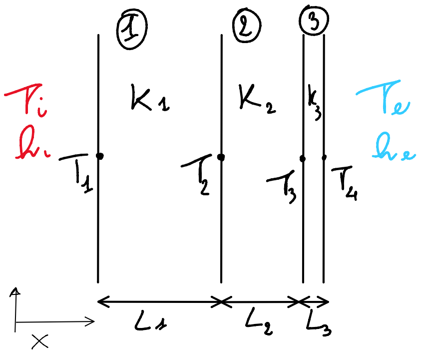
ESE 1

lunedì 8 novembre 2021 16:35



*** DATI *** Cond. stazionaria; monodimensionale

$S = 15\,m^2$ $T_i = 900°C$ $h_i = h_e = 10\ \frac{W}{m^2K}$

Parete ① → $L_1 = 60\ cm$ mattoni refrattari
$K_1 = 3\ W/mK$

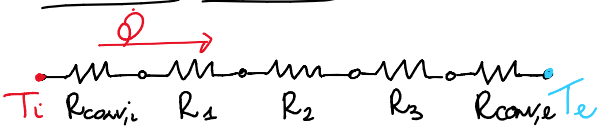
Parete ② → $L_2 = 30\ cm$ isolante
$K_2 = 0{,}1\ W/mK$

Parete ③ → $L_3 = 2\ cm$ acciaio
$K_3 = 20\ W/mK$

*** Richieste ***

$R_{TOT} = ?$ $\dot{Q} = ?$ $T(x) = ?$ $T_{max,2} = ?$

*** Analogia elettrica ***



5 resistenze in serie $R_{TOT} = \sum_J R_J$

Resistenza convettiva $R_{conv} = \frac{1}{hS}$

$R_{conv,i} = 0{,}0067\ K/W$
$R_{conv,e} = 0{,}067\ K/W$

stessa S
stesso R

Resistenza conduttiva $R_{cond} = \frac{L}{KS}$

$R_1 = 0{,}0133\ K/W$
$R_2 = 0{,}2\ K/W$
$R_3 = 6{,}67 \cdot 10^{-7}\ K/W$

$$R_{TOT} = R_{conv,i} + R_1 + R_2 + R_3 + R_{conv,e} = 0{,}2268\ K/W$$

$$\dot{Q} = \frac{\Delta T_{ie}}{R_{TOT}} = \frac{T_i - T_e}{R_{TOT}} = 3880\ W$$

Distribuzione di $T(x)$ (parete; monodimensionale; stazionario; $q_{GEN} = 0 \rightarrow T(x)$ lineare)

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \cancel{\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)} + \cancel{\frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right)} + \cancel{\dot{q}_{GEN}} = \cancel{\rho c \frac{\partial T}{\partial t}}$$ EQ. GENERALE CONDUZIONE

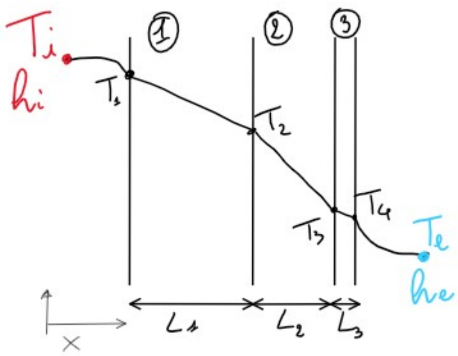
$$\frac{d}{dx}\left(K\frac{dT}{dx}\right) = 0 \longrightarrow K\frac{dT}{dx} = C_1 \xrightarrow{K\,cost} T(x) = \frac{C_1}{K}x + C_2$$ (vedi lezione)

$T(x)$ lineare

Cond. contorno (per determinare C_1, C_2)

es. temperature agli estremi

$$\left.\begin{aligned} x = 0 &\rightarrow T(0) = T_{S,1} \\ x = L &\rightarrow T(L) = T_{S,2} \end{aligned}\right\} \quad T(x) = \frac{T_{S2} - T_{S1}}{L} \cdot x + T_{S1}$$



× CALCOLARE IL ΔT PER OGNI STRATO $\Delta T_i = R_i \dot{\Phi}$ → UGUALE IN OGNI STRATO

es.

