



Chaos Synchronization via Predictive Schemes

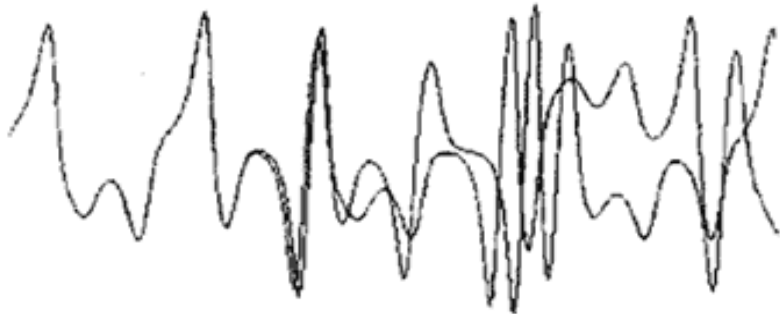
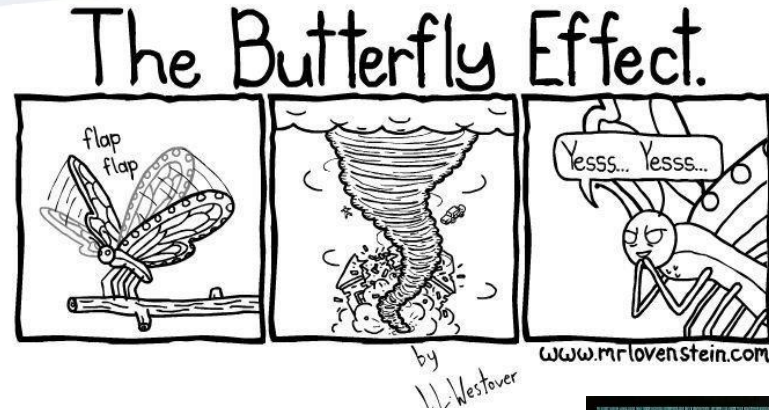
Giovanni Licitra

07/08/2015 Freiburg

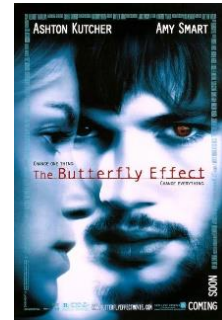
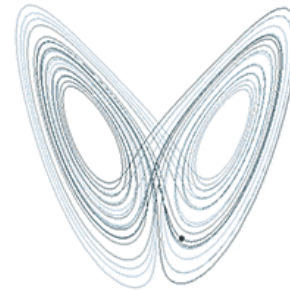
Does the Flap of a Butterfly's Wings in Brazil set off a Tornado in Texas?



Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions and to the parameters of the system. A response popularly referred to as the butterfly effect.



*Lorenz's experiment:
the difference between
the starting values of
these curves is only
.000127*



How to synchronize two chaotic systems

Master system

$$\begin{cases} \dot{x}_1 = p[y_1 - \frac{(2x_1^3 - x_1)}{7}] \\ \dot{y}_1 = x_1 - y_1 + z_1 \\ \dot{z}_1 = -qy_1 \end{cases}$$

Slave system

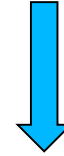
$$\begin{cases} \dot{x}_2 = p[y_2 - \frac{(2x_2^3 - x_2)}{7}] + u_x \\ \dot{y}_2 = x_2 - y_2 + z_2 + u_y \\ \dot{z}_2 = -qy_2 + u_z \end{cases}$$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \end{bmatrix}$$

$C(e)$

Synchronization condition

$$\lim_{t \rightarrow \infty} \|\vec{e}\| = 0$$



$$\min \int \|\vec{e}\|_2^2 dt$$

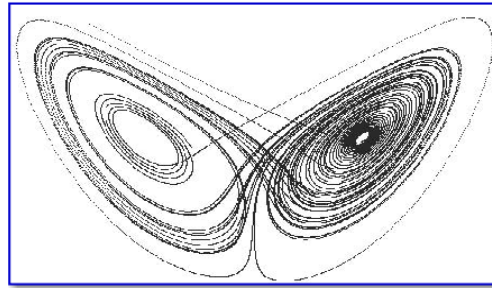
How to synchronize two chaotic systems (2)

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}}_C \underbrace{\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \end{bmatrix}}_{\tilde{x}}$$

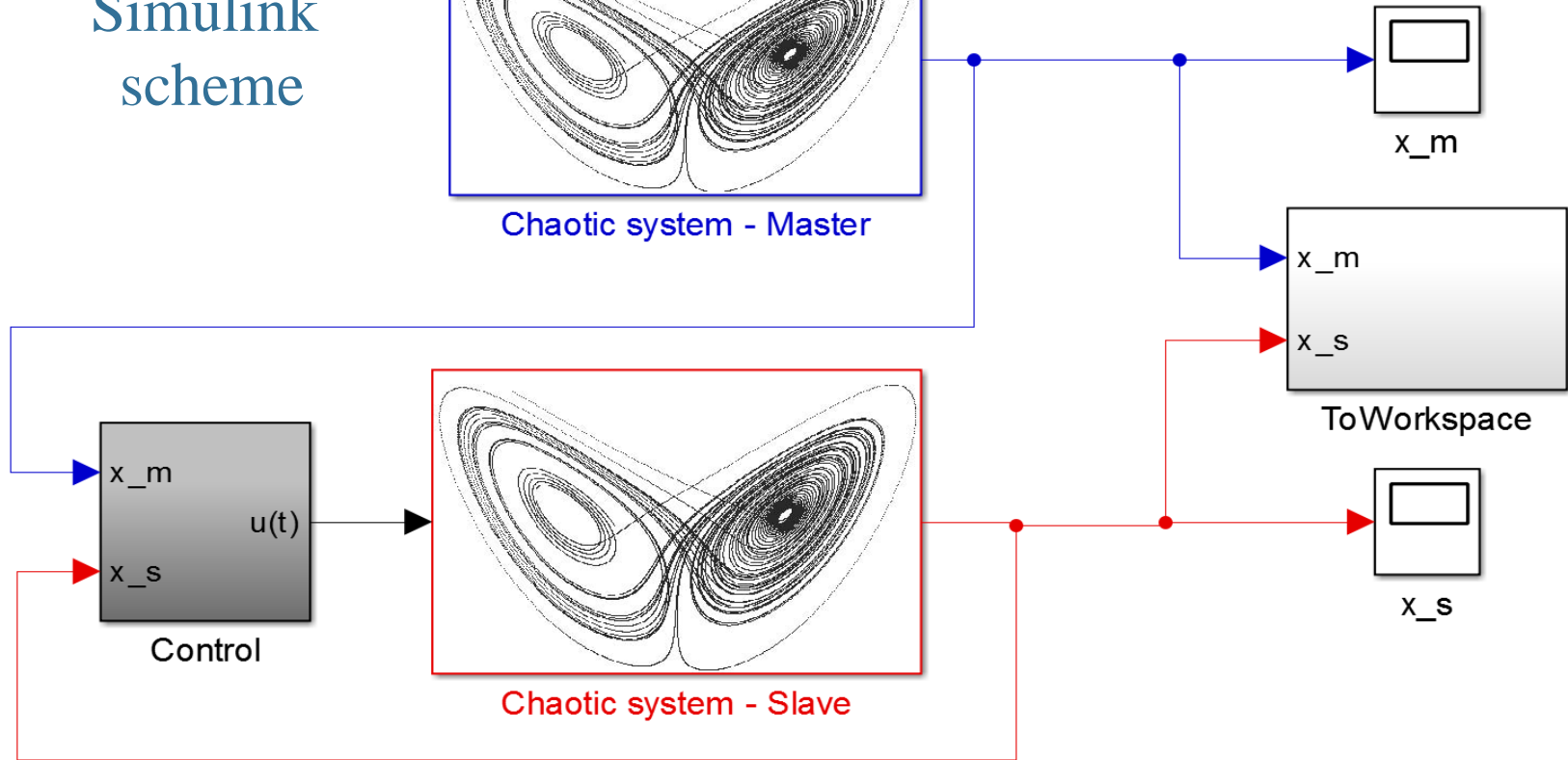
$$\|\vec{e}\|_2^2 = \vec{e}^T \vec{e} = \tilde{x}^T C^T C \tilde{x} = \tilde{x}^T Q \tilde{x}$$

