Mix 2 - Gauss

Tuesday, 2 August 2022 19:44

APPLICAZIONI TEOREMA DI GAUSS

TRATTAZIONE TEORICA

Σ SUPERFICIE CHIUSA

$$\phi_\Sigma(\vec{E}) = \oiint \vec{E}\cdot\hat{m}\, dS = \frac{Q_{INT}}{\varepsilon_0}$$

SFERA PIENA, CARICA UNIFORME ρ

CAMPO ELETTRICO

PER SIMMETRIA:

$\vec{E} = E(r)\,\vec{U_r}$

$$\phi = \iint \vec{E}\cdot\hat{m}\, dS = \iint \vec{E}\cdot\vec{U_R}\, dS = \iint E(r)\, dS$$

ESSENDO E COSTANTE $\forall$ SUPERFICIE SFERICA

$$= E(r)\iint_\Sigma dS = \underline{E(r)\, 4\pi r^2}$$

CASO 1 $r < R$

$$\phi = E\, 4\pi r^2 = \frac{Q_{INC}}{\varepsilon_0}$$

$$Q_{INC} = \iiint_V \rho\, dV = \frac{4}{3}\pi r^3 \rho$$

$$E\,\cancel{4\pi r^2} = \frac{\cancel{4\pi} r^{\cancel{3}}\rho}{3\varepsilon_0} \to E(r) = \frac{\rho}{3\varepsilon_0} r$$

CASO 2 $r > R$

$$\phi = E\, 4\pi r^2 = \frac{Q_{INC}}{\varepsilon_0}$$

$$Q_{INC} = \iiint_V \rho\, dV = \frac{4}{3}\pi \underline{\underline{R}}^3 \rho$$

$$E(r) = \frac{Q_{INC}}{4\pi\varepsilon_0 r^2} = \frac{1}{\cancel{4\pi}\varepsilon_0 r^2}\,\frac{\cancel{4\pi} R^3\rho}{3} = \frac{R^3\rho}{3\varepsilon_0 r^2}$$

IL CAMPO ESTERNO DELLA SFERA SI COMPORTA COME SE TUTTA LA CARICA FOSSE NEL CENTRO

CAMPO ELETTRICO

$$\vec{E} = \begin{cases} \frac{\rho}{3\varepsilon_0} r\, \vec{U_r} & r < R \\ \frac{Q}{4\pi\varepsilon_0 r^2}\, \vec{U_r} & r > R \end{cases}$$

RICAVO IL POTENZIALE DAL CAMPO

$\vec{E} = -\text{GRAD } V$

$dV = -E\cdot dl$

CASO 2 $r > R$

$$\int_{V(\infty)}^{V(r)} dV = \int_\infty^r -E\, dr$$

$$V(r) = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}\, dr = +\frac{Q}{4\pi\varepsilon_0 r}$$

CASO 1 $r < R$

$$V(r) = -\int_\infty^R E_2(r)\, dr - \int_R^r E_1(r)\, dr$$

$$= -\frac{Q}{4\pi\varepsilon_0 r} - \int_R^r \frac{\rho r}{3\varepsilon_0}\, dr$$

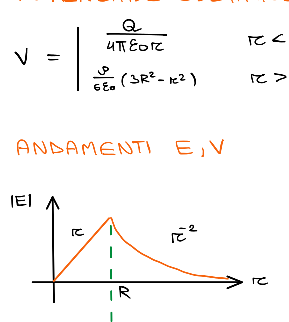
$$= -\frac{Q}{4\pi\varepsilon_0 r} - \frac{\rho}{3\varepsilon_0}\left(\frac{r^2}{2} - \frac{R^2}{2}\right)$$

$$= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$$

POTENZIALE ELETTRICO

$$V = \begin{cases} \frac{Q}{4\pi\varepsilon_0 r} & r < R \\ \frac{\rho}{6\varepsilon_0}(3R^2 - r^2) & r > R \end{cases}$$

ANDAMENTI E, V



FILO INFINITO CARICATO OMOGENEAMENTE (λ)

CALCOLA V, E

CAMPO ELETTRICO

$$\phi = \cancel{\iint_1 E\,\hat{m}_1\, dS} + \iint_2 E\,\hat{m}_2\, dS + \cancel{\iint_3 E\,\hat{m}_3\, dS}$$

CONTRIBUTI 1, 2 $\to$ 0 POICHE' $\vec{E} \perp \hat{m}_2 = \vec{U_R}$

MODULO $\vec{E}$ COSTANTE SU SUPERFICIE LATERALE DEL CILINDRO

$$\phi = \iint_2 E(r)\, \cancel{\vec{U_R}\cdot\vec{m_2}}\, dS = E(r)\, 2\pi r h$$

$Q_{INT} = \lambda h$ POICHE' OMOGENEA

GAUSS $E\, 2\pi r \cancel{h} = \frac{\lambda \cancel{h}}{\varepsilon_0} \to \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{U_r}$

