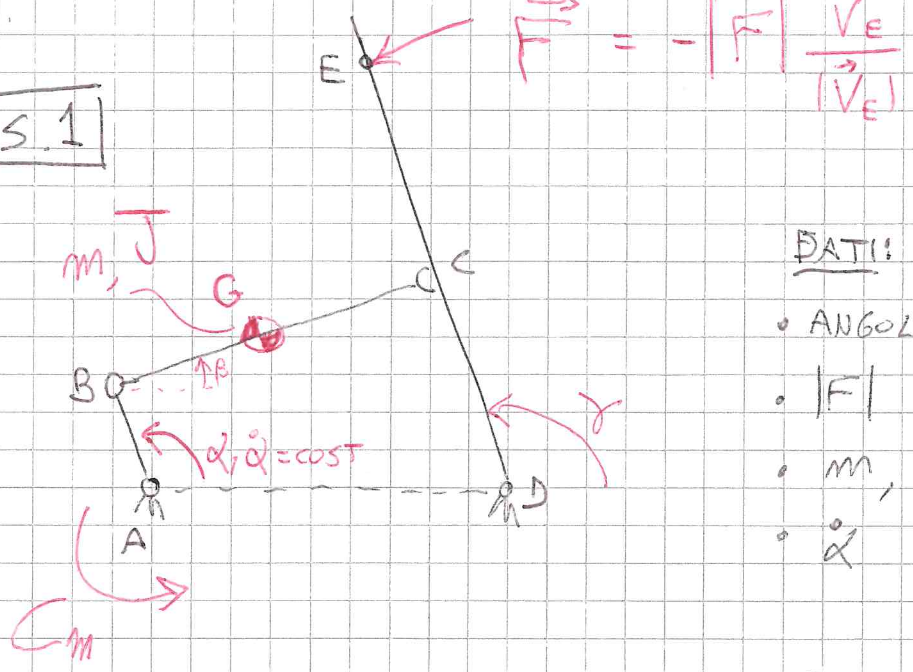
ES. 1



DATI:

- ANGOLI e LUNGHEZZE ASTE
- $|F|$
- $m, \bar{J}$
- $\dot{\alpha}$

→ CALCOLARE IL VALORE DI C_m NELL'ATTO DI MOTO RAFFIGURATO

- ~~SNE~~ APPLICO IL TEOREMA DELL'ENERGIA CINETICA

$$\frac{dE_c}{dt} = W$$

CON

$$E_c = \frac{1}{2} m V_G^2 + \frac{1}{2} \bar{J} \omega_{BC}^2$$

$$\frac{dE_c}{dt} = m V_G \underset{\substack{\updownarrow \\ \text{ACCELERAZIONE TANGENZIALE}}}{\dot{V}_G} + \bar{J} \dot{\omega}_{BC} \omega_{BC}$$

N.B. $\vec{V}_G = V_G \vec{\tau}$, $\vec{a}_G = \dot{V}_G \vec{\tau} + \dfrac{V_G^2}{\rho} \vec{n}$

$V_G^2 = \vec{V}_G \cdot \vec{V}_G$ $\qquad \dfrac{dV_G^2}{dt} = 2 \vec{V}_G \cdot \vec{a}_G = 2 V_G \dot{V}_G$

$$\frac{dE_c}{dt} = m\,\vec{V}_G \cdot \vec{a}_G + J\,\vec{\omega}_{BC} \cdot \dot{\vec{\omega}}_{BC}$$

$$= m\,V_G\,\underbrace{\dot{V}_G}_{\text{ACC TANGENZIALE} \neq \text{MODULO DI } \vec{a}_G} + J\,\omega_{BC}\,\dot{\omega}_{BC}$$

INCOGNITE: $\vec{V}_G$, $\vec{a}_G$, $\vec{\omega}_{BC}$, $\dot{\vec{\omega}}_{BC}$

$$W = \vec{C}_m \cdot \dot{\vec{\alpha}} + \vec{F} \cdot \vec{V}_E$$

$$= C_m \vec{k} \cdot \dot{\alpha} \vec{k} + -|F| \frac{\vec{V}_E}{|\vec{V}_E|} \cdot \vec{V}_E =$$

