

Assignment:- 1

AI1110: Probability and Random Variables

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NCERT(12.13.6.6)

Question. In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $\frac{5}{6}$. What is the probability that he will knock down fewer than 2 hurdles?

Answer: 0.48451

Solution:

Probability of clearing a hurdle $\Pr(E) = \frac{5}{6}$

Probability of knocking down a hurdle $\Pr(E') =$

$$1 - \frac{5}{6} = \frac{1}{6}$$

Let,

X be the number of hurdles the player knocks down.

The probability of knocking down fewer than 2 hurdles is the sum of the probabilities of knocking down 0 hurdles and 1 hurdle:

Using Cumulative Distribution Function(cdf),

$$F_X(r) = \Pr(X \leq r) \quad (1)$$

Here, $r = 1$

$$F_X(1) = \Pr(X \leq 1) = \Pr(X = 0) + \Pr(X = 1) \quad (2)$$

Using the binomial distribution formula,

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (3)$$

where,

$${}^nC_k = \frac{n!}{k!(n-k)!}$$

$$p = \Pr(E') = \frac{1}{6}$$

$$\Pr(X = 0) = {}^{10}C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^{10} = 0.161505 \quad (4)$$

$$\Pr(X = 1) = {}^{10}C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^9 = 0.323011 \quad (5)$$

Therefore,

$$\Pr(X < 2) = \Pr(X = 0) + \Pr(X = 1) \quad (6)$$

$$= 0.1615 + 0.3230 \quad (7)$$

$$= 0.48451 \quad (8)$$

So, the probability that the player will knock down fewer than 2 hurdles is 0.48451 or approximately 48.45%.