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CS325 – HW6

**Problem 1:**

**a) shortest path from g to c = 16**

```
max dc
st
    dg = 0
    da - df <= 5
    da - dh <= 4
    db - da <= 8
    db - df <= 7
    db - dh <= 9
    dc - db <= 4
    dc - df <= 3
    dd - dc <= 3
    dd - de <= 9
    dd - dg <= 2
    de - db <= 10
    de - dd <= 25
    de - df <= 2
    df - da <= 10
    df - dd <= 18
    dg - de <= 7
    dh - dg <= 3
END
```

**b) shortest path to each:**

DA	7.000000
DB	12.000000
DC	16.000000
DD	2.000000
DE	19.000000
DF	17.000000
DH	3.000000

max da + db + dc + dd + de + df + dg + dh

ST

dg = 0  
da - df <= 5  
da - dh <= 4  
db - da <= 8  
db - df <= 7  
db - dh <= 9  
dc - db <= 4  
dc - df <= 3  
dd - dc <= 3  
dd - de <= 9  
dd - dg <= 2  
de - db <= 10  
de - dd <= 25  
de - df <= 2  
df - da <= 10  
df - dd <= 18  
dg - de <= 7  
dh - dg <= 3

END

## Problem 2:

### OBJECTIVE FUNCTION VALUE

120196.0

VARIABLE	VALUE	
X1	7000.000000	Silk
X2	13625.000000	Polyester
X3	13100.000000	Blend 1
X4	8500.000000	Blend 2

Max  $(6.70 - .75 - (20 \cdot .125 + 6 \cdot 0 + 9 \cdot 0))x_1 + (3.55 - .75 - (20 \cdot 0 + 6 \cdot .08 + 9 \cdot 0))x_2 + (4.31 - .75 - (20 \cdot 0 + 6 \cdot .05 + 9 \cdot .05))x_3 + (4.81 - .75 - (20 \cdot 0 + 6 \cdot .03 + 9 \cdot .07))x_4$

*Simplified:*

Max  $3.45x_1 + 2.32x_2 + 2.81x_3 + 3.25x_4$

ST

$.125x_1 \leq 1000$   
 $.08x_2 + .05x_3 + .03x_4 \leq 2000$   
 $.05x_3 + .07x_4 \leq 1250$   
 $x_1 \leq 7000$   
 $x_1 \geq 6000$

$x_2 \leq 14000$   
 $x_2 \geq 10000$   
 $x_3 \leq 16000$   
 $x_3 \geq 13000$   
 $x_4 \leq 8500$   
 $x_4 \geq 6000$   
 $x_1 \geq 0$   
 $x_2 \geq 0$   
 $x_3 \geq 0$   
 $x_4 \geq 0$   
 END

$\text{Max } \sum_{i=1}^n (s_i - l_i - (\sum_{j=1}^n c_{ij} * a_{ij}))x_i$  (sum of the selling price of product i minus the labor cost of i minus the sum of the cost of material j in i times amount of material i in j all times the amount of product i)

S.T.

$b_i \leq x_i \leq c_i$  (amount between min b of product i and max c of product i)

$\sum_{i=1}^n a_{ij} \leq d_j$  (sum of material j in each product i less than or equal to amount of material j available)

$x_i \geq 0$

### Problem 3:

i = plant number

j = warehouse number

k = retailer number

$x_{ij}$  = amount shipped from plant i to warehouse j

$y_{jk}$  = amount shipped from warehouse j to retailer k

n = number of i

m = number of j available to i

c = cost,  $c_{ij}$  = cost plant i to warehouse j,  $c_{jk}$  = cost warehouse to retailer

l = number of k available to j

$b_i$  = supply of i

$b_k$  = demand of k

$\text{max } \sum_{i=1}^n (\sum_{j=1}^m (c_{ij}x_{ij}) + \sum_{k=1}^l (c_{jk} * y_{jk}))$  (the sum of all the plant's sum of costs of shipping to each warehouse added together with the sum of the shipping costs of the corresponding warehouses to each corresponding available retailer.)

S.T.

