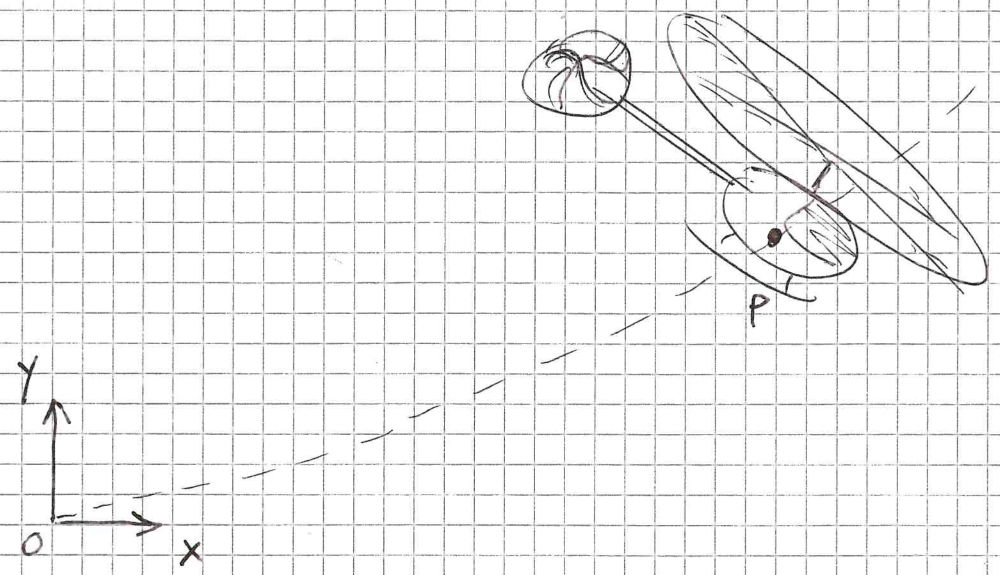
# ESERCITAZIONE 1

# CINEMATICA DEL PUNTO

## PROBLEMA 1




DURANTE LA PRIMA FASE DI DECOLLO, LA POSIZIONE DELL'ELICOTTERO P É NOTA IN FORMA PARAMETRICA

$$\begin{cases} x = 2t^2 & [m] \\ y = 0.04t^3 & [m] \end{cases}$$

con $t$ (tempo) IN SECONDI

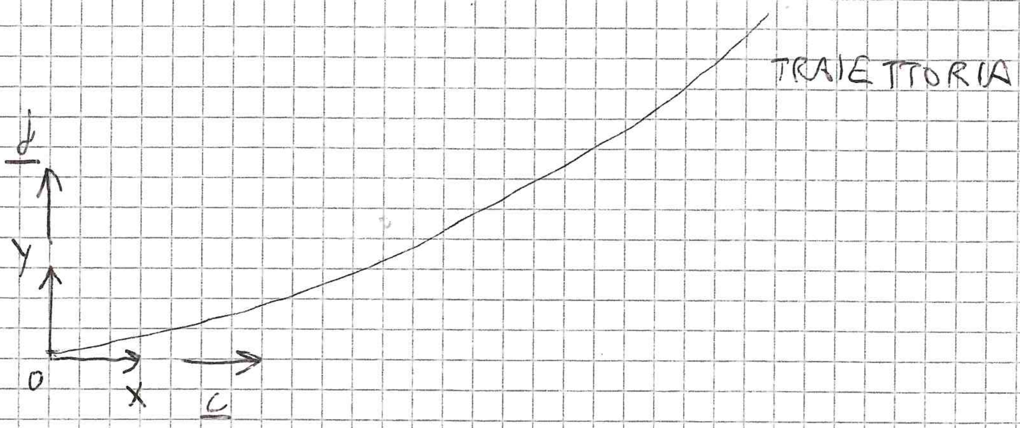
### DOMANDE

- CALCOLARE LA TRAIETTORIA $y(x)$
- AL TEMPO $t = 10\ s$ DETERMINARE:
  - VETTORE POSIZIONE $(P-O)$
  - VETTORE VELOCITA' $\underline{v}_P$
  - VETTORE ACCELERAZIONE $\underline{a}_P$

## TRAIETTORIA

$$\begin{cases} x = 2t^2 \\ y = 0.04\,t^3 \end{cases} \longrightarrow t = \sqrt{\frac{x}{2}}$$

$$y(x) = 0.04\left(\frac{x}{2}\right)^{\frac{3}{2}}$$



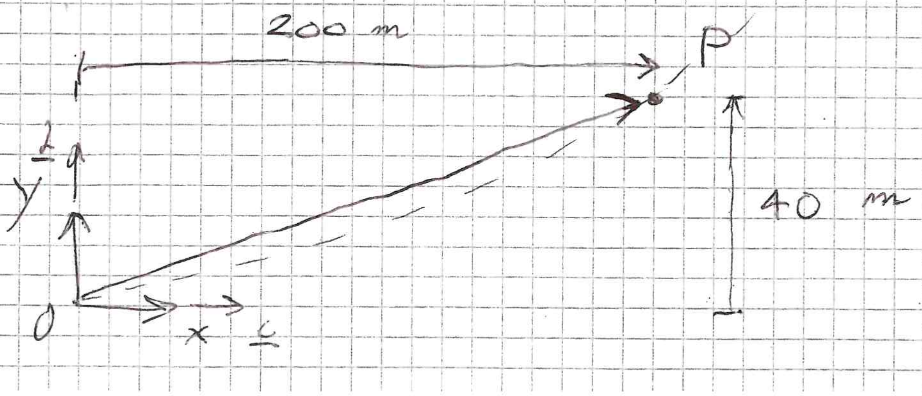

## POSIZIONE $(t = 10\ s)$

$$(P-O) = x(10)\,\underline{i} + y(10)\,\underline{j} \qquad \left[\underline{i},\underline{j} : \text{VERSORI ASSI } x, y\right]$$

