MFE5130 Project Report: Predicting the Compound Call Options

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I. ABSTRACT

A. Background and Methodology

This project mainly uses the binary tree model and the Monte Carlo method to study the option pricing problem. The selected underlying asset is SSE 50. The main research objectives of the project are

(1) Estimating the historical volatility, implied volatility, and dividend yields of the underlying asset and finding a risk-free interest rate.

(2) Using the n-step forward tree and the estimated values to estimate the prices of European and American options, respectively.

(3) Using the Monte Carlo method to estimate the prices of European options and American options, respectively.

(4) Calculating δ, γ, θ, and ν of European and American options.

(5) Measuring the implied volatility of European call options.

B. Conclusion

(1) The American style in-the-money compound call options tend to have a higher value than European, however, for at-the-money and out-of-money options, they are almost the same.

(2) The compound cal options have a positive value of theta. As the time to expiration increases, the value of compound call options will rise instead of drop as normal call options

(3) The vega of compound cal options have a positive value. It means that these options can be used as risk mitigation tools

II. INTRODUCTION

A. Background

In our report, we mainly focus on solving the option pricing problem of the Shanghai Stock Exchange 50 Index. Initially, we will introduce options, the SSE 50 Index, and the background knowledge of option pricing. An option is a contract, originated in the American and European markets in the late eighteenth century that gives the holder the right to buy or sell an asset at a fixed price on or before a specific date. Option pricing is the fixed price agreed in the option contract at which the option holder buys or sells the underlying asset. Moreover, an option contract's value can be divided into intrinsic value and time value. The value of an option contract is equal to the sum of its intrinsic value and time value. Intrinsic value refers to the present value of the gain that a long can receive if they exercise an option, which is the difference between the asset's market price and the strike price. Time value refers to the value implied by the possibility that the price fluctuation of the underlying asset will benefit the option holder during the validity period of the option.

A compound option has an option as the underlying asset and has two exercise dates and two strike prices. Compound options are primarily used in the money market and fixed income market, which can reduce the uncertainty of the risk protection ability of options. There are four basic compound options: Call on a put, Call on a call, Put on a put, and Put on a call. Take the European call on a call as an example. On the first exercise date, the holder of the compound option can purchase a call option at the first strike price. Buying this call option enables the holder to purchase the underlying asset at the second strike price on the second exercise date. The above is a case of 1-fold compound options. In reality, an n-fold compound option has n+1 exercise dates and strike prices.

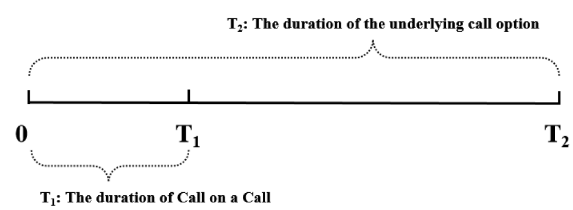

Fig. 1. The time dimension of the Call on a Call

Many models are used to solve option pricing problems, including the binomial tree model, the Black-Scholes model, and the Monte Carlo method model. The binomial option pricing model is an option pricing model proposed by J.C.Cox, S.A. Ross, M.Rubinstein, and Sharpe, or CRR, which is mainly implemented to calculate the value of American options value. Its advantage is that it is relatively intuitive and straightforward. The BS model is derived from the principle of risk-free arbitrage. If two assets in the market can obtain the same return after the holding period expires, then their prices must be equal. Suppose the prices of the two are different. In that case, investors will long undervalued assets and, at the same time, short overvalued assets and carry out arbitrage transactions until the price difference between the two disappears. This process makes the prices of the two assets eventually balance. The Monte Carlo method is a mathematical method used to simulate random phenomena. The mechanism of the Monte Carlo method for option pricing is to simulate the underlying asset's price change according to the given price of the underlying asset's movement process. When the simulation times reach a specific number, the expectation is obtained by taking the average value. According to the law of large numbers, the Monte Carlo simulation results finally meet the convergence. The Monte Carlo method is suitable for the pricing problem of high-dimensional derivative securities. It is superior to the binary tree method in problems with path-dependence pricing.

In this project, we choose SSE 50 Index as the underlying asset. Based on scientific and objective methods, the Shanghai Stock Exchange 50 Index selects the most representative 50 stocks with large scale and good liquidity in the Shanghai stock market to form sample stocks. It comprehensively reflects the overall situation of a group of leading enterprises with the most market influence in the Shanghai stock market. The Shanghai Stock Exchange 50 Index has been officially released since January 2, 2004. Its goal is to establish an investment index with active transactions and large scale, mainly used as the basis of derivative financial instruments.

B. Structure

This report can be divided into five parts.

1) Introduction part is mainly focusing on the background of our underlying assets and compound call options.

2) Methodology part is related with the model we used, such as CRR binomial tree and BSM models. We use plenty of formulas to introduce our intuitive thinking and deviation process.

3) Result part summarized the output in predicting these compound call options.

4) Discussion part will introduce the defects of our models and points out the future way to improve them.

5) The last part is the conclusion parts.

