1 - Vettori

Sunday, 15 August 2021 17:52

1

$A = (1, 2, 3)$ $\qquad \vec{r}_1 = \hat{u}_x + 2\hat{u}_y + 3\hat{u}_z$

$B = (1, 3, 1)$ $\qquad \vec{r}_2 = \hat{u}_x + 3\hat{u}_y + \hat{u}_z$

$\Delta\vec{r}\,,\ \|\Delta\vec{r}\| = ?$

$$\Delta\vec{r} = \vec{r}_1 - \vec{r}_2 = \begin{bmatrix}1\\2\\3\end{bmatrix} - \begin{bmatrix}1\\3\\1\end{bmatrix} = \begin{bmatrix}0\\-1\\2\end{bmatrix}$$

$$\Delta\vec{r} = -\hat{u}_y + 2\hat{u}_z$$

$$\|\Delta\vec{r}\| = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

2

$\vec{a} = (3, 4, -5)$

$\vec{b} = (-1, 2, 6)$

DETERMINA θ

$$\boxed{\vec{a} \cdot \vec{b} = \|a\|\,\|b\| \cos\theta}$$

$$a \cdot b = \begin{bmatrix}3\\4\\-5\end{bmatrix} \cdot \begin{bmatrix}-1\\2\\6\end{bmatrix} = -3 + 8 - 30 = -25$$

$$\|a\| = \sqrt{9 + 16 + 25} = 5\sqrt{2}$$

$$\|b\| = \sqrt{1 + 4 + 36} = \sqrt{41}$$

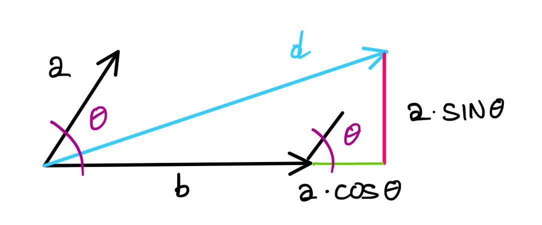
$$\theta = \text{ARCCOS}\left(\frac{-25}{5\sqrt{2}\sqrt{41}}\right) = 123.5°$$

VETTORI $\vec{a}, \vec{b}$ FORMANO UN ANGOLO θ **3**

CALCOLA IL MODULO DI

$c = \vec{a} - \vec{b}\ ,\quad d = \vec{a} + \vec{b}$

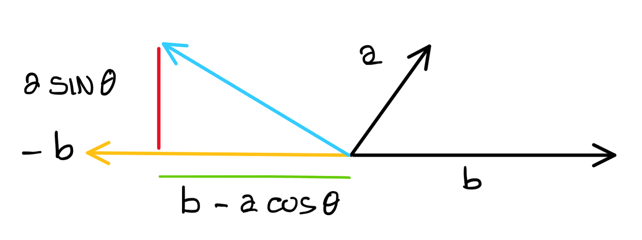
IN FUNZIONE DI a, b, θ



$$d^2 = (a \sin\theta)^2 + (b + a\cos\theta)^2$$

$$= \underline{a^2 \sin^2\theta} + b^2 + \underline{a^2\cos^2\theta} + 2ab\cos\theta$$

$$= a^2 + b^2 + 2\,ab\cos\theta$$



$$c^2 = (a\sin\theta)^2 + (b - a\cos\theta)^2$$

$$= \underline{a^2\sin^2\theta} + b^2 + \underline{a^2\cos^2\theta} - 2ab\cos\theta$$

$$= a^2 + b^2 - 2ab\cos\theta$$

IN CONCLUSIONE

$$\|a \pm b\| = \sqrt{\|a\|^2 + \|b\|^2 \pm 2\|a\|\,\|b\|\cos\theta}$$

4

$\vec{a} = (3, 4, -5)$

$\vec{b} = (-1, 2, 6)$

CALCOLA $\underline{a \times b}$

$$a \times b = \begin{vmatrix} i & j & k \\ 3 & 4 & -5 \\ -1 & 2 & 6 \end{vmatrix} = \begin{bmatrix}24+10\\-18+5\\6+4\end{bmatrix} = \begin{bmatrix}34\\-13\\10\end{bmatrix}$$

$$a \times b = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{matrix} +i\,(a_y b_z - a_z b_y) \\ -j\,(a_x b_z - a_z b_x) \\ +k\,(a_x b_y - a_y b_x) \end{matrix}$$