oppure $\underline{p}$

$$\begin{cases} x(10) = 200\ m \\ y(10) = 40\ m \end{cases}$$

$$(P-O) = 200\,\underline{i} + 40\,\underline{j}$$



# VELOCITÀ

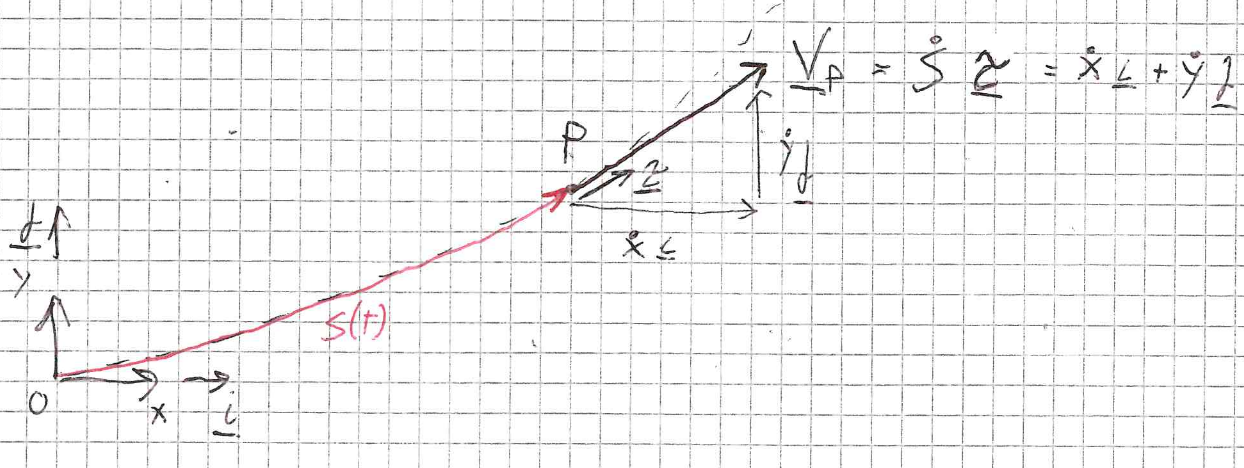
$$\underline{V}_P = \frac{d}{dt}(P-O) = \dot{x}(t)\,\underline{i} + \dot{y}(t)\,\underline{j}$$

$$\dot{x} = \frac{dx}{dt}$$
$$\dot{y} = \frac{dy}{dt}$$

oppure

$$\underline{V}_P = \dot{s}\,\underline{\tau}$$ CON $\underline{\tau}$ : VERSORE TANGENTE ALLA TRAIETTORIA

$$\dot{s} = \frac{ds}{dt}$$ , $s(t)$ ASCISSA CURVILINEA




→ COMPONENTI CARTESIANE

$$\begin{cases} \dot{x} = 2 \cdot 2t \overset{t=10}{=} 40 \ m/s \\ \dot{y} = 0.04 \cdot 3t^2 = 12 \ m/s \end{cases}$$

$$\underline{V}_P(10) = 40\,\underline{i} + 12\,\underline{j}$$

$$V_P = |\underline{V}_P| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1744} = 41.8 \ m/s$$

→ COMPONENTE TANGENZIALE

$$\underline{V}_P = \mathring{S}\,\underline{\tau}$$

$$|\underline{V}_P| = |\mathring{S}| = 41.8\ m/s \longrightarrow \mathring{S} = 41.8\ m/s$$

$$\underline{\tau} = \frac{\underline{V}_P}{|\underline{V}_P|} = \frac{40}{41.8}\underline{i} + \frac{12}{41.8}\underline{j} = 0.957\,\underline{i} + 0.287\,\underline{j}$$

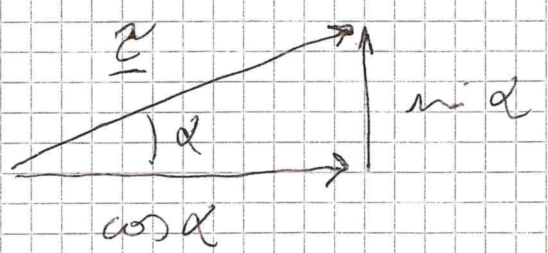
VERIFICA CHE $\underline{\tau}$ È TANGENTE

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\mathring{y}}{\mathring{x}} = \frac{12}{40} = 0.3$$