III. Methodology

A. Parameter Estimation

The historical volatility of the underlying asset is the standard deviation of continuously compounded returns calculated from the historical data.

$$R_T = \ln \frac{P_t}{P_{t-1}} \tag{1}$$

$$\sigma^2 = \frac{1}{T-1} \sum_{t=1}^{T} \left(R_t - \bar{R}\right)^2 \tag{2}$$

Where P_t is the daily closing price of the SSE 50 Index, R_t is the logarithmic return rate of the SSE 50 Index, $\bar{R}$ is the mean return rate. σ^2 is the variance of return estimation, and σ is the corresponding standard deviation volatility. The estimation of the historical volatility is daily volatility. By multiplying $\sqrt{252}$ we can also calculate the yearly volatility.

In this project, the dividend yield is 0. And we choose the one-year fixed deposit interest rate as risk-free interest rate.

B. Binomial Tree Model

We use Cox-Ross-Rubinstein Binomial Tree or CRR to predict the value of European and American compound call options. According to Geske (1979), compound call options can be seen as a call option on another call option. It has three layers, underlying stocks, first and second options. In this project, we need to predict the price of the third layer, which is the top level. The following figure shows the structure of compound call options and the parameters each layer needs.

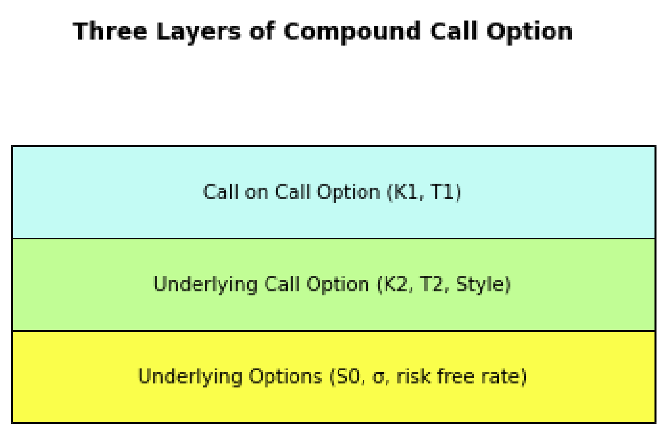

Fig. 2. Three Layers of Compound Call Option

S_0 represents the initial value of underlying stocks, and σ denotes the volatility of underlying stocks. As for the second layer, K_2 and T_2 represent the strike price and time to expiration of the first option level. Similarly, in the third layer, K_1 and T_1 represent the strike price and time to expiration of the second option level. The following figure explicitly explains the relationship between these parameters.

If the holders of compound call-on-call options want to use their rights, they must decide whether long the underlying options or the first options we have already mentioned, at time T_1, are in a specific price K_1. The underlying options have the time to expiration T_2 and strike price K_2. The underlying options can be European and American styles, but call-on-call options are European styles.

For European call options, the payoff formula is as follows:

$$Payoff = Max\left\{S_T - K, 0\right\} \tag{3}$$

S_T denotes the value of the underlying asset at time T, and K represents the strike price of these call options. In this research, the strike price and corresponding α are as follows:

In this project, we cannot directly set the strike price of underlying options K_2 at time zero. Instead, we could use α as a coefficient to calculate K_2. The procedure is first based on underlying assets' initial value and historical volatilities. Considering the risk-free rate, we need to use the CRR binomial model to predict the price movement of underlying assets. After analyzing all possible prices, we could combine the probability of each possible result and the respected price, which can be called the expected price, at time T_1. The formula is

$$Expected\ Price = \sum_{i=1}^{N} P(Node_i) Payoff(Node_i) \tag{4}$$

Then, K_2 is

$$K_2 = \alpha \times ExpectedPrice \tag{5}$$

Thus, combining the payoff formula and calculated K2, we could predict the values of underlying options and call-on-call options. The following part will introduce the procedure of the CRR binomial model.

The binomial tree method assumes that the value of assets can either increase or decrease by a certain amount within a single period. In other words, asset value change has only two cases: upper or lower nodes. According to Hoek and Elliott (2006), based on the non-arbitrage theorem, suppose the risk-free rate is r_f. It should fulfill the following inequalities:

$$0 < S\left(1,0\right) < \left(1+r\right)S\left(0\right) < S\left(1,1\right) \tag{6}$$

r_f is the risk-free rate. $S\left(1,1\right)$ represents the upper node value of underlying assets at time 1, and $S\left(1,0\right)$ represents the lower node value of underlying assets at time 1. If this is violated, it could produce an arbitrage opportunity.

In order to model the price of assets, the probability of whether the price will increase or decrease also needs to concern. Similarly, according to Hoek and Elliott (2006), based on the no-arbitrage theorem, π, which represents the probability of increasing price, should follow this equation (5):

$$\begin{aligned} Expected\ Payoff &= \left(1+r\right)S\left(0\right) \\ &= \pi S\left(1,1\right) + \left(1-\pi\right)S\left(1,0\right) \end{aligned} \tag{7}$$

After solving this equation, π should equal to

$$\pi = \frac{\left(1+r\right)S\left(0\right) - S\left(1,0\right)}{S\left(1,1\right) - S\left(1,0\right)} \tag{8}$$

As for the CRR model, the following notation will be used:

$$S\left(0\right) = S > 0 \tag{9}$$

$$S\left(1,1\right) = uS \tag{10}$$

$$S\left(1,0\right) = dS \tag{11}$$

where

$$u = e^{\sigma\sqrt{T}} \tag{12}$$

$$d = e^{-\sigma\sqrt{T}} \tag{13}$$

u and d represent the percentage of upward and downward movement. Considering the equation (5) and (6), these can be rewritten as

$$\left(1+r\right)S = \pi uS + \left(1-\pi\right)dS \tag{14}$$

$$\pi = \frac{\left(1+r\right) - d}{u - d} \tag{15}$$

In our project, the time to expiration is 29 days. Therefore, we assume that the value of underlying assets can only change once daily, so we have 29 steps to predict asset prices.

