

Esercizi - Conduzione

Tuesday, 11 January 2022 11:25

ES 1 FORNO INDUSTRIALE $S = 15 \text{ m}^2$

3 STRATI

$$T_i = 500^\circ\text{C} \quad T_e = 20^\circ\text{C}$$

$$L_1 = 0.6 \text{ m} \quad K_1 = 3 \text{ W/mK} \quad \text{MATTONE}$$

$$L_2 = 0.3 \text{ m} \quad K_2 = 0.1 \text{ W/mK} \quad \text{ISOLANTE}$$

$$L_3 = 0.02 \text{ m} \quad K_3 = 20 \text{ W/mK} \quad \text{ACCIAIO}$$

PARETE IN CONDIZIONE STAZIONARIA

$$\text{SCAMBIO CONNETTIVO } h_1 = h_3 = 10 \text{ W/m}^2\text{K}$$

$$R_{\text{TOT}}, Q, T(x), T_{\text{MAX},2} ?$$

ANALOGIA ELETTRICA

$$R_{\text{COND},i} = R_i = \frac{1}{h_i S} = 0.0067 \text{ K/W}$$

$$R_{\text{COND},e} = R_e = \frac{1}{h_e S} = 0.0067 \text{ K/W}$$

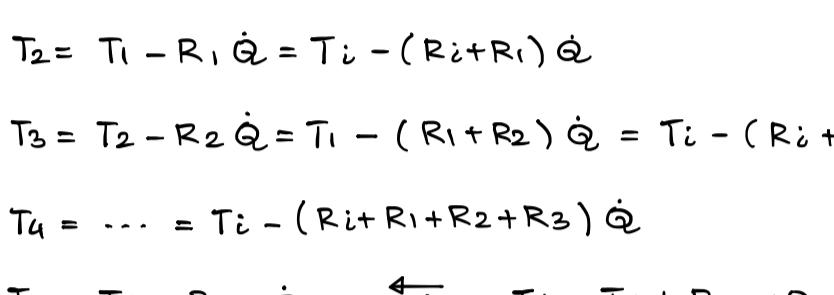
$$R_{\text{COND},1} = R_1 = \frac{L_1}{K_1 S} = 0.0133 \text{ K/W}$$

$$R_{\text{COND},2} = R_2 = \frac{L_2}{K_2 S} = 0.2 \text{ K/W}$$

$$R_{\text{COND},3} = R_3 = \frac{L_3}{K_3 S} = 6.67 \cdot 10^{-7} \text{ K/W}$$



RESISTENZA TERMICA COMPLESSIVA



$$R_{\text{TOT}} = \sum_k R_k = R_i + \dots + R_e = 0. \quad 8 \text{ K/W}$$

POTENZA TERMICA TRASMESSA

$$Q = \frac{(T_i - T_e)}{R_{\text{TOT}}} = \frac{(T_i - T_1)}{R_1} = \frac{(T_2 - T_3)}{R_2} = \frac{(T_3 - T_e)}{R_3} \quad \text{LOCALMENTE}$$

$$Q = \frac{\Delta T_{\text{TOT}}}{R_{\text{TOT}}} = \frac{(T_i - T_e)}{\sum R_k} = 3880 \text{ W}$$

DISTRIBUZIONE DELLA TEMPERATURA

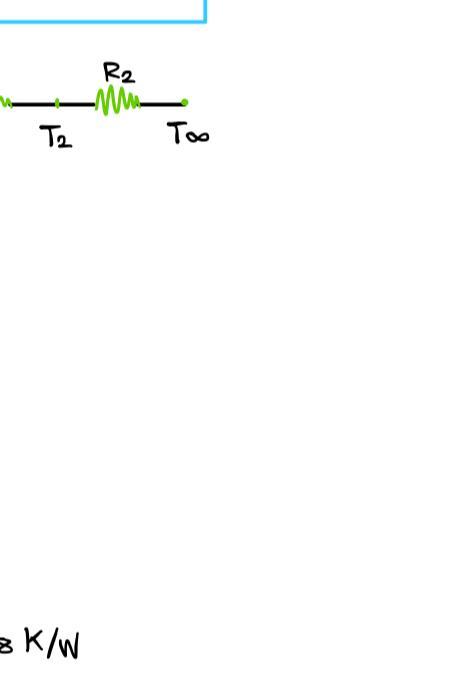
$$\frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(K \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(K \frac{\partial T}{\partial z} \right) + Q_{\text{GEN}} = \rho c \frac{\partial T}{\partial t}$$

