Esercizi - Termodinamica

Friday, 7 January 2022 14:32

ES 1

SFERA DI RAME

$d = 20\,cm \quad T_{Cu} = 55°C$

$\rho_{Cu} = 8938 \frac{kg}{m^3} \quad C_{Cu} = 385.37 \frac{J}{kg\,K}$

RECIPIENTE

$m_{H2O} = 50\,kg \quad T_{H2O} = 20°C$

$\rho_{H2O} = 1000\,kg/m^3 \quad C_{H2O} = 4186 \frac{J}{kg\,K}$

TEMPERATURA FINALE ?

SISTEMA

$\Delta U = Q + L$

ADIABATICO $\rightarrow Q = 0$

NO LAVORO $\rightarrow L = 0$

$\Delta U_{TOT} = 0 \rightarrow \Delta U_{TOT} = \Delta U_{H2O} + \Delta U_{Cu}$

MASSA DELLA PALLA DI RAME

$V_{Cu} = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = 4.19 \cdot 10^{-3}\,m^3$

$m_{Cu} = V_{Cu} \cdot \rho_{Cu} = 37.435\,kg$

CONSERVAZIONE DELL' ENERGIA

NESSUN RISCHIO DI EBOLLIZIONE

$C_{Cu}\, m_{Cu}\, \Delta T_{Cu} = C_{H2O}\, m_{H2O}\, \Delta T_{Cu}$

$C_{Cu}\, m_{Cu}\,(T_{Cu} - T_E) = C_{H2O}\, m_{H2O}\,(T_E - T_{H2O})$

$T_E\,(C_{H2O} m_{H2O} + C_{Cu} m_{Cu}) = C_{Cu} m_{Cu} T_{Cu} + C_{H2O} m_{H2O} T_{H2O}$

$$T_E = \frac{C_{Cu} m_{Cu} T_{Cu} + C_{H2O} m_{H2O} T_{H2O}}{C_{Cu} m_{Cu} + C_{H2O} m_{H2O}} = 24.88\,°C$$

AUMENTO DELL' ENTROPIA ?

II PRINCIPIO TERMODINAMICA

$\Delta S_{UNI} = \Delta S_{SISTEMA} + \Delta S_{AMBIENTE} \geq 0$ (ADIABATICO)

$\Delta S_{SISTEMA} = \Delta S_{Cu} + \Delta S_{H2O} = \Delta S_{UNI}$

$$\Delta S_{UNI} = m_{Cu} C_{Cu} \ln\left(\frac{T_E}{T_{Cu}}\right) + m_{H2O} C_{H2O} \ln\left(\frac{T_E}{T_{H2O}}\right) = 378 \frac{J}{K}$$

ES 2

RECIPIENTE (ARIA)

$V = 3\,m^3 \quad T_a = 25°C \quad P = 101325\,Pa$

$MM_a = 28.84\,kg/Kmol$

CALCOLA R* CONDIZIONI PTS

ARIA E' GAS IDEALE

$PV = mRT \rightarrow PV = m\,MM \frac{R}{MM} T \rightarrow PV = m R^* T$

$$R^* = R_a = \frac{R}{MM_a} = \frac{8314 \frac{J}{Kmol\,K}}{28.84 \frac{kg}{Kmol}} = 288.3 \frac{J}{kg\,K}$$

DENSITA'

$P v = \frac{P}{\rho} = R^* T \rightarrow \rho = \frac{P}{R^* T} = 1.18 \frac{kg}{m^3}$