OPPURE DERIVO $y(x)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(0.04\left(\frac{x}{2}\right)^{\frac{3}{2}}\right) = 0.04\,\frac{3}{2}\,\frac{1}{2}\left(\frac{x}{2}\right)^{\frac{1}{2}} =$$

$$= 0.04\,\frac{3}{4}\left(\frac{200}{2}\right)^{\frac{1}{2}} = 0.3$$




$$tg\,\alpha = \frac{dy}{dx} = 0.3$$

$$\alpha = 16.7° \longrightarrow \cos\alpha = 0.957$$

$$\sin\alpha = 0.287$$

CHE SONO LE COMPONENTI LUNGO $\underline{i}$ e $\underline{j}$ DI $\underline{\tau}$

# ACCELERAZIONE

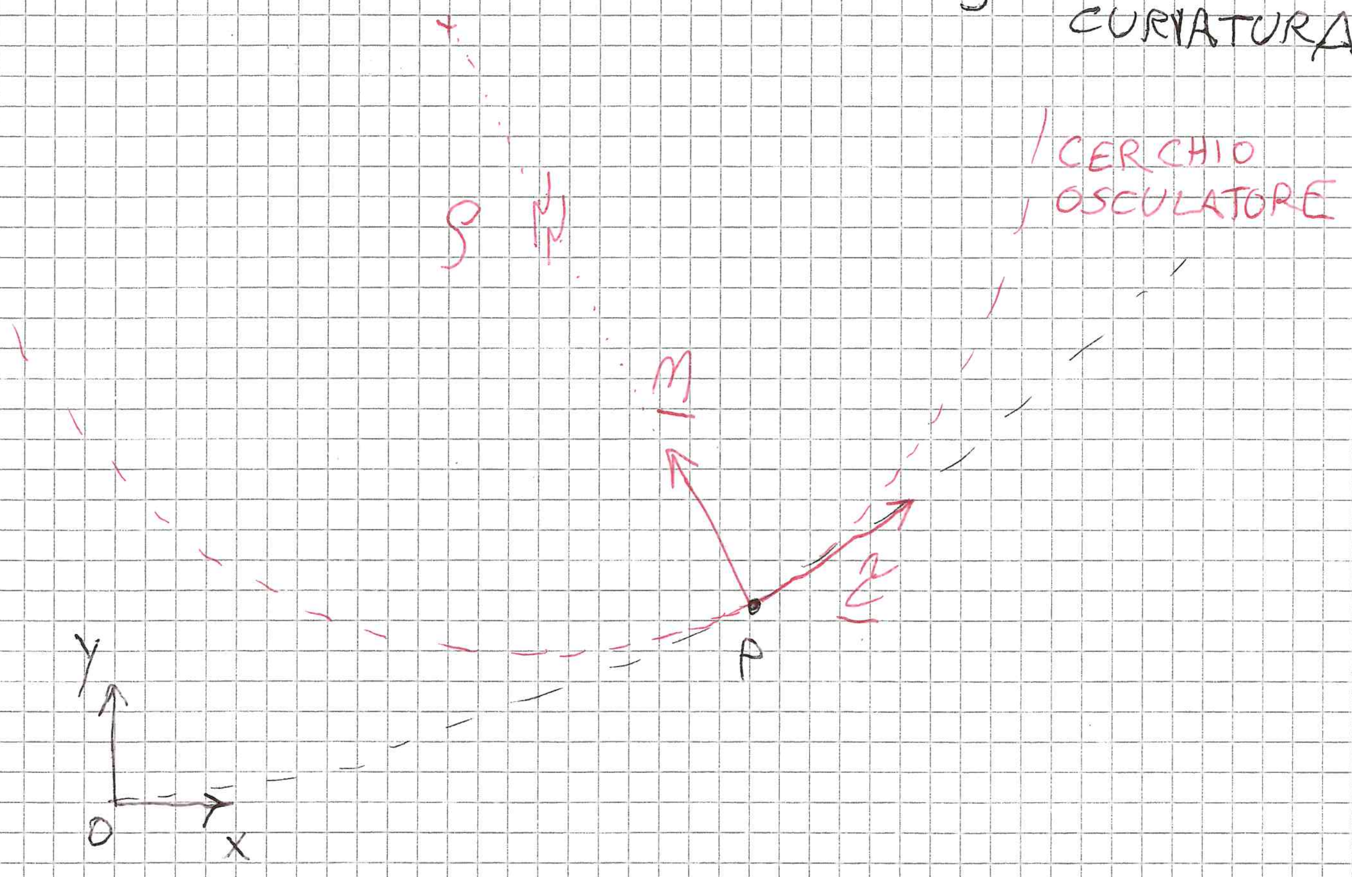
$$\underline{a}_P = \frac{d\underline{V}_P}{dt} = \ddot{x}\,\underline{i} + \ddot{y}\,\underline{j}$$

OPPURE

$$\underline{a}_P = \ddot{s}\,\underline{\tau} + \frac{\dot{s}^2}{\rho}\,\underline{n}$$

$\underline{n}$ : VERSORE NORMALE

$\rho$ : RAGGIO DI CURVATURA




$$\underline{a}_P = \underbrace{\ddot{s}\,\underline{\tau}}_{\text{ACC. TANGENZIALE}} + \underbrace{\frac{\dot{s}^2}{\rho}\,\underline{n}}_{\text{ACC. NORMALE}}$$

