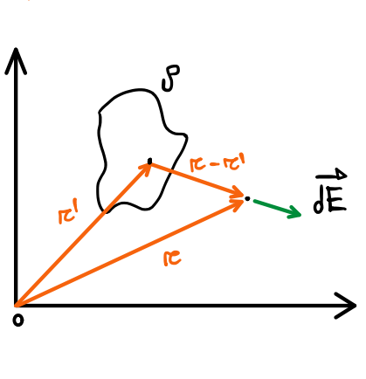
Mix 1 - Distribuzioni

Saturday, 30 July 2022 22:59

CALCOLO DIRETTO DI CAMPO E POTENZIALE ELETTROSTATICO

TRATTAZIONE TEORICA

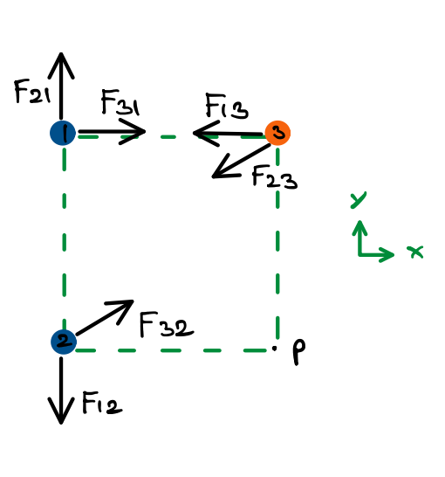


$$dq = \rho(r')\,dV$$

$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{1}{|r-r'|^2}\cdot\boxed{\frac{r-r'}{|r-r'|}}\ \vec{u_R}$$

$$dV = \frac{dq}{4\pi\varepsilon_0}\frac{1}{|r-r'|} \qquad V(\infty)=0$$

$$\vec{E} = \iiint d\vec{E} \qquad V = \iiint dV \qquad \vec{E} = -\nabla V$$



$q_1 = q_2 = +2\ mC$

$q_3 = -3\ nC$

$L = 1\ cm$

FORZE AGENTI SU OGNI CARICA

$$F_{21} = -F_{12} = \frac{K q_1 q_2}{L^2}\vec{U_y}$$

$$F_{31} = -F_{31} = \frac{-K q_1 q_2}{L^2}\vec{U_x}$$

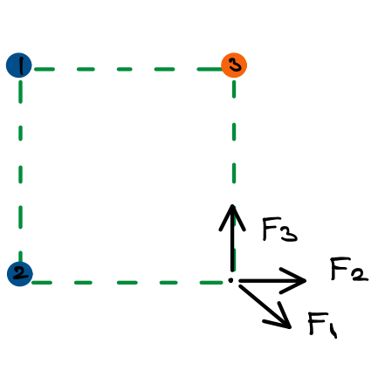
$$F_{32} = -F_{23} = -\frac{K q_2 q_3}{(L\sqrt{2})^2}\left(\sin\alpha\,\vec{U_x} + \cos\alpha\,\vec{U_y}\right)$$

$$F_1 = \begin{cases} x: & F_{31} = 539\ \mu N \\ y: & F_{21} = 359\ \mu N \end{cases}$$

$$F_2 = \begin{cases} x: & F_{32}\cos(\alpha) = 150\ \mu N \\ y: & -F_{21} + F_{32}\sin(\alpha) = -165\ \mu N \end{cases}$$

$$F_3 = \begin{cases} x: & -F_{31} - F_{32}\cos(\alpha) = -730\ \mu N \\ y: & -F_{32}\sin(\alpha) = -150\ \mu N \end{cases}$$

