Assignment:- 1

AI1110: Probability and Random Variables
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NCERT(12.13.6.6)

Question. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?
Answer: 0.48451

Solution: In a hurdle race, a player has to cross 10 hurdles. Let,

E: The probability of clearing a hurdle.

$$\Pr(E) = \frac{5}{6} \tag{1}$$

E': The probability of knocking down a hurdle.

$$\Pr(E') = 1 - \frac{5}{6} = \frac{1}{6} \tag{2}$$

Let,

X be the number of hurdles the player knocks down.

X follows a binomial distribution with parameters $n = 10$ (the number of trials, i.e., the number of hurdles).

The probability of knocking down fewer than 2 hurdles is the sum of the probabilities of knocking down 0 hurdles and 1 hurdle:

$$\Pr(X < 2) = \Pr(X = 0) + \Pr(X = 1) \tag{3}$$

Using the binomial distribution formula, we can calculate each term:

$$\Pr(X = k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{4}$$

where,

${}^{n}C_k = \frac{n!}{k!(n-k)!}$ is the binomial coefficient.

$p = \Pr(E') = \frac{1}{6}$

$$\Pr(X = 0) = {}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} = 0.161505 \tag{5}$$

$$\Pr(X = 1) = {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^{9} = 0.323011 \tag{6}$$

Therefore,

$$\Pr(X < 2) = \Pr(X = 0) + \Pr(X = 1) \tag{7}$$
$$= 0.1615 + 0.3230 \tag{8}$$
$$= 0.48451 \tag{9}$$

So, the probability that the player will knock down fewer than 2 hurdles is 0.48451 or approximately 48.45%.