$[\hat J_i, \hat J_j] = i\,\epsilon_{ijk}\hat J_k$

$\hat J^2|jm\rangle = j(j+1)\hbar^2|jm\rangle$

$j = 0, \frac12, 1, \frac32 \ldots$

$\hat J_z|jm\rangle = m\hbar|jm\rangle$

$m = -j, -j+1, \ldots, j-1, j$

$\# = 2j+1$

$\hat J_+ = \hat J_x + i\hat J_y$ | $\hat J_- = (\hat J_+)^\dagger$

$\hat J_- = \hat J_x - i\hat J_y$

$\langle j'm'|jm\rangle = \delta_{jj'}\delta_{mm'}$

$\langle j'm'|\hat J^2|jm\rangle = j(j+1)\hbar^2\delta_{jj'}\delta_{mm'}$

$\langle j'm'|\hat J_z|jm\rangle = m\hbar\,\delta_{jj'}\delta_{mm'}$

$\langle j'm'|\hat J_+|jm\rangle = ?$

$$\hat J_-\hat J_+ = (\hat J_x - i\hat J_y)(\hat J_x + i\hat J_y)$$
$$= \hat J^2 - \hat J_z^2 + i(\hat J_x\hat J_y - \hat J_y\hat J_x)$$
$$= \hat J^2 - \hat J_z^2 - \hbar\hat J_z$$

$$\langle jm|\hat J_-\hat J_+|jm\rangle = \langle jm|\hat J^2 - \hat J_z^2 - \hbar\hat J_z|jm\rangle$$
$$= \hbar^2(j^2 - m^2 + j - m) = \hbar^2(j-m)(j+m+1)$$

$\hat J_+|jm\rangle = c_{jm}|jm+1\rangle$

$|c_{jm}|^2 = \langle jm|(\hat J_+)^\dagger\hat J_+|jm\rangle = \hbar^2(j-m)(j+m+1)$

$c_{jm} = \hbar\sqrt{(j-m)(j+m+1)}$

$\hat J_+|jm\rangle = \hbar\sqrt{(j-m)(j+m+1)}\,|jm+1\rangle$

$\hat J_-|jm\rangle = c_{jm}|jm-1\rangle = \hbar\sqrt{(j+m)(j-m+1)}\,|jm\rangle$

$$|c_{jm}|^2 = \langle jm|\hat J_+\hat J_-|jm\rangle = \langle jm|\hat J^2 - \hat J_z^2 + \hbar\hat J_z|jm\rangle$$
$$= \hbar^2(j(j+1) - m^2 + m)|jm\rangle$$

p.85 voorbeelden eens lezen
Hij schrijft de komma's niet
Maar staat wel in cursus

$$\langle j'm'|\hat J_+|jm\rangle = \hbar\sqrt{(j-m)(j+m+1)}\,\langle j'm'|jm+1\rangle$$
$$= \hbar\sqrt{(j-m)(j+m+1)}\,\delta_{jj'}\delta_{m',m+1}$$

$$\left\{\begin{array}{l}\hat J_+ = \hat J_x + i\hat J_y \\ \hat J_- = \hat J_x - i\hat J_y\end{array}\right. \to \begin{array}{l}\hat J_x = \frac12(\hat J_+ + \hat J_-) \\ \hat J_y = -\frac{i}{2}(\hat J_+ - \hat J_-)\end{array}$$

$\vec\varphi = \varphi\vec n$

$\hat R(\vec\varphi) = \exp\left(-\frac{i}{\hbar}(\hat{\vec J}\cdot\vec n)\varphi\right)$