CAMPO ELETTRICO IN P

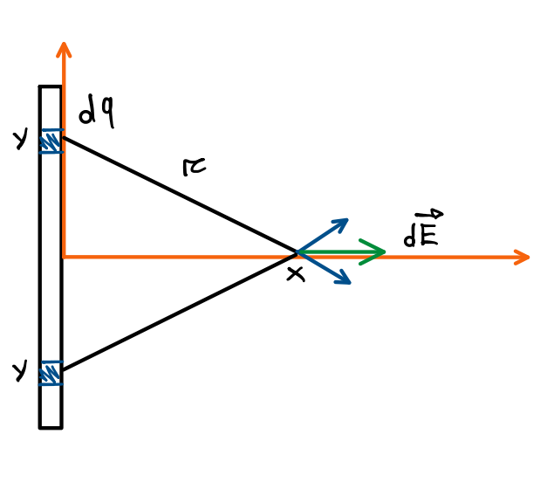


$$F_P = \begin{cases} x: & F_2 + F_1\sin\alpha \\ y: & F_3 - F_1\sin\alpha \end{cases}$$

$$E_x = \frac{1}{\cancel{4}}\left(\frac{k\cancel{4}q_2}{L^2} + \frac{k\cancel{4}q_1}{(\sqrt{2}L)^2}\cos\alpha\right) = 243\ K\frac{N}{C}$$

$$E_y = \frac{1}{\cancel{4}}\left(\frac{k\cancel{4}q_3}{L^2} - \frac{k\cancel{4}q_1}{(\sqrt{2}L)^2}\sin\alpha\right) = 206\ K\frac{N}{C}$$

CARICA DISTRIBUITA UNIFORMEMENTE



$$d\vec{E} = \frac{dq}{4\pi\varepsilon_0}\frac{1}{|r-r'|^2}\vec{U_R}$$

$$\vec{E} = E(x)\,\vec{U_x}$$

CAMPO ELETTRICO ASSIALE

DENSITÀ LINEARE DI CARICA $\quad \lambda = \frac{Q}{2L}$

$dq = \lambda\,dy$

$r = \sqrt{x^2+y^2}$

$dE = 2\cdot\frac{\lambda dy\cos\alpha}{4\pi\varepsilon_0 r^2}$

$\cos\alpha = \frac{x}{r}$

$$\longrightarrow \quad E = \int_0^L \frac{2\lambda x\,dy}{4\pi\varepsilon_0 r^3}$$

$$E = \int_0^L \frac{2\lambda x\,dy}{4\pi\varepsilon_0(x^2+y^2)^3} = \frac{2\lambda x}{4\pi\varepsilon_0}\int_0^L \frac{dy}{(x^2+y^2)^3}$$

METODO 1 $\quad y = x\sinh t$

$\cosh t^2 - \sinh^2 t = +1$

METODO 2 $\quad y = x\tan\alpha$

$$dy = \frac{x}{\cos^2\alpha}d\alpha$$

$$\cos\alpha = \frac{x}{r}$$

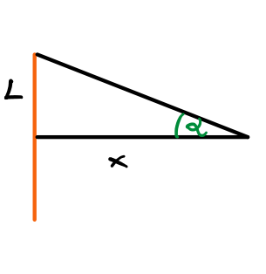
$$dE = \frac{2\lambda x\,dy}{4\pi\varepsilon_0 r^3}$$

$$= \frac{2\lambda\cancel{x}}{4\pi\varepsilon_0}\cdot\frac{\cos^{3}\alpha}{x^{\cancel{3}}}\cdot\frac{\cancel{x}}{\cancel{\cos^2\alpha}}d\alpha$$

$$= \frac{2\lambda\cos\alpha}{4\pi\varepsilon_0 x}d\alpha$$

α (A DIFFERENZA DI t) HA VALORE FISICO

$$E = \int_0^{\alpha_{MAX}} \frac{2\lambda\cos\alpha}{4\pi\varepsilon_0 x}d\alpha$$



$$\sin(\alpha_{MAX}) = \frac{L}{\sqrt{L^2+x^2}}$$

$$E = \frac{2\lambda}{4\pi\varepsilon_0 x}\int_0^{\alpha_{MAX}}\cos\alpha\,d\alpha$$

$$= \frac{2\lambda}{4\pi\varepsilon_0 x}\left|\sin\alpha\right|_0^{\alpha_{MAX}}$$

$$= \frac{2\lambda}{4\pi\varepsilon_0 x}\sin\left(\arcsin\left(\frac{L}{\sqrt{L^2+x^2}}\right)\right)$$