IPOTESI

- MONODIMENSIONALITÀ $T(x,y,z,t) = T(x,t)$
- STAZIONARITÀ $T(x,t) = T(x)$ $\rightarrow \frac{\partial T}{\partial t} = 0$
- NO GENERAZIONE $Q_{\text{GEN}} = 0$

$$\frac{\partial}{\partial x} \left(K \frac{\partial T}{\partial x} \right) = 0 \quad \rightarrow \quad K \frac{\partial^2 T}{\partial x^2} = C_1$$

$$dT = \frac{C_1}{K} dx \quad T(x) = \frac{C_1}{K} x + C_2$$



CONDIZIONI AL CONTORNO

$$T(x=0) = T_i \quad \rightarrow \quad T(x) = \frac{T_b - T_i}{L} x + T_i$$

$$T(x=L) = T_b$$

TEMPERATURA DEI PARETI

$$T_1 = T_i - R_i Q$$

$$T_2 = T_i - R_1 Q = T_i - (R_i + R_1) Q$$

$$T_3 = T_2 - R_2 Q = T_i - (R_i + R_1 + R_2) Q = T_i - (R_i + R_1 + R_2 + R_3) Q$$

$$T_4 = \dots = T_i - (R_i + R_1 + R_2 + R_3) Q$$

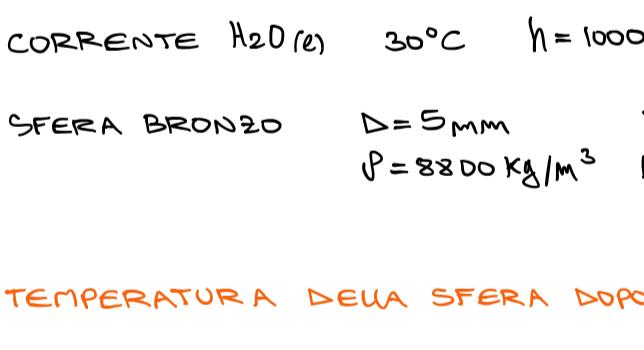
$$T_e = T_i - R_{\text{TOT}} Q \quad \leftrightarrow \quad T_i = T_e + R_{\text{TOT}} Q$$

ES 2

$$\text{TUBO } L = 1 \text{ m} \quad r_{\text{INT}} = 8 \text{ mm} \quad s_t = 2 \text{ mm} \quad k_t = 20 \text{ W/mK}$$

$$\text{ISOLANTE } s_i = 10 \text{ mm} \quad k_i = 0.16 \text{ W/mK}$$

$$\text{TEMPERATURA COSTANTE} \quad T_i = 80^\circ\text{C} \quad T_2 = 10^\circ\text{C}$$



ANALOGIA ELETTRICA

$$R_{\text{COND},\text{CILINDRICA}} = \frac{\log \left(\frac{r_{\text{OUT}} + s_t}{r_{\text{INT}}} \right)}{2\pi k L}$$

$$R_{\text{COND},\text{INT}} = \frac{\log \left(\frac{r_{\text{OUT}} + s_t + s_i}{r_{\text{INT}}} \right)}{2\pi k_i L} = R_1$$

$$R_{\text{COND},\text{ISO}} = \frac{\log \left(\frac{s_i}{r_{\text{OUT}} + s_t + s_i} \right)}{2\pi k_i L} = R_2$$

$$R_{\text{TOT}} = \sum R_i = R_1 + R_2 = 0.5562 \text{ K/W}$$

CALORE IN TRANSITO

$$Q = \frac{\Delta T}{R_{\text{TOT}}} = \frac{T_i - T_3}{R_{\text{TOT}}} = 73.5 \text{ W}$$

$$T_2 = T_i - R_1 Q = T_i + R_{\text{ISO}} Q = 73.5^\circ\text{C}$$

ES 3

$$\text{CONDUTTORE ELETTRICO } L = 5 \text{ m} \quad D = 3 \text{ mm} \quad k_c = 350 \text{ W/mK}$$

