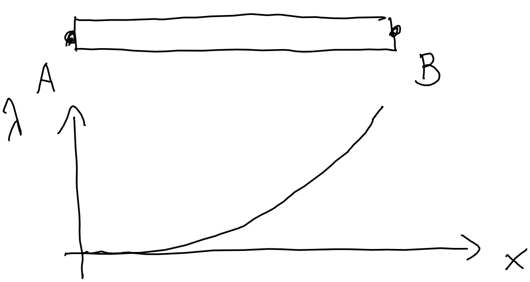
ESERCIZIO 1

$m = 200\,g$ $\quad L = 1\,m$ $\quad S = 3\,cm^2$

$\lambda(x) = Hx^2$



a) calcolo del centro di massa e H

$$X_{cm} = \frac{\int_0^L \lambda x\,dx}{m} = \int_0^L \frac{Hx^3}{m}dx = \frac{HL^4}{4m}$$

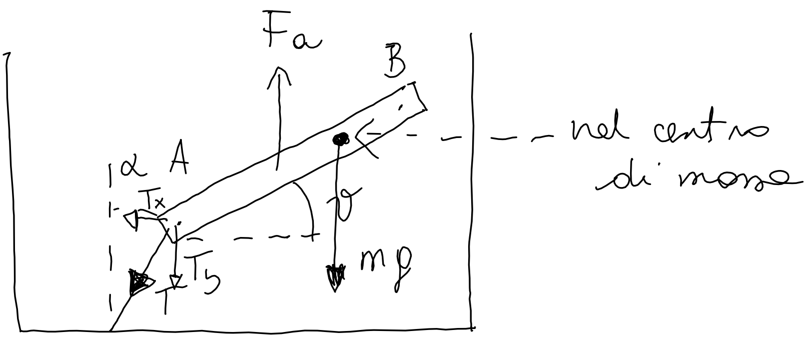
$$m = \int_0^L \lambda\,dx = \int_0^L Hx^2\,dx = HL^3/3$$

$$\hookrightarrow \quad H = \frac{3m}{L^3} = \frac{3 \cdot 0.2\,kg}{(1m)^3} = 0.6\,\frac{kg}{m^3}$$

$$X_{cm} = \frac{HL^4}{4m} = \frac{3\cancel{m}}{\cancel{L^3}}\frac{L^4}{4\cancel{m}} = \frac{3}{4}L = 75\,cm$$

b)

b)



Eq Newton lungo x $\Rightarrow$ $T_x = 0$

$\Rightarrow$ $\alpha = \text{atan}\left(\frac{T_x}{T_y}\right) = 0$

Eq Newton lungo y

$F_a - mg - T = 0$ $\qquad (T = T_y)$

$T = F_a - mg = \rho V g - mg = g(\rho V - m)$

dato che $T>0$, $\rho > \frac{m}{V}$ $\boxed{\text{densità media del corpo}}$

c) 2ª eq cardinale

$F_a \frac{L}{2} \cos\theta - F_g \frac{3}{4} L \cos\theta = 0$ $\qquad$ EQUILIBRIO

$\cos\theta \frac{L}{2} \left(\rho V - \frac{3}{2} m\right) g = 0$

SOLUZIONI: $\theta = 0$, $\theta = \pi$ sono sempre punti di equilibrio $\forall$ valore di ρ.

$\forall \theta$, $\rho V = \frac{3}{2} m$ $\qquad$ EQUILIBRIO

Se $\rho = \frac{3m}{V} = \frac{3m}{2SL}$, allora si ha equilibrio statico $\forall \vartheta$.

$$\rho = \frac{3m}{2SL} = \frac{3 \cdot 0.2\,kg}{2 \cdot 3 \cdot 10^{-4}\,m^2 \cdot 1\,m} = 10^3 \frac{kg}{m^3}$$

In tal caso:

$$T = g(\rho V - m) = \frac{3}{2} mg - mg = \frac{1}{2} mg = 0.1\,kg \cdot 9.81 \frac{m}{s^2} \simeq 1N$$

ESERCIZIO 2

$r = 20\,cm = 0.2\,m$

$n = 4\,mol$ biatomico

$p = 1\,atm$ $T_1 = 20°C$

a) $pV = nRT = pSh_0 = p\pi r^2 h_0$

$$h_0 = \frac{nRT_1}{\pi r^2 p} = \frac{4\,\cancel{mol} \cdot 8.314 \frac{J\ \cancel{Nm}}{\cancel{mol}\,\cancel{K}} \cdot 293.15\,\cancel{K}}{\pi (0.2)^2\,\cancel{m^2} \cdot 101325\,\cancel{Pa}\,\frac{\cancel{N}}{\cancel{m^2}}}$$

$$= 0.766\,m = 76.6\,cm$$

b) trasf. rev. isobara

$T_2 = 0°C = 273.15\,K$

$$h_2 = \frac{nRT_2}{\pi r^2 p} = \frac{h_0}{T_1} \cdot T_2 = 76.6\,cm \cdot \frac{273.15}{293.15} =$$

$$= 0.713\,m = 71.3\,cm \qquad \Delta h = -5.3\,cm$$

$\Delta S_{univ} = 0$ (trasf. rev.)