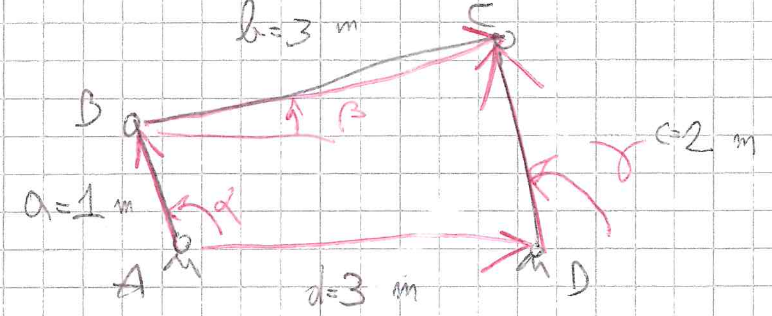
$$= C_m \dot{\alpha} - |F||\vec{V}_E|$$

INCOGNITE $\vec{V}_E$, C_m

$\vec{V}_G$, $\vec{a}_G$, $\vec{\omega}_{BC}$, $\dot{\vec{\omega}}_{BC}$, $\vec{V}_E$ SONO PERÒ NOTE DAI LEGAMI CINEMATICI IN FUNZIONE DI α, $\dot{\alpha}$

$\rightarrow$ L'UNICA INCOGNITA È C_m

CINEMATICA (GIA' VISTA ALL'ESERCITAZIONE 2)



$$\alpha = \frac{\pi}{2}$$

$$(C-B) + (B-A) = (C-D) + (D-A)$$

EQ. DI CHIUSURA

$(C-B)$	$(B-A)$	$(C-D)$	$(D-A)$
MOD. COST.	MODULO COST	MODULO COSTANTE	MODULO COSTANTE
ANG. $\beta(t)$	ANG. $\alpha(t)$ NOTO	ANGOLO $\gamma(t)$	ANGOLO COSTANTE $(=0)$

$$\begin{cases} BC\cos\beta + AB\cos\alpha = CD\cos\gamma + AD \\ BC\sin\beta + AB\sin\alpha = CD\sin\gamma \end{cases} \rightarrow \begin{aligned} \beta &= 0.337 \text{ rad} \\ \gamma &= 1.655 \text{ rad} \end{aligned}$$

$$\Downarrow \frac{d}{dt}$$

$$\begin{bmatrix} -BC\sin\beta & +CD\sin\gamma \\ BC\cos\beta & -CD\cos\gamma \end{bmatrix} \begin{bmatrix} \dot\beta \\ \dot\gamma \end{bmatrix} = \begin{bmatrix} AB\sin\alpha \\ -AB\cos\alpha \end{bmatrix} \dot\alpha$$

$$[A]\begin{bmatrix} \dot\beta \\ \dot\gamma \end{bmatrix} = [B]\dot\alpha \Longrightarrow \begin{bmatrix} \dot\beta \\ \dot\gamma \end{bmatrix} = [A]^{-1}[B]\dot\alpha = \begin{matrix} -0.03 \ \text{rad/s} \\ 0.487 \ \text{rad/s} \end{matrix}$$

$$\downarrow \frac{d}{dt}$$

$$[A]\begin{bmatrix} \ddot\beta \\ \ddot\gamma \end{bmatrix} + \underbrace{\left(\frac{d}{dt}[A]\right)\begin{bmatrix} \dot\beta \\ \dot\gamma \end{bmatrix}}_{\downarrow} = \left(\frac{d}{dt}[B]\right)\dot\alpha \qquad (\dot\alpha = \text{COST})$$

$$[A]\begin{bmatrix} \ddot\beta \\ \ddot\gamma \end{bmatrix} + \begin{bmatrix} -BC\cos\beta\cancel{\dot\beta} & CD\cos\gamma\cancel{\dot\gamma} \\ -BC\sin\beta\cancel{\dot\beta} & CD\sin\gamma\cancel{\dot\gamma} \end{bmatrix}\begin{bmatrix} \dot\beta^2 \\ \dot\gamma^2 \end{bmatrix} = \begin{bmatrix} AB\cos\alpha \\ -AB\sin\alpha \end{bmatrix}\dot\alpha^2 \rightarrow \begin{bmatrix} \ddot\beta \\ \ddot\gamma \end{bmatrix} = \begin{matrix} 0.1 \\ 0.11 \end{matrix} \ \text{rad/s}^2$$

