

CALCOLO DEI GDL

$$2 \text{ C.R.} \times 3 = 6 \text{ gdl}$$

$$\text{FUNDE} = -1$$

$$\text{PATTINO} = -2$$

$$\text{ROTOLAMENTO} = -2$$

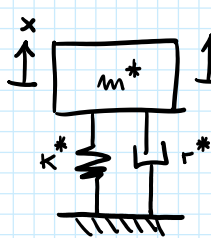
1

SYS AD 1 COORDINATA LIBERA

C.L. \dot{x}

OVVERO LA TRASLAZIONE VERTICALE DEL C.R. A "L"

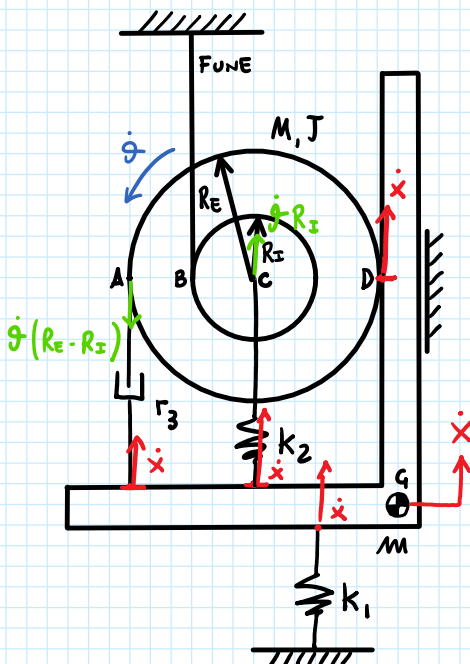
OBIETTIVO \rightarrow MODELLIZZAZIONE DEL SYS (SISTEMA)



QUESTO SI CHIAMA
SYS RIDOTTO EQUIVALENTE
DESCRITTO DALL'EQUAZIONE DIFFERENZIALE

$$m^* \ddot{x} + r^* \dot{x} + k^* x = F^*$$

LEGAMI CINEMATICI (VELOCITÀ)



1) IL C.R. A "L" TRASLA QUINDI TUTTI I PUNTI HANNO VELOCITÀ UGUALE \dot{x}

2) CONSIDERANDO IL DISCO RIGIDO

I) IL P.TO "B" HA VELOCITÀ $\bar{v}_B = 0$

PERCHÉ È TRATTENUTO (VINCOLO) DALLA FUNE
QUINDI $\bar{v}_B = 0$ e P.TO B = C.I.R.

II) IL P.TO "D" HA VELOCITÀ $\bar{v}_D = \dot{x}$

APPLICANDO RIVALS (TRA P.TI B e D)

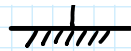
$$\bar{v}_D = \bar{v}_B + \dot{\theta} (R_I + R_E)$$

RAGGIO TOTALE DI ROTAZIONE

$$= \dot{\theta} (R_I + R_E)$$

$$3) \bar{v}_C = \dot{\theta} R_I = R_I \cdot \frac{\dot{x}}{(R_I + R_E)} = \frac{R_I}{(R_I + R_E)} \cdot \dot{x}$$

$$\bar{v}_C = \dot{\theta} (R_E - R_I) = (R_E - R_I) \cdot \dot{\theta}$$



$$3) \quad V_c = \dot{\theta} R_I = \dot{\theta} \cdot \frac{R_I}{(R_I + R_E)} = \frac{R_I}{(R_I + R_E)} \cdot \dot{x}$$

$$\bar{V}_A = \dot{\theta} (R_E - R_I) = \frac{(R_E - R_I)}{(R_I + R_E)} \cdot \dot{x}$$

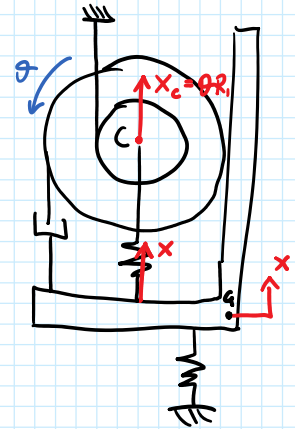
4) VELOCITÀ \rightarrow SPOSTAMENTO

$$\bar{V}_c = \dot{\theta} R_I \Rightarrow \left(\frac{R_I}{R_I + R_E} \right) \cdot \dot{x}$$

LEGAMI LINEARI INTEGRO

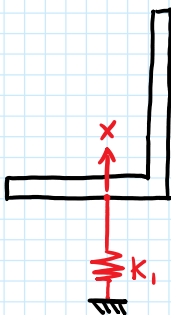
$\neq 0 \Rightarrow$ Conservazione $= 0$

$$\cancel{C_1} + x_c = \cancel{C_2} + \left(\frac{R_I}{R_I + R_E} \right) \cdot x \Rightarrow x_c = \left(\frac{R_I}{R_I + R_E} \right) x$$



5) ALLUNGAMENTI DI MOLLE E SMORZATORI
PER CONVENZIONE: " $\Delta \ell > 0$ = ALLUNGAMENTO"