$\sum_{j=1}^m (x_{ij}) = b_i$  (the sum of items shipped out of each plant must be equal to its supply)

$\sum_{j=1}^m (y_{jk}) \leq b_k$  (the sum of all products arriving at a retailer must be greater than or equal to demand)

$x_{ij}, y_{jk} \geq 0$  (non-negativity constraints)

a)

min  $10x_{11} + 15x_{12} + 11x_{21} + 8x_{22} + 13x_{31} + 8x_{32} + 9x_{33} + 14x_{42} + 8x_{43} + 5y_{11} + 6y_{12} + 7y_{13} + 10y_{14} + 12y_{23} + 8y_{24} + 10y_{25} + 14y_{26} + 14y_{34} + 12y_{35} + 12y_{36} + 6y_{37}$

ST

$$x_{11} + x_{12} = 150$$

$$x_{21} + x_{22} = 450$$

$$x_{31} + x_{32} + x_{33} = 250$$

$$x_{42} + x_{43} = 150$$

$$y_{11} \geq 100$$

$$y_{12} \geq 150$$

$$y_{13} + y_{23} \geq 100$$

$$y_{14} + y_{24} + y_{34} \geq 200$$

$$y_{25} + y_{35} \geq 200$$

$$y_{26} + y_{36} \geq 150$$

$$y_{37} \geq 100$$

$$x_{11} \geq 0$$

$$x_{12} \geq 0$$

$$x_{21} \geq 0$$

$$x_{22} \geq 0$$

$$x_{31} \geq 0$$

$$x_{32} \geq 0$$

$$x_{33} \geq 0$$

$$x_{42} \geq 0$$

$$x_{43} \geq 0$$

Y11 >= 0

Y12 >= 0

Y13 >= 0

Y14 >= 0

Y23 >= 0

Y24 >= 0

Y25 >= 0

Y26 >= 0

Y34 >= 0

Y35 >= 0

Y36 >= 0

Y37 >= 0

END

OBJECTIVE FUNCTION VALUE

1) 16400.00

VARIABLE	VALUE	REDUCED COST
X11	150.000000	0.000000
X12	0.000000	5.000000
X21	0.000000	3.000000
X22	450.000000	0.000000
X31	0.000000	5.000000
X32	250.000000	0.000000
X33	0.000000	1.000000
X42	0.000000	6.000000
X43	150.000000	0.000000
Y11	100.000000	0.000000

Y12	150.000000	0.000000
Y13	100.000000	0.000000
Y14	0.000000	2.000000
Y23	0.000000	5.000000
Y24	200.000000	0.000000
Y25	200.000000	0.000000
Y26	0.000000	2.000000
Y34	0.000000	6.000000
Y35	0.000000	2.000000
Y36	150.000000	0.000000
Y37	100.000000	0.000000

**b)**

It is still possible because both warehouse 1 and 3 are able to receive from all the plants and able to ship to all the retailers.

$\min 10X_{11} + 11X_{21} + 13X_{31} + 9X_{33} + 8X_{43} + 5Y_{11} + 6Y_{12} + 7Y_{13} + 10Y_{14} + 14Y_{34} + 12Y_{35} + 12Y_{36} + 6Y_{37}$

ST

$$X_{11} = 150$$

$$X_{21} = 450$$

$$X_{31} + X_{33} = 250$$

$$X_{43} = 150$$

$$Y_{11} \geq 100$$

$$Y_{12} \geq 150$$

$$Y_{13} \geq 100$$

$$Y_{14} + Y_{34} \geq 200$$

$$Y_{35} \geq 200$$

$$Y_{36} \geq 150$$

$$Y_{37} \geq 100$$

$$X_{11} \geq 0$$

X21 >= 0

X31 >= 0

X33 >= 0

X43 >= 0

Y11 >= 0

Y12 >= 0

Y13 >= 0

Y14 >= 0

Y34 >= 0

Y35 >= 0

Y36 >= 0

Y37 >= 0

END

OBJECTIVE FUNCTION VALUE

1) 18800.00

VARIABLE	VALUE	REDUCED COST
X11	150.000000	
X21	450.000000	
X31	0.000000	
X33	250.000000	
X43	150.000000	

Y11 100.000000

Y12 150.000000

Y13 100.000000

Y14 200.000000

Y34 0.000000

Y35 200.000000

Y36 150.000000

Y37 100.000000

c)