$T_i \xrightarrow[R_1]{\dot{\Phi}} T_1$ $\quad T_1 = T_i - R_1 \dot{\Phi}$

• ESSENDO IL PROFILO LINEARE → $T_{MAX,2} = T_2$

$T_1 \xrightarrow[R_2]{\dot{\Phi}} T_2$ $\quad T_2 = T_1 - R_2 \dot{\Phi} = 822{,}38\,°C$

STESSO RISULTATO

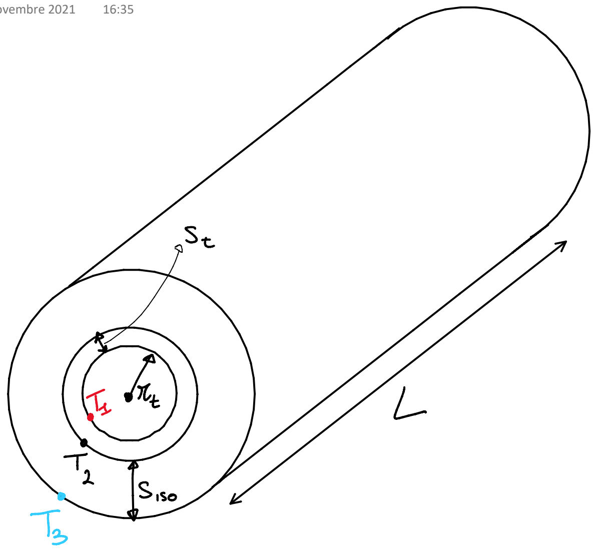
$T_i \xrightarrow{R_{conv,i}} \xrightarrow{R_1} T_2$ $\quad T_2 = T_i - (R_{conv,i} + R_1)\dot{\Phi} = 822{,}38\,°C$

STESSO RISULTATO

$T_2 \xrightarrow{R_2} T_3 \xrightarrow{R_3} T_4 \xrightarrow{R_{conv,e}} T_e$ $\quad T_2 = T_e + \dot{\Phi}\left(R_2 + R_3 + R_{conv,e}\right) = 822{,}38\,°C$

ESE 2

lunedì 8 novembre 2021 16:35



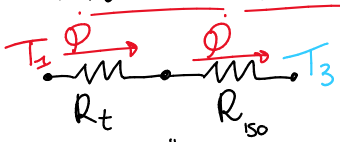
DATI

$r_t = 8\,mm \quad S_t = 2\,mm \quad K_t = 29\,W/mK \quad L = 1\,m$

$S_{iso} = 10\,mm \quad K_{iso} = 0{,}116\,W/mK$

$T_1 = 80°C \quad T_2 = 10°C \quad \dot{Q} = ? \quad T_2 = ?$

* ANALOGIA ELETTRICA *



2 RESISTENZE CONDUTTIVE IN SERIE $R_{TOT} = R_1 + R_2$

es. $\dot{Q} = \dfrac{T_1 - T_2}{R_1 + R_2} \Longrightarrow T_2 = T_1 - \dot{Q}(R_1 + R_2)$

- GEOMETRIA CILINDRICA $R_{COND} = \dfrac{\ln \frac{r_e}{r_i}}{2\pi K L}$ (RESISTENZA CONDUTTIVA; FLUSSO TERMICO IN DIREZIONE RADIALE)

$R_{cond,t} = \dfrac{\ln\left[\dfrac{(r_t + s_t)}{r_t}\right]}{2 K_t L} = 0{,}001225\,K/W$

$R_{cond,iso} = \dfrac{\ln\left[\dfrac{(r_t + S_t + S_{iso})}{r_t + S_t}\right]}{2 K_{iso} L} = 0{,}9510\,K/W$