$$= \frac{2\lambda}{4\pi\varepsilon_0 x}\cdot\frac{L}{\sqrt{L^2+x^2}}$$

TUTTE LE DISTRIBUZIONI LIMITATE
VISTE A GRANDI DISTANZE SEMBRANO
SIMILI A CARICHE PUNTIFORMI

CAMPO ELETTRICO ASINTOTICO

SE $x >> 0 \rightarrow \lim_{x\to\infty}\sqrt{L^2+x^2} \sim |x|$

$$x\to\infty \qquad E(x) \sim \frac{2\lambda L}{4\pi\varepsilon_0 x|x|} = \frac{Q}{4\pi\varepsilon_0 x^2}$$

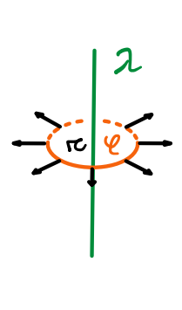
TUTTE LE DISTRIBUZIONI LIMITATE
VISTE A PICCOLE DISTANZE SEMBRANO
SIMILI A LASTRE DI LUNGHEZZA INFINITA

CAMPO ELETTRICO INFINITESIMO

SE $x << 1 \rightarrow \lim_{x\to 0}\sqrt{L^2+x^2} \sim |L|$

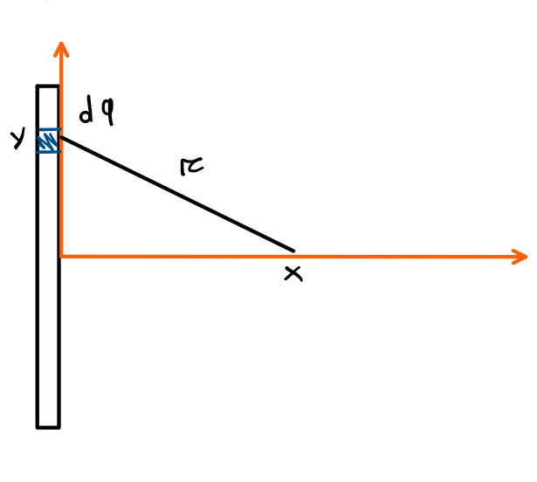
$$x\to 0 \qquad E(x) = \frac{2\lambda}{4\pi\varepsilon_0 x}\cdot\frac{\cancel{L}}{|\cancel{L}|} = \frac{\lambda}{2\pi\varepsilon_0 x}$$

GENERALIZZAZIONE IN 3D



$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{U_r} \qquad \forall\varphi$$

POTENZIALE ELETTRICO ASSIALE



$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

$$dq = \lambda\,dy$$

$$r = \sqrt{x^2+y^2}$$

$$V = 2\int_0^L \frac{\lambda}{4\pi\varepsilon_0}\cdot\frac{dy}{\sqrt{x^2+y^2}}$$

SOSTITUISCO $\quad y = x\sinh t \qquad dy = x\cosh t\,dt$

IDENTITÀ $\quad \cosh^2 t - \sinh^2 t = +1$

$$= 2\int \frac{\lambda x\cosh t\,dt}{4\pi\varepsilon_0\sqrt{x^2+x^2\sinh^2 t}}$$

$$x^2 + x^2\sinh^2 x = x^2\cosh^2 x$$

$$= 2\int \frac{\lambda\cancel{x\cosh t}\,dt}{4\pi\varepsilon_0\cancel{|x\cosh t|}}$$

$$= 2\int \frac{\lambda}{4\pi\varepsilon_0}dt \longrightarrow V(t) = \frac{2\lambda t}{4\pi\varepsilon_0}$$

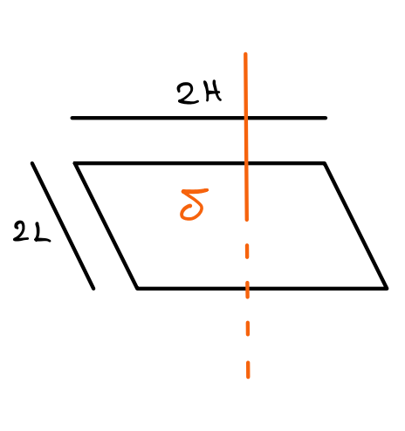
TORNANDO IN FUNZIONE DI x

$$V(x) = \frac{2\lambda}{4\pi\varepsilon_0}\operatorname{settsinh}\left(\frac{y}{x}\right)\Big|_0^L$$

