Robust IRP considering budget violation under demand uncertainty

Appendix: Values of some parameters in the case study

Table A1 lists the distances between the nodes. Table A2 lists the initial inventory levels and maximum inventory capacities of retailers. Table A3 lists the representative demand points and their probabilities of retailers. Table A4 lists the expectations, lower bounds, upper bounds, and upper bounds of MAD of demands of retailers.

Table A1

The distances (km) between vendor and retailers.

Distance	1	2	3	4	5	6	7	8	9	10
0	14	8.5	17.5	13.6	7.4	12.1	12.5	9.3	9.4	16
1	0	5.5	5.3	4.4	8.4	3	2.6	5.5	5.3	1.9
2	5.5	0	10.9	3.9	2.7	2.8	3	3.3	6.3	6.9
3	5.3	10.9	0	9.3	13.5	7.5	7.2	10.8	9.4	3.8
4	4.4	3.9	9.3	0	6.8	6.1	5.1	6.8	9.8	3.3
5	8.4	2.7	13.5	6.8	0	6	6.4	4.7	6	11.1
6	3	2.8	7.5	6.1	6	0	1.5	3.1	4.4	4.9
7	2.6	3	7.2	5.1	6.4	1.5	0	3.2	5.5	4.2
8	5.5	3.3	10.8	6.8	4.7	3.1	3.2	0	3.1	6.9
9	5.3	6.3	9.4	9.8	6	4.4	5.5	3.1	0	6.2
10	1.9	6.9	3.8	3.3	11.1	4.9	4.2	6.9	6.2	0

Table A2

The initial inventory levels and maximum inventory capacities of retailers.

Distance	1	2	3	4	5	6	7	8	9	10
$W_{0,i}$	35	35	39	33	37	45	27	36	34	27
C_i	216	216	210	232	211	245	225	210	212	210

Table A3

Representative demand points and their probabilities of retailers.

	$\hat{d}_{i,1}$	$P(\hat{d}_{i,1})$	$\hat{d}_{i,2}$	$P(\hat{d}_{i,2})$	$\hat{d}_{i,3}$	$P(\hat{d}_{i,3})$		$\hat{d}_{i,1}$	$P(\hat{d}_{i,1})$	$\hat{d}_{i,2}$	$P(\hat{d}_{i,2})$	$\hat{d}_{i,3}$	$P(\hat{d}_{i,3})$
R_1	105	0.49	126	0.26	85	0.25	R_6	161	0.38	129	0.47	96	0.15
R_2	106	0.38	81	0.32	131	0.3	R_7	82	0.42	56	0.3	106	0.28
R_3	93	0.26	146	0.17	119	0.57	R_8	112	0.36	85	0.34	137	0.3
R_4	98	0.49	128	0.26	74	0.25	R_9	99	0.49	71	0.17	125	0.34
R_5	83	0.24	132	0.28	110	0.48	R_{10}	85	0.45	58	0.3	110	0.25

Note: $R_{i,i\epsilon\{1,\cdots,10\}}$ represents the ith retailer.

Table A4

The expectations, lower bounds, upper bounds, and upper bounds of MAD of demands of retailers.

	1	2	3	4	5	6	7	8	9	10
Expectations of demands of retailers										
$t = 1$	105	102	113	96	110	137	80	103	103	82
$t = 2$	105	104	116	100	106	135	79	113	105	84
$t = 3$	106	110	123	104	113	136	84	115	101	84
Upper bounds of MAD of demands of retailers										
$t = 1$	13	20	10	18	17	19	19	17	15	16
$t = 2$	15	15	17	16	15	18	17	20	16	19
$t = 3$	11	17	16	16	11	23	16	17	18	16
Upper bounds of demands of retailers										
$t = 1$	139	154	132	153	161	180	111	143	157	130
$t = 1$	138	137	160	145	149	180	118	151	147	149
$t = 1$	151	152	183	141	141	200	149	151	139	116
Lower bounds of demands of retailers										
$t = 1$	71	61	79	41	68	59	33	64	53	36
$t = 1$	70	68	64	62	75	83	35	67	67	25
$t = 1$	73	56	74	58	56	85	44	64	59	47

The demand scenarios and nominal probabilities after normalizing

Demand scenarios of retailers based on $S = 3$

$d^1_{t,i}$ = [[105, 81, 119, 98, 110, 129, 82, 112, 99, 85], [105, 106, 119, 98, 110, 129, 82, 112, 99, 58], [105, 106, 119, 98, 83, 129, 82, 85, 125, 85]];

$d^2_{t,i}$ = [[105, 106, 146, 128, 110, 129, 56, 112, 71, 85], [126, 106, 119, 98, 110, 161, 82, 112, 99, 85], [105, 106, 119, 98, 110, 129, 82, 112, 99, 110]];

$d^3_{t,i}$ = [[105, 106, 93, 98, 83, 129, 82, 85, 99, 85], [105, 81, 119, 98, 110, 129, 56, 112, 99, 85], [126, 106, 119, 128, 110, 96, 82, 112, 99, 85]];

Table A5

Nominal value of probability vector after normalizing based on $S = 3$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P^0_{i,s_1}	0.4851	0.3137	0.57	0.4851	0.25	0.47	0.4118	0.3269	0.34	0.3
P^0_{i,s_2}	0.2574	0.3725	0.17	0.2574	0.5	0.38	0.2941	0.3462	0.17	0.25
P^0_{i,s_3}	0.2574	0.3137	0.26	0.2574	0.25	0.15	0.2941	0.3269	0.49	0.45

Demand scenarios of retailers based on $S = 6$

$d^1_{t,i}$ = [[105, 106, 119, 98, 110, 129, 82, 112, 99, 85], [105, 106, 119, 98, 83, 129, 56, 112, 99, 58], [105, 106, 119, 98, 110, 96, 82, 112, 71, 110]];

$d^2_{t,i}$ = [[105, 106, 146, 98, 132, 161, 56, 112, 71, 85], [126, 81, 119, 128, 110, 129, 82, 85, 71, 85], [105, 81, 119, 98, 83, 129, 82, 85, 99, 85]];

$d^3_{t,i}$ = [[105, 106, 119, 98, 110, 129, 56, 112, 99, 85], [105, 81, 119, 128, 132, 96, 82, 137, 99, 58], [85, 106, 146, 128, 110, 129, 56, 112, 99, 85]];

$d^4_{t,i}$ = [[105, 106, 93, 128, 110, 129, 82, 85, 99, 85], [85, 106, 119, 98, 110, 96, 82, 112, 125, 58], [105, 131, 119, 128, 110, 96, 82, 112, 71, 58]];

$d^5_{t,i}$ = [[105, 106, 146, 74, 83, 129, 82, 112, 71, 58], [105, 131, 119, 98, 110, 129, 82, 85, 99, 85], [126, 131, 93, 98, 110, 129, 56, 112, 99, 85]];

$d^6_{t,i}$ = [[85, 81, 119, 128, 83, 129, 82, 112, 99, 85], [126, 106, 146, 98, 110, 161, 106, 112, 71, 85], [105, 106, 119, 98, 83, 161, 82, 85, 99, 58]];

Table A6

Nominal value of probability vector after normalizing based on $S = 6$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P^0_{i,s_1}	0.2983	0.2081	0.4021	0.319	0.16	0.0997	0.1654	0.1799	0.1444	0.0971
P^0_{i,s_2}	0.1583	0.1476	0.1199	0.1693	0.0933	0.2525	0.1654	0.1605	0.0501	0.2621
P^0_{i,s_3}	0.1522	0.1752	0.1199	0.0898	0.1867	0.0997	0.1181	0.1499	0.4163	0.1748
P^0_{i,s_4}	0.1522	0.1643	0.1834	0.0898	0.32	0.0318	0.2315	0.1699	0.1002	0.1165
P^0_{i,s_5}	0.1583	0.1297	0.0547	0.1628	0.16	0.3123	0.1654	0.1699	0.1444	0.1748
P^0_{i,s_6}	0.0808	0.1752	0.1199	0.1693	0.08	0.2041	0.1543	0.1699	0.1444	0.1748