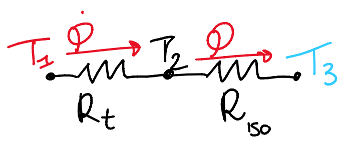
$R_{TOT} = 0{,}9552\,K/W$

$\dot{Q} = \dfrac{(T_1 - T_3)}{R_{TOT}} = 73{,}5\,W$ (POTENZA TERMICA)

$T_1 - T_2 = R_{cond,t} \cdot \dot{Q} \Longrightarrow T_2 = T_1 - R_{cond,t} \cdot \dot{Q} = 79{,}9°C$

ANALOGO

$T_2 = T_3 + R_{cond,iso} \cdot \dot{Q} = 79{,}9°C$



ESE 3

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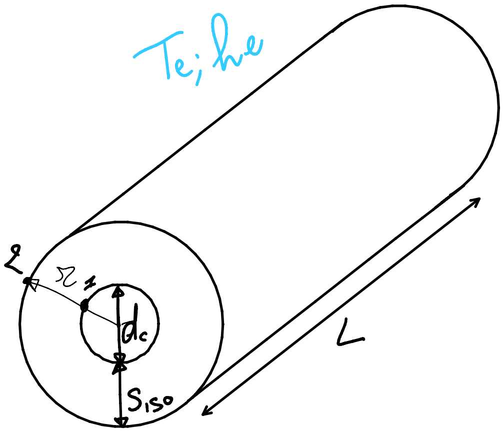


* DATI * Cond. Stazionarie

$L = 5\,m$; $d_c = 3\,mm$; $S_{iso} = 2\,mm$

$K_c = 390\,W/mK$ (Rame); $K_{iso} = 0{,}15\,\frac{W}{mK}$

$\dot{q}_{DISS} = 400\,W/m^2$ $T_e = 30°C$ (aria)

$h_e = 12\,W/m^2K$ $T_1 = ?$ $T_{max,c}$?

Potenza Termica Dissipata dal Conduttore

$$\dot{Q} = \dot{q}_{DISS}\left[\frac{W}{m^2}\right] \cdot \underbrace{(\pi d_c L)}_{\text{SUP. LATERALE CONDUTTORE}}[m^2] = 18{,}85\,W$$

• Considero Spessore Isolante (Senza Generazione) + Convezione Esterna

$$T_1 \xrightarrow{\dot{Q}} R_{iso} \; T_2 \xrightarrow{\dot{Q}} R_{conv,e} \; T_e$$

(Analogia Elettrica)

$$R_{iso} = \frac{\ln(r_2/r_1)}{2\pi K_{iso} L} = 0{,}1788\,K/W \qquad R_{conv,e} = \frac{1}{h_e S_e} = \frac{1}{h_e 2\pi r_{EST}} = 0{,}7578\,K/W$$

$$R_{TOT} = R_{iso} + R_{conv,e} = 0{,}9377\,K/W$$

$$\dot{Q} = \frac{(T_1 - T_e)}{R_{TOT}} \Longrightarrow T_1 = T_e + \dot{Q} \cdot R_{TOT} = 47{,}67\,°C$$

• Calcolo Tmax all'interno del Conduttore

↳ Necessario determinare $T(r)$ ⟶ Geometria Cilindrica

$$\frac{1}{r}\frac{\partial}{\partial r}\left(Kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(K\frac{\partial T}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\frac{\partial T}{\partial t}$$

Eq. Generale Conduzione in Coordinate Cilindriche

- REGIME STAZIONARIO $\frac{\partial T}{\partial t} = 0$
- MONODIMENSIONALE $\frac{\partial T}{\partial z} = \frac{\partial T}{\partial \varphi} = 0$

$$\frac{1}{r}\frac{d}{dr}\left(K r \frac{dT}{dr}\right) = -\dot{q}$$

↳ GENERAZIONE INTERNA $\left[\frac{W}{m^3}\right]$

$$\dot{q} = \frac{\dot{\Phi}}{V} = \frac{\dot{\Phi}}{\pi r_c^2 L} = 533346 \ \frac{W}{m^3}$$

