Universidade Federal de Uberlândia

FEELT – Faculdade de Engenharia Elétrica

SISTEMAS E CONTROLE

Roteiro 04a – Modelagem com a TL

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SUMÁRIO

Introdução

Esta semana utilizamos a Transformada de Laplace para modelar sistemas dinâmicos. Esse método facilita a modelagem no domínio complexo e permite obter funções de transferência diretamente na variável s.

Atividade 01

Assistir os vídeos de 1 a 3, da seguinte lista:
Sistemas de Controle – Professor Aniel
https://www.youtube.com/playlist?list=PLjhzxDly7tNQp2CkUHvAKPciOsnuYk_f_

Atividade 02

Da lista de problemas do livro, disponibilizado no arquivo "Nise - cap2 - Lista de Exercícios", resolva as seguintes sequências:

a) Faça os exercícios de 17 até 20, modelando os sistemas elétricos.
b) Faça os exercícios de 26 até 29, modelando os sistemas mecânicos lineares.
c) Faça os exercícios de 32 até 34, modelando os sistemas mecânicos rotativos.

Resolução A - 17

Find the transfer function G(s) = Vo(s)/Vi(s), for each network shown in Figure P2.3. [Section: 2.4]

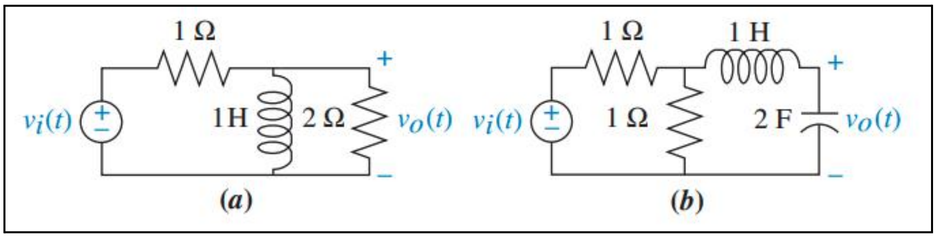
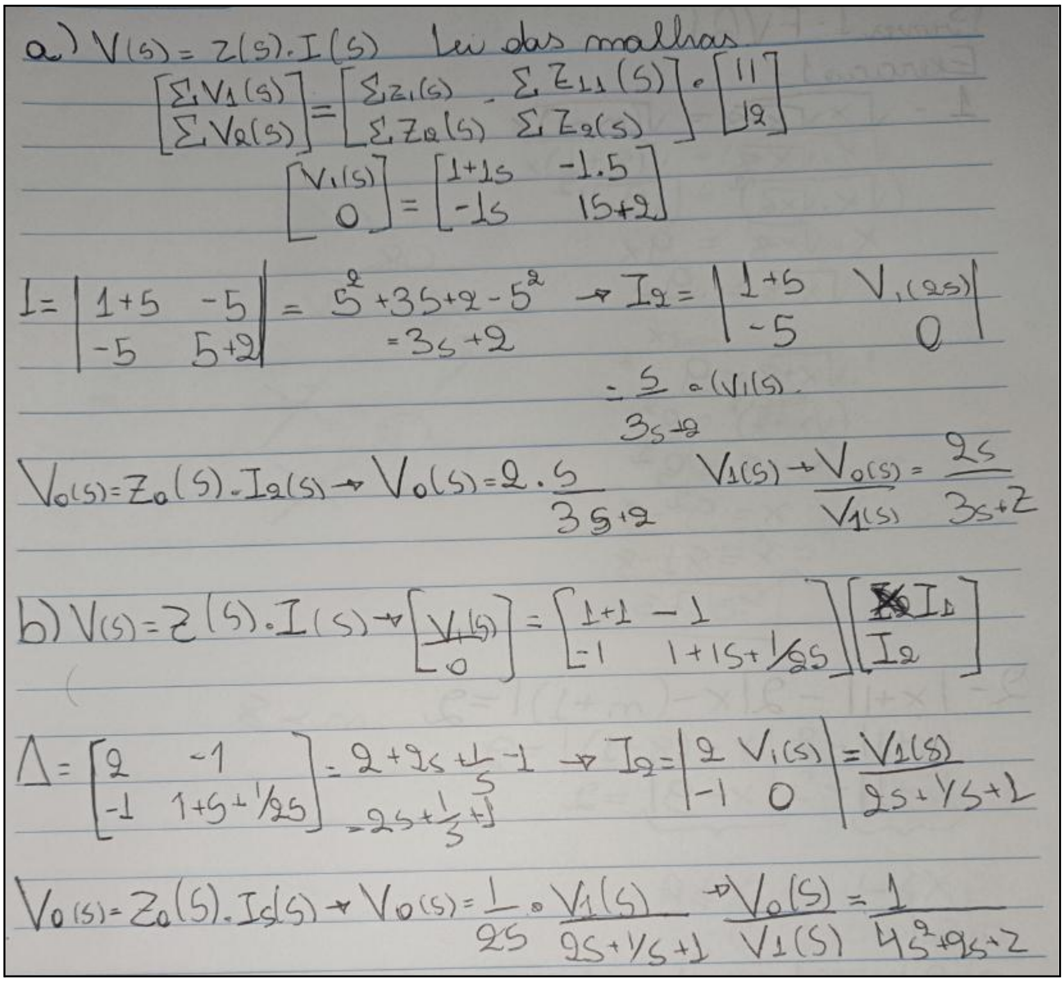