$$\text{PLASTICA } s_p = 2 \text{ mm} \quad k_p = 0.15 \text{ W/mK}$$

$$\text{POTENZA DISSIPATA } Q = 400 \text{ W/m}^2$$

$$\text{ARIA } T_\infty = 30^\circ\text{C} \quad h = 12 \text{ W/m}^2\text{K}$$

$$T_1, T_2 \text{ IN CONDIZIONI STAZIONARIE}$$



CALORE DISSIPATO

$$\dot{Q} = Q_{\text{DISS}} \cdot 2\pi \frac{L}{2} = 18.85 \text{ W}$$

ANALOGIA ELETTRICA

$$R_{\text{COND}} = \frac{\log \left(\frac{r_{\text{OUT}} + s_p}{r_{\text{INT}}} \right)}{2\pi k L} = 0.1753 \text{ K/W}$$

$$R_2 = R_{\text{CONNESSIONE}} = \frac{1}{S_h h} = \frac{1}{h 2\pi k_{\text{EST}} L} = 0.7578 \text{ K/W}$$

$$\text{CON } k_{\text{EST}} = \frac{D}{2} + s_p$$

$$R_{\text{TOT}} = \sum R_i = R_1 + R_2 = 0.9377 \text{ K}$$

TEMPERATURE SUPERFICIALI

$$T_1 = T_\infty + R_{\text{TOT}} Q = 47.67^\circ\text{C}$$

$$T_2 = T_\infty + R_2 Q = 44.28^\circ\text{C}$$

TEMPERATURA MAX ALL'INTERNO DEL CONDUTTORE

$$\frac{1}{R} \frac{\partial}{\partial R} \left(K_R \frac{\partial T}{\partial R} \right) + \frac{1}{R^2} \frac{\partial}{\partial R} \left(K \frac{\partial T}{\partial R} \right) + \frac{\partial}{\partial Z} \left(K \frac{\partial T}{\partial Z} \right) + \dot{q} = \rho c \frac{\partial T}{\partial Z} \quad \text{MONODIMENSIONALE}$$

NO ACCUMULO REGIME STAZIONARIO

$$\frac{1}{R} \frac{d}{dR} \left(K_R \frac{dT}{dR} \right) = - \dot{q} \quad \text{GENERAZIONE INTERNA}$$

$$\text{DOVE } \dot{q} = \frac{Q}{V} = \frac{Q}{\pi R^2 L} = 533316 \text{ W/m}^3$$

$$\int \frac{d}{dR} \left(K_R \frac{dT}{dR} \right) = \int - \dot{q} R$$

$$KR \frac{dT}{dR} = - \frac{\dot{q} R^2}{2} + C_1 \quad dT = \left(- \frac{\dot{q} R}{2K} + \frac{C_1}{KR} \right) dR$$

$$T(R) = - \frac{\dot{q} R^2}{4K} + C_1 \ln R + C_2$$

CONDIZIONI AL CONTORNO

$$T(x=0) = T_i$$

$$T(x=L) = T_\infty$$

$$\rightarrow T(R) = T_i + \frac{Q R c}{4K} - \frac{\dot{q} R^2}{4K}$$

$$T(0) = T_i + \frac{Q R c}{4K} = 47.68^\circ\text{C} \approx T_i$$

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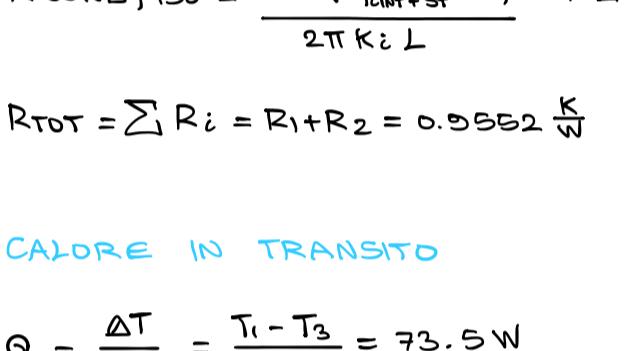
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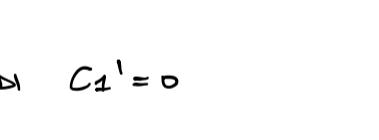
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