$$\frac{d}{dr}\left(K r \frac{dT}{dr}\right) = -r\dot{q} \xrightarrow{\text{INTEGRO}} K r \frac{dT}{dr} = -\dot{q}\frac{r^2}{2} + C_1$$

$$\xrightarrow{\text{INTEGRO}} T = -\frac{\dot{q}}{4K} r^2 + C_1' \ln r + C_2$$

PER DETERMINARE LE COSTANTI DI INTEGRAZIONE ⟶ AGGIUNGO LE COND. AL CONTORNO

$r = 0 \longrightarrow C_1' = 0$ POICHÉ $T(0) \neq \infty$

$r = r_c \longrightarrow T = T_1 \quad T_1 = -\frac{\dot{q}}{2K} r_c^2 + C_2 \longrightarrow C_2 = T_1 + \frac{\dot{q} r_c^2}{4K}$

⇓ PROFILO DI TEMPERATURA

$$T(r) = -\frac{\dot{q} r^2}{4K} + T_1 + \frac{\dot{q} r_c^2}{4K}$$

- PER DETERMINARE LA T_{MAX} NEL CONDUTTORE: $dT/dr = 0$ per $r = 0$ (CENTRO)

$$\Downarrow$$

$$T_{max} = T_1 + \frac{\dot{q} r_c^2}{4K} = 47{,}68\,°C \simeq T_1$$

CASO b ⟶ SPESSORE ISOLANTE RADDOPPIATO $S_{iso} = 4\ mm$

$$R_{cond,\,iso} = \frac{\ln\left(\frac{r_t + S_t + S_{iso}}{r_t + S_t}\right)}{2\pi K L} = 0{,}2757 \ \frac{K}{W}$$ (AUMENTA RISPETTO AL CASO a)

$$R_{conv,\,e} = \frac{1}{h_e 2\pi (r_t + S_t + S_{iso})} = 0{,}4823 \ \frac{K}{W}$$ (DIMINUISCE RISPETTO AL CASO b)

$$\ldots \frac{\ldots}{h_e\, 2\pi (r_e + s_e + s_{iso})} = 0{,}40\ldots \frac{\ldots}{W} \;(\ldots)$$

SE $S_{iso} \uparrow \longrightarrow R_{cond,iso} \uparrow \; ; \; R_{conv,e} \downarrow \Longrightarrow R_{TOT} \uparrow\downarrow \,?$

RAGGIO CRITICO DI ISOLAMENTO

$$\dot{\Phi} = \frac{\Delta T}{R_{TOT}} = \frac{\Delta T}{2\pi L} \left(\frac{\ln\left(\frac{r_{EST}}{r_{INT}}\right)}{k} + \frac{1}{h_e r_{EST}} \right)^{-1}$$

$\downarrow$ TROVO IL RAGGIO ESTERNO CHE MASSIMIZZA $\dot{\Phi}$
MINIMIZZA R_{TOT}

$\downarrow$

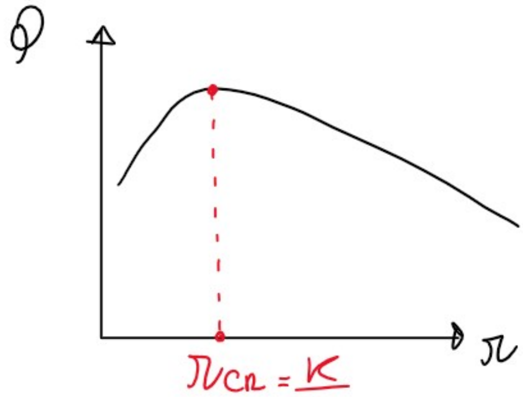
$$\frac{d\dot{\Phi}}{dr_e} = 0 = -\Delta T \left[\left(\frac{\ln\left(\frac{r_{EST}}{r_{INT}}\right)}{K} + \frac{1}{h_e \cdot r_{EST}} \right) \frac{1}{2\pi L} \right]^{-2} \cdot$$

$$\cdot \frac{1}{2\pi L} \left(\frac{1}{K} \cdot \frac{1}{r_{EST}/r_{INT}} \cdot \frac{1}{r_{INT}} - \frac{1}{h\, r_{EST}^2} \right)$$

