Negations

Negations

- A ***proposition*** is a statement that is either true or false.
  - Sentences that are questions or commands are not propositions.
- Some examples:
  - If n is an even integer, then n^2 is an even integer.
  - $\varnothing = \mathbb{R}$.
  - Moonlight is a good movie.
- The ***negation*** of a proposition X is a proposition that is true whenever X is false and is false whenever X is true.
- For example, consider the statement "it is snowing outside."
  - Its negation is "it is not snowing outside."
  - Its negation is *not* "it is sunny outside." ⚠

How do you find the negation of a statement?

The negation of the ***universal*** statement

Every *P* is a *Q*

is the *existential* statement

There is a *P* that is not a *Q*.

The negation of the ***universal*** statement

For all x, $P(x)$ is true.

is the *existential* statement

There exists an x where $P(x)$ is false.

The negation of the ***existential*** statement

There exists a *P* that is a *Q*

is the *universal* statement

Every *P* is not a *Q*.

The negation of the ***existential*** statement

There exists an *x* where *P*(*x*) is true

is the *universal* statement

For all *x*, *P*(*x*) is false.

Puppy Logic

- Consider the statement

I love all puppies.

Puppy Logic

- Consider the statement

  I love all puppies.

What is the negation?

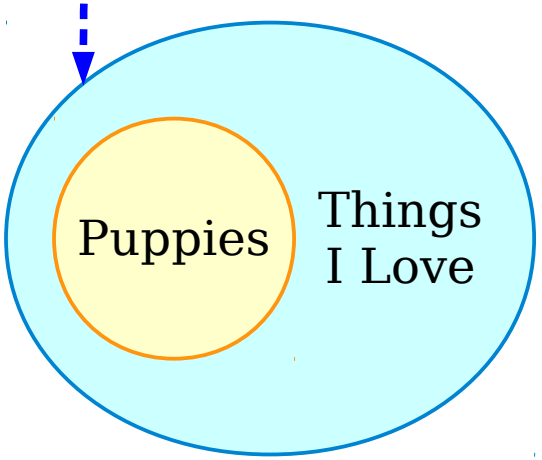
A. I don’t love any puppies.
B. I love some puppies and not others.
C. There is at least one puppy I don’t love.

Answer at **PollEv.com/cs103** or
text **CS103** to **22333** once to join, then **A**, **B**, or **C**.

Puppy Logic

- Consider the statement

I love all puppies.



“I love all puppies.”

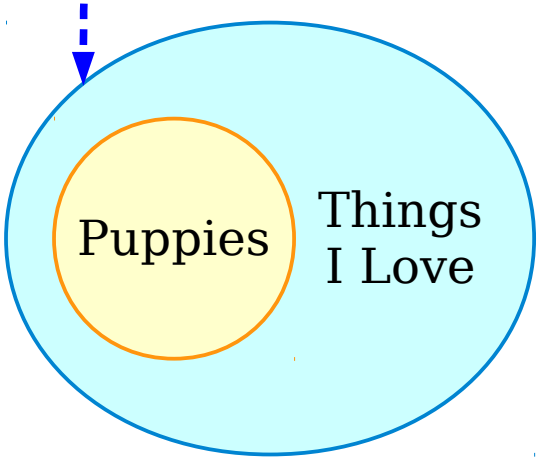
Puppy Logic

- Consider the statement

  I love all puppies.

- The following statement is ***not*** the negation of the original statement:

  ⚠ **I don't love *any* puppies.** ⚠



"I love all puppies."

