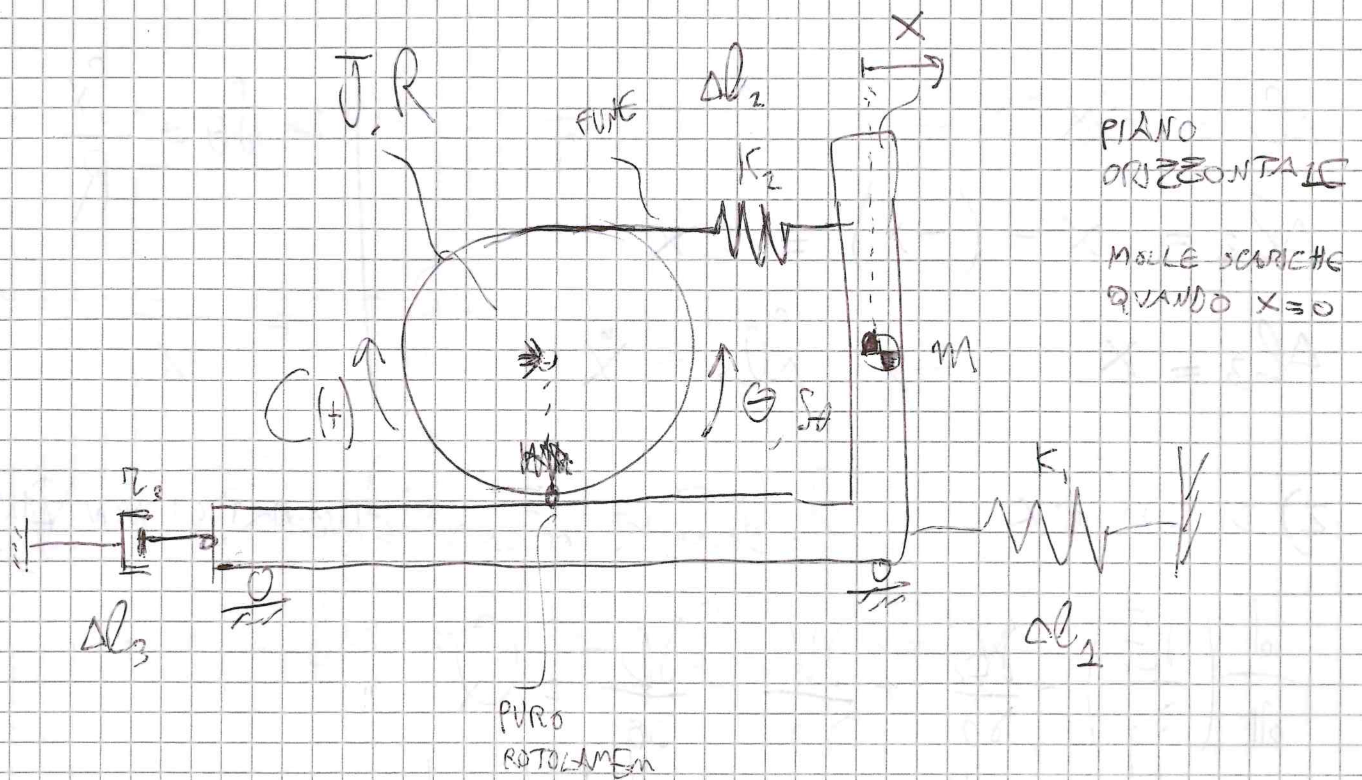
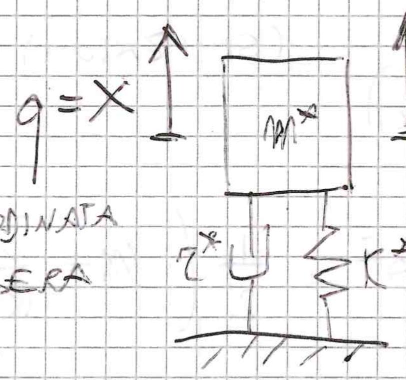


