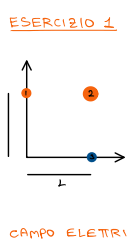
1 - Elettrostatica

Monday, 1 August 2022 09:22

ESERCIZIO 1

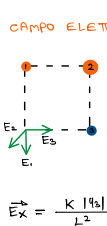


$q_1 = q$

$q_2 = 2q$

$q_3 = -q$

CAMPO ELETTRICO $\vec{E}$ IN (0,0)



$$E = \begin{array}{c|l} x & E_3 - E_2 \cos\alpha \\ y & -E_1 - E_2 \sin\alpha \end{array}$$

QUADRATO → $\alpha = 45°$

$$\vec{E}_x = \frac{k|q_3|}{L^2} - \frac{k|q_2|}{(\sqrt{2}L)^2}\cos 45$$
$$= \frac{k|q_3|}{L^2} - \frac{k|q_2|}{2L^2}\frac{\sqrt{2}}{2}$$
$$= \frac{kq}{L^2} - \frac{k\,\cancel{2}q}{\cancel{2}L^2}\frac{\sqrt{2}}{2}$$
$$= \frac{kq}{L}\left(1 - \frac{\sqrt{2}}{2}\right)\vec{U_x}$$

$$\vec{E}_y = -\frac{k|q_1|}{L^2} - \frac{k|q_2|}{(\sqrt{2}L)^2}\sin 45$$
$$= -\frac{kq}{L^2} - \frac{k\,\cancel{2}q}{\cancel{2}L^2}\frac{\sqrt{2}}{2}$$
$$= -\frac{kq}{L}\left(+1 + \frac{\sqrt{2}}{2}\right)\vec{U_y}$$

MODULO E DIREZIONE DI $\vec{E}$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 L^2}\left[\left(1 - \frac{1}{\sqrt{2}}\right)\vec{U_x} - \left(1 + \frac{1}{\sqrt{2}}\right)\vec{U_y}\right]$$

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \frac{q\sqrt{3}}{4\pi\varepsilon_0 L^2}$$

$$\gamma = \arctan\left(\frac{E_y}{E_x}\right) = \arctan(3+2\sqrt{2}) \cong 80.3°$$

POTENZIALE ELETTRICO V IN (0,0)

$V = V_1 + V_2 + V_3$ SOVRAPPOSIZIONE EFFETTI

$$V = \frac{kq_1}{L} + \frac{kq_2}{\sqrt{2}L} + \frac{kq_3}{L} =$$
$$= \cancel{\frac{kq}{L}} + \frac{2}{\sqrt{2}}\frac{kq}{L} - \cancel{\frac{kq}{L}}$$
$$= \sqrt{2}\,\frac{kq}{L} = \frac{\sqrt{2}\,q}{4\pi\varepsilon_0 L}$$

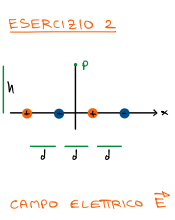
FORZA ELETTROSTATICA $\vec{F}$ IN (0,0)

$$\vec{F} = \vec{E}\cdot q_0 = \frac{q\cdot q_0}{4\pi\varepsilon_0 L^2}\left[\left(1 - \frac{1}{\sqrt{2}}\right)\vec{U_x} - \left(1 + \frac{1}{\sqrt{2}}\right)\vec{U_y}\right]$$

ENERGIA POTENZIALE ELETTRICA U IN (0,0)

$$U = Vq_0 = \frac{\sqrt{2}\,q\,q_0}{4\pi\varepsilon_0 L}$$

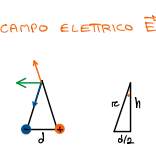
ESERCIZIO 2



TUTTE LE CARICHE $|q| = 10\,mC$

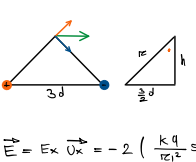
$d = 2\,cm$ $h = 3\,cm$

CAMPO ELETTRICO $\vec{E}$ IN (0,P)



$r_1 = \sqrt{h^2 + (d/2)^2} \cong 3.16\,cm$

$\alpha_1 = \arcsin\left(\frac{d/2}{r}\right) = 9.27°$



$r_2 = \sqrt{h^2 + (\frac{3}{2}d)^2} \cong 4.24\,cm$

$\alpha_2 = \arcsin\left(\frac{3d}{2r}\right) = 45°$

$$\vec{E} = E_x\vec{U_x} = -2\left(\frac{kq}{r_1^2}\sin\alpha_1\right) + 2\left(\frac{kq}{r_2^2}\sin\alpha_2\right)$$
$$= \frac{q}{2\pi\varepsilon_0}\left(\frac{\sin\alpha_2}{r_2^2} - \frac{\sin\alpha_1}{r_1^2}\right)\vec{U_x}$$
$$= 47921\,\vec{U_x}\ N/C$$

POTENZIALE ELETTRICO V IN (0,P)

$V = V_1 + V_2 + V_3 + V_4$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{r_2} - \frac{1}{r_1}\right) = 0$$

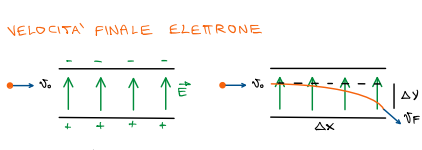
ESERCIZIO 3

$$e^- \left|\begin{array}{l} m = 9.11\cdot 10^{-31}\,kg \\ v_0 = 3\cdot 10^6\,m/s \end{array}\right.$$

$\vec{E} = 500\,\vec{U_y}\ \frac{N}{C}$ UNIFORME

$\Delta x = 5\,cm$

VELOCITA' FINALE ELETTRONE



$$\vec{v}(t) = \begin{array}{c|l} x & v_{0x} \text{ COSTANTE} \\ y & \cancel{v_{0y}} + a t \end{array}$$

$t \in R^+$

ACCELERAZIONE $\Sigma F = ma$ $a = \frac{eE}{m}$

TEMPO $t = \frac{\Delta x}{v_0}$

$$\vec{v}_F = v_0\vec{U_x} + \frac{eE\Delta x}{m\,v_0}\vec{U_y}$$

MODULO E VERSO

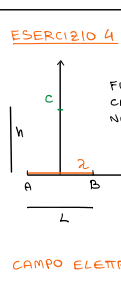
$$|\vec{v}_F| = \sqrt{v_{Fx}^2 + v_{Fy}^2} = 3.34\cdot 10^6\ m/s$$

$$\gamma = \arctan\left(\frac{v_{Fy}}{v_{Fx}}\right) = 26°$$

SPOSTAMENTO Δy

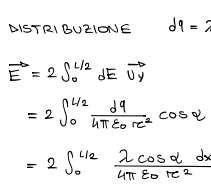
$$\Delta y = \cancel{y_0} + \cancel{v_{0y}t} + \frac{1}{2}at^2$$
$$= \frac{1}{2}\frac{eE}{m}\left(\frac{\Delta x}{v_0}\right)^2 = 1.22\,cm$$

ESERCIZIO 4



FILO CARICO UNIFORMEMENTE λ
CALCOLA CAMPO ELETTRICO
NEI PUNTI C, D

CAMPO ELETTRICO IN C



$r = \sqrt{h^2 + x^2}$

$\cos\alpha = \frac{ADJ}{HYP} = \frac{h}{r}$

$\tan\alpha = \frac{OPP}{ADJ} = \frac{x}{h}$

DISTRIBUZIONE $dq = \lambda\,dx$

$$\vec{E} = 2\int_0^{L/2} dE\ \vec{U_y}$$
$$= 2\int_0^{L/2}\frac{dq}{4\pi\varepsilon_0 r^2}\cos\alpha$$
$$= 2\int_0^{L/2}\frac{\lambda\cos\alpha\,dx}{4\pi\varepsilon_0 r^2}$$