According to equation (3), the expected payoff of the European call option at time T is

$$\begin{aligned} ExpectedPayoff &= \pi Max\left\{uS_{T-1} - K, 0\right\} \\ &\quad + \left(1-\pi\right)Max\left\{dS_{T-1} - K, 0\right\} \end{aligned} \tag{16}$$

In this project, we assume that the risk-free rate is also the discount rate so that the prices of European call options are

$$Price = ExpectedPayoff \times e^{-r\times T} \tag{17}$$

Similarly, the American call option has the same property as the European call option. However, it has extra early exercise rights. The time to expiration T_1 and T_2 are 10 and 29 days. Therefore, we assume that the value of underlying assets can only change once each day. Besides, holders of American call-on-call options could only decide to early exercises between T_1 and T_2 since it is meaningless to long already exercised American options at time T_1.

According to the equation, the expected payoff of the American call option at time t is

$$Payoff = Max\left\{S_t - K, 0\right\} \tag{18}$$

In order to model the American call option, we assume that Holders are entirely rational investors. They will execute immediately as they find early exercising earns more profit by predicting the payoff at time $t+1$ like the following inequation.

$$\begin{aligned} Price_t > &[\pi Max\left\{uS_t - K, 0\right\} \\ &+ (1-\pi)\, Max\left\{dS_t - K, 0\right\}] \times e^{-r} \end{aligned} \tag{19}$$

In order to analyze whether it is profitable for early exercising, we choose to evaluate the payoff at each node from the back and decide whether it is needed for early exercises.

In conclusion, the price formula of European and American call-on-call options are

$$P^E = Max\left(C^E\left(S_0, \alpha S_{T_1}, \sigma, r_f, T_2\right) - K_1, 0\right) \tag{20}$$

$$P^A = Max\left(C^A\left(S_0, \alpha S_{T_1}, \sigma, r_f, T_1, t\right) - K_1, 0\right) \tag{21}$$

$$where\ T_1 \le t \le T_2 \tag{22}$$

C. Black-Scholes model and Monte Carlo

In this section, we use the Monte Carlo method to estimate option values on ETF, which has zero dividend-paying. The assumption of risk neutrality is used to evaluate the option price by Cox and Ross (1976) and Boyle (1977). This assumption implies that the equilibrium rate of return on all assets (stocks (ETF) in particular in this case) is equal to the risk-free rate.

The expected return on the stock (ETF) is given by

$$E\left(S_T/S_t\right) = e^{r_f(T-t)} \tag{23}$$

In addition, it is assumed that the ratio of successive stocks (ETF) values follows a log-normal distribution, and so the ratio S_{t+1}/S_t has a log-normal distribution with a mean equal to e^{r_f}

$$E\left(S_T/S_t\right) = exp\left(r_f\right) = exp\left[\left(r_f - \frac{1}{2}\sigma^2\right)^2 + \frac{1}{2}\sigma^2\right] \tag{24}$$

Using the properties of the log-normal distribution, we can generate the distribution of stocks (ETF) prices for one period hence by forming the random variables,

$$S_{t+1} = S_t exp\left[r_f - \frac{1}{2}\sigma^2 + \sigma\omega\right] \tag{25}$$

$$Where\ \omega \sim N\left(0, 1\right) \tag{26}$$

We set up simulations by generating S_{T+t},

$$S_{T+t} = S_t exp\left[(r_f - \frac{1}{2}\sigma^2)T + \sigma\omega\sqrt{T}\right] \tag{27}$$

and then we could generate a series of payoff C_t

$$C_{T+t} = Max\left\{S_{T+t} - K, 0\right\} \tag{28}$$

In particular, the price of stock at time T_1 and T_2 is given by

$$S_{T_1} = S_0 exp\left[(r_f - \frac{1}{2}\sigma^2)T_1 + \sigma\omega\sqrt{T_1}\right] \tag{29}$$

$$S_{T_2} = S_0 exp\left[(r_f - \frac{1}{2}\sigma^2)T_2 + \sigma\omega\sqrt{T_2}\right] \tag{30}$$

The payoff of the underlying option at its expiration time T_2 is

$$C_{T_2} = Max\left\{S_{T_2} - \alpha S_{T_1}, 0\right\} \tag{31}$$

By calculating the expectation of the discounted expected value of payoff T_2,we have evaluated the price of option at time T_1,

$$C_{T_1} = Max\left\{S_{T_2} - \alpha S_{T_1}, 0\right\} D_{T_2 - T_1} \tag{32}$$

where $D_{T_2-T_1}$ is the discount factor, and it is given by

$$D_{T_2-T_1} = exp\left(-r_f\left(T_2 - T_1\right)\right) \tag{33}$$

So, at the expiration time of derivative X(which is time T_1), the value of derivative X is

$$V_{T_1} = E\left[Max\left\{C_{T_1} - K_1, 0\right\}\right]$$
$$= E\left[Max\left\{Max\left\{S_{T_2} - \alpha S_{T_1}, 0\right\} D_{T_2 - T_1} - K_1, 0\right\}\right] \quad (34)$$

and the value of derivative X at time zero is

$$V_0 = D_{T_1} V_{T_1} = V_{T_1} exp(-r_f T_1) \quad (35)$$

D. Greek Letters

As for the definition of Greek letters, the following table summarizes them as

Greek Letter	Definition
Delta	Delta (Δ) represents the rate of change between the option's price and a $1 change in the underlying asset's price.
Gamma	Gamma (Γ) represents the rate of change between an option's delta and the underlying asset's price. This is called second-order (second-derivative) price sensitivity.
Theta	Theta (Θ) represents the rate of change between the option price and time, or time sensitivity.
Vega	Vega (ν) represents the rate of change between an option's value and the underlying asset's implied volatility.