$\Downarrow$

$$\frac{d\dot{\Phi}}{dr_{EST}} = 0 \Longrightarrow r_{EST} = \frac{K}{h}$$

RAGGIO CRITICO

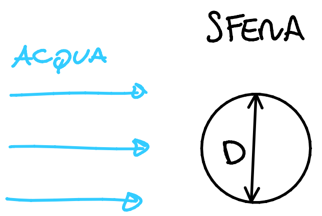


$r_{CR} = \frac{K}{h}$

- SE $r < r_{CR}$: L'AGGIUNTA DI ISOLANTE AUMENTA LA POTENZA TERMICA TRASMESSA

ESE 4

lunedì 8 novembre 2021 16:35



*** DATI ***

$D = 5\,mm \quad T_i(t=0s) = 200°C \; ; \; h = 1000\,W/m^2K$

$K = 52\,W/mK \quad c = 420\,J/Kg/K$ (BRONZO)

$T_{H_2O} = 30°C \quad T(t=15\,s) = ?$

REGIME NON STAZIONARIO

$\downarrow$

È POSSIBILE USARE L'APPROCCIO A PARAMETRI CONCENTRATI ?

$\downarrow$

LUNGHEZZA CARATTERISTICA $L_c = \dfrac{V}{A_{scambio}} = \dfrac{r}{3}$ (× SFERA)

n° Biot $\quad Bi = \dfrac{h L_c}{K} = \underbrace{0{,}016 < 0{,}1}$

OK APPROCCIO A PARAMETRI CONCENTRATI

↳ T UNIFORME NEL CORPO $T(t)$

• **CALCOLO $T(t)$**

$\cancel{\dot{Q}_{in}} + \cancel{\dot{Q}_{GEN}} - \dot{Q}_{OUT} = \dot{U}_{ACC}$ (BILANCIO ENERGETICO GENERALE)

$\Downarrow$

$$-h A_{scambio} \left(T(t) - T_{ACQUA} \right) = \rho_s c_s \frac{dT}{t}$$

$\Downarrow$ INTEGRAZIONE CON SOSTITUZIONE DI VARIABILI $\theta = T(t)\,T_{ACQUA}$

↳ SVOLTO A LEZIONE

$\downarrow$

$$t - t_i = -\frac{\rho c V}{h A} \ln \frac{T(t) - T_{ACQUA}}{T_i - T_{ACQUA}} \qquad \left(\frac{\theta}{\theta_i} = e^{-Bi\,Fo} \right)$$

t_i: 0 sec $\quad T_i$: 200°C $\quad T_{ACQUA}$: 30°C

$\Downarrow$

$$T(15\,s) = 31{,}3°C$$

ESE 5

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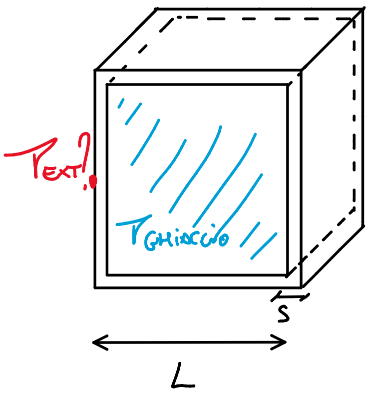

* DATI *

$T_{GHIACCIO} = 0°C$; $T_{EXT} = 20°C$

$L = 200\ mm$ $s = 1\ mm$ $K_S = 0{,}05\ W/mK$

$\rho_{GHIACCIO} = 920\ kg/m^3$ $\Delta h_{f\,GHIACCIO} = 334\ kJ/kg$

$t_{FUSIONE}$?