Figura P2.3.



a) $V(s) = Z(s).I(s)$ Lei das malhas

$$\begin{bmatrix}\Sigma V_1(s)\\ \Sigma V_2(s)\end{bmatrix} = \begin{bmatrix}\Sigma Z_1(s) & \Sigma Z_{11}(s)\\ \Sigma Z_2(s) & \Sigma Z_2(s)\end{bmatrix} \cdot \begin{bmatrix}I_1\\ I_2\end{bmatrix}$$

$$\begin{bmatrix}V_1(s)\\ 0\end{bmatrix} = \begin{bmatrix}1+1s & -1.s\\ -1s & 1s+2\end{bmatrix}$$

$$\Delta = \begin{vmatrix}1+s & -s\\ -s & s+2\end{vmatrix} = s^2+3s+2-s^2 = 3s+2 \rightarrow I_2 = \begin{vmatrix}1+s & V_1(2s)\\ -s & 0\end{vmatrix} = \frac{s}{3s+2} \cdot (V_1(s))$$

$$V_o(s) = Z_o(s).I_2(s) \rightarrow V_o(s) = 2 . \frac{s}{3s+2} V_1(s) \rightarrow \frac{V_o(s)}{V_1(s)} = \frac{2s}{3s+2}$$

b) $V(s) = Z(s).I(s) \rightarrow \begin{bmatrix}V_1(s)\\ 0\end{bmatrix} = \begin{bmatrix}1+1 & -1\\ -1 & 1+1s+1/2s\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix}$

$$\Delta = \begin{bmatrix}2 & -1\\ -1 & 1+s+1/2s\end{bmatrix} = 2+2s+\frac{1}{s}-1 = 2s+\frac{1}{s}+1 \rightarrow I_2 = \begin{vmatrix}2 & V_1(s)\\ -1 & 0\end{vmatrix} = \frac{V_1(s)}{2s+1/s+1}$$

$$V_o(s) = Z_o(s).I_2(s) \rightarrow V_o(s) = \frac{1}{2s} \cdot \frac{V_1(s)}{2s+1/s+1} \rightarrow \frac{V_o(S)}{V_1(S)} = \frac{1}{4s^2+2s+2}$$

Figura 1 - Resolução 17 itens A e B.

Resolução A - 18

Find the transfer function, G(s) = VL(s)/V(s), for each network shown in Figure P2.4. [Section: 2.4]

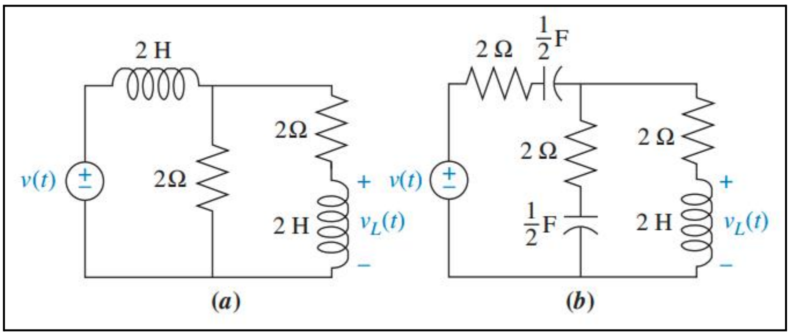


Figura P2.4.

