The Art of Translation

Using the predicates

- $Person(p)$, which states that p is a person, and
- $Loves(x, y)$, which states that x loves y,

write a sentence in first-order logic that means "everybody loves someone else."

$$
\begin{aligned}
&\forall p.\ (Person(p) \to \\
&\quad \exists q.\ (Person(q) \land p \neq q \land \\
&\qquad Loves(p, q) \\
&\quad) \\
&)
\end{aligned}
$$

How many of the following first-order logic statements are correct translations of “everyone loves someone else?”

$$
\begin{array}{l}
\forall p.\ (Person(p) \to \\
\quad \exists q.\ (Person(q) \land \\
\qquad Loves(p, q) \\
\quad) \\
)
\end{array}
$$

$$
\begin{array}{l}
\forall p.\ (Person(p) \land \\
\quad \exists q.\ (Person(q) \land p \neq q \land \\
\qquad Loves(p, q) \\
\quad) \\
)
\end{array}
$$

$$
\begin{array}{l}
\forall p.\ (Person(p) \to \\
\quad \exists q.\ (Person(q) \land p \neq q \to \\
\qquad Loves(p, q) \\
\quad) \\
)
\end{array}
$$

$$
\begin{array}{l}
\exists p.\ (Person(p) \to \\
\quad \forall q.\ (Person(q) \land p \neq q \land \\
\qquad Loves(p, q) \\
\quad) \\
)
\end{array}
$$

Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then **0**, **1**, **2**, **3**, or **4**.

Using the predicates

- $Person(p)$, which states that p is a person, and
- $Loves(x, y)$, which states that x loves y,

write a sentence in first-order logic that means “there is a person that everyone else loves.”

$$
\begin{aligned}
&\exists p.\ (Person(p)\ \land \\
&\qquad \forall q.\ (Person(q)\ \land\ p \neq q \to \\
&\qquad\qquad Loves(q, p) \\
&\qquad) \\
&)
\end{aligned}
$$

Combining Quantifiers

- Most interesting statements in first-order logic require a combination of quantifiers.
- Example: "Everyone loves someone else."

$$\forall p.\ (Person(p) \to \exists q.\ (Person(q) \land p \neq q \land Loves(p, q)))$$

For every person,

there is some person

who isn't them

that they love.

Combining Quantifiers

- Most interesting statements in first-order logic require a combination of quantifiers.
- Example: “There is someone everyone else loves.”

$$\exists p.\ (Person(p) \land \forall q.\ (Person(q) \land p \neq q \to Loves(q, p)))$$

There is some person

who everyone

who isn't them

loves.

For Comparison

$$\forall p.\ (Person(p) \to \exists q.\ (Person(q) \land p \neq q \land Loves(p, q)))$$

For every person,

there is some person

who isn't them

that they love.

$$\exists p.\ (Person(p) \land \forall q.\ (Person(q) \land p \neq q \to Loves(q, p)))$$