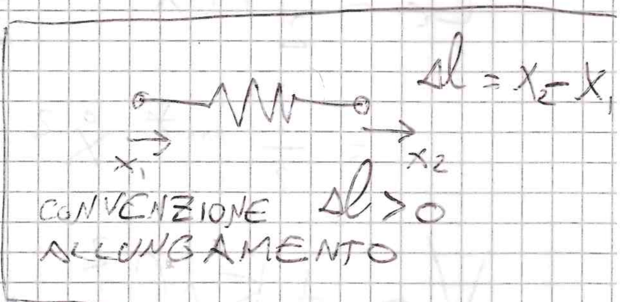
PIANO ORIZZONTALE

MOLLE SCARICHE QUANDO X=0

EQUAZIONE DI MOTO

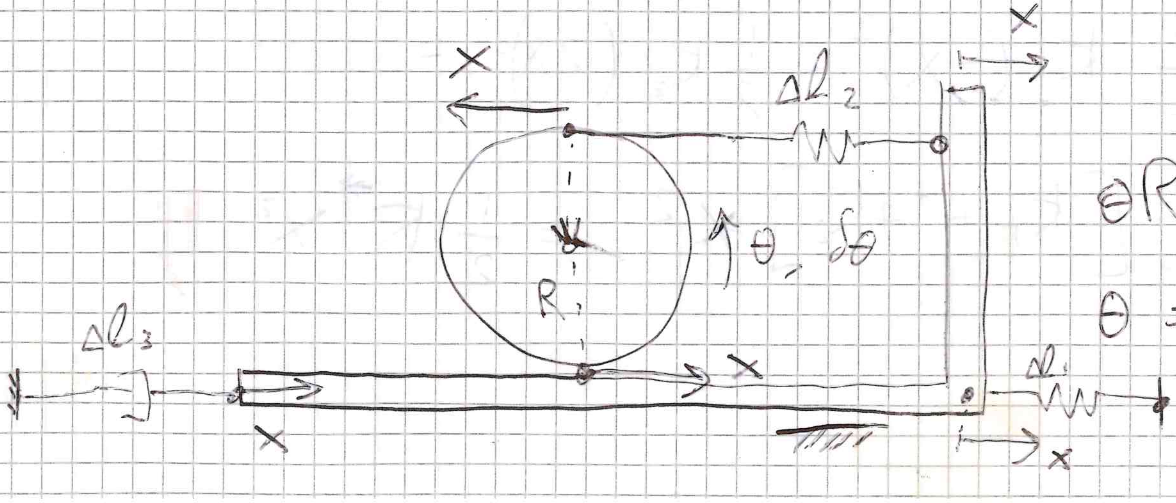


$$m^*\ddot{q}+r^*\dot{q}+k^*q=F^*$$



1) CINEMATICA

COORDINATA LIBERA: X



$$\theta R = X$$

$$\theta = \frac{X}{R}$$

$\Delta l_1 = -x$

$\Delta l_2 = x - (-x) = 2x$

$\Delta l_3 = x \rightarrow \Delta \dot{l}_3 = \dot{x}$

$\delta\theta = \dfrac{\delta x}{R}$

2) DINAMICA

SOLO SE LEGAMI CINEMATICI NON LINEARI $V = V(q\,\dot q)$

$$\underbrace{\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot q}\right) - \cancel{\frac{\partial E_c}{\partial q}}}_{\text{FORZE DI INERZIA}} + \underbrace{\frac{\partial V}{\partial q}}_{\text{FORZE POTENZIALI}} + \underbrace{\frac{\partial D}{\partial \dot q}}_{\text{FORZE VISCOSE}} = \underbrace{Q}_{\text{ALTRE FORZE}}$$

FORZE DI INERZIA

FORZE POTENZIALI (ELASTICHE, GRAVITAZIONALI)

FORZE VISCOSE (SMORZATORI)