18 a) $V(s) = I(s) \cdot Z(s) \rightarrow \begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+2 & -2 \\ -2 & 2+2+2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$\Delta = \begin{vmatrix} 2s+2 & -2 \\ -2 & 4+2s \end{vmatrix} = 4s^2+12s+4 \rightarrow I_2 = \dfrac{\begin{vmatrix} 2s+2 & V(s) \\ -2 & 0 \end{vmatrix}}{4s^2+12s+4} = \dfrac{V(s)}{2s^2+6s+2}$

$V_L(s) = Z_L(s) \cdot I_2 = \dfrac{2s \cdot V(s)}{2s^2+6s+2} \rightarrow \dfrac{V_L(s)}{V(s)} = \dfrac{s}{s^2+3s+1}$

b) $V(s) = Z(s) \cdot I(s) \rightarrow \begin{bmatrix} V(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2+2/s+2+\frac{2}{s} & -2-2/s \\ -2-2/s & 2+2/s+2+2s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$\Delta = \begin{vmatrix} 4+4/s & -2-2/s \\ -2-2/s & 4+4/s \end{vmatrix} = 12 + \frac{2s}{s} + \frac{12}{s^2} \rightarrow I_2 = \begin{vmatrix} 4+4/s & V(s) \\ -2-2/s & 0 \end{vmatrix}$

$V_L(s) = \dfrac{2s \cdot (s^2+s) V(s)}{6s^2+16s+6} \rightarrow \dfrac{V_L(s)}{V(s)} = \dfrac{s^3+s^2}{3s^2+7s+3}$

Figura 2 - Resolução 18 itens A e B.

Resolução A - 19

Find the transfer function, G(s) = Vo(s)/Vi(s), for each network shown in Figure P2.5. Solve the problem using mesh analysis. [Section: 2.4]

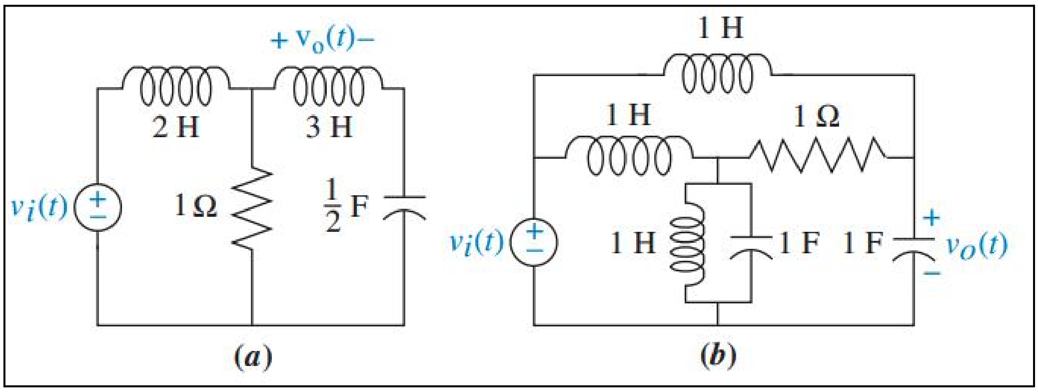


Figura P2.5.

19-a) $\begin{bmatrix} V_1(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -1 \\ -1 & 1+3s+2/s \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$ $\quad I_2 = \dfrac{s.V_i(s)}{6s^3+5s^2+4s+2}$

b) $\begin{bmatrix} V_i(s) \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} s+s/(s^2+1) & -s/(s^2+1) & -s \\ -s/(s^2+1) & s/s^2+1+1+1/s & -1 \\ -s & -1 & s+s+1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$

$I_2 = \dfrac{(s^3+2s^2+2s)V_i}{(s^4+2s^3+3s+2)} \rightarrow V_o(s) = \dfrac{1}{s} \cdot I_2 \rightarrow \dfrac{V_o(s)}{V_1(s)} = \dfrac{s^2+2s+2}{s^4+2s^3+3s^3+3s+2}$

Figura 3 - Resolução 19 itens A e B.

Resolução A - 20

Repeat Problem 19 using nodal equations. [Section: 2.4]

20) a) $V_2 = \dfrac{(3s^2+2)V_i}{(2s+1)(3s^2+2)+2s^2}$ $\quad I_2 = \dfrac{V_o}{V_i} = \dfrac{3s^2}{6s^3+5s^2+4s+2}$

b) $V_3 = \dfrac{(s^2+2s+2)V_i}{(s^4+2s+3/s^2+3s+2)} \rightarrow I_3 = \dfrac{V_3}{1/s} = sV_3$

$V_o = 1/s \cdot I_3 = V_3 = \dfrac{V_o}{V_1} = \dfrac{s^2+2s+2}{(s^4+2s^3+3s^2+3s+2)}$

Figura 4 - Resolução 20 itens A e B.