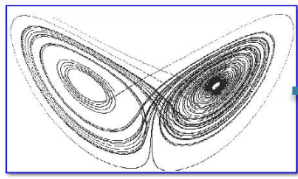
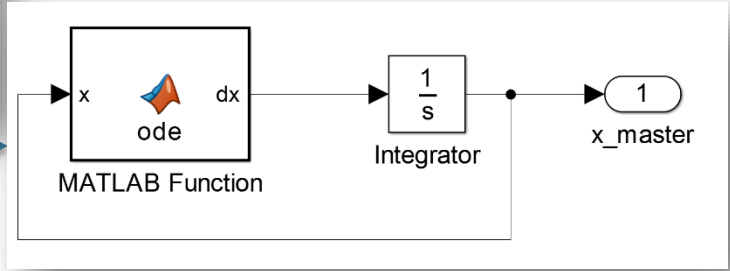
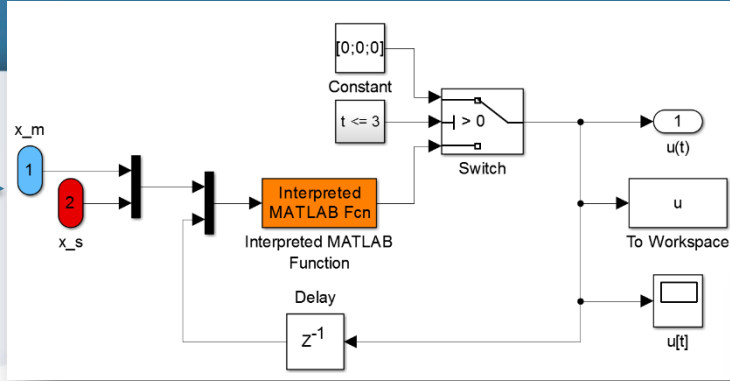
This is nothing else that a regulation problem!!!

Simulink scheme

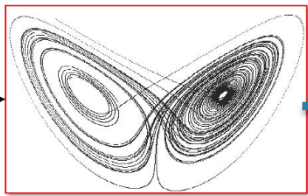


Chaotic system - Master

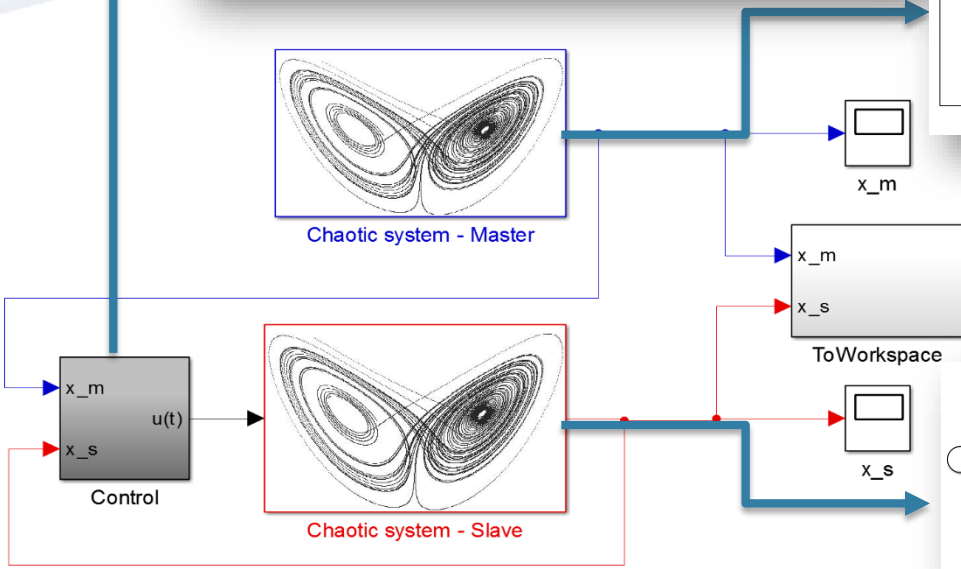
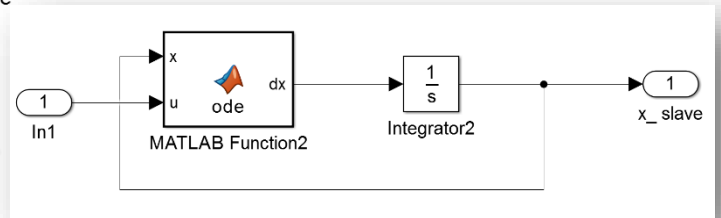