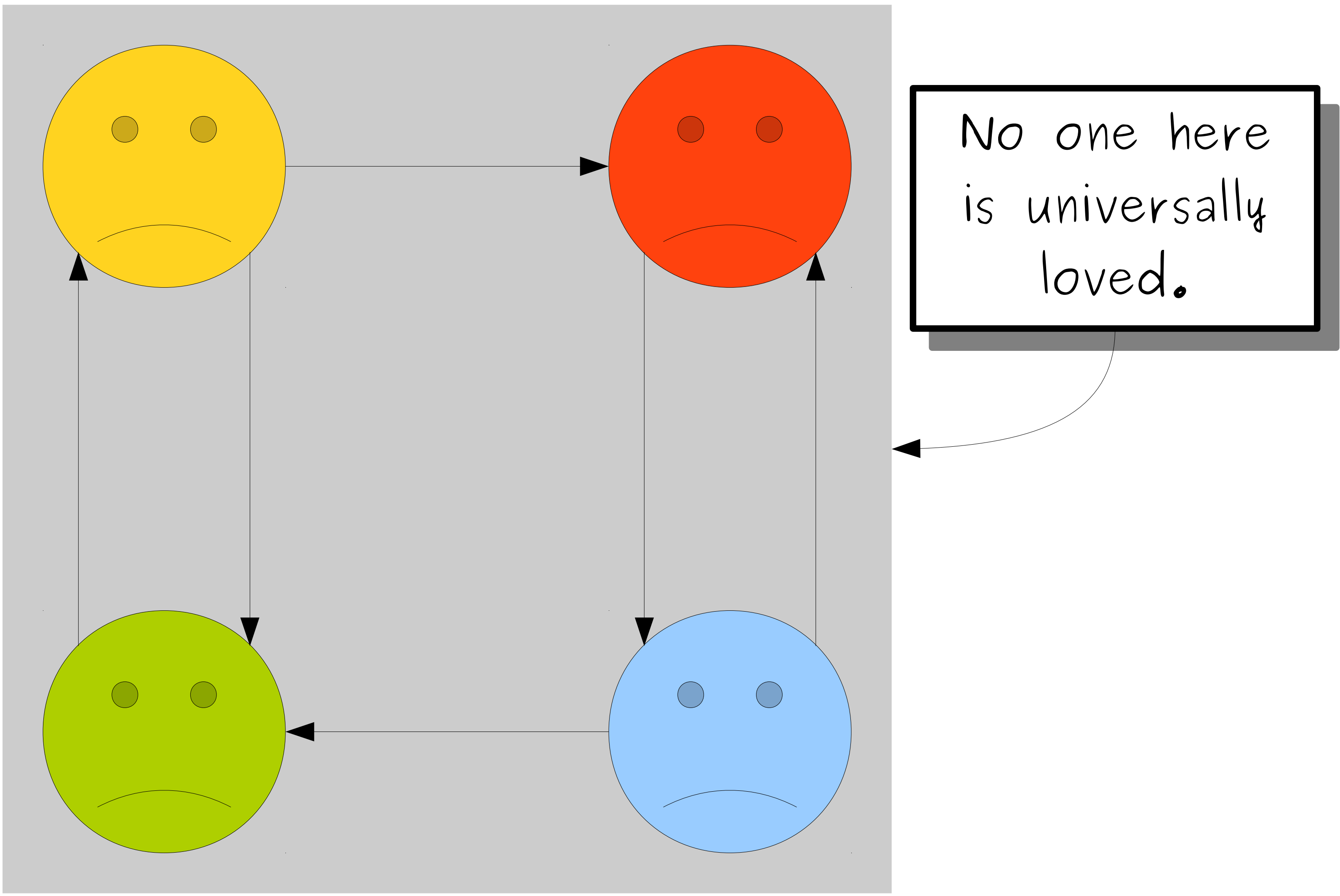
There is some person

who everyone