$\Delta S_{sorgenti} + \Delta S_{gas} = 0$

$\delta Q_{isobara} = nc_p\,dT$

$\delta S = \frac{\delta Q}{T} = \frac{nc_p\,dT}{T}$ $\Delta S_{gas} = nc_p \ln\left(\frac{T_2}{T_1}\right)$

$$\Delta S_{gas} = n \cdot \frac{7}{2} R \ln\left(\frac{T_2}{T_1}\right) =$$

$$= \frac{4 \cancel{mol} \cdot 7}{2} \cdot 8.314 \frac{J}{\cancel{mol}\, K} \ln\left(\frac{273.15}{293.15}\right)$$

$$= -8.225\ J/K$$

$$\Delta S_{sorgente} = -\Delta S_{gas} = 8.225\ J/K$$

(c) ISOCORA IRREVERSIBILE

$$p_f = \frac{nRT_1}{\pi r^2 h_2} = \frac{\cancel{n}\cancel{R}T_1}{\cancel{\pi r^2}} \cdot \frac{\cancel{\pi r^2}\, p}{\cancel{n}\cancel{R}T_2} =$$

$$= 1\,atm\, \frac{293.15}{273.15} = 1.0732\,atm$$

$$\Delta S_{univ} = \Delta S_{gas} + \Delta S_{sorg}$$

$$\Delta S_{gas} = n c_v \ln\left(\frac{T_1}{T_2}\right) = n \frac{5}{2} R \ln\left(\frac{T_1}{T_2}\right)$$

$$\Delta S_{sorg} = \frac{Q_1}{T_1}$$ Q_1 calore fornito da T_1 al gas, pari a $Q_1 = n c_v \Delta T$

$$= n c_v \left(\frac{T_1 - T_2}{T_1}\right) = n c_v \left(1 - \frac{T_2}{T_1}\right)$$

$$\Delta S_{surr} = n\frac{5}{2}R\left[\ln\left(\frac{T_1}{T_2}\right) + 1 - \frac{T_2}{T_1}\right] =$$

$$= \frac{83.14\,J}{K}\left[\ln\left(\frac{293.15}{273.15}\right) + 1 - \frac{273.15}{293.15}\right] =$$

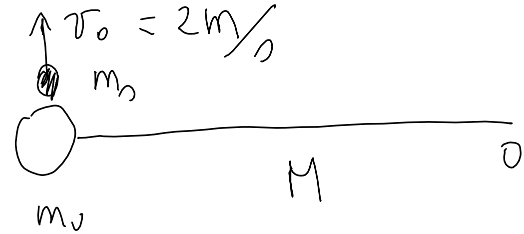
$$= \frac{83.14\,J}{K} \cdot 0.1389 = 11.55\,\frac{J}{K} > 0$$

ESERCIZIO 3

$m_u = 80\ kg$

$M = 210\ kg$

$R = 1.5\ m$



(a) CONS. MOMENTO ANGOLARE → rispetto al polo in O

$$-m_s v_0 R + m_u \omega R R + \underbrace{I^{O}_{asta}}_{\frac{1}{12}MR^2 + M\left(\frac{R}{2}\right)^2 = \frac{MR^2+3MR^2}{12} = \frac{1}{3}MR^2} \omega = 0$$

$$\omega\left(m_u R^{\cancel{2}} + \frac{1}{3} M R^{\cancel{2}}\right) = m_s v_0 \cancel{R}$$

$$\omega = \frac{m_s v_0}{R\left(m_u + \frac{M}{3}\right)} = \frac{15\ kg \cdot \frac{2m}{s}}{1.5m\,(80kg + 70kg)} = \frac{1}{5\ 1.5}\frac{1}{s} = \frac{2}{15}\frac{rad}{s}$$

(b) $v_0 = \frac{2m}{s}$

$$v_u = \omega R = \frac{2 \cdot 1.5 m}{15}\frac{rad}{s} = \frac{0.2\ m}{s}$$

la velocità relativa è $2.2 \frac{m}{s}$

(c) Si conserva il momento angolare

$$m R^2 \omega + I^0_{asta} \omega = \omega_f I^0_{asta}$$

$$\omega_f = \omega + \frac{m R^2 \omega}{I^0_{asta}} = \omega \left(1 + \frac{m R^2}{I^0_{asta}}\right)$$

$$= \omega \left(1 + \frac{m \cancel{R^2}}{\frac{1}{3} M \cancel{R^2}}\right) = \omega \left(1 + \frac{3m}{M}\right) =$$

$$= \omega \left(1 + \frac{240 kg}{210 kg}\right) =$$

$$= \omega \left(1 + \frac{8}{7}\right) = \frac{15}{7} \omega$$

$$= \frac{\cancel{15}}{7} \frac{2}{\cancel{15}} \frac{rad}{s} = \frac{2}{7} \frac{rad}{s}$$

(d) lavoro che devo compiere per spostarmi

$$L = \Delta E_k$$

$$E_k^{iniz} = \frac{1}{2}\left(m_v R^2 + I^O_{asta}\right)\omega^2 =$$

$$E_k^{fin} = \frac{1}{2} I^O_{asta}\, \omega_f^2 =$$

$$= \frac{1}{2} I^O_{asta}\left[\omega^2 + \frac{m_v^2 R^4 \omega^2}{I^{O\,2}_{asta}} + \frac{2\omega^2 m_v R^2}{I^O_{asta}}\right] =$$

$$= \frac{1}{2} I^O_{asta}\,\omega^2 + \frac{1}{2}\frac{m_v^2 R^4 \omega^2}{I^O_{asta}} + \omega^2 m_v R^2$$

$$\Delta E_k = E_k^{fin} - E_k^{iniz} = \frac{1}{2} m_v R^2 \omega^2 + \frac{1}{2} m_v R^2 \left(\frac{m_v R^2}{I^O_{asta}}\right)\omega^2$$

$$= \frac{1}{2} m_v R^2\, \omega\, \omega_f =$$

$$= 40\,\text{kg} \cdot (1.5)^2\,\text{m}^2 \cdot \frac{2}{15}\,\frac{2}{7}\,\frac{1}{s^2} =$$

$$= \boxed{3.43\ \text{J}}$$