DOVE $\operatorname{settsinh}(k) = \log\left(x + \sqrt{1+x^2}\right)$

$$V(x) = \frac{2\lambda}{4\pi\varepsilon_0}\log\left(\frac{L}{x} + \sqrt{1+\left(\frac{L}{x}\right)^2}\right)$$

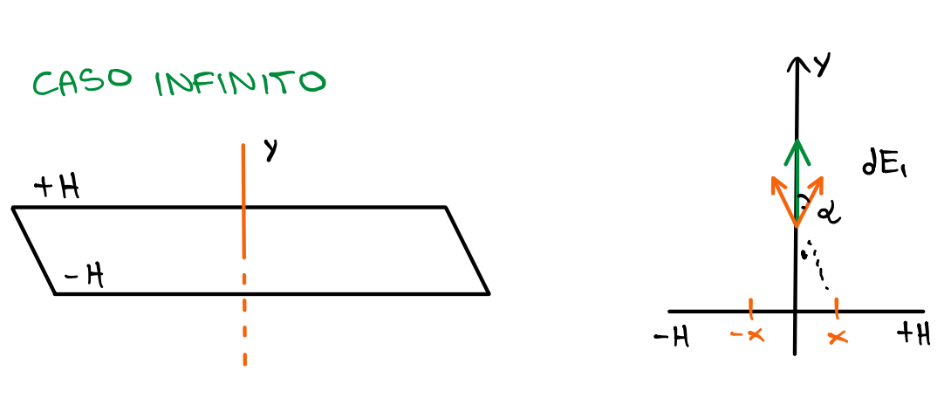
$$= \frac{Q}{4\pi\varepsilon_0 L}\log\left(\frac{L}{x} + \sqrt{1+\left(\frac{L}{x}\right)^2}\right)$$



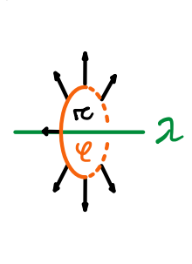
CARICA Q DISTRIBUITA
UNIFORMEMENTE

$$\sigma = \frac{Q}{4LH}$$

CASO INFINITO

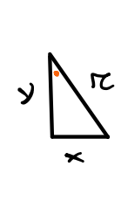


CONSIDERIAMO LA LASTRA COME DISTRIBUZIONE
CONTINUA DI BARRETTE OMOGENEE (SPAGHETTO)



$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\vec{U_r} \qquad \forall\varphi$$

$$\vec{E} = E(y)\,\vec{U_z} \qquad |d\vec{E_1}| = \frac{\sigma\,dx}{2\pi\varepsilon_0 r}$$



$$\cos\alpha = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

$$d\vec{E} = 2\,d\vec{E_1}\cos\alpha = \frac{\cancel{2}\sigma y\,dx}{\cancel{2}\pi\varepsilon_0 r^2} = \frac{\sigma y\,dx}{\pi\varepsilon_0(x^2+y^2)}$$

$$\vec{E} = \int_0^H \frac{\sigma y\,dx}{\pi\varepsilon_0(x^2+y^2)}dx$$

$$= \frac{\sigma\cancel{y}}{\pi\varepsilon_0}\cdot\int_0^H \frac{dx}{1+\left(\frac{x}{y}\right)^2}\frac{1}{y^{\cancel{2}}}$$

