(t.23) $\langle j_1 j_2; m_1 m_2 | \hat{R} | j_1 j_2; m_1' m_2' \rangle = \sum_{jm} \sum_{j'm'} \langle j_1 m_1 j_2 m_2 | jm \rangle \langle j_1 m_1' j_2 m_2' | j'm' \rangle \langle j_1 j_2; jm | \hat{R} | j_1 j_2; j'm' \rangle$

$\longrightarrow \delta_{jj'} D^{(j')}_{mm'}$

$= \langle j_1 m_1 | \hat{R} | j_1 m_1' \rangle \langle j_2 m_2 | \hat{R} | j_2 m_2' \rangle$

$= D^{(j_1)}_{m_1 m_1'} D^{(j_2)}_{m_2 m_2'}$

$\hat{R} = \exp(-i \hat{\vec{J}} \cdot \vec{n} \varphi)$

$\hookrightarrow \hat{\vec{J}}_1 + \hat{\vec{J}}_2$

$= \exp(-i \hat{\vec{J}}_1 \cdot \vec{n} \varphi) \exp(-i \hat{\vec{J}}_2 \cdot \vec{n} \varphi)$

$| j_1 j_2; m_1 m_2 \rangle = \sum_{jm} \langle j_1 m_1 j_2 m_2 | jm \rangle | j_1 j_2; jm \rangle$

$|j_1 - j_2| \ldots j_1 + j_2$

$\hat{\vec{L}} = \hat{\vec{r}} \times \hat{\vec{p}}$

$|\vec{r}\rangle = |xyz\rangle = |r \theta \varphi\rangle$

$\langle \vec{r} | \psi \rangle = \psi(\vec{r})$

$\langle r \theta \varphi | \psi \rangle = \psi(r, \theta, \varphi)$

$\hat{L}_x = y \hat{p}_z - z \hat{p}_y$, etc.

$= -i\hbar \left(y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right)$

?: in termen van $\frac{\partial}{\partial r}, \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \varphi}$

$x^2 + y^2 = r^2 \sin^2 \theta$

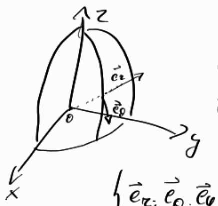
$r \sin \theta = \sqrt{x^2 + y^2}$

$\begin{cases} x = r \sin\theta \cos\varphi \\ y = r \sin\theta \sin\varphi \\ z = r \cos\theta \end{cases}$ $\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \theta = \operatorname{Atan}\left(\frac{\sqrt{x^2+y^2}}{z} \right) \\ \varphi = \operatorname{Atan}\left(\frac{y}{x} \right) \end{cases}$

$\frac{\partial}{\partial r} = \left(\frac{\partial x}{\partial r} \right) \left(\frac{\partial}{\partial x} \right) + \left(\frac{\partial y}{\partial r} \right) \frac{\partial}{\partial y} + \left(\frac{\partial z}{\partial r} \right) \left(\frac{\partial}{\partial z} \right) = \sin\theta \cos\varphi \frac{\partial}{\partial x} + \sin\theta \sin\varphi \frac{\partial}{\partial y} + \cos\theta \frac{\partial}{\partial z}$

$\frac{1}{r} \frac{\partial}{\partial \theta} = \cos\theta \cos\varphi \frac{\partial}{\partial x} + \cos\theta \sin\varphi \frac{\partial}{\partial y} - \sin\theta \frac{\partial}{\partial z}$

$\frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi} = -\sin\varphi \frac{\partial}{\partial x} + \cos\varphi \frac{\partial}{\partial y}$



$\vec{e}_r = (\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta)$

$\vec{e}_\theta = (\cos\theta \cos\varphi, \cos\theta \sin\varphi, -\sin\theta)$

$\vec{e}_\varphi = (-\sin\varphi, \cos\varphi, 0)$

$\{\vec{e}_r, \vec{e}_\theta, \vec{e}_\varphi\}$ rechtshandig o.b.

$\vec{e}_x \times \vec{e}_y = \vec{e}_z$

$\vec{e}_r \times \vec{e}_\theta = \vec{e}_\varphi$

$\frac{\partial}{\partial r} = \vec{e}_r \cdot \vec{\nabla}$

$\frac{1}{r} \frac{\partial}{\partial \theta} = \vec{e}_\theta \cdot \vec{\nabla}$

$\frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi} = \vec{e}_\varphi \cdot \vec{\nabla}$

$\vec{\nabla} = \vec{e}_r (\vec{e}_r \cdot \vec{\nabla}) + \vec{e}_\theta (\vec{e}_\theta \cdot \vec{\nabla}) + \vec{e}_\varphi (\vec{e}_\varphi \cdot \vec{\nabla})$

$= \vec{e}_r \frac{\partial}{\partial r} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{e}_\varphi \frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi}$

$\hat{\vec{L}} = \vec{r} \times \hat{\vec{p}}$

$= -i\hbar (\vec{r} \times \vec{\nabla})$

$= -i\hbar r (\vec{e}_r \times \vec{\nabla})$

$= -i\hbar r \, \vec{e}_r \times \left(\cancel{\vec{e}_r \frac{\partial}{\partial r}} + \vec{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \vec{e}_\varphi \frac{1}{r \sin\theta} \frac{\partial}{\partial \varphi} \right)$

$= -i\hbar \cancel{r} \left(\frac{1}{\cancel{r}} \vec{e}_\varphi \frac{\partial}{\partial \theta} + \vec{e}_\theta \frac{1}{\cancel{r} \sin\theta} \frac{\partial}{\partial \varphi} \right)$

$\hat{L}_x = -i\hbar \left(-\sin\varphi \frac{\partial}{\partial \theta} - \cot\theta \cos\varphi \frac{\partial}{\partial \varphi} \right)$

$\hat{L}_y = -i\hbar \left(\cos\varphi \frac{\partial}{\partial \theta} - \cot\theta \sin\varphi \frac{\partial}{\partial \varphi} \right)$

$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi}$

$\hat{L}^2 = -\hbar^2 (\vec{r} \times \nabla)^2$

$\hat{\vec{L}} = -i\hbar \left(\vec{e}_\varphi \frac{\partial}{\partial \theta} - \vec{e}_\theta \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} \right)$

$\hat{\vec{L}}^2 = -\hbar^2 \left(\vec{e}_\varphi \frac{\partial}{\partial \theta} - \vec{e}_\theta \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} \right) \cdot \left(\vec{e}_\varphi \frac{\partial}{\partial \theta} - \vec{e}_\theta \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} \right)$

$\vec{e}_\varphi \frac{\partial}{\partial \theta} \cdot \vec{e}_\varphi \frac{\partial}{\partial \theta} = \cancel{\vec{e}_\varphi \cdot \left(\frac{\partial \vec{e}_\varphi}{\partial \theta} \right) \frac{\partial}{\partial \theta}} + \underbrace{\vec{e}_\varphi \cdot \vec{e}_\varphi}_{=1} \frac{\partial^2}{\partial \theta^2}$

$\overset{\shortparallel}{0}$

$= \frac{\partial^2}{\partial \theta^2}$

$\vec{e}_\theta \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} \cdot \vec{e}_\theta \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} = \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \varphi^2}$

$-\vec{e}_\theta \frac{1}{\sin\theta} \frac{\partial}{\partial \varphi} \cdot \vec{e}_\varphi \frac{\partial}{\partial \theta} =$