ESPRIMIAMO TUTTO TRAMITE α

$x : [0, L/2] \rightarrow \alpha : [0, \alpha_{MAX}]$

$x(\alpha) = h\tan\alpha$

$r(\alpha) = \frac{h}{\cos\alpha}$ $dx = \frac{h\,d\alpha}{\cos^2\alpha}$

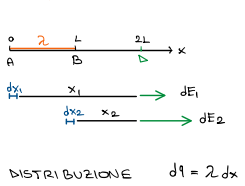
$$= 2\int_0^{\alpha_M}\frac{\lambda\cos\alpha}{4\pi\varepsilon_0}\,\frac{\cancel{\cos^2\alpha}}{h^{\cancel{2}}}\,\frac{\cancel{h}\,d\alpha}{\cancel{\cos^2\alpha}}$$
$$= 2\int_0^{\alpha_M}\frac{\lambda}{4\pi\varepsilon_0 h}\cos\alpha\,d\alpha$$
$$= \frac{\lambda}{2\pi\varepsilon_0 h}\sin\alpha\Big|_0^{\alpha_M}$$

ANGOLO MASSIMO α_M

$r\sin\alpha = L/2$ $\alpha_{MAX} = \arcsin\left(\frac{L/2}{\sqrt{h^2 + L^2/4}}\right)$

$$\vec{E}(C) = \frac{\lambda\sin(\arcsin(\cdots))}{2\pi\varepsilon_0 h} = \frac{\lambda L}{2\pi\varepsilon_0 h\sqrt{4h^2 + L^2}}\vec{U_y}$$

CAMPO ELETTRICO IN D



$E(D) = E_x\vec{U_x}$

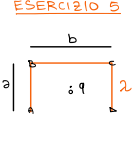
DISTRIBUZIONE $dq = \lambda\,dx$

$$dE = \frac{dq}{4\pi\varepsilon_0(2L - x)^2} = \frac{\lambda\,dx}{4\pi\varepsilon_0(2L-x)^2}$$

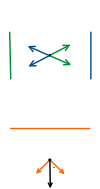
$$E = \int_0^L dE =$$
$$= \int_0^L\frac{\lambda\,dx}{4\pi\varepsilon_0(2L-x)^2}$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\int_0^L\frac{1}{(2L-x)^2}$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{(2L-x)}\right]_0^L$$
$$= \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{L} - \frac{1}{2L}\right)$$
$$= \frac{\lambda}{8\pi\varepsilon_0 L}\vec{U_x}$$

$$\frac{\partial}{\partial x}(2L-x)^{-1} =$$
$$= -(2L-x)^{-2}\frac{\partial}{\partial x}(-x)$$
$$= +(2L-x)^{-2}$$

ESERCIZIO 5



DETERMINA LA FORZA
ELETTROSTATICA AGENTE
SULLA CARICA q

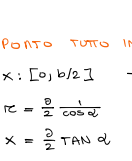


I CAMPI GENERATI DA $\overline{AB}$
E $\overline{CD}$ HANNO CONTRIBUTI OPPOSTI
LA LORO SOVRAPPOSIZIONE E' ZERO

IL PROBLEMA SI RIDUCE A
QUESTA DISTRIBUZIONE

$\vec{E} = E_y\vec{U_y}$

DISTRIBUZIONE $dq = \lambda\,dx$



$r = \sqrt{x^2 + (a/2)^2}$

$r\cos\alpha = \frac{a}{2}$

$\tan\alpha = \frac{2x}{a}$

$$E = 2\int_0^{b/2}\frac{\lambda\,dx}{4\pi\varepsilon_0 r^2}\cos\alpha$$

PORTO TUTTO IN FUNZIONE DI α

$x : [0, b/2] \rightarrow \alpha : [0, \alpha_M]$

$r = \frac{a}{2}\frac{1}{\cos\alpha}$

$x = \frac{a}{2}\tan\alpha$

$dx = \frac{a}{2}\frac{1}{\cos^2\alpha}d\alpha$

ANGOLO MASSIMO

$r_{MAX}\sin\alpha = \frac{b}{2}$

$$\alpha_{MAX} = \arcsin\left(\frac{b}{2r_{MAX}}\right) = \arcsin\left(\frac{b}{\sqrt{a^2+b^2}}\right)$$

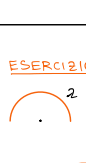
$$E = \frac{\lambda}{2\pi\varepsilon_0}\int_0^{b/2}\frac{1}{r^2}\cos\alpha\,dx$$
$$= \frac{\lambda}{2\pi\varepsilon_0}\int_0^{\alpha_M}\frac{4}{a^2}\cancel{\cos^2\alpha}\cos\alpha\,\frac{a}{2}\frac{1}{\cancel{\cos^2\alpha}}d\alpha$$
$$= \frac{\lambda}{\pi a\varepsilon_0}\int_0^{\alpha_M}\cos\alpha\,d\alpha$$
$$= \frac{\lambda}{\pi a\varepsilon_0}\sin(\alpha_{MAX})$$
$$= \frac{\lambda}{\pi a\varepsilon_0}\cdot\frac{b}{\sqrt{a^2+b^2}}\vec{U_y}$$

INOLTRE $\lambda = \frac{Q}{l_{TOT}} = \frac{Q}{2a+b}$

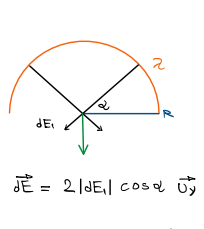
FORZA ELETTROSTATICA DI q

$$\vec{F} = q\vec{E} = \frac{b}{\pi a\varepsilon_0}\cdot\frac{Qq}{2a+b}\cdot\frac{1}{\sqrt{a^2+b^2}}\vec{U_y}$$

ESERCIZIO 6



FILO λ OMOGENEO



$dq = \lambda\,dl$

$|dE_1| = \frac{dq}{4\pi\varepsilon_0 R^2}$

$$d\vec{E} = 2|dE_1|\cos\alpha\ \vec{U_y}$$

$$d\vec{E} = 2\,\frac{\lambda\cos\alpha\,dl}{4\pi\varepsilon_0 R^2}$$

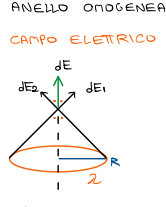
DOVE $dl = R\,d\alpha$

$$\vec{E} = \int_0^{\pi/2}\frac{2\lambda R}{4\pi\varepsilon_0 R^2}\cos\alpha\,d\alpha$$
$$= \frac{\lambda}{2\pi\varepsilon_0 R}\sin\alpha\Big]_0^{\pi/2}$$
$$= \frac{\lambda}{2\pi\varepsilon_0 R}\vec{U_y}$$

ESERCIZIO 7

ANELLO OMOGENEAMENTE CARICATO

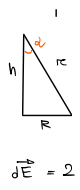
CAMPO ELETTRICO LUNGO L'ASSE



$\lambda = \frac{Q}{2\pi R}$

$dq = \lambda\,dl$ $dl = R\,d\alpha$

$|d\vec{E}_1| = \frac{dq}{4\pi\varepsilon_0 r^2}$



$r = \sqrt{h^2 + R^2}$

$\cos\alpha = \frac{ADJ}{HYP} = \frac{h}{\sqrt{h^2+R^2}}$

$$d\vec{E} = 2|d\vec{E}_1|\cos\alpha$$

$$\vec{E} = \int_0^{\pi R}\frac{2\lambda h\,dl}{4\pi\varepsilon_0 r^3} = \frac{\lambda h R}{2\varepsilon_0(R^2+h^2)^{3/2}}\vec{U_y}$$

MOTO DI UNA PARTICELLA ATTORNO AL CENTRO

AL CENTRO $h \ll R$

$$\vec{E} \sim \frac{\lambda h R}{2\varepsilon_0 R^3} = \frac{\lambda h}{2\varepsilon_0 R^2}\vec{U_y}$$

$$\vec{F} = -q\vec{E} = -\frac{\lambda q}{2\varepsilon_0 R^2}\,y\,\vec{U_y}$$

EQUAZIONE DI MOTO

$$m\ddot{y} = -\underbrace{\boxed{\frac{\lambda q}{2\varepsilon_0 R^2}}}_{k}\,y$$

