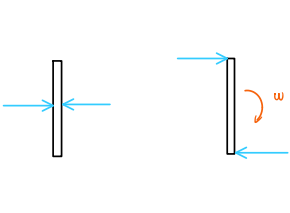
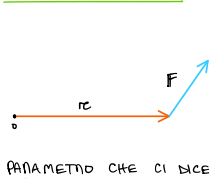
8 - Corpo Rigido

Wednesday, 25 August 2021 08:01

CORPO ESTESO ⟶ È IMPORTANTE DOVE APPLICO LE FORZE



MOMENTO DI FORZA (RISPETTO AD UN POLO O)

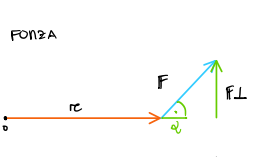


$$\boxed{\vec{M}_{(o)} = \vec{r} \times \vec{F}}$$

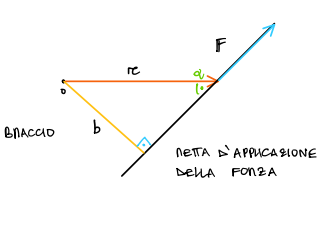
ENTRANTE O USCENTE DAL PIANO

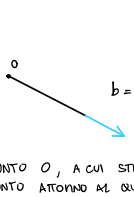
PARAMETRO CHE CI DICE SE UNA FORZA PUÒ CAUSARE ROTAZIONE

$\| M_{(o)} \| = r \cdot (F \sin\alpha) = r \cdot F_\perp$



$\| M_{(o)} \| = (r \sin\alpha)\, F = b \cdot F$

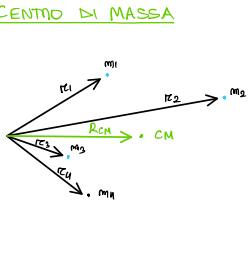




$b = 0 \longrightarrow M_{(o)} = 0$

PUNTO O, A CUI STIAMO INCHIODANDO IL CORPO
PUNTO ATTORNO AL QUALE AVVIENE LA ROTAZIONE

CENTRO DI MASSA



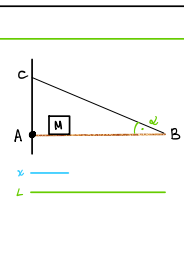
$$\boxed{\vec{R}_{CDM} = \frac{\sum \vec{r}_i m_i}{\sum m_i}}$$

SE CORPO ESTESO (O CONTINUO)

$$\boxed{\vec{R}_{CDM} = \frac{\int \vec{r}_i m_i}{\int m_i} = \frac{\int_{corpo} r_i m_i}{M}}$$

ES 1 MANZONI

ASTA ATTACCATA AD UN PERNO



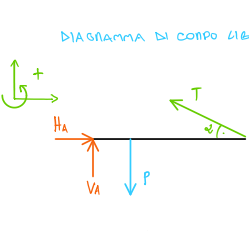
$T(x) = ?$ COME CAMBIA LA TENSIONE CON LA POSIZIONE DI M

PERNO NON È SEMPLICE APPOGGIO

UN'ASTA LIBERA HA 3 GRADI DI LIBERTÀ
IL PERNO NE CONSENTE SOLO 1 (ROTAZIONE)

VINCOLA 2 GRADI DI LIBERTÀ
↓
ESERCITA 2 FORZE

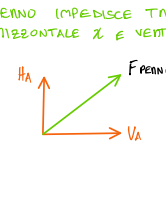
DIAGRAMMA DI CORPO LIBERO



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

CONDIZIONI DI STATICA

IL PERNO IMPEDISCE TRASLAZIONE ORIZZONTALE x E VERTICALE y



$\sum F_x = 0$ $\quad H_A - T\cos\alpha = 0$
$\sum F_y = 0$ $\quad T\sin\alpha + V_A - mg = 0$
$\sum M_A = 0$ $\quad -mg\,x + T\sin\alpha\, L = 0$

3 INCOGNITE H_A, V_A, T
2 EQUAZIONI
NON BASTA

SCEGLIENDO IL POLO IN A
H_A, V_A NON HANNO MOMENTO

$L T \sin\alpha = mg x$ $\quad \boxed{T = \frac{mg\,x}{L \sin\alpha}}$

$H_A = T\cos\alpha$ $\quad \boxed{H_A = \frac{mg\,x}{L \tan\alpha}}$

$V_A = mg - T\sin\alpha$ $\quad \boxed{V_A = mg - \frac{mg\,x}{L} = mg\left(1 - \frac{x}{L}\right)}$

POLO ENTITÀ PURAMENTE MATEMATICA
NON DEVE NECESSARIAMENTE RUOTARE

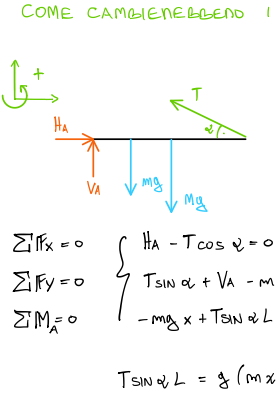
$+ T\sin\alpha$ USCENTE
$- mg$ ENTRANTE

T MASSIMA CHE LA FUNE PUÒ SOPPORTARE : $T_{MAX} \longrightarrow x$ PER EVITARE CHE FUNE NON SI SPEZZI

$\boxed{T < T_{MAX}} \longrightarrow \frac{mg\,x}{L\sin\alpha} < T_{MAX}$ $\quad \boxed{x < \frac{L \sin\alpha\, T_{MAX}}{mg}}$

È MEGLIO SPOSTARE L'OGGETTO VICINO AL MURO

COME CAMBIEREBBERO I CONTI SE L'ASTA AVESSE MASSA ?



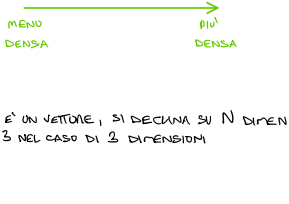
$\sum F_x = 0$ $\quad H_A - T\cos\alpha = 0$
$\sum F_y = 0$ $\quad T\sin\alpha + V_A - mg - Mg = 0$
$\sum M_A = 0$ $\quad -mg\,x + T\sin\alpha\, L - Mg\frac{L}{2} = 0$

$T \sin\alpha\, L = g\left(mx + M\frac{L}{2}\right)$

$$\boxed{T = \frac{g\,m\,x}{\sin\alpha\, L} + \frac{g M}{2 \sin\alpha}} = \frac{g}{\sin\alpha}\left(\frac{mx}{L} + \frac{M}{2}\right)$$

ES 2 MANZONI

BARRA CON DENSITÀ LINEARE NON UNIFORME



$H = 2\,g/cm^2$

X_{CDM} ?

$$\boxed{\vec{R}_{CDM} = \frac{\int \vec{r}\, dm}{\int dm}}$$

$\vec{R}$ È UN VETTORE, SI DECLINA SU N DIMENSIONI
3 NEL CASO DI 3 DIMENSIONI

$X_{CDM} = \frac{\int x\, dm}{\int dm}$ $\quad \boxed{dm = \lambda(x)\, dx}$ $\quad \lambda = \frac{dm}{dx}$ DEFINIZIONE DI DENSITÀ LINEARE

$M = \int dm = \int_{corpo} Hx\, dx = \left[H\frac{x^2}{2}\right]_0^L = H\frac{L^2}{2}$

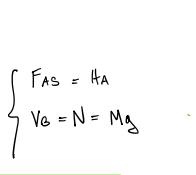
$\int x\, dm = \int_{corpo} Hx^2\, dx = \left[H\frac{x^3}{3}\right]_0^L = H\frac{L^3}{3}$

$$\boxed{X_{CDM} = \frac{HL^3}{3} \cdot \frac{2}{HL^2} = \frac{2}{3}L}$$

ES 3 MANZONI

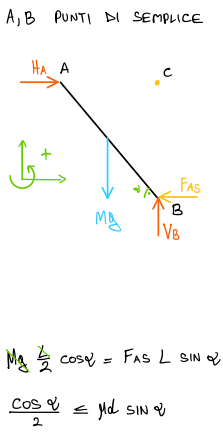
$L = 2m$
$m = 7\,kg$

ATTRITO
$\mu_s = 0.38$



α PER CUI SCALA NON SCIVOLA

A, B PUNTI DI SEMPLICE APPOGGIO



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases} \quad \begin{cases} F_{AS} = H_A \\ V_B = N = Mg \end{cases}$$

$\boxed{Mg\frac{L}{2}\cos\alpha = F_{AS} L \sin\alpha}$ (M_C) ⟵ MEGLIO QUESTO CONFLUENZA DI 2 INCOGNITE

$Mg\frac{L}{2}\cos\alpha = H_A L \cos\alpha$ (M_B)