Chaotic system - Master

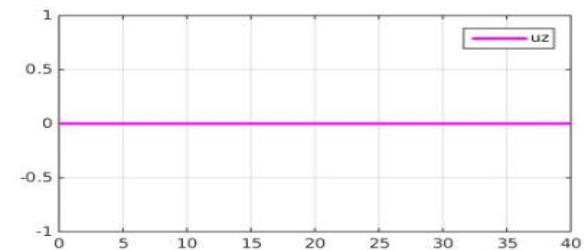
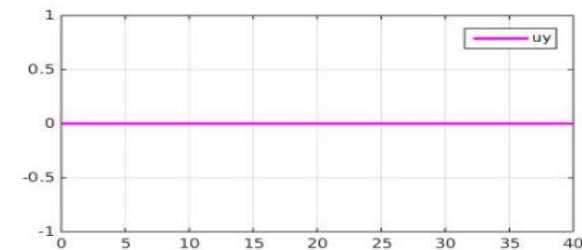
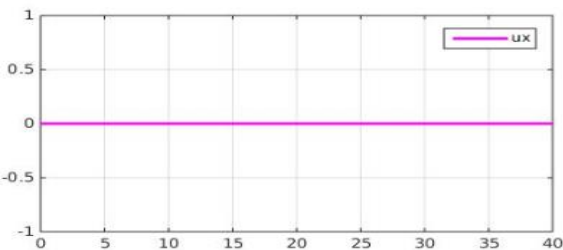
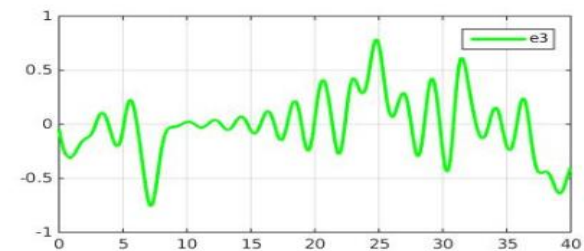
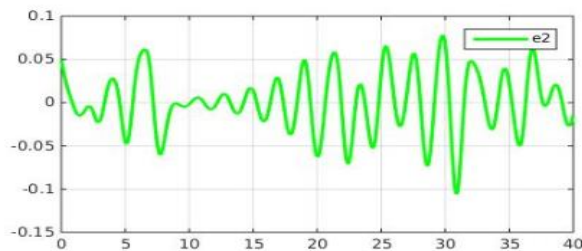
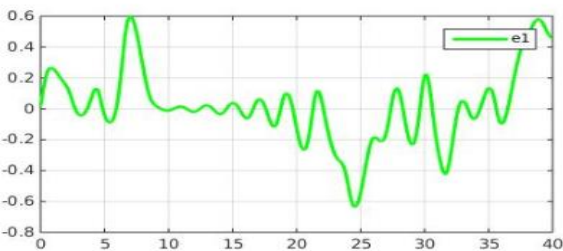
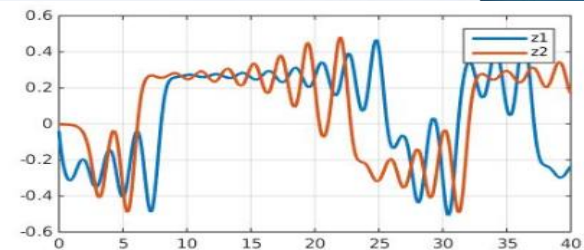
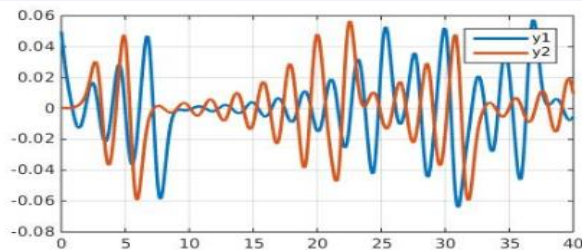
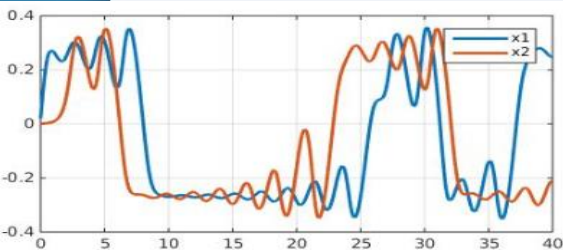


Chaotic system - Slave



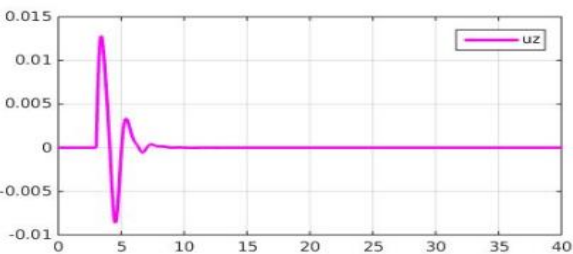
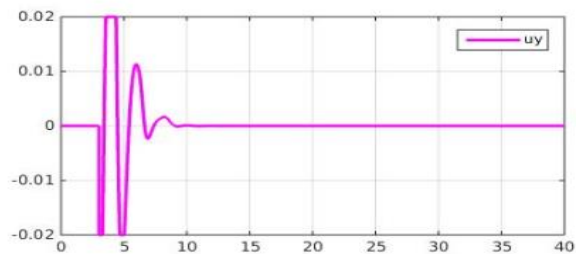
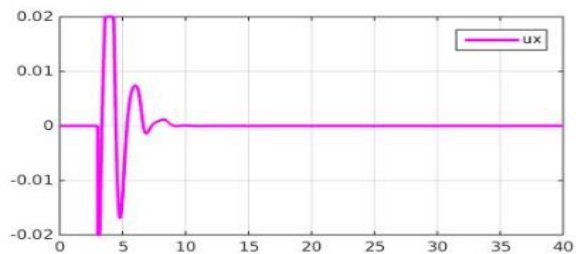
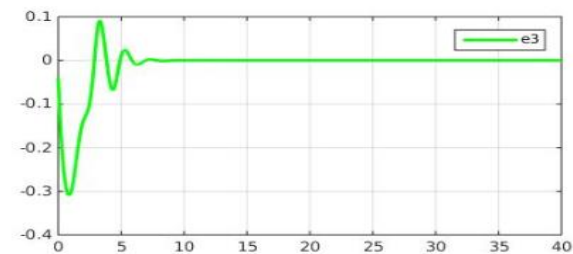
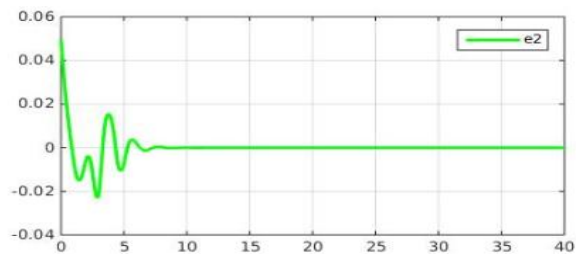
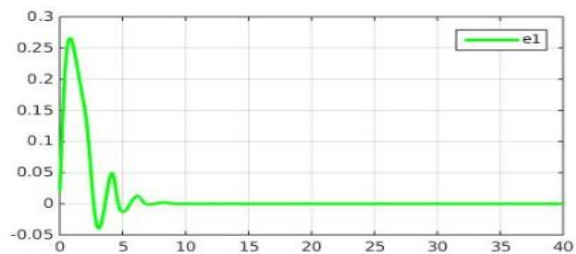
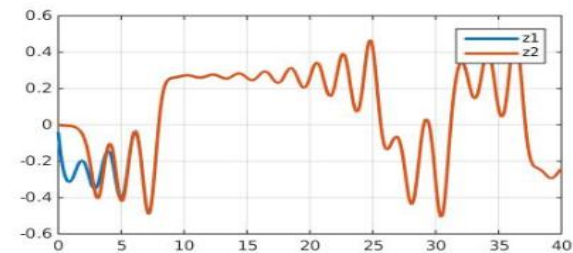
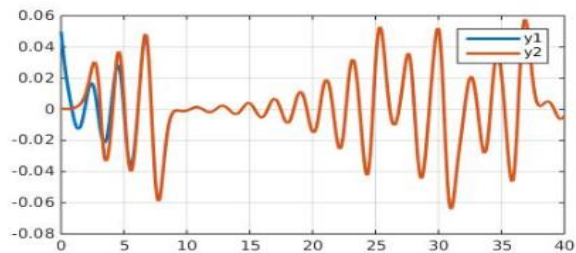
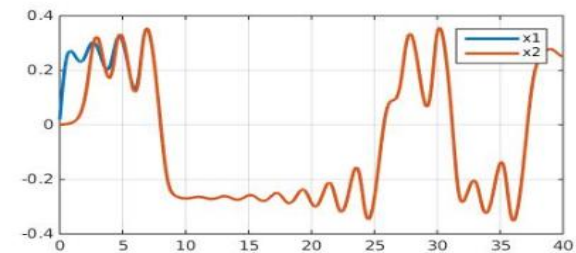
No Synchronization

