



POLITECNICO
MILANO 1863



Production 4.0 – Teaching Lab at DMecc Advanced Manufacturing Processes

Lab 1 – Revision 1

Scheduling of laboratory activities

Lab 0 – Guidelines

- Introduction to lab Production 4.0
- Presentation of lab set up
- Presentation of case study

Today

Lab 1 – Revision 1

- Moving heat source theory
- Implementation of thermal model in MATLAB
- Experimental data provided for efficiency calibration

27th September

28th September

Deadline for group registration

Lab 2 – Revision 2

- Revision of MATLAB code
- Definition of process parameters
- Gcode generation for testing optimised process parameters

25th October

Lab 3 – Revision 3

- Revision of manufactured workpieces
- Critical project considerations and comments

3rd December

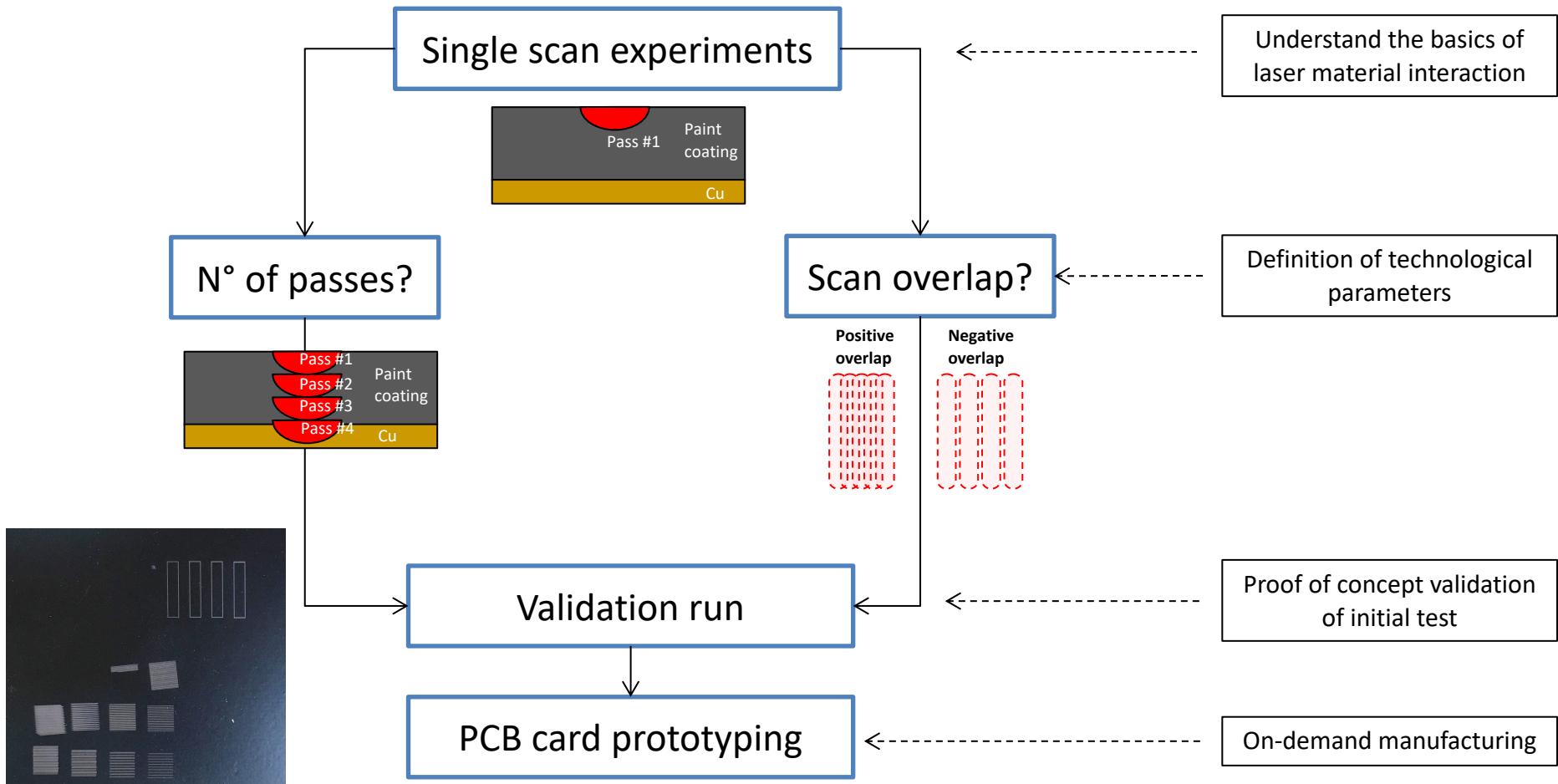
31st December

Forms hand-in for lab work evaluation

On-demand manufacturing of custom PCB boards

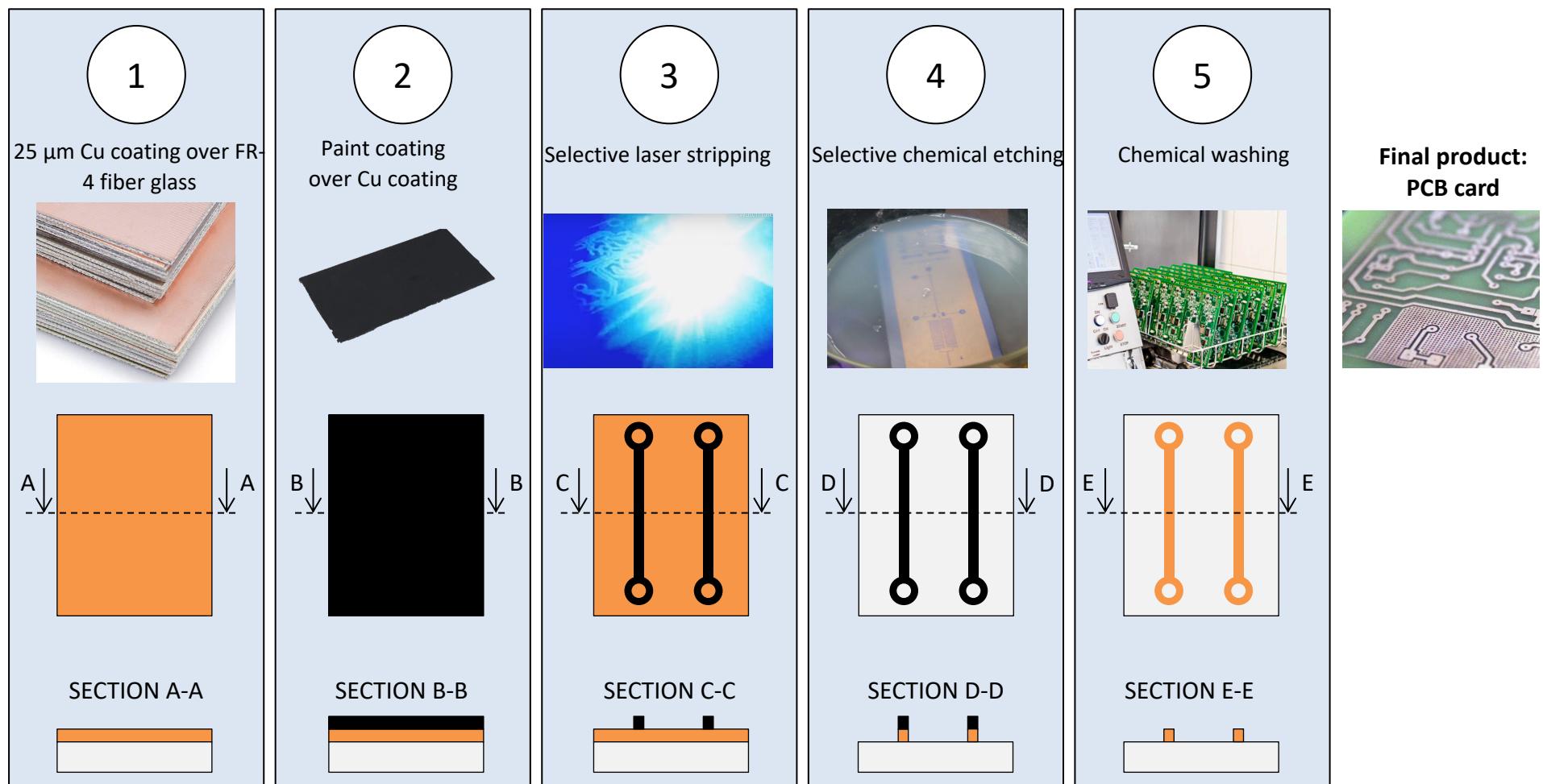
Aim of the project:

Design the manufacturing process for custom PCB board production



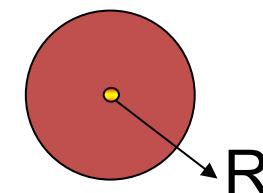
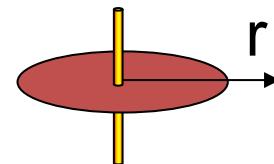
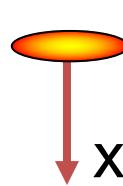
On-demand manufacturing of custom PCB boards

PCB prototyping process



Common analytical solutions of thermal models

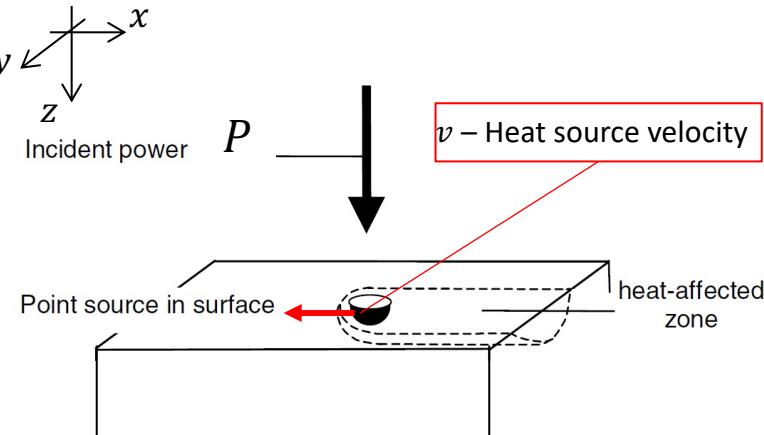
Source type – Temperature gradient			
Beam movement	Planar source Only in thickness x	Line source Normal to the beam plane – Material surface x and y	Point source Every direction x, y and, z
Motionless beam	Time dependant $T(x,t)$	Time dependant $T(x,y,t)$	Time dependant $T(x,y,z,t)$
Moving beam	Quasi-stationary $T(x,v)$	Quasi-stationary $T(x,y)$	Quasi-stationary $T(x,y,z)$