$Mg\frac{L}{2}\cos\alpha = F_{AS} L \sin\alpha \le Mg\,\mu_s L \sin\alpha$

$\frac{\cos\alpha}{2} \le \mu_s \sin\alpha$ $\quad \frac{1}{2\mu_s} \le \tan\alpha$

$\boxed{\alpha_{MAX} = \arctan\left(\frac{1}{2\mu_s}\right)} = 55°$

GLI ESERCIZI SUI MOMENTI SI RISOLVONO SEMPRE CON 3 EQN
DI CUI UNA ALMENO DI ROTAZIONE E UNA ALMENO DI TRASLAZIONE

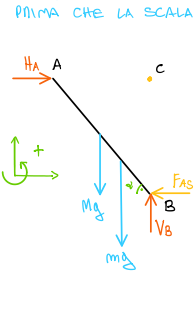
$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_C = 0 \end{cases} \quad \begin{cases} \sum F_x = 0 \\ \sum M_A = 0 \\ \sum M_B = 0 \end{cases} \quad \text{ETC ...}$$

NO 3 TRASLAZIONI
O 3 ROTAZIONI

PER AGGIUNGERE INFORMAZIONI LE EQUAZIONI DEVONO ESSERE LINEARMENTE INDIPENDENTI

SALE UN UOMO DI MASSA $m = 80\,kg$
CON SCALA $\alpha = 60°$

MASSIMA DISTANZA CHE L'UOMO PUÒ PERCORRERE PRIMA CHE LA SCALA SCIVOLI ?



$\sum F_x = 0$ $\quad H_A = F_{AS}$
$\sum F_y = 0$ $\quad V_B = g(M + m)$
$\sum M_C = 0$ $\quad Mg\frac{L}{2}\cos\alpha + mg\,x\cos\alpha = F_{AS} L \sin\alpha$

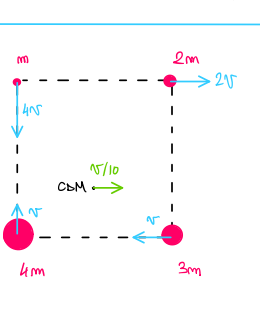
$mg\,x\cos\alpha = (m + M) g L \sin\alpha\, \mu_s - Mg\frac{L}{2}\cos\alpha$

$x = \frac{(m + M) L \sin\alpha\, \mu_s - M\frac{L}{2}\cos\alpha}{m \cos\alpha} = 1{,}28\,m$ DISTANZA DA INIZIO SCALA

$d = x\cos\alpha = 0{,}61\,m$ DISTANZA DALLA BASE DELLA SCALA

ES 1 SCHEDA

4 PUNTI MATERIALI
$m, 2m, 3m, 4m$



$\vec{R}_{CDM} = ?$ $\quad \vec{P}_{CDM} = ?$ $\quad \vec{V}_{CDM} = ?$

POSIZIONE INIZIALE DEL CENTRO DI MASSA

$m : (0, L)$
$2m : (L, L)$
$3m : (L, 0)$
$4m : (0, 0)$

$X_{CDM} = \frac{0 \cdot m + L \cdot 2m + L \cdot 3m + 0 \cdot 4m}{10m} = \frac{1}{2}L$

$Y_{CDM} = \frac{L \cdot m + L \cdot 2m + 0 \cdot 3m + 0 \cdot 4m}{10m} = \frac{3}{10}L$

$M = 10m$ (MASSA TOTALE DEL SISTEMA)

$\vec{R}_{CDM} = \left(\frac{\sum m_i x_i}{\sum m_i}, \frac{\sum m_i y_i}{\sum m_i}\right)$ $\quad \boxed{CDM = \left(\frac{1}{2}, \frac{3}{10}\right)L}$

VELOCITÀ DEL CENTRO DI MASSA

$\vec{v}(m) : (0, -4v)$
$\vec{v}(2m) : (2v, 0)$
$\vec{v}(3m) : (-v, 0)$
$\vec{v}(4m) : (0, v)$

$\vec{V}_{CDM\,x} = \frac{2m \cdot 2v + 3m(-v)}{10m} = \frac{v}{10}$

$\vec{V}_{CDM\,y} = \frac{m(-4v) + 4m\, v}{10m} = 0$

$\vec{V}_{CDM} = \left(\frac{\sum m_i v_{xi}}{\sum m_i}, \frac{\sum m_i v_{yi}}{\sum m_i}\right)$ $\quad \boxed{\vec{V}_{CDM} = \left(\frac{1}{10}, 0\right)v}$

MOMENTO DEL CENTRO DI MASSA

$\vec{P}_{CDM} = (M_{TOT} \cdot V_{CDM\,x}, M_{TOT} \cdot V_{CDM\,y})$ $\quad \boxed{\vec{P}_{CDM} = (m v, 0)}$

SE $\nexists F_{ESTERNE}$ COME SI MUOVE IL CDM ?

MOTO RETTILINEO UNIFORME SULLA x CON VELOCITÀ $V_x = \frac{v}{10}$ (VERSO DX)

DOPO QUANTO TEMPO IL CDM ESCE DAL QUADRATO ?

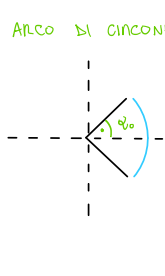
CDM $\left(\frac{1}{2}, \frac{3}{10}\right)L$ $\quad \vec{V}_{CDM} \left(\frac{1}{10}, 0\right)v$

$v = \frac{\Delta x}{\Delta t}$ ⟶ $\boxed{\Delta t = \frac{\frac{1}{2}L}{v_x} = \frac{1}{2}L \cdot \frac{10}{v} = 5\frac{L}{v}}$

ES 3

A) ARCO DI CIRCONFERENZA R, ANGOLO θ, SIMMETRIA RISPETTO A x
B) SETTORE DI CERCHIO R, ANGOLO θ, SIMMETRIA RISPETTO x
C) PAC-MAN BIDIMENSIONALE, CENTRO A CIRCONFERENZA R
D) TRIANGOLO ISOSCELE, BASE SU x, CENTRO BASE IN (0,0)
E) CONO ALTO h CON BASE SU PIANO xy, CENTRO BASE IN $\vec{0}$

ARCO DI CIRCONFERENZA



$x = R\cos\alpha$
$y = R\sin\alpha$

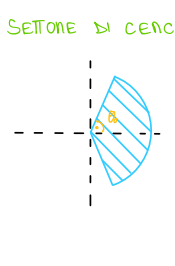
$\boxed{\vec{r}_{CDM} = \frac{\int \vec{r}\, dm}{\int dm}}$

$Y_{CDM} = 0$ PER SIMMETRIA $\quad \boxed{dm = R\, d\alpha}$

- $M = 2\int_0^{\alpha_0} R\, d\alpha = 2R\left[\alpha\right]_0^{\alpha_0} = 2R\alpha_0$
- $\int \vec{r}\, dm = 2\int_0^{\alpha_0} R\cos\alpha\,(R\, d\alpha) = 2R^2\int_0^{\alpha_0}\cos\alpha\, d\alpha = 2R^2\left[\sin\alpha\right]_0^{\alpha_0} = 2R^2\sin\alpha_0$

$\boxed{X_{CDM} = \frac{2R^2\sin\alpha_0}{2R\alpha_0} = \frac{R\sin\alpha_0}{\alpha_0}}$ $\quad CDM = \left(\frac{R\sin\alpha_0}{\alpha_0}, 0\right)$

SETTORE DI CERCHIO



$x = \rho\cos\theta$
$y = \rho\sin\theta$

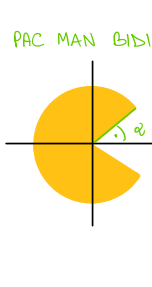
$\boxed{\vec{r}_{CDM} = \frac{\int \vec{r}\, dm}{\int dm}}$

$Y_{CDM} = 0$ PER SIMMETRIA $\quad \boxed{dm = \rho\, d\theta\, d\rho}$

- $M = \int_0^R\int_{-\theta_0}^{+\theta_0} \rho\, d\theta\, d\rho = 2\int_0^R\int_0^{\theta_0} \rho\, d\theta\, d\rho = 2\theta_0\int_0^R \rho\, d\rho = 2\theta_0\left[\frac{\rho^2}{2}\right]_0^R = \frac{2\theta_0 R^2}{2} = \theta_0 R^2$
- $\int \vec{r}\, dm = \int_0^R\int_{-\theta_0}^{+\theta_0} \rho\cos\theta \cdot \rho\, d\theta\, d\rho = 2\int_0^R\int_0^{\theta_0} \rho^2\cos\theta\, d\theta\, d\rho = 2\int_0^{\theta_0}\left[\frac{\rho^3}{3}\right]_0^R\cos\theta\, d\theta = \frac{2}{3}R^3\int_0^{\theta_0}\cos\theta\, d\theta = \frac{2}{3}R^3\left[\sin\theta\right]_0^{\theta_0} = \frac{2}{3}R^3\sin\theta_0$