5

$\vec{a} = (2, 1, -3)$

$\vec{b} = (1, -2, 1)$

C TALE CHE

$$\begin{cases} c \perp a \\ c \perp b \\ \|c\| = 5 \end{cases}$$

$$\begin{vmatrix}2\\1\\-3\end{vmatrix} \cdot \begin{vmatrix}c_x\\c_y\\c_z\end{vmatrix} = 0 \qquad \underline{2c_x + c_y - 3c_z = 0}$$

$$\begin{vmatrix}1\\-2\\1\end{vmatrix} \cdot \begin{vmatrix}c_x\\c_y\\c_z\end{vmatrix} = 0 \qquad \underline{c_x - 2c_y + c_z = 0}$$

$$\|c\| = 5 \qquad \underline{c_x^2 + c_y^2 + c_z^2 = 25}$$

$$\begin{cases} 2c_x + c_y - 3c_z = 0 \\ c_x - 2c_y + c_z = 0 \\ c_x^2 + c_y^2 + c_z^2 = 25 \end{cases} \qquad \begin{cases} c_y = 3c_z - 2c_x \\ c_x - 6c_z + 4c_x + c_z = 0 \\ \text{—} \end{cases}$$

$$\begin{cases} c_y = 3c_z - 2c_x \\ 5c_x = 5c_z \\ \text{—} \end{cases} \qquad \underline{\underline{c_x = c_y = c_z}}$$

$$3c_x^2 = 25 \qquad \boxed{c_x = c_y = c_z = \pm\frac{5}{\sqrt{3}}}$$

$$c = \pm\frac{5}{\sqrt{3}}(1, 1, 1) = \pm\frac{5}{\sqrt{3}}(\hat{u}_x, \hat{u}_y, \hat{u}_z)$$

6

$\vec{a} = (-1, 3, 2)$

CALCOLA

- $\|a\|$ $\qquad \|a\| = \sqrt{1 + 9 + 4} = \sqrt{14}$
- θ FORMATO CON ASSE Z

ASSE Z (VERSORE)

$\hat{u}_z = (0, 0, 1)$ $\qquad \underline{\|u_z\| = 1}$

$$\vec{a} \cdot \hat{u}_z = \|a\|\,\|u_z\| \cos\theta$$

$$\theta = \text{ARCCOS}\left(\frac{\begin{bmatrix}-1\\3\\2\end{bmatrix} \cdot \begin{bmatrix}0\\0\\1\end{bmatrix}}{\sqrt{14}}\right) = \text{ARCCOS}\left(\frac{2}{\sqrt{14}}\right) = 57{,}68°$$

7

- CALCOLA IL MODULO $\|P\|$ DEL VETTORE $\vec{a} = (-1, 3, 2)$ NEL PIANO XY
- CALCOLA ANGOLO φ CON PIANO Y

P PROIEZIONE SU XY $\begin{bmatrix}-1\\3\\0\end{bmatrix}$

$\|P\| = \sqrt{1+9} = \sqrt{10}$ $\qquad \|a\| \neq \|P\| = \sqrt{14}$

$\hat{u}_y = (0, 1, 0)$ $\qquad \|\hat{u}_y\| = 1$

VERSORE ASSE Y

$$\varphi = \text{ARCCOS}\left(\frac{\begin{bmatrix}-1\\3\\2\end{bmatrix} \cdot \begin{bmatrix}0\\1\\0\end{bmatrix}}{\sqrt{14}}\right) = \text{ARCCOS}\left(\frac{+3}{\sqrt{10}}\right) = 36{,}69°$$

8

$\vec{a} = (-1, 0, 3)$

$\vec{b} = (2, 1, -1)$

CALCOLA: $\begin{cases} a \cdot b \\ a \times b \\ b \times a \end{cases}$

$$a \cdot b = \begin{bmatrix}-1\\0\\3\end{bmatrix} \cdot \begin{bmatrix}2\\1\\-1\end{bmatrix} = -2 + 0 - 3 = \underline{\underline{-5}}$$

$$a \times b = \begin{vmatrix} i & j & k \\ -1 & 0 & 3 \\ 2 & 1 & -1 \end{vmatrix} = \begin{bmatrix}-3\\5\\-1\end{bmatrix}$$

IL PRODOTTO VETTORIALE INVERSO E' <u>ANTIPARALLELO</u>

$$\boxed{a \times b = -b \times a}$$

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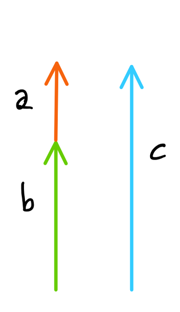
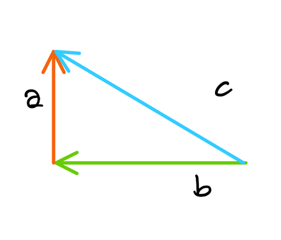
3 VETTORI a, b, c TALI CHE $c = a + b$ **9**

- CHE SUCCEDE SE $\|c\| = \|b\| + \|a\|$?
- CHE SUCCEDE SE $\|c\|^2 = \|b\|^2 + \|a\|^2$



$$\boxed{\begin{matrix}\text{SE} & \|c\| = \|a\| + \|b\| \\ \text{ALLORA} & a \parallel b\end{matrix}}$$



$$\boxed{\begin{matrix}\text{SE} & \|c\|^2 = \|a\|^2 + \|b\|^2 \\ \text{ALLORA} & a \perp b\end{matrix}}$$