Demand scenarios of retailers based on $S = 9$

$d^1_{t,i}$ = [[105, 131, 93, 128, 83, 161, 56, 112, 99, 85], [105, 106, 119, 98, 110, 161, 56, 112, 99, 85], [105, 106, 146, 74, 83, 129, 82, 112, 125, 85]];

$d^2_{t,i}$ = [[126, 131, 93, 98, 110, 129, 56, 112, 99, 85], [126, 131, 119, 74, 110, 129, 82, 112, 125, 58], [105, 106, 119, 128, 110, 96, 56, 85, 99, 85]];

$d^3_{t,i}$ = [[105, 106, 119, 98, 110, 96, 106, 85, 125, 85], [105, 81, 93, 128, 110, 129, 82, 112, 99, 110], [126, 81, 119, 98, 83, 129, 82, 137, 71, 58]];

$d^4_{t,i}$ = [[105, 106, 119, 98, 83, 129, 82, 85, 99, 58], [105, 106, 146, 98, 110, 129, 82, 85, 99, 85], [85, 131, 146, 98, 110, 129, 56, 112, 71, 58]];

$d^5_{t,i}$ = [[126, 106, 146, 128, 110, 161, 56, 85, 71, 85], [105, 106, 119, 98, 83, 129, 82, 112, 125, 58], [126, 106, 93, 98, 110, 129, 106, 85, 99, 110]];

$d^6_{t,i}$ = [[126, 106, 146, 128, 110, 129, 56, 112, 71, 110], [85, 131, 119, 98, 110, 161, 82, 85, 99, 85], [105, 106, 146, 128, 132, 129, 82, 112, 71, 85]];

$d^7_{t,i}$ = [[105, 131, 119, 74, 83, 96, 82, 137, 71, 58], [126, 81, 119, 98, 83, 161, 82, 112, 71, 58], [105, 106, 119, 128, 83, 129, 106, 85, 99, 58]];

$d^8_{t,i}$ = [[126, 81, 119, 98, 110, 129, 82, 85, 125, 58], [105, 106, 146, 98, 132, 96, 82, 112, 99, 85], [105, 106, 119, 128, 110, 129, 82, 112, 125, 110]];

$d^9_{t,i}$ = [[105, 106, 119, 98, 132, 96, 82, 112, 99, 58], [85, 106, 119, 74, 83, 129, 56, 112, 99, 85], [105, 81, 146, 98, 110, 161, 56, 137, 99, 85]];

Table A7

Nominal value of probability vector after normalizing based on $S = 9$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P^0_{i,s_1}	0.2249	0.112	0.046	0.0645	0.0577	0.1378	0.0884	0.1246	0.1762	0.2109
P^0_{i,s_2}	0.0633	0.0884	0.1542	0.0645	0.2308	0.0673	0.0884	0.1177	0.1762	0.1406
P^0_{i,s_3}	0.1193	0.1006	0.1542	0.1265	0.1154	0.0673	0.1156	0.0981	0.0611	0.0781
P^0_{i,s_4}	0.1148	0.112	0.0301	0.2383	0.1154	0.2108	0.1238	0.1112	0.0881	0.0938
P^0_{i,s_5}	0.0633	0.1418	0.046	0.1265	0.1154	0.1704	0.0825	0.1112	0.0611	0.0781
P^0_{i,s_6}	0.0609	0.112	0.0301	0.0671	0.1346	0.1704	0.1238	0.1177	0.0306	0.1172
P^0_{i,s_7}	0.1193	0.0943	0.338	0.0645	0.0288	0.0544	0.1156	0.0981	0.0306	0.0625
P^0_{i,s_8}	0.1193	0.1194	0.1008	0.1265	0.1346	0.0673	0.1733	0.1177	0.1223	0.0781
P^0_{i,s_9}	0.1148	0.1194	0.1008	0.1216	0.0673	0.0544	0.0884	0.1038	0.2539	0.1406

Demand scenarios of retailers based on $S = 12$

$d^1_{t,i}$ = [[105, 106, 146, 98, 110, 129, 56, 137, 99, 85], [85, 131, 146, 128, 110, 161, 82, 112, 99, 85], [85, 106, 119, 74, 83, 129, 56, 137, 125, 85]];

$d^2_{t,i}$ = [[126, 106, 119, 98, 110, 129, 106, 112, 99, 110], [126, 81, 119, 74, 110, 96, 106, 85, 99, 85], [105, 106, 146, 128, 110, 96, 82, 85, 71, 85]];

$d^3_{t,i}$ = [[105, 131, 146, 128, 110, 129, 82, 112, 99, 85], [126, 106, 119, 98, 132, 129, 56, 85, 125, 110], [105, 106, 146, 74, 110, 96, 82, 112, 125, 85]];

$d^4_{t,i}$ = [[105, 106, 146, 98, 110, 161, 82, 85, 71, 85], [105, 106, 93, 98, 83, 161, 82, 112, 99, 85], [126, 131, 119, 98, 110, 129, 56, 112, 71, 58]];

$d^5_{t,i}$ = [[126, 106, 119, 98, 83, 96, 56, 112, 99, 58], [105, 106, 146, 74, 110, 129, 82, 112, 125, 85], [126, 81, 119, 98, 110, 96, 106, 85, 71, 85]];

$d^6_{t,i}$ = [[126, 106, 146, 128, 132, 161, 82, 85, 99, 110], [105, 81, 146, 98, 132, 161, 56, 85, 71, 58], [85, 81, 146, 98, 110, 96, 106, 112, 125, 85]];

$d^7_{t,i}$ = [[105, 106, 146, 74, 83, 129, 56, 85, 71, 58], [105, 106, 119, 98, 110, 129, 82, 112, 125, 110], [105, 106, 119, 98, 132, 129, 82, 85, 71, 110]];

$d^8_{t,i}$ = [[85, 131, 119, 98, 83, 129, 106, 112, 99, 85], [105, 131, 93, 128, 110, 96, 82, 137, 125, 85], [105, 106, 146, 98, 83, 129, 82, 112, 99, 110]];

$d^9_{t,i}$ = [[85, 131, 93, 98, 110, 161, 56, 137, 71, 58], [126, 106, 119, 98, 83, 129, 106, 85, 71, 85], [105, 131, 119, 128, 83, 96, 56, 112, 99, 110]];

$d^{10}_{t,i}$ = [[105, 81, 93, 98, 132, 96, 82, 112, 99, 58], [126, 106, 119, 128, 83, 96, 106, 112, 99, 58], [126, 81, 146, 128, 110, 129,82, 112, 99, 85]];

$d^{11}_{t,i}$ = [[105, 81, 119, 128, 132, 129, 56, 137, 71, 110], [105, 131, 119, 128, 83, 129, 56, 112, 71, 85], [85, 106, 119, 74, 132, 161, 82, 85, 125, 58]];

$d^{12}_{t,i}$ = [[126, 106, 119, 128, 110, 96, 82, 112, 99, 85], [105, 81, 93, 128, 110, 161, 82, 137, 71, 58], [105, 131, 119, 98, 132, 161, 82, 137, 99, 58]];

