APL Final Exam

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Introduction

Given, a long wire carries a current I = $\frac{4\pi}{\mu_o}\cos(\phi)exp(j\omega t)$ through a loop, where ϕ is the angle in the polar coordinates. The wire is of radius 10 is on the x-y plane centered at origin. We need to compute and plot the magnetic field $\vec{B}$ along the z-axis from z = 1cm to z = 1000cm and plot it then fit the data to $\vec{B}$ = cz^b.

Assignment Questions

Pseudo code

A detailed pseudo code to solve this problem is given by:

```
'''
This program plots the magnetic field vector along the z-axis from a loop antenna
and fit the data into an exponential.
Define a meshgrid of size 3 by 3 by 1000
k=1/radius , lambda = 2pi/k
Break the loop into 100 sections and find out the phi values of the centres
of the sections
Calculate I=4picos(phi)/mu_0 at the above points and plot them.
r' = radius*c_[cos(phi), sin(phi), zeros_like(phi)]
dl' = c_[-sin(phi), cos(phi), zeros_like(phi)]*lambda/100
Function calc(l):
R[i,j,k,l]=|r_ijk-r'_l|
A_(x,y)[i,j,k] = sum_l(cos(phi[l])*exp(-1j*k*R[i,j,k,l])*dl'_(x,y)[l])
Calculate B using the given equation
plot B vs z
Fit B as c*(z^b)
i.e; log(B)=log(c)+b*log(z)
fit [1 log(z)]*[log(c) b] = log(B)

'''
```

Current elements

We are required to break the volume into a 3 by 3 by 1000 meshgrid. We take the loop antenna between -1 and 1 in x and y directions and we take z values from 1 to 1000 .

```
x=np.linspace(0,2,3)
y=np.linspace(0,2,3)
z=np.linspace(1,1000,1000)
X,Y,Z=np.meshgrid(x,y,z)
```

We break the circumference of the loop into 100 sections and find out the phi values of the centre points of each of those loops.

```
sections=100
phi=np.linspace(0,2*pi,sections+1)
phi=phi[:-1]
```

We find the current elements at those points as shown below:

```
x=radius*np.cos(phi)
y=radius*np.sin(phi)
current_x =-np.sin(phi)*np.cos(phi)*4*pi/mu_0
current_y = np.cos(phi)*np.cos(phi)*4*pi/mu_0

plt.quiver(x,y,current_x,current_y)
```

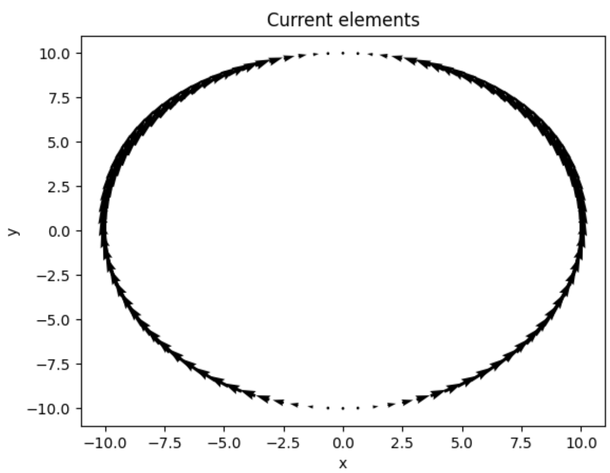

Figure 1: Current elements plot

Vector Potential and Magnetic field

r' is a point on the loop. dl' is the unit tangent vector at that point on the loop.

```
r_prime = np.array([x,y, np.zeros(len(phi))]).T
dl_prime = np.array([-2*pi*y/sections,2*pi*x/sections, np.zeros(len(phi))]).T
```

We need to find the R vector inside a function and then extend the function to calculate the various terms in the equation 1 .

```
def calc(l):
R = r - r_prime[l]
R = np.sqrt(np.sum(R**2, axis=-1))
temp = np.cos(phi[l])*np.exp(-1j*k*R)/R
temp = np.expand_dims(temp, axis=-1)
term = temp*dl_prime[l]
return term
```

We first find out A by the below equation.

$$\vec{A}(r,\phi,z) = \frac{\mu_o}{4\pi}\int \frac{I(\phi)\hat{\phi}e^{-jkR}ad\phi}{R}$$

But since the wire is a loop, this integral can be reduced to a sum:

$$\vec{A}ijk = \sum l = 0^{N-1}\frac{cos(\phi' l)exp(-jkRijkl)\vec{dl}'}{R_{ijkl}}$$

