

$$|j_1 j_2 j m\rangle = \sum_{j_1, m_1, j_2, m_2} \langle j_1 j_2 j m | j_1 m_1 j_2 m_2 \rangle |j_1 m_1 j_2 m_2\rangle$$

$$m_1 = j_1, \dots, -j_1$$

$$m_2 = -j_2, \dots, j_2$$

$$\text{Coefficients: } \langle j_1 m_1 j_2 m_2 | j m \rangle$$

$$= \langle j_1 j_2 j m \rangle$$

j_1 heeltallig, j_2 heeltallig $\rightarrow j$ heeltallig
 j_1 halfstallig, j_2 heeltallig $\rightarrow j$ halfstallig
 j_1 halfstallig, j_2 halfstallig $\rightarrow j$ heeltallig

$$j_1 = 2, j_2 = \frac{3}{2} \rightarrow |j_1 - j_2| = 2 - \frac{3}{2} = \frac{1}{2}$$

$$j = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$$

$$j_1 + j_2 = 2 + \frac{3}{2} = \frac{7}{2}$$

$$\sum_{j=j_1-j_2}^{j_1+j_2} (2j+1) = 2 \left\{ \frac{1}{2} (j_1+j_2)(j_1+j_2+1) - \frac{1}{2} (j_1-j_2)(j_1-j_2-1) \right\} + j_1 + j_2 - j_1 + j_2 + 1$$

$$= (j_1+j_2)^2 - (j_1-j_2)^2 + j_1 + j_2 + j_1 - j_2 + j_1 + j_2 + 1$$

$$= 4j_1 j_2 + 2j_1 + 2j_2 + 1 = (2j_1+1)(2j_2+1)$$

Inverse transformatie: $|j_1 j_2 j m\rangle = \sum_{j_1, m_1} \langle j_1 j_2 j m | j_1 m_1 j_2, m_2 \rangle |j_1 m_1 j_2, m_2\rangle$

$$= \sum_{j_1, m_1} \langle j_1 m_1 j_2 m_2 | j m \rangle |j_1 m_1 j_2, m_2\rangle$$

$$\sum_{m_1, m_2} \delta_{m, m_1+m_2} |j_1 j_2 j m\rangle$$

$$= \sum_{j_1, m_1} \langle j_1 m_1 j_2 m_2 | j m \rangle \sum_{m_2} \langle j_1 m_1 j_2 m_2 | j m \rangle |j_1 j_2 j m\rangle$$

$$\sum_{m_1, m_2} = \langle j_1 m_1 j_2 m_2 | j m \rangle \langle j_1 m_1 j_2 m_2 | j m \rangle = \delta_{m, m_1+m_2} \delta_{m, m_1+m_2} = \sum_{j_1, m_1} \langle j_1 m_1 j_2 m_2 | j m \rangle \langle j_1 m_1 j_2 m_2 | j m \rangle$$

$$\hat{J}_{\pm} |j_1 j_2 j m\rangle = \hbar \sqrt{(j \mp m)(j \pm m + 1)} |j_1 j_2 j m \pm 1\rangle$$

Maakt: $\hat{J}_{\pm} = \hat{J}_{1\pm} + \hat{J}_{2\pm}$

$$\hat{J}_{\pm} |j_1 j_2 j m\rangle = (\hat{J}_{1\pm} + \hat{J}_{2\pm}) \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | j m \rangle |j_1 j_2 j m_1 m_2\rangle$$

$$= \hbar \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | j m \rangle \sqrt{(j_1 \mp m_1)(j_1 \pm m_1 + 1)} |j_1 j_2 j m_1 \pm 1, m_2\rangle$$

$$+ \hbar \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | j m \rangle \sqrt{(j_2 \mp m_2)(j_2 \pm m_2 + 1)} |j_1 j_2 j m_1, m_2 \pm 1\rangle$$

Sluit met $\langle j_1 j_2 j m_1 m_2 |$

$$\hat{J}_{\pm} \hat{J}_{\pm} = \hat{J}_{\pm}^2$$

$$m = 0 \rightarrow m = 0$$

$$m = 1 \rightarrow m = -1, 0, 1$$

$$\sqrt{(j \mp m)(j \pm m + 1)} \langle j_1 m_1 j_2 m_2 | j m \pm 1 \rangle$$

$$\sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | j m \rangle \sqrt{(j_1 \mp m_1)(j_1 \pm m_1 + 1)} \delta_{m_1, m_1 \pm 1} \delta_{m_2, m_2}$$

$$+ \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | j m \rangle \sqrt{(j_2 \mp m_2)(j_2 \pm m_2 + 1)} \delta_{m_1, m_1} \delta_{m_2, m_2 \pm 1}$$

$$= \langle j_1 m_1 \mp 1, j_2 m_2 | j m \rangle \sqrt{(j_1 \mp m_1 + 1)(j_1 \pm m_1)}$$

$$+ \langle j_1 m_1, j_2 m_2 \mp 1 | j m \rangle \sqrt{(j_2 \mp m_2 + 1)(j_2 \pm m_2)}$$

$$\langle j_1 j_2 j_2 j_2 | j_1 j_2 j_1 j_2 \rangle = 1$$

$$\langle j_1 - j_1, j_2 - j_2 | j_1 + j_2, -j_1 - j_2 \rangle = 1$$