Q NECESSARIO PER RISCALDARE A V COSTANTE ARIA FINO A $T_F = 500°C$

MASSA ARIA

$m_a = \rho V = 3.54\,kg$

I PRINCIPIO TERMODINAMICA

$\Delta U = Q + L \rightarrow \Delta U = Q$

ARIA COME GAS PERFETTO BIATOMICO

$Q = m\, C_V\, \Delta T = m_a\left(\frac{5}{2}R^*\right)(T_F - T_i) = 1211\,KJ$

PRESSIONE FINALE DOPO IL RISCALDAMENTO

$P_i v = R^* T_i$

$P_F v = R^* T_F \rightarrow P_F = \rho R^* T_F = 263\,KPa$

ES 3

ARIA $m = 0.01\,kg \quad P = P_{ATM} \quad R^* = 288.3 \frac{J}{kg\,K}$

CILINDRO VERTICALE ADIABATICO $S = 0.01\,m^2 \quad h = 1\,m$

L PER COMPRIMERE ARIA MEDIANTE TRASFORMAZIONE ADIABATICA REV FINO A $V_F = V/2$?

ADIABATICA REV $\rightarrow$ ISOENTROPICA

PV^γ cost

$P_1 V_1^\gamma = P_2 V_2^\gamma \quad P_{ATM} V^\gamma = P_2\left(\frac{V}{2}\right)^\gamma = P_2 V^\gamma 2^{-\gamma}$

$P_2 = 2^{+\gamma} P_{ATM} = 267.39\,KPa$

CON $\gamma = \frac{7}{5}$ POICHE' ARIA BIATOMICA (78% N_2, 21% O_2)

I TERMODINAMICA

$\Delta U = Q + L = L \rightarrow L = m_a C_V (T_2 - T_1)$ (Q = 0: ADI)

VOLUME INIZIALE

$V_1 = Sh = 0.01\,m^3$

TEMPERATURA INIZIALE

$P_1 V_1 = m_a R^* T_1 \rightarrow T_1 = \frac{P_1 V_1}{m_a R^*} = 351.5\,K$

TEMPERATURA FINALE

$P_2 V_2 = m_a R^* T_2 \rightarrow T_2 = \frac{P_2 V_2}{m_a R^*} = 463\,K$

LAVORO NECESSARIO

$L = \Delta U = m\, C_V\, \Delta T = m\, \frac{5}{2} R^* (T_2 - T_1) = 803\,J$

ES 4

CILINDRO ASSE ORIZZONTALE

NO ATTRITO, ADIABATICA

AZOTO $MM(N_2) = 28.013 \quad m(N_2) = 1\,kg$

$T_1 = 20°C$ (293.15 K) $\quad V_1 = 0.855\,m^3 \quad V_2 = 0.2746\,m^3$

T_{GAS} A FINE COMPRESSIONE ?

L SCAMBIATO CON ESTERNO ?

RICAVO COSTANTE R^*

$R^* = \frac{R}{MM} = 296.8 \frac{J}{kg\,K}$

PRESSIONE INIZIALE

$P_1 V_1 = m R^* T_1 \rightarrow P_1 = \frac{m R^* T_1}{V_1} = 101.25\,KPa$

TRASFORMAZIONE ISOENTROPICA

$P_1 V_1^\gamma = P_2 V_2^\gamma \quad P_2 = P_1\left(\frac{V_1}{V_2}\right)^\gamma = 500\,KPa$

CON $\gamma = \frac{7}{5}$ $\quad N_2$ BIATOMICO

TEMPERATURA FINALE

$P_2 V_2 = m R^* T_2 \rightarrow T_2 = \frac{P_2 V_2}{m R^*} = 462\,K$

LAVORO SCAMBIATO

$\Delta U = Q + L$ (Q = 0: ADI) $\quad L = \Delta U = m\, C_V\, \Delta T \quad C_V = \frac{5}{2} R^*$ BI

$L = m\, \frac{5}{2} R^* (T_2 - T_1) = 125.3\,KJ$

INTRODUCIAMO RAME

$m_{Cu} = 1\,kg \quad C_{Cu} = 385 \frac{J}{kg\,K}$

PAGLIETTA FINISSIMA — EQUILIBRIO TERMICO

DETERMINA $C_{SPECIFICO}$, $V_{SPECIFICO}$, T_F, L

$\Delta S_{UNI} = \Delta S_{MIX} = \Delta S_{N2} + \Delta S_{Cu}$

$\Delta S_{Cu} = m_{Cu} C_{Cu} \ln \frac{T_2}{T_1}$

$\Delta S_{N2} = m_{N2} C_{N2} \ln \frac{T_2}{T_1} - R^* m_{N2} \ln \frac{P_2}{P_1}$