i.c. ($x_1 = 0.02$ $x_2 = 0.0002$, $y_1 = 0.05$ $y_2 = 0.0005$, $z_1 = 0.04$ $z_2 = 0.0004$)

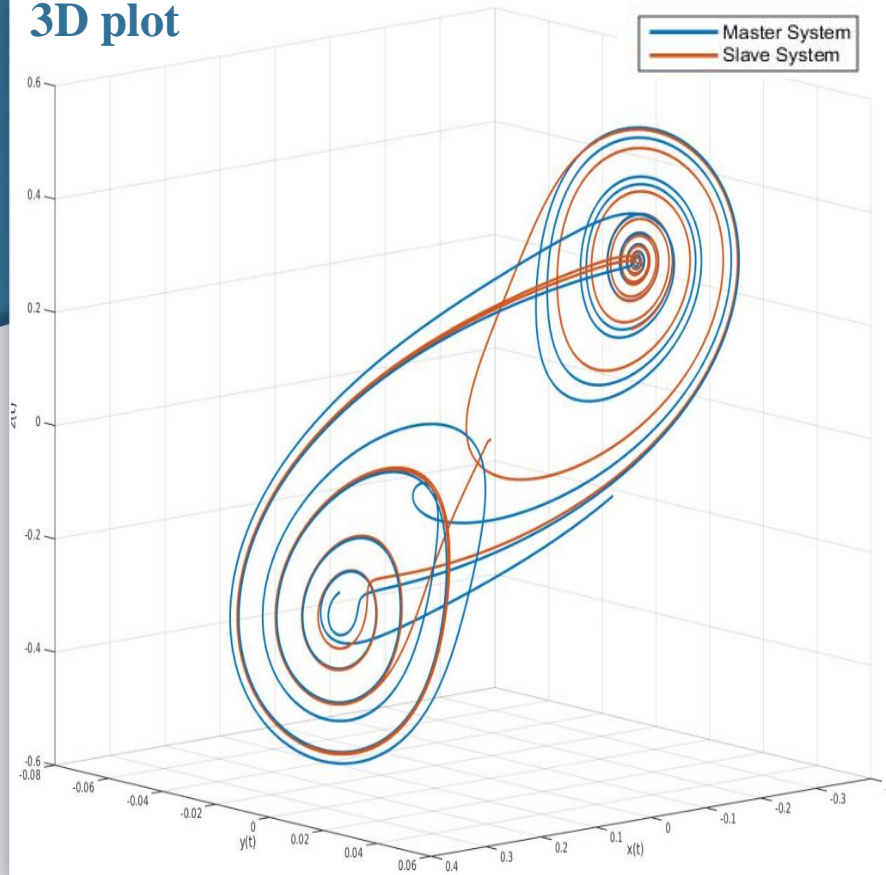


Synchronization via MPC (MPT toolbox)

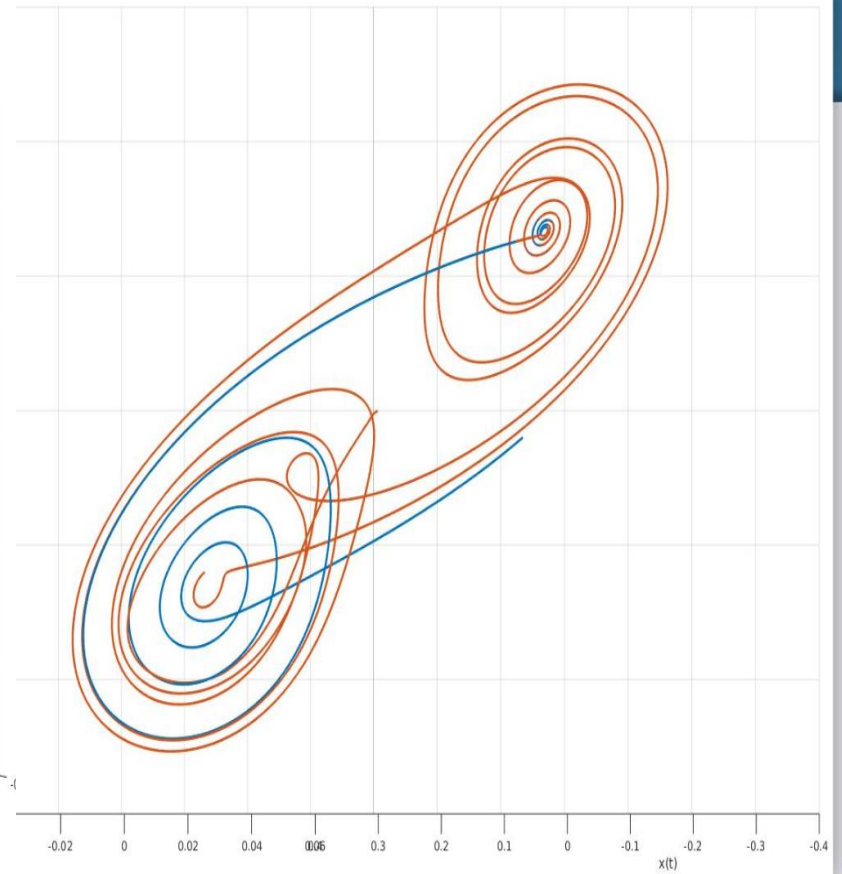
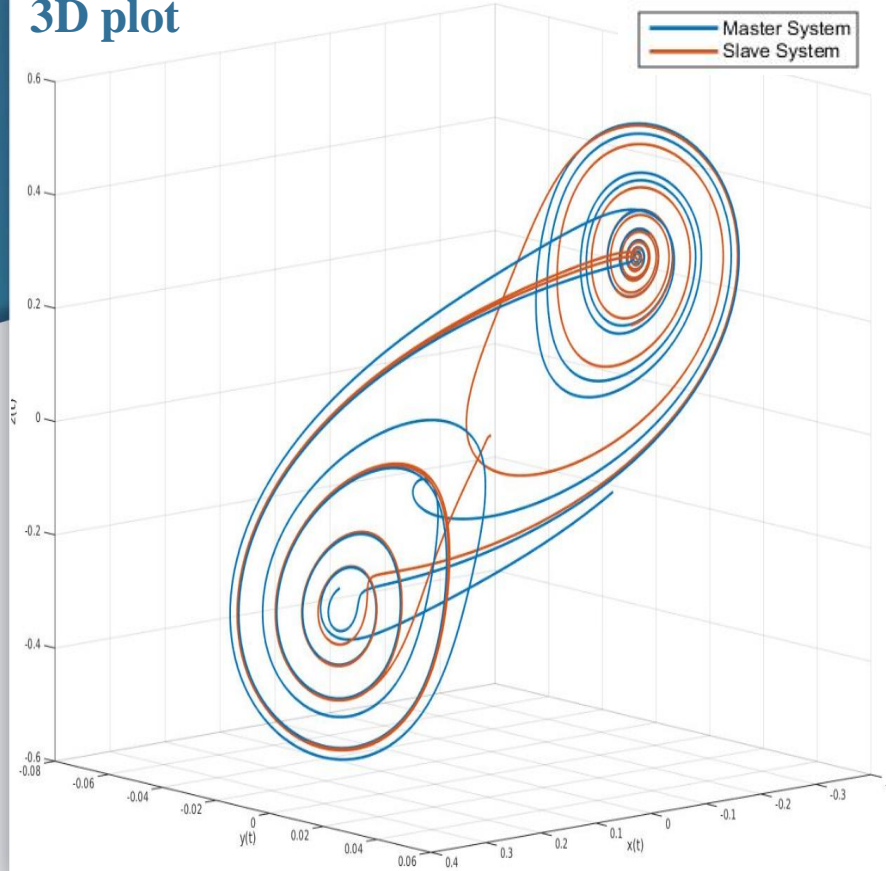
Control stabilizes starting from $t = 3$ s