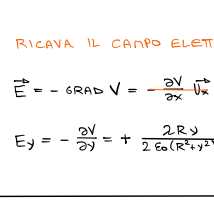
MOTO ARMONICO SEMPLICE
LUNGO ASSE y

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\lambda q}{2\varepsilon_0 R^2 m}}$$

ESERCIZIO 8

ANELLO OMOGENEAMENTE CARICATO

POTENZIALE ELETTRICO LUNGO L'ASSE



$\lambda = \frac{Q}{2\pi R}$

$dq = \lambda\,dl$

$r = \sqrt{h^2 + R^2}$

$dV = \frac{\lambda\,dl}{4\pi\varepsilon_0 r}$ $V = \int_\lambda dV$

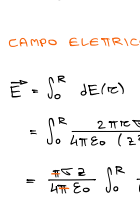
$$V = \int_0^{2\pi R}\frac{\lambda\,dl}{4\pi\varepsilon_0 r} = \frac{\lambda R}{2\varepsilon_0\sqrt{h^2+R^2}}$$

RICAVA IL CAMPO ELETTRICO DA V

$$\vec{E} = -\text{GRAD}\,V = -\frac{\partial V}{\partial x}\vec{U_x} - \frac{\partial V}{\partial y}\vec{U_y}$$

$$E_y = -\frac{\partial V}{\partial y} = +\frac{\lambda R y}{2\varepsilon_0(R^2+y^2)^{3/2}}$$

ESERCIZIO 9



IL DISCO E' UNA DISTRIBUZIONE
CONTINUA DI ANELLI R-VARIABILI

CAMPO ELETTRICO GENERATO SULL'
ASSE DI UN ANELLO DI RAGGIO r

$$\vec{E}(r) = \frac{Q\,z}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}\vec{U_z}$$

CARICA SU ANELLO SPESSO dr

$$\sigma = \frac{dq}{dS} = \frac{dq}{2\pi r\,dr}$$ $\lambda = \sigma z$

CAMPO ELETTRICO

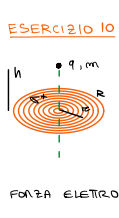
$$\vec{E} = \int_0^R dE(r)$$
$$= \int_0^R\frac{2\pi r\sigma z\,dr}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}$$
$$= \frac{\pi\sigma z}{4\pi\varepsilon_0}\int_0^R\frac{2r\,dr}{(z^2+r^2)^{3/2}}$$
$$= \frac{\sigma z}{4\varepsilon_0}\frac{(-2)}{\sqrt{z^2+r^2}}\Big|_0^R$$
$$= \frac{\sigma}{2\varepsilon_0}z\left(\frac{1}{|z|} - \frac{1}{\sqrt{z^2+R^2}}\right)\vec{U_z}$$

POTENZIALE ELETTRICO

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

$$V = \int_0^R dV(r)$$
$$= \int_0^R\frac{2\pi r\sigma\,dr}{4\pi\varepsilon_0(z^2+r^2)^{1/2}}$$
$$= \frac{\sigma}{2\varepsilon_0}\left|\sqrt{z^2+r^2}\right|_0^R$$
$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - |z|\right)$$

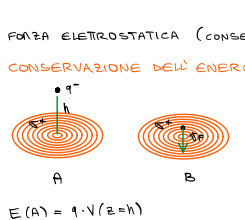
ESERCIZIO 10



SE CARICA PARTE FERMA
DA $h = 3R$, TROVA v_F

FORZA ELETTROSTATICA (CONSERVATIVA)

CONSERVAZIONE DELL'ENERGIA



$E(A) = q\cdot V(z=h)$

$E(B) = \frac{m v_F^2}{2} + q\cdot V(z=0)$

$E(A) = E(B)$

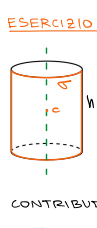
$$q\cdot\frac{\sigma}{2\varepsilon_0}\left(\sqrt{h^2+R^2} - h\right) = \frac{m v_F^2}{2} + q\cdot\frac{\sigma}{2\varepsilon_0}R$$

$$\frac{q\sigma}{\varepsilon_0}R(\sqrt{10} - 3) = m v_F^2 + \frac{q\sigma}{\varepsilon_0}R$$

$$m v_F^2 = \frac{q\sigma}{\varepsilon_0}R(\sqrt{10} - 4)$$

$$v_F = +\sqrt{\frac{q\sigma R}{\varepsilon_0 m}(\sqrt{10} - 4)}$$

ESERCIZIO 11

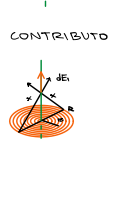


DISTRIBUZIONE CARICA OMOGENEA

AREA LATERALE + 1 BASE

CAMPO ELETTRICO IN C

CONTRIBUTO AREA LATERALE $\vec{E} = 0$ (SIMMETRIA)

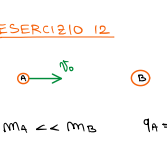


IL PROBLEMA SI SEMPLIFICA AL
CAMPO ASSIALE GENERATO DA
UN DISCO CARICO OMOGENEAMENTE

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\vec{U_z}$$

DOVE $\sigma = \frac{Q}{A} = \frac{Q}{(2\pi R h) + \pi R^2}$

ESERCIZIO 12



$m_A \ll m_B$ $q_A = q_B$

TROVA Δx AFFINCHE' $v_F = v_0/2$

CONSERVAZIONE ENERGIA

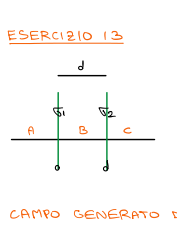
$$\frac{m v_0^2}{2} = \frac{m}{2}\left(\frac{v_0}{2}\right)^2 + \frac{kq^2}{\Delta x}$$

$$\frac{m v_0^2}{2} - \frac{m v_0^2}{8} = \frac{kq^2}{\Delta x}$$

$$\frac{3}{8}m v_0^2 = \frac{kq^2}{\Delta x}$$

$$\Delta x = \frac{8}{3}\frac{kq^2}{m v_0^2} = \frac{2q^2}{3\pi\varepsilon_0 m v_0^2}$$

ESERCIZIO 13



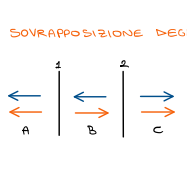
LASTRE INFINITE DISTANTI d
E CARICATE RISPETTIVAMENTE $\sigma_{1,2}$

CALCOLA $\vec{E}, V$ $(V(x=0) = 0)$

CAMPO GENERATO DA UN SOLO PIANO

$$\Phi_\Sigma(\vec{E}) = 2EA = \frac{\sigma A}{\varepsilon_0} \longrightarrow E_x = \frac{\sigma_x}{2\varepsilon_0}$$

SOVRAPPOSIZIONE DEGLI EFFETTI



$$\vec{E}(A) = \left(-\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}\right)\vec{U_x}$$

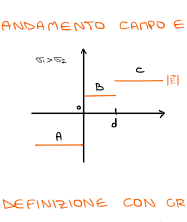
$$\vec{E}(B) = \left(+\frac{\sigma_1}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0}\right)\vec{U_x}$$

$$\vec{E}(C) = \left(+\frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}\right)\vec{U_x}$$

CAMPO ELETTRICO

$$\vec{E}(x) = \begin{cases} -\frac{\sigma_1+\sigma_2}{2\varepsilon_0}\vec{U_x} & x < 0 \\ \frac{\sigma_1-\sigma_2}{2\varepsilon_0}\vec{U_x} & 0 < x < d \\ \frac{\sigma_1+\sigma_2}{2\varepsilon_0}\vec{U_x} & x > d \end{cases}$$