ACC. TANGENZIALE

[VARIAZIONE MODULO DELLA VELOCITÀ $\underline{V}_P$]

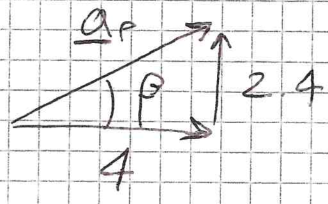
ACC. NORMALE

[VARIAZIONE DIREZIONE DI $\underline{V}_P$]

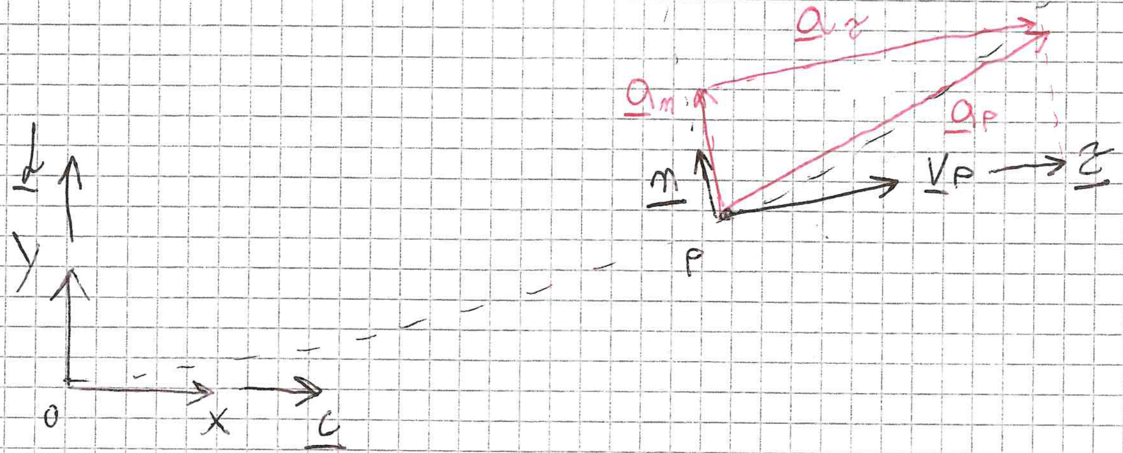
COMPONENTI CARTESIANE

$$\begin{cases} \ddot{x} = 4 \quad m/s^2 \\ \ddot{y} = 0.12 \cdot 2T = 2.4 \quad m/s^2 \end{cases}$$

$$\underline{a}_P = 4\,\underline{i} + 2.4\,\underline{j}$$




$\beta \simeq 31°$




CALCOLIAMO $\underline{a}_\tau$ e $\underline{a}_n$

$$\begin{cases} \underline{a}_\tau = \ddot{s}\,\underline{\tau} = a_\tau\,\underline{\tau} \\ \underline{a}_n = \dfrac{\dot{s}^2}{\rho}\,\underline{n} = a_n\,\underline{n} \end{cases}$$

→ CALCOLO COMPONENTE TANGENZIALE:

$$\underline{a}_P = \ddot{s}\,\underline{\tau} + \frac{\dot{s}^2}{\rho}\,\underline{n}$$

NOTI: $\underline{a}_P$, $\underline{\tau}$, $\dot{s}$

INCOGNITE: $\underline{n}$, $\rho$, $\ddot{s}$

MOLTIPLICO SCALARMENTE PER $\underline{\tau}$

$$\underline{a}_P \cdot \underline{\tau} = \ddot{s}\,\underbrace{\underline{\tau}\cdot\underline{\tau}}_{1} + \frac{\dot{s}^2}{\rho}\,\underbrace{\underline{n}\cdot\underline{\tau}}_{0}$$

$$\ddot{s} = \underline{a}_P \cdot \underline{\tau}$$

$$= (4\,\underline{i} + 2.4\,\underline{j})(0.957\,\underline{i} + 0.287\,\underline{j})$$

$$= 4 \cdot 0.957 + 2.4 \cdot 0.287 = 4.52\ m/s^2$$

OPPURE:

CALCOLO $s(t) = \sqrt{\dot{x}^2(t) + \dot{y}^2(t)} \rightarrow \ddot{s} = \frac{d\,s(t)}{dt} = \ldots$

$$\Rightarrow \underline{a}_\tau = 4.52\,\underline{\tau}$$

→ CALCOLO COMPONENTE NORMALE

OPZIONE 1

$$|\underline{a}_P|^2 = \ddot{x}^2 + \ddot{y}^2 = 21.76$$

$$|\underline{a}_P|^2 = \ddot{s}^2 + \left(\frac{\dot{s}^2}{\rho}\right)^2 \Rightarrow \frac{\dot{s}^2}{\rho} = 1.15\ m/s^2 = a_n$$

$$\underline{a}_n = 1.15\,\underline{n}$$

DEVO TROVARE $\underline{n}$

$$\underline{a}_P = a_\tau \underline{\tau} + a_n \underline{n} \rightarrow \underline{n} = \frac{\underline{a}_P - a_\tau \underline{\tau}}{a_n}$$

