

Logical Implication

Implications

- An ***implication*** is a statement of the form

If P is true, then Q is true.

- Some examples:
 - Math: If n is an even integer, then n^2 is an even integer.
 - Set Theory: If $A \subseteq B$ and $B \subseteq A$, then $A = B$.
 - Queen Bey: If you like it, then you should put a ring on it.

What Implications Mean

- Consider the simple statement
If I put fire near cotton, it will burn.
- Some questions to consider:
 - Does this apply to all fire and all cotton, or just some types of fire and some types of cotton? (*Scope*)
 - Does the fire cause the cotton to burn, or does the cotton burn for another reason? (*Causality*)
- These are significantly deeper questions than they might seem.
- To mathematically study implications, we need to formalize what implications really mean.

Understanding Implications

**“If there's a rainbow in the sky,
then it's raining somewhere.”**

- In mathematics, implication is *directional*.
 - The above statement doesn't mean that if it's raining somewhere, there has to be a rainbow.
- In mathematics, implications only say something about the consequent when the antecedent is true.
 - If there's no rainbow, it doesn't mean there's no rain.
- In mathematics, implication says nothing about *causality*.
 - Rainbows do not cause rain. ☺

What Implications Mean

- In mathematics, a statement of the form
For any x , if $P(x)$ is true, then $Q(x)$ is true
means that any time you find an object x where $P(x)$ is true, you will see that $Q(x)$ is also true (for that same x).
- There is no discussion of causation here. It simply means that if you find that $P(x)$ is true, you'll find that $Q(x)$ is also true.

Implication, Diagrammatically