Fig. 3. Summary of the Definition of Greek Letters

Based on the definition, to calculate the Greek letters, the following notations will be used

$$\Delta = \frac{\partial V\left(S, K, \sigma, r, T, \delta\right)}{\partial S} \quad (36)$$

$$\Gamma = \frac{\partial^2 V\left(S, K, \sigma, r, T, \delta\right)}{\partial^2 S} \quad (37)$$

$$\theta = \frac{\partial V\left(S, K, \sigma, r, T - t, \right)}{\partial t} \quad (38)$$

$$\nu = \frac{\mathrm{d} V\left(S, K, \sigma, r, T, \delta\right)}{\mathrm{d}\sigma} \quad (39)$$

However, the compound call option has no exact calculation formula for Greek letters, which means that the calculation procedure for Greek letters should be modified, so there is no direct way to evaluate them. Zhang, Wan, Lim, and Man (2013) provided a method to solve this problem. Intuitively, derivative means the rate of change of a function concerning a variable, and it can be written as

$$\frac{\partial f\left(x\right)}{\partial x} \approx \frac{f\left(x + \Delta x\right) - f\left(x\right)}{\Delta x} \quad (40)$$

In the CRR binomial tree model, the payoff of the call option at each time t can be written as

$$P\left(t, 0\right) = Max\left\{uS_{t-1} - K, 0\right\} \quad (41)$$

$$P\left(t, 1\right) = Max\left\{dS_{t-1} - K, 0\right\} \quad (42)$$

$P\left(t, 0\right)$ and $P\left(t, 1\right)$ represent the value at the upper and lower node at time t. Suppose we rewrite x and Δx as

$$x = dS_{t-1} \quad (43)$$

$$\Delta x = uS_{t-1} - dS_{t-1} = S_{t-1}\left(u - d\right) \quad (44)$$

So that the payoff can also be rewritten as

$$P\left(t, 0\right) = Max\left\{x + \Delta x - K, 0\right\} \quad (45)$$

$$P\left(t, 1\right) = Max\left\{x - K, 0\right\} \quad (46)$$

Considering the above equation and treating $P\left(t, 0\right)$ and $P\left(t, 1\right)$ are functions with respect to S, we note that the delta formula is

$$\Delta = \frac{P\left(t, 0\right) - P\left(t, 1\right)}{uS_{t-1} - dS_{t-1}}$$
$$= \frac{Max\left\{uS_{t-1} - K, 0\right\} - Max\left\{dS_{t-1} - K, 0\right\}}{uS_{t-1} - dS_{t-1}} \quad (47)$$

Furthermore, Gamma is the second derivative function. It can be rewritten as

$$\Gamma = \frac{\frac{P(t+1,0) - P(t+1,1)}{u^2 S_{t-1} - udS_{t-1}} - \frac{P(t+1,1) - P(t+1,2)}{udS_{t-1} - d^2 S_{t-1}}}{\frac{u^2 S_{t-1} - d^2 S_{t-1}}{2}} \quad (48)$$

$P\left(t + 1, 0\right)$, $P\left(t + 1, 1\right)$, and $P\left(t + 1, 2\right)$ represent the payoff at top, middle and down nodes at time t+1.

As for theta, in compound call options, there are two types of time to expiration, so it is important to discuss them separately. Similar to delta, the calculation procedure is

$$\Theta_1 = \frac{V(S,K,\sigma,r,T_1+\Delta t,T_2) - V(S,K,\sigma,r,,T_1,T_2)}{\Delta t} \tag{49}$$

$$\Theta_2 = \frac{V(S,K,\sigma,r,T_1,T_2+\Delta t) - V(S,K,\sigma,r,,T_1,T_2)}{\Delta t} \tag{50}$$

In this project, we assume Δt is one day.

As for Vega, the formula is similar to theta.

$$\nu = \frac{V(S,K,\sigma+\Delta\sigma,r,T_1,T_2) - V(S,K,\sigma,r,,T_1,T_2)}{\Delta\sigma} \tag{51}$$

E. Implied Volatility

Unlike underlying asset volatility, which is derived from the log return of underlying assets, implied volatility is derived from the real market price of call options.

The option of Shanghai stock 50 ETF is European style, so we will only use these volatilities to price the European call option.