Puppy Logic

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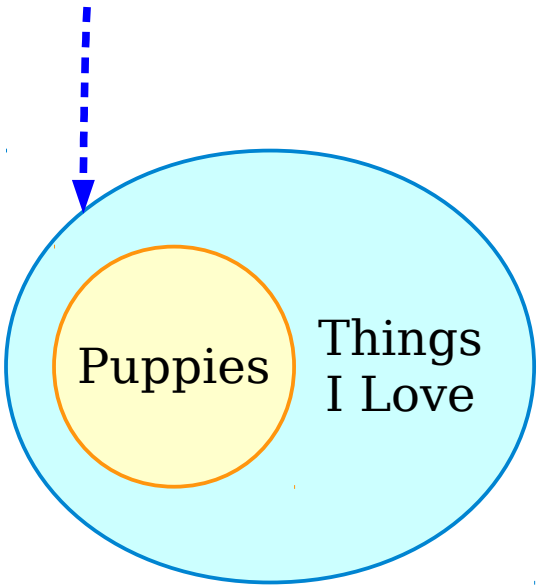
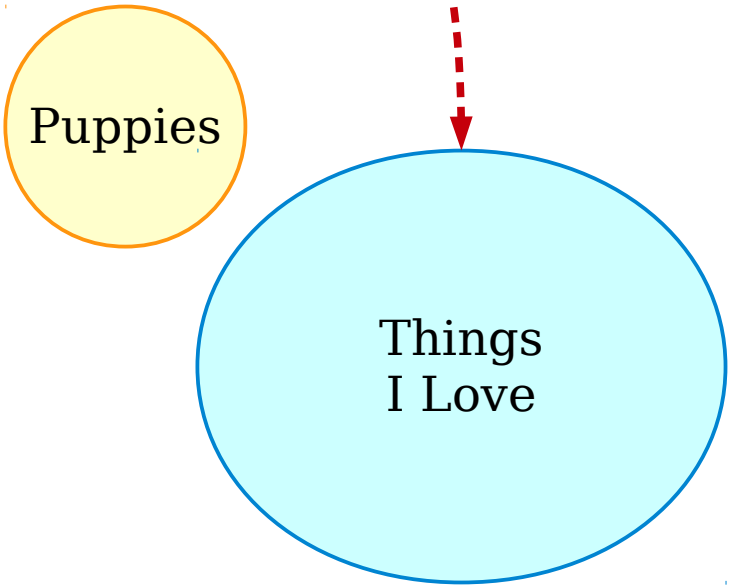


“I love all puppies.”



“I don't love *any* puppies.”

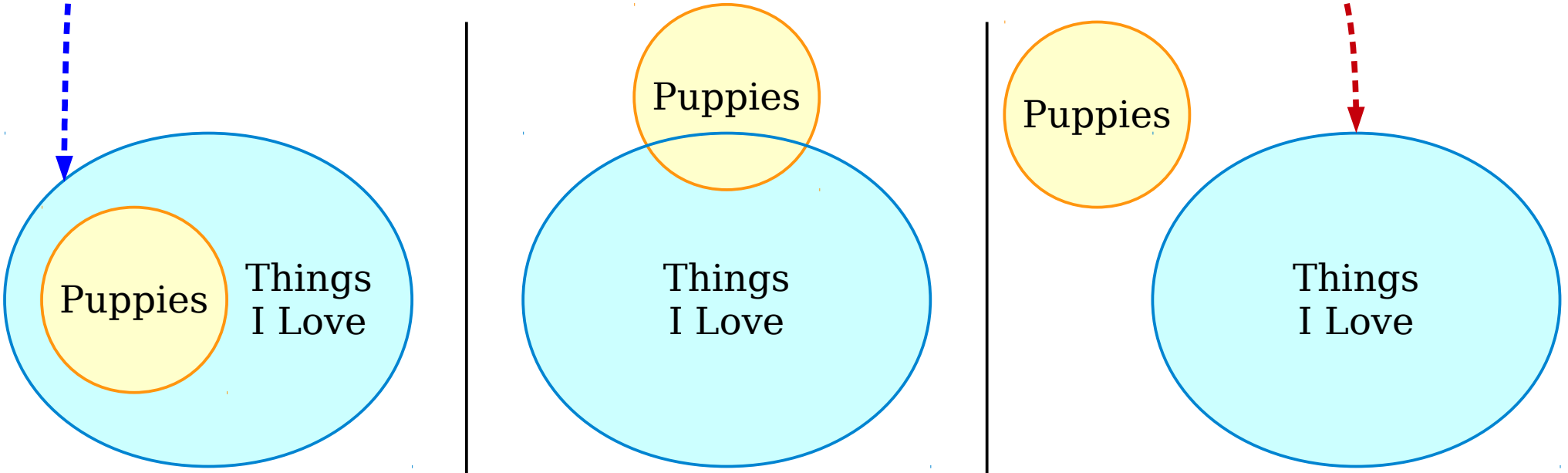
Puppy Logic

- Consider the statement

I love all puppies.