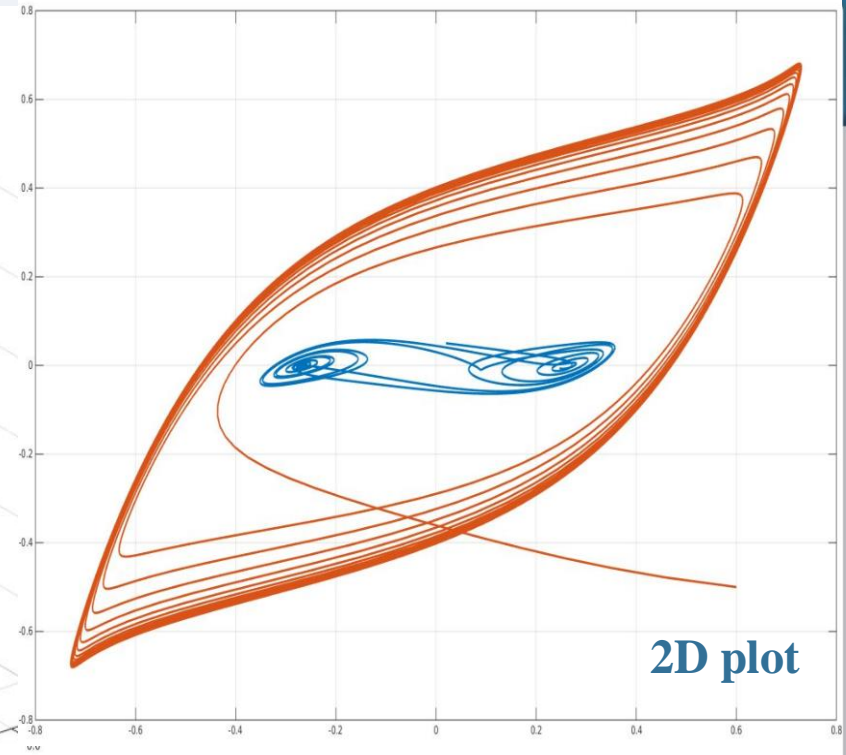
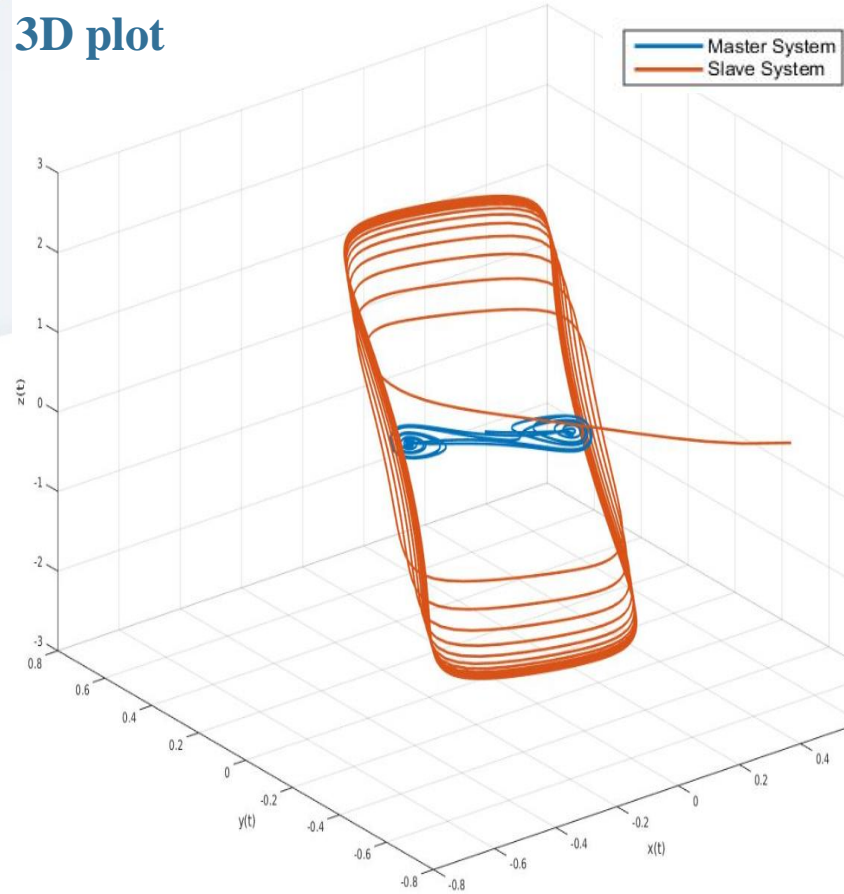
3D plot