CALORE PER SCIOGLIERE IL GHIACCIO

$$Q = M_{GHIACCIO}\,\Delta h_f = \underbrace{\rho_{GHIACCIO}\,\underbrace{(L-2s)^3}_{V_{GHIACCIO}}}_{\text{massa ghiaccio } 7{,}141\ kg}\,\Delta h_f = 2385\ J$$

POTENZA TERMICA SCAMBIATA ATTRAVERSO LE PARETI

$$\dot{Q} = \frac{\Delta T}{R_{TOT}} = \frac{T_{EXT} - T_{GHIACCIO}}{R_{COND,S}} = 240\ W$$

$T_{GHIACCIO}$ COSTANTE DURANTE LA FUSIONE

$$R_{COND,S} = \frac{s}{K_s A} = \frac{s}{K_s 6L^2} = 0{,}0833\ \frac{K}{W}$$

(assunto monodimensionale su ogni faccia)

TEMPO NECESSARIO

$$t = \frac{Q}{\dot{Q}} = 9937{,}5\ s$$

ESE 6

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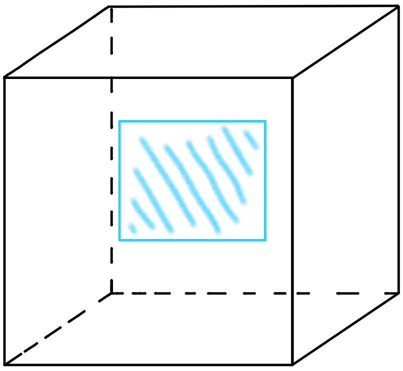


*** Dati ***

$A_F = 1{,}6\,m^2$; $A_M = 39\,m^2$

$T_i = 20°C$; $T_e = -5°C$; $S_{vetro} = 5\,mm$

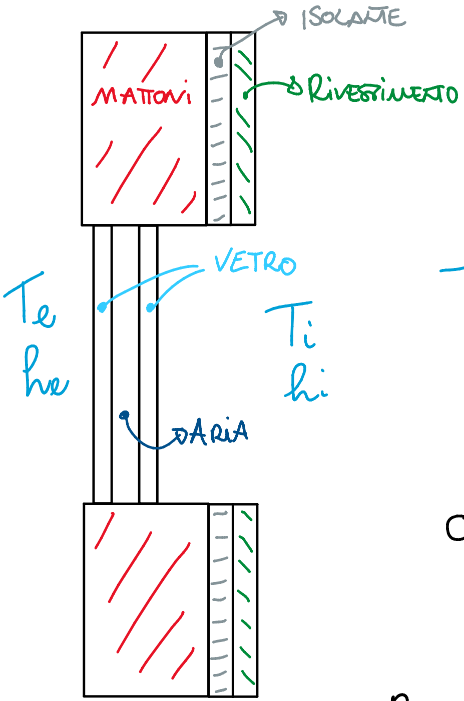
a) Doppi vetri
$S_{int} = 1\,cm$
Aria ferma

b) Doppia finestra
$S_{int} = 20\,cm$
Aria in moto

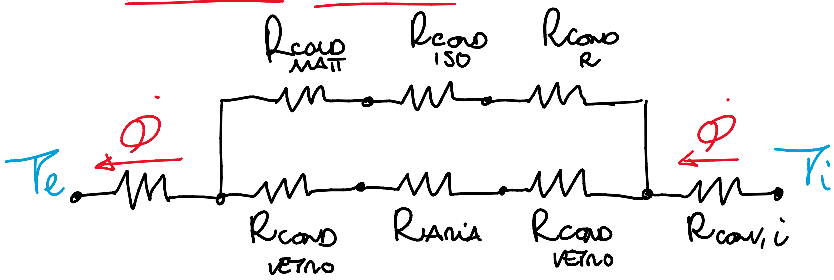
	Spessore [cm]	K [W/m/K]
Mattoni	30	0.065
Isolante	2	0.04
Rivestimento interno	3	0.8
Vetro	0.5	1.28