We will develop simple and opportunistic models for estimating process behaviour

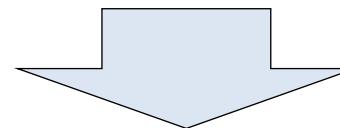
Thermal model – Problem formulation

Point heat source model



Dowden, The mathematics of thermal modeling

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{Heat equation}$$



$$T(x, y, z, t) = ?$$

Solution workflow

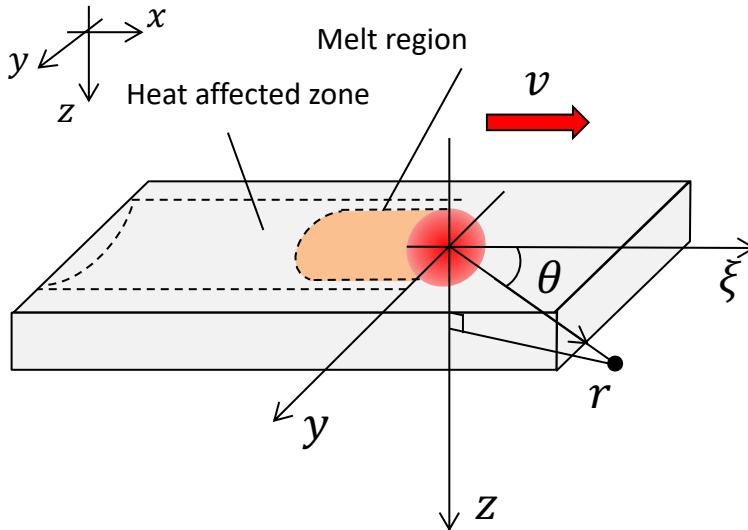
- Define process conditions
- Determine hypothesis
- Impose boundary conditions
- Solve mathematical problem

Main PARAMETERS

- κ – Thermal conductivity (W/mK)
- α – Thermal diffusivity (m^2/s)
- v – Heat source velocity (m/s)
- P – Incident power (W)

Thermal model – Hypothesis & BCs

Point heat source model - Hypothesis

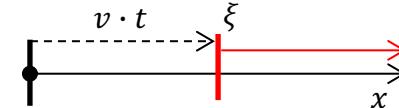


Relative coordinates

Observer moving with heat source at speed v

→ Stationary problem

$$\xi = x - v \cdot t$$



No convection or radiation heat transfer

Steady state conditions

Constant ambient temperature

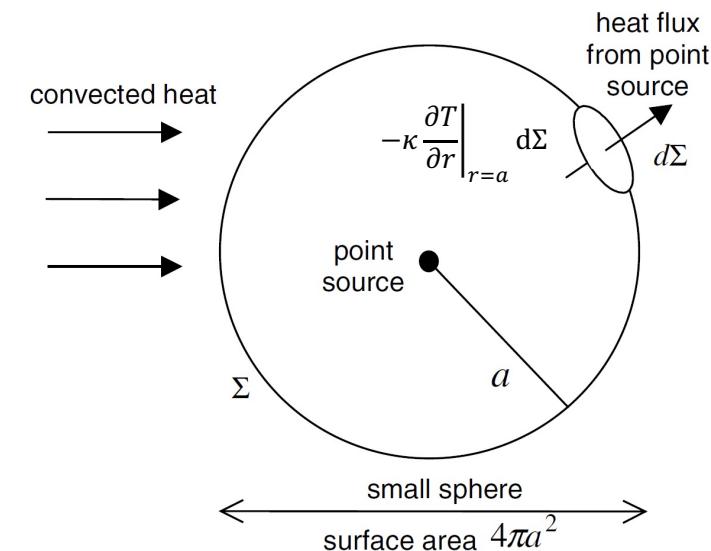
Homogeneous & isotropic material

Constant thermal properties

Point source boundary conditions

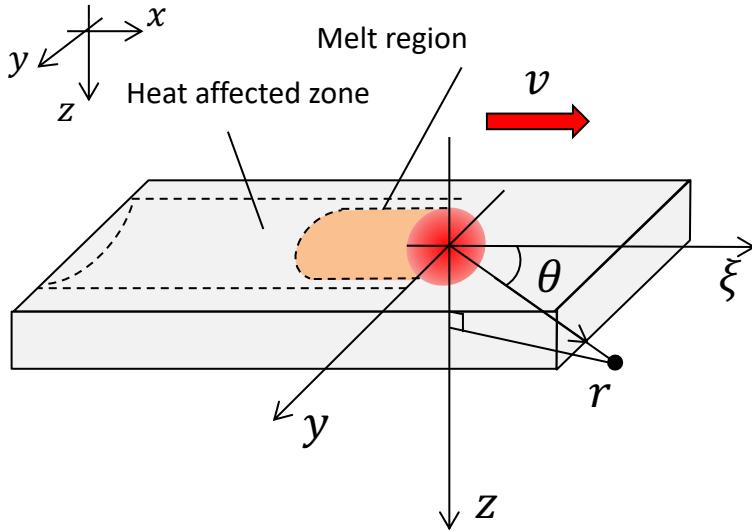
→ Assume point source is at the interior of material

→ All power input flows through an infinitesimally small sphere of radius a



Thermal model – Solution

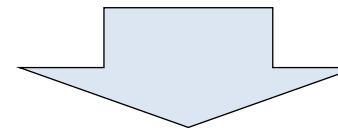
Point heat source model



$$T(x, y, z, t) = ?$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Heat equation



$$T(\xi, y, z) = T_0 + \eta \cdot \frac{P}{2\pi\kappa r} e^{-\frac{v}{2\alpha}(\xi+r)}$$

$$T(\xi, y, z) = T_0 + \eta \cdot f \text{ (Process parameters, material properties)}$$

$$\alpha = \frac{\kappa}{\rho c_p}$$

$$r = \sqrt{\xi^2 + y^2 + z^2}$$

η – Process efficiency

κ – Thermal conductivity (W/mK)

α – Thermal diffusivity (m^2/s)

c_p – Heat capacity (J/kgK)

x – Spatial x-coordinate (m)

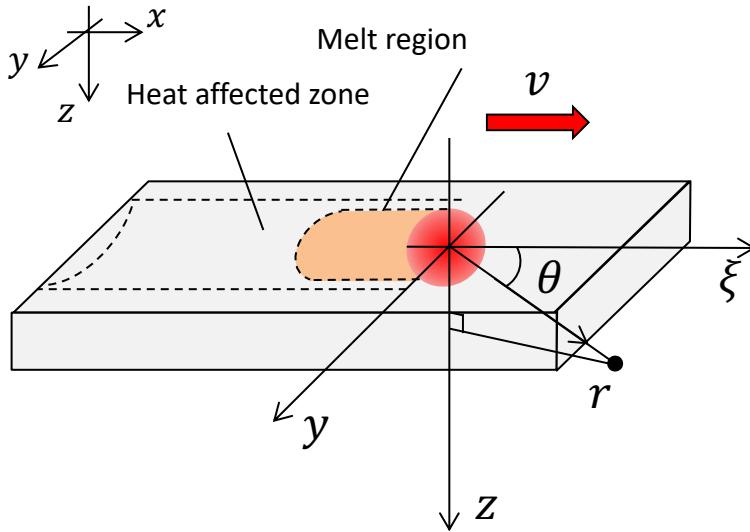
r – Radius from heat source origin (m)

v – velocity of heat source (m/s)

T_0 – Ambient temperature

Thermal model – Solution

Point heat source model



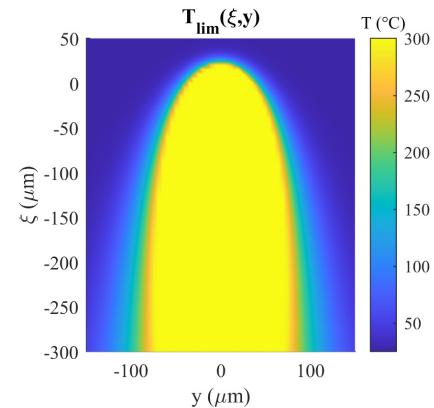
$$T(\xi, y, z) = T_0 + \eta \cdot \frac{P}{2\pi\kappa r} e^{-\frac{v}{2\alpha}(\xi+r)}$$

$$r = \sqrt{\xi^2 + y^2 + z^2}$$

What does the process efficiency coefficient η represent?