Resolução B - 26

Find the transfer function, G(s) = X2(s)/F(s), for the translational mechanical system shown in Figure P2.11. (Hint: place a zero mass at x2(t).) [Section: 2.5]

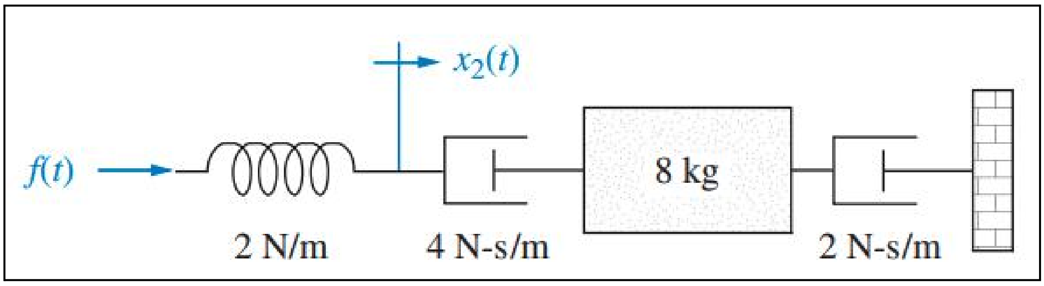
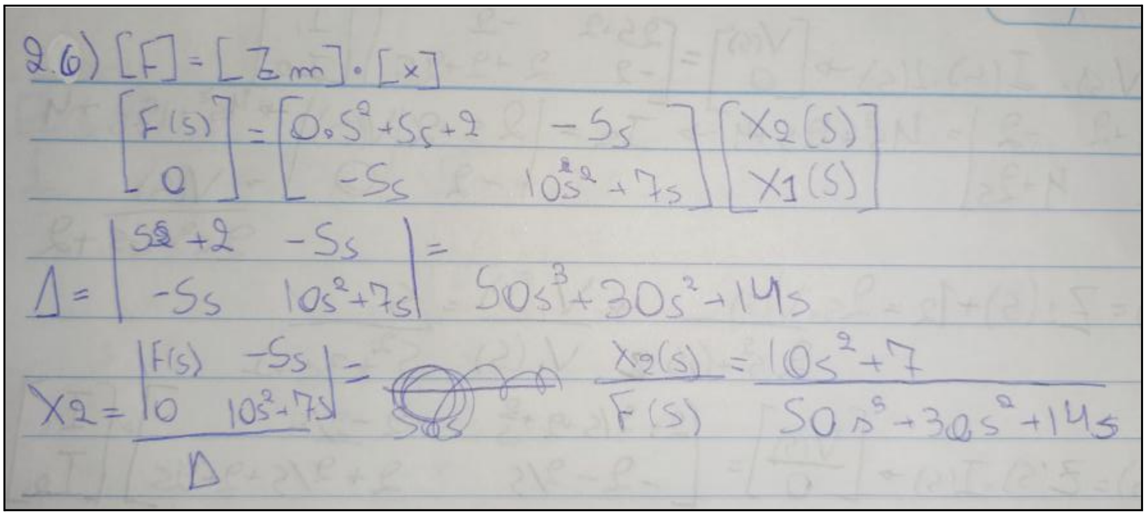


Figura P2.11.



$$2.6)\ [F] = [Z_m]\cdot[x]$$

$$\begin{bmatrix} F(s) \\ 0 \end{bmatrix} = \begin{bmatrix} 0\cdot s^2+5s+2 & -5s \\ -5s & 10s^2+7s \end{bmatrix}\begin{bmatrix} X_2(s) \\ X_1(s) \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5s+2 & -5s \\ -5s & 10s^2+7s \end{vmatrix} = 50s^3+30s^2+14s$$

$$X_2 = \frac{\begin{vmatrix} F(s) & -5s \\ 0 & 10s^2+7s \end{vmatrix}}{\Delta} = \qquad \frac{X_2(s)}{F(s)} = \frac{10s^2+7}{50s^3+30s^2+14s}$$

Figura 5 - Resolução 26.

