MLAI Week 6 Exercise: Logistic regression & PyTorch for deep learning

Note: An indicative mark is in front of each question. The total mark is 13. You may mark your own work when we release the solutions.

4   1. Figure 1 shows the COVID test results at a centre. What are the observed odds of COVID positive for the age group of 20–29? *Hint: You may follow a similar example in Lecture 6.*

Age	COVID	Age	COVID	Age	COVID
9	1	41	0	54	1
10	1	42	1	55	0
15	0	46	0	58	1
17	1	47	1	60	1
23	1	48	1	60	0
25	1	49	1	62	1
28	0	49	0	65	0
30	0	50	1	67	1
33	0	51	1	71	1
33	1	51	0	77	1
38	0	52	1	81	1

Figure 1: Age and COVID test results: $0 = $ negative, $1 = $ positive.

2   2. Derive π from $\log \frac{\pi}{1-\pi} = \mathbf{w}^\top \mathbf{x}$ (slide 22), i.e. derive the logistic function from the logit function.

2   3. The last equation on slide 23 writes the log likelihood in terms of π_i. Rewrite the equation in terms of the weight vector $\mathbf{w}$ and input vector $\mathbf{x}$.

2   4. In a binary (two-class) logistic regression model, the weight vector $\mathbf{w} = [4, \text{-}2, 5, \text{-}3, 11, 9]$. We apply it to some object that we'd like to classify; the vectorized feature representation of this object is $\mathbf{x} = [6, 8, 2, 7, \text{-}3, 5]$. What is the probability, according to the model, that this instance belongs to the positive class?

3   5. Consider the fully connected neural network (multilayer perceptron) on slide 33. If we insert two new hidden layers between the old Hidden Layer (4 neurons) and the Output Layer (2 neurons), i.e., New Layer 1 after old Hidden layer, New Layer 2 after New layer 1, and Output Layer after New Layer 2, with full connections between all adjacent layers and no other connections. The same activation function sigma (sigmoid) is used in the new hidden layers. How many learnable parameters in total are there for this three-hidden-layer neural network?