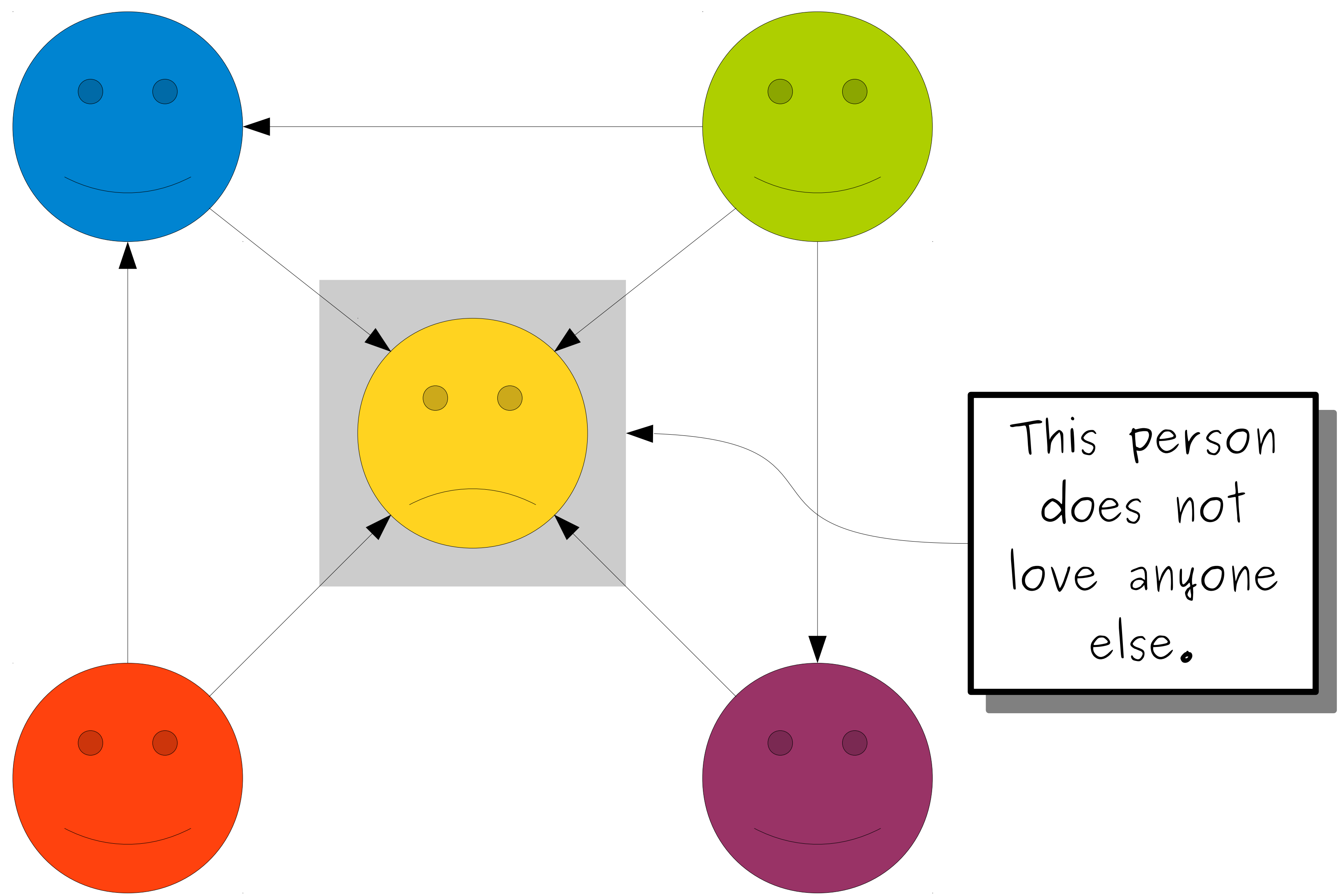
who isn't them

loves.