Resolução B - 27

For the system of Figure P2.12 find the transfer function, G(s) = X1(s)/F(s) [Section: 2.5]

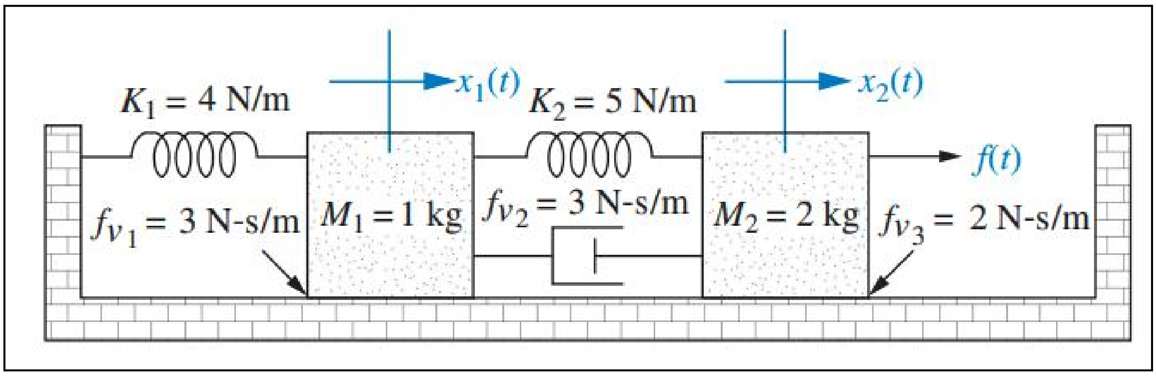


Figura P2.12.

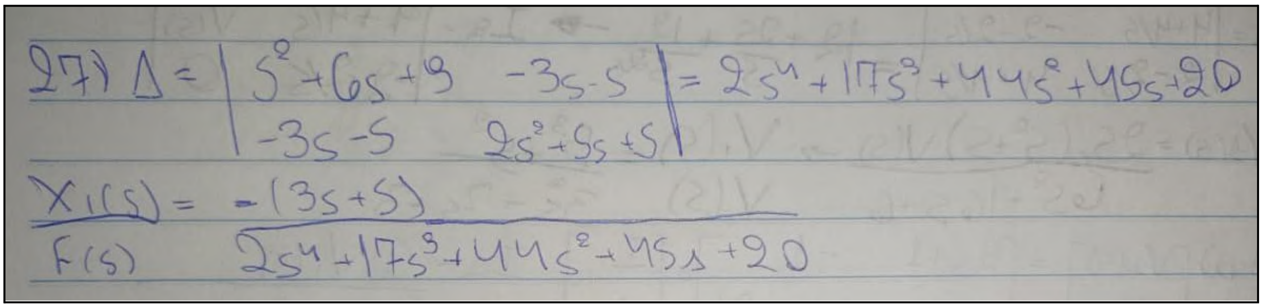


Figura 6 - Resolução 27.

Resolução B - 28

Find the transfer function, G(s) = X3(s)/F(s), for the translational mechanical system shown in Figure P2.13. [Section: 2.5]

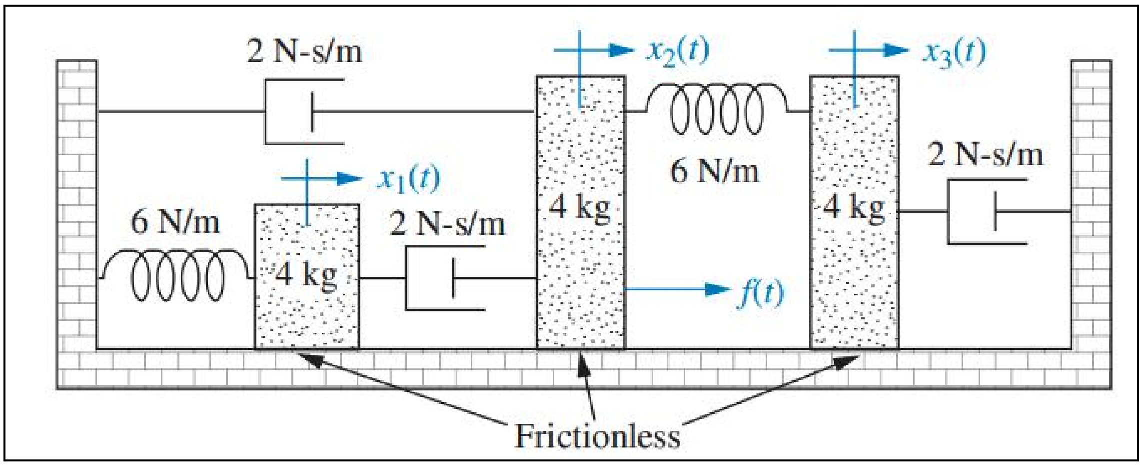


Figura P2.13.