If calibrated allows to take into account modelling inaccuracies:

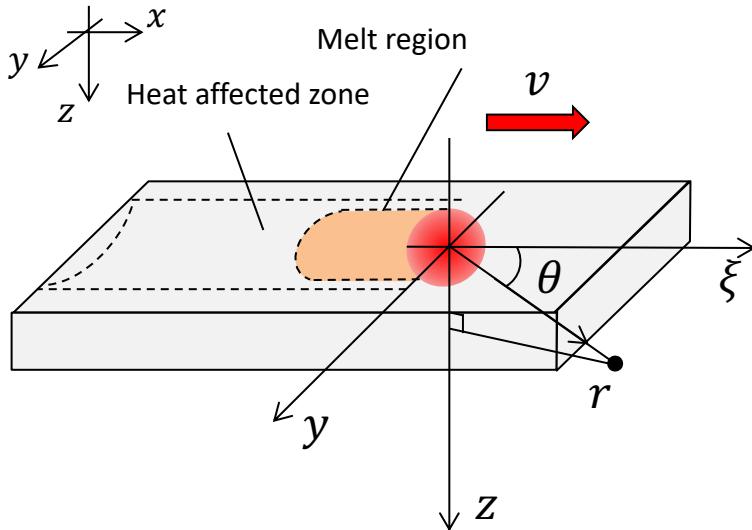
- Reflection of the incoming light
- Fluid dynamic aspects
- Other means of heat transfer
- Effect of other process parameters



“All models are wrong, but some are useful”
George E.P. Box (famous statistician)

Thermal model – Modified Solution

Point heat source model



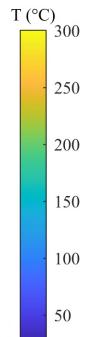
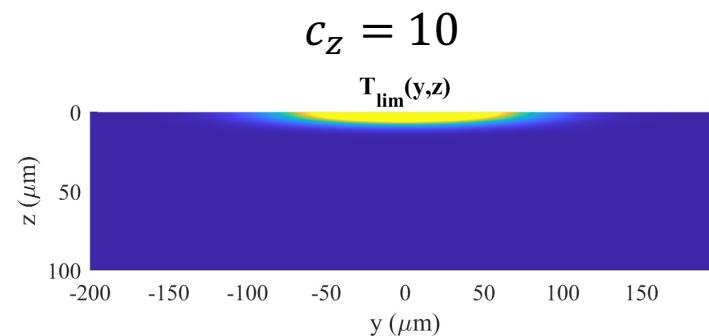
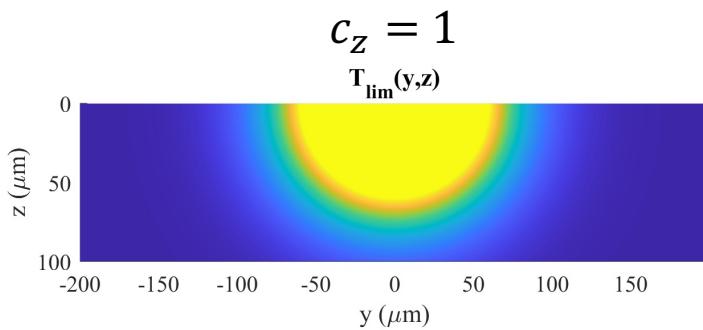
Modified solution

$$T(\xi, y, z) = T_0 + \eta \cdot \frac{P}{2\pi\kappa r} e^{-\frac{v}{2\alpha}(\xi+r)}$$

$$r = \sqrt{\xi^2 + y^2 + (c_z \cdot z)^2}$$

c_z - Geometrical correction factor

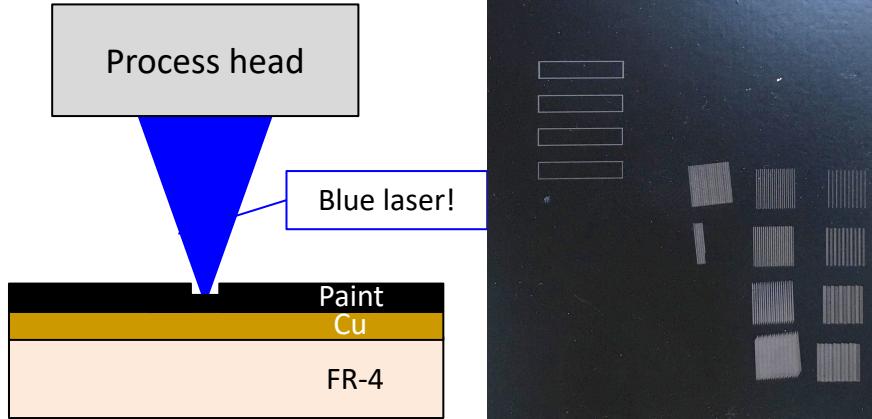
Empirical coefficient which indicates different heat diffusion in the workpiece



From past experiments, in the case of paint stripping $c_z = 10$

Laboratory 01 - Tasks

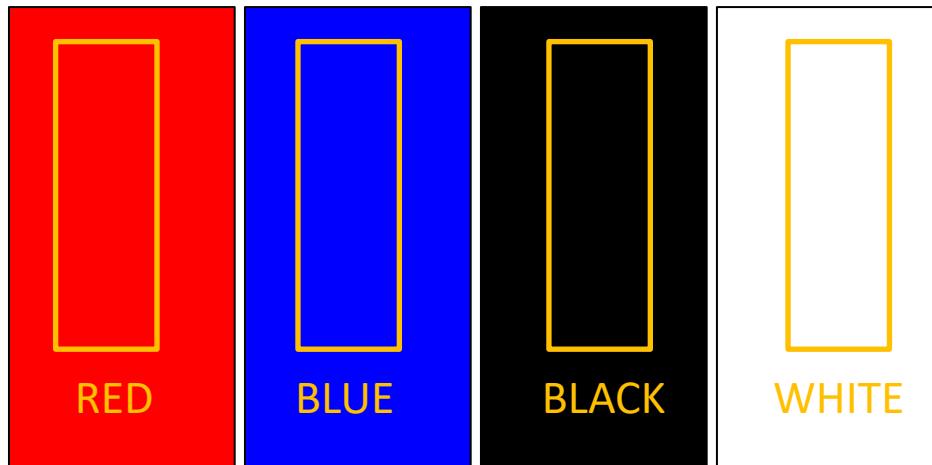
Process



Experimental design:

Fixed factors	
Power, P (mW)	1600
Emission wavelength, λ (nm)	445
Focus position, f (mm)	0
N° of passes	1
Feed rate, v (mm/min)	800
Variable factors	
Paint coating colour	RED – BLUE – BLACK - WHITE

Single tracks cannot be drawn on Snapmaker Luban
→ Rectangles with only outer outline



Characterization:

Optical microscope measurement of track width

Aim of the work:

Calculate temperature distribution with $\eta=0.3$

Estimate track width with $\eta=0.3$

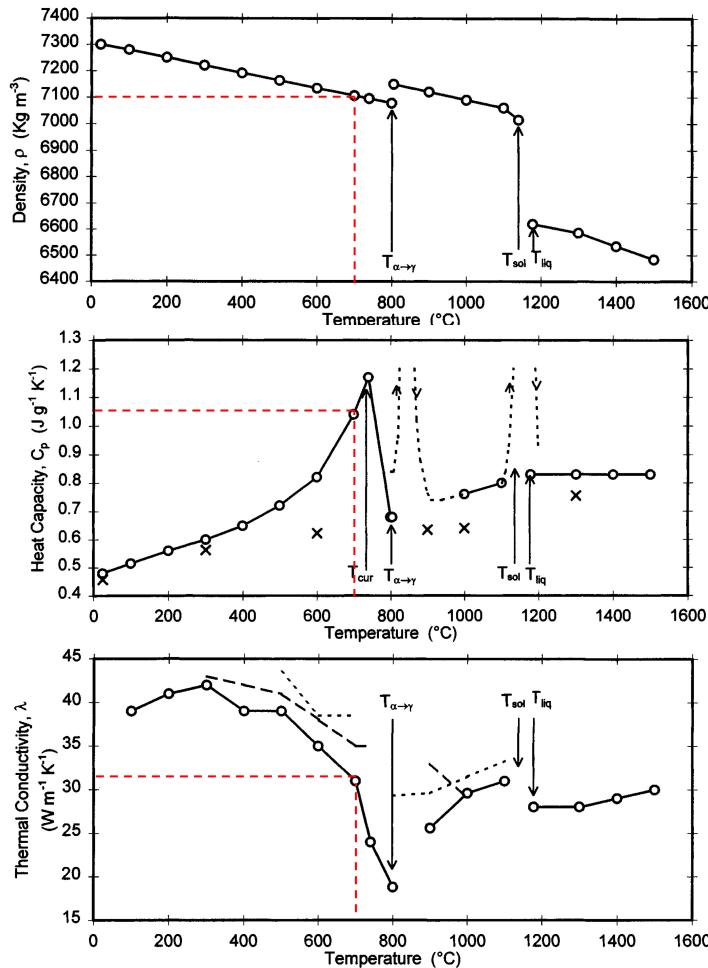
Calibrate efficiency using experimental data

Estimate penetration depth → n° of passes required?

Define hatch distance to ensure etching of large areas

Thermal model – Example of material properties

Material properties of Ductile iron



Material properties are temperature dependent

We take the material properties at 700 °C

Parameter	Symbol	Value	Units
Density	ρ	7100	kg m^{-3}
Specific heat capacity	c_p	1000	$\text{J kg}^{-1} \text{K}^{-1}$
Thermal conductivity	k	32	$\text{W m}^{-1} \text{K}^{-1}$
Solidus temperature	$T_{\text{vaporization}}$	1140	°C
Liquidus temperature	T_{liquidus}	1235	°C
Austenite formation T	$T_{\text{austenitization}}$	805	°C

Mills, K. C. (2002). Recommended values of thermophysical properties for selected commercial alloys. Woodhead Publishing.