ALTRE FORZE

$$E_c = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2 = \frac{1}{2}\left(m + \frac{J}{R^2}\right)\dot{x}^2$$

$$= \frac{1}{2} m^* \dot{x}^2$$

$$V = \frac{1}{2} K_1 \Delta l_1^2 + \frac{1}{2} K_2 \Delta l_2^2 =$$

$$= \frac{1}{2} K_1 (-x)^2 + \frac{1}{2} K_2 (2x)^2 =$$

$$= \frac{1}{2}\left[K_1 + 4K_2\right] x^2 = \frac{1}{2} K^* x^2$$

$$D = \frac{1}{2} \eta_3 \Delta \dot{l}_3^2 = \frac{1}{2} \eta_3 \dot{x}^2 = \frac{1}{2} \eta^* \dot{x}^2$$

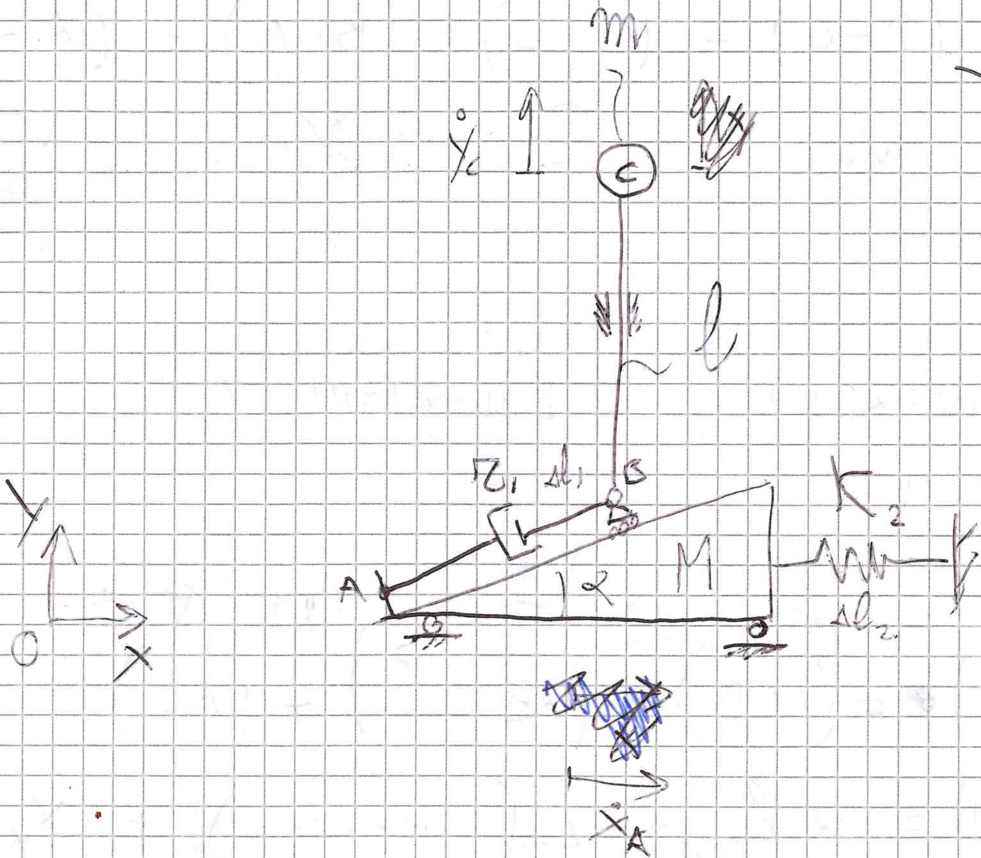
$$Q = \frac{\delta L}{\delta x} = -\frac{C(t)}{R}$$

$$\delta L = \vec{C} \cdot \delta \vec{\Theta} = -C(t) \vec{K} \cdot \delta\Theta \vec{K} =$$