$$\vec{V}_G = \vec{V}_B + \vec{\omega}_{BC} \wedge (G-B) = V_{Gx}\,\vec{i} + V_{Gy}\,\vec{j}$$

$$\vec{V}_B = \vec{\omega}_{AB} \wedge (B-A)$$

$$\vec{\omega}_{AB} = \dot{\alpha}\,\vec{k}$$

$$(B-A) = AB\,(\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j})$$

$$\vec{\omega}_{BC} = \dot{\beta}\,\vec{k}$$

$$(G-B) = \cancel{BC}\left(\frac{BC}{2}\right)(\cos\beta\,\vec{i} + \sin\beta\,\vec{j})$$

$$\vec{a}_G = \vec{a}_B + \dot{\vec{\omega}}_{BC} \wedge (G-B) - \omega_{BC}^2\,(G-B) = a_{Gx}\,\vec{i} + a_{Gy}\,\vec{j}$$

$$\vec{a}_B = \cancel{\dot{\vec{\omega}}_{AB} \wedge (B-A)} - \omega_{AB}^2\,(B-A)$$

$$\dot{\omega}_{AB} = \ddot{\alpha} = 0$$

$$\dot{\omega}_{BC} = \ddot{\beta}\,\vec{k}$$

$$\vec{V}_E = \vec{\omega}_{CD} \wedge (E-D)$$

$$\vec{\omega}_{CD} = \dot{\gamma}\,K$$

$$(E-D) = DE\,(\cos\gamma\,\vec{i} + \sin\gamma\,\vec{j})$$

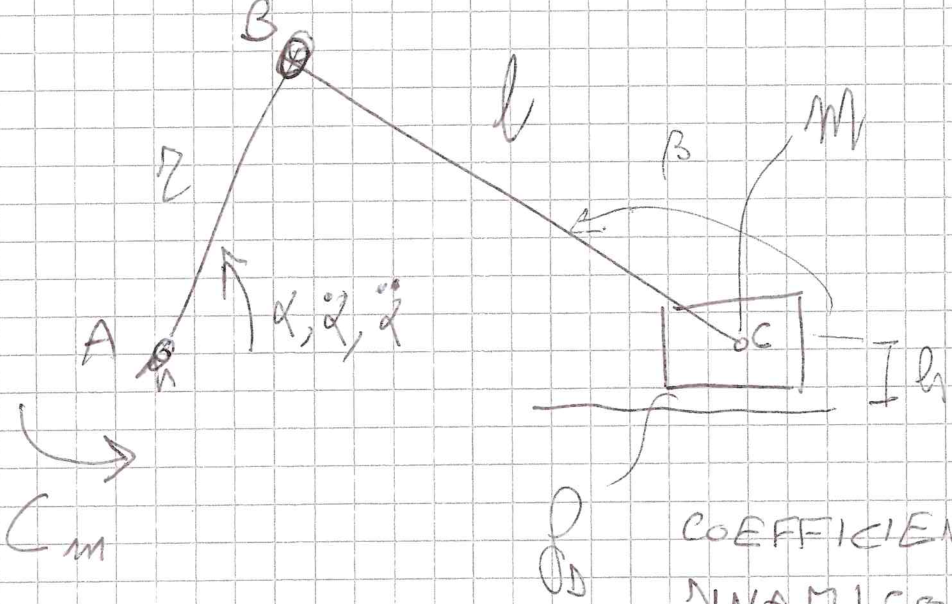
$$\frac{dE_c}{dt} = W$$

$$m\,\vec{a}_G \cdot \vec{V}_G + J\,\dot{\vec{\omega}}_{BC} \cdot \vec{\omega}_{BC} = C_m\,\dot{\alpha} - |F||V_E|$$

$$m\left(a_{Gx}V_{Gx} + a_{Gy}V_{Gy}\right) + J\,\ddot{\beta}\,\dot{\beta} = C_m\,\dot{\alpha} - |F||\dot{\gamma}\,DE|$$

DA CUI RICAVO C_m UNICA INCOGNITA.