Table A8

Nominal value of probability vector after normalizing based on $S = 12$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P^0_{i,s_1}	0.0495	0.0873	0.026	0.0528	0.095	0.1653	0.0705	0.0674	0.1558	0.1611
P^0_{i,s_2}	0.0535	0.0931	0.0872	0.0528	0.1901	0.0208	0.0614	0.0866	0.0779	0.0895
P^0_{i,s_3}	0.1009	0.0873	0.026	0.0528	0.1109	0.0653	0.0987	0.0916	0.1081	0.0895
P^0_{i,s_4}	0.1009	0.0873	0.0398	0.1949	0.095	0.1337	0.0987	0.0916	0.027	0.1074
P^0_{i,s_5}	0.0535	0.0931	0.0872	0.0994	0.095	0.0208	0.0658	0.0916	0.0541	0.1074
P^0_{i,s_6}	0.0515	0.0784	0.0078	0.1034	0.0647	0.0427	0.0658	0.0866	0.0541	0.0597
P^0_{i,s_7}	0.1902	0.1105	0.0872	0.0994	0.0554	0.2045	0.0987	0.0866	0.0188	0.0332
P^0_{i,s_8}	0.097	0.0689	0.0398	0.1034	0.0475	0.0653	0.0922	0.0809	0.1558	0.0895
P^0_{i,s_9}	0.0515	0.0689	0.1334	0.1034	0.0475	0.0528	0.047	0.0764	0.027	0.0597
$P^0_{i,s_{10}}$	0.0535	0.0784	0.0398	0.0549	0.0554	0.0208	0.0922	0.097	0.2246	0.0716
$P^0_{i,s_{11}}$	0.097	0.0735	0.2924	0.028	0.0323	0.1653	0.0705	0.0764	0.0188	0.0597
$P^0_{i,s_{12}}$	0.1009	0.0735	0.1334	0.0549	0.1109	0.0427	0.1382	0.0674	0.0779	0.0716

Demand scenarios of retailers based on $S = 15$

$d^1_{t,i}$ = [[85, 106, 119, 128, 110, 129, 82, 112, 71, 85], [126, 81, 93, 128, 83, 161, 82, 112, 99, 58], [105, 106, 119, 128, 110, 96, 106, 112, 125, 85]];

$d^2_{t,i}$ = [[105, 81, 93, 128, 83, 161, 82, 137, 99, 110], [85, 106, 93, 98, 132, 129, 106, 112, 99, 110], [105, 106, 119, 128, 110, 129, 56, 85, 71, 85]];

$d^3_{t,i}$ = [[105, 131, 146, 74, 132, 96, 82, 85, 99, 110], [105, 131, 93, 98, 132, 161, 82, 137, 125, 85], [126, 106, 119, 74, 110, 129, 82, 85, 71, 58]];

$d^4_{t,i}$ = [[85, 131, 93, 98, 110, 96, 82, 85, 125, 58], [105, 106, 146, 98, 83, 96, 56, 85, 125, 85], [85, 81, 119, 128, 83, 129, 82, 112, 99, 85]];

$d^5_{t,i}$ = [[126, 131, 93, 128, 110, 129, 82, 112, 71, 85], [105, 131, 146, 74, 110, 96, 82, 85, 71, 85], [105, 81, 146, 74, 83, 96, 56, 137, 125, 58]];

$d^6_{t,i}$ = [[105, 81, 119, 98, 110, 129, 106, 137, 71, 85], [105, 106, 119, 128, 110, 96, 56, 112, 125, 110], [105, 131, 119, 74, 110, 161, 56, 137, 99, 58]];

$d^7_{t,i}$ = [[105, 106, 146, 128, 132, 129, 82, 85, 71, 58], [85, 106, 119, 98, 83, 129, 56, 137, 99, 85], [85, 106, 119, 98, 132, 96, 56, 112, 99, 110]];

$d^8_{t,i}$ = [[126, 131, 146, 98, 83, 129, 56, 85, 125, 85], [105, 81, 119, 98, 83, 129, 56, 112, 71, 58], [126, 106, 146, 98, 110, 129, 82, 112, 99, 58]];

$d^9_{t,i}$ = [[85, 131, 119, 98, 83, 161, 56, 85, 99, 85], [85, 81, 119, 128, 132, 161, 82, 85, 125, 85], [85, 131, 146, 98, 83, 129, 82, 85, 99, 85]];

$d_{t,i}^{10}$ = [[105, 81, 93, 98, 83, 129, 106, 85, 99, 85], [85, 106, 93, 98, 110, 161, 82, 112, 71, 85], [126, 81, 146, 74, 132, 129, 106, 85, 99, 110]];

$d_{t,i}^{11}$ = [[105, 81, 146, 128, 110, 161, 56, 112, 125, 85], [126, 131, 146, 98, 132, 96, 56, 112, 99, 110], [105, 131, 146, 74, 132, 129, 56, 85, 99, 110]];

$d_{t,i}^{12}$ = [[85, 81, 146, 74, 83, 96, 106, 112, 71, 110], [126, 131, 146, 74, 110, 129, 106, 137, 99, 85], [85, 81, 119, 74, 83, 129, 82, 85, 71, 85]];

$d_{t,i}^{13}$ = [[85, 106, 119, 98, 110, 129, 82, 112, 99, 110], [126, 106, 146, 74, 83, 161, 106, 85, 71, 85], [126, 81, 146, 128, 132, 161, 82, 85, 125, 110]];

$d_{t,i}^{14}$ = [[126, 81, 119, 128, 83, 129, 82, 112, 71, 58], [105, 81, 146, 128, 110, 96, 106, 112, 71, 85,], [85, 106, 93, 98, 110, 129, 106, 137, 99, 58]];

$d_{t,i}^{15}$ = [[126, 106, 93, 98, 110, 161, 82, 85, 99, 58], [85, 81, 119, 128, 132, 129, 56, 112, 99, 58], [85, 81, 119, 128, 110, 96, 106, 137, 99, 85]];

Table A9

Nominal value of probability vector after normalizing based on $S = 15$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P_{i,s_1}^0	0.0514	0.0809	0.1295	0.0275	0.0876	0.0405	0.0786	0.0793	0.0443	0.0891
P_{i,s_2}^0	0.0968	0.0809	0.0591	0.0518	0.0511	0.1268	0.0561	0.0624	0.0638	0.0413
P_{i,s_3}^0	0.1007	0.0599	0.0386	0.0479	0.0596	0.0405	0.1178	0.0589	0.0443	0.0495
P_{i,s_4}^0	0.0494	0.0639	0.0386	0.0975	0.0438	0.016	0.0842	0.0707	0.0885	0.0891
P_{i,s_5}^0	0.1007	0.0504	0.0115	0.0254	0.0876	0.016	0.0842	0.0624	0.0154	0.0891
P_{i,s_6}^0	0.1898	0.0639	0.284	0.0498	0.1753	0.0405	0.0401	0.0551	0.0443	0.0495
P_{i,s_7}^0	0.0494	0.0961	0.0847	0.0975	0.0298	0.0501	0.0601	0.0624	0.0638	0.0495
P_{i,s_8}^0	0.0534	0.0639	0.0253	0.1838	0.0438	0.1568	0.0601	0.0749	0.0443	0.0594
P_{i,s_9}^0	0.0252	0.0504	0.0847	0.0975	0.0256	0.1025	0.0842	0.0668	0.1276	0.1337
$P_{i,s_{10}}^0$	0.0514	0.0681	0.0176	0.0938	0.0511	0.1268	0.0524	0.0707	0.0638	0.0743
$P_{i,s_{11}}^0$	0.1007	0.0504	0.0075	0.0498	0.0596	0.0405	0.0429	0.0749	0.1276	0.0413
$P_{i,s_{12}}^0$	0.0262	0.0538	0.0253	0.0244	0.0438	0.0501	0.0524	0.0624	0.0221	0.0743
$P_{i,s_{13}}^0$	0.0273	0.0809	0.0253	0.0498	0.0511	0.1025	0.0786	0.0707	0.0443	0.0413
$P_{i,s_{14}}^0$	0.0514	0.0681	0.0386	0.0518	0.0876	0.0501	0.0524	0.0661	0.0221	0.0594
$P_{i,s_{15}}^0$	0.0262	0.0681	0.1295	0.0518	0.1023	0.0405	0.0561	0.0624	0.1839	0.0594