This equation is valid for any x_i, y_j, z_k , and is summed over the current elements in the loop.
Now, Magnetic field $\vec{B}$ is just the curl of vector $\vec{A}$. Along the z axis this becomes:

$$B_z(z) = \frac{A_y(\Delta x,0,z) - A_x(0,\Delta y,z) - A_y(-\Delta x,0,z) + A_x(0,-\Delta y,z)}{2\Delta x\Delta y}$$

```
A = np.zeros_like(r, dtype=np.complex128)
for l in range(dl_prime.shape[0]):
    A += calc(l)

B_z = (A[2,1,:,1]-A[1,2,:,0]-A[0,1,:,1]+A[1,0,:,0])/4
plt.loglog(z,abs(B_z))
```

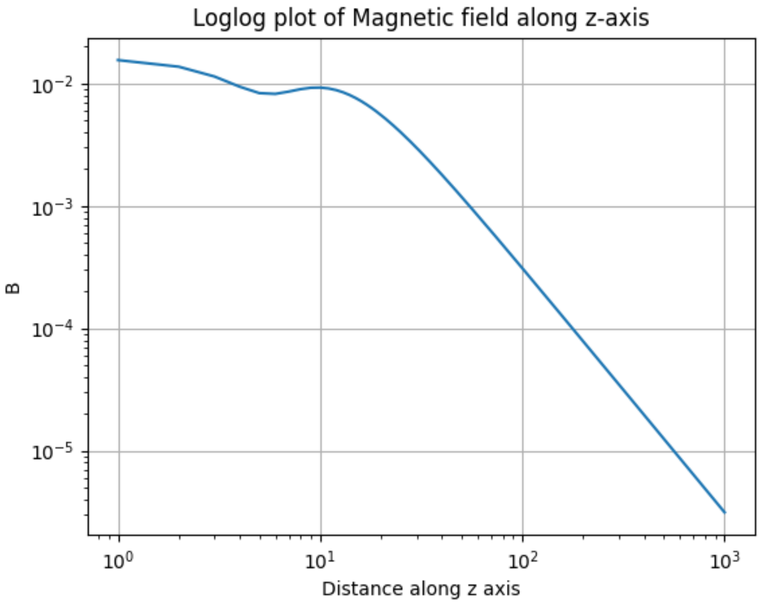

Figure 2: Magnetic field vs. z plot

Least squares estimation

We need to fit the B vector into the form cz^b . To do this, we need to use the lstsq function in the numpy.linalg module.

```
M = c_[np.ones(990), np.log(z[10:])]
B_matrix = np.log(np.abs(B_z[10:]))
ans, residue, rank, s = lstsq(M, B_matrix,rcond=-1)
c = np.exp(ans[0])
b = ans[1]
```

After doing this, we get the output:

```
c= 2.3787866850449175
b= -1.9555117906435737
```

Also, we compare the original magnetic field with the estimate.

This happens because of the sinusoidal variation of the current in both space and time. For a static magnetic field, we expect the magnetic field to fall a $1/z^3$ i.e; Expected decay rate = 3 The original value of the decay rate we got was, c= 2.379 Hence, we can say that B_z fell as expected but with a slight variance. This difference arose due to the non-linearity of the loglog plot of the Magnetic field plot .

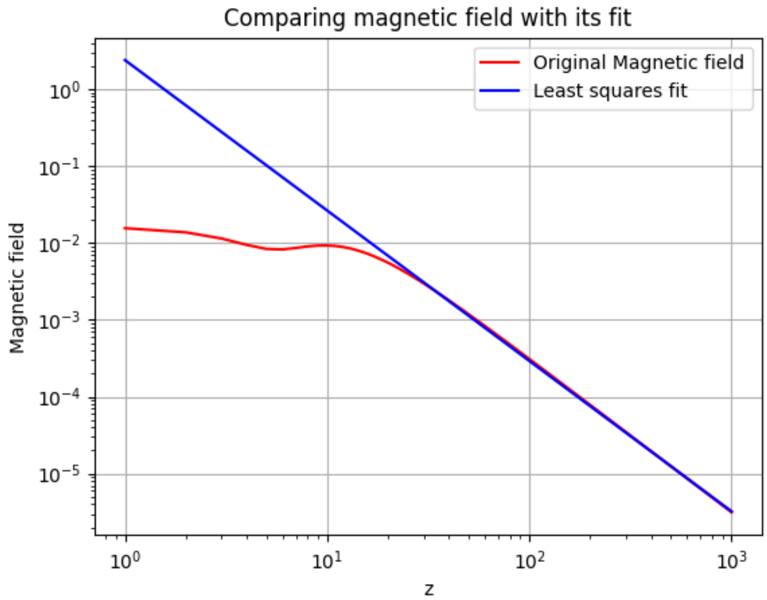

Figure 3: Comparing original magnetic field and its lstsq estimate

Conclusion

We took a loop wire carrying a current I = $\frac{4\pi}{\mu_o}\cos(\phi)exp(j\omega t)$, divided into 100 sections , calculated the current elements in each of those sections and plotted them. We calculated the vector potential of the loop and also the magnetic field using the given equations. We fit the vector into a cz^b type and compared it with the original magnetic field.