$\boxed{X_{CDM} = \frac{2}{3}\frac{R^3\sin\theta_0}{\theta_0 R^2} = \frac{2}{3}\frac{R\sin\theta_0}{\theta_0}}$ $\quad CDM = \left(\frac{2}{3}\frac{R\sin\theta_0}{\theta_0}, 0\right)$

PAC MAN BIDIMENSIONALE



$x = \rho\cos\theta$
$y = \rho\sin\theta$

$\boxed{\vec{r}_{CDM} = \frac{\int \vec{r}\, dm}{\int dm}}$

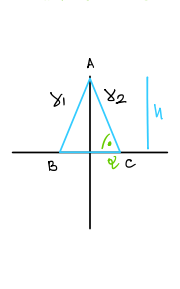
$Y_{CDM} = 0$ PER SIMMETRIA $\quad \boxed{dm = \rho\, d\theta\, d\rho}$

- $M = \int_0^R\int_{\theta_0}^{2\pi-\theta_0} \rho\, d\theta\, d\rho = 2\int_0^R\int_{\theta_0}^{\pi} \rho\, d\theta\, d\rho = 2(\pi - \theta)\int_0^R \rho\, d\rho = 2(\pi - \theta_0)\left[\frac{\rho^2}{2}\right]_0^R = (\pi - \theta_0) R^2$
- $\int \vec{r}\, dm = \int_0^R\int_{\theta_0}^{2\pi-\theta_0} \rho\cos\theta \cdot \rho\, d\theta\, d\rho = 2\int_0^R\int_{\theta_0}^{\pi} \rho^2\cos\theta\, d\theta\, d\rho = 2\int_{\theta_0}^{\pi}\left[\frac{\rho^3}{3}\right]_0^R\cos\theta\, d\theta = \frac{2}{3}R^3\int_{\theta_0}^{\pi}\cos\theta\, d\theta = \frac{2}{3}R^3\left[\sin\theta\right]_{\theta_0}^{\pi} = \frac{2}{3}R^3(\cancel{\sin\pi} - \sin\theta_0) = -\frac{2}{3}R^3\sin\theta_0$

$\boxed{X_{CDM} = -\frac{2}{3}\frac{R^3\sin\theta_0}{(\pi - \theta_0)R^2} = -\frac{2}{3}\frac{R\sin(\theta_0)}{(\pi - \theta_0)}}$ $\quad CDM = \left(-\frac{2}{3}\frac{R\sin(\theta_0)}{(\pi - \theta_0)}, 0\right)$

TRIANGOLO ISOSCELE

X_{CDM} PER SIMMETRIA SI TROVA IN 0



$A = (0, h)$
$B = \left(\frac{h}{\tan\alpha}, 0\right)$
$C = \left(-\frac{h}{\tan\alpha}, 0\right)$

$\boxed{Y_{CDM} = \frac{h}{3}}$

SEMPLICE, SENZA USARE INTEGRALI

$y_1 : y = \tan\alpha \cdot x + h$
$y_2 : y = -\tan\alpha \cdot x + h$

$D = \left[0, \frac{h}{\tan\alpha}\right] \times \left[-\tan\alpha \cdot x + h\right]$ "x SEMPLICE"

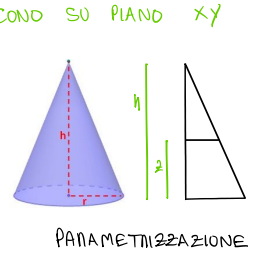
METODO "DIO È MORTO"

$M = \frac{h^2}{2\tan\alpha}$ $\quad \boxed{dm = dx\, dy}$

$\iint_D y\, dx\, dy = \int_0^{\frac{h}{\tan\alpha}}\int_0^{-\tan\alpha \cdot x + h} y\, dy\, dx = \int_0^{\frac{h}{\tan\alpha}}\left[\frac{y^2}{2}\right]_0^{-\tan\alpha \cdot x + h} dx =$
$= \frac{1}{2}\int_0^{\frac{h}{\tan\alpha}} \tan^2\alpha\, x^2 + h^2 - 2xh\tan\alpha\, dx = \frac{1}{2}\left[\tan^2\alpha\frac{x^3}{3} + h^2 x - \frac{2x^2}{2}h\tan\alpha\right]_0^{\frac{h}{\tan\alpha}} =$
$= \frac{1}{2}\left(\frac{\tan^2\alpha}{3}\frac{h^3}{\tan^3\alpha} + h^2\frac{h}{\tan\alpha} - \frac{h^2}{\tan^2\alpha}h\tan\alpha\right) = \frac{h^3}{2\tan\alpha}\left(\frac{1}{3} + 1 - 1\right) = \frac{h^3}{6\tan\alpha}$

$Y_{CDM} = \frac{\iint_D y\, dx\, dy}{\iint_D dx\, dy} = \frac{h^3}{6\tan\alpha} \cdot \frac{2\tan\alpha}{h^2} = \boxed{\frac{h}{3}}$

CONO SU PIANO xy



$Z_{CDM} = \frac{\iiint_D z\, dx\, dy\, dz}{\iiint_D dx\, dy\, dz}$

$X_{CDM}, Y_{CDM} = 0$ PER SIMMETRIA

PARAMETRIZZAZIONE
$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \\ z = z \end{cases}$$

COORDINATE CILINDRICHE
$\rho \in \left[0, \frac{h - z}{h}R\right]$
$\theta \in [0, 2\pi]$
$z \in [0, h]$

$J_{CILINDRICHE} = \int \rho\, d\rho\, d\theta\, dz$

$V = \int_0^h\int_0^{\frac{h-z}{h}R}\int_0^{2\pi} \rho\, d\theta\, d\rho\, dz = 2\pi\int_0^h\left[\frac{\rho^2}{2}\right]_0^{\frac{h-z}{h}R} dz =$
$= 2\pi\int_0^h \frac{(h - z)^2}{2h^2}R^2\, dz = \frac{\pi R^2}{h^2}\int_0^h (h^2 + z^2 - 2hz)\, dz =$
$= \frac{\pi R^2}{h^2}\left[h^2 z + \frac{z^3}{3} - 2\frac{hz^2}{2}\right]_0^h = \frac{\pi R^2}{h^2}\left(h^3 + \frac{h^3}{3} - h^3\right) = \frac{\pi R^2 h}{3}$

$\iiint_D z\, dx\, dy\, dz = \int_0^h\int_0^{\frac{h-z}{h}R}\int_0^{2\pi} z\,\rho\, d\theta\, d\rho\, dz = 2\pi\int_0^h\left[\frac{\rho^2}{2}\right]_0^{\frac{h-z}{h}R} z\, dz =$
$= \frac{\pi R^2}{h^2}\int_0^h (h - z)^2 z\, dz = \frac{\pi}{h^2}R^2\int_0^h h^2 z + z^3 - 2hz^2\, dz =$
$= \frac{\pi R^2}{h^2}\left[h^2\frac{z^2}{2} + \frac{z^4}{4} - \frac{2hz^3}{3}\right]_0^h = \pi R^2 h^2\left(\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right) = \frac{\pi R^2 h^2}{12}$

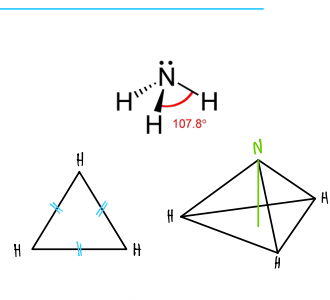
$Z_{CDM} = \frac{\iiint_D z\, dx\, dy\, dz}{\iiint_D dx\, dy\, dz} = \frac{\pi R^2 h^2}{12} \cdot \frac{3}{\pi R^2 h} = \boxed{\frac{h}{4}}$

ES 6

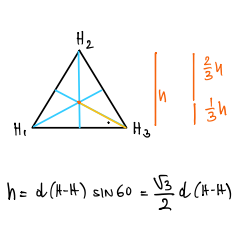
MOLECOLA D'AMMONIACA NH_3
FORMA TETRAEDRICA

$m(N) = 14\, m(H)$
$d(H\text{-}H) = 1{,}628\,Å$
$d(N\text{-}H) = 1{,}014\,Å$

CDM (NH_3) ?