Thermal model – Material properties of paint

Material properties of paint coating

Paint coating applied:

Resin-based paint

Reference from literature



Optik - International Journal for Light and Electron Optics 174 (2018) 46–55



Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.com/locate/ijleo



Original research article

Removal of paint layer by layer using a 20 kHz 140 ns quasi-continuous wave laser



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Xinyang Li^{a,*}

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^b Laboratory of All-Solid-State Light Source, Institute of Semiconductors, Chinese Academy of Sciences, Beijing 100083, People's Republic of China

^c University of Chinese Academy of Sciences, Beijing 100049, People's Republic of China

Coating thickness: 20 µm

Parameter	Symbol	Value	Units
Density	ρ	1450	kg m^{-3}
Specific heat capacity	c_p	2500	$\text{J kg}^{-1} \text{K}^{-1}$
Thermal conductivity	k	0.3	$\text{W m}^{-1} \text{K}^{-1}$
Vaporization temperature	T_{vap}	300	°C
Absorption (at $\lambda=1064$ nm)	A	0.3	-

Thermal model – Software

Introduction to MATLAB

- Advanced programming software
- Freely available for Polimi students

How to download Matlab:

[Mathworks Matlab – Servizio software di Ateneo \(polimi.it\)](#)

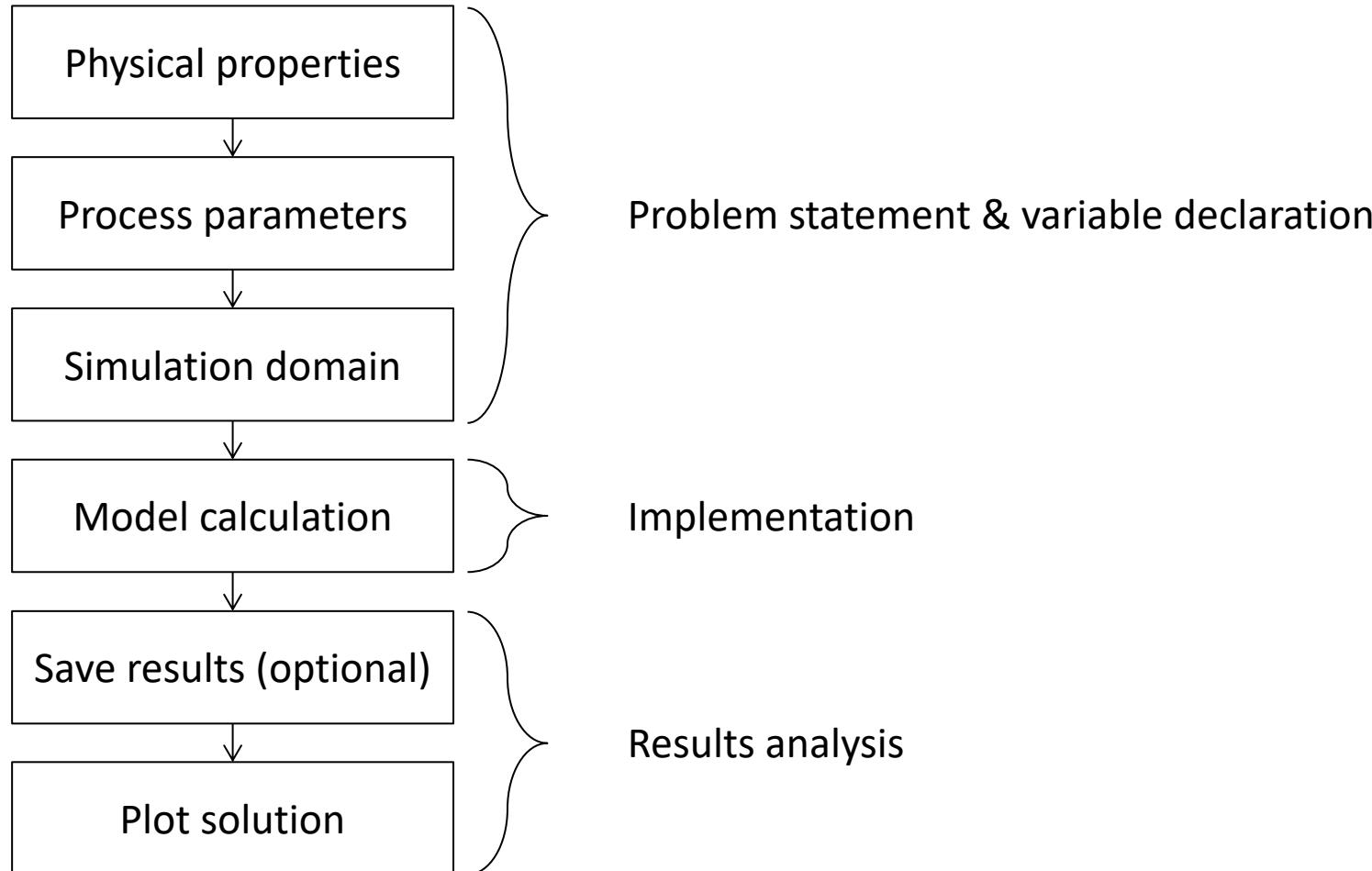
For help/suggestions use Matlab documentation

<https://it.mathworks.com/help/matlab/>



Thermal model – Implementation

Example of coding structure in Matlab

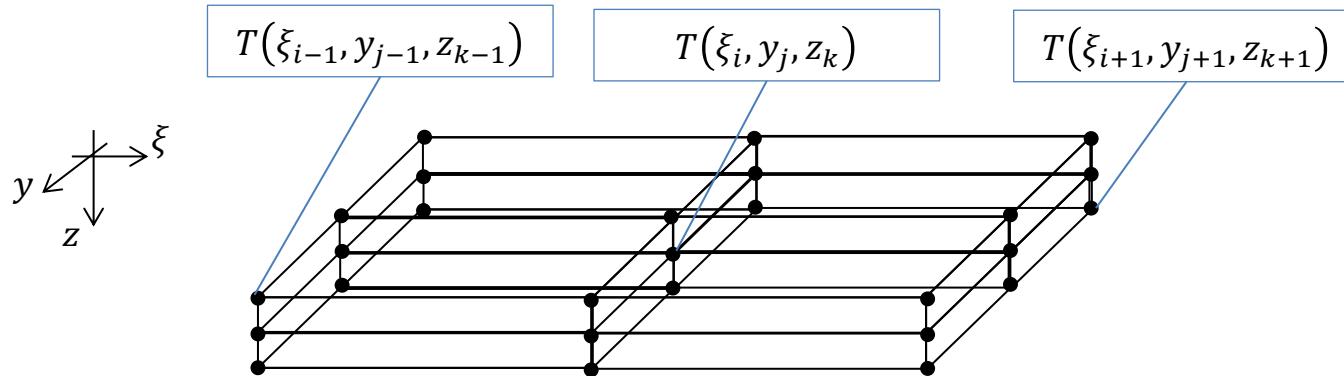


Thermal model – Implementation

How to implement analytical model in a numerical code?

We need to calculate the temperature in different positions of our domain

$$T(\xi_i, y_j, z_k) \forall i, j, k$$



→ I need to initialize different vectors which determine all the positions in the domain where I want to solve the equation

Thermal model – Implementation

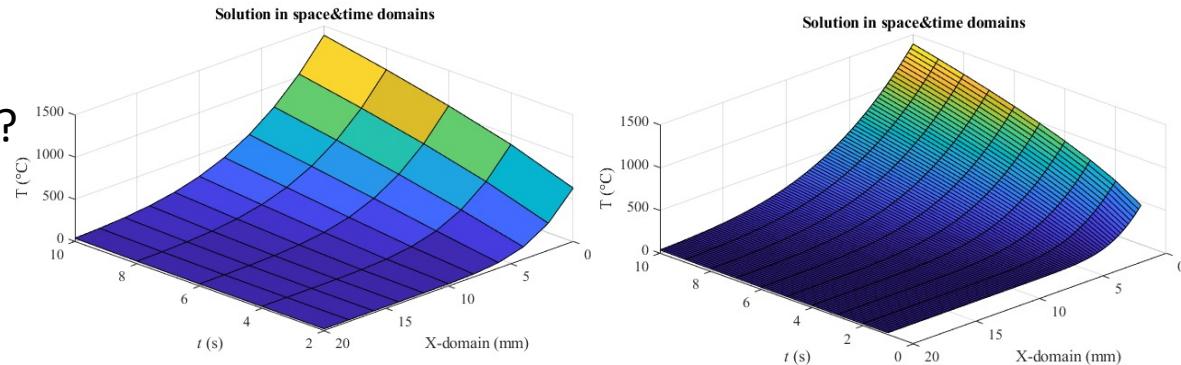
2 main variables

- Extent of domain
- Resolution of domain

Appropriate choice required → Computational efficiency vs accuracy

What is the expected spatial extent?

What is the resolution required?



- From experience we can expect spatial domain in the order of 1 mm.
- Resolution usually 2 orders of magnitude smaller.
- Useful to start with fast simulation then increase computational load.
- Restrict domain to required needs (i.e. do I need to calculate for all dimensions?)