$$= -C \, \delta\Theta = \left(-\frac{C}{R}\right) \delta x = Q \, \delta x$$

$$\Downarrow$$

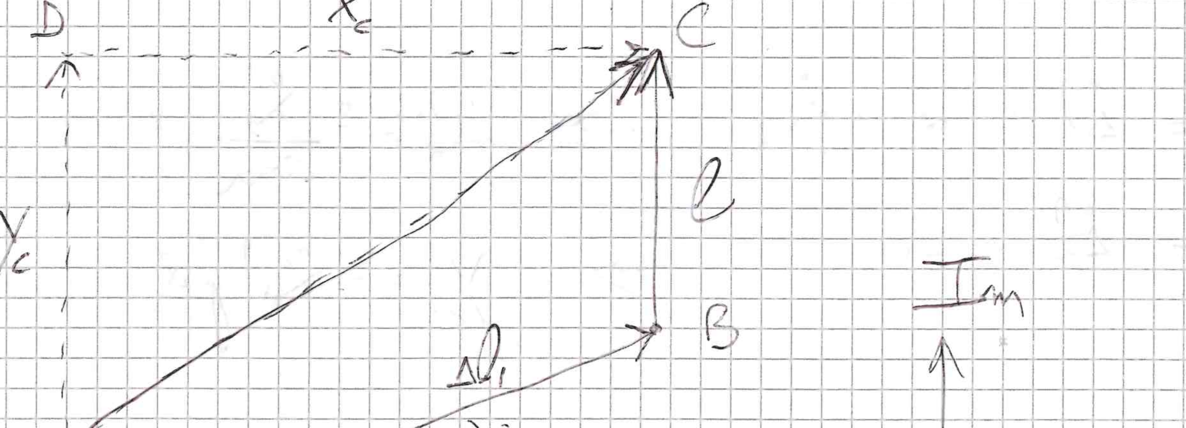
$$m^* \ddot{x} + \eta^* \dot{x} + K^* x = Q(t)$$



PIANO ORIZZONTALE

① CINEMATICA

COORDINATA LIBERA: x_A $\qquad$ $\Delta l_2 = -x_A$



$$(C-O) = (C-B) + (B-A) + (A-O)$$

	$(C-O)$	$(C-B)$	$(B-A)$	$(A-O)$
MODULO	VARIABILE	COSTANTE	V.	V.
ANGOLO	V.	C.	C.	C.

$$(C-D) + (D-O) = (C-B) + (B-A) + (A-O)$$

MOD:	C.	V.	C.	V.	V.
ANG:	C.	C.	C.	C.	C.

3 COORD VARIABILI , 2 EQUAZIONI
(DI CUI 1 COORD LIBERA)

$$(CD) + (i\, DO) = (i\, CB) + (AB\, e^{i\alpha}) + (AO)$$

$$x_c + i \cdot y_c(t) = i\, l + \Delta l(t)(\cos\alpha + i \sin\alpha) + x_A(t)$$

$$\Big\downarrow \, d/dt$$

$$i\, \dot{y}_c = \dot{\Delta l}(\cos\alpha + i\sin\alpha) + \dot{x}_A$$

$$\begin{cases} 0 = \dot{\Delta l}\cos\alpha + \dot{x}_A \\ \dot{y}_c = \dot{\Delta l}\sin\alpha \end{cases} \quad \begin{aligned} &\rightarrow \dot{\Delta l} = -\frac{\dot{x}_A}{\cos\alpha} \\ &\rightarrow \dot{y}_c = -\dot{x}_A \operatorname{tg}\alpha \end{aligned}$$

2) DINAMICA

$$E_c = \frac{1}{2} M \dot{x}_A^2 + \frac{1}{2} m \dot{y}_C^2 =$$

$$= \frac{1}{2}\left(M + m(\operatorname{tg}\alpha)^2\right) \dot{x}_A^2 = \frac{1}{2} m^* \dot{x}_A^2$$

$$D = \frac{1}{2} r_1 \dot{\Delta l}_1^2 = \frac{1}{2}\left(\frac{r_1}{\cos\alpha^2}\right) \dot{x}_A = \frac{1}{2} r^* \dot{x}_A^2$$

$$V = \frac{1}{2} K_2 \Delta l_2^2 = \frac{1}{2}\left(K_2\right) x_A^2 = \frac{1}{2} K^* x_A^2$$