ES. 2

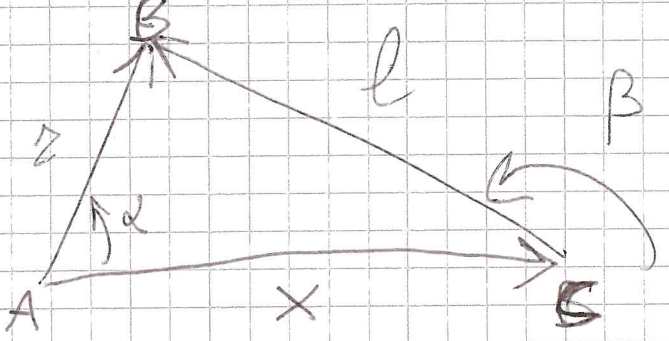


NOTI
r, l, h
$\alpha, \dot{\alpha}, \ddot{\alpha}$
m
f_D

TROVARE
C_m

COEFFICIENTE DI ATTRITO DINAMICO

CINEMATICA (GIÀ VISTA A LEZIONE)



$$(B-C)+(C-A)=(B-A) \longrightarrow \begin{matrix} x(\alpha) \\ \beta(\alpha) \end{matrix}$$

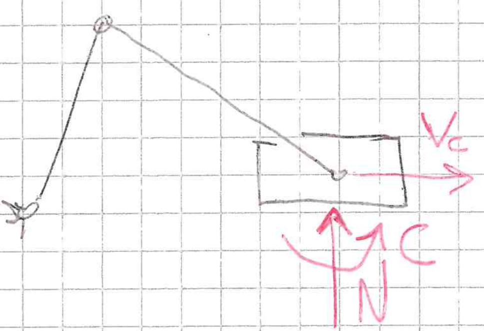
$$\xrightarrow{d/dt} \begin{matrix} \dot{x}(\alpha,\dot{\alpha}) \\ \dot{\beta}(\alpha,\dot{\alpha}) \end{matrix} \xrightarrow{d/dt} \begin{matrix} \ddot{x}(\alpha,\dot{\alpha},\ddot{\alpha}) \\ \ddot{\beta}(\alpha,\dot{\alpha},\ddot{\alpha}) \end{matrix}$$

BILANCIO DI POTENZE

$$\frac{dE_c}{dt} = W_{ATTIVE} + W_{ATTRITO}$$

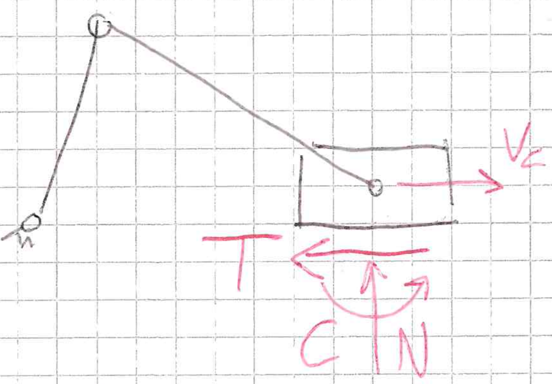
INFATTI:

CASO IDEALE (NO ATTRITO)



$W_{ATTRITO} = 0$

CASO REALE (ATTRITO)



$$\vec{T} = -|N| f_D \frac{\vec{V}_c}{|\vec{V}_c|}$$

$$W_{ATTR} = \vec{T} \cdot \vec{V}_c = -|N| f_D |\vec{V}_c|$$

$$W_{ATTIVE} = \vec{C}_m \cdot \vec{\omega}_{AB} = C_m \dot{\alpha}$$

$$E_c = \frac{1}{2} m \vec{V}_c \cdot \vec{V}_c = \frac{1}{2} m V_c^2 \qquad \vec{V}_c = \dot{x}\,\vec{i}$$