POTENZIALE ELETTRICO

RIFERIMENTO $V(RIF) = V(R_0) = 0$

$$dV = -\vec{E}\cdot\vec{dl} = -E(r)\, dr$$

$$\int_{V(R_0)}^r dV = \int_{R_0}^r -\frac{\lambda}{2\pi\varepsilon_0}\cdot\frac{dr}{r}$$

$$V(r) = -\frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{r}{R_0}\right) + \underline{V(R_0)}$$ COSTANTE ADDITIVA

PIANO INFINITO CON DENSITÀ DI CARICA σ

CAMPO ELETTRICO

$\vec{E} = E(z)\,\vec{U_z}$

PER LA SIMMETRIA (INFINITI FILI ORTOGONALI) TUTTO IL CAMPO SI SVILUPPA SULLA z

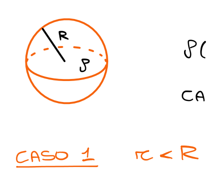
$$\phi(\vec{E}) = \iint_1 \vec{E}\,\hat{m}_1\, dS + \cancel{\iint_2 \vec{E}\,\hat{m}_2\, dS} + \iint_3 \vec{E}\,\hat{m}_3\, dS$$

$$= 2\iint E(z)\, dS = 2E(z)\pi R^2$$

$Q_{INT} = \pi R^2\sigma$

GAUSS $2E\cancel{A} = \frac{\cancel{A}\sigma}{\varepsilon_0}$ $\qquad \vec{E} = \frac{\sigma}{2\varepsilon_0}\vec{U_z}$

ANDAMENTI E, V



ESTENDO AD $\mathbb{R}^-$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\,\text{SIGN}(z)\,\vec{U_z} = \frac{\sigma}{2\varepsilon_0}\cdot\frac{z}{|z|}\vec{U_z}$$

$E = -\text{GRAD}V$

$V = -\frac{\sigma}{2\varepsilon_0}|z|$ $\qquad (V(0) = 0)$

SFERA CON CARICA NON UNIFORME

$\rho(r) = \rho_0\left(\frac{r}{R}\right)$

CALCOLA $\vec{E}$, V

CASO 1 $r < R$

$\phi(E) = 4\pi r^2 E$

$$Q_{INC} = \iiint \rho(r)\, dV = \int_0^r \rho_0\frac{r}{R}\, 4\pi r^2\, dr = \rho_0\pi r^4$$

GAUSS $4\cancel{\pi r^2}E = \frac{\rho_0\cancel{\pi} r^{\cancel{4}\,2}}{\varepsilon_0}$ $\qquad \vec{E} = \frac{\rho_0 r^2}{4\varepsilon_0}\vec{U_r}$

CASO 2 $r > R$

$\phi(E) = 4\pi r^2 E$

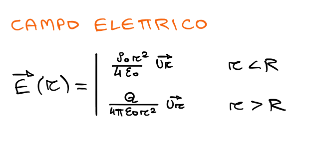
$$Q_{INC} = \iiint \rho(r)\, dV = \int_0^R \rho_0\frac{r}{R}\, 4\pi r^2\, dr = \rho_0\pi R^4$$

GAUSS $4\cancel{\pi}r^2 E = \frac{\rho_0\cancel{\pi}R^4}{\varepsilon_0}$ $\qquad \vec{E} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\vec{U_r} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{U_r}$

CAMPO ELETTRICO

$$\vec{E}(r) = \begin{cases} \frac{\rho_0 r^2}{4\varepsilon_0}\vec{U_r} & r < R \\ \frac{Q}{4\pi\varepsilon_0 r^2}\vec{U_r} & r > R \end{cases}$$

ANDAMENTO E, V



RICAVO IL POTENZIALE DA $\vec{E}$

$\vec{E} = -\text{GRAD } V$

$dV = -E\, dr$

CASO 2 $r > R$

$$\int_{V(\infty)}^{V(r)} dV = -\int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}\, dr$$

$$V(r) - \cancel{V(\infty)} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \cancel{\frac{1}{\infty}}\right)$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$ OVVIAMENTE

CASO 1 $r < R$

$$\int_{V(\infty)}^{V(r)} dV = -\int_\infty^r E\, dr$$

$$V(r) - \cancel{V(\infty)} = -\int_\infty^R E_2\, dr - \int_R^r E_1\, dr$$

$$V(r) = +\frac{Q}{4\pi\varepsilon_0 R} - \frac{\rho_0}{4\varepsilon_0}\left(\frac{r^3}{3}\right)_R^r$$

$$= \frac{Q}{4\pi\varepsilon_0 R} - \frac{\rho_0}{12\varepsilon_0}(r^3 - R^3)$$

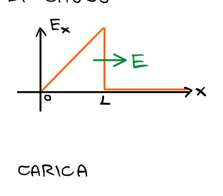
POTENZIALE ELETTRICO

$$V(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 R} - \frac{\rho_0}{12\varepsilon_0}(r^3 - R^3) & r < R \\ \frac{Q}{4\pi\varepsilon_0 r} & r > R \end{cases}$$

