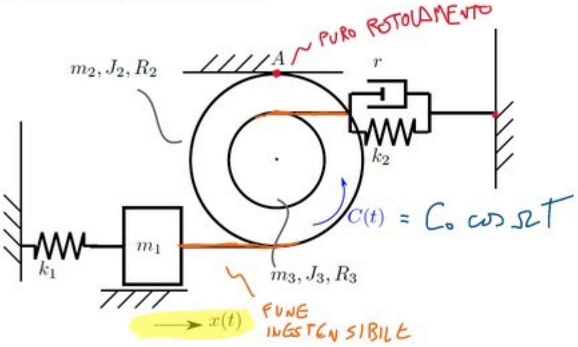
Esercizio 2

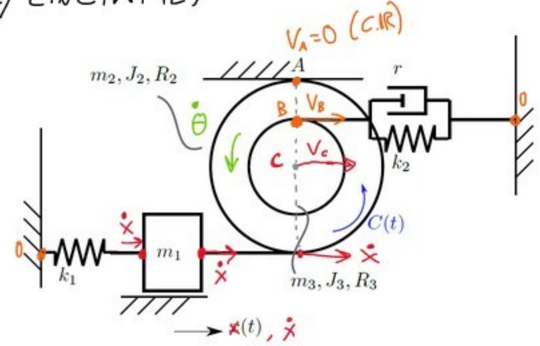
lunedì 21 dicembre 2020
09:34

Si chiede di calcolare:

1. l'equazione di moto del sistema, usando come coordinata libera $x(t)$.
2. la pulsazione propria del sistema non smorzato ω_0 ed il coefficiente di smorzamento h
3. l'integrale particolare $x_P(t)$



① CINEMATICA



$$\delta\theta = \frac{\delta x}{2R_2}$$

$$\dot\theta\, 2R_2 = \dot x \quad \Rightarrow \quad \dot\theta = \frac{\dot x}{2R_2}$$

$$V_B = (R_2 - R_1)\dot\theta = \left(\frac{R_2 - R_1}{2R_2}\right)\dot x$$

$$V_c = R_2\dot\theta = \frac{R_2}{2R_2}\dot x = \frac{\dot x}{2}$$

$$\Delta l_1 = x$$

$$\Delta l_2 = -\left(\frac{R_2 - R_1}{2R_2}\right)x \quad , \quad \Delta \dot l_2 = -\left(\frac{R_2 - R_1}{2R_2}\right)\dot x$$

② DINAMICA $\rightarrow$ $$\frac{d}{dT}\left(\frac{\partial E_c}{\partial \dot x}\right) - \frac{\partial E_c}{\partial x} + \frac{\partial D}{\partial \dot x} + \frac{\partial V}{\partial x} = \frac{\delta L}{\delta x}$$

$$E_c = \frac{1}{2}m_1\dot x^2 + \frac{1}{2}(m_2 + m_3)V_c^2 + \frac{1}{2}(J_2 + J_3)\dot\theta^2 =$$

$$= \frac{\dot x^2}{2}\underbrace{\left(m_1 + \frac{(m_2 + m_3)}{4} + \frac{(J_2 + J_3)}{4R_2^2}\right)}_{m^* = 9.54\ [Kg]} = \frac{1}{2}m^*\dot x^2$$

$$\frac{d}{dT}\left(\frac{\partial E}{\partial \dot x}\right) - \frac{\partial E_c}{\partial x} = m^*\ddot x$$

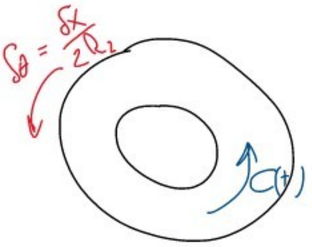
$$D = \frac{1}{2}r\Delta \dot l_2^2 = \frac{1}{2}\underbrace{r\left[-\left(\frac{(R_2 - R_1)}{2R_2}\right)\right]^2}_{r^* = 3.95\ \left[\frac{Ns}{m}\right]}\dot x^2 = \frac{1}{2}r^*\dot x^2$$

$$\frac{\partial D}{\partial \dot x} = r^*\dot x$$

$$V = \frac{1}{2} K_1 \Delta l_1^2 + \frac{1}{2} K_2 \Delta l_2^2 = \frac{x^2}{2} \underbrace{\left[K_1 (-1)^2 + K_2 \left(- \frac{R_2 - R_1}{2R_2} \right)^2 \right]}_{K^* = 8160 \; [N/m]}$$

$$\frac{\partial V}{\partial x} = K^* x$$

$$\delta L = \vec{C}(t) \cdot \delta \vec{\theta} = \left[\frac{C(t)}{2R_2} \right] \delta x$$



$$\left(\frac{C_0}{2R_2} \right) \cos \Omega t$$

$$\underbrace{F_0^*}_{25.9 \; [N]} \cos \Omega t$$

EQ DI MOTO

$$m^* \ddot{x} + r^* \dot{x} + K^* x = F_0^* \cos \Omega t$$

DOMANDA 2

PULSAZIONE SISTEMA NON SMORZATO

$$\omega_0 = \sqrt{\frac{K^*}{m^*}} = 29.2 \; rad/s$$

COEFF. DI SMORZAMENTO h (OPPURE ξ)

$$h = \frac{r^*}{2 m^* \omega_0} = 0.007 < 1$$

DOMANDA 3 : INTEGRALE PARTICOLARE

$$m^* \ddot{x} + r^* \dot{x} + K^* x = F_0^* \cos \Omega t$$

(NON SCRIVO PIU' GLI ASTERISCHI PER SNELLEZZA DI NOTAZIONE

$$m \ddot{x} + r \dot{x} + K x = F_0 \cos \Omega t = F_0 e^{i \Omega t}$$

$$x = X_0 e^{i\Omega t}, \quad \dot{x} = (i\Omega) X_0 e^{i\Omega t}, \quad \ddot{x} = -\Omega^2 e^{i\Omega t}$$

$$(-m\Omega^2 + i \Omega r + K) X_0 \cancel{e^{i\Omega t}} = F_0 \cancel{e^{i\Omega t}}$$

$$X_0 = \frac{F_0}{-m\Omega^2 + i\Omega\eta + K} = 3.638\,E\text{-}3 - 2.1\,E\text{-}5\, i$$

$$X_0 = |X_0|\, e^{i\varphi} \qquad |X_0| = 3.638\,E\text{-}3 \quad [m]$$

$$\varphi = \operatorname{atan}\left(\frac{I_m}{R_e}\right) = -5.77\,E\text{-}3 \quad [rad]$$

$$x = X_0\, e^{i\Omega T} = |X_0|\, e^{i\varphi}\, e^{i\Omega T} = |X_0|\, e^{i(\Omega T + \varphi)}$$

$$\stackrel{Re}{=} \underbrace{|X_0|} \cos(\Omega T + \underbrace{\varphi})$$