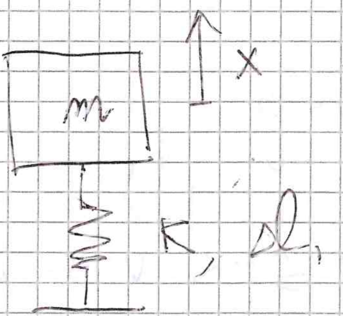
$$Q = 0$$

$$m^* \ddot{x}_A + r^* \dot{x}_A + K^* x_A = 0$$

EFFETTO FORZA PESO

1. PRECARICO MOLLE
2. RIGIDEZZA (EFFETTO DEL 2° ORDINE)

1



$\Delta l_1 = 0$ per $x = 0$

$$E_c = \frac{1}{2} m \dot{x}^2$$

$$V = \frac{1}{2} K \Delta l_1^2 + mgh = \frac{1}{2} K x^2 + mgx$$

$$\boxed{m\ddot{x} + Kx + mg = 0}$$

$$m\ddot{x} + Kx = -mg$$

SOLUZIONE STATICA $\quad x =$ COSTANTE $\quad , \ddot{x} = 0$

$$\bar{x} = -\frac{mg}{K}$$

$$K\bar{x} = -mg$$

CAMBIO DI COORDINATA

$$X = \bar{X} + \tilde{X}$$

$\tilde{X}$ → VIBRAZIONE RISPETTO ALLA STATICA

$$\tilde{X} = X - \bar{X}$$
$$\dot{X} = \dot{\tilde{X}}$$
$$\ddot{X} = \ddot{\tilde{X}}$$

$$m\ddot{\tilde{X}} + K(\bar{X} + \tilde{X}) = -mg$$

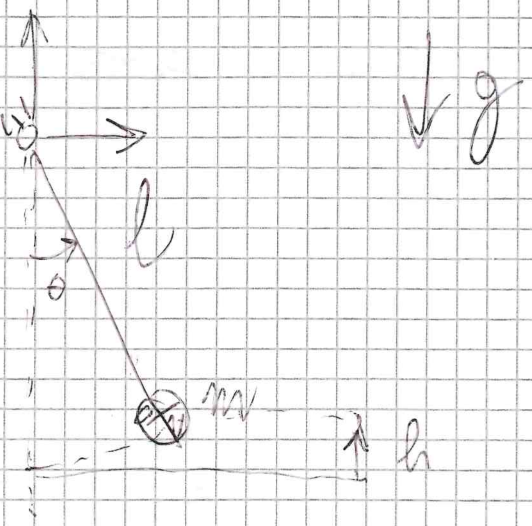
$$\boxed{m\ddot{\tilde{X}} + K\tilde{X} = 0}$$

IL SISTEMA VIBRA ATTORNO ALLA POS. DI EQ. STATICO

L'UNICO EFFETTO DELLA FORZA PESO È DI PRECARICARE LA MOLLA (NON C'È NELL'EQ. DI MOTO)

2 RIGIDEZZA

ESEMPIO: PENDOLO



$$E_c = \frac{1}{2} m (l\dot{\theta})^2 = \frac{1}{2}(ml^2)\dot{\theta}^2$$

$$V = mgh = mg(l - l\cos\theta)$$

$$\frac{\partial V}{\partial \theta} = mgl \sin\theta$$

$$ml^2\ddot{\theta} + mgl\sin\theta = 0$$

se θ piccolo $\rightarrow \sin\theta \approx \theta$

$$\underbrace{(ml^2)}_{m^*}\ddot{\theta} + \underbrace{(mgl)}_{K^*}\theta = 0$$

$$\omega = \sqrt{\frac{K^*}{m^*}} = \sqrt{\frac{mgl}{ml^2}} = \sqrt{\frac{g}{l}}$$