$$\frac{dE_c}{dt} = m \dot{x} \ddot{x}$$

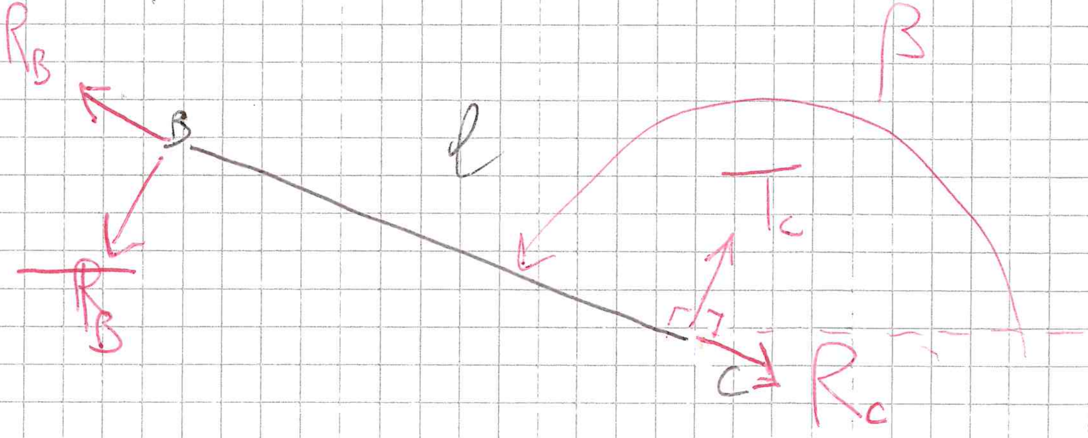
A $\boxed{m \ddot{x} \dot{x} = C_m \dot{\alpha} - |N| f_D |\dot{x}|}$

APPARE 1 INCOGNITA IN PIÙ N

ESSENDO UNA REAZIONE VINCOLARE

LA POSSO CALCOLARE FACENDO DEGLI EQUILIBRI DINAMICI

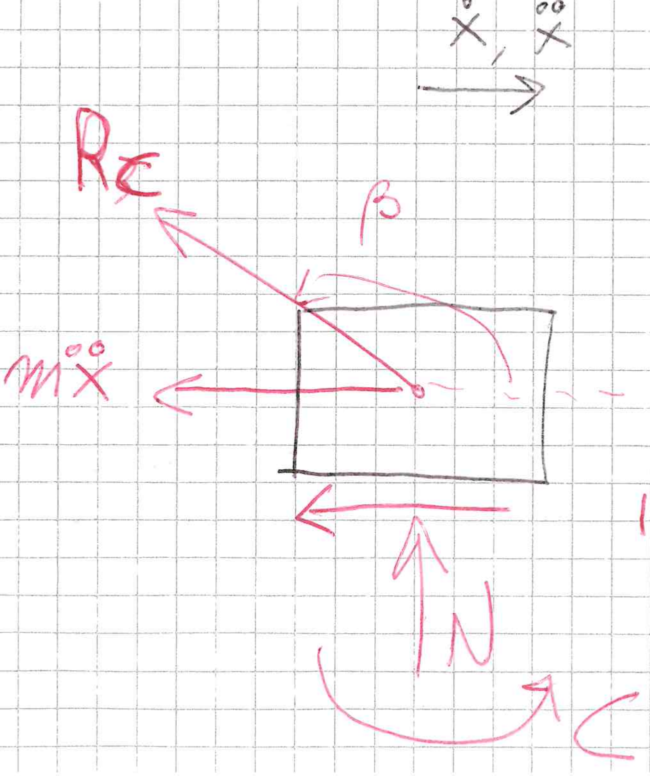
• ISOLO BC



$$\begin{cases} T_C = T_B \\ R_C = R_B \\ T_B \ell = 0 \end{cases} \longrightarrow \begin{cases} T_C = 0 \\ R_C = R_B \\ T_B = 0 \end{cases}$$

LA BIELLA (SCARICA) TRASMETTE SOLO AZIONE ASSIALE

• ISOLO CORSOIO



$$\begin{cases} -m\ddot{x} - |N| f_d \, sign(\dot{x}) + R_C \cos\beta = 0 \\ R_C \sin\beta + N = 0 \end{cases}$$

$\longrightarrow R_C$

$N \longrightarrow$ LO INSERISCO IN (A)