$\vec{L} = \vec{r} \times \vec{p}$

$[\hat{r}_i, \hat{p}_j] = i\hbar\delta_{ij}$

$[\hat{L}_i, \hat{L}_j] = i\hbar\varepsilon_{ijk}\hat{L}_k$

$\hat{\vec{\mu}} = \gamma\hat{\vec{S}}$ Energie $= -\vec{\mu}\cdot\vec{B}$

$\hookrightarrow -g\frac{e}{2m_e}$

$g \simeq 2$ voor elektronen

$\mathcal{H} = \mathcal{L}_2 \otimes \mathcal{H}_S$

$\hat{R}(\delta\vec{\varphi}) = \hat{1} - \frac{i}{\hbar}(\hat{\vec{L}} + \hat{\vec{S}})\cdot\delta\vec{\varphi}$

$\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}}$

$[\hat{S}_i, \hat{S}_j] = i\hbar\varepsilon_{ijk}\hat{S}_k$

$\Rightarrow [\hat{J}_i, \hat{J}_j] = i\hbar\varepsilon_{ijk}\hat{J}_k$

Spectrum van $\hat{\vec{J}}$?

$\hat{\vec{J}}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2$

$[\hat{J}^2, \hat{J}_x] = [\hat{J}_x^2, \hat{J}_x] + [\hat{J}_y^2, \hat{J}_x] + [\hat{J}_z^2, \hat{J}_x]$

$= \hat{J}_y[\hat{J}_y, \hat{J}_x] + [\hat{J}_y, \hat{J}_x]\hat{J}_y + \hat{J}_z[\hat{J}_z, \hat{J}_x] + [\hat{J}_z, \hat{J}_x]\hat{J}_z$

$= \hat{J}_y(-i\hbar\hat{J}_z) + (-i\hbar\hat{J}_z)\hat{J}_y + \hat{J}_z(i\hbar\hat{J}_y) + (i\hbar\hat{J}_y)\hat{J}_z$

$= 0 = [\hat{J}^2, \hat{J}_y] = [\hat{J}^2, \hat{J}_z]$

Gemeenschappelijk eigenbasis van $\hat{J}_z, \hat{J}^2$

$\hat{J}^2|a,b\rangle = a|a,b\rangle$

$\hat{J}_z|a,b\rangle = b|a,b\rangle$

Ladderoperatoren: $\hat{J}_+ = \hat{J}_x + i\hat{J}_y$

$\hat{J}_- = \hat{J}_x - i\hat{J}_y = (\hat{J}_+)^\dagger$

$\hat{J}_+\hat{J}_- = (\hat{J}_x + i\hat{J}_y)(\hat{J}_x - i\hat{J}_y) = \hat{J}_x^2 + \hat{J}_y^2 - i(\hat{J}_x\hat{J}_y - \hat{J}_y\hat{J}_x)$

$= \hat{J}_x^2 + \hat{J}_y^2 - i[\hat{J}_x, \hat{J}_y]$

$\hat{J}_-\hat{J}_+ = \hat{J}_x^2 + \hat{J}_y^2 + i[\hat{J}_x, \hat{J}_y]$

$\frac{1}{2}(\hat{J}_+\hat{J}_- + \hat{J}_-\hat{J}_+) = \hat{J}_x^2 + \hat{J}_y^2 = J^2 - \hat{J}_z^2$

Nu: $\langle\psi|\hat{J}_+\hat{J}_-|\psi\rangle \geq 0 \rightarrow \langle\psi|\hat{J}_-^\dagger\hat{J}_-|\psi\rangle = \langle\hat{J}_-\psi|\hat{J}_-\psi\rangle \geq 0$

$\langle\psi|\hat{J}_-\hat{J}_+|\psi\rangle \geq 0$

$\Rightarrow (\hat{J}^2 - \hat{J}_z^2)$ positieve operator

$(\hat{J}^2 - \hat{J}_z^2)|a,b\rangle = (a - b^2)|a,b\rangle$

$\Rightarrow a \geq b^2 \geq 0$

$[\hat{J}_z, \hat{J}_\pm] = [\hat{J}_z, \hat{J}_x \pm i\hat{J}_y]$

$= i\hbar\hat{J}_y \pm i(-i\hbar\hat{J}_x)$

$= i\hbar\hat{J}_y \pm \hbar\hat{J}_x$

$= \pm\hbar(\hat{J}_x \pm i\hat{J}_y) = \pm\hbar\hat{J}_\pm$

$[\hat{J}_+, \hat{J}_-] = [\hat{J}_x + i\hat{J}_y, \hat{J}_x - i\hat{J}_y]$

$= i(-i\hbar\hat{J}_z) - i(i\hbar\hat{J}_z)$

$= 2\hbar\hat{J}_z$

$\hat{J}_z\hat{J}_\pm|a\,b\rangle = ([\hat{J}_z, \hat{J}_\pm] + \hat{J}_\pm\hat{J}_z)|a\,b\rangle$

$= \pm\hbar\hat{J}_\pm|a\,b\rangle + b\hat{J}_\pm|a\,b\rangle$

$= (b \pm \hbar)\hat{J}_\pm|a\,b\rangle$

$\Rightarrow \hat{J}_\pm|a\,b\rangle$ is eigenvector $\hat{J}_z$!

$\rightarrow a \geq b^2 \geq 0$, stel $b_{min} \leq b \leq b_{max}$

$\hat{J}_+|a\,b_{max}\rangle = 0$ $\hat{J}_-|a\,b_{min}\rangle$

$\hat{J}_-\hat{J}_+|a\,b_{max}\rangle = 0$

$(\hat{J}^2 - \hat{J}_z^2 + i\,i\hbar\hat{J}_z)|a\,b_{max}\rangle = 0$

$(a - b_{max}^2 - \hbar b_{max})$

$\hat{J}_+\hat{J}_-|a\,b_{min}\rangle = 0$

$(\hat{J}^2 - \hat{J}_z^2 + \hbar\hat{J}_z)|a\,b_{min}\rangle = 0$

$a - b_{min}^2 + \hbar b_{min} = 0$

$a = b_{max}^2 + \hbar b_{max} = b_{max}(b_{max} + \hbar)$

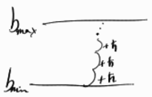
$a = b_{min}^2 - \hbar b_{min} = b_{min}(b_{min} - \hbar)$

$a = b_{min}^2 - \hbar b_{min} = b_{min}(b_{min} - \hbar)$

$b_{max}^2 - b_{min}^2 + \hbar(b_{max} + b_{min}) = 0$

$(b_{max} + b_{min})(b_{max} - b_{min} + \hbar) = 0$

$b_{min} = -b_{max}$ of $\cancel{b_{min} = b_{max} + \hbar}$



$b_{max} = -b_{max} + n\hbar,\ n \in \mathbb{N}$

$b_{max} = \frac{n\hbar}{2}$

Notatie: $j = \frac{n}{2}$ en $b = m\hbar$

$-b_{max} \leq b \leq b_{max}$

$-j \leq m \leq j$

$\hat{J}_z|j\,m\rangle = m\hbar|j\,m\rangle$

$a = b_{max}(b_{max} + \hbar)$

$= j(j+1)\hbar^2$

$j = 0$ $m = 0$

$j = \frac{1}{2}$; $m = -\frac{1}{2}, +\frac{1}{2}$

$j = 1$; $m = -1, 0, 1$

$j = \frac{3}{2}$; $m = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}$