Assignment 1

AI1110: Probability and Random Variables
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NCERT(12.13.6.6)

Question. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?
Answer: 0.48451

Solution: In a hurdle race, a player has to cross 10 hurdles.
Let,

E: The probability of clearing a hurdle.

$$P(E) = \frac{5}{6}$$

$\bar{E}$: The probability of knocking down a hurdle.

$$P(\bar{E}) = 1 - \frac{5}{6} = \frac{1}{6}$$

We need to find the probability that he will knock down fewer than 2 hurdles.

Let, X be the number of hurdles the player knocks down. Then X follows a binomial distribution with parameters $n = 10$ (the number of trials, i.e., the number of hurdles).

The probability of knocking down fewer than 2 hurdles is the sum of the probabilities of knocking down 0 hurdles and 1 hurdle:

$$P(X < 2) = P(X = 0) + P(X = 1)$$

Using the binomial distribution formula, we can calculate each term:

$$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1 - p)^{n-k}$$

where,

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the binomial coefficient.

$p = P(\bar{E}) = \frac{1}{6}$

$$P(X = 0) = \binom{10}{0} \cdot \left(\frac{1}{6}\right)^0 \cdot \left(\frac{5}{6}\right)^{10} = 0.161505$$

$$P(X = 1) = \binom{10}{1} \cdot \left(\frac{1}{6}\right)^1 \cdot \left(\frac{5}{6}\right)^9 = 0.323011$$

Therefore,

$$\begin{aligned} P(X < 2) &= P(X = 0) + P(X = 1) \\ &= 0.1615 + 0.3230 \\ &= 0.48451 \end{aligned}$$

So, the probability that the player will knock down fewer than 2 hurdles is 0.48451 or approximately 48.45%.