3D plot

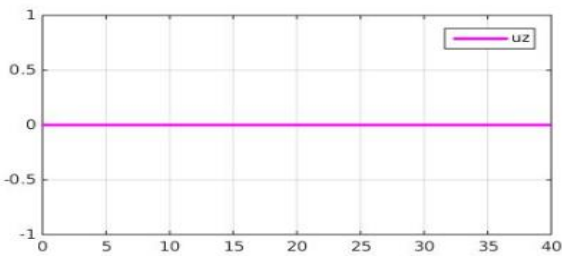
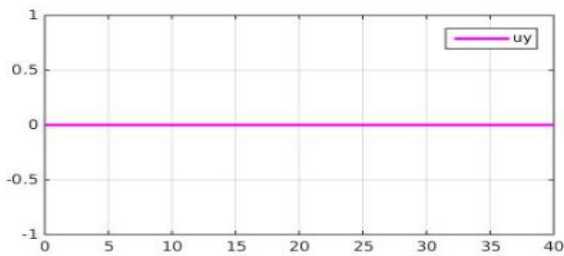
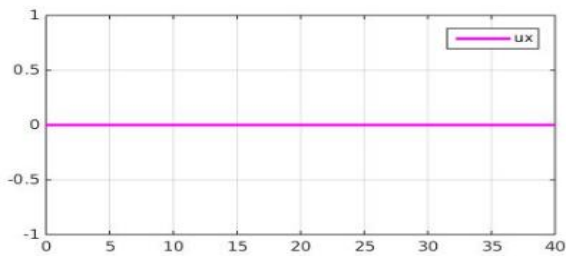
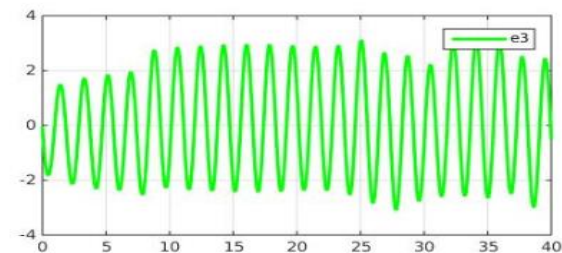
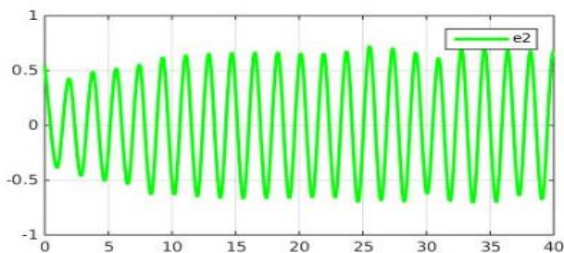
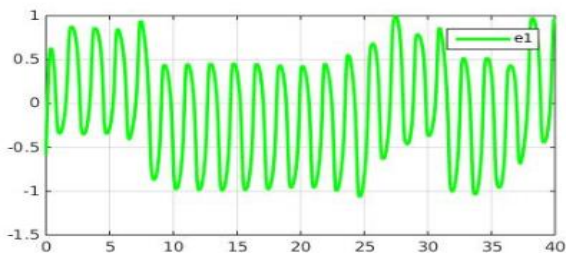
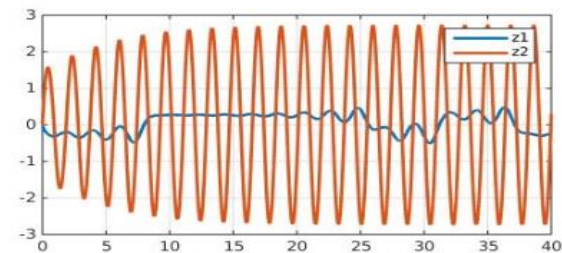
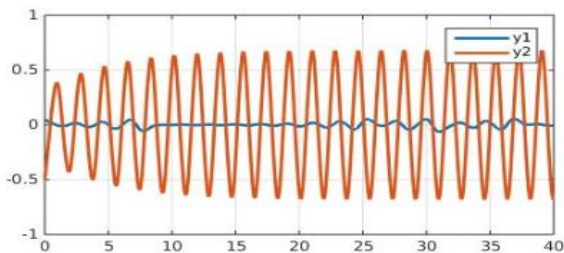
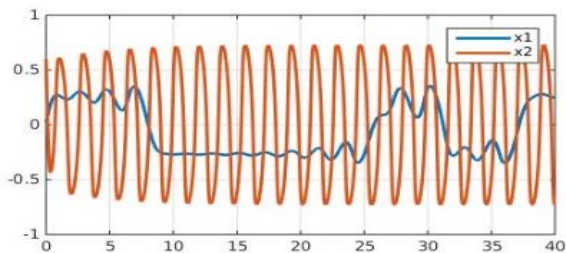