Demand scenarios of retailers based on $S = 18$

$d_{t,i}^{1}$ = [[105, 81, 146, 98, 132, 129, 82, 85, 99, 85], [105, 81, 93, 98, 132, 96, 82, 85, 99, 85], [85, 106, 119, 128, 132, 161, 106, 112, 71, 110]];

$d_{t,i}^{2}$ = [[105, 106, 119, 98, 83, 129, 56, 85, 99, 85], [85, 131, 146, 128, 110, 129, 82, 137, 99, 58], [126, 106, 119, 98, 110, 96, 82, 137, 125, 58]];

$d^3_{t,i}$ = [[105, 131, 93, 98, 132, 96, 56, 112, 99, 85], [105, 81, 119, 98, 83, 161, 56, 137, 71, 110], [105, 106, 119, 98, 110, 129, 56, 85, 99, 58]];

$d^4_{t,i}$ = [[85, 81, 119, 128, 83, 161, 106, 112, 99, 58], [126, 131, 93, 74, 132, 129, 56, 112, 71, 58], [105, 131, 146, 74, 110, 129, 56, 137, 71, 58]];

$d^5_{t,i}$ = [[105, 81, 119, 128, 132, 96, 82, 85, 71, 110], [105, 106, 93, 98, 110, 161, 56, 137, 125, 58], [126, 106, 119, 128, 83, 96, 56, 85, 99, 110]];

$d^6_{t,i}$ = [[85, 81, 146, 128, 110, 96, 106, 137, 99, 85], [126, 131, 146, 128, 132, 129, 56, 85, 71, 58], [126, 106, 93, 98, 83, 96, 106, 85, 125, 85]];

$d^7_{t,i}$ = [[105, 106, 93, 128, 110, 129, 82, 137, 99, 85], [126, 106, 119, 98, 132, 96, 56, 112, 125, 85], [126, 131, 146, 98, 132, 129, 106, 112, 99, 58]];

$d^8_{t,i}$ = [[126, 106, 146, 98, 110, 161, 82, 85, 125, 58], [126, 106, 146, 74, 83, 161, 82, 112, 71, 58], [105, 81, 146, 74, 110, 96, 56, 137, 99, 110]];

$d^9_{t,i}$ = [[126, 106, 93, 74, 110, 129, 56, 112, 99, 58], [105, 81, 146, 128, 110, 161, 106, 85, 99, 85], [105, 131, 93, 128, 110, 129, 82, 112, 99, 58]];

$d^{10}_{t,i}$ = [[85, 106, 93, 98, 83, 129, 82, 112, 99, 58], [105, 81, 146, 128, 110, 161, 106, 112, 125, 58,], [85, 106, 146, 128, 83, 96, 82, 85, 71, 85]];

$d^{11}_{t,i}$ = [[126, 106, 119, 74, 110, 129, 56, 137, 71, 110], [126, 131, 119, 128, 132, 129, 106, 85, 125, 85], [126, 131, 146, 98, 110, 129, 106, 137, 71, 58]];

$d^{12}_{t,i}$ = [[85, 81, 119, 98, 132, 129, 106, 112, 71, 110], [85, 106, 119, 74, 83, 96, 106, 112, 71, 85], [105, 81, 119, 98, 132, 96, 56, 112, 99, 85]];

$d^{13}_{t,i}$ = [[105, 131, 146, 128, 83, 161, 106, 85, 99, 58], [85, 106, 119, 98, 132, 161, 56, 112, 125, 85], [105, 106, 146, 74, 132, 161, 82, 85, 125, 85]];

$d^{14}_{t,i}$ = [[105, 106, 93, 74, 110, 129, 82, 112, 71, 58], [85, 106, 93, 98, 110, 161, 106, 85, 71, 85], [85, 106, 93, 74, 132, 161, 56, 137, 125, 110]];

$d^{15}_{t,i}$ = [[105, 131, 146, 74, 110, 96, 56, 137, 125, 85], [126, 131, 93, 74, 110, 96, 82, 137, 71, 85], [85, 81, 146, 98, 83, 129, 56, 137, 71, 85]];

$d^{16}_{t,i}$ = [[105, 131, 146, 98, 110, 161, 82, 85, 71, 110], [126, 131, 146, 98, 83, 161, 56, 137, 99, 110], [105, 131, 119, 74, 83, 129, 82, 112, 71, 110]];

$d^{17}_{t,i}$ = [[85, 81, 119, 74, 132, 96, 82, 85, 71, 85], [126, 131, 146, 74, 110, 96, 82, 112, 99, 58], [85, 81, 146, 74, 132, 96, 82, 112, 99, 110]];

$d^{18}_{t,i}$ = [[126, 131, 93, 74, 132, 96, 106, 112, 125, 85], [105, 106, 93, 128, 132, 129, 82, 137, 71, 110], [126, 81, 119, 74, 110, 129, 82, 137, 125, 85]];

Table A10

Nominal value of probability vector after normalizing based on $S = 18$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P^0_{i,s_1}	0.0787	0.0559	0.0368	0.0838	0.0288	0.0379	0.0686	0.0615	0.0614	0.0661
P^0_{i,s_2}	0.0418	0.0622	0.0807	0.0838	0.0725	0.0468	0.0734	0.0452	0.1228	0.0529
P^0_{i,s_3}	0.1543	0.0524	0.1234	0.1579	0.0423	0.0379	0.0375	0.0543	0.0614	0.0441
P^0_{i,s_4}	0.0418	0.0413	0.0368	0.0218	0.0423	0.1187	0.035	0.0575	0.0213	0.0353
P^0_{i,s_5}	0.0819	0.0663	0.1234	0.0445	0.0423	0.0121	0.0525	0.0513	0.0426	0.0245
P^0_{i,s_6}	0.0222	0.0524	0.011	0.0445	0.0423	0.0149	0.0326	0.0513	0.0426	0.0793
P^0_{i,s_7}	0.0434	0.0622	0.0368	0.0838	0.0494	0.0468	0.049	0.0575	0.1228	0.0793
P^0_{i,s_8}	0.0434	0.0663	0.0072	0.0411	0.0725	0.0306	0.0734	0.0543	0.0426	0.0294
P^0_{i,s_9}	0.0819	0.0524	0.0168	0.0227	0.1451	0.1187	0.049	0.0651	0.1769	0.0529
$P^0_{i,s_{10}}$	0.0402	0.0663	0.011	0.0445	0.0363	0.0379	0.0686	0.0651	0.0426	0.0529
$P^0_{i,s_{11}}$	0.0231	0.0491	0.0807	0.0428	0.0846	0.1468	0.0326	0.0452	0.0148	0.0441
$P^0_{i,s_{12}}$	0.0402	0.0559	0.2705	0.0806	0.0247	0.0149	0.0326	0.0689	0.0213	0.0661
$P^0_{i,s_{13}}$	0.0787	0.0622	0.0241	0.0428	0.0247	0.0776	0.049	0.0615	0.0852	0.0793
$P^0_{i,s_{14}}$	0.0402	0.0788	0.0257	0.0411	0.0846	0.0959	0.049	0.0543	0.0148	0.0441
$P^0_{i,s_{15}}$	0.0418	0.0413	0.011	0.0411	0.0725	0.0149	0.0525	0.0399	0.0148	0.119
$P^0_{i,s_{16}}$	0.0819	0.0388	0.0241	0.0806	0.0363	0.0959	0.0734	0.0543	0.0213	0.0204
$P^0_{i,s_{17}}$	0.0213	0.0441	0.0241	0.021	0.0494	0.0048	0.1028	0.0651	0.0614	0.0441
$P^0_{i,s_{18}}$	0.0434	0.0524	0.0563	0.0218	0.0494	0.0468	0.0686	0.0479	0.0296	0.0661