ANDAMENTO CAMPO ELETTRICO



DEFINIZIONE CON GRADIENTE

$$\vec{E} = -\text{GRAD}\,V = -\frac{\partial V}{\partial x}\vec{U_x}$$

$\int dV = -\int E\,dx$ $\underline{V(0) = 0}$

CASO 1 $x < 0$

$$\int_0^{V(x)} dV = -\int_0^x -\frac{\sigma_1+\sigma_2}{2\varepsilon_0}dx$$

$$V(x) = \frac{\sigma_1+\sigma_2}{2\varepsilon_0}x$$

CASO 2 $x \in (0,d)$

$$\int_0^{V(x)} dV = -\int_0^x\frac{\sigma_1-\sigma_2}{2\varepsilon_0}dx$$

$$V(x) = -\frac{\sigma_1-\sigma_2}{2\varepsilon_0}x$$

CASO 3 $x = d$

$$V(x=d) = -\frac{\sigma_1-\sigma_2}{2\varepsilon_0}d$$

CASO 4 $x > d$

$$\int_{V(d)}^{V(x)} dV = -\int_d^x\frac{\sigma_1+\sigma_2}{2\varepsilon_0}dx$$

$$V(x) - V(d) = -\frac{\sigma_1+\sigma_2}{2\varepsilon_0}(x-d)$$

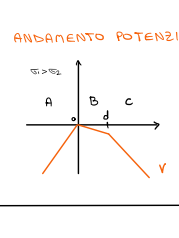
$$V(x) = -\frac{\sigma_1+\sigma_2}{2\varepsilon_0}(x-d) - \frac{\sigma_1-\sigma_2}{2\varepsilon_0}d$$

$$V(x) = \frac{\sigma_2 d}{\varepsilon_0} - \frac{\sigma_1+\sigma_2}{2\varepsilon_0}$$

POTENZIALE ELETTRICO

$$V(x) = \begin{cases} \frac{\sigma_1+\sigma_2}{2\varepsilon_0}x & x < 0 \\ -\frac{\sigma_1-\sigma_2}{2\varepsilon_0}x & 0 < x < d \\ \frac{\sigma_2 d}{\varepsilon_0} - \frac{\sigma_1+\sigma_2}{2\varepsilon_0} & x > d \end{cases}$$

ANDAMENTO POTENZIALE ELETTRICO

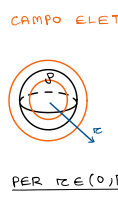


ESERCIZIO 14

SFERA DI RAGGIO R, CALCOLA $\vec{E}, V$

CARICA Q^+ UNIFORMEMENTE DISTRIBUITA

CAMPO ELETTRICO



DISTRIBUZIONE $Q = \rho\frac{4}{3}\pi r^3$

TEOREMA DI GAUSS

$$\Phi_\Sigma(\vec{E}) = \frac{Q}{\varepsilon_0} = \frac{4\pi\rho r^3}{3\varepsilon_0}$$

PER $r \in (0,R)$

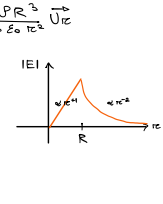
$$4\pi r^2 E = \frac{4\pi\rho r^3}{3\varepsilon_0} \qquad \vec{E} = \frac{\rho r}{3\varepsilon_0}\vec{U_r}$$

PER $r > R$

$$4\pi r^2 E = \frac{4\pi\rho R^3}{3\varepsilon_0} \qquad \vec{E} = \frac{\rho R^3}{3\varepsilon_0 r^2}\vec{U_r}$$

CAMPO ELETTRICO

$$\vec{E}(r) = \begin{cases} \frac{\rho r}{3\varepsilon_0}\vec{U_r} & r \in (0,R) \\ \frac{\rho R^3}{3\varepsilon_0 r^2}\vec{U_r} & r \in [R,\infty) \end{cases}$$



RICAVO IL POTENZIALE DA $\vec{E}$

$$E = -\text{GRAD}\,V = -\frac{\partial V}{\partial r}\vec{U_r}$$

PER $r > R$

$$\int_0^{V(r)} dV = -\frac{Q}{4\pi\varepsilon_0}\int_\infty^r\frac{dr}{r^2}$$

$$V(r) = +\frac{Q}{4\pi\varepsilon_0 r} = \frac{R^3\rho}{3\varepsilon_0 r}$$

PER $r \in (0,R)$

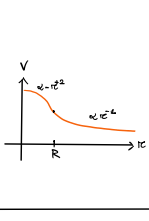
$$\int_{V(r)}^{V(R)} dV = -\frac{\rho}{3\varepsilon_0}\int_r^R r\,dr$$

$$V(R) - V(r) = -\frac{\rho}{3\varepsilon_0}\left(\frac{R^2}{2} - \frac{r^2}{2}\right)$$

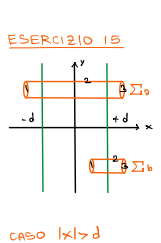
$$V(r) = \frac{\rho}{6\varepsilon_0}(R^2 - r^2) + V(R)$$
$$= \frac{\rho}{6\varepsilon_0}(R^2 - r^2) + \frac{\rho R}{3\varepsilon_0}$$
$$= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$$

POTENZIALE ELETTRICO

$$\vec{E}(r) = \begin{cases} \frac{\rho}{6\varepsilon_0}(3R^2 - r^2) & r \in (0,R) \\ \frac{R^3\rho}{3\varepsilon_0 r} & r \in [R,\infty) \end{cases}$$



ESERCIZIO 15



CARICA Q^+ DISTRIBUITA
TRA PIANI $x = \pm d$

CALCOLA $\vec{E}, V$ ASSUMENDO
$V(RIF) = V(0) = 0$

CASO $|x| > d$
SUPERFICIE Σ_a

$$\Phi_\Sigma(\vec{E}) = 2\iint_1\vec{E}\cdot\hat{n}\,dS + \cancel{\iint_2\vec{E}\cdot\hat{n}_2\,dS} = 2E(d)\pi R^2$$

$$Q_{INT} = \iiint_\tau d\tau = 2(\pi R^2 d)\rho$$

GAUSS $\quad 2E(d)\pi R^2 = \frac{2\pi R^2 d\rho}{\varepsilon_0} \qquad \vec{E}(x) = \frac{\rho d}{\varepsilon_0}\text{SIGN}(x)\vec{U_x}$

CASO $|x| < d$
SUPERFICIE Σ_b

$$\Phi_\Sigma(\vec{E}) = \iint_1\vec{E}\cdot\hat{n}\,dS + \cancel{\iint_2\vec{E}\cdot\hat{n}_2\,dS} + \iint_3\vec{E}\cdot\hat{n}_3\,dS$$
$$= E(x)\pi R^2 + E(d)\pi R^2$$
$$= E(x)\pi R^2 + \frac{\rho d}{\varepsilon_0}\pi R^2$$

$Q_{INT} = \iiint_\tau d\tau = (\pi R^2 x)\rho$

GAUSS $\quad E(x)A + \frac{\rho d}{\varepsilon_0}A = \frac{A\,x\,\rho}{\varepsilon_0}$

$$\vec{E}(x) = \frac{\rho}{\varepsilon_0}(x_0 - d)\vec{U_x} = \frac{\rho}{\varepsilon_0}x\,\text{SIGN}(x)\vec{U_x}$$

NELLA REGIONE INTERNA $x = d - x_0$

CAMPO ELETTRICO

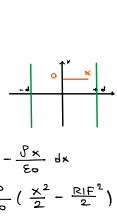
$$\vec{E}(x) = \begin{cases} \frac{\rho}{\varepsilon_0}x\,\text{SIGN}(x)\vec{U_x} & |x| < d \\ \frac{\rho d}{\varepsilon_0}\text{SIGN}(x)\vec{U_x} & |x| > d \end{cases}$$