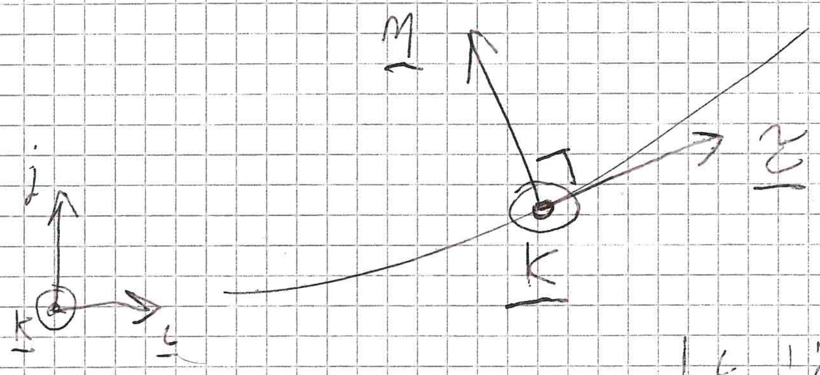
$$\underline{n} = -0.287\, \underline{i} + 0.958\, \underline{j}$$

VOLENDO SAPERE IL VALORE DI $\rho$

$$\frac{\dot{s}^2}{\rho} = 1.15 \rightarrow \rho = \frac{\dot{s}^2}{1.15} = 1517.3\ m$$

OPZIONE 2

CALCOLO $\underline{n}$, USANDO IL PRODOTTO VETTORIALE E INTRODUCENDO IL VERSORE USCENTE DAL PIANO $\underline{k}$



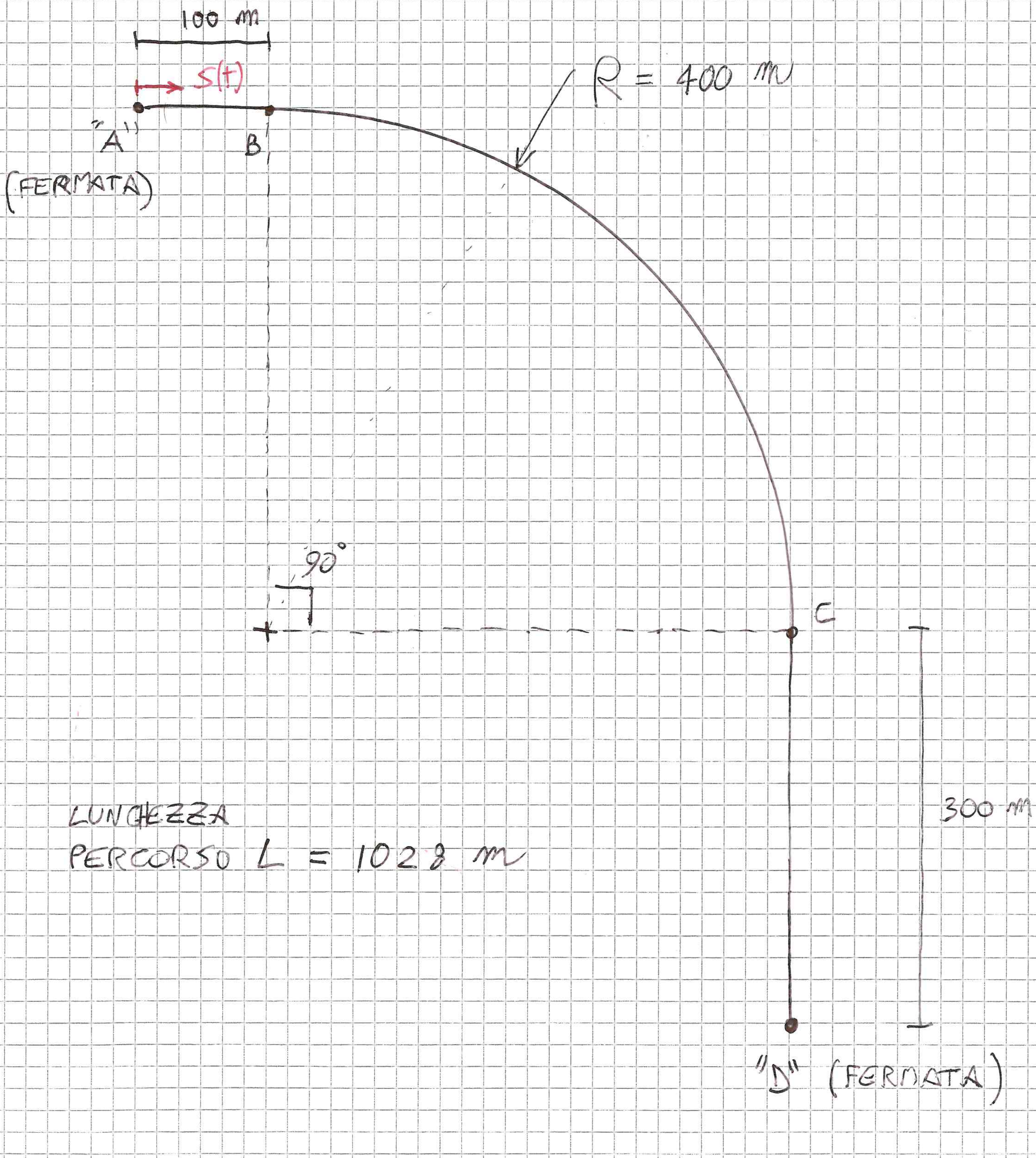

$$\Rightarrow \underline{n} = \underline{k} \wedge \underline{\tau} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & 1 \\ 0.958 & 0.287 & 0 \end{vmatrix} = -0.287\, \underline{i} + 0.958\, \underline{j}$$

$$a_n = \underline{a}_P \cdot \underline{n} = 1.15\ m/s^2$$

$$\underline{a}_n = 1.15\, \underline{n}$$

# PROBLEMA 2

UN TRAM VIAGGIA LUNGO IL SEGUENTE PERCORSO PER ANDARE DALLA FERMATA "A" ALLA FERMATA "D"