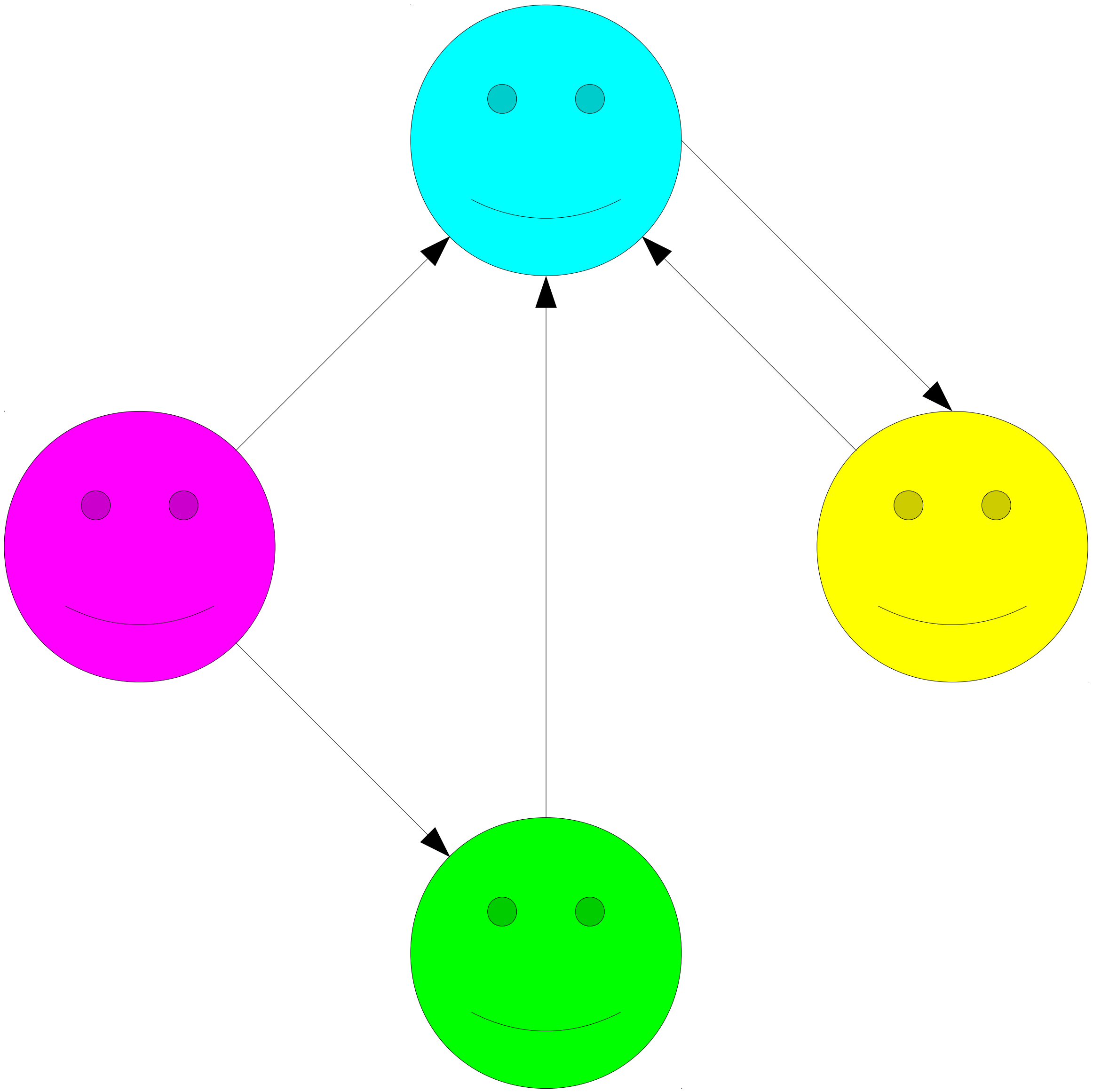
Everyone Loves Someone Else



There is Someone Everyone Else Loves



Everyone Loves Someone Else ***and*** There is Someone Everyone Else Loves



$$\forall p.\ (Person(p) \to \exists q.\ (Person(q) \land p \neq q \land Loves(p, q)))$$

For every person,

there is some person

who isn't them

that they love.

$$\land$$

$$\exists p.\ (Person(p) \land \forall q.\ (Person(q) \land p \neq q \to Loves(q, p)))$$

There is some person

who everyone

who isn't them

loves.

Quantifier Ordering

- The statement

  $$\forall x.\ \exists y.\ P(x, y)$$

  means "for any choice of x, there's some choice of y where $P(x, y)$ is true."
- The choice of y can be different every time and can depend on x.