RICAVO IL POTENZIALE

$\vec{E} = -\text{GRAD}\,V$

$dV = -E\,dx$

CASO a $|x| < d$

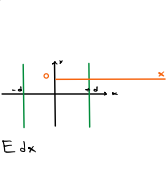


$$\int_{V(RIF)}^{V(x)} dV = \int_{RIF}^x -\frac{\rho x}{\varepsilon_0}dx$$

$$V(x) - V(RIF) = -\frac{\rho}{\varepsilon_0}\left(\frac{x^2}{2} - \frac{RIF^2}{2}\right) \qquad V(RIF) = V(0) = 0$$

$$V(x) = -\frac{\rho x^2}{2\varepsilon_0}$$

CASO b $|x| > d$



$$\int_{V(RIF)}^x dV = -\int_{RIF}^x E\,dx$$

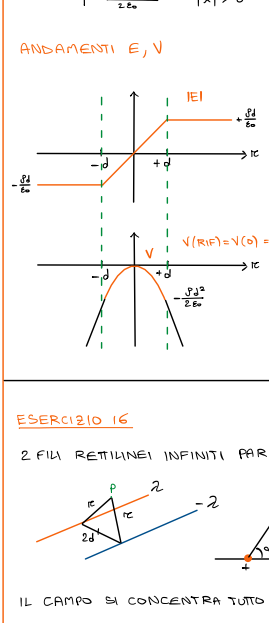
$$V(x) - \cancel{V(RIF)} = -\int_{RIF}^d E_a - \int_d^x E_b$$

$$V(x) = -\frac{\rho x^2}{2\varepsilon_0} - \int_d^x\frac{\rho d}{\varepsilon_0}dx \qquad V(RIF) = V(0) = 0$$
$$= -\frac{\rho x^2}{2\varepsilon_0} - \frac{\rho d}{\varepsilon_0}(x - d)$$
$$= -\frac{\rho d(2x - d)}{2\varepsilon_0}$$

POTENZIALE ELETTRICO

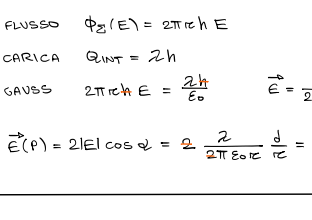
$$V(x) = \begin{cases} -\frac{\rho x^2}{2\varepsilon_0} & |x| < d \\ -\frac{\rho d(2x-d)}{2\varepsilon_0} & |x| > d \end{cases}$$

ANDAMENTI E, V



ESERCIZIO 16

2 FILI RETTILINEI INFINITI PARALLELI CARICHI λ



$\vec{E} = E(x)\vec{U_x}$

$\cos\alpha = d/r$

IL CAMPO SI CONCENTRA TUTTO SULLA x

OGNI SINGOLO FILO EMETTE UN CAMPO A SIMMETRIA CILINDRICA

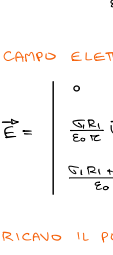
FLUSSO $\Phi_\Sigma(E) = 2\pi r h E$

CARICA $Q_{INT} = \lambda h$

GAUSS $\quad 2\pi r h E = \frac{\lambda h}{\varepsilon_0} \qquad \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{U_r}$

$$\vec{E}(P) = 2|E|\cos\alpha = 2\,\frac{\lambda}{2\pi\varepsilon_0 r}\,\frac{d}{r} = \frac{\lambda d}{\pi\varepsilon_0 r^2}\vec{U_x}$$

ESERCIZIO 17



2 CILINDRI INFINITI
COASSIALI CARICHI
UNIFORMEMENTE σ_1, σ_2

$r < R_1$ $\quad Q_{INC} = 0 \rightarrow E = 0$

$R_1 < r < R_2$

$\Phi_\Sigma(E) = E(r)\,2\pi r h$

$Q_{INC} = \sigma S = \sigma_1 2\pi R_1 h$

GAUSS $\quad E(r)\,2\pi r h = \frac{\sigma_1 2\pi R_1 h}{\varepsilon_0} \qquad \vec{E}(r) = \frac{\sigma_1 R_1}{\varepsilon_0 r}\vec{U_r}$

$r > R_2$

$\Phi_\Sigma(E) = E(r)\,2\pi r h$

$Q_{INC} = \sigma_1 S_1 + \sigma_2 S_2 = \sigma_1(2\pi R_1 h) + \sigma_2(2\pi R_2 h)$

GAUSS $\quad E(r)\,2\pi r h = \frac{2\pi h(\sigma_1 R_1 + \sigma_2 R_2)}{\varepsilon_0}$

$$\vec{E}(r) = \frac{\sigma_1 R_1 + \sigma_2 R_2}{\varepsilon_0 r}\vec{U_r}$$

CAMPO ELETTRICO

$$\vec{E} = \begin{cases} 0 & r < R_1 & \\ \frac{\sigma_1 R_1}{\varepsilon_0 r}\vec{U_r} & r \in (R_1, R_2) & \propto r^{-1} \\ \frac{\sigma_1 R_1 + \sigma_2 R_2}{\varepsilon_0 r}\vec{U_r} & r > R_2 & \propto r^{-1} \end{cases}$$

RICAVO IL POTENZIALE

$\vec{E} = -\vec{\nabla}V = -\text{GRAD}\,V$

$dV = -E\,dr$

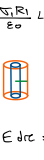
CONSIDERANDO $V_{RIF} = V(R_2) = 0$

$R_1 < r < R_2$

$$\int_{V(R_2)}^{V(r)} dV = \int_{R_2}^r -\frac{\sigma_1 R_1}{\varepsilon_0 r}dr$$

$$V(r) - \cancel{V(R_2)} = -\frac{\sigma_1 R_1}{\varepsilon_0}\log\left(\frac{r}{R_2}\right)$$

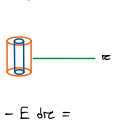
$r < R_1$



$$\int_{V(R_2)}^{V(r)} dV = \int_{R_2}^r -E\,dr = V(R_1) - \cancel{\int_{R_1}^r 0}$$

$$V(r) - \cancel{V(R_2)} = -\frac{\sigma_1 R_1}{\varepsilon_0}\log\left(\frac{R_1}{R_2}\right) \quad \text{COSTANTE}$$

$r > R_2$



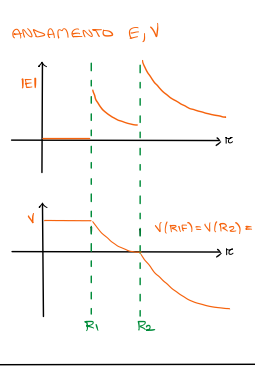
$$\int_{V(R_2)}^{V(r)} dV = \int_{R_2}^r -E\,dr =$$

$$V(r) - \cancel{V(R_2)} = -\frac{\sigma_1 R_1 + \sigma_2 R_2}{\varepsilon_0}\log\left(\frac{r}{R_2}\right)$$

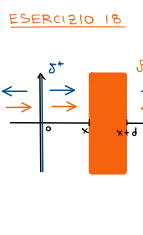
POTENZIALE ELETTRICO

$$V = \begin{cases} -\frac{\sigma_1 R_1}{\varepsilon_0}\log\left(\frac{R_1}{R_2}\right) & r < R_1 & \text{COST} \\ -\frac{\sigma_1 R_1}{\varepsilon_0}\log\left(\frac{r}{R_2}\right) & r \in (R_1, R_2) & \log(r) \\ -\frac{\sigma_1 R_1 + \sigma_2 R_2}{\varepsilon_0}\log\left(\frac{r}{R_2}\right) & r > R_2 & \log(r) \end{cases}$$