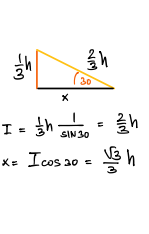
$H_1 \left(0 \quad \frac{2}{3}h \quad 0\right)$
$H_2 \left(+\frac{\sqrt{3}}{3}h \quad -\frac{1}{3}h \quad 0\right)$
$H_3 \left(-\frac{\sqrt{3}}{3}h \quad -\frac{1}{3}h \quad 0\right)$
$N \left(0 \quad 0 \quad H\right)$



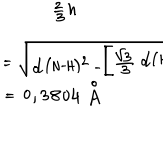
$h = d(H\text{-}H)\sin 60 = \frac{\sqrt{3}}{2}d(H\text{-}H)$

$\boxed{Z_{CDM} = \frac{14\, m(H) \cdot H}{17\, m(H)} = \frac{14}{17}H}$

PER SIMMETRIA IL CDM È DA CALCOLARE SOLO SU z



$I = \frac{1}{3}h\frac{1}{\sin 30} = \frac{2}{3}h$
$x = I\cos 30 = \frac{\sqrt{3}}{3}h$



$H = \sqrt{d(N\text{-}H)^2 - \left[\frac{\sqrt{3}}{3}d(H\text{-}H)\right]^2} = 0{,}3804\,Å$

MATRICE DELLE DISTANZE AGGIORNATA

$H_1 \left(0 \quad \frac{\sqrt{3}}{3}d(H\text{-}H) \quad 0\right)$
$H_2 \left(+\frac{d(H\text{-}H)}{2} \quad -\frac{\sqrt{3}}{6}d(H\text{-}H) \quad 0\right)$
$H_3 \left(-\frac{d(H\text{-}H)}{2} \quad -\frac{\sqrt{3}}{6}d(H\text{-}H) \quad 0\right)$
$N \left(0 \quad 0 \quad \sqrt{d(N\text{-}H)^2 - \left[\frac{\sqrt{3}}{3}d(H\text{-}H)\right]^2}\right)$

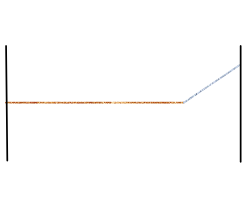
QUANTO DISTA IL CDM DALL'ATOMO DI AZOTO (N) ?

$\boxed{d = H - Z_{CDM} = 0{,}067\,Å}$

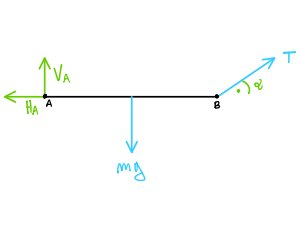
ES 7 APPENDIABITI

$m_{ASTA} = 2\,kg$
$L_{ASTA} = L$
$\alpha = 30°$

TROVA TUTTE LE FORZE DELLA CERNIERA E FUNE



$\sum F_x = 0$ $\quad H_A = T\cos\alpha$
$\sum F_y = 0$ $\quad V_A + T\sin\alpha = mg$
$\sum M_A = 0$ $\quad mg\frac{L}{2} = T L \sin\alpha$



$V_A = mg - T\sin\alpha = mg - \frac{mg}{2}$
$H_A = T\cos\alpha$

$\boxed{T = \frac{mg}{2\sin\alpha}} = 19{,}6\,N$

$\boxed{V_A = \frac{mg}{2}} = 9{,}81\,N$

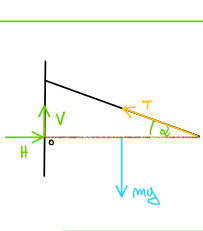
$\boxed{H_A = \frac{mg}{2\tan\alpha}} = 17\,N$

PROBLEMI SFUSI DALL'ESERCIZIARIO

ES 8.6

ASTA OMOGENEA AB
$M = 10\,kg$ $\quad \alpha = 30°$
INCERNIERATA IN A

T, V, H ?



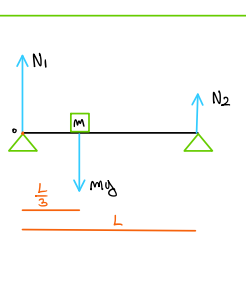
$\sum F_x$ $\quad V + T\sin\alpha = mg$
$\sum F_y$ $\quad H = T\cos\alpha$
$\sum M_0$ $\quad mg\frac{L}{2} = T\sin\alpha\, L$

$$\boxed{\begin{aligned} T &= \frac{mg}{2\sin\alpha} \\ H &= T\cos\alpha = \frac{mg}{2\tan\alpha} \\ V &= mg - T\sin\alpha = \frac{mg}{2} \end{aligned}}$$

ES 8.7

2 FULCRI LISCI

CALCOLA TUTTE LE FORZE D'APPOGGIO

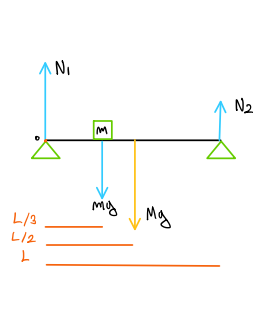


$\sum F_y$ $\quad N_1 + N_2 = mg$
$\sum M_0$ $\quad mg\frac{L}{3} = N_2 L$

$\boxed{N_2 = \frac{mg}{3}} \longrightarrow \boxed{N_1 = \frac{2}{3}mg}$

CHE SUCCEDE NEL CASO LA TRAVE FOSSE OMOGENEA ED AVESSE MASSA NON TRASCURABILE ?

CALCOLA N_1, N_2



$\sum F_y$ $\quad N_1 + N_2 = mg + Mg$
$\sum M_0$ $\quad mg\frac{L}{3} + Mg\frac{L}{2} = N_2 L$

$$\boxed{\begin{aligned} N_2 &= \frac{mg}{3} + \frac{Mg}{2} \\ N_1 &= mg + Mg - N_2 = \frac{2}{3}\frac{mg}{3} + \frac{Mg}{2} \end{aligned}}$$

NOTARE COME LA F_P DELLA TRAVE, ESSENDO CONCENTRATA AL CENTRO (CDM) SI DISTRIBUISCE EGUALMENTE TRA I 2 FULCRI

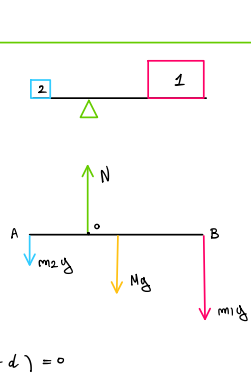
ES 8.8

ASTA OMOGENEA $M = 10\,kg$ $\quad L = 1\,m$
APPOGGIATA SU FULCRO LISCIO

$d = 0.2\,m$ (DISTANZA $\overline{AO}$)

ASTA IN EQUILIBRIO

SE $m_2 = 5\,kg$ CALCOLA m_1

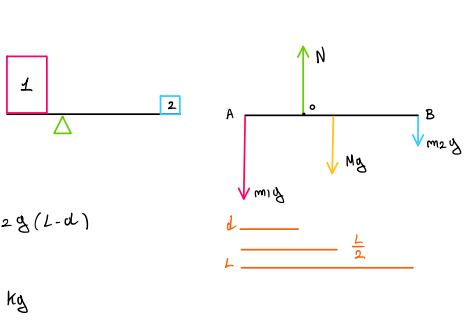


$\sum F_y$ $\quad (m_2 + m_1 + M)g = N$
$\sum M_0$ $\quad + m_2 g\, d - Mg\left(\frac{L}{2} - d\right) - m_1 g(L - d) = 0$

$m_1 = \frac{m_2 d - M\left(\frac{L}{2} - d\right)}{L - d} = -2{,}5\,kg$ IMPOSSIBILE NON ESISTE MASSA NEGATIVA

QUESTA CONFIGURAZIONE NON È STABILE ∀ VALORI DI m_1

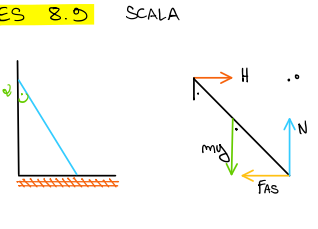
CAMBIAMO CONFIGURAZIONE



$\sum F_y$ $\quad (m_2 + m_1 + M)g = N$
$\sum M_0$ $\quad m_1 g\, d = Mg\left(\frac{L}{2} - d\right) + m_2 g(L - d)$

$m_1 = \frac{M\left(\frac{L}{2} - d\right) + m_2(L - d)}{d} = 35\,kg$

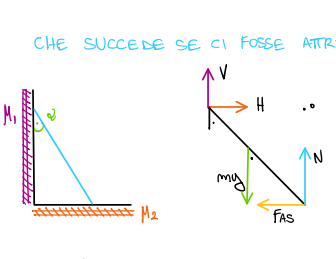
ES 8.9 SCALA



$\sum F_x$ $\quad N = mg$
$\sum F_y$ $\quad H = F_{AS}$
$\sum M_0$ $\quad mg\frac{L}{2}\sin\alpha = F_{AS} L \cdot \cos\alpha$

$mg\frac{L}{2}\sin\alpha = F_{AS} L\cos\alpha$
$F_{AS} = \frac{mg}{2}\tan\alpha \le mg\mu_s$ $\quad \tan\alpha \le 2\mu_s$ $\quad \boxed{\alpha_{MAX} = \arctan(2\mu_s)}$

CHE SUCCEDE SE CI FOSSE ATTRITO ANCHE LUNGO LA PARETE ?