3D plot

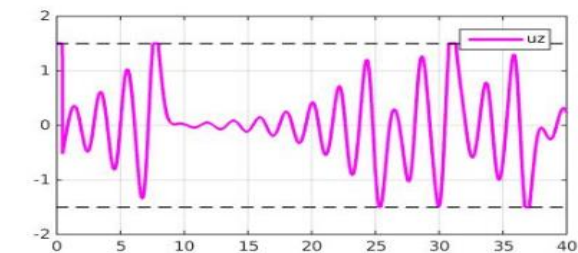
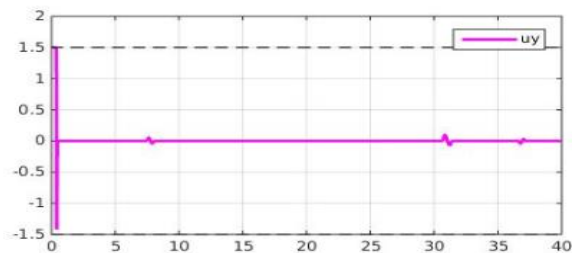
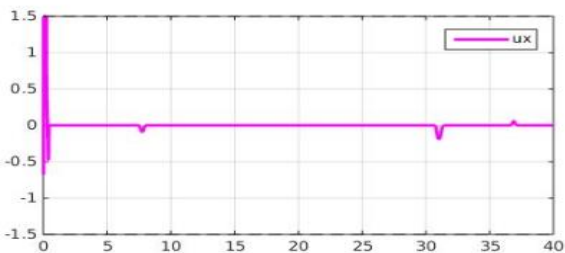
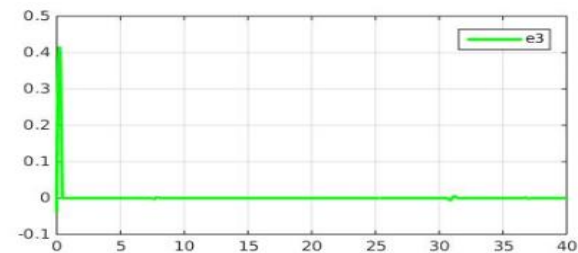
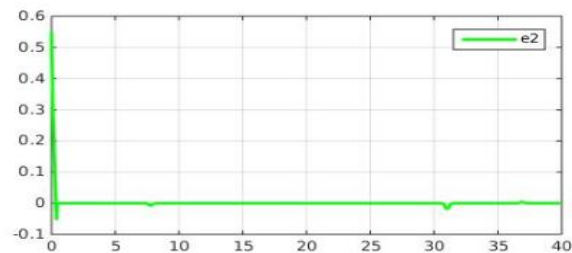
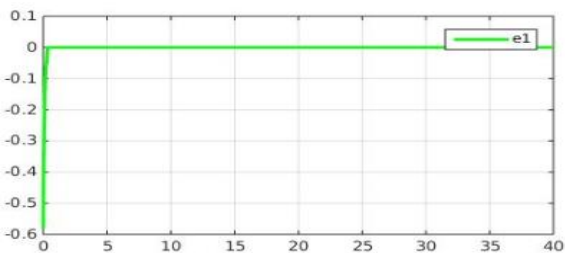
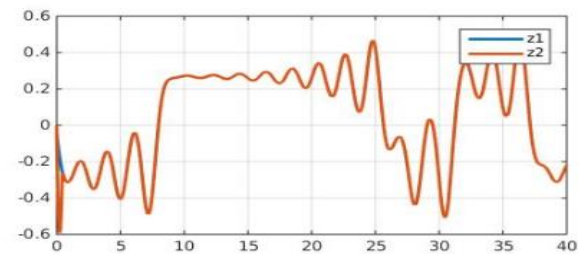
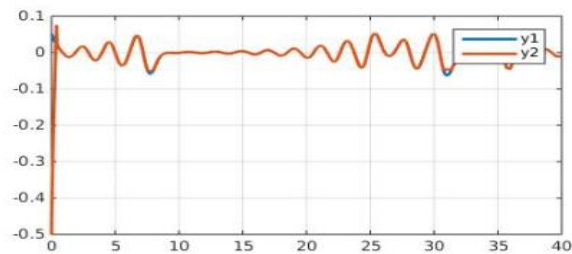
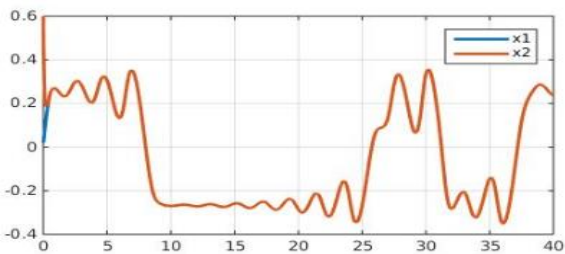


No Synchronization

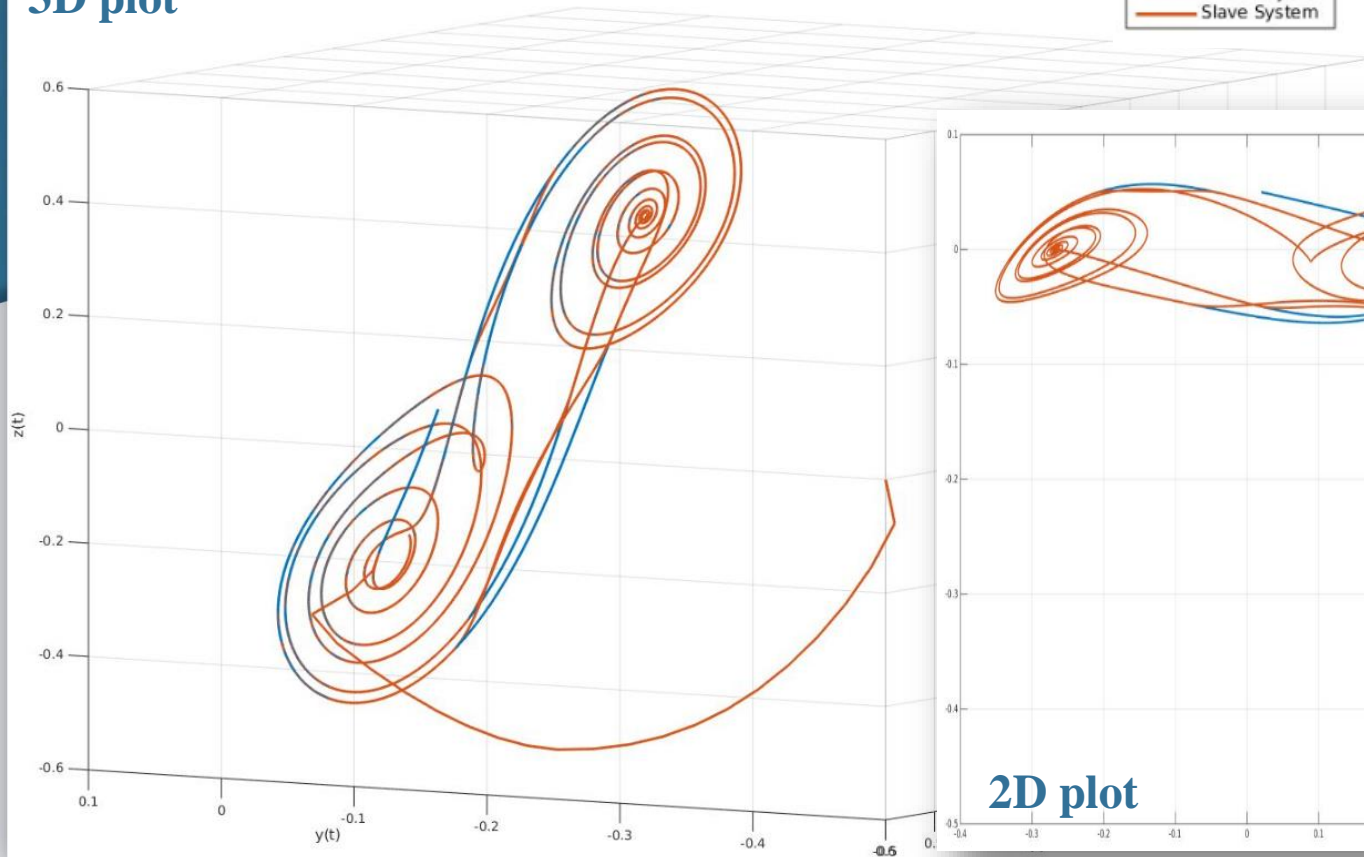
i. c. ($x_1 = 0.02$, $x_2 = 0.6$, $y_1 = 0.05$, $y_2 = -0.5$, $z_1 = -0.04$, $z_2 = 0.0004$)



