Proof by Contrapositive

To prove the statement

“If *P* is true, then *Q* is true,”

you could choose to instead prove the equivalent statement

“If *Q* is false, then *P* is false.”

(if that seems easier).

This is called a ***proof by contrapositive***.

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We're starting this proof by telling the reader that it's a proof by contrapositive. This helps cue the reader into what's about to come next.

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Here, we're explicitly writing out the contrapositive. This tells the reader what we're going to prove. It also acts as a sanity check by forcing us to write out what we think the contrapositive is.

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We've said that we're going to prove this new implication, so let's go do it! The rest of this proof will look a lot like a standard direct proof.

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From this, we see that there is an integer m (namely, $2k^2 + 2k$) such that $n^2 = 2m + 1$.

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The general pattern here is the following:

1. Start by announcing that we're going to use a proof by contrapositive so that the reader knows what to expect.

2. Explicitly state the contrapositive of what we want to prove.

3. Go prove the contrapositive.