Matrixelementen in $|jm\rangle$-basis

$\langle j'm'|\hat R(\vec\varphi)|jm\rangle \sim \delta_{jj'} \to D^{(j)}_{m'm}(R) = \langle jm'|\hat R(\varphi\vec n)|jm\rangle$

$$\begin{bmatrix} \boxed{D^{(0)}} & 0 & 0 & \\ 0 & \boxed{D^{(1/2)}} & 0 & \\ 0 & 0 & \boxed{D^{(1)}} & \\ & & & \ddots \end{bmatrix}$$

$$\langle jm'|\hat R(R_2R_1)|jm\rangle = \langle jm'|\hat R(R_2)\underbrace{}_{\hat 1 = \sum_{m''}|jm''\rangle\langle jm''|}\hat R(R_1)|jm\rangle$$
$$= \sum_{m''}\langle jm'|\hat R(R_2)|jm''\rangle\langle jm''|\hat R(R_1)|jm\rangle$$

$$D^{(j)}_{m'm}(R_2R_1) = \sum_{m''} D^{(j)}_{m'm''}(R_2)\,D^{(j)}_{m''m}(R_1)$$

$$D^{(j)}(R_2R_1) = D^{(j)}(R_2)\,D^{(j)}(R_1)$$

p.88 blokdiagonaalstructuur

$j = 1 \to 3\times3$ matrices

$j(j+1) = 2$

$[\hat J^2] = 2\hbar\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$

$[\hat J_z] = \hbar\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}$ vgl. (2.158)

eigenvectoren: $\{|jj\rangle, |jj-1\rangle, \ldots, |j-j\rangle\}$

$\{|11\rangle, |10\rangle, |1-1\rangle\}$

$|11\rangle \to \begin{bmatrix}1\\0\\0\end{bmatrix}$

$|10\rangle \to \begin{bmatrix}0\\1\\0\end{bmatrix}$

$|1-1\rangle \to \begin{bmatrix}0\\0\\-1\end{bmatrix}$

$[\hat J_+] = \hbar\sqrt2\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}$

$[\hat J_-] = \hbar\sqrt2\begin{bmatrix}0&0&0\\1&0&0\\0&1&0\end{bmatrix}$ $\hat J_+|11\rangle = 0$

$\hat J_x = \frac12(\hat J_+ + \hat J_-)$

$[\hat J_x] = \frac{\hbar}{\sqrt2}\begin{bmatrix}0&1&0\\1&0&1\\0&1&0\end{bmatrix}$

(V_x, V_y, V_z)

$V_0 = V_z$

$V_{\pm1} = \frac{\mp1}{\sqrt2}(V_x \mp iV_y)$

$V_{+1} = (-V_x + iV_y)\frac{1}{\sqrt2}$

$V_{-1} = (V_x + iV_y)\frac{1}{\sqrt2}$

$\begin{bmatrix}V_{+1}\\V_0\\V_{-1}\end{bmatrix} = U\begin{bmatrix}V_x\\V_y\\V_z\end{bmatrix}$

$[U] = \begin{bmatrix}-\frac{1}{\sqrt2} & \frac{i}{\sqrt2} & 0\\ 0 & 0 & 1\\ \frac{1}{\sqrt2} & \frac{i}{\sqrt2} & 0\end{bmatrix}$

$[\hat J^c_i] = U^\dagger[\hat J_z]U$

$\boxed{J = \frac12}$

$[\hat J_z] = \hbar\begin{bmatrix}\frac12 & 0\\ 0 & -\frac12\end{bmatrix}$

Volgorde: $\{|\frac12\,\frac12\rangle, |\frac12\,-\frac12\rangle\}$

$[\hat J^2] = \hbar^2\frac34\begin{bmatrix}1&0\\0&1\end{bmatrix}$

$[\hat J_+] = \hbar\begin{bmatrix}0&1\\0&0\end{bmatrix}$

$[\hat J_-] = \hbar\begin{bmatrix}0&0\\1&0\end{bmatrix}$

$[\hat J_x] = \frac{\hbar}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix}$

$[\hat J_y] = -\frac{i}{2}\hbar\begin{bmatrix}0&1\\-1&0\end{bmatrix}$

$= \frac{\hbar}{2}\begin{bmatrix}0&-i\\i&0\end{bmatrix}$

$\hat{\vec J} = \frac{\hbar}{2}\hat{\vec\sigma}$

$\sigma_x = \begin{bmatrix}0&1\\1&0\end{bmatrix}$; $\sigma_y = \begin{bmatrix}0&-i\\i&0\end{bmatrix}$

$\sigma_z = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$

$\sigma_x\sigma_y = i\sigma_z$; $\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = 1$

$\sigma_x\sigma_y = -\sigma_y\sigma_z$