According toWunkaew, Liu, and Golubnichiy (2022), the Newton-Raphson method could be implemented to estimate implied volatilities. The algorithm can be written as the formula shown below:

$$\sigma_{n+1} = \sigma_n - \frac{V(\sigma_n)}{V'(\sigma_n)} \tag{52}$$

$$V'(\sigma_n) \approx \frac{V(\sigma_n+\Delta\sigma) - V(\sigma_n)}{\Delta\sigma} \tag{53}$$

In this project, initial volatility σ_0 is 0.5, the threshold is 10^{-5}, and the maximum iteration time is 100. Only if the difference $\sigma_{n+1} - \sigma_n$ is less than 10^{-5} or the iteration steps are larger than 100, this algorithm will not stopped. After estimating the implied volatilities, we use them to measure the value of the European call option through CRR binomial tree method.

In the real market, options related to Shanghai 50 ETF have strike prices of 2.5, 2.6, and 2.7, which are already defined. Therefore, in this research, suppose T_1 is 10 and T_2 is 29. The strike price and corresponding α are as following:

	K=2.5	K=2.6	K=2.7
α	0.96154	1	1.03846

Fig. 4. The α and related Strike Prices

IV. Results

A. Values of Parameters

We choose SSE 50 Index as underlying asset. The values of parameters in this project are shown in figure 5.

Notation	Description	Value
S_0	Initial price of SEE 50 Index at 11/17/2022	2.60
K_1	Option strike of SSE 50 Index	2.50
K_2	Option strike of SSE 50 Index	2.60
K_3	Option strike of SSE 50 Index	2.70
T_1	Maturity of the underlying option	29
T_2	Maturity of compound option	10
r	Risk-free interest rate(daily)	0.000041
σ	Volatility(daily)	0.0136

Fig. 5. Parameters of this project

B. CRR Binomial Tree Model

We assume that the strike price of second layer options K_1 is 0.05, the time to expiration T_1 and T_2 are 10 and 29 days. Based on CRR binomial tree model, the European and American call-on-call prices should be

	α=0.9	α=1	α=1.1
European Call	0.2189	0.0389	0.0006
American Call	0.2877	0.0394	0.0006

Fig. 6. The European and American Call-on-Call prices

The above table shows the values of the American call option are always higher than the European call option with the same strike price. One of the reasons is American call option has extra early exercise rights so that it may have more premium in the pricing procedure.

Additionally, we simulated the option price concerning different underlying asset values and time to expiration.

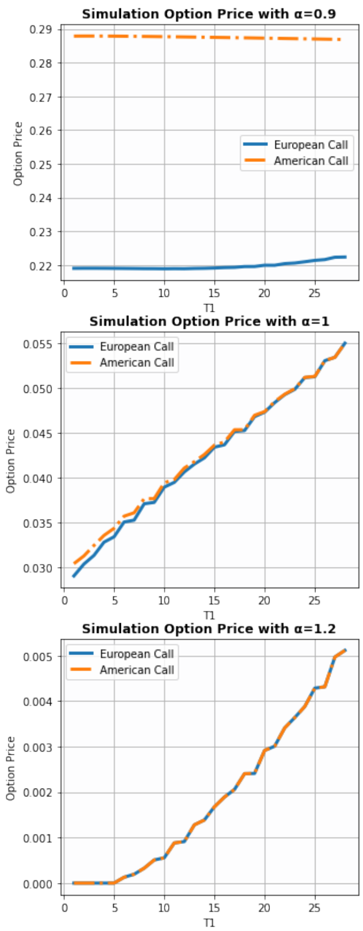

Fig. 7. The Relationship between T_1 and Option Prices

Two points should be concerned in figure 7. First, no matter the alpha value, the values of call-on-call options tend to be positively correlated with T_1, especially when alpha is larger than or equal to one, which means at time T_1 the underlying option is an at-the-money or out-of-money option.Two points should be concerned. First, no matter the alpha value, the values of call-on-call options tend to be positively correlated with T_1, especially when alpha is larger than or equal to one, which means at time T_1 the underlying option is an at-the-money or out-of-money option. Considering the definition of theta, it should always be positive. It can be explained that as T_1 increases, the time interval between 0 and T_1 will be expanded. Holders of the call-on-call option will benefit from it. Once this interval is extended, the unknown total price movement will be limited, and the payoff of call-on-call options tends to be more stable.

Besides, when alpha is less than 1, which means that the strike price of underlying options is lower than the expected value of underlying stocks, the price of American call-on-call option price is always higher than European. Extra early exercise rights produce more profits because the holder of underlying American call options tend to early exercise if the value of stocks is higher than the strike prices. In this project, we did a simulation to test the relationship between the alpha and option price. It clearly shows that as α is less than one, American call-on-call options have higher price than the European.

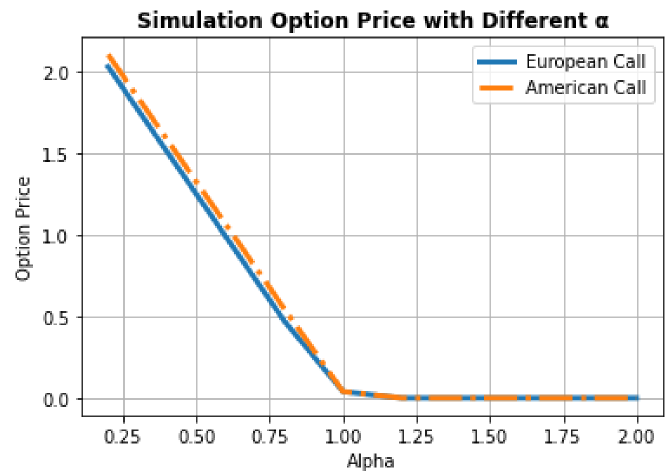

Fig. 8. The Relationship between α and Option Prices

As for the relationship between K_1 and call-on-call option price, in figure 9, a higher strike price obviously means less payoff for call options. The third layer is also a European call option, so the total value tends to decrease as its strike price increases. Besides, once alpha is less than 1, American call-on-call options tend to have higher value, which has been proved.