# DOMANDA

CALCOLARE LA LEGGE DI MOTO CHE MINIMIZZA IL TEMPO DI PERCORRENZA TRA A e D

NOTI:

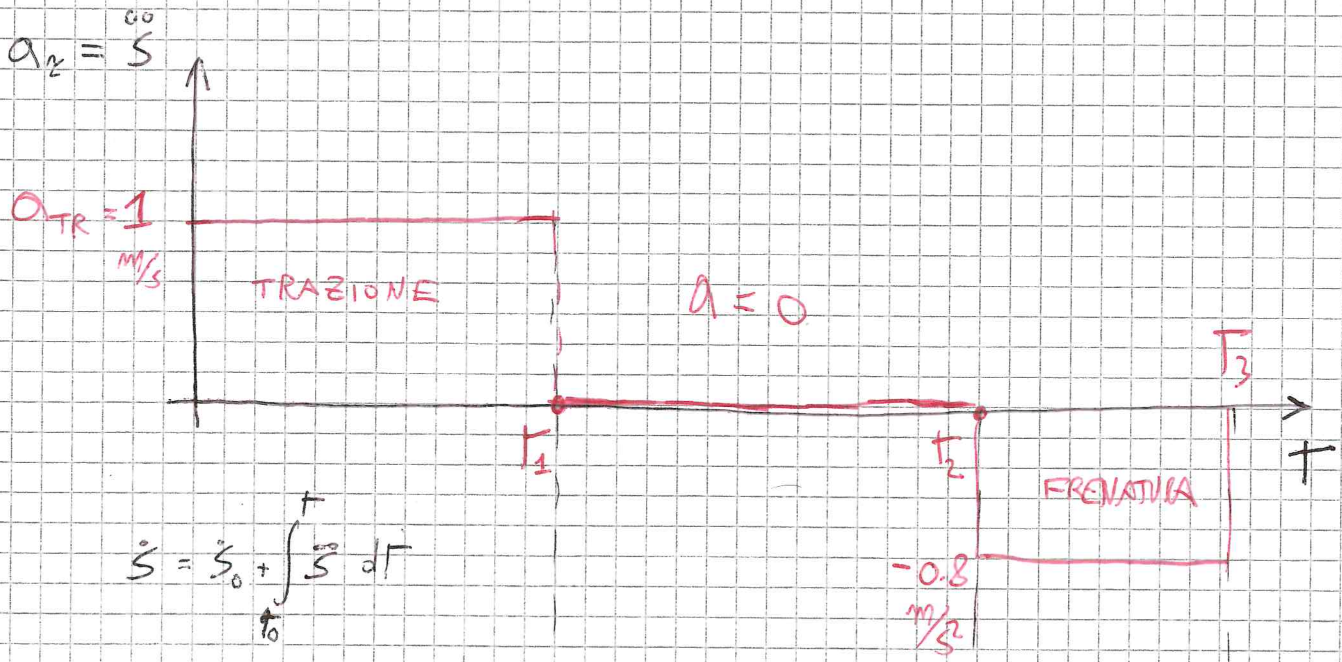
$V_{MAX} = 60 \ Km/h$

$$\left.\begin{array}{l} a_{TRAZIONE} \ 1 \ m/s^2 \\ a_{FRENATA} = -0.8 \ m/s^2 \end{array}\right\} \text{ACC MAX TANGENZIALI}$$

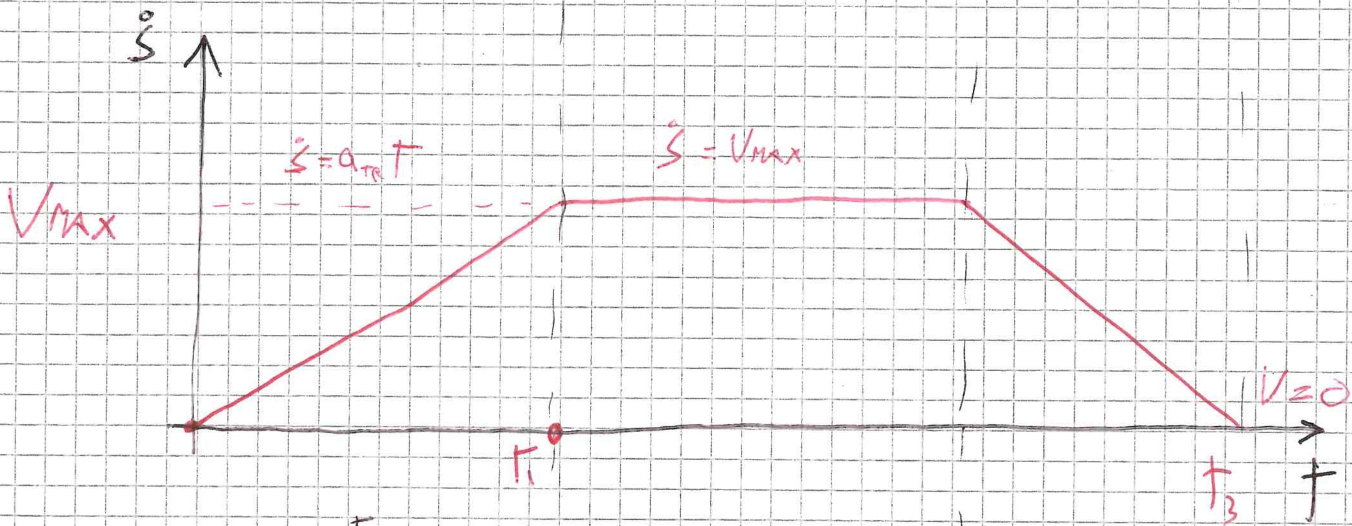
RISPETTANDO LIMITE DI COMFORT PASSEGGERO

