Combinatorial Game Theory Ideas

Kartik Sabharwal

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1 Five After Zero

1.1 Motivation

Let $N(i)$ denote the nimber of the game with i pins, where $i > 0$.
It so happens that for any i such that $N(i) = 0$, $N(i+1) = 5$.

1.2 Patterns

For the purposes of this section, we can turn this observation into a claim.
What would we need to show to prove the claim?

1. we can derive a state s from i such that $N(s) = 0$
2. we can derive a state s from i such that $N(s) = 1$
3. we can derive a state s from i such that $N(s) = 2$
4. we can derive a state s from i such that $N(s) = 3$
5. we can derive a state s from i such that $N(s) = 4$
6. we can derive no state s from i such that $N(s) = 5$

So consider a number i such that $N(i) = 0$. We want to show that $i + 1$ satisfies properties 1 through 6.
Is there a pattern in how situations where $N(n) = 5$ achieve states that satisfy properties 1 through 5 above?

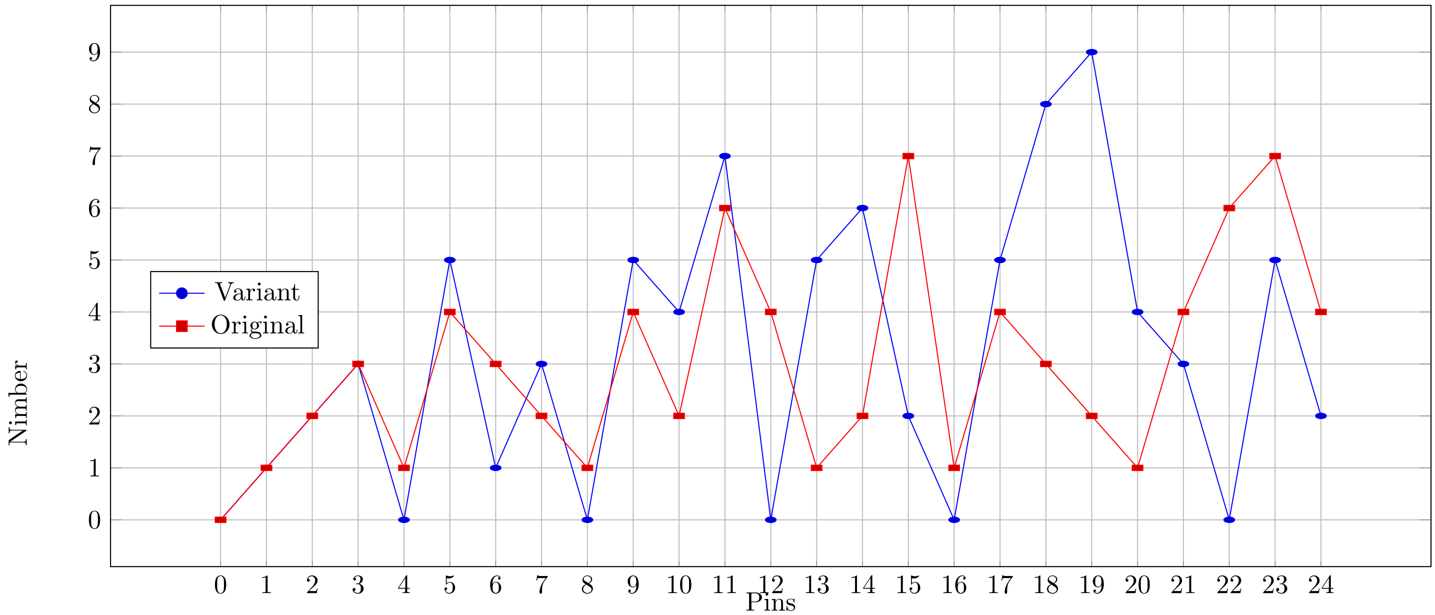
	0	1	2	3	4
5	4	2.2	2	3	3.1
9	8	6	1.7,2+4	7	4.4,1+5
13	12	4+6	2+8	6.6	10,4.8,1+9
17	16	6+8	2+12,8.8	6.10	1+13,3+11,4+10
23	22	?	9+11	21	20