Fig. 9. The Relationship between K_1 and Option Prices

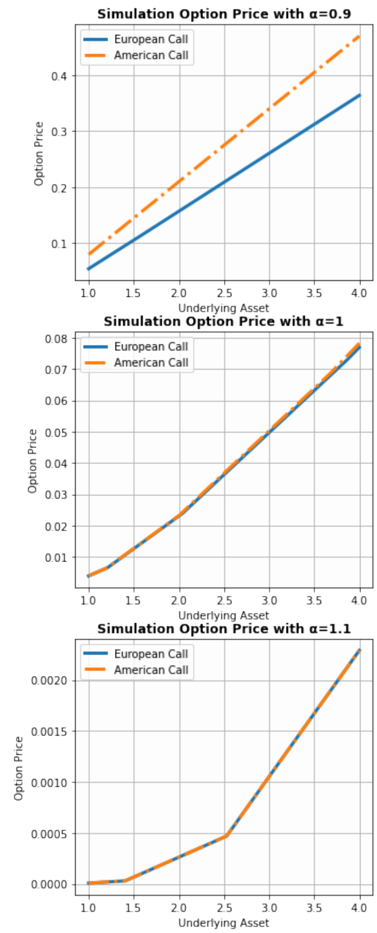

Fig. 10. The Relationship between S_0 and Option Prices

As for the initial value of underlying stocks, the simulation results show that it is positively correlated with option price. It means that the Delta and Gamma should always be positive. Additionally, option prices seem to follow a piece-wise nonlinear model.

C. BSM and Monte Carlo

By using the Monte Carlo method, we could generate several stock-price process, figure 11 shows the simulations for stock-price process of 30 trials.

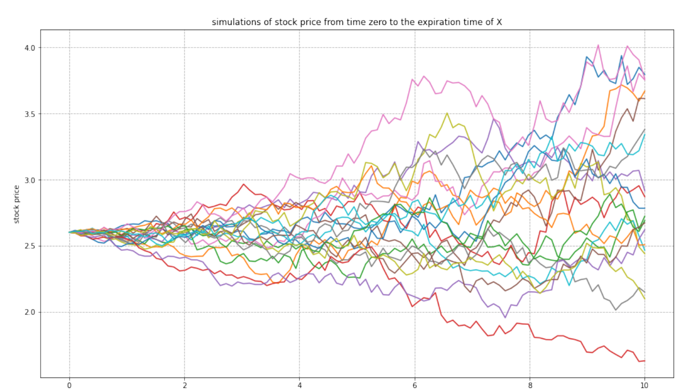

Fig. 11. Simulations of Stock Price from time zero to the Expiration Time of X

European call option values with several trials

Strike price K	α	Number of simulations	Option values by Monte Carlo	Option values by binary tree
0	0.1	500	2.3290	2.3403
	0.5	500	1.2930	1.3015
	1.0	500	0.0924	0.0789
	1.5	500	0	0
	0.1	1000	2.3441	2.3403
	0.5	1000	1.2960	1.3015
	1.0	1000	0.0931	0.0789
	1.5	1000	0	0
1	0.1	500	1.3475	1.3407
	0.5	500	0.3113	0.302
	1.0	500	0	0
	1.5	500	0	0
	0.1	1000	1.3477	1.3407
	0.5	1000	0.3011	0.302
	1.0	1000	0	0
	1.5	1000	0	0

Fig. 12. European call option values with several trials

From figure 12, we can see that CRR binomial tree model and Monte Carlo model produce similar results.

Besides, we simulated the influence of α for the option price, with Monte Carlo simulation, their relationship is similar to figure 8.

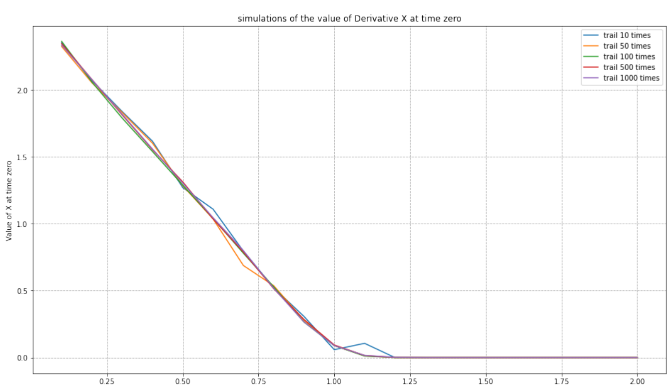

Fig. 13. simulations to evaluate the price of derivative X under different multipliers of α

D. Greek Letters

The following table summarizes the Delta, Gamma, theta, and vega for European and American call options with the different strike prices.

