

$\delta S = \frac{\delta Q}{T} = \frac{nc_p\,dT}{T}$ $\Delta S_{gas} = nc_p \ln\left(\frac{T_2}{T_1}\right)$

$$\Delta S_{gas} = n \cdot \frac{7}{2} R \ln\left(\frac{T_2}{T_1}\right) =$$

$$= \frac{4\cancel{mol}}{2} \cdot 7 \cdot 8.314 \frac{J}{\cancel{mol}\,K} \ln\left(\frac{273.15}{293.15}\right)$$

$$= -8.225\ J/K$$

$$\Delta S_{sorgente} = -\Delta S_{gas} = 8.225\ J/K$$

(c) ISOCORA IRREVERSIBILE

$$P_f = \frac{nRT_1}{\pi r^2 h_2} = \frac{\cancel{n}RT_1}{\cancel{\pi r^2}} \frac{\cancel{\pi r^2}\, P}{\cancel{nR}T_2} =$$

$$= 1\,atm\ \frac{293.15}{273.15} = 1.0732\,atm$$

$$\Delta S_{univ} = \Delta S_{gas} + \Delta S_{sorg}$$

$$\Delta S_{gas} = n c_v \ln\left(\frac{T_1}{T_2}\right) = n \frac{5}{2} R \ln\left(\frac{T_1}{T_2}\right)$$

$$\Delta S_{sorg} = \frac{\hat{Q}_1}{T_1}$$ $\hat{Q}_1$ calore fornito da T_1 al gas, pari a $\hat{Q}_1 = -n c_v \Delta T$

$$= -n c_v \left(\frac{T_1 - T_2}{T_1}\right) = n c_v \left(-1 + \frac{T_2}{T_1}\right)$$

$$\Delta S_{surr} = n\frac{5}{2}R\left[\ln\left(\frac{T_1}{T_2}\right) - 1 + \frac{T_2}{T_1}\right] =$$

$$= \frac{83.14\ J}{K}\left[\ln\left(\frac{293.15}{273.15}\right) - 1 + \frac{273.15}{293.15}\right] =$$

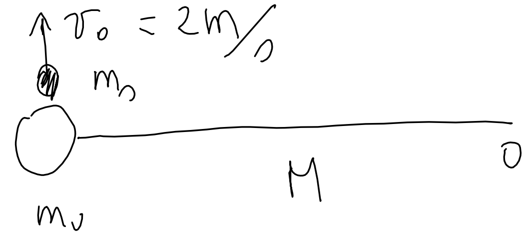
$$= 0.2\ \frac{J}{K} > 0$$

ESERCIZIO 3

$m_U = 80\,kg$

$M = 210\,kg$

$R = 1.5\,m$



(a) CONS. MOMENTO ANGOLARE → rispetto al polo in O

$$-m_s v_0 R + m_U \omega R R + \underbrace{I^{O}_{aste}}_{\frac{1}{12}MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{MR^2+3MR^2}{12} = \frac{1}{3}MR^2}\omega = 0$$

$$\omega\left(m_U R^{\cancel{2}} + \frac{1}{3}MR^{\cancel{2}}\right) = m_s v_0 \cancel{R}$$

$$\omega = \frac{m_s v_s}{R\left(m_U + \frac{M}{3}\right)} = \frac{15\,kg \cdot \frac{2m}{s}}{1.5\,m\,(80\,kg + 70\,kg)} = \frac{1}{5 \cdot 1.5}\,\frac{1}{s} = \frac{2}{15}\,\frac{rad}{s}$$

(b) $v_0 = \frac{2m}{s}$

$$v_U = \omega R = \frac{2 \cdot 1.5\,m}{15}\,\frac{rad}{s} = 0.2\,\frac{m}{s}$$

la velocità relativa è $2.2\,\frac{m}{s}$

(c) Si conserva il momento angolare

$$mR^2\omega + I^0_{asta}\omega = \omega_f I^0_{asta}$$

$$\omega_f = \omega + \frac{mR^2\omega}{I^0_{asta}} = \omega\left(1 + \frac{mR^2}{I^0_{asta}}\right)$$

$$= \omega\left(1 + \frac{m\cancel{R^2}}{\frac{1}{3}M\cancel{R^2}}\right) = \omega\left(1 + \frac{3m}{M}\right) =$$

$$= \omega\left(1 + \frac{240\,kg}{210\,kg}\right) =$$

$$= \omega\left(1 + \frac{8}{7}\right) = \frac{15}{7}\omega$$

$$= \frac{\cancel{15}}{7}\,\frac{2}{\cancel{15}}\,\frac{rad}{s} = \frac{2}{7}\,\frac{rad}{s}$$

(d) lavoro che deve compiere per spostarmi

$$L = \Delta E_k$$

$$E_k^{iniz} = \frac{1}{2}\left(m_v R^2 + I^0_{asta}\right)\omega^2 =$$

$$E_k^{fin} = \frac{1}{2} I^0_{asta}\, \omega_f^2 =$$

$$= \frac{1}{2} I^0_{asta}\left[\omega^2 + \frac{m_v^2 R^4 \omega^2}{I^{0\,2}_{asta}} + \frac{2\omega^2 m_v R^2}{I^0_{asta}}\right] =$$

$$= \frac{1}{2} I^0_{asta}\omega^2 + \frac{1}{2}\frac{m_v^2 R^4 \omega^2}{I^0_{asta}} + \omega^2 m_v R^2$$

$$\Delta E_k = E_k^{fin} - E_k^{iniz} = \frac{1}{2} m_v R^2 \omega^2 + \frac{1}{2} m_v R^2 \left(\frac{m_v R^2}{I^0_{asta}}\right)\omega^2$$

$$= \frac{1}{2} m_v R^2\, \omega\, \omega_f =$$

$$= 40\,kg \cdot (1.5)^2\, m^2 \cdot \frac{2}{15}\; \frac{2}{7}\; \frac{1}{s^2} =$$

$$= \boxed{3.43 \ J}$$