$\Delta S_{Cu} = -\Delta S_{N2}$

$m_{Cu} C_{Cu} \ln \frac{T_2}{T_1} = -m_{N2}\left(C_{PN2} \ln \frac{T_2}{T_1} - R^* \ln \frac{P_2}{P_1}\right)$

$(m_{Cu} C_{Cu} + m_{N2} C_{N2}) \ln \frac{T_2}{T_1} = m_{N2} R^* \ln \frac{P_2}{P_1}$

$\frac{P_2}{P_1} = \frac{m R^* T_2}{V_2} \cdot \frac{V_1}{m R^* T_1} = \frac{T_2 V_1}{V_2 T_1}$

$(m_{Cu} C_{Cu} + m_{N2} C_{N2}) \ln \frac{T_2}{T_1} = m_{N2} R^* \ln \frac{T_2 V_1}{T_1 V_2}$

$\left(\frac{T_2}{T_1}\right)^{\frac{m_{Cu} C_{Cu} + m_{N2} C_{N2}}{a}} = \left(\frac{T_2 V_1}{T_1 V_2}\right)^{m_{N2} R^*}$

$\frac{T_2}{T_1} = \left(\frac{T_2 V_1}{T_1 V_2}\right)^{\frac{m_{N2} R^*}{a}}$

$\left(\frac{T_2}{T_1}\right)^{\frac{a - m_{N2} R^*}{a}} = \left(\frac{V_1}{V_2}\right)^{\frac{m_{N2} R^*}{a}}$

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{m_{N2} R^*}{a} \cdot \frac{a}{a - m_{N2} R^*}} = \left(\frac{V_1}{V_2}\right)^{\frac{m_{N2} R^*}{a - m_{N2} R^*}}$

LAVORO

$L = \Delta U = m_{N2} C_{N2} \Delta T + m_{Cu} C_{Cu} \Delta T$

INDICE POLITROPICA

$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{\frac{R'/a}{1 - R'/a}} \quad PV = m R' T$

$\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} \rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R'/a}{1 - R'/a} + 1}$

ES 5

ARIA $m = 1\,kg \quad R^* = 288.3 \frac{J}{kg\,K}$

$P_1 = 10^5\,Pa \quad T_1 = 298.15\,K$

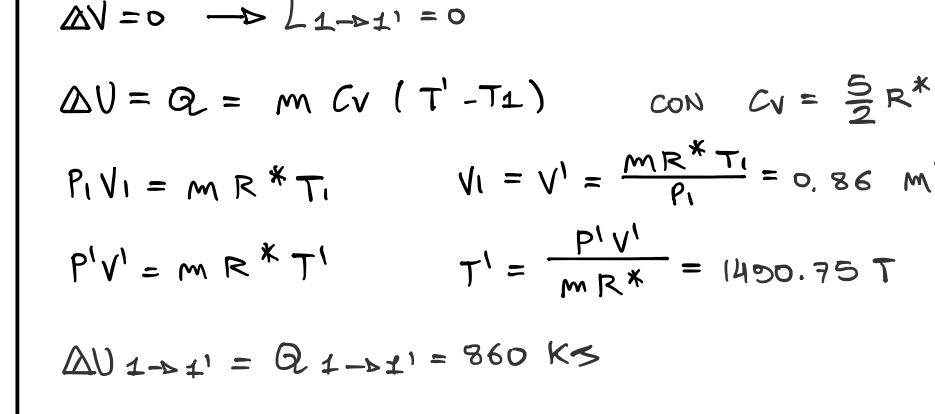
$P_2 = 5 \cdot 10^5\,Pa \quad T_2 = 298.15\,K$