ANDAMENTO E, V



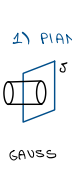
ESERCIZIO 18



PIANO E PARALLELEPIPEDO
INFINITI LUNGO y, z

TROVA σ, ρ AFFINCHE
$\vec{E} \neq 0$ PER $0 < x < x_0 + d$

1) PIANO σ

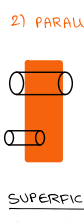


$\Phi_\Sigma(\vec{E}) = 2AE$

$Q_{INC} = A\sigma$

GAUSS $\quad 2AE = \frac{A\sigma}{\varepsilon_0} \qquad \vec{E} = \frac{\sigma}{2\varepsilon_0}\text{SIGN}(x)\vec{U_x}$

2) PARALLELEPIPEDO ρ



SUPERFICIE Σ_1 $\quad x < 0 \vee x > x_0 + d$

$\Phi_\Sigma(E) = 2EA$

$Q_{INT} = \rho A d \quad \rightarrow \quad \vec{E} = \frac{\rho d}{2\varepsilon_0}\vec{U_x}$

SUPERFICIE Σ_2

$$\Phi_{\Sigma_2}(\vec{E}) = E(\Sigma)\pi R^2 + E(\Sigma_2)\pi R^2$$

$$Q_{INT} = \pi R^2(x_0 - x)$$

$$\frac{\rho d}{2\varepsilon_0}\cancel{\pi R^2} + E\,\cancel{\pi R^2} = \frac{\cancel{\pi R^2}(x_0 - x)\rho}{\varepsilon_0}$$

$$E = \frac{(x_0 - x)\rho}{\varepsilon_0} - \frac{\rho d}{2\varepsilon_0}$$

$$\vec{E} = \frac{\rho}{\varepsilon_0}\left(x_0 - x - \frac{d}{2}\right)\vec{U_x}$$

SOVRAPPOSIZIONE EFFETTI

$E = 0$ IN $\quad x < 0,\ x > x_0 + d$

$$\frac{\sigma}{2\varepsilon_0} = \frac{\rho d}{2\varepsilon_0} \longrightarrow \sigma = \rho d$$

CAMPO ELETTRICO

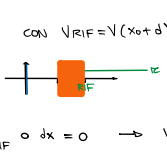
$$\vec{E} = \begin{cases} 0 & x < 0 \\ \frac{\sigma}{2\varepsilon_0} + \frac{\rho d}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}\vec{U_x} & x \in (0, x_0) \\ \frac{\sigma}{2\varepsilon_0} + \frac{\rho}{\varepsilon_0}(\cdots) = \frac{\sigma}{\varepsilon_0}\left(1 - \frac{x - x_0}{d}\right)\vec{U_x} & x \in (x_0, x_0+d) \\ 0 & x > x_0 + d \end{cases}$$

RICAVO IL POTENZIALE ELETTRICO

$\vec{E} = -\text{GRAD}\,V$

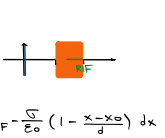
$dV = \int E\,dx$ CON $V_{RIF} = V(x_0 + d) = 0$

$x > x_0 + d$



$$\int_{RIF}^x dV = \int_{RIF}^x 0\,dx = 0 \longrightarrow V(x) = 0$$

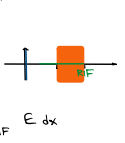
$x \in (x_0, x_0 + d)$



$$\int_{RIF}^x dV = \int_{RIF}^x -\frac{\sigma}{\varepsilon_0}\left(1 - \frac{x - x_0}{d}\right)dx$$

$$V(x) = \frac{\sigma}{\varepsilon_0 d}\int_{RIF}^x(x - x_0 - d)\,dx$$
$$= \frac{\sigma}{\varepsilon_0 d}\left[\frac{x^2}{2} - (x_0 + d)x\right]_{RIF}^x$$
$$= \frac{\sigma}{\varepsilon_0 d}\left(\frac{x^2}{2} - (x_0+d)x - \frac{(x_0+d)^2}{2} + (x_0+d)(x_0+d)\right)$$
$$= \frac{\sigma}{2d\varepsilon_0}\left(x^2 - 2(x_0+d)x + (x_0+d)^2\right)$$

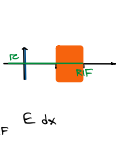
$x \in (0, x_0)$



$$\int_{RIF}^x dV = \int_{RIF}^x E\,dx$$

$$V(x) = V(x_0) - \int_{x_0}^x\frac{\sigma}{\varepsilon_0}dx$$
$$= \frac{\sigma}{2\varepsilon_0 d}d^2 - \frac{\sigma}{\varepsilon_0}(x - x_0)$$
$$= \frac{\sigma}{\varepsilon_0}\left(\frac{d}{2} - x + x_0\right)$$

$x \in (0, x_0)$



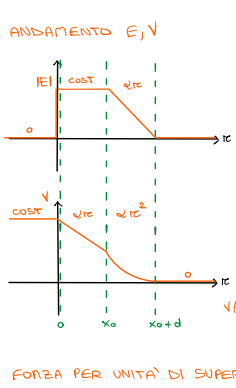
$$\int_{RIF}^x dV = \int_{RIF}^x E\,dx$$

$$V(x) = V(0) - \cancel{\int_0^x 0\,dx} = V(0)$$

POTENZIALE ELETTRICO

$$\vec{E} = \begin{cases} \frac{\sigma}{\varepsilon_0}\left(\frac{d}{2} + x_0\right) & x < 0 \\ \frac{\sigma}{\varepsilon_0}\left(\frac{d}{2} - x + x_0\right) & x \in (0, x_0) \\ \frac{\sigma}{2d\varepsilon_0}\left(x^2 - 2(x_0+d)x + (x_0+d)^2\right) & x \in (x_0, x_0+d) \\ 0 & x > x_0 + d \end{cases}$$

ANDAMENTO E, V



FORZA PER UNITA' DI SUPERFICIE

MOLTO VICINO A dS $\quad E \sim \frac{\sigma}{2\varepsilon_0}$

$$dF = dq\,E = \sigma\,dS\,E_{TOT - dS} = \sigma\,dS\,\frac{\sigma}{2\varepsilon_0}$$

$$\vec{F} = \frac{\sigma^2}{2\varepsilon_0}\vec{U_x}$$

ESERCIZIO 19

CILINDRO DI LUNGHEZZA INFINITA
CARICATO UNIFORMEMENTE ρ

CALCOLA E, V ASSUMENDO $V(r=0) = 0$

$r < R$

$\Phi_\Sigma(\vec{E}) = 2\pi r h E$

$Q_{INC} = \pi r^2 h\rho$

GAUSS $\quad 2\pi r h E = \frac{\pi r^2 h\rho}{\varepsilon_0}$

$$\vec{E} = \frac{\rho}{2\varepsilon_0}r\,\vec{U_r}$$

$r > R$

$\Phi_\Sigma(\vec{E}) = 2\pi r h E$

$Q_{INC} = \pi R^2 h\rho$

GAUSS $\quad 2\pi r h E = \frac{\pi R^2 h\rho}{\varepsilon_0}$

$$\vec{E} = \frac{R^2\rho}{2\varepsilon_0 r}\vec{U_r}$$

CAMPO ELETTRICO

$$\vec{E}(r) = \begin{cases} \frac{\rho}{2\varepsilon_0}r\,\vec{U_r} & r < R \\ \frac{R^2\rho}{2\varepsilon_0 r}\vec{U_r} & r > R \end{cases}$$