$\sum F_x$ $\quad V + N = mg$
$\sum F_y$ $\quad H = F_{AS}$
$\sum M_0$ $\quad mg\frac{L}{2}\sin\alpha = F_{AS} L \cdot \cos\alpha$

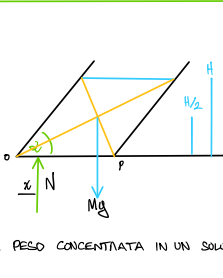
$mg\frac{L}{2}\sin\alpha = F_{AS} L\cos\alpha$
$F_{AS} = \frac{mg}{2}\tan\alpha \le (mg - V)\mu_2$
$\frac{mg}{2\mu_2}\tan\alpha \le mg - V$
$V \le mg - \frac{mg}{2\mu_s}\tan\alpha \le$

BOH

ES 8.10

RECIPIENTE CILINDRICO NON RETTO
$R = 0{,}3\,m$ $\quad \alpha = 70°$
RIEMPITO FINO AD ALTEZZA H

H MAX AFFINCHÉ EQUILIBRIO



$G\left(R + \frac{H}{2}\cot\alpha, \frac{H}{2}\right)$ TUTTA LA FORZA PESO CONCENTRATA IN UN SOLO PUNTO

$\sum M_0 = 0 \longrightarrow$ $\quad Mg\, x = Mg\left(R + \frac{H}{2\tan\alpha}\right)$

$x = R + \frac{H}{2\tan\alpha} \le 2R$

$\boxed{H \le 2R\tan\alpha \simeq 1{,}65\,m}$

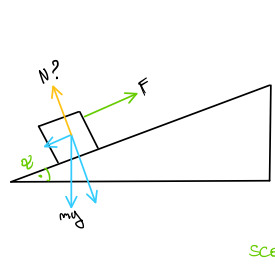
ES 8.11

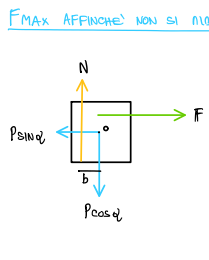
BLOCCO OMOGENEO M
CUBO DI LATO ℓ

PIANO LISCIO (NO ATTRITO)

F APPLICATA d DA BASE

F MAX AFFINCHÉ NON SI RIBALTI





$\sum F_x$ $\quad F - mg\sin\alpha = ma$
$\sum F_y$ $\quad mg\cos\alpha = N$
$\sum M_0$ $\quad N b = F\left(d - \frac{\ell}{2}\right)$

SCEGLIAMO COME POLO IL CENTRO DI MASSA POICHÉ ELIMINA I MOMENTI DELLE COMPONENTI DELLA FORZA PESO

$N b = mg\cos\alpha\, b = F\left(d - \frac{\ell}{2}\right)$ ⟶ $\boxed{b = \frac{F}{mg\cos\alpha}\left(d - \frac{\ell}{2}\right)}$

LA FORZA NORMALE È INTERNA AL BLOCCO SE $\boxed{|b| \le \frac{\ell}{2}}$

$\frac{F}{mg\cos\alpha}\left(d - \frac{\ell}{2}\right) \le \frac{\ell}{2}$

$F \le \frac{\ell}{2}mg\cos\alpha \cdot \left(\frac{2d - \ell}{2}\right)^{-1}$ ⟶ $\boxed{F \le \frac{mg\,\ell\cos\alpha}{2d - \ell}}$

OTTENIAMO COSÌ TUTTE LE INCOGNITE DEL PROBLEMA

$F_{MAX} = \frac{mg\,\ell\cos\alpha}{2d - \ell}$

$a_{MAX} = \frac{F_{MAX}}{m} - g\sin\alpha = g\left(\frac{\ell\cos\alpha}{d - \ell} - \sin\alpha\right)$

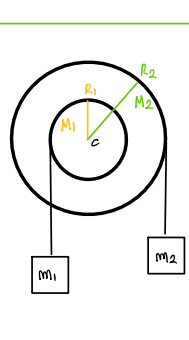
$N = mg\cos\alpha$

ES 8.12

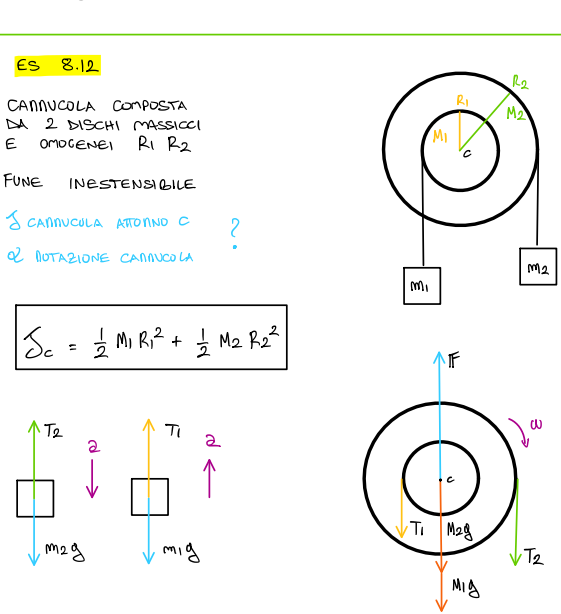
CARRUCOLA COMPOSTA DA 2 DISCHI MASSICCI E OMOGENEI R_1 R_2

FUNE INESTENSIBILE

J CARRUCOLA ATTORNO C ?
α ROTAZIONE CARRUCOLA



$$\boxed{J_C = \frac{1}{2}M_1 R_1^2 + \frac{1}{2}M_2 R_2^2}$$



$\Sigma 1$ $\quad T_1 - m_1 g = m_1 a$
$\Sigma 2$ $\quad m_2 g - T_2 = m_2 a$

$T_1 = m_1(g + a)$
$T_2 = m_2(g - a)$

$\sum F_x$ $\quad$ —
$\sum F_y$ $\quad F = M_1 g + M_2 g + T_1 + T_2$
$\sum M_C$ $\quad T_2 R_2 - T_1 R_1 = J_C\,\alpha$

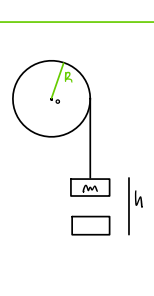
$m_2(g - a)R_2 - m_1(g + a)R_1 = J_C\,\alpha$
$m_2 g R_2 - m_2 a R_2 - m_1 g R_1 - m_1 a R_1 = J_C\,\alpha$
$m_2 g R_2 - m_2\alpha R_2^2 - m_1 g R_1 - m_1\alpha R_1^2 = J_C\,\alpha$ $\quad$ CON $a_x = \alpha R_x$
$\alpha\left(J_C + m_1 R_1^2 + m_2 R_2^2\right) = g(m_2 R_2 - m_1 R_1)$

$$\boxed{\alpha = g\frac{m_2 R_2 - m_1 R_1}{J_C + m_1 R_1^2 + m_2 R_2^2}}$$

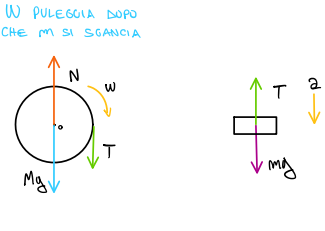
UNA ACCELERAZIONE DIPENDE SEMPRE DALL'INERZIA TOTALE, AL DENOMINATORE CI DEVE ESSERE UNA SOMMA MAI UNA DIFFERENZA

ES 8.13

$m = 2\,kg$
$M = 10\,kg$
$R = 0{,}5\,m$
$h = 2\,m$



ω PULEGGIA DOPO CHE m SI SCARICA



$mg - T = ma$
$T = m(g - a) = mg - m\alpha R$

$\sum F_y$ $\quad N = Mg + T$
$\sum M_0$ $\quad TR = J_0\,\alpha$ $\qquad (mg - m\alpha R)R = J_0\,\alpha$

$mgR - m\alpha R^2 = J_0\,\alpha$ $\quad \alpha(J_0 + mR^2) = mgR$ $\quad \boxed{\alpha = \frac{mgR}{J_0 + mR^2}}$

MOTO UNIFORMEMENTE ACCELERATO

$Y_F = Y_0 + V_0 t + \frac{1}{2}at^2 \longrightarrow y(t) = \frac{1}{2}at^2$

IN QUANTO TEMPO IL BLOCCO SCENDE DI h ?