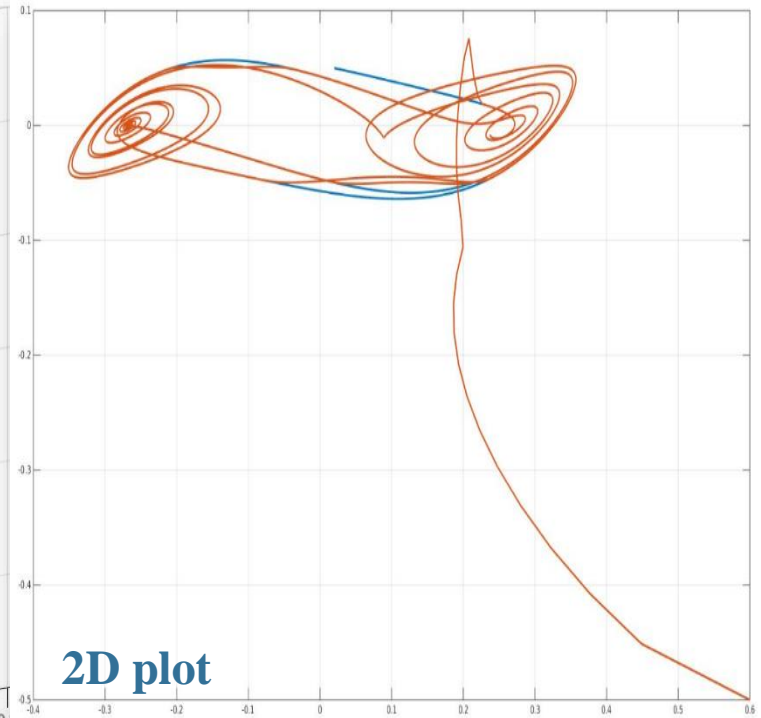
Synchronization via NMPC (ACADO Toolkit)



3D plot



2D plot

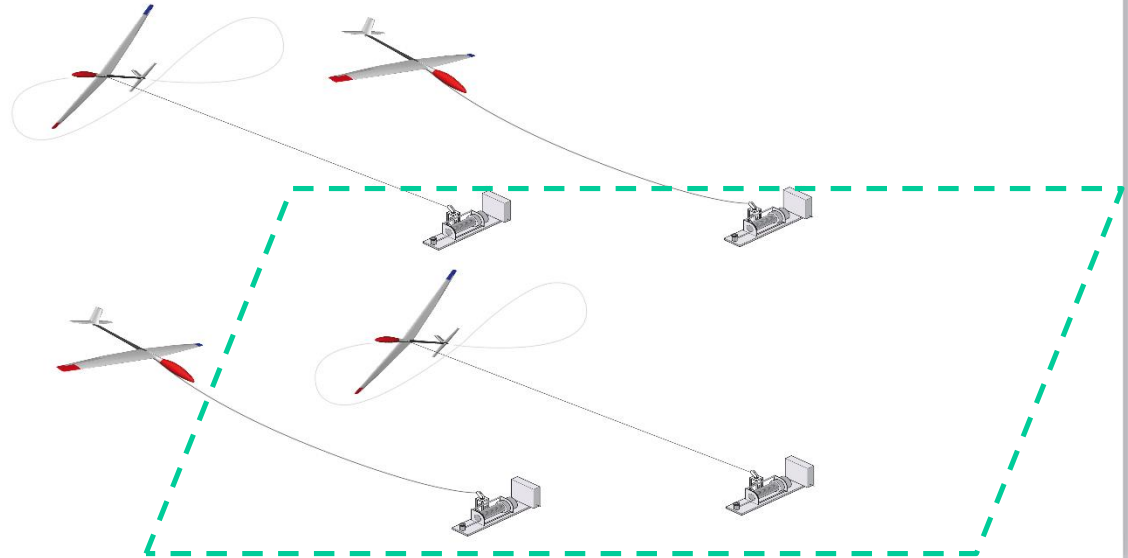


**If it is possible to synchronize chaotic systems, it should
be simpler to synchronize deterministic ones...**

What's Ampyx Power?



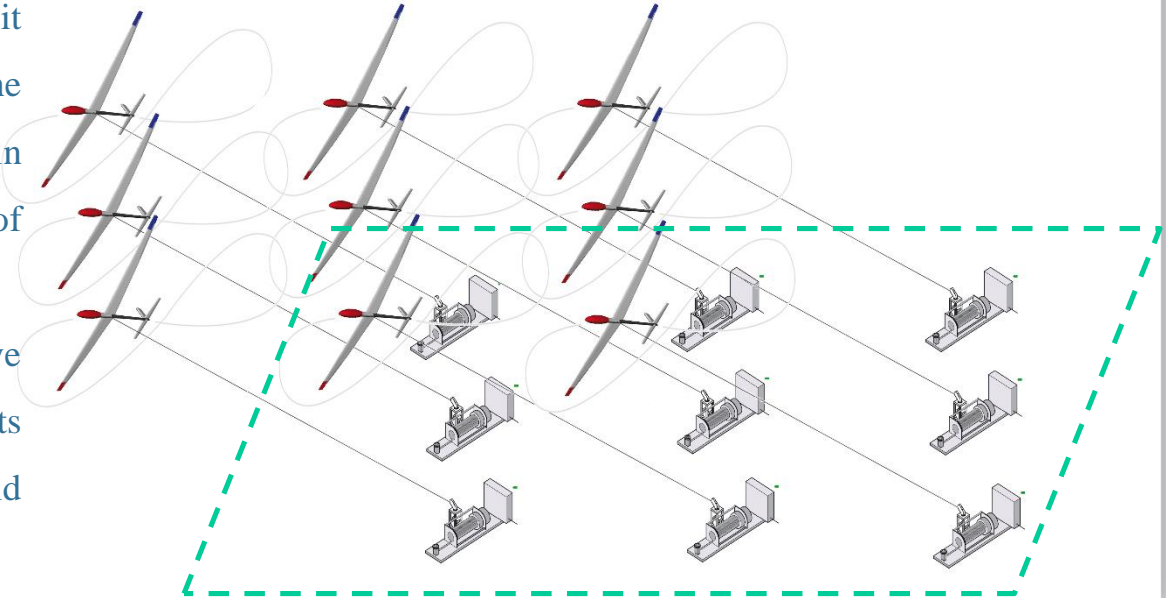
Ampyx Power is a start-up company developing a novel wind energy technology. Its product, the PowerPlane®, a tethered high-strength autonomously controlled glider. The basic principle of power generation uses a “pumping cycle” that uses strong tether tension during roll-out to drive a generator at the ground



Why this concept could be useful for Ampyx Power?

Given a fixed amount of land, it would be possible minimize the distance between these system in order to increase the number of them.

Moreover applying predictive schemes, geometric constraints could be enforced in order to avoid any crash.





Thanks for your
attention