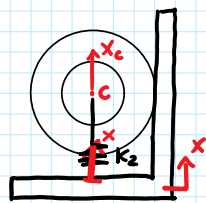
MOLLA 1



$$\Delta \ell_1 = x$$

SI ALLUNGA

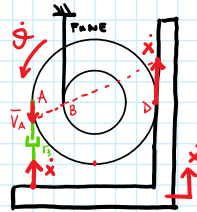
MOLLA 2



$$\begin{aligned} \Delta \ell_2 &= x_c - x \\ &= \left(\frac{R_I}{R_I + R_E} \right) x - x \\ &= \left(\frac{R_I}{R_I + R_E} - 1 \right) x \\ &= \left(\frac{R_I - R_I - R_E}{R_I + R_E} \right) x \\ &= - \frac{R_E}{R_I + R_E} \cdot x \end{aligned}$$

SI ACCORCIA

SMORZATORE



$$\begin{aligned} \Delta \ell_3 &= -\dot{x} - V_A \\ &= -\dot{x} - \left(\frac{R_E - R_I}{R_E + R_I} \right) \dot{x} \\ &= - \left(1 + \frac{R_E - R_I}{R_E + R_I} \right) \dot{x} \\ &= - \frac{2R_E}{R_E + R_I} \dot{x} \end{aligned}$$

SI ACCORCIA

6) EQ. DI MOTO (LAGRANGE)

$$m^* \ddot{x} + r^* \dot{x} + k^* x = F^*$$

$$\left[\frac{d}{dt} \left(\frac{\partial E_c}{\partial \dot{x}} \right) - \frac{\partial E_c}{\partial x} \right] + \frac{\partial D}{\partial \dot{x}} + \frac{\partial V}{\partial x} = \frac{\partial L}{\partial x} \quad \leftarrow \text{COORD. LIBERA}$$

$$\begin{aligned} E_c &= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \left(\frac{R_I}{R_I + R_E} \right)^2 \dot{x}^2 + \frac{1}{2} J \frac{1}{(R_I + R_E)^2} \dot{x}^2 \\ &= \frac{1}{2} \left[m + M \left(\frac{R_I}{R_I + R_E} \right)^2 + \frac{J}{(R_I + R_E)^2} \right] \dot{x}^2 = \frac{1}{2} m^* \dot{x}^2 \end{aligned}$$

m^*

$$\left[\frac{d}{dt} \left(\frac{\partial E_c}{\partial \dot{x}} \right) - \frac{\partial E_c}{\partial x} \right] + \frac{\partial D}{\partial \dot{x}} + \frac{\partial V}{\partial x} = \frac{\partial L}{\partial x} \quad \leftarrow \text{COORD. LIBERA}$$

$$E_c = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \left(\frac{R_1}{R_1 + R_E} \right)^2 \dot{x}^2 + \frac{1}{2} J \frac{1}{(R_1 + R_E)^2} \dot{x}^2$$

$$= \frac{1}{2} \left[m + M \left(\frac{R_1}{R_1 + R_E} \right)^2 + \frac{J}{(R_1 + R_E)^2} \right] \dot{x}^2 = \frac{1}{2} m^* \dot{x}^2$$

m^*

$$\frac{\partial E_c}{\partial x} = 0 ; \quad \frac{\partial E_c}{\partial \dot{x}} = m^* \dot{x} ; \quad \frac{d}{dt} \left(\frac{\partial E_c}{\partial \dot{x}} \right) = m^* \ddot{x}$$

$$D = \frac{1}{2} r_3 \dot{\Delta \ell_3}^2 = \frac{1}{2} r_3 \left(- \frac{2 R_E}{R_E + R_1} \right)^2 \dot{x}^2$$

$$= \frac{1}{2} \left[r_3 \left(- \frac{2 R_E}{R_E + R_1} \right)^2 \right] \dot{x}^2 = \frac{1}{2} r^* \dot{x}^2$$

r^*

$$\frac{\partial D}{\partial \dot{x}} = r^* \dot{x}$$

$$V = \frac{1}{2} k_1 \Delta \ell_1^2 + \frac{1}{2} k_2 \Delta \ell_2^2 \quad (\text{No } g \text{ in questo ES.})$$

$$= \frac{1}{2} k_1 x^2 + \frac{1}{2} k_2 \left(- \frac{R_E}{R_1 + R_E} \right)^2 x^2$$

$$= \frac{1}{2} \left[k_1 + k_2 \left(- \frac{R_E}{R_1 + R_E} \right)^2 \right] x^2$$

k^*

$$\delta L = C(t) \cdot \delta \vartheta \quad \text{MA} \quad \delta \vartheta = \frac{\delta x}{R_1 + R_E} \quad \text{DA CUI}$$

$$\delta L = C(t) \cdot \frac{\delta x}{R_1 + R_E} = \frac{C(t)}{\underbrace{R_1 + R_E}_{F^*}} \delta x$$

F^*

QUINDI L'EQ DI MOTO È

$$m^* \ddot{x} + r^* \dot{x} + k^* x = F^* \quad (\text{Eq. diff. ordinaria II ordine})$$