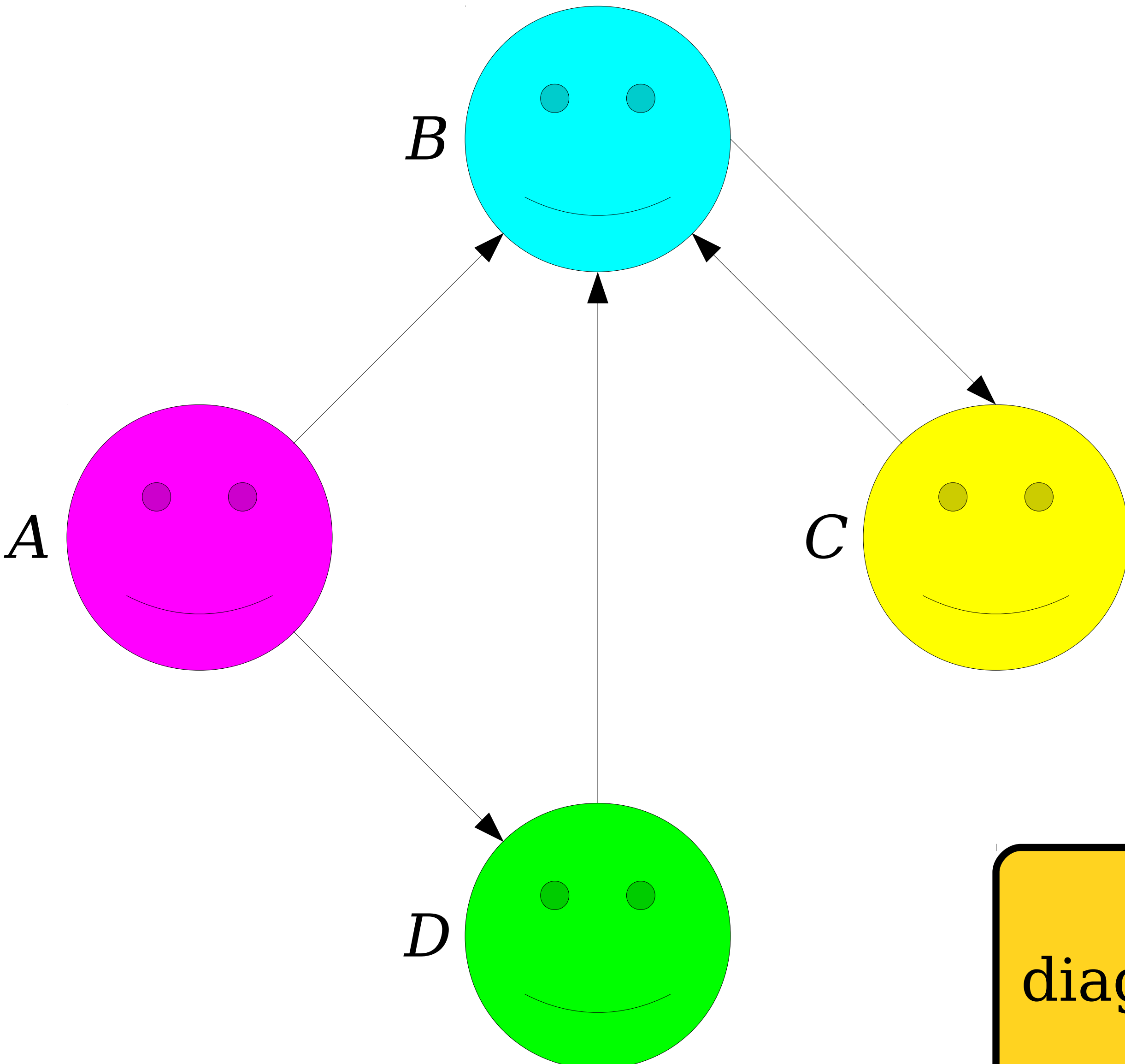
Quantifier Ordering

- The statement

$$\exists x.\ \forall y.\ P(x, y)$$

  means "there is some x where for any choice of y, we get that $P(x, y)$ is true."
- Since the inner part has to work for any choice of y, this places a lot of constraints on what x can be.

Order matters when mixing existential and universal quantifiers!



Which person in this diagram do you most aspire to be?

Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then **A**, **B**, **C**, or **D**.

Set Translations

Using the predicates

- $Set(S)$, which states that S is a set, and
- $x \in y$, which states that x is an element of y,

write a sentence in first-order logic that means "the empty set exists."

First-order logic doesn't have set operators or symbols "built in." If we only have the predicates given above, how might we describe this?

$$\exists S. (Set(S) \land \neg \exists x. x \in S)$$

$$\exists S. (Set(S) \land \forall x. x \notin S)$$

Both of these translations are correct. Just like in propositional logic, there are many different equivalent ways of expressing the same statement in first-order logic.

Using the predicates

- $Set(S)$, which states that S is a set, and
- $x \in y$, which states that x is an element of y,

write a sentence in first-order logic that means "two sets are equal if and only if they contain the same elements."

$$
\begin{aligned}
&\forall S.\ (Set(S) \rightarrow \\
&\quad \forall T.\ (Set(T) \rightarrow \\
&\qquad (S = T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T)) \\
&\quad) \\
&)
\end{aligned}
$$

$$
\begin{aligned}
&\forall S.\ (Set(S) \to \\
&\quad \forall T.\ (Set(T) \to \\
&\quad\quad (S = T \leftrightarrow \forall x.\ (x \in S \leftrightarrow x \in T)) \\
&\quad) \\
&)
\end{aligned}
$$

You sometimes see the universal quantifier pair with the ↔ connective. This is especially common when talking about sets because two sets are equal when they have precisely the same elements.