$h_i = 8\,W/m^2K$; $h_e = 23\,W/m^2K$

$h_{int,b} = 14\,W/m^2K$



*** Analogia elettrica ***

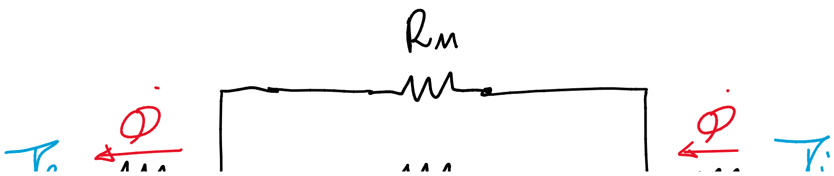


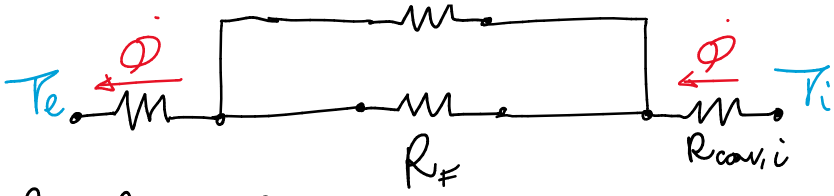
Caso a) R_a → Resistenza conduttiva

Caso b) R_a → Resistenza convettiva

• Geometria piana
- Res. conduttiva: $R_{cond} = \frac{L}{KA}$
- Res. convettiva: $R_{conv} = \frac{1}{hA}$

• Resistenza equivalente
- Res. in serie $R_{TOT} = \sum_i R_i$
- Res. in parallelo $R_{TOT} = \sum_i \frac{1}{R_i}$





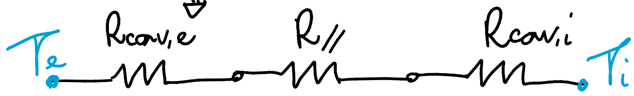
$$R_M = R_{cond_M} + R_{cond_{ISO}} + R_{cond_R}$$

$$R_F = R_{cond,V} + R_A + R_{cond,V}$$

$\Downarrow$

CALCOLO LA RESISTENZA EQUIVALENTE $R_{//} = \frac{1}{R_M} + \frac{1}{R_F}$ (Res. in Parallelo)

$\Downarrow$



$\Downarrow$

$R_{TOT} = R_{conv,e} + R_{//} + R_{conv,i}$ (Res. in Serie)

→ Risoluzione Numerica

$A_{TOT} = A_M + A_F = 40{,}6\ m^2 \qquad A_{PARETE} = \frac{A_{TOT}}{4} = 10{,}15\ m^2$

$R_{conv,e} = \frac{1}{h_e(A_M + A_F)} = 0{,}0043\ K/W \qquad R_{cond_M} = \frac{S_M}{K_M A_M} = 0{,}5388\ K/W$

$R_{conv,i} = \frac{1}{h_i(A_M + A_F)} = 0{,}0123\ K/W \qquad R_{cond_i} = \frac{S_{iso}}{K_{ISO} A_{iso}} = 0{,}0585\ K/W$

$R_{conv_{Aria\ b)}} = \frac{1}{h_f A_F} = 0{,}0446\ K/W \qquad R_{cond_R} = \frac{S_R}{K_R A_R} = 0{,}0044\ K/W$

$R_{cond_V} = \frac{S_V}{K_V A_F} = 0{,}0024\ K/W$

$R_{cond,\ Aria\ a)} = \frac{S_{Aria}}{K_{Aria} A_F} = 0{,}2648\ K/W$

R_{TOT} → a) 0,2028 K/W; b) [illegible] K/W $\Big\}$ $R_{TOT,a)} > R_{TOT\ b)} \qquad \dot{Q}_{a)} < \dot{Q}_B$

$$R_{TOT} \begin{cases} a)\ 0{,}2028\ K/W \\ b)\ 0{,}0623\ K/W \end{cases} \quad R_{TOT,a)} > R_{TOT\,b)} \quad \underline{\dot{Q}_{a)}} < \dot{Q}_B$$

a) soluzione migliore

$$\Downarrow$$

$$\dot{Q}_A = \frac{\Delta T}{R_{TOT,a}} = \frac{T_e - T_i}{R_{TOT\,a)}} = 123\ W$$