$|\underline{a}| < 1.5 \ m/s^2$

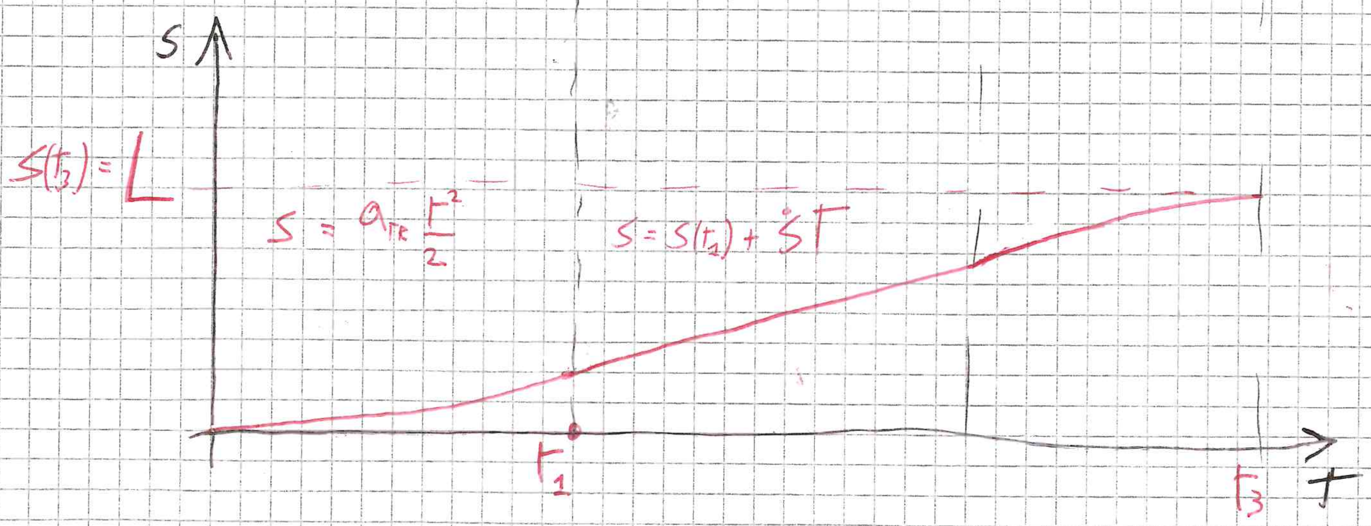
# LEGGE DI MOTO CHE MINIMIZZA IL TEMPO DI PERCORRENZA




$$\dot{s} = \dot{s}_0 + \int_{t_0}^{t} \ddot{s}\, dt$$




$$s = s_0 + \int_{t_0}^{t} \dot{s}\, dt$$



BISOGNA CALCOLARE $T_1$, $T_2$, $T_3$ E VERIFICARE IL COMFORT

$\boxed{\text{TRA} \quad 0 < T \leq T_1}$ PARTO DA FERMO E ACCELERO FINO A $V_{MAX}$