$h = \frac{1}{2}at^2$ $\quad t_{CADUTA} = \sqrt{\frac{2h}{a}}$

COME CAMBIA LA VELOCITÀ ANGOLARE IN QUESTO TEMPO ?

$\omega_F = \omega_0 + \alpha t \longrightarrow \omega(t) = \alpha t$

$\omega_F = \omega(t_{CADUTA}) = \alpha\sqrt{\frac{2h}{\alpha R}} = \sqrt{\frac{2h\alpha}{R}}$ $\quad \boxed{\omega_F = \sqrt{\frac{2h\,mg}{J_0 + mR^2}}}$

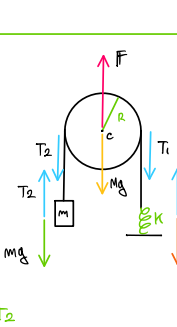
OPPURE PIÙ SEMPLICEMENTE POSSIAMO USARE L'ENERGIA

$mgh = \frac{1}{2}J_0\omega^2 + \frac{1}{2}m v^2$ $\quad$ LEGATE DA VINCOLO CINEMATICO

$2mgh = J_0\omega^2 + m\omega^2 R^2 \longrightarrow \omega^2 = \sqrt{\frac{2hmg}{J_0 + mR^2}}$

ES 8.14

$m = 1\,kg$
$K = 30\,N/m$
$M = 10\,kg$
$R = 0{,}5\,m$



Δx MOLLA PER EQ ?

A PRIORI NON POSSIAMO DIRE CHE $T_1 = T_2$

$T_1 = T_2$ SOLO NEL CASO DI EQUILIBRIO

$\Sigma 1$ $\quad T_1 = Kx$
$\Sigma 2$ $\quad mg - T_2 = ma$
$\sum F_y$ $\quad Mg + T_1 + T_2 = F$
$\sum M_0$ $\quad (T_2 - T_1)R = J_0\,\alpha$

EQUILIBRIO : $a, \alpha = 0 \longrightarrow T_1 = T_2 \longrightarrow mg = K\Delta x$ $\quad \boxed{\Delta x_{EQ} = \frac{mg}{K}}$

m VIENE SPOSTATA DA EQUILIBRIO
MOSTRA CHE IL MOTO È ARMONICO
CALCOLA PERIODO OSCILLAZIONI

$\Sigma 2$ $\quad T_2 = mg - ma$
ΣM $\quad (T_2 - T_1)\cancel{R} = J_0\,\alpha = \frac{1}{2}MR^{\cancel{2}}\frac{a}{\cancel{R}}$

$T_2 = mg - ma$
$T_2 = Kx + \frac{Ma}{2}$

$mg - ma = Kx + \frac{Ma}{2}$

$a\left(\frac{M}{2} + m\right) = mg - Kx$

$\ddot{x} = \frac{2mg}{M + 2m} - \frac{2Kx}{M + 2m}$

$\boxed{\ddot{x} - \Omega^2 x = \frac{2mg}{M + 2m} \quad \text{CON} \quad \Omega = \sqrt{\frac{2K}{M + 2m}}}$

EQUAZIONE CARATTERISTICA DI UN OSCILLATORE ARMONICO DI PULSAZIONE PARI A Ω E PERIODO T

$T = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{M + 2m}{2K}} \simeq 2.81\,s$

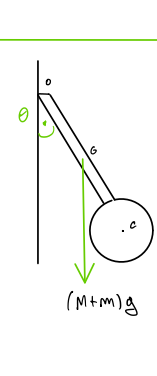
CALCOLA MASSIMA ω ROTAZIONE PULEGGIA
$\Delta x = 0{,}4\,m$

$S(t) = A\cos(\omega t + \varphi) + X_{EQ}$
$V(t) = -A\omega\sin(\omega t + \varphi)$
V_{MAX} QUANDO $\sin(\omega t + \varphi) = 1$
$V_{MAX} = -A\omega = -\Delta x\,\Omega$
$\omega_{MAX} = -\frac{\Delta x\,\Omega}{R} \simeq 1.79\,rad/s$

ES 8.15

PENDOLO FISICO = ASTA RIGIDA + DISCO

CALCOLA PERIODO DELLE PICCOLE OSCILLAZIONI



EQUAZIONE DEL MOTO RISPETTO AD O

$J_0\,\alpha = -(M + m)g L\sin\theta$

PER PICCOLE OSCILLAZIONI $\sin\theta \approx \theta$

$J_0\,\alpha = -(M + m)g\, d\,\theta$

$\alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$ $\qquad \ddot{\theta} = -\frac{(M + m)g\, d}{J_0}\theta$

$\boxed{\ddot{\theta} = -\Omega^2\theta \quad \text{CON} \quad \Omega = \sqrt{\frac{(M + m)g\, d}{J_0}}}$

$T = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{J_0}{(M + m)g L}}$

DOBBIAMO CALCOLARE:

LUNGHEZZA d DEL SEGMENTO $\overline{OG}$
(DISTANZA DEL BARICENTRO DALL'ESTREMO)

$d = \frac{\frac{mL}{2} + M(L + R)}{m + M}$

MOMENTO TOTALE DEL CORPO

$J_0 = J_{0\,ASTA} + J_{0\,DISCO}$
$= \frac{1}{3}mL^2 + \frac{1}{2}ML^2 + M(R + L)^2$

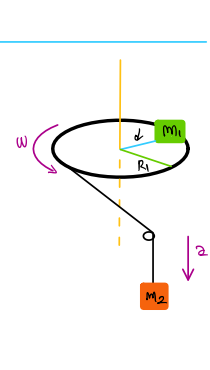
ASTA RISPETTO AD UNA ESTREMITÀ
RISPETTO AL CDM
TEOREMA DEGLI ASSI PARALLELI

ES 8.16

PULEGGIA R, M
RUOTA SENZA ATTRITO
SU UN PIANO ORIZZONTALE

m_1 SU PIANO SCABRO μ_s
A DISTANZA $d < R$

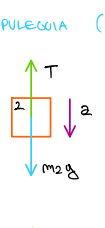
PULEGGIA MESSA IN ROTAZIONE DA FERMA CON UN PESO m_2



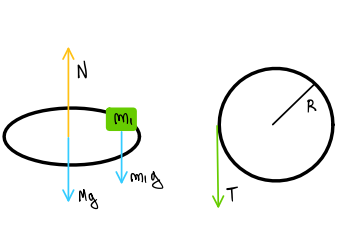
α PULEGGIA

Δt IN CUI m_1 SI STACCA DAL PIANO

α PULEGGIA (SENZA m_1)



$\boxed{m_2 g - T = m_2 a}$



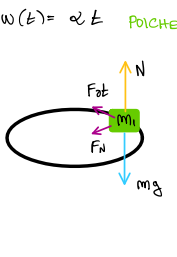
$N = m_1 g + Mg$
$TR = J_0\,\alpha$
CON $J_0 = \frac{1}{2}MR^2 + m_1 d^2$

$\Sigma 2$ $\quad T = m_2(g - a) = m_2(g - \alpha R)$
ΣM $\quad TR = J_0\,\alpha$

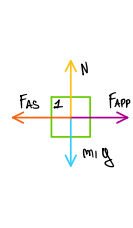
$m_2 g R - m_2\alpha R^2 = J_0\,\alpha$ ⟶ $\boxed{\alpha = \frac{m_2 g R}{m_2 R^2 + m_1 d^2 + \frac{1}{2}MR^2}}$

MOTO DELLA CARRUCOLA È UNIFORMEMENTE ACCELERATO

$\omega(t) = \alpha t$ POICHÉ PARTE DA FERMA



$$\begin{cases} F_{AT} = m_1\alpha d \\ F_{AN} = m_1\omega^2 d \end{cases} \longrightarrow |F_a| = \sqrt{F_{AT}^2 + F_{AN}^2}$$



$|F_a| = m_1\sqrt{\alpha^2 d^2 + \omega^4 d^2} = m_1 d\alpha\sqrt{1 + \alpha^4 t^4}$

$|F_a| \le \mu_s N$ $\qquad \alpha d\sqrt{1 + \alpha^4 t^4} \le \mu_s g$

RISOLVENDO PER t OTTENGO IL TEMPO DI DISTACCO

$$\boxed{t_{DISTACCO} = \frac{1}{\alpha}\left[\left(\frac{\mu_s g}{d\alpha}\right)^2 - 1\right]^{1/4}}$$

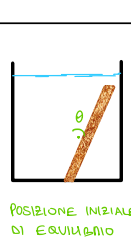
COME CAMBIA α QUANDO m_1 SI STACCA ?