	European Call-on-Call	American Call-on-Call
Delta	0.0268	0.027
Gamma	0	0
Theta_T1	0.0005	0.0003
Theta_T2	0	0.0002
Vega	5.0527	5.0686

Fig. 14. The result of Greek Letters

The above figure shows the Greek letters of at-the-money options with K_1 equal to 0.05. The American and European options have similar performances based on Greek Letters. The Gamma is close to zero, which means that at-the-money call-on-call options follow a linear equation concerning initial stock values. As for theta, both of them are positive. It is the same as we analyzed before. Positive Vega means that the call-on-call option value is positively correlated with underlying stock volatility.
To further study the property of Greek letters, this project also simulated them.

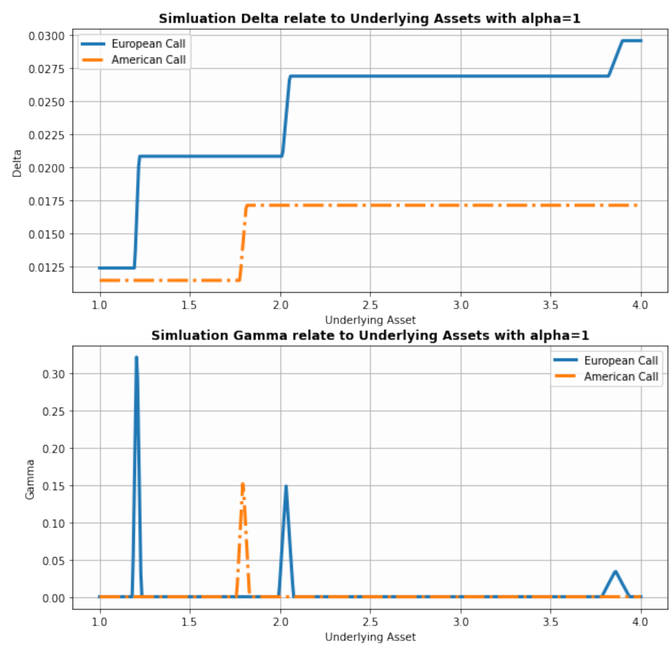

Fig. 15. The simulation results of Delta and Gamma with $\alpha = 1$

Firstly, the simulation results of Delta show that Delta dramatically increases at specific underlying stock values. Additionally, the Delta of American options tends to be lower than European. As for Gamma, although the distribution of both European and American are similar, Americans typically have a lower number of peaks. According to the above figure

10, the movement of American and European call-on-call options are not perfectly correlated, and both of them have no constant changing rate. Furthermore, if we pay attention to the influence of α, the distribution of Delta and Gamma will change.

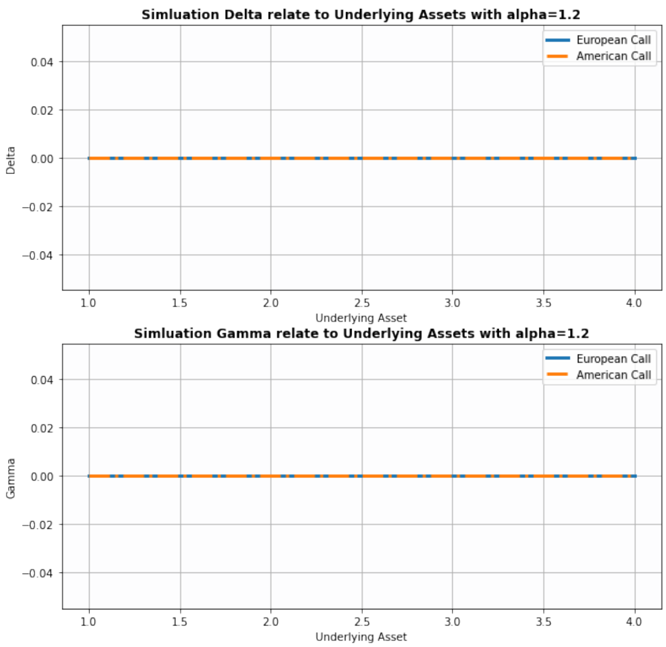

Fig. 16. The simulation results of Delta and Gamma with $\alpha = 1.2$

Suppose the α is 1.2, meaning that the call-on-call option is an out-out-money option, and Delta and Gamma are constantly zero. Besides, the value of this option is also zero, no matter what the underlying price is. It can be explained that higher alpha means a higher strike price of the underlying option and a poorer payoff.

The following figure 17 shows the relationship between Delta, Gamma, and alpha. Regarding alpha, the American and European deltas are close, but Gamma is significantly different. However, as the alpha increases, both Delta and Gamma converge to 0, similar to simple out-of-money call options.