$$V(T) = \dot{S}(T) = \cancel{0} + \int_0^T a_{TR}\, dT = a_{TR} T$$

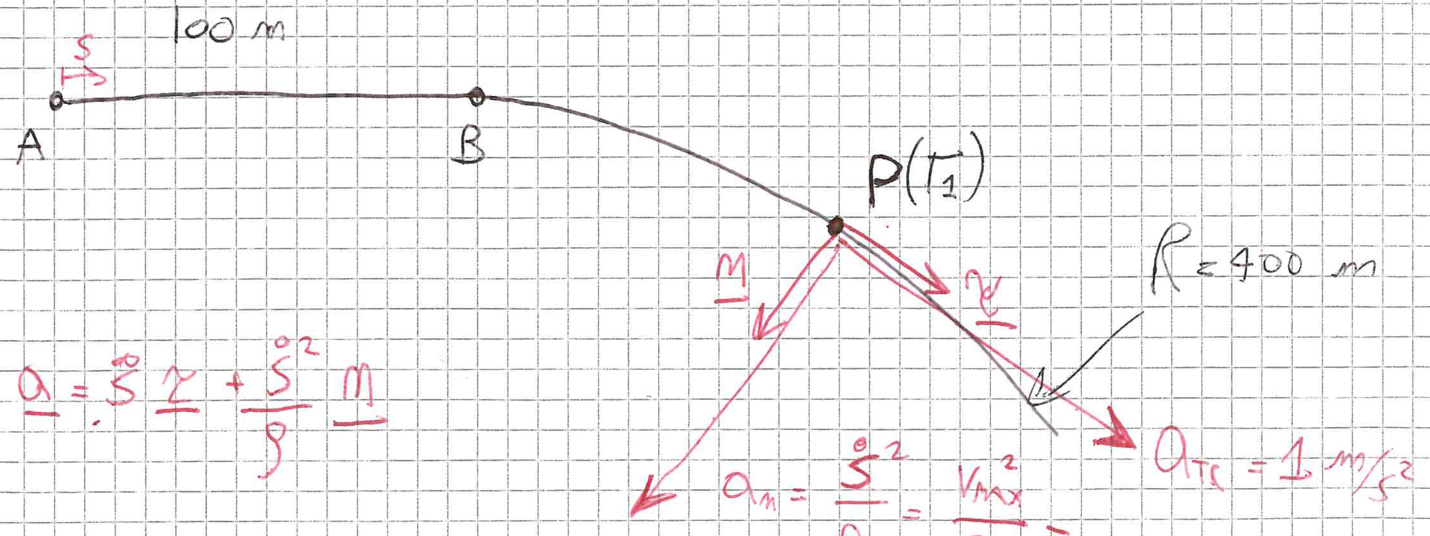
$$\dot{S}(T_1) = V_{MAX} = a_{TR} T_1 \rightarrow T_1 = \frac{V_{MAX}}{a_{TR}} = \frac{(60/3.6)\ m/s}{1}$$

$$= 16.7\ s$$

<u>VERIFICA COMFORT</u>

$$S(T) = \underbrace{\cancel{S_0}}_{=0} + \int_0^T \dot{S}\, dT = \frac{a_{TR}}{2} T^2$$

$$S(T_1) = 139.4\ m$$




$$\underline{a} = \ddot{S}\,\underline{\tau} + \frac{\dot{S}^2}{\rho}\,\underline{n}$$

$$a_{TR} = 1\ m/s^2$$

$$a_n = \frac{\dot{S}^2}{R} = \frac{V_{MAX}^2}{R} = 0.7\ m/s^2$$

$$|\underline{a}| = \sqrt{1^2 + 0.7^2} = \sqrt{1.49} = 1.22 \ m/s^2 < 1.5 \ m/s^2$$

$\Rightarrow$ COMFORT VERIFICATO

$$\boxed{t_2 < t < t_3}$$

TRAM A $V_{MAX}$ INIZIA A FRENARE E SI FERMA IN $S(t_3) = L = 1028 \ m$

$$V(t) = V_{MAX} + \int_{t_2}^{t} a_{FR} \, dt = V_{MAX} + a_F (t - T_2)$$

$$= 16.7 - 0.8 (t - T_2)$$

$$V(t_3) = 0$$

$$\rightarrow 0 = 16.7 - 0.8 (t_3 - t_2) \Rightarrow (t_3 - t_2) = 20.9 \ s$$

$$S(t) = S(t_1) + \int_{t_1}^{t_2} V_{MAX} \, dt + \int_{t_2}^{t} \left( V_{MAX} + a_{FR}(t - T_2) \right) dt$$

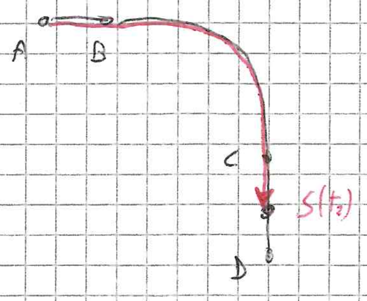
$$S(t_3) = 1028 = 139.4 + 16.7 (t_2 - 16.7) + 16.7 (t_3 - t_2) +$$

$$\ldots - 0.8 \frac{(t_3 - t_2)^2}{2}$$

$$\Rightarrow t_2 = 59.5 \ s$$

$$t_3 = 80.4 \ s$$

$$s(t_2) = 139.4 + 16.7\,(59.5 - 16.7) = 854 \text{ m}$$




QUANDO INIZIA A FRENARE IL TRAM HA UNA TRAIETTORIA RETTILINEA

$$\Rightarrow \underline{a}_n = 0$$

$$|\underline{a}| = |a_t| = 0.8 < 1.5 \text{ m/s}^2$$

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$$\begin{cases} v(t) \\ s(t) \end{cases} \longrightarrow \quad v(s) = ?$$

CONSIDERIAMO $0 < t < t_1$

$$v(t) = a_{TR}\, t$$

$$s(t) = a_{TR} \frac{t^2}{2} \longrightarrow t = \sqrt{\frac{2s}{a_{TR}}}$$

$$v(s) = a_{TR} \sqrt{\frac{2s}{a_{TR}}} = \sqrt{2 s\, a_{TR}} = \sqrt{2s}$$

AD ESEMPIO IN $s = 98$ m

$$v = \sqrt{2 \cdot 98} = \sqrt{196} = 14 \text{ m/s}$$