$$\boxed{\alpha = \left.\frac{m_2 g R}{m_2 R^2 + m_1 d^2 + \frac{1}{2}MR^2}\right|_{m_1 = 0} = \frac{m_2 g R}{m_2 R^2 + \frac{1}{2}MR^2} = \frac{2m_2 g}{R(2m_2 + M)}}$$

ES 8.17

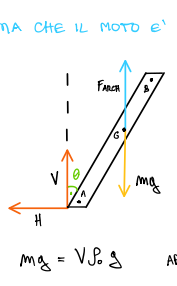
ASTA OMOGENEA ρ_0 LUNGA L
ESTREMO INCERNIERATO
FLUIDO $\rho_f > \rho_0$

ASTA SPOSTATA DI θ DA VERTICALE



POSIZIONE INIZIALE DI EQUILIBRIO

MOSTRA CHE IL MOTO È PERIODICO



$\sum F_x$ $\quad H = ma$
$\sum F_y$ $\quad V + F_{ARCH} = mg$
$\sum M$ $\quad (F_{ARCH} - mg)\frac{L}{2}\sin\theta = J_0\,\alpha$

$mg = V\rho_0 g$ $\quad$ APPLICATA AL BARICENTRO G

ASTA OMOGENEA COMPLETAMENTE IMMERSA NEL FLUIDO
⟶ $F_{ARCH} = V\rho_f g$ $\quad$ APPLICATA AL BARICENTRO G

$Vg(\rho_f - \rho_0)\frac{L}{2}\sin\theta = -\alpha \cdot \frac{1}{3}ML^2$

$Vg(\rho_f - \rho_0)\frac{L}{2}\sin\theta = -\alpha\frac{1}{3}V\rho_0 L^2$

$\alpha = -\frac{3g(\rho_f - \rho_0)}{2\rho_0 L}\sin\theta$

PER PICCOLE OSCILLAZIONI POSSIAMO ASSUMERE $\sin\theta \simeq \theta$

$\boxed{\ddot{\theta} = -\Omega^2\theta \quad \text{CON} \quad \Omega = \sqrt{\frac{3g(\rho_f - \rho_0)}{2\rho_0 L}}}$

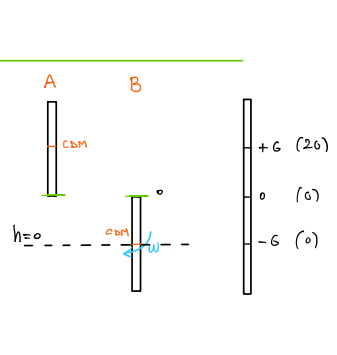
EQUAZIONE STD DI UN OSCILLATORE ARMONICO DI PULSAZIONE Ω

$\boxed{T = \frac{2\pi}{\Omega} = 2\pi\sqrt{\frac{3g(\rho_f - \rho_0)}{2\rho_0 L}}}$

ES 8.18

ASTA RETTILINEA OMOGENEA

EQUILIBRIO STABILE ED INSTABILE



TEOREMA DI KÖNIG

CONSERVAZIONE DELL'ENERGIA

$E(A) = mg\,2\cdot\frac{L}{2} = mgL$

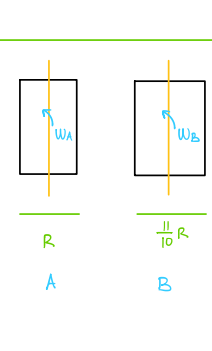
$E(B) = \frac{1}{2}J_0\omega^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$

$mgL = \frac{1}{6}mL^2\omega^2$ $\quad \boxed{\omega = \sqrt{\frac{6g}{L}}}$

ES. 8.19

CILINDRO MASSICCIO E OMOGENEO
RUOTA ω_1 ATTORNO ASSE SIMMETRIA

IL CILINDRO VIENE SCALDATO E IL RAGGIO AUMENTA DEL 10%



ω_2 DOPO ESPANSIONE TERMICA ?

PER LA CONSERVAZIONE DEL MOMENTO ANGOLARE :

$L(A) = L(B)$ $\qquad J_A\,\omega_A = J_B\,\omega_B$

DOVE $\quad J_A = \frac{1}{2}MR_A^2$ $\qquad J_B = \frac{1}{2}MR_B^2$

$\omega_B = \omega_A\frac{J_A}{J_B} = \omega_A\frac{\frac{1}{2}MR^2}{\frac{1}{2}M\frac{121}{100}R^2}$ ⟶ $\boxed{\omega_B = \frac{100}{121}\omega_A}$

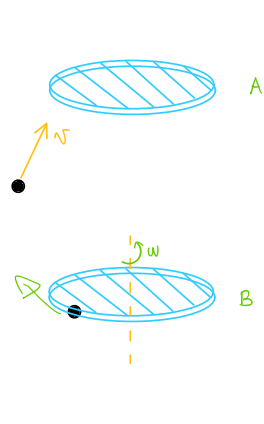
CON L'AUMENTARE DEL RAGGIO LA VELOCITÀ ANGOLARE ω È DIMINUITA DEL 17.4%

ES 8.20

PROIETTILE
$m = 0{,}05\,kg$
$v_0 = 100\,m/s$

DISCO MASSICCIO DI LEGNO
$M = 10\,kg$ $\quad R = 0{,}5\,m$

IL DISCO RUOTA SENZA ATTRITO
IL PROIETTILE SI CONFICCA



$\omega_0 = 0\,rad/s$ $\qquad \omega_F = ?$

PER LA CONSERVAZIONE DEL MOMENTO ANGOLARE

$L(A) = L(B)$
$L(A) = m v R$
$L(B) = J_{0\,TOTALE} \cdot \omega$

$J_{0\,TOTALE} = J_{0\,DISCO} + mR^2 = \frac{1}{2}MR^2 + mR^2$

$\omega = \frac{m v R}{J_{0\,TOTALE}} = \frac{2m v R}{R^2(M + 2m)}$ ⟶ $\boxed{\omega = \frac{2m v}{R(M + 2m)}} = 1{,}98\,rad/s$

NOTARE CHE L'URTO È ANELASTICO

CALCOLA L'ENERGIA DISSIPATA

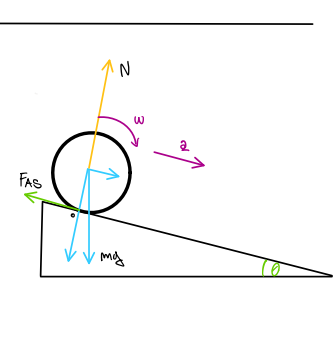
$E(A) = \frac{1}{2}m v^2$
$E(B) = \frac{1}{2}J_{TOT}\,\omega^2$

$\Delta E = E(A) - E(B) = \frac{1}{2}m v_0^2\left(1 - \frac{mR^2}{J_0}\right)$

ES 8.21

DISCO OMOGENEO M, R
PURO ROTOLAMENTO
PIANO INCLINATO SCABRO μ_s, θ

- α CADUTA DEL DISCO
- F_a ESERCITATA (PIANO-DISCO)
- θ_{MAX} CHE CONSENTE ANCORA ROTOLAMENTO



$\sum F_x$ $\quad mg\sin\theta - F_{AS} = ma = m\alpha R$
$\sum F_y$ $\quad N = mg\cos\theta$
$\sum M_0$ $\quad mg\sin\theta\, R = J_0\,\alpha$

$J_0 = \frac{1}{2}mR^2 + (R)^2 m = \frac{3}{2}mR^2$

$\cancel{m}g\sin\theta\,\cancel{R} = \frac{3}{2}\cancel{m}R^{\cancel{2}}\alpha$

$g\sin\theta = \frac{3}{2}R\alpha$ ⟶ $\boxed{\alpha = \frac{2g\sin\theta}{3R}}$

CALCOLO DELLA FORZA D'ATTRITO

$mg\sin\theta - F_{AS} = ma = m\alpha R$

$F_{AS} = m(g\sin\theta - \alpha R) = mg\sin\theta\left(1 - \frac{2}{3}\right)$ ⟶ $\boxed{F_{AS} = \frac{1}{3}mg\sin\theta}$

θ_{MAX} CHE CONSENTE ANCORA ROTOLAMENTO ?