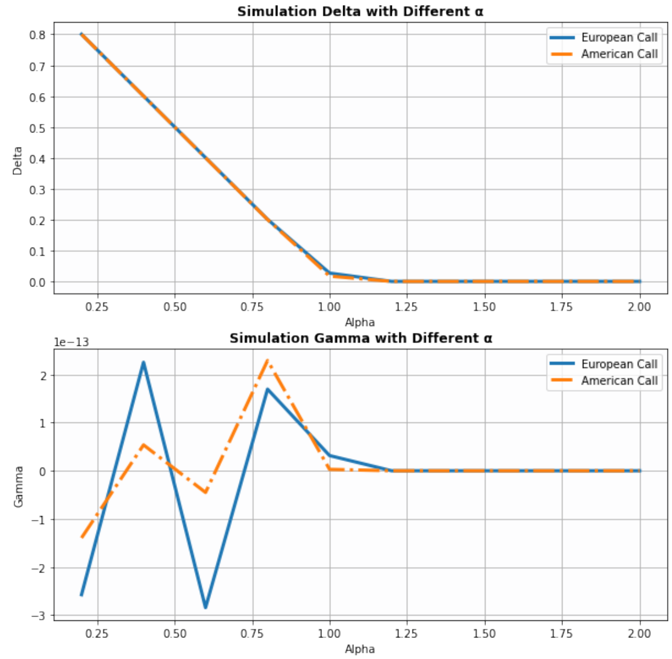

Fig. 17. The simulation results of Delta and Gamma with different α

As for theta, both are always positive, which is already referred to above figure 7. However, the value of both two thetas significantly fluctuated. The influence of time to expiration is not constant. Besides, the American and European theta of T2 are almost the same, but T1 is slightly different. The figure has already shown that.

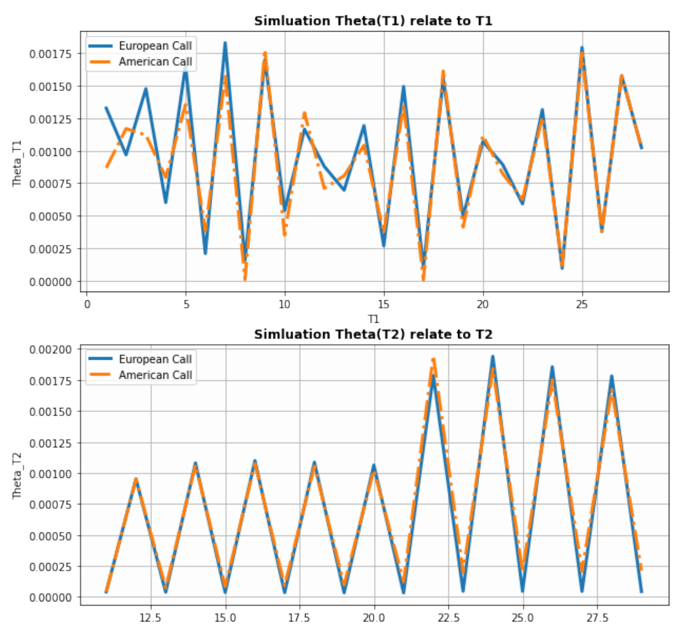

Fig. 18. The simulation results of Delta and Gamma with different T_1 and T_2

E. Implied Volatility

As we have already mentioned, to measure the implied volatility, we use the Newton-Raphson method to calculate it. The following table 19 shows the optimal implied volatility.

	K=2.5	K=2.6	K=2.7	Asset
Volatility	0.0478	0.0337	0.0276	0.0136

Fig. 19. Implied Volatility

The implied volatility seems far from the asset return volatility. The alpha and implied volatility are negatively correlated, and both are larger than the asset volatility. Considering the Vega we have already calculated is positive, the call-on-call options should have higher option values with higher volatility.

	K=2.5	K=2.6	K=2.7
European Call-on-Call (Implied)	0.3179	0.1698	0.0812
European Call-on-Call (Asset)	0.138	0.0567	0.0172

Fig. 20. Price based on Implied Volatility

V. Discussion

1) In the section on using Monte Carlo method to evaluate the value of derivative X, an option on option, there remains three points to go further. First, for the convenience of acquiring data, we use ETF with no dividend instead of a dividend-paying stocks, for it is hard to find a standard stock option in China's derivatives market, for the next time we may choose a stock with both paying dividends and a standardized option. Second, we only evaluate the option price for one day, and we need more empirical research on a series of stock prices threaded by time with rolling window methods, which is more meaningful in practical works. Third, it is also meaningful to double-check the result with Black-Scholes-Merton model, and we will continue in this progress.

2) In this project, we use the approximation method to calculate the Greek letters. However, it may be far from the real value. Besides, this numerical analysis highly depends on the price of options based on the CRR binomial tree. This model may need to measure the real value of call-on-call options correctly. Due to the lack of real market data, it is impossible to test its accuracy.

VI. Conclusions

In this project, we research options pricing by the binary trees model and the Monte Carlo model. The underlying asset we selected is the Shanghai Stock Exchange 50 Index, a sample of the top 50 stocks that are comprehensively ranked according to factors such as total market value and transaction amount to reflect the overall situation of a group of leading enterprises with the most market influence in the Shanghai securities market. We first estimated the historical volatility and elaborated on the dividend yield and risk-free interest rate.

We use CRR binomial tree model to measure the price of compound call options. We found that American in-the-money call-on-call options tend to have a higher value than European. However, as for at-the-money and out-of-money call options, European and American options have similar prices. Besides, the strike price of the second option K_1 is negatively correlated with the option prices.

The Greek letters of compound options are quite different from the simple call options. Firstly, the delta and gamma of compound call options have multiple peaks, while normal call options tend to have one peak. Additionally, the two types of theta of compound call options are positive, however, a normal call option should have a negative value of theta.

In the section we use Monte Carlo method to evaluate the value, with the number of trials increasing, the evaluation of derivative X is closer to the evaluation made by n-step forward tree, and with the value of α increasing, the value of derivative X sharply decreases to zero, and it is not very sensitive to the number of trials

The result of implied volatility is higher than asset volatility. Besides, considering positive vega, higher volatility means higher profit, which means compound call options are good tools for risk mitigation.

VII. Appendix

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