Demand scenarios of retailers based on $S = 21$

$d^1_{t,i}$ = [[85, 106, 119, 98, 110, 161, 56, 112, 99, 85], [126, 106, 93, 98, 110, 161, 106, 112, 71, 110], [105, 106, 119, 98, 83, 96, 82, 112, 125, 58]];

$d^2_{t,i}$ = [[105, 131, 119, 74, 83, 129, 82, 137, 71, 58], [85, 81, 146, 128, 110, 96, 82, 137, 99, 110], [105, 106, 93, 128, 83, 161, 106, 85, 99, 110]];

$d^3_{t,i}$ = [[105, 131, 119, 74, 110, 129, 82, 112, 99, 85], [126, 131, 146, 98, 83, 161, 82, 112, 99, 58], [126, 106, 146, 128, 110, 161, 56, 85, 99, 85]];

$d^4_{t,i}$ = [[85, 106, 146, 128, 110, 96, 56, 85, 99, 85], [105, 131, 146, 128, 83, 129, 56, 85, 125, 110], [85, 131, 146, 128, 83, 96, 82, 85, 99, 85]];

$d^5_{t,i}$ = [[126, 131, 146, 98, 132, 129, 106, 85, 125, 85], [85, 106, 93, 128, 83, 129, 56, 85, 71, 58], [105, 131, 146, 74, 83, 129, 106, 137, 99, 58]];

$d^6_{t,i}$ = [[105, 81, 146, 98, 132, 96, 82, 85, 99, 58], [105, 131, 119, 74, 132, 96, 82, 137, 71, 85], [105, 81, 146, 98, 110, 129, 82, 112, 71, 58]];

$d^7_{t,i}$ = [[105, 81, 146, 128, 83, 161, 82, 112, 99, 58], [85, 106, 119, 98, 83, 129, 56, 137, 125, 85], [85, 131, 119, 98, 132, 129, 82, 85, 125, 85]];

$d^{8}_{t,i}$ = [[126, 81, 119, 74, 83, 161, 82, 85, 99, 58], [126, 131, 119, 128, 83, 96, 106, 137, 71, 58], [105, 106, 93, 98, 110, 96, 106, 85, 99, 58]];

$d^{9}_{t,i}$ = [[85, 106, 93, 98, 132, 161, 106, 85, 71, 58], [85, 81, 119, 98, 132, 129, 82, 85, 99, 110], [85, 81, 93, 128, 83, 161, 56, 112, 71, 58]];

$d^{10}_{t,i}$ = [[105, 131, 146, 98, 110, 161, 106, 137, 71, 85], [85, 106, 119, 98, 132, 129, 56, 85, 125, 85], [126, 106, 93, 74, 83, 96, 82, 112, 99, 58]];

$d^{11}_{t,i}$ = [[105, 106, 146, 98, 110, 96, 106, 112, 71, 85], [126, 106, 146, 128, 110, 129, 82, 85, 71, 85], [105, 81, 119, 128, 110, 129, 82, 85, 99, 110]];

$d^{12}_{t,i}$ = [[85, 106, 93, 74, 83, 129, 56, 85, 99, 110], [126, 131, 93, 74, 132, 96, 82, 137, 125, 58], [85, 81, 146, 98, 132, 96, 106, 137, 71, 110]];

$d^{13}_{t,i}$ = [[85, 106, 93, 128, 132, 129, 56, 112, 71, 85], [126, 106, 93, 98, 110, 96, 56, 137, 71, 85], [126, 131, 119, 128, 83, 129, 106, 112, 125, 85]];

$d^{14}_{t,i}$ = [[126, 106, 146, 98, 132, 161, 56, 137, 99, 110], [105, 81, 93, 128, 83, 96, 82, 112, 99, 110], [85, 131, 119, 98, 110, 161, 82, 112, 125, 58]];

$d^{15}_{t,i}$ = [[126, 131, 119, 98, 83, 96, 106, 85, 125, 110], [85, 81, 119, 74, 83, 96, 56, 112, 125, 85], [85, 81, 119, 74, 83, 161,56, 112, 99, 110]];

$d^{16}_{t,i}$ = [[105, 131, 93, 128, 132, 96, 82, 112, 125, 58], [105, 106, 119, 74, 83, 161, 56, 85, 125, 58], [126, 81, 119, 128, 132, 161, 106, 112, 125, 110]];

$d^{17}_{t,i}$ = [[126, 81, 119, 128, 83, 129, 106, 137, 71, 110], [105, 106, 119, 128, 110, 161, 82, 137, 71, 85], [105, 106, 146, 74, 132, 96, 106, 137, 71, 85]];

$d^{18}_{t,i}$ = [[85, 131, 93, 128, 110, 129, 82, 137, 99, 85], [105, 131, 146, 98, 132, 129, 56, 112, 99, 58], [126, 131, 119, 74, 110, 96, 56, 137, 71, 110]];

$d^{19}_{t,i}$ = [[126, 106, 119, 74, 83, 129, 56, 112, 71, 58], [85, 81, 146, 128, 110, 129, 56, 137, 99, 110], [126, 106, 119, 74, 110, 161, 56, 137, 125, 85]];

$d^{20}_{t,i}$ = [[85, 81, 93, 128, 132, 161, 56, 85, 71, 110], [105, 81, 119, 74, 110, 161, 106, 112, 125, 110], [105, 106, 146, 98, 110, 161, 106, 85, 125, 110]];

$d^{21}_{t,i}$ = [[126, 81, 146, 128, 83, 161, 106, 85, 125, 58], [105, 106, 146, 128, 132, 96, 106, 112, 99, 58], [126, 81, 93, 98, 110, 129, 56, 137, 125, 85]];

Table A11

Nominal value of probability vector after normalizing based on $S = 21$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P^0_{i,s_1}	0.0385	0.068	0.0951	0.1419	0.07	0.0277	0.0442	0.0591	0.0341	0.0412
P^0_{i,s_2}	0.0726	0.0452	0.0284	0.0204	0.035	0.0342	0.0619	0.0387	0.0491	0.0229
P^0_{i,s_3}	0.0401	0.0424	0.0185	0.0384	0.07	0.0867	0.0664	0.0558	0.1416	0.0741
P^0_{i,s_4}	0.037	0.0424	0.0055	0.0212	0.035	0.0135	0.0474	0.0498	0.0982	0.0617
P^0_{i,s_5}	0.0385	0.0424	0.0085	0.0384	0.0204	0.1326	0.0295	0.0439	0.0341	0.0494
P^0_{i,s_6}	0.1423	0.0381	0.0185	0.0724	0.0476	0.0135	0.0929	0.0465	0.017	0.0494
P^0_{i,s_7}	0.037	0.0452	0.0622	0.0753	0.0204	0.1072	0.0664	0.0465	0.0682	0.0741
P^0_{i,s_8}	0.0401	0.0452	0.0951	0.0384	0.035	0.0109	0.0413	0.0439	0.0491	0.0329
P^0_{i,s_9}	0.0189	0.0482	0.0434	0.0753	0.0238	0.0867	0.0442	0.0527	0.017	0.0274
$P^0_{i,s_{10}}$	0.0385	0.0537	0.0284	0.0724	0.0408	0.0342	0.0442	0.0465	0.0341	0.0741
$P^0_{i,s_{11}}$	0.0755	0.0573	0.0185	0.0399	0.14	0.0423	0.0619	0.0527	0.017	0.0617
$P^0_{i,s_{12}}$	0.0197	0.0452	0.0129	0.0369	0.0238	0.0135	0.0442	0.0387	0.0341	0.0229
$P^0_{i,s_{13}}$	0.0204	0.0537	0.0434	0.0399	0.0408	0.0423	0.0316	0.0492	0.0118	0.1111
$P^0_{i,s_{14}}$	0.0385	0.0452	0.0284	0.0753	0.0408	0.0277	0.0664	0.0492	0.0982	0.0229
$P^0_{i,s_{15}}$	0.0197	0.0381	0.2085	0.0369	0.0175	0.0109	0.0316	0.0558	0.0682	0.0343
$P^0_{i,s_{16}}$	0.0755	0.0452	0.0951	0.0204	0.0238	0.0277	0.0442	0.0558	0.0473	0.0274
$P^0_{i,s_{17}}$	0.0755	0.0573	0.0622	0.0204	0.0408	0.0342	0.0413	0.0342	0.0059	0.0617
$P^0_{i,s_{18}}$	0.0385	0.0335	0.0284	0.0384	0.0817	0.0423	0.0474	0.041	0.0491	0.0412
$P^0_{i,s_{19}}$	0.0204	0.0573	0.0622	0.0196	0.07	0.1072	0.0339	0.041	0.0341	0.0412
$P^0_{i,s_{20}}$	0.0726	0.0482	0.0284	0.0384	0.0817	0.0701	0.0295	0.0527	0.0236	0.0191
$P^0_{i,s_{21}}$	0.0401	0.0482	0.0085	0.0399	0.0408	0.0342	0.0295	0.0465	0.0682	0.0494