RICAVO IL POTENZIALE DA E

$\vec{E} = -\text{GRAD}\,V$

$dV = -E\,dr$

$r < R$

$$\int_{V(0)}^{V(r)} dV = \int_0^r\frac{\rho}{2\varepsilon_0}r\,dr$$

$$V(r) - \cancel{V(0)} = -\frac{\rho}{2\varepsilon_0}\left(\frac{r^2}{2}\right)$$

$$V(r) = -\frac{\rho r^2}{4\varepsilon_0}$$

$r > R$

$$\int_{V(R)}^{V(r)} dV = \int_0^r E\,dr$$

$$V(r) - V(R) = V(R) - \int_R^r\frac{R^2\rho}{2\varepsilon_0 r}dr$$

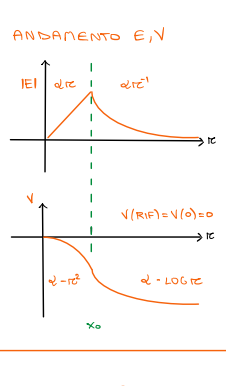
$$V(r) = -\frac{\rho R^2}{4\varepsilon_0} - \frac{\rho R^2}{2\varepsilon_0}\log\left(\frac{r}{R}\right)$$

$$V(r) = -\frac{\rho R^2}{4\varepsilon_0}\left(1 + 2\log\frac{r}{R}\right)$$

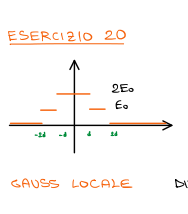
POTENZIALE ELETTRICO

$$V(r) = \begin{cases} -\frac{\rho r^2}{4\varepsilon_0} & r < R \\ -\frac{\rho R^2}{4\varepsilon_0}\left(1 + 2\log\frac{r}{R}\right) & r > R \end{cases}$$

ANDAMENTO E, V



ESERCIZIO 20



DISTRIBUZIONE DI CARICA
CHE CREA IL SEGUENTE
CAMPO ELETTRICO?

GAUSS LOCALE $\quad \text{DIV}\,E = \frac{\rho}{\varepsilon_0}$

$\rho = \varepsilon_0\,\text{DIV}\,E = \varepsilon_0\frac{\partial E_x}{\partial x} = 0$ POICHE' COSTANTE

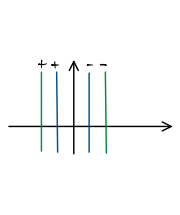
DISCONTINUITA' $\quad (E^+ - E^-)\,\hat{n} = \frac{\sigma}{\varepsilon_0}$

$\sigma(-2d) = \varepsilon_0(E_0 - 0) = \varepsilon_0 E_0$

$\sigma(-d) = \varepsilon_0(2E_0 - E_0) = \varepsilon_0 E_0$

$\sigma(+d) = \varepsilon_0(E_0 - 2E_0) = -\varepsilon_0 E_0$

$\sigma(+2d) = \varepsilon_0(0 - E_0) = -\varepsilon_0 E_0$



CAMPO ELETTRICO

$$\vec{E}(x) = \begin{cases} 0 & x < -2d \\ E_0 & x \in (-2d, d) \\ 2E_0 & x \in (-d, +d) \\ E_0 & x \in (+d, +2d) \\ 0 & x > 2d \end{cases}$$

RICAVO IL POTENZIALE

$\vec{E} = -\text{GRAD}\,V$

$dV = -E\,dx$

$V_{RIF} = V(0) = 0$

$x \in (0, d)$

$$\int_{V(0)}^{V(x)} dV = \int_0^x 2E_0\,dx$$

$$V(x) - \cancel{V(0)} = -2E_0 x$$

$x \in (d, 2d)$

$$\int_{V(0)}^{V(x)} dV = \int_0^x -E(x)\,dx$$

$$V(x) - \cancel{V(0)} = V(d) - \int_d^x E_0\,dx$$

$$V(x) = -2E_0 d - E_0(x - d)$$

$$V(x) = -E_0(x + d)$$

$x \in (2d, \infty)$

$$\int_{V(0)}^{V(x)} dV = \int_0^x -E(x)\,dx$$

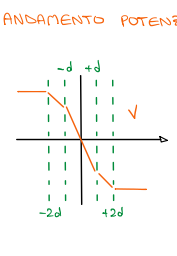
$$V(x) - V(0) = V(2d) - \cancel{\int_{2d}^x 0\,dx}$$

$$V(x) = V(2d) = -E_0(3d)$$

POTENZIALE ELETTRICO

$$V(x) = \begin{cases} +3dE_0 & x < -2d \\ -E_0(x - d) & x \in (-2d, -d) \\ -2E_0 x & x \in (-d, +d) \\ -E_0(x + d) & x \in (+d, +2d) \\ -3dE_0 & x > 2d \end{cases}$$

ANDAMENTO POTENZIALE



VELOCITA' DI UNA CARICA LIBERA IN $x = 2d$

ASSUMENDO q^+ TENDERA' VERSO IL
MINIMO POTENZIALE (ENERGIA)

$$E_1 = qV(2d) + \cancel{\frac{m v_0^2}{2}} = -3dE_0\cdot q$$

$$E_2 = qV(\infty) + \frac{m v_F^2}{2} = \frac{m v_F^2}{2}$$

$$E_1 = E_2 \longrightarrow v_F = +\sqrt{\frac{-6dE_0 q}{m}}$$

ESERCIZIO 21

CARICA Q DISTRIBUITA IN SFERA R.

DENSITA' DI VOLUME (RADIALE) $\rho = \alpha r$

PARAMETRO α

$$Q_{TOT} = \iiint\rho(r)\,dV = \int_0^R\rho(r)\,dr$$
$$= \int_0^R\alpha r\cdot 4\pi r^2\,dr = 4\pi\alpha\left(\frac{r^4}{4}\right)_0^R = \pi\alpha R^4$$

$$\rightarrow \alpha = \frac{Q}{\pi R^4}$$

$r < R$

$\Phi(E) = E\,4\pi r^2$

$$Q_{INT} = \int_0^r\rho(r)\,dr = \pi\alpha r^4 = \frac{\pi Q r^4}{\pi R^4} = Q\left(\frac{r}{R}\right)^4$$

GAUSS $\quad E\,\cancel{4\pi r^2} = \frac{\pi\alpha r^{\cancel{4}\,2}}{\varepsilon_0} \qquad \vec{E} = \frac{\alpha r^2}{4\varepsilon_0}\vec{U_r} = \frac{Q r^2}{4\pi\varepsilon_0 R^4}\vec{U_r}$

$r > R$

$\Phi(E) = E\,4\pi r^2$

$Q_{INT} = \pi\alpha R^4$

GAUSS $\quad E\,4\pi r^2 = \frac{\pi\alpha R^4}{\varepsilon_0} \qquad \vec{E} = \frac{\alpha R^4}{4\varepsilon_0 r^2}\vec{U_r} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{U_r}$

CAMPO ELETTRICO

$$\vec{E}(r) = \begin{cases} \frac{Q r^2}{4\pi\varepsilon_0 R^4}\vec{U_r} & r < R \\ \frac{Q}{4\pi\varepsilon_0 r^2}\vec{U_r} & r > R \end{cases}$$

ESERCIZIO 22

$$U = \frac{1}{2}\int_S\sigma V\,dS$$

CARICA DISTRIBUITA SU SUPERFICE SFERICA
ENERGIA ELETTROSTATICA DISTRIBUZIONE

CAMPO ELETTRICO

$r < R$ $\quad \vec{E} = 0$ POICHE' $Q_{INT} = 0$ GAUSS

$r > R$

$\Phi(E) = 4\pi r^2 E_0$

$Q_{INT} = 4\pi R^2\sigma$

$$4\pi r^2 E = \frac{4\pi R^2\sigma}{\varepsilon_0}$$

$$\vec{E} = \frac{\sigma R^2}{\varepsilon_0 r^2}\vec{U_r} = \frac{Q}{4\pi\varepsilon_0 r^2}\vec{U_r}$$

RICAVO IL POTENZIALE DA $\vec{E}$

$V_{RIF} = V(\infty) = 0$

$\vec{E} = -\text{GRAD}\,V$

$r > R$

$$\int_0^{V(r)} dV = -\int_\infty^r\frac{Q}{4\pi\varepsilon_0 r^2}$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

