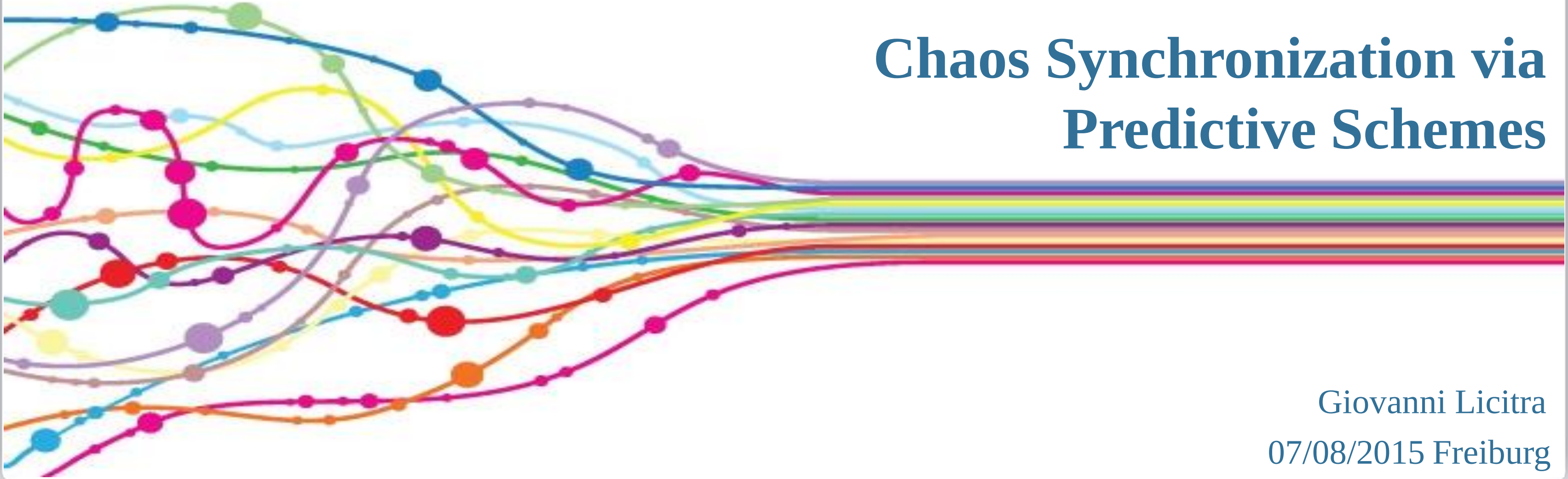
Chaos Synchronization via Predictive Schemes



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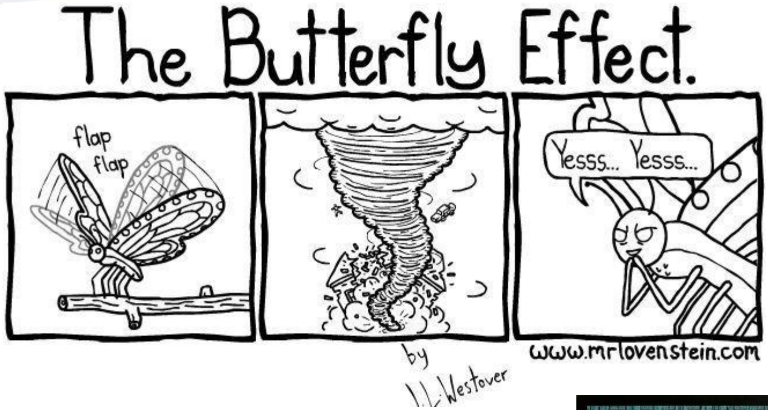
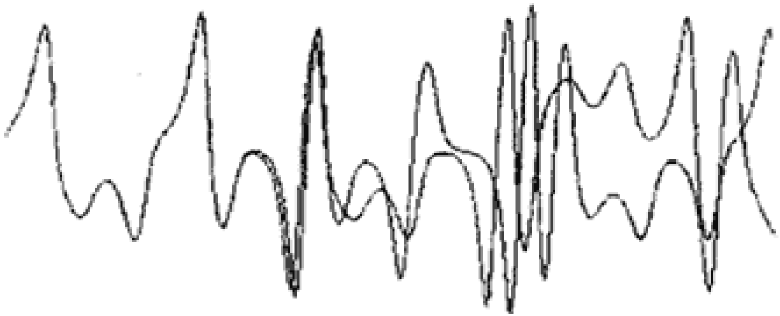
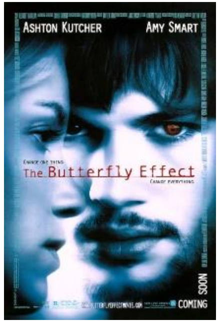
Does the Flap of a Butterfly’s Wings in Brazil set off a Tornado in Texas?

Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions and to the parameters of the system. A response popularly referred to as the butterfly effect.





Lorenz's experiment: the difference between the starting values of these curves is only .000127



How to synchronize two chaotic systems

Ampyx Power

Master system

$$\begin{cases} \dot{x}_1 = p[y_1 - \frac{(2x_1^3 - x_1)}{7}] \\ \dot{y}_1 = x_1 - y_1 + z_1 \\ \dot{z}_1 = -qy_1 \end{cases}$$

$$\vec{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - x_2 \\ y_1 - y_2 \\ z_1 - z_2 \end{bmatrix}$$

$$\boldsymbol{C(e)}$$

Slave system

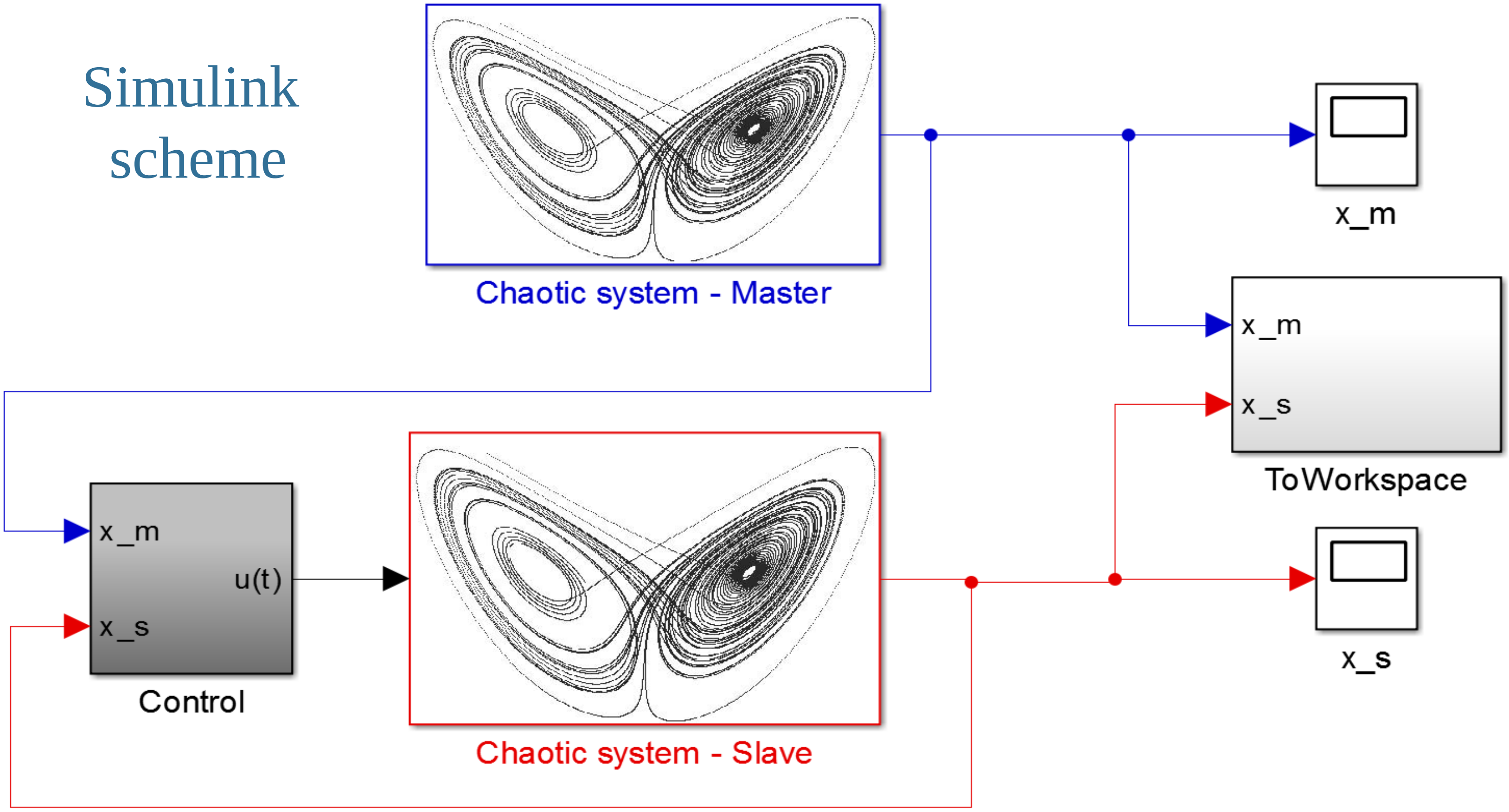
$$\begin{cases} \dot{x}_2 = p\left[y_2 - \frac{(2x_2^3 - x_2)}{7}\right] + \mathrm{u_x} \\ \dot{y}_2 = x_2 - y_2 + z_2 + \mathrm{u_y} \\ \dot{z}_2 = -qy_2 + \mathrm{u_z} \end{cases}$$