LAVORO L, CALORE Q, ENTROPIA ΔS

a) comp ISOCORA REV + CALORE A P cost

b) comp ISOTERMA

c) comp ISOENTROPICA + CALORE A V cost



CASO a

1 → 1' ISOCORA

$\Delta V = 0 \rightarrow L_{1\to1'} = 0$

$\Delta U = Q = m\, C_V (T' - T_1) \quad$ CON $C_V = \frac{5}{2} R^*$ BIATOMICO

$P_1 V_1 = m R^* T_1 \quad V_1 = V' = \frac{m R^* T_1}{P_1} = 0.86\,m^3$

$P' V' = m R^* T' \quad T' = \frac{P' V'}{m R^*} = 1490.75\,T$

$\Delta U_{1\to1'} = Q_{1\to1'} = 860\,KJ$

1' → 2

$L_{1'\to2} = P_2(V_2 - V') = -P_2(V_1 - V_2) = -344\,KJ$

$P_2 V_2 = m R^* T_2 \rightarrow V_2 = \frac{m R^* T_2}{P_2} = 0.172\,m^3$

$\Delta U = m\, C_V (T_2 - T') = -860\,KJ$

$\Delta Q = m\, C_P (T_2 - T') = \Delta U + L = -1204\,KJ$

TRASFORMAZIONE a

$\Delta U = \Delta U_{1\to1'} + \Delta U_{1'\to2} = 0 \quad$ RAGIONEVOLE POICHE' $\Delta T = 0$

$L = L_{1\to1'} + L_{1'\to2} = -344\,KJ \quad$ PERCORSO IN SENSO INVERSO (←)

$Q = Q_{1\to1'} + Q_{1'\to2} = -344\,KJ$

$\Delta S = ~~m C_P \ln \frac{T_2}{T_1}~~ - m R^* \ln \frac{P_2}{P_1} = -464\,J/K$

ENTROPIA (ΔS) E ENERGIA INTERNA (ΔU) SONO FUNZIONI DI STATO, SI CALCOLANO SOLO CON I PUNTI INIZIALI E FINALI (NON SERVE SPEZZARE IL PERCORSO)

- $\Delta S = -464\,J/K$
- $\Delta U = 0$
- $L = -344\,KJ$ (+)
- $Q = -344\,KJ$

CASO b - ISOTERMA REVERSIBILE

$\Delta T = 0 \rightarrow \Delta U = 0$

$L = -Q = mRT \ln \frac{V_2}{V_1} = m R^* T \ln \frac{V_2}{V_1} = -138.3\,KJ$

$\Delta S = m\, C_P \ln \frac{T_2}{T_1} - m R^* \ln \frac{P_2}{P_1} = -464\,J/K$

- $\Delta S = -464\,J/K$
- $\Delta U = 0$
- $L = -138.3\,KJ$ (+)
- $Q = -138.3\,KJ$

CASO c

1 → 1' ADIABATICA REV (ISO-S)

$P_1 V_1^\gamma = P' V'^\gamma \quad$ CON $V' = V_2$, $\gamma = \frac{7}{5}$ BI

$V_1 = \frac{m R^* T_1}{P_1} = 0.86\,m^3$

$V_2 = V' = \frac{m R^* T_2}{P_2} = 0.172\,m^3$

$P' = P_1\left(\frac{V_1}{V'}\right)^\gamma = 9.5 \cdot 10^5\,Pa$

$T' = \frac{V' P'}{m R^*} = 566.8\,K$

$\Delta U = \frac{5}{2} R^* (T' - T_1) = 193.6\,KJ$

$Q = 0 \rightarrow$ ADIABATICA

$L = -\Delta U = -193.6\,KJ$

1' → 2 ISOCORA

$L = 0$

$\Delta U = Q = C_V\, m\,(T_2 - T') = -193.6\,KJ$

TRASFORMAZIONE c

- $\Delta S = -464\,KJ$
- $\Delta U = 0$
- $Q = -193.6\,KJ$
- $L = -193.6\,KJ$ (+)