21-499 Problems

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Recall that in the *point-selection game* we take turns selecting previously unchosen points of $\mathbb{R}^2$. The goal is to build a congruent copy of some goal set in one's own points.

Question 1. *Polite chocolate* is played as with an $n \times k$ grid of squares (a chocolate bar). Players eat according to the following polite protocol: they choose a square (i, j) in the bar, and eat all squares not below or to the left of that square; that is, they eat all squares (i', j') with $i' \geq i$ and $j' \geq j$.

Of course, the (very polite) players lose the game if they finish the chocolate bar.

Show that unless $n = k = 1$, the first player has a winning strategy.

$3 \times 3 \times 3$ tic-tac-toe is played on a $3 \times 3 \times 3$ board (of 27 squares). Otherwise, the rules are the same as normal tic-tac-toe: the first to get 3 in-a-row wins. (Note that there are many lines in this game. For example, each corner now participates in 7 lines.)

Question 2. Find an explicit winning strategy (with proof) for P1 in $3 \times 3 \times 3$ tic-tac-toe. (You should be able to win in a very small number of moves. . . .)