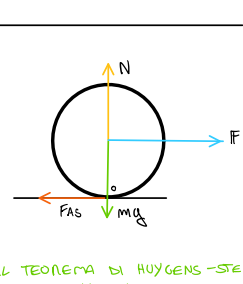
$\boxed{F_{AS} \le N\mu_s}$ ⟶ $\frac{mg\sin\theta}{3} \le mg\cos\theta\,\mu_s$

$\tan\theta \le 3\mu_s$ $\qquad \boxed{\theta_{MAX} = \arctan(3\mu_s)}$

ES 8.22

RUOTA, DISCO $10\,kg$
PIANO ORIZZONTALE SCABRO $\mu_s = 0.1$
F ORIZZONTALE COSTANTE

F AFFINCHÉ RUOTA NON SLITTI



$\sum F_x$ $\quad N = mg$
$\sum F_y$ $\quad F - F_{AS} = ma$
$\sum M_0$ $\quad F \cdot R = J_0\,\alpha$

PER IL TEOREMA DI HUYGENS-STEINER DEGLI ASSI PARALLELI:

$J_0 = \frac{1}{2}mR^2 + (R)^2 m = \frac{3}{2}mR^2$

$F \cdot R = \frac{3}{2}mR^2\alpha$ ⟶ $F = \frac{3}{2}m\alpha R$ $\qquad \boxed{\alpha = \frac{2F}{3mR}}$

$\boxed{F_{AS} = F - ma \le N\mu_s}$

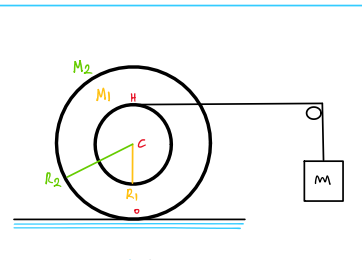
$F - m\alpha R \le mg\mu_s$ $\qquad F - mR\left(\frac{2F}{3mR}\right) \le mg\mu_s$ $\qquad \boxed{F \le 3mg\mu_s}$

ES 8.23

CORPO RIGIDO

2 DISCHI MASSICCI SALDATI

IL CORPO RUOTA SOTTO IL PIANO ORIZZONTALE SCABRO SOTTO L'AZIONE DELLA MASSA m



IMPORTANTE

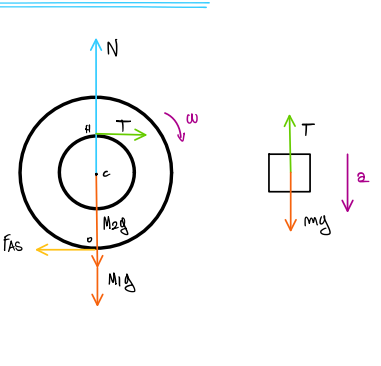
IL BLOCCO M ACCELERA CON LA STESSA α DEL PUNTO H NELLA RUOTA PICCOLA

J₀ CENTRO DI ISTANTANEA ROTAZIONE

ACCELERAZIONE CADUTA DI m

MINIMO μ_s PER EVITARE STRISCIAMENTO

$mg - T = ma = m(R_1 + R_2)\alpha$ $\quad$ ACCELERAZIONE NEL PUNTO H



$\sum F_x$ $\quad T - F_{AS} = (M_1 + M_2)a = (M_1 + M_2)\alpha R_2$
$\sum F_y$ $\quad N = g(M_1 + M_2)$
$\sum M_0$ $\quad T(R_1 + R_2) = J_0\,\alpha$

$J_C = \frac{1}{2}M_1 R_1^2 + \frac{1}{2}M_2 R_2^2$
$J_0 = \frac{1}{2}M_1 R_1^2 + \frac{1}{2}M_2 R_2^2 + (M_1 + M_2)R_2^2$ $\quad$ PER IL TEOREMA DEGLI ASSI PARALLELI
$J_0 = \frac{1}{2}M_1 R_1^2 + \frac{1}{2}M_2 R_2^2 + M_1 R_2^2 + M_2 R_2^2$

$$\boxed{J_0 = \frac{1}{2}M_1\left(R_1^2 + 2R_2^2\right) + \frac{3}{2}M_2 R_2^2}$$

$$\begin{cases} T = m(g - a) = m\left[g - (R_1 + R_2)\alpha\right] \\ T(R_1 + R_2) = J_0\,\alpha \end{cases}$$

$\left(mg - m(R_1 + R_2)\alpha\right)(R_1 + R_2) = J_0\,\alpha$
$mg(R_1 + R_2) - m(R_1 + R_2)^2\alpha = J_0\,\alpha$
$\alpha\left(J_0 + m(R_1 + R_2)^2\right) = mg(R_1 + R_2)$

$$\boxed{\alpha = \frac{mg(R_1 + R_2)}{J_0 + m(R_1 + R_2)^2}}$$

CONOSCENDO α ORA POSSIAMO OTTENERE T

$T(R_1 + R_2) = \alpha J_0$

$T = \frac{\alpha J_0}{R_1 + R_2} = \frac{mg\,\cancel{(R_1 + R_2)}}{J_0 + m(R_1 + R_2)^2} \cdot \frac{J_0}{\cancel{R_1 + R_2}}$

$$\boxed{T = \frac{mg J_0}{J_0 + m(R_1 + R_2)^2}}$$

COME DETERMINO INVECE N E F_{AS} ?

$\boxed{N = g(M_1 + M_2)}$

$F_{AS} = T - (M_1 + M_2)\alpha R_2 = \frac{mg J_0}{J_0 + m(R_1 + R_2)^2} - (M_1 + M_2)R_2 \cdot \frac{mg(R_1 + R_2)}{J_0 + m(R_1 + R_2)^2}$

$$\boxed{F_{AS} = \frac{mg}{J_0 + m(R_1 + R_2)^2}\left(J_0 - (M_1 + M_2)(R_1 + R_2)R_2\right)}$$

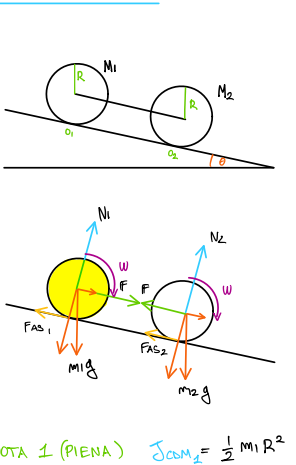
$F_{AS} \le \mu_s N$ $\quad$ ETC

ES. 8.24

DISCO MASSICCIO R, M_1
COLLEGATO A RUOTA BICICLETTA R, M_2

PURO ROTOLAMENTO (NO STRISCIAMENTO)
PIANO INCLINATO SCABRO

CALCOLA ACCELERAZIONE DEI 2 CORPI E FORZA ASSIALE



RUOTA 1

$\sum F_x$ $\quad m_1 g\sin\theta + F - F_{AS1} = m_1 a$
$\sum F_y$ $\quad m_1 g\cos\theta = N$ $\quad$ (EQ.)
$\sum M_0$ $\quad (m_1 g\sin\theta + F)R = J_{0,1}\,\alpha$

RUOTA 1 (PIENA) $\quad J_{CDM_1} = \frac{1}{2}m_1 R^2$
RUOTA 2 (VUOTA) $\quad J_{CDM_2} = m_2 R^2$

MOMENTI D'INERZIA CALCOLATI IN O

$J_{0,1} = J_{CDM,1} + m_1 R^2 = \frac{3}{2}m_1 R^2$
$J_{0,2} = J_{CDM,2} + m_2 R^2 = 2m_2 R^2$

RUOTA 2

$\sum F_x$ $\quad m_2 g\sin\theta - F - F_{AS2} = m_2 a$
$\sum F_y$ $\quad m_2 g\cos\theta = N$ $\quad$ (EQ.)
$\sum M_0$ $\quad (m_2 g\sin\theta - F)R = J_{0,2}\,\alpha$

SIA α CHE F DEVONO ESSERE UGUALI IN ENTRAMBI I SISTEMI

$$\begin{cases} 1 \quad m_1 g\sin\theta + F = \frac{1}{R}J_{0,1}\,\alpha \\ 2 \quad m_2 g\sin\theta - F = \frac{1}{R}J_{0,2}\,\alpha \end{cases}$$

$\frac{1}{R}J_{0,1}\,\alpha - m_1 g\sin\theta = m_2 g\sin\theta - \frac{1}{R}J_{0,2}\,\alpha$

$\frac{1}{R}\alpha\left(J_{0,1} + J_{0,2}\right) = (m_1 + m_2)g\sin\theta$

$\alpha\left(\frac{3}{2}m_1 R^2 + 2m_2 R^2\right) = (m_1 + m_2)gR\sin\theta$

$\alpha = \frac{(m_1 + m_2)gR\sin\theta}{\left(\frac{3}{2}m_1 R^2 + 2m_2 R^2\right)}$ ⟶ $\boxed{\alpha = \frac{2(m_1 + m_2)g\sin\theta}{R(3m_1 + 4m_2)}}$

CONOSCENDO L'ACCELERAZIONE POSSO OTTENERE LA FORZA

$(m_1 g\sin\theta + F)R = J_{0,1}\,\alpha$
$(m_2 g\sin\theta - F)R = J_{0,2}\,\alpha$

$\boxed{F = -\frac{M_1 M_2\, g\sin\theta}{3M_1 + 4M_2}}$ $\quad$ T ASSIALE DI COMPRESSIONE