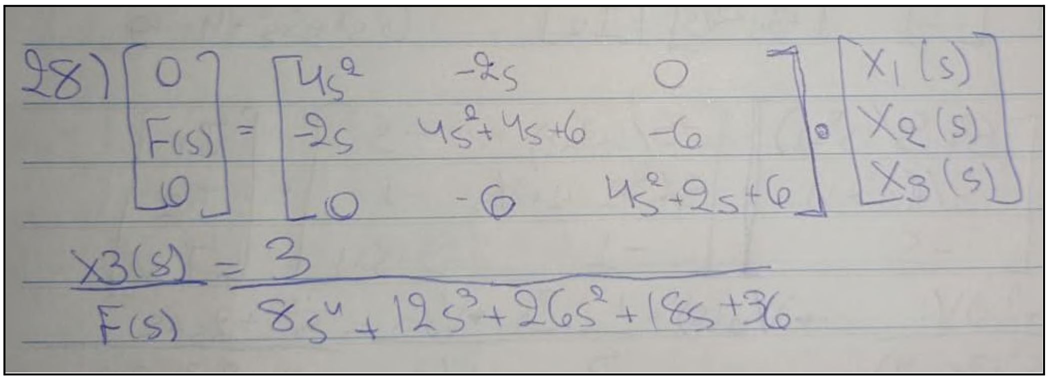


Figura 7 - Resolução 28.

Resolução C - 32

For each of the rotational mechanical systems shown in Figure P2.17, write, but do not solve, the equations of motion. [Section: 2.6]

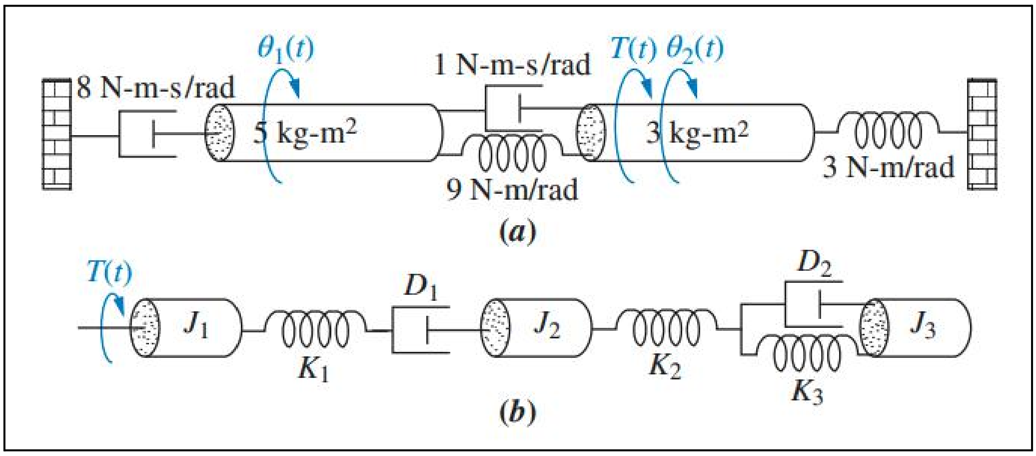


Figura P2.17.

32) $[T(s)] = [Z_m(s)][\Theta(s)]$

$$\begin{bmatrix} T(s) \\ 0 \end{bmatrix} = \begin{bmatrix} s^2+2s+1 & -s-1 \\ -s-1 & 2s+1 \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix} \qquad \frac{\theta_2(s)}{T(s)} = \frac{1}{2s^2+2s}.$$

Figura 8 - Resolução 32.