Demand scenarios of retailers based on $S = 24$

$d^1_{t,i}$ = [[85, 131, 119, 98, 132, 129, 56, 112, 99, 85], [126, 106, 93, 128, 110, 161, 56, 137, 99, 85], [105, 106, 146, 98, 83, 129, 56, 112, 125, 110]];

$d^2_{t,i}$ = [[105, 106, 119, 98, 83, 161, 56, 112, 71, 58], [85, 131, 93, 74, 110, 129, 106, 85, 99, 85], [105, 81, 119, 74, 132, 161, 56, 85, 99, 85]];

$d^3_{t,i}$ = [[105, 81, 93, 74, 83, 161, 56, 85, 99, 58], [126, 131, 146, 98, 83, 96, 106, 112, 71, 58], [126, 131, 119, 128, 83, 96, 82, 112, 125, 58]];

$d^4_{t,i}$ = [[85, 81, 146, 128, 132, 161, 56, 85, 99, 110], [105, 106, 119, 98, 83, 129, 82, 137, 99, 58], [85, 106, 146, 128, 110, 96, 106, 85, 71, 58]];

$d^5_{t,i}$ = [[126, 81, 146, 98, 83, 129, 106, 85, 125, 85], [85, 131, 119, 128, 132, 161, 82, 112, 125, 58], [105, 106, 119, 74, 83, 96, 106, 137, 71, 58]];

$d^6_{t,i}$ = [[105, 106, 93, 98, 110, 129, 106, 137, 125, 110], [105, 81, 146, 74, 132, 129, 82, 137, 99, 58], [85,

106, 93, 98, 110, 96, 82, 85, 99, 110]];

$d^{7}_{t,i}$ = [[105, 131, 93, 128, 110, 96, 82, 85, 71, 85], [85, 81, 93, 74, 83, 161, 56, 137, 71, 58], [85, 81, 119, 98, 83, 129, 56, 112, 71, 110]];

$d^{8}_{t,i}$ = [[126, 106, 146, 128, 132, 161, 106, 112, 71, 85], [126, 106, 146, 128, 110, 161, 56, 85, 99, 85], [126, 106, 146, 98, 110, 129, 82, 137, 71, 58]];

$d^{9}_{t,i}$ = [[85, 106, 119, 128, 83, 96, 56, 137, 99, 58], [85, 131, 119, 98, 132, 96, 56, 137, 125, 110], [85, 131, 119, 98, 132, 129, 82, 112, 99, 85]];

$d^{10}_{t,i}$ = [[105, 106, 93, 74, 110, 96, 82, 137, 71, 85], [85, 106, 119, 128, 83, 96, 106, 137, 71, 110], [126, 131, 119, 128, 132, 161, 82, 137, 125, 85]];

$d^{11}_{t,i}$ = [[105, 106, 146, 128, 83, 129, 106, 85, 125, 58], [126, 81, 146, 74, 110, 96, 106, 85, 125, 110], [105, 131, 119, 74, 83, 129, 82, 112, 125, 110]];

$d^{12}_{t,i}$ = [[85, 131, 93, 128, 110, 129, 82, 112, 71, 110], [126, 131, 119, 74, 83, 129, 106, 112, 125, 110], [85, 106, 93, 128, 110, 129, 106, 85, 99, 58]];

$d^{13}_{t,i}$ = [[85, 131, 119, 74, 110, 129, 82, 112, 71, 110], [126, 106, 146, 74, 132, 161, 56, 112, 99, 58], [126, 131, 146, 98, 83, 161, 106, 137, 125, 85]];

$d^{14}_{t,i}$ = [[126, 81, 93, 98, 110, 96, 106, 85, 99, 85], [105, 106, 119, 128, 132, 129, 106, 85, 71, 110], [85, 81, 146, 128, 132, 96, 106, 137, 71, 58]];

$d^{15}_{t,i}$ = [[126, 131, 146, 74, 132, 161, 106, 137, 71, 58], [85, 131, 146, 98, 132, 96, 56, 112, 71, 110], [85, 131, 93, 74, 83, 129, 106, 85, 99, 58]];

$d^{16}_{t,i}$ = [[105, 106, 119, 74, 83, 161, 56, 112, 125, 85], [105, 106, 146, 98, 83, 161, 82, 137, 99, 58], [126, 81, 93, 98, 132,96, 82, 85, 71, 85]];

$d^{17}_{t,i}$ = [[126, 81, 119, 98, 83, 96, 56, 137, 125, 85], [105, 81, 119, 74, 110, 129, 82, 112, 99, 85], [105, 106, 93, 128, 110, 161, 56, 137, 125, 85]];

$d^{18}_{t,i}$ = [[85, 81, 119, 128, 110, 129, 82, 137, 99, 110], [105, 106, 119, 128, 110, 129, 106, 85, 125, 85], [126, 131, 146, 128, 110, 161, 56, 112, 71, 85]];

$d^{19}_{t,i}$ = [[126, 131, 146, 98, 83, 161, 82, 112, 99, 58], [85, 81, 93, 98, 132, 96, 82, 85, 125, 85], [126, 106, 119, 98, 110, 161, 56, 112, 125, 110]];

$d^{20}_{t,i}$ = [[85, 131, 93, 98, 110, 129, 82, 112, 125, 58], [105, 81, 93, 98, 110, 96, 56, 112, 125, 85], [105, 131, 146, 128, 83, 96, 82, 112, 99, 58]];

$d^{21}_{t,i}$ = [[126, 106, 93, 128, 132, 161, 56, 85, 99, 110], [105, 131, 146, 128, 132, 161, 56, 85, 99, 110], [126, 106, 146, 74, 110, 161, 106, 85, 99, 110]];

$d^{22}_{t,i}$ = [[105, 81, 146, 74, 110, 161, 82, 85, 125, 58], [105, 81, 93, 128, 110, 129, 82, 112, 71, 58], [105, 81, 93, 74, 132, 129, 106, 85, 71, 85]];

$d^{23}_{t,i}$ = [[126, 81, 119, 74, 83, 96, 106, 137, 99, 58], [126, 81, 146, 74, 83, 96, 106, 112, 71, 58], [85, 131,

119, 74, 110, 96, 56, 112, 99, 110]];

$d_{t,i}^{24}$ = [[126, 131, 146, 98, 132, 96, 82, 137, 71, 110], [126, 106, 93, 98, 132, 129, 82, 85, 125, 85], [105, 81, 146, 74, 132, 129, 82, 85, 125, 110]];