$$= \frac{\sigma}{\pi\varepsilon_0}\arctan\left(\frac{x}{y}\right)\Big|_0^H$$

$$= \frac{\sigma}{\pi\varepsilon_0}\arctan\left(\frac{H}{y}\right)\vec{U_z}$$

PER $y\to\infty$

DA LONTANO LA LASTRA DI LUNGHEZZA
INFINITA APPARE COME UNO SPAGHETTO

$$\lim_{y\to\infty}\arctan\left(\frac{H}{y}\right) \sim \frac{H}{y}$$

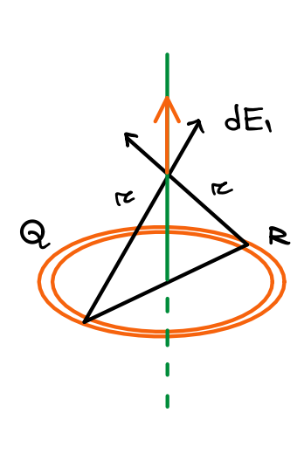
$$\vec{E} \sim \frac{\sigma}{\pi\varepsilon_0}\frac{H}{y}\cdot\frac{2}{2} = \frac{\lambda}{2\pi\varepsilon_0 y}\vec{U_R}$$

PER $H\to\infty$

LA LASTRA DIVENTA UN PIANO INFINITO

$$\lim_{H\to\infty}\arctan\left(\frac{H}{y}\right) = \frac{\pi}{2}$$

$$\vec{E} = \frac{\sigma}{\pi\varepsilon_0}\frac{\pi}{2} = \frac{\sigma}{2\varepsilon_0}\vec{U_z} \qquad \underline{\text{GAUSS}}$$

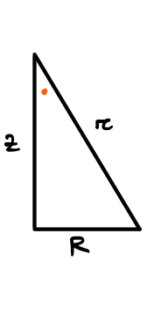


CONTRIBUTI OPPOSTI (α, $\alpha+\pi$)
IL CAMPO È SOLO SU z

$\vec{E} = E(z)\,\vec{U_z}$

TUTTI I PUNTI IN ANELLO CONDIVIDONO r

DENSITÀ DI CARICA $\quad \lambda = \frac{Q}{2\pi R}$



$$\cos\alpha = \frac{ADJ}{HYP} = \frac{z}{\sqrt{R^2+z^2}}$$

$$r = \sqrt{R^2+z^2}$$

POTENZIALE ELETTRICO

$$dV = \frac{\lambda\cdot dl}{4\pi\varepsilon_0 r} = \frac{\lambda\cdot dl}{4\pi\varepsilon_0(z^2+R^2)^{1/2}}$$

$$V = \int_0^{2\pi R} dV = \frac{\lambda\,2\pi R}{4\pi\varepsilon_0(z^2+R^2)^{1/2}} = \frac{Q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}$$

CAMPO ELETTRICO

METODO 1 - GRADIENTE

$$\vec{E} = -\nabla V = \cancel{-\frac{\partial V}{\partial x}\vec{U_x}} \cancel{-\frac{\partial V}{\partial y}\vec{U_y}} - \frac{\partial V}{\partial z}\vec{U_z}$$

$$= -\frac{\partial V}{\partial z}\vec{U_z}$$

$$= -\frac{\partial}{\partial z}\left(\frac{Q}{4\pi\varepsilon_0\sqrt{z^2+R^2}}\right)\vec{U_z}$$

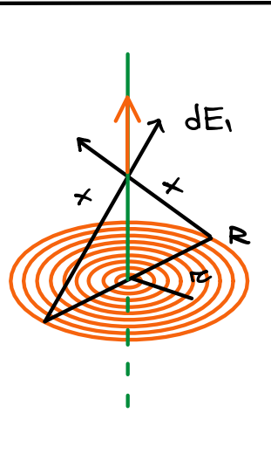
$$= \frac{Qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\vec{U_z}$$

METODO 2 - DEFINIZIONE

$$d\vec{E} = 2|d\vec{E_1}|\cos\alpha = \frac{2\lambda\,dl\,z}{4\pi\varepsilon_0 r^3}$$

$$\vec{E} = \int_0^{\pi R} d\vec{E} = \frac{Qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}$$

ASINTOTICO $\quad z\to\infty \qquad E \sim \frac{Q}{4\pi\varepsilon_0 z^2}$



IL DISCO È UNA DISTRIBUZIONE
CONTINUA DI ANELLI VARIABILI

CONTRIBUTO DI UN ANELLO DI RAGGIO r
AD ALTEZZA ASSIALE z

$$\vec{E}(r) = \frac{Qz}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}\vec{U_z}$$