$\min 10X_{11} + 15X_{12} + 11X_{21} + 8X_{22} + 13X_{31} + 8X_{32} + 9X_{33} + 14X_{42} + 8X_{43} + 5Y_{11} + 6Y_{12} + 7Y_{13} + 10Y_{14} + 12Y_{23} + 8Y_{24} + 10Y_{25} + 14Y_{26} + 14Y_{34} + 12Y_{35} + 12Y_{36} + 6Y_{37}$

ST

$$X_{11} + X_{12} = 150$$

$$X_{21} + X_{22} = 450$$

$$X_{31} + X_{32} + X_{33} = 250$$

$$X_{42} + X_{43} = 150$$

$$X_{12} + X_{22} + X_{32} + X_{42} = 100$$

$$Y_{11} \geq 100$$

$$Y_{12} \geq 150$$

$$Y_{13} + Y_{23} \geq 100$$

$$Y_{14} + Y_{24} + Y_{34} \geq 200$$

$$Y_{25} + Y_{35} \geq 200$$

$$Y_{26} + Y_{36} \geq 150$$

$$Y_{37} \geq 100$$

$$X_{11} \geq 0$$

$$X_{12} \geq 0$$

$$X_{21} \geq 0$$

$$X_{22} \geq 0$$

$$X_{31} \geq 0$$



X32 >= 0  
X33 >= 0  
X42 >= 0  
X43 >= 0  
Y11 >= 0  
Y12 >= 0  
Y13 >= 0  
Y14 >= 0  
Y23 >= 0  
Y24 >= 0  
Y25 >= 0  
Y26 >= 0  
Y34 >= 0  
Y35 >= 0  
Y36 >= 0  
Y37 >= 0  
END

#### OBJECTIVE FUNCTION VALUE

1) 17700.00

VARIABLE	VALUE	REDUCED COST
X11	150.000000	
X12	0.000000	
X21	350.000000	
X22	100.000000	
X31	0.000000	

X32	0.000000
X33	250.000000
X42	0.000000
X43	150.000000
Y11	100.000000
Y12	150.000000
Y13	100.000000
Y14	0.000000
Y23	0.000000
Y24	200.000000
Y25	200.000000
Y26	0.000000
Y34	0.000000
Y35	0.000000
Y36	150.000000
Y37	100.000000

**Problem 4:**

$$\max \sum_{i=1}^n x_i$$

ST

$$\sum_{i=1}^n c_i x_i = d$$

$$x_i \geq 0$$

$$x_i \in \mathbb{Z}$$

Minimize:  $A + B + C + D + E + F$

ST

$$1.00A + .50B + .25C + .10D + .05E + .01F = \text{Desired Change}$$

$$A, B, C, D, E, F \geq 0$$

$$A, B, C, D, E, F = \text{Integers}$$

$x_i$  = amount of currency  $i$

$c_i$  = value of currency  $i$

$Z$  = set of integers

$A$  = Amount for which to find change

$$\min \sum_{i=1}^n x_i$$

ST

$$\sum_{i=1}^n c_i x_i = A$$

$$x_i \geq 0$$

$$x_i \in Z$$

a)

$$\min a + b + c + d$$

ST

$$1a + 5b + 10c + 25d = 202$$

$$a \geq 0$$

$$b \geq 0$$

$$c \geq 0$$

$$d \geq 0$$

END

GIN a

GIN b

GIN c

GIN d

OBJECTIVE FUNCTION VALUE

1) 10.000000

VARIABLE	VALUE
A	2.000000
B	0.000000
C	0.000000

D 8.000000

**b)**

min  $a + b + c + d + e$

ST

$1a + 3b + 7c + 12d + 27e = 293$

$a \geq 0$

$b \geq 0$

$c \geq 0$

$d \geq 0$

END

GIN a

GIN b

GIN c

GIN d

GIN e

OBJECTIVE FUNCTION VALUE

1) 14.000000

VARIABLE	VALUE
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A	0.000000
---	----------

B	0.000000
---	----------

C	2.000000
---	----------

D	3.000000
---	----------

E	9.000000
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