- The following statement is ***not*** the negation of the original statement:

⚠ **I don’t love *any* puppies.** ⚠



“I love all puppies.” “It's complicated.” **“I don't love *any* puppies.”**

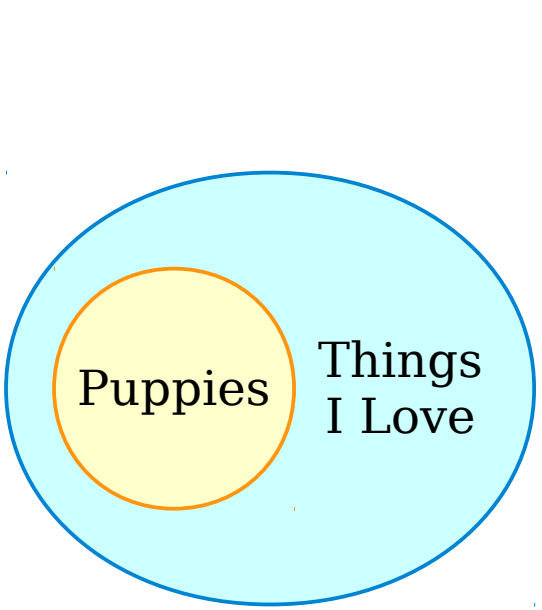
Puppy Logic

- Consider the statement

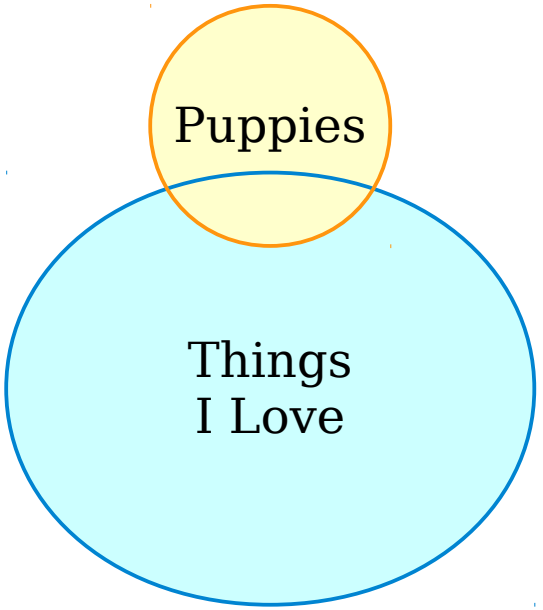
I love all puppies.

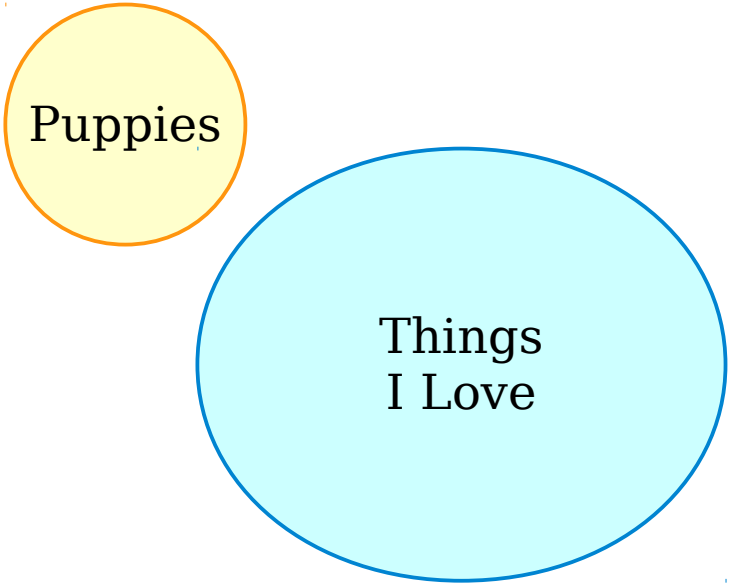
- Here's the proper negation of our initial statement about puppies:

There's *at least* one puppy I don't love.



“I love all puppies.”





“There's *at least* one puppy I don't love.”

How do you negate an implication?

Let’s look at:

- Negation of an implication
- A close relative of negation: the Contrapositive

The negation of the statement

“If *P* is true,
then *Q* is true”

is the statement

“P is true,
and Q is false.”

The negation of an implication
is not an implication!

“If your March Madness bracket is perfect, then you get an A in CS103.”

Which of the following is inconsistent with the above statement?

(A) Your bracket was terrible, and you got an A.
(B) Your bracket was terrible, and you got a B+.
(C) Your bracket was perfect, and you got a B+.
(D) Both (A) and (C)

The negation of the statement

“If your March Madness bracket is perfect, then you get an A in CS103.”

is the statement

“Your March Madness bracket is perfect, and you still didn’t get an A in CS103.

The negation of an implication is not an implication!

The negation of the statement

"For any x, if $P(x)$ is true, then $Q(x)$ is true"

is the statement

"There is at least one x where $P(x)$ is true and $Q(x)$ is false."

The negation of an implication is not an implication!