$r < R$

$$\int_0^{V(r)} dV = V(R) - \cancel{\int_R^r 0\,dr}$$

$$V(r) = V(R) = \frac{Q}{4\pi\varepsilon_0 R}$$

POTENZIALE E CAMPO ELETTRICO

$$\vec{E}(r) = \begin{cases} 0 & r < R & \text{COST} \\ \frac{Q}{4\pi\varepsilon_0 r^2} & r > R & \propto r^2 \end{cases}$$

$$V(r) = \begin{cases} \frac{Q}{4\pi\varepsilon_0 R} & r < R & \text{COST} \\ \frac{Q}{4\pi\varepsilon_0 r} & r > R & \propto r^{-1} \end{cases}$$

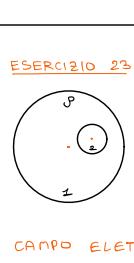
ENERGIA ELETTROSTATICA

$$U = \frac{1}{2}\sigma V\int_S dS = \frac{Q^2}{8\pi\varepsilon_0 R}$$

OPPURE POTEVANO APPLICARE

$$U = \frac{1}{2}\int_\tau\varepsilon_0 E^2\,d\tau = \frac{\varepsilon_0}{2}\int_R^\infty\left(\frac{Q}{4\pi\varepsilon_0 r^2}\right)^2 4\pi r^2\,dr$$
$$= \frac{1}{2}\varepsilon_0\frac{Q^2}{16\pi^2\varepsilon_0^2}\int_R^\infty\frac{4\pi r^2}{r^4}dr = \frac{Q^2}{8\pi\varepsilon_0 R}$$

ESERCIZIO 23



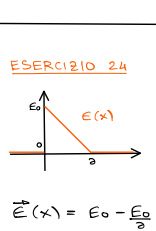
SFERA CON FORO SFERICO

PUO' ESSERE VISTA COME
SOVRAPPOSIZIONE SFERA 1 (+ρ)
E SFERA 2 (−ρ)

CAMPO ELETTRICO NEL FORO

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{r_1} - \frac{\rho}{3\varepsilon_0}\vec{r_2} = \frac{\rho}{3\varepsilon_0}(\vec{r_1} - \vec{r_2}) = \frac{\rho}{3\varepsilon_0}\vec{R}$$

ESERCIZIO 24



DETERMINA
DISTRIBUZIONE
DI CARICA

$$\vec{E}(x) = E_0 - \frac{E_0}{a}x$$

DISTRIBUZIONE

$$\rho = \varepsilon_0\,\text{DIV}\,E = \varepsilon_0\frac{\partial}{\partial x}E_x = -\frac{E_0\varepsilon_0}{a}$$

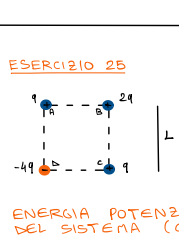
DISCONTINUITA'

$$(E^+ - E^-)\,\hat{n} = \frac{\sigma}{\varepsilon_0}$$

$$(E_0 - 0) = \frac{\sigma}{\varepsilon_0} \qquad \sigma = \varepsilon_0 E_0$$

PARALLELEPIPEDO VOLUMETRICAMENTE CARICO ρ
CON FOGLIETTO CARICO σ A x = 0

ESERCIZIO 25



SISTEMA RIGIDO DI
CARICHE PUNTIFORMI

**ENERGIA POTENZIALE ELETTROSTATICA
DEL SISTEMA (CONFIGURAZIONE)**

A) PORTO CARICA q IN A, ALTRE 3 CARICHE
A DISTANZA INFINITA

$F_A = 0 \rightarrow L_A = 0$

B) PRESENTE 1 CARICA

$$L_B = \int_\infty^r\frac{-2q^2\,dr}{4\pi\varepsilon_0 r^2} = \frac{-q^2}{2\pi\varepsilon_0}\left(-\frac{1}{r} + \cancel{\frac{1}{\infty}}\right) = \frac{q^2}{2\pi\varepsilon_0 r}$$

C) PRESENTI 2 CARICHE

$$L_C = \int_\infty^r -\frac{2q^2\,dr}{4\pi\varepsilon_0 r^2} + \int_\infty^{r\sqrt{2}} -\frac{q^2\,dr}{4\pi\varepsilon_0 r^2}$$
$$= \frac{-q^2}{2\pi\varepsilon_0}\left(-\frac{1}{r} + \cancel{\frac{1}{\infty}}\right) - \frac{q^2}{4\pi\varepsilon_0}\left(-\frac{1}{r\sqrt{2}} + \cancel{\frac{1}{\infty}}\right)$$
$$= \frac{q^2}{2\pi\varepsilon_0 r} + \frac{q^2\sqrt{2}}{8\pi\varepsilon_0 r}$$

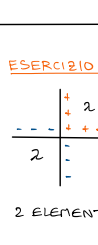
D) PRESENTI 3 CARICHE

$$L_D = +2\int_\infty^r\frac{4q^2\,dr}{4\pi\varepsilon_0 r^2} + \int_\infty^{r\sqrt{2}}\frac{8q^2\,dr}{4\pi\varepsilon_0 r^2}$$
$$= \frac{2q^2}{\pi\varepsilon_0}\left(-\frac{1}{r} + \frac{1}{\infty}\right) + \frac{2q^2}{\pi\varepsilon_0}\left(-\frac{1}{r\sqrt{2}} + \frac{1}{\infty}\right)$$
$$= -\frac{2q^2}{\pi\varepsilon_0 r} - \frac{q^2\sqrt{2}}{\pi\varepsilon_0 r}$$

LAVORO TOTALE (SISTEMA)

$$U_{SYS} = L_A + L_B + L_C + L_D = -\frac{q^2(8 + 7\sqrt{2})}{8\pi\varepsilon_0 r}$$

ESERCIZIO 26



MOMENTO DI DIPOLO?

2 ELEMENTI ALLA VOLTA $\quad d\vec{P} = (2x)\lambda\,dx\,\vec{U_x}$

$$\vec{P_1} = 2\lambda\vec{U_x}\int_0^d 2x\,dx = 2d^2\vec{U_x}$$

$$\vec{P_2} = 2\lambda\vec{U_y}\int_0^d 2y\,dy = 2d^2\vec{U_y}$$

SOVRAPPOSIZIONE

$$\vec{P} = \vec{P_1} + \vec{P_2} = 2d^2(\vec{U_x} + \vec{U_y})$$

ESERCIZIO 27

PARTICELLA q, m

IN SFERA CARICA OMOGENEAMENTE ρ

CAMPO ELETTRICO E_0 → PARTICELLA SI SPOSTA DI $r < R$

CAMPO ELETTRICO INTERNO SFERA

$\Phi_S(E) = 4\pi r^2 E$

$Q_{INT} = \frac{4}{3}\pi r^3\rho$

$$4\pi r^2 E = \frac{4\pi r^3\rho}{3\varepsilon_0} \qquad \vec{E} = \frac{\rho}{3\varepsilon_0}r\,\vec{U_r}$$

EQUILIBRIO PARTICELLA

$F(E_0) - F(E_{SFERA}) = 0$

$$\frac{\rho r}{3\varepsilon_0} = -E_0 \longrightarrow r = -\frac{3\varepsilon_0 E_0}{\rho} = \frac{4\pi\varepsilon_0 R^3}{Q}E_0$$

MOMENTO DI DIPOLO

$$\vec{P} = Q\vec{r} = 4\pi\varepsilon_0 R^3 E_0$$

MOTO SE E_0 VIENE TOLTO

OSCILLA ATTORNO A O CON $A = r$
IN MOTO PERIODICO (ARMONICO SEMPLICE)

$F = QE$

$$m\ddot{r} = -\frac{Q^2}{4\pi\varepsilon_0 R^3}r$$

$$\omega = \sqrt{\frac{Q^2}{4\pi\varepsilon_0 R^3}} \quad \text{OSCILLAZIONI}$$