2 Record-breaking Nimbers?

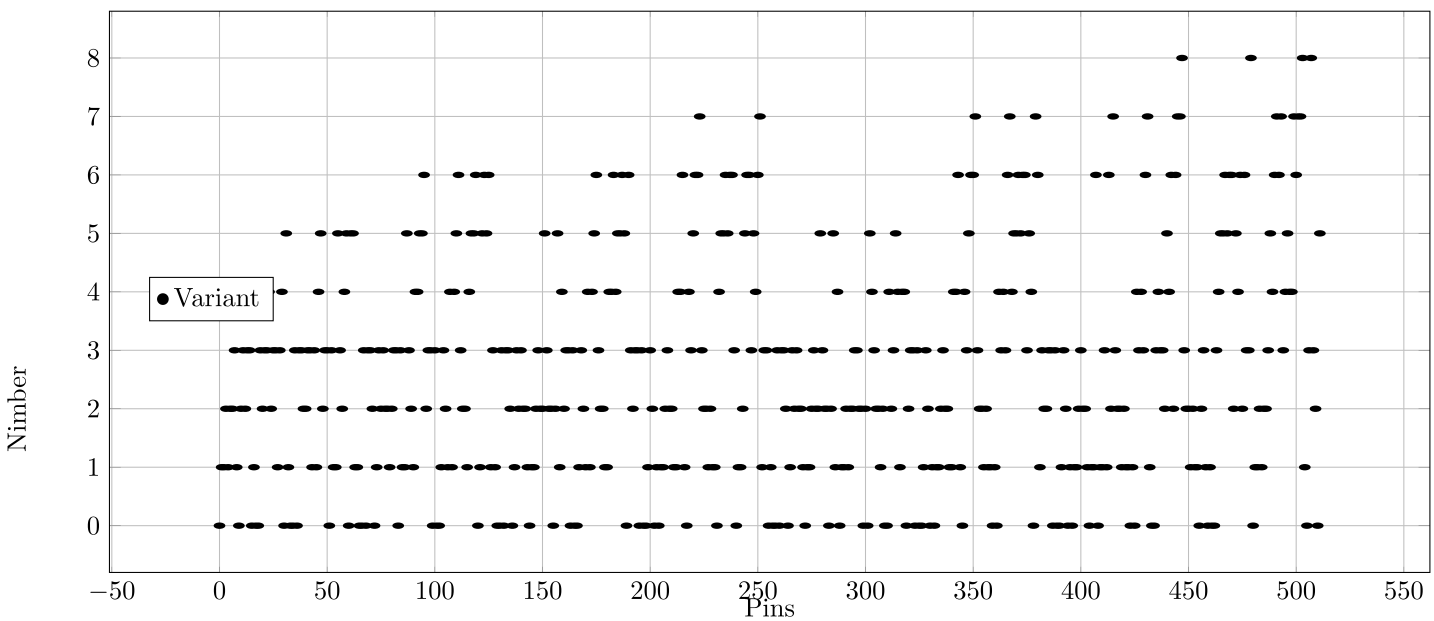
Also notice that $\{i : N(i) = \max_{j \in [i]}(N(j))\} = \{1, 2, 3, 5, 11, 19\}$ which may mean that if a given number of pins corresponds to a nimber higher than the ones before it, the number is likely to be prime.

3 Nimber-Pin Graph

pins	vari	orig
0	0	0
1	1	1
2	2	2
3	3	3
4	0	1
5	5	4
6	1	3
7	3	2
8	0	1
9	5	4
10	4	2
11	7	6
12	0	4
13	5	1
14	6	2
15	2	7
16	0	1
17	5	4
18	8	3
19	9	2
20	4	1
21	3	4
22	0	6
23	5	7
24	2	4



4 Expanded Nimber Graph


Variant
Nimber
Pins
0
1
2
3
4
5
6
7
8
−50
0
50
100
150
200
250
300
350
400
450
500
550