Table A12

Nominal value of probability vector after normalizing based on S = 24.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P_{i,s_1}^0	0.0346	0.048	0.0274	0.0679	0.0348	0.089	0.0295	0.0429	0.0866	0.0556
P_{i,s_2}^0	0.0652	0.0405	0.0919	0.0333	0.0348	0.072	0.0275	0.0459	0.0433	0.0668
P_{i,s_3}^0	0.036	0.0319	0.0274	0.0346	0.0149	0.0091	0.0385	0.0486	0.0301	0.0297
P_{i,s_4}^0	0.0332	0.0513	0.0179	0.036	0.0348	0.0284	0.0385	0.0383	0.0433	0.0247
P_{i,s_5}^0	0.0346	0.0405	0.0601	0.0346	0.0174	0.0284	0.0359	0.0405	0.0209	0.0445
P_{i,s_6}^0	0.0652	0.0513	0.0125	0.0652	0.0696	0.0351	0.0539	0.0338	0.0866	0.0206
P_{i,s_7}^0	0.0332	0.0341	0.0419	0.0346	0.0298	0.0284	0.0413	0.0405	0.0052	0.0371
P_{i,s_8}^0	0.0191	0.0609	0.0053	0.036	0.0696	0.072	0.0385	0.0405	0.015	0.0668
P_{i,s_9}^0	0.017	0.0379	0.2016	0.0679	0.0203	0.0112	0.0413	0.0357	0.0866	0.0371
$P_{i,s_{10}}^0$	0.0346	0.048	0.0919	0.0184	0.0348	0.0091	0.0539	0.0298	0.0104	0.0556
$P_{i,s_{11}}^0$	0.0678	0.0405	0.0179	0.0177	0.0298	0.0351	0.0359	0.0459	0.0417	0.0206
$P_{i,s_{12}}^0$	0.0176	0.0379	0.0419	0.0184	0.0596	0.1101	0.0359	0.0486	0.0301	0.0206
$P_{i,s_{13}}^0$	0.0183	0.0379	0.0179	0.0333	0.0348	0.072	0.0385	0.0429	0.0301	0.0371
$P_{i,s_{14}}^0$	0.0346	0.0432	0.0274	0.036	0.0406	0.0112	0.024	0.0383	0.015	0.0371
$P_{i,s_{15}}^0$	0.0176	0.0299	0.0082	0.0333	0.0203	0.0284	0.0257	0.0405	0.015	0.0247
$P_{i,s_{16}}^0$	0.0678	0.0513	0.0274	0.0652	0.0174	0.023	0.0578	0.0405	0.0301	0.0668
$P_{i,s_{17}}^0$	0.0678	0.0432	0.0919	0.0346	0.0596	0.0284	0.0413	0.0357	0.0601	0.1001
$P_{i,s_{18}}^0$	0.0346	0.0405	0.0601	0.0191	0.1192	0.089	0.0385	0.0405	0.0301	0.0556
$P_{i,s_{19}}^0$	0.0183	0.0405	0.0274	0.1279	0.0348	0.023	0.0578	0.0486	0.0601	0.0371
$P_{i,s_{20}}^0$	0.0652	0.0319	0.0125	0.0679	0.0596	0.0112	0.0578	0.0515	0.0601	0.0445
$P_{i,s_{21}}^0$	0.036	0.048	0.0082	0.0184	0.0406	0.0582	0.0275	0.0434	0.1249	0.0172
$P_{i,s_{22}}^0$	0.1277	0.0363	0.0125	0.0177	0.0696	0.089	0.0539	0.0459	0.0104	0.0445
$P_{i,s_{23}}^0$	0.0183	0.0341	0.0601	0.017	0.0298	0.0036	0.0257	0.0429	0.0433	0.0247
$P_{i,s_{24}}^0$	0.036	0.0405	0.0082	0.0652	0.0237	0.0351	0.0809	0.0383	0.0209	0.0309

Demand scenarios of retailers based on S = 27

$d_{t,i}^{1}$ = [[105, 81, 119, 98, 83, 129, 106, 137, 125, 58], [105, 106, 93, 74, 110, 161, 82, 112, 99, 58], [105, 106, 146, 98, 83, 161, 106, 112, 125, 58]];

$d_{t,i}^{2}$ = [[126, 131, 119, 74, 110, 96, 82, 85, 71, 110], [85, 81, 119, 98, 132, 129, 56, 85, 71, 85], [126, 106, 93, 98, 110, 96, 106, 137, 125, 85]];

$d_{t,i}^{3}$ = [[85, 106, 146, 74, 83, 129, 106, 137, 125, 58], [85, 81, 146, 128, 110, 96, 56, 112, 125, 85], [126, 106, 146, 98, 110, 161, 56, 137, 71, 110]];

$d_{t,i}^{4}$ = [[85, 131, 93, 128, 110, 96, 106, 137, 71, 85], [105, 131, 146, 98, 83, 161, 106, 85, 125, 58], [126, 131, 119, 74, 83, 161, 106, 85, 99, 85]];

$d_{t,i}^{5}$ = [[105, 131, 93, 74, 83, 96, 56, 137, 99, 58], [85, 131, 93, 98, 132, 96, 56, 137, 99, 110], [105, 106, 119, 128, 110, 129, 56, 137, 99, 85]];

$d_{t,i}^{6}$ = [[126, 131, 146, 128, 132, 96, 106, 112, 99, 110], [126, 106, 119, 128, 83, 96, 56, 112, 71, 110], [105, 81, 93, 74, 132, 161, 106, 112, 71, 85]];

$d^{7}_{t,i}$ = [[126, 106, 119, 98, 83, 161, 56, 137, 125, 110], [126, 131, 93, 98, 83, 161, 106, 85, 71, 58], [126, 81, 119, 98, 132, 96, 82, 112, 71, 110]];

$d^{8}_{t,i}$ = [[85, 81, 146, 128, 132, 161, 82, 85, 71, 85], [85, 81, 93, 74, 110, 129, 82, 137, 99, 110], [85, 131, 146, 98, 132, 161, 106, 112, 71, 85]];

$d^{9}_{t,i}$ = [[105, 106, 93, 128, 110, 129, 82, 137, 125, 85], [126, 106, 93, 98, 83, 96, 106, 137, 71, 58], [126, 106, 146, 128, 132, 129, 82, 85, 125, 58]];

$d^{10}_{t,i}$ = [[85, 81, 93, 74, 132, 161, 56, 85, 99, 58], [126, 81, 119, 74, 110, 161, 82, 112, 71, 110], [85, 106, 146, 74, 83, 129, 56, 137, 125, 58]];

$d^{11}_{t,i}$ = [[126, 106, 93, 98, 132, 129, 106, 85, 71, 58], [85, 131, 146, 98, 132, 129, 82, 112, 71, 110], [85, 131, 146, 128, 83, 161, 56, 85, 99, 110]];

$d^{12}_{t,i}$ = [[105, 106, 146, 128, 110, 129, 106, 85, 71, 58], [85, 81, 93, 128, 83, 129, 56, 137, 125, 85], [85, 81, 93, 98, 110, 129, 82, 137, 71, 85]];

$d^{13}_{t,i}$ = [[105, 106, 146, 128, 132, 96, 56, 112, 99, 85], [126, 81, 146, 98, 110, 129, 106, 85, 99, 85], [105, 131, 119, 98, 110, 161, 106, 85, 71, 110]];

$d^{14}_{t,i}$ = [[126, 81, 146, 128, 110, 129, 82, 112, 71, 110], [105, 81, 119, 128, 132, 161, 106, 137, 125, 110], [126, 81, 119, 128, 83, 129, 56, 137, 125, 110]];