Example

Fixed simulation domain

$$\xi \in [-1000, 50]$$

$$y \in [-200, 200]$$

$$z \in [0, 20]$$

Case 1

$$\Delta\xi, \Delta y, \Delta z = 10\mu m$$

$$t_{calc} = 1.497219 \text{ s}$$

Case 2

$$\Delta\xi, \Delta y, \Delta z = 1\mu m$$

$$t_{calc} = 2.436115 \text{ s}$$

Thermal model implementation

Calculation of track width

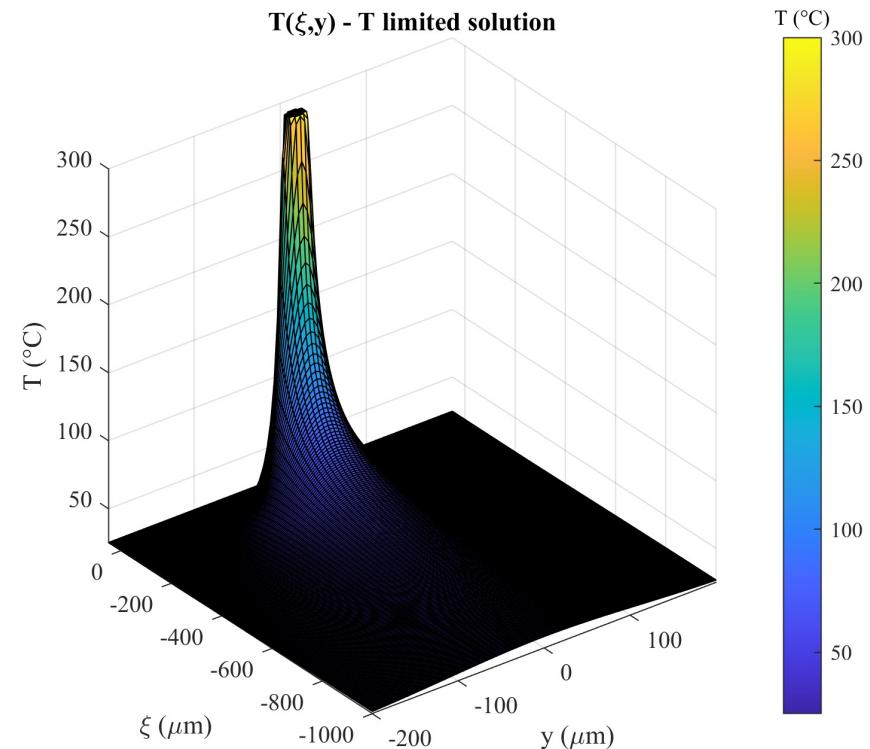
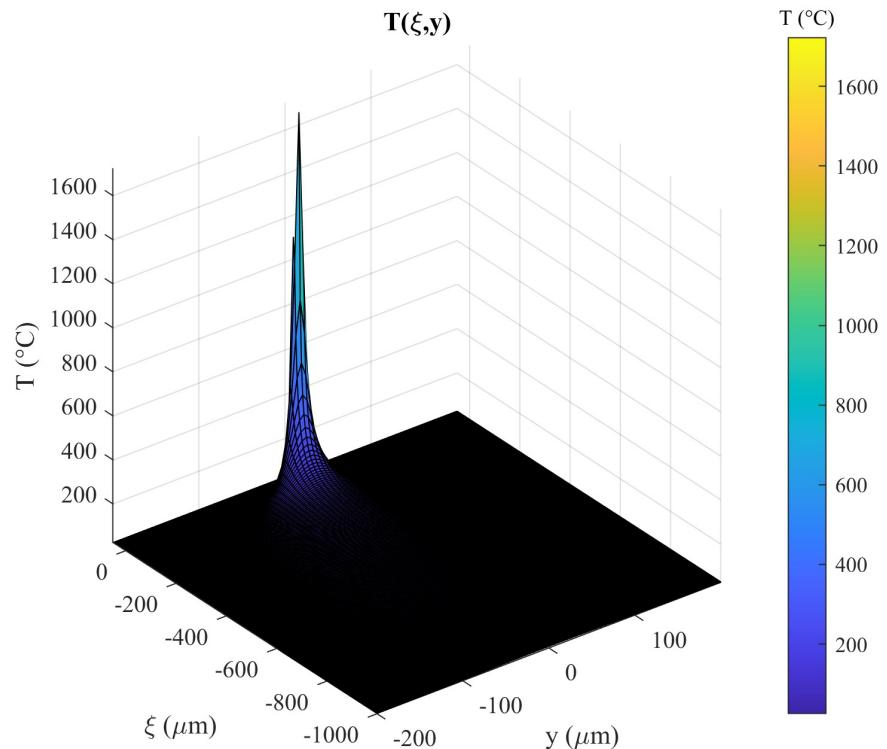
Useful Matlab functions:

for [for loop to repeat specified number of times - MATLAB for - MathWorks Italia](#)

surf [Surface plot - MATLAB surf - MathWorks Italia](#)

Thermal model – Example results

Example of calculated temperature fields

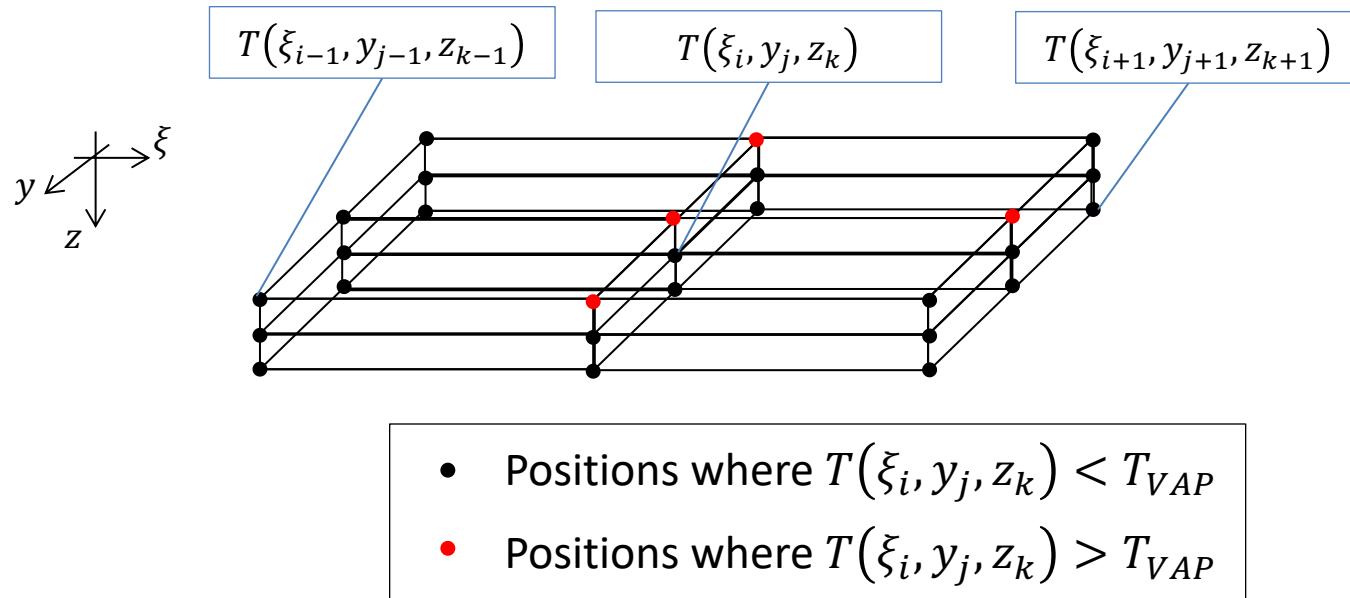


Thermal model implementation

Calculation of track width

From our Temperature matrix we can estimate the vaporized material
 $T(\bar{\xi}, \bar{y}, \bar{z}) \rightarrow w_{VAP}$ where $T(\xi_i, y_j, z_k) > T_{VAP}$

Example:



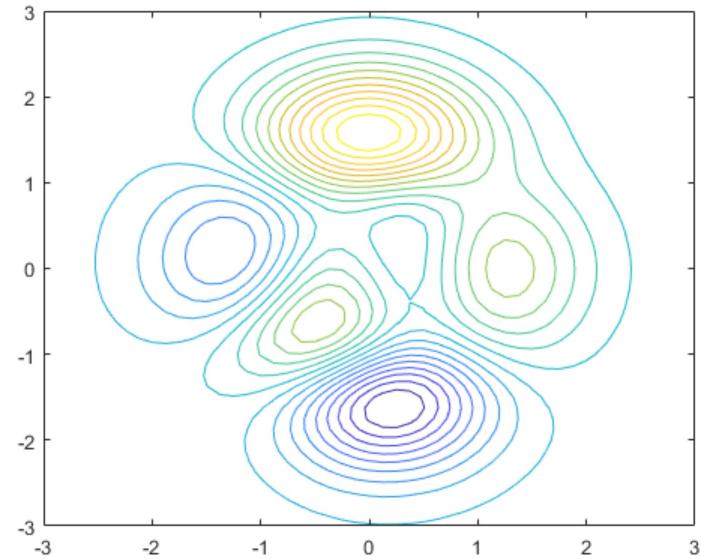
Thermal model implementation

Calculation of track width

Useful Matlab functions:

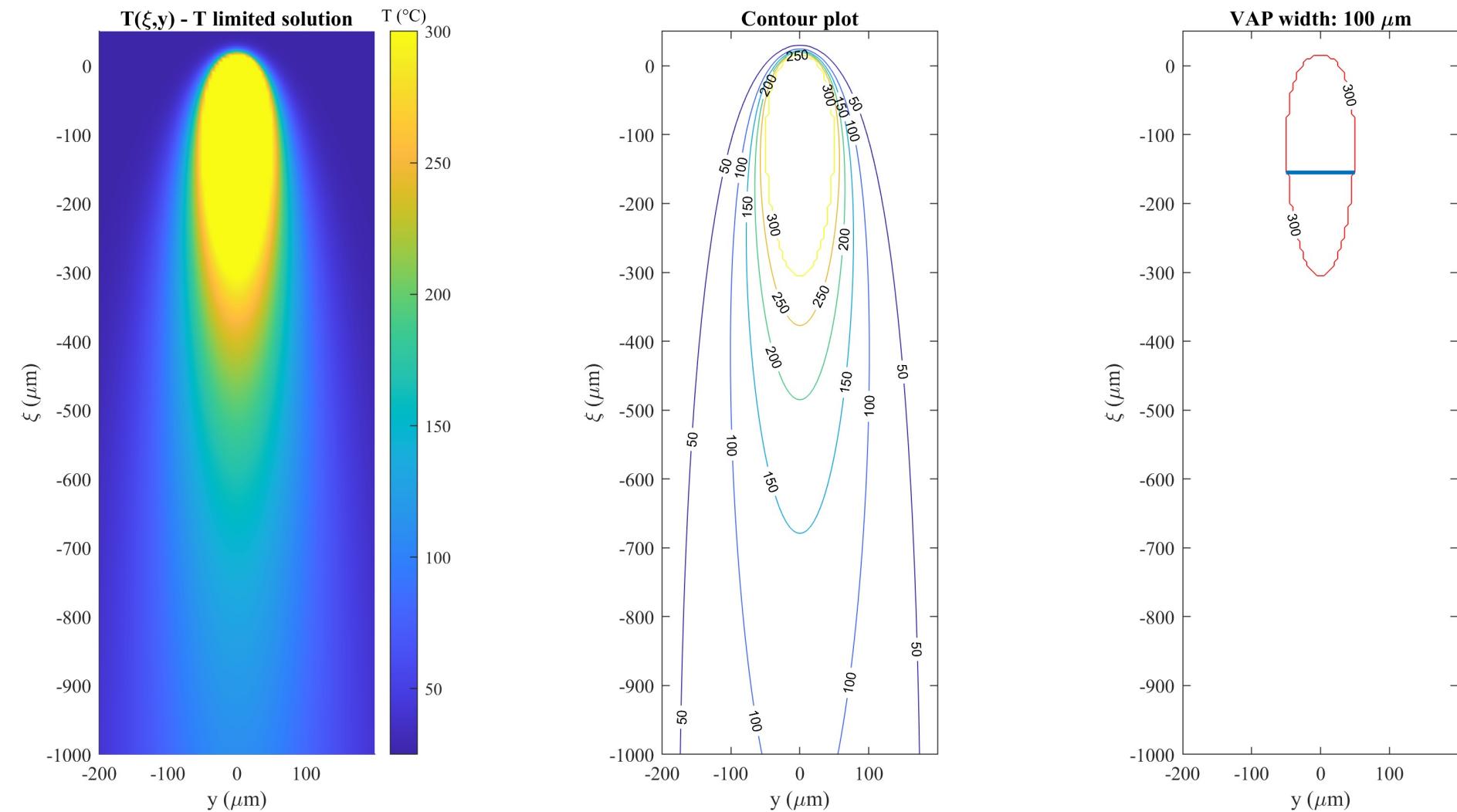
contour [Contour plot of matrix - MATLAB contour - MathWorks Italia](#)