CARICA SU ANELLO SPESSO dr

$$\sigma = \frac{dq}{dS} = \frac{dq}{2\pi r\,dr}$$

CAMPO ELETTRICO TOTALE

$$\vec{E} = \int_0^R dE(r)$$

$$= \int_0^R \frac{2\pi r\sigma z\,dr}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}$$

$$= \frac{\cancel{\pi}\sigma z}{4\cancel{\pi}\varepsilon_0}\int_0^R \frac{2r\,dr}{(z^2+r^2)^{3/2}}$$

$$= \frac{\sigma z}{4\varepsilon_0}\frac{(-2)}{\sqrt{z^2+r^2}}\Big|_0^R$$

$$= \frac{\sigma}{2\varepsilon_0}z\left(\frac{1}{|z|} - \frac{1}{\sqrt{z^2+R^2}}\right)\vec{U_z}$$

POTENZIALE ELETTRICO

$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$

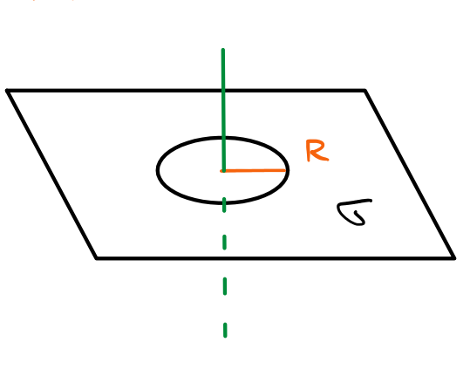
$$V = \int_0^R dV(r)$$

$$= \int_0^R \frac{2\pi r\sigma z\,dr}{4\pi\varepsilon_0(z^2+r^2)^{1/2}}$$

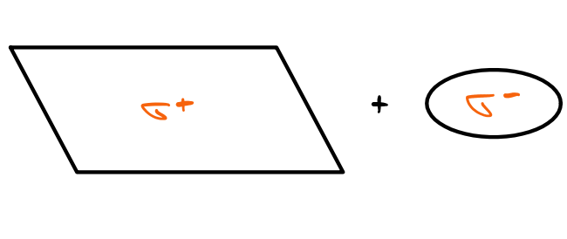
$$= \frac{\sigma z}{2\varepsilon_0}\left|\sqrt{z^2+r^2}\right|_0^R$$

$$= \frac{\sigma z}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - |z|\right)$$

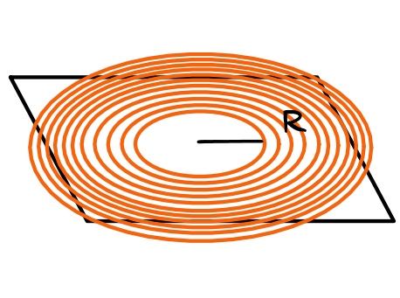
PIANO CON BUCO CIRCOLARE



METODO 1 - SOVRAPPOSIZIONE EFFETTI



METODO 2 - INTEGRALE IMPROPRIO



UGUALE A CAMPO ELETTRICO
DI UN DISCO MA CAMBIANO
I CONTRIBUTI

$$\vec{E} = \int_R^\infty d\vec{E}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left|\frac{1}{\sqrt{z^2+r^2}}\right|_\infty^R$$

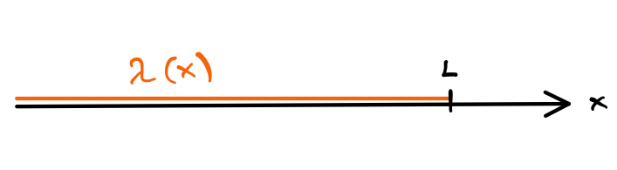
$$= \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\vec{U_z}$$