Resolução C - 33

For the rotational mechanical system shown in Figure P2.18, find the transfer function G(s) = θ2(s)/T(s) [Section: 2.6]

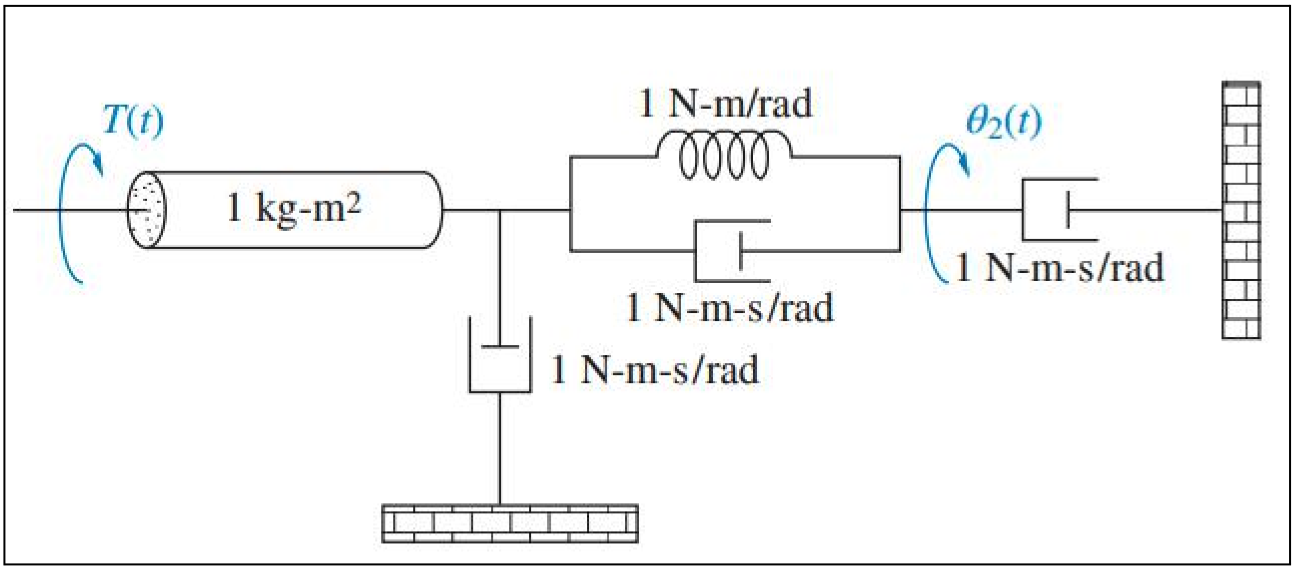
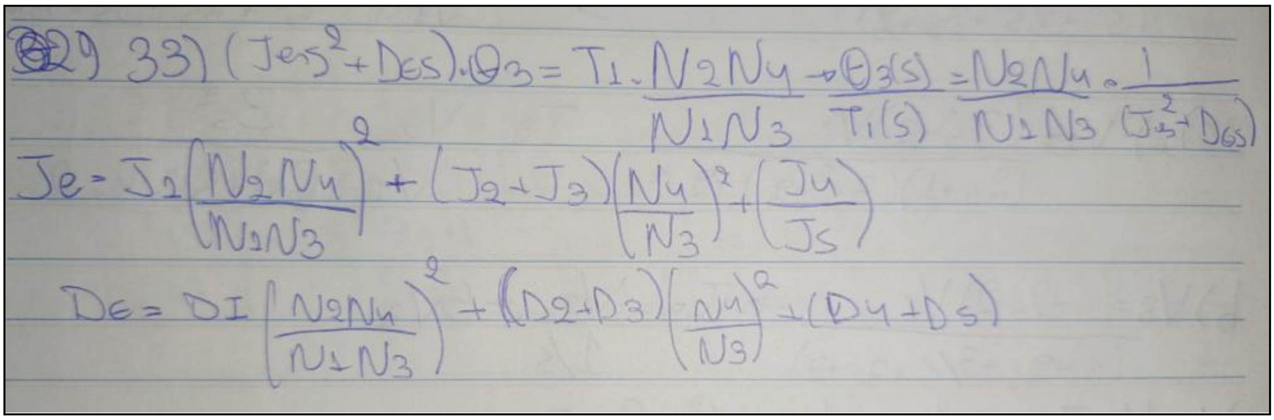


Figura P2.18.



33) $(J_e s^2 + D_e s)\cdot\theta_3 = T_1 \cdot \frac{N_2 N_4}{N_1 N_3} \rightarrow \frac{\theta_3(s)}{T_1(s)} = \frac{N_2 N_4}{N_1 N_3} \cdot \frac{1}{(J_e s^2 + D_e s)}$

$J_e = J_1\left(\frac{N_2 N_4}{N_1 N_3}\right)^2 + (J_2+J_3)\left(\frac{N_4}{N_3}\right)^2 + \left(\frac{J_4}{J_5}\right)$

$D_e = D_1\left(\frac{N_2 N_4}{N_1 N_3}\right)^2 + (D_2+D_3)\left(\frac{N_4}{N_3}\right)^2 + (D_4+D_5)$

Figura 9 - Resolução 33.