The contour function can also give in output the values of a certain contour value
(for example the vaporisation temperature...)



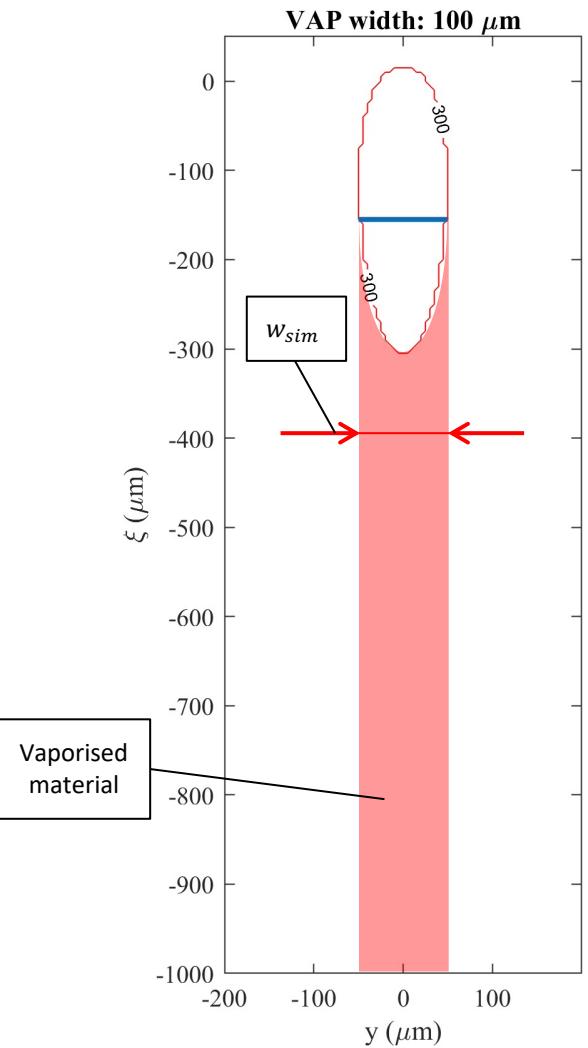
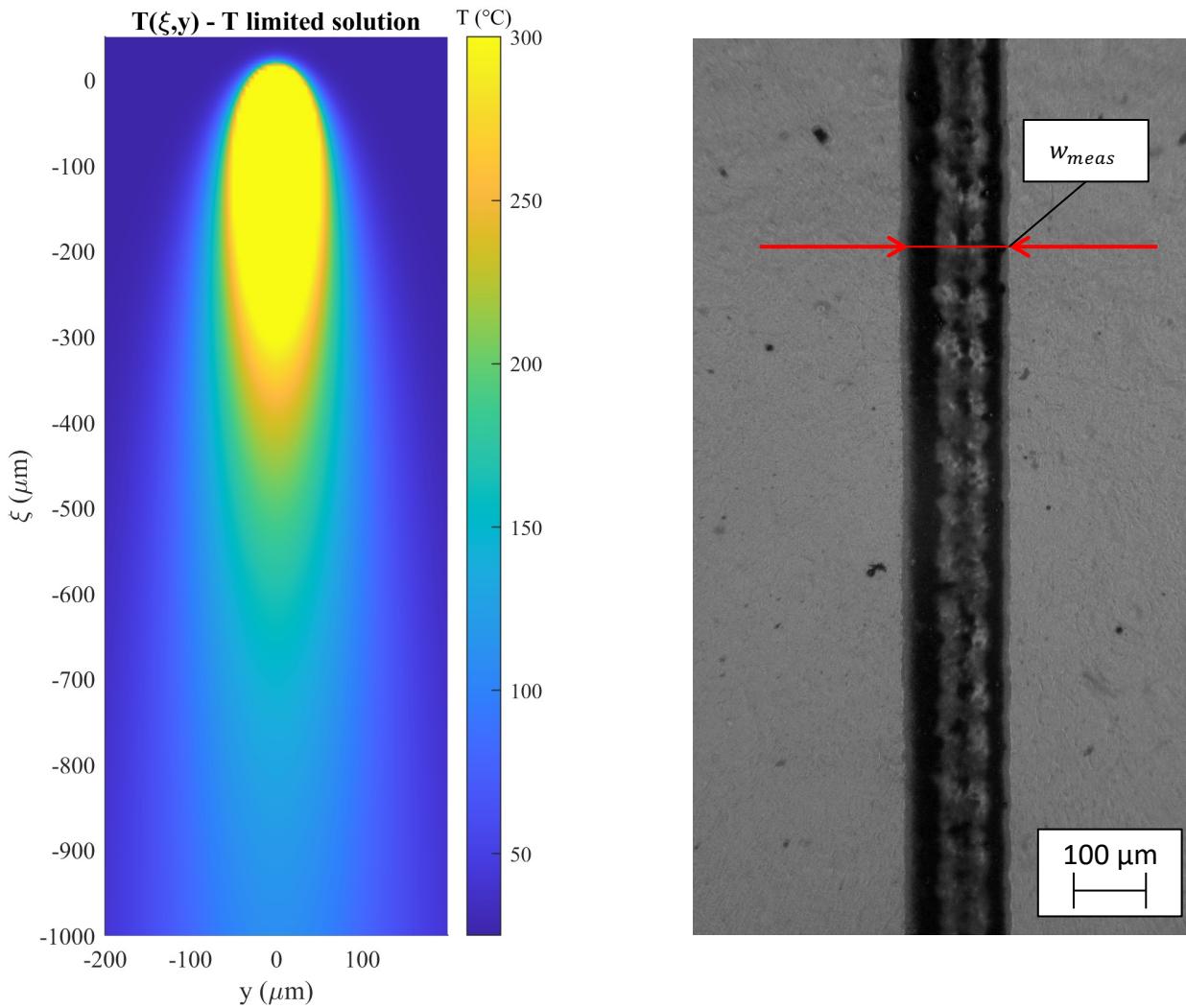
Thermal model – Example results

Calculation of track width



Thermal model – Example results

Calculation of track width vs measured track width



Thermal model implementation

Calibration of process efficiency

It is clear that the value of η we use affects our mathematical solution

→ we can use experimental data to calibrate it!

With the analytical model we can calculate parameters that otherwise we would not be able to measure.

For example:

- Penetration depth 
- Cooling rate $\frac{\partial T}{\partial t}$
- Cooling gradients $\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}, \frac{\partial T}{\partial z}$

Useful for the
laboratory project!

Thermal model implementation

Comparison with laboratory experiments

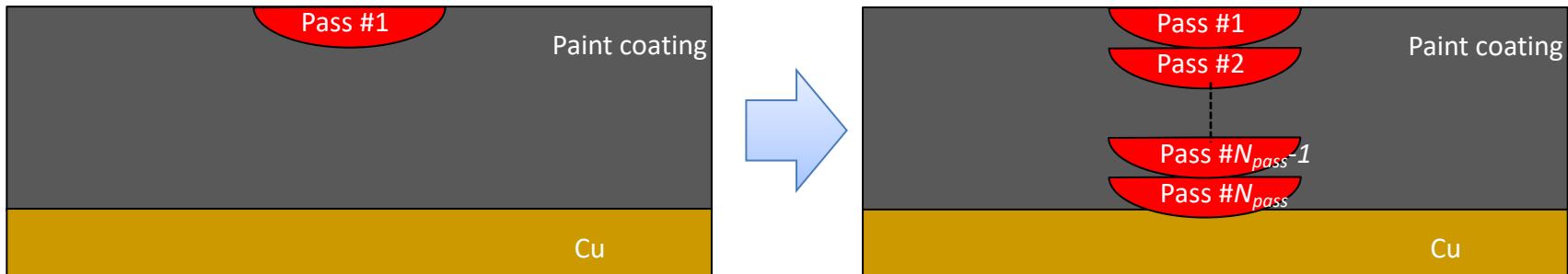
Track width smaller than theoretical value!

$$\rightarrow \eta < 1$$

Minimisation problem to calibrate efficiency coefficient based on experimental data