DISTRIBUZIONI CONTINUE DI CARICA

DENSITÀ DI CARICA UNIFORME $\rightarrow \lambda(x) = \lambda = cost$

$\lambda = dq/dx \qquad dq = \lambda\,dx$

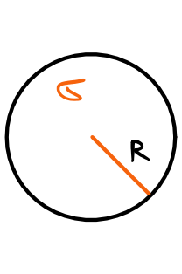
$Q = \int dq = \int_0^L \lambda\,dx = \lambda L$



$$\lambda(x) = \lambda_0\frac{x}{L}$$

CALCOLA LA CARICA TOTALE

$$Q = \int_0^L \lambda(x)\,dx = \frac{\lambda_0}{L}\int_0^L x\,dx = \frac{\lambda_0}{L}\left(\frac{x^2}{2}\right)_0^L = \frac{L}{2}\lambda_0$$



$$\sigma(r) = \sigma_0\frac{r}{R}$$

$$dS = r\,dr\,d\varphi$$

CALCOLA LA CARICA TOTALE

$$Q = \iint dq = \iint \sigma\,dS$$

$$= \int_0^R\int_0^{2\pi}\left(\frac{\sigma_0 r}{R}\right)(r\,dr\,d\varphi)$$

$$= \int_0^{2\pi}d\varphi\int_0^R \frac{\sigma_0}{R}r^2\,dr$$

$$= \frac{2\pi\sigma_0}{R}\left(\frac{R^3}{3} - 0\right)$$

$$= \frac{2}{3}\pi\sigma_0 R^2$$

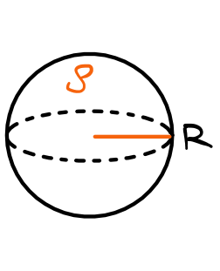
SEMPLICE IN r

$S(r) = \pi R^2 \rightarrow dS(r) = 2\pi r\,dr$

$$Q = \int \sigma(r)\cdot dS(r) =$$

$$= \int_0^R \sigma_0\frac{r}{R}\,2\pi r\,dr$$

$$= \frac{2}{3}\pi\sigma_0 R^2$$



$$\rho(r) = \rho_0\frac{r}{R}$$

$$dV = r^2\sin\theta\,dr\,d\theta\,d\varphi$$

CALCOLA LA CARICA TOTALE

$$Q = \iiint dq = \iiint_V \rho\,dV$$

$$= \int_0^R\int_0^\pi\int_0^{2\pi}\left(\rho_0\frac{r}{R}\right)\left(r^2\sin\theta\,dr\,d\theta\,d\varphi\right)$$

$$= \int_0^{2\pi}d\varphi\cdot\int_0^\pi\sin\theta\,d\theta\cdot\int_0^R\frac{\rho_0}{R}r^3\,dr$$

$$= \cancel{2}\pi\cdot\cancel{2}\cdot\frac{\rho_0}{\cancel{R}}\,\frac{R^{\cancel{4}\,3}}{\cancel{4}}$$

$$= \pi\rho_0 R^3$$

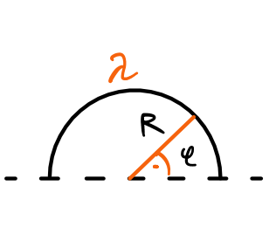
SEMPLICE IN R

$V(R) = \frac{4}{3}\pi R^3 \rightarrow dV = 4\pi R^3$

$$Q = \int_0^R \rho(R)\cdot dV(R)$$

$$= \int_0^R \rho_0\frac{r}{R}4\pi R^3$$

$$= \pi\rho_0 R^3$$



$$\lambda(\varphi) = \lambda_0\sin^2\varphi$$

$$dL = R\,d\varphi$$

CALCOLA LA CARICA TOTALE

$$Q = \int dq = \int \lambda\,dL$$

$$= \int \lambda_0\sin^2\varphi\,R\,d\varphi$$

$$= \lambda_0 R\int_0^\pi \sin^2\varphi\,d\varphi$$

$$= \frac{\lambda_0 R}{2}\int_0^\pi(1-\cos(2\varphi))\,d\varphi$$

$$= \frac{\lambda_0 R}{2}\left(\int_0^\pi d\varphi + \cancel{\int_0^\pi\cos(2\varphi)\,d\varphi}\right)$$

$$= \frac{\lambda_0 R\pi}{2}$$