TEOREMA DELLA DIVERGENZA

FORMA LOCALE DEL TEOREMA DI GAUSS

$$E = \begin{cases} Kx\,\hat{U_x} & x \in (0, L) \\ 0 & \text{ALTROVE} \end{cases}$$



CALCOLA LA DISTRIBUZIONE DI CARICA

LE ESPRESSIONI IN FORMA LOCALE NON VEDONO LE DENSITÀ DI CARICA LINEARI E SUPERFICIALI

GAUSS LOCALE $\nabla\cdot E = \frac{\rho}{\varepsilon_0}$

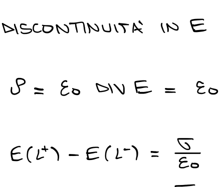
$(E_1 - E_2)\,\hat{N} = \frac{\sigma}{\varepsilon_0}$ COMPONENTE NORMALE DISCONTINUA

DISCONTINUITÀ IN E $\to \exists\,\sigma$ DISTRIBUZIONE

$$\rho = \varepsilon_0\,\text{DIV } E = \varepsilon_0\frac{\partial E}{\partial x} = \varepsilon_0 K$$ DENSITÀ CARICA VOLUMETRICA

$$E(L^+) - E(L^-) = \frac{\sigma}{\varepsilon_0}$$

$$0 - KL = \frac{\sigma}{\varepsilon_0} \to \sigma = -\varepsilon_0 KL$$



DISTRIBUZIONE

PARALLELEPIPEDO ILLIMITATO IN y, z SPESSO L AD L E' APPOGGIATO UN FOGLIO CARICO σ

CILINDRO INDEFINITO DI RAGGIO R

CASO 1 $r < R$

$\phi(E) = E(r)\cdot 2\pi r h$

$Q_{IN} = \pi R^2 h\rho$

$$\cancel{2\pi r h}E = \frac{\cancel{\pi r^2 h}\rho}{\varepsilon_0} \qquad \vec{E}(r) = \frac{\rho r}{2\varepsilon_0}\vec{U_r}$$

CASO 2 $r > R$

$\phi(E) = E(r)\cdot 2\pi r h$

$Q_{IN} = \pi R^2 h\rho$

$$2\cancel{\pi} r\cancel{h} E = \frac{\cancel{\pi}R^2\cancel{h}\rho}{\varepsilon_0} \qquad \vec{E}(r) = \frac{\rho R^2}{2\varepsilon_0 r}\vec{U_r}$$

CAMPO ELETTRICO

$$\vec{E} = \begin{cases} \frac{\rho r}{2\varepsilon_0}\vec{U_r} & r < R \\ \frac{\rho R^2}{2\varepsilon_0 r}\vec{U_r} & r > R \end{cases}$$

RICAVO IL POTENZIALE DA $\vec{E}$

$E = -\text{GRAD } V$

$dV = -E\, dr$

CASO 2 $r > R$

$$\int_{V(RIF)}^{V(r)} dV = -\int_{RIF}^r \frac{\rho R^2}{2\varepsilon_0}\cdot\frac{dr}{r}$$

$$V(r) - V(RIF) = -\frac{\rho R^2}{2\varepsilon_0}\left(\log r - \log(RIF)\right)$$

$$V(r) = -\frac{\rho R^2}{2\varepsilon_0}\log\left(\frac{r}{RIF}\right) \cancel{+V(RIF)}$$ $V_{RIF} = 0$

CASO 1 $r < R$

$$\int_{V(RIF)}^{V(r)} dV = -\int_{RIF}^r E\, dr$$

$V_{RIF} = 0$

$$V(r) = V(RIF) - \int_{RIF}^R E_2 - \int_R^r E_1$$

$$= \cancel{V(RIF)} - \frac{\rho R^2}{2\varepsilon_0}\log\left(\frac{r}{RIF}\right) + \cancel{V(RIF)} - \int_R^r \frac{\rho}{2\varepsilon_0}(r\, dr)$$

$$= V(R) - \frac{\rho}{2\varepsilon_0}\left(\frac{r^2}{2} - \frac{R^2}{2}\right)$$

$$= \frac{\rho}{4\varepsilon_0}(R^2 - r^2)$$

POTENZIALE ELETTRICO

$$V(r) = \begin{cases} \frac{\rho}{4\varepsilon_0}(R^2 - r^2) & r < R \\ -\frac{\rho R^2}{2\varepsilon_0}\log\left(\frac{r}{RIF}\right) & r > R \end{cases}$$ $\qquad V(RIF) = 0$