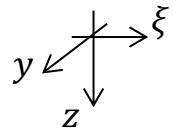
Once the model is calibrated

→ possible to estimate penetration depth & number of passes required



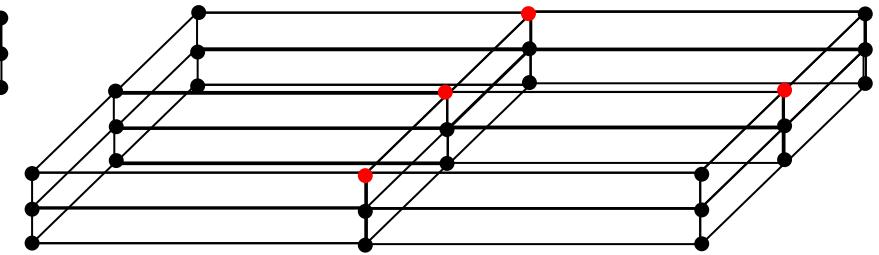
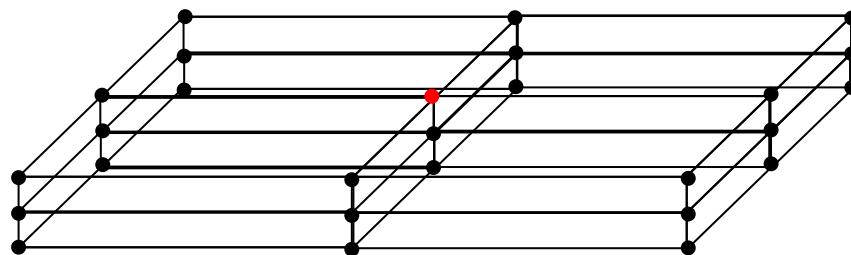
Thermal model implementation

Calibration of process efficiency



$$\eta = 0.1$$

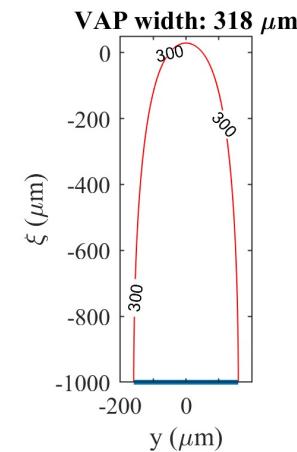
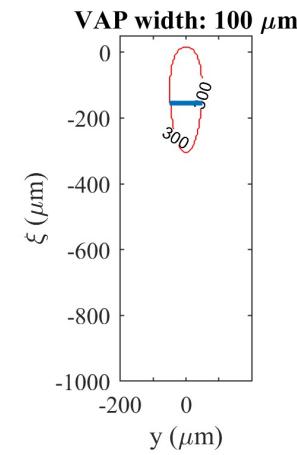
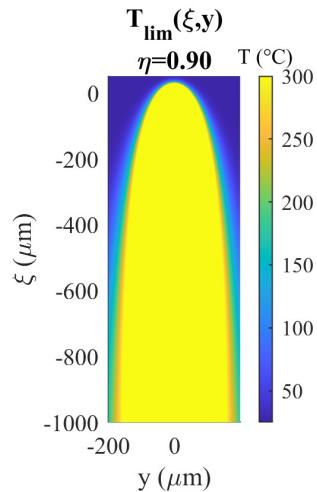
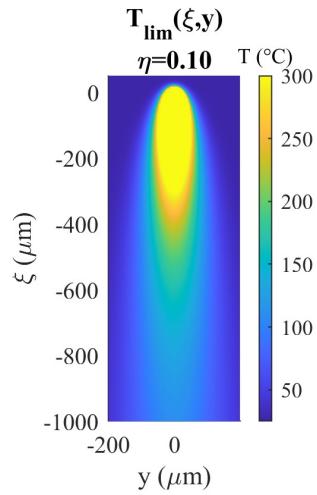
$$\eta = 0.9$$



- Positions where $T(\xi_i, y_j, z_k) < T_{VAP}$
- Positions where $T(\xi_i, y_j, z_k) > T_{VAP}$

Thermal model – Example results

Calibration of process efficiency



Thermal model implementation

Calibration of process efficiency

We need to approximate our solution to our experimental measurements

$$w_{simulation} \neq w_{measured}$$

→ implement an error minimization algorithm

A numerical approach is effective since we do not have a linear relationship between η and $w_{simulation}$

Let's define the relative error:

$$err(\%) = \frac{|w_{simulation} - w_{measured}|}{w_{measured}}$$

We define also our temperature distribution matrix independent of the ambient T:

$$T^*(\bar{x}, \bar{y}, \bar{z}) = T(\bar{x}, \bar{y}, \bar{z}) - T_0$$

Thermal model implementation

Calibration of process efficiency

For increasing values of η calculate the track width:

$\forall i$

$$T_{tmp}(\bar{x}, \bar{y}, \bar{z}) = T^*(\bar{x}, \bar{y}, \bar{z}) \cdot \eta_i + T_0$$

$\rightarrow w_{sim,i}$

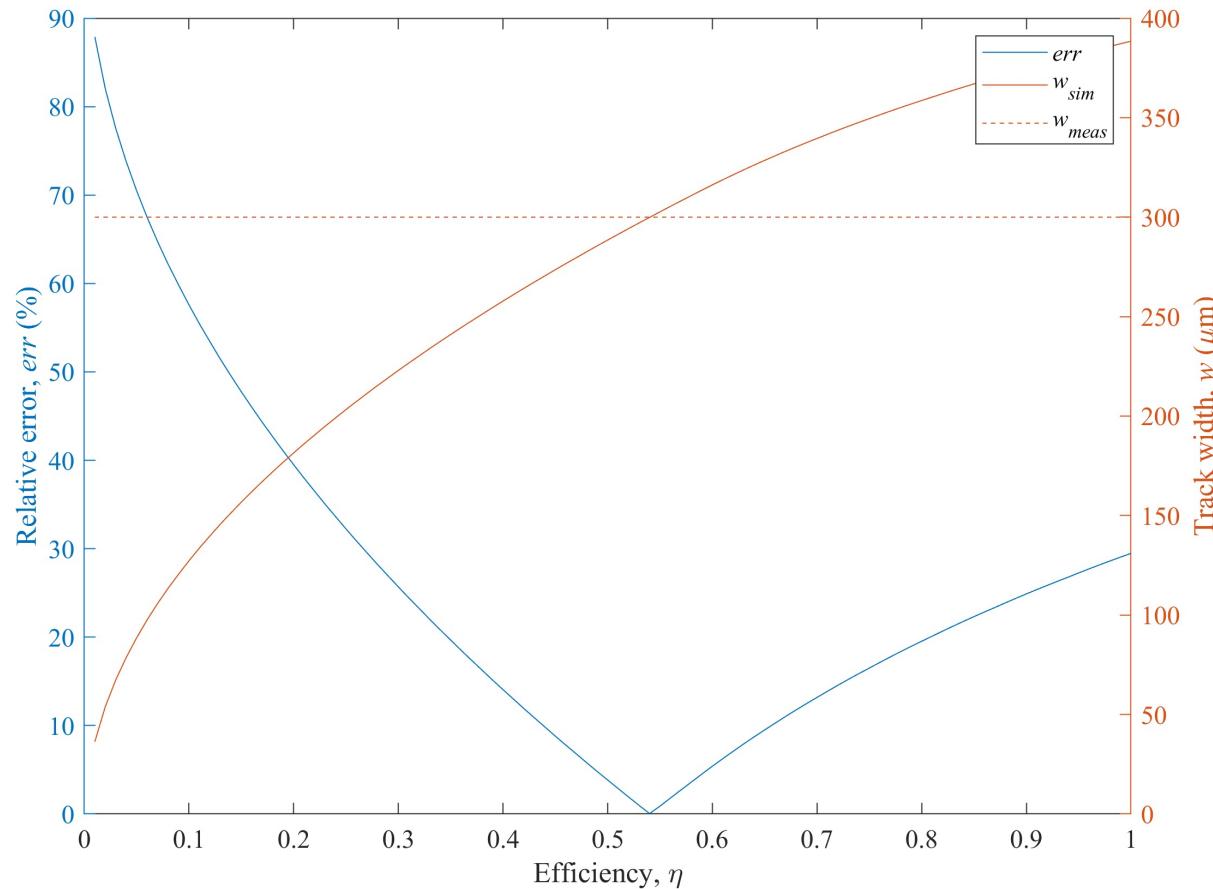
$\rightarrow e_i$

Once obtained the error vector apply an algorithm to numerically identify the minimum (we can even consider \min [Minimum elements of an array - MATLAB](#) [min - MathWorks Italia](#))

Tip: Once obtained the temperature matrix $T^*(\bar{x}, \bar{y}, \bar{z})$ you are not required to calculate all over again the code but you may calculate a temporary temperature matrix $T_{tmp}(\bar{x}, \bar{y}, \bar{z})$