$d^{15}_{t,i}$ = [[126, 106, 146, 74, 132, 161, 56, 137, 99, 85], [126, 106, 146, 128, 132, 96, 56, 85, 125, 85], [85, 131, 93, 128, 110, 96, 82, 137, 125, 58]];

$d^{16}_{t,i}$ = [[105, 131, 119, 98, 110, 96, 82, 85, 99, 85], [105, 106, 119, 128, 110, 161, 82, 112, 99, 58], [85, 106, 146, 74, 110, 129, 56, 112, 125, 110]];

$d^{17}_{t,i}$ = [[85, 81, 146, 74, 83, 96, 82, 137, 71, 85], [105, 131, 119, 128, 83, 161, 56, 112, 71, 110], [105, 81, 146, 74, 110, 96, 82, 85, 71, 58]];

$d^{18}_{t,i}$ = [[105, 131, 119, 98, 132, 129, 82, 85, 99, 85], [85, 106, 119, 74, 132, 161, 56, 85, 125, 110], [126, 131, 119, 74, 132, 96, 56, 85, 99, 110]];

$d^{19}_{t,i}$ = [[126, 131, 119, 98, 83, 161, 106, 112, 99, 110], [105, 81, 93, 98, 132, 96, 82, 137, 125, 85], [85, 81, 93, 74, 83, 161, 82, 85, 71, 110]];

$d^{20}_{t,i}$ = [[85, 81, 119, 98, 83, 161, 56, 112, 71, 110], [105, 131, 146, 128, 83, 129, 82, 112, 99, 85], [85, 131, 119, 128, 83, 129, 106, 137, 99, 58]];

$d^{21}_{t,i}$ = [[105, 106, 93, 74, 110, 129, 82, 85, 71, 85], [126, 106, 146, 74, 132, 96, 106, 85, 99, 85], [85, 81, 93, 98, 132, 96, 106, 112, 125, 85]];

$d^{22}_{t,i}$ = [[126, 106, 119, 128, 132, 161, 56, 112, 125, 110], [105, 131, 146, 74, 83, 129, 56, 112, 99, 110], [105, 106, 93, 128, 83, 96, 106, 85, 99, 58]];

$d^{23}_{t,i}$ = [[85, 81, 93, 98, 132, 161, 82, 85, 99, 110], [126, 106, 119, 128, 83, 161, 82, 137, 71, 58], [126, 131, 93, 98, 110, 161, 82, 85, 99, 85]];

$d_{t,i}^{24}$ = [[126, 131, 146, 128, 110, 96, 56, 112, 125, 58], [85, 81, 93, 74, 110, 96, 82, 137, 125, 58], [105, 131, 119, 74, 83, 96, 82, 112, 99, 110]];

$d_{t,i}^{25}$ = [[85, 81, 119, 74, 110, 161, 106, 137, 125, 58], [85, 131, 146, 98, 110, 96, 106, 137, 99, 85], [105, 106, 146, 74, 132, 129, 56, 112, 71, 58]];

$d_{t,i}^{26}$ = [[85, 81, 93, 98, 83, 129, 106, 112, 125, 110], [126, 106, 93, 74, 132, 129, 106, 85, 125, 58], [105, 81, 93, 128, 132, 96, 82, 137, 125, 58]];

$d_{t,i}^{27}$ = [[105, 131, 93, 74, 83, 96, 56, 112, 125, 58], [105, 131, 119, 74, 110, 129, 106, 85, 71, 58], [126, 81, 119, 128, 132, 129, 56, 112, 99, 85]];

Table A13

Nominal value of probability vector after normalizing based on $S = 27$.

	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}
P_{i,s_1}^0	0.1176	0.0462	0.0252	0.06	0.0276	0.0679	0.0329	0.0389	0.0566	0.027
P_{i,s_2}^0	0.0169	0.0365	0.0845	0.06	0.0645	0.0106	0.0353	0.0347	0.0098	0.0506
P_{i,s_3}^0	0.0163	0.0462	0.0049	0.0319	0.0553	0.0268	0.0252	0.0324	0.0197	0.0338
P_{i,s_4}^0	0.0319	0.027	0.0252	0.0319	0.0276	0.0217	0.022	0.0347	0.0283	0.0608
P_{i,s_5}^0	0.06	0.0342	0.0385	0.0319	0.0323	0.0106	0.027	0.027	0.1176	0.0338
P_{i,s_6}^0	0.0331	0.0365	0.0252	0.0169	0.0188	0.0086	0.0235	0.0467	0.0142	0.0281
P_{i,s_7}^0	0.0176	0.0365	0.0845	0.1176	0.0161	0.0217	0.0353	0.0367	0.0098	0.0188
P_{i,s_8}^0	0.0156	0.0307	0.0075	0.0319	0.0376	0.0679	0.0494	0.0367	0.0142	0.0506
P_{i,s_9}^0	0.0331	0.0549	0.0115	0.0331	0.0323	0.0331	0.0494	0.0306	0.0197	0.0405
$P_{i,s_{10}}^0$	0.0163	0.0389	0.0252	0.0156	0.0323	0.0679	0.0378	0.0367	0.0283	0.0225
$P_{i,s_{11}}^0$	0.0163	0.0342	0.0075	0.0624	0.0188	0.0839	0.0353	0.0416	0.0142	0.0188
$P_{i,s_{12}}^0$	0.0306	0.0389	0.0115	0.0331	0.0553	0.1038	0.0353	0.0306	0.0098	0.0608
$P_{i,s_{13}}^0$	0.0624	0.0365	0.0165	0.0624	0.0645	0.0268	0.0235	0.0416	0.0408	0.0506
$P_{i,s_{14}}^0$	0.0331	0.0328	0.0552	0.0176	0.0323	0.0839	0.0353	0.0324	0.0197	0.0156
$P_{i,s_{15}}^0$	0.0169	0.0433	0.0075	0.0169	0.0376	0.0086	0.0378	0.0306	0.0566	0.0608
$P_{i,s_{16}}^0$	0.06	0.0433	0.0552	0.0319	0.1106	0.0268	0.0529	0.0441	0.0816	0.0338
$P_{i,s_{17}}^0$	0.06	0.0307	0.0165	0.0163	0.0276	0.0086	0.0529	0.0367	0.0049	0.0338
$P_{i,s_{18}}^0$	0.0319	0.0342	0.1852	0.0306	0.022	0.0268	0.0378	0.0393	0.0816	0.0281
$P_{i,s_{19}}^0$	0.0319	0.0307	0.0385	0.06	0.0161	0.0217	0.0494	0.0367	0.0283	0.0281
$P_{i,s_{20}}^0$	0.0306	0.0288	0.0552	0.0331	0.0138	0.0839	0.0353	0.0389	0.0408	0.0338
$P_{i,s_{21}}^0$	0.0319	0.0462	0.0115	0.0306	0.0376	0.0106	0.0329	0.0416	0.0283	0.0911
$P_{i,s_{22}}^0$	0.0624	0.0433	0.0252	0.0169	0.0161	0.0268	0.0252	0.0441	0.0816	0.0188
$P_{i,s_{23}}^0$	0.0169	0.0365	0.0385	0.0624	0.0323	0.0549	0.0741	0.0347	0.0408	0.0338
$P_{i,s_{24}}^0$	0.0319	0.0288	0.0252	0.0163	0.0553	0.0034	0.0529	0.0389	0.0566	0.0225
$P_{i,s_{25}}^0$	0.0306	0.0365	0.0165	0.0306	0.0645	0.0268	0.0235	0.0324	0.0283	0.0405
$P_{i,s_{26}}^0$	0.0319	0.0389	0.0176	0.0319	0.0188	0.0331	0.0329	0.0367	0.0393	0.0225
$P_{i,s_{27}}^0$	0.0624	0.0288	0.0845	0.0163	0.0323	0.0331	0.0252	0.0441	0.0283	0.0405