Resolução C - 34

Find the transfer function, θ1(s)/T(s) , for the system shown in Figure P2.19.

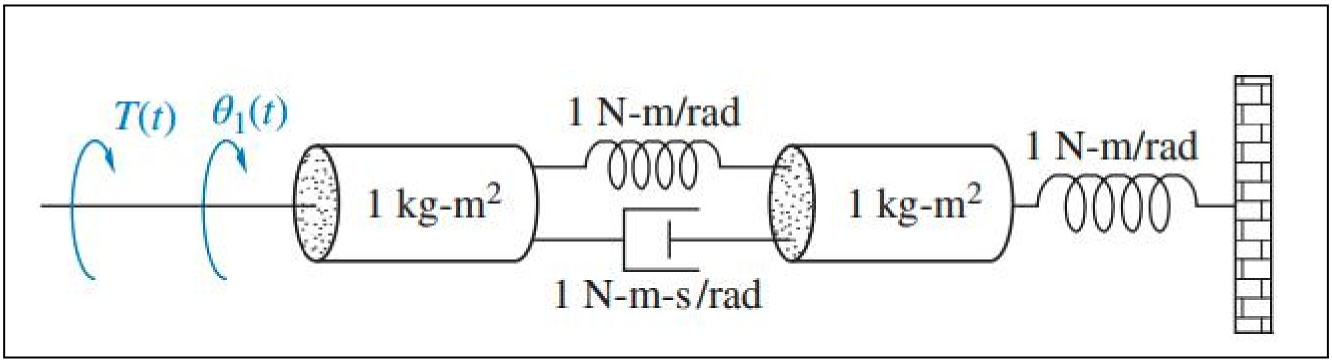
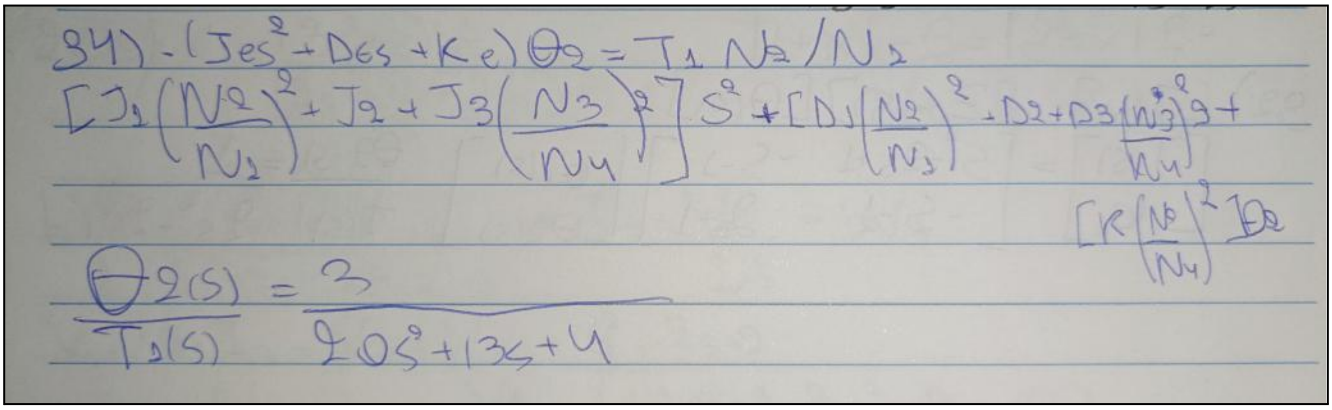


Figura P2.19.



34) $(J_e s^2 + D_e s + K_e)\theta_2 = T_1 N_2/N_1$

$\left[J_1\left(\frac{N_2}{N_1}\right)^2 + J_2 + J_3\left(\frac{N_3}{N_4}\right)^2\right]s^2 + \left[D_1\left(\frac{N_2}{N_1}\right)^2 + D_2 + D_3\left(\frac{N_3}{N_4}\right)^2\right]s + \left[K\left(\frac{N_3}{N_4}\right)^2\right]\theta_2$

$\frac{\theta_2(s)}{T_1(s)} = \frac{3}{20s^2 + 13s + 4}$

Figura 10 - Resolução 34.