Thermal model – Example results

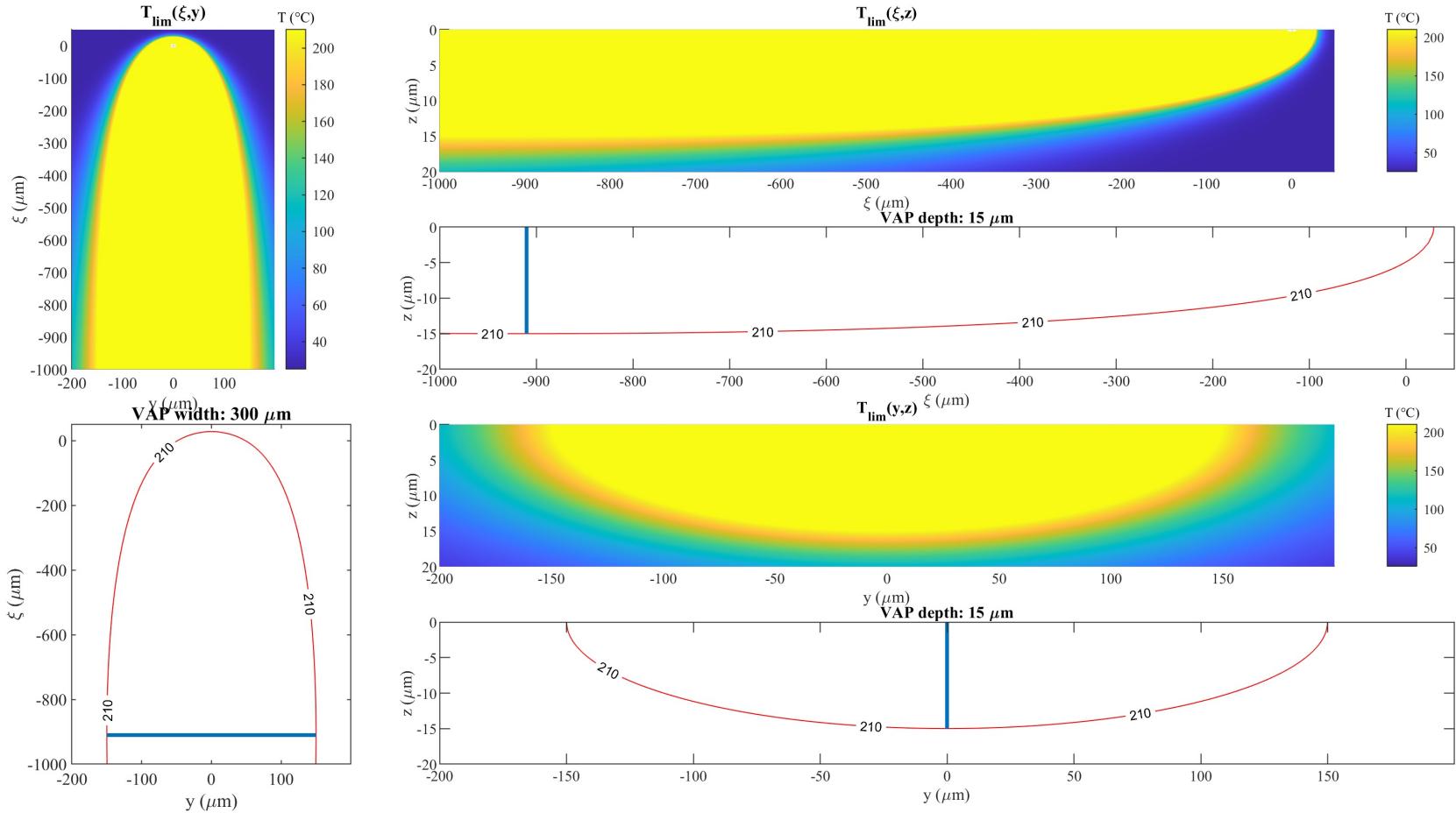
Calibration of process efficiency



→Once η is calibrated we can estimate the penetration depth using the same approach employed for the track width calculation

Thermal model – Example results

Estimation of penetration depth

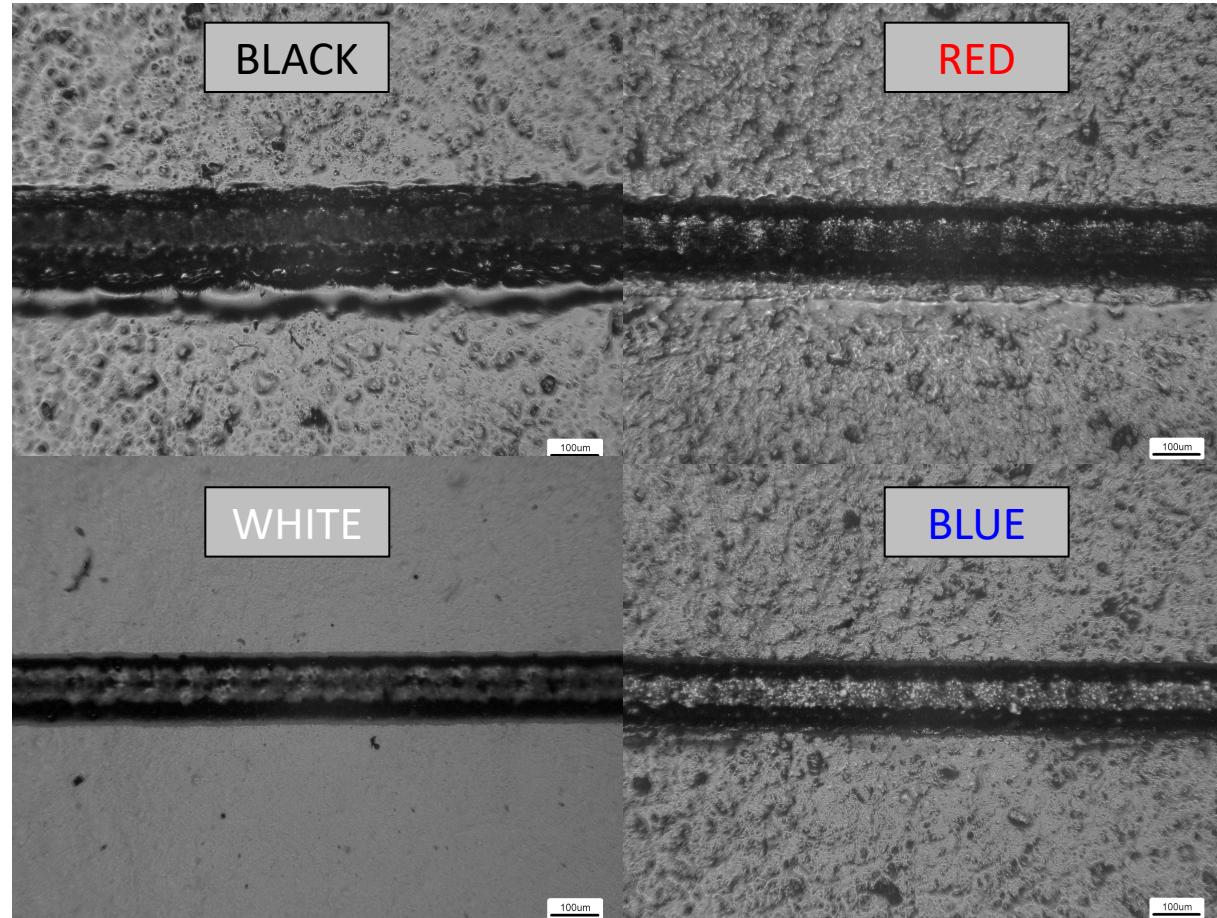


Experimental data

Fixed factors	
Power, P (mW)	1600
Emission wavelength, λ (nm)	445
Focus position, f (mm)	0
N° of passes	1
Feed rate, v (mm/min)	800
Variable factors	
Paint coating colour	RED – BLUE – BLACK - WHITE

Results from single track experiments

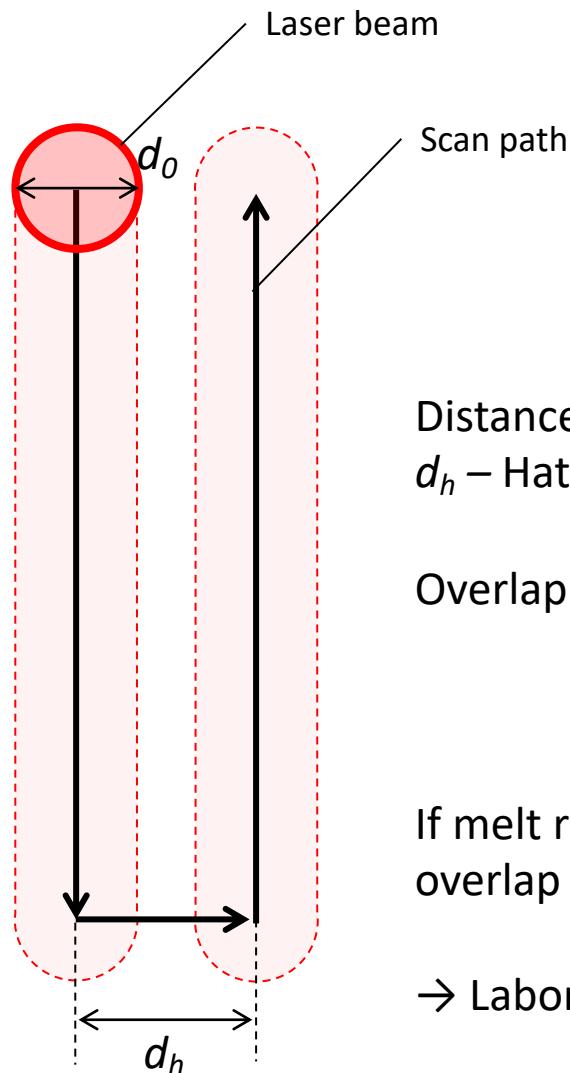
5 track width measurements (w_{meas}) per single track



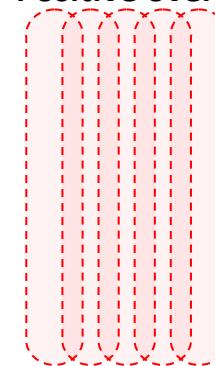
Meas #	Track width, w_{meas} (μm)			
	Black	White	Red	Blue
1	172	133	136	129
2	163	129	144	132
3	177	135	138	124
4	177	135	132	128
5	165	128	137	122

Large area processing

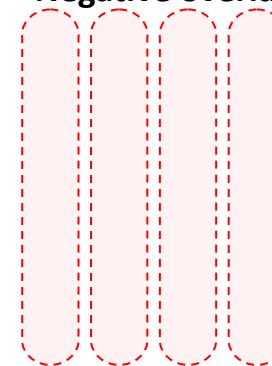
Theoretical concept



Positive overlap



Negative overlap



Distance between adjacent scan tracks:
 d_h – Hatch distance

Overlap O_h

$$O_h(\%) = \frac{d_0 - d_h}{d_0}$$

If melt region > beam waist diameter d_0 we can have negative overlap to process large areas

→ Laboratory experiments required to determine optimal overlap

Contact details

Questions?



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