Synchronization condition

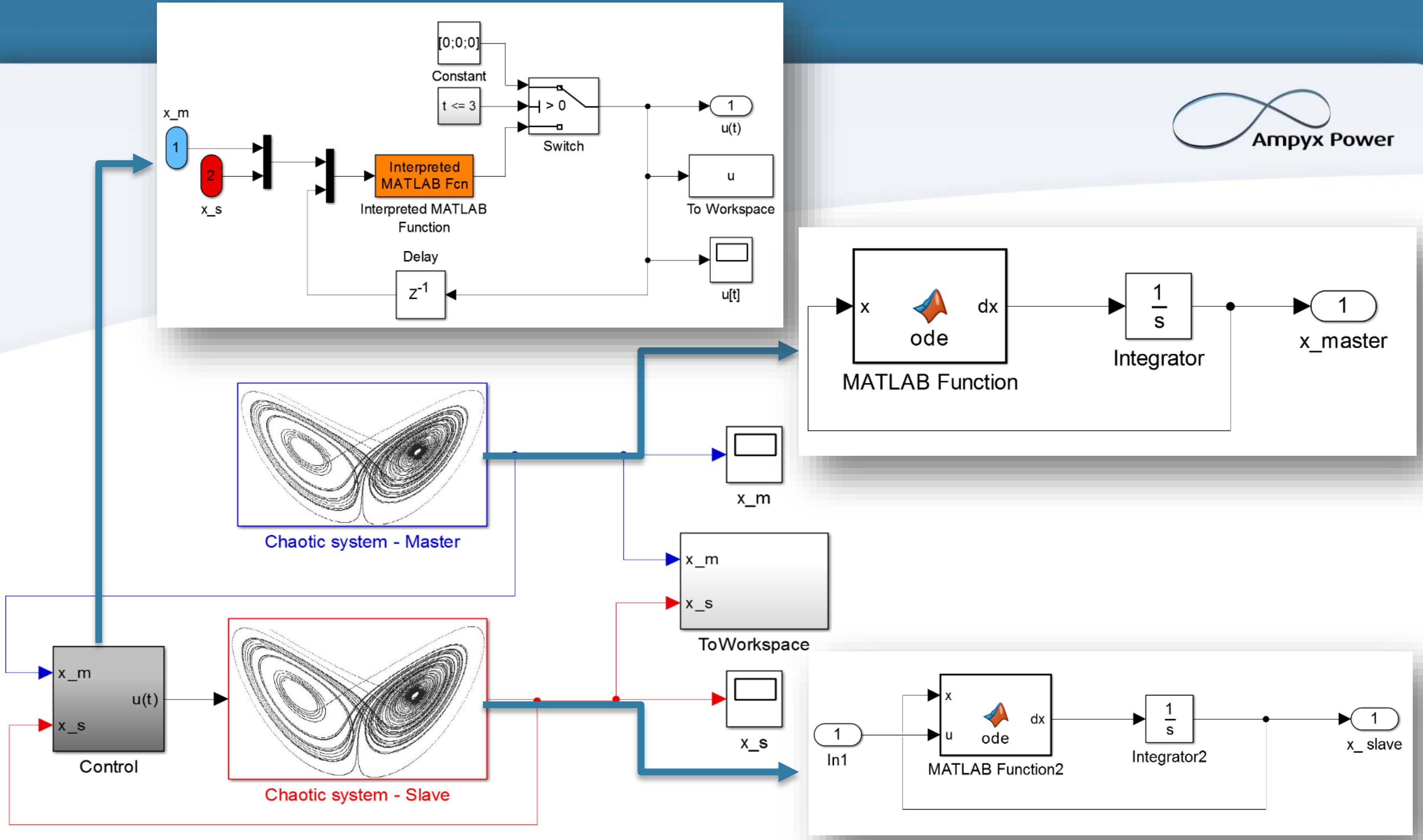
$$\lim_{t \to \infty} ||\vec{e}|| = 0$$

$$\min \int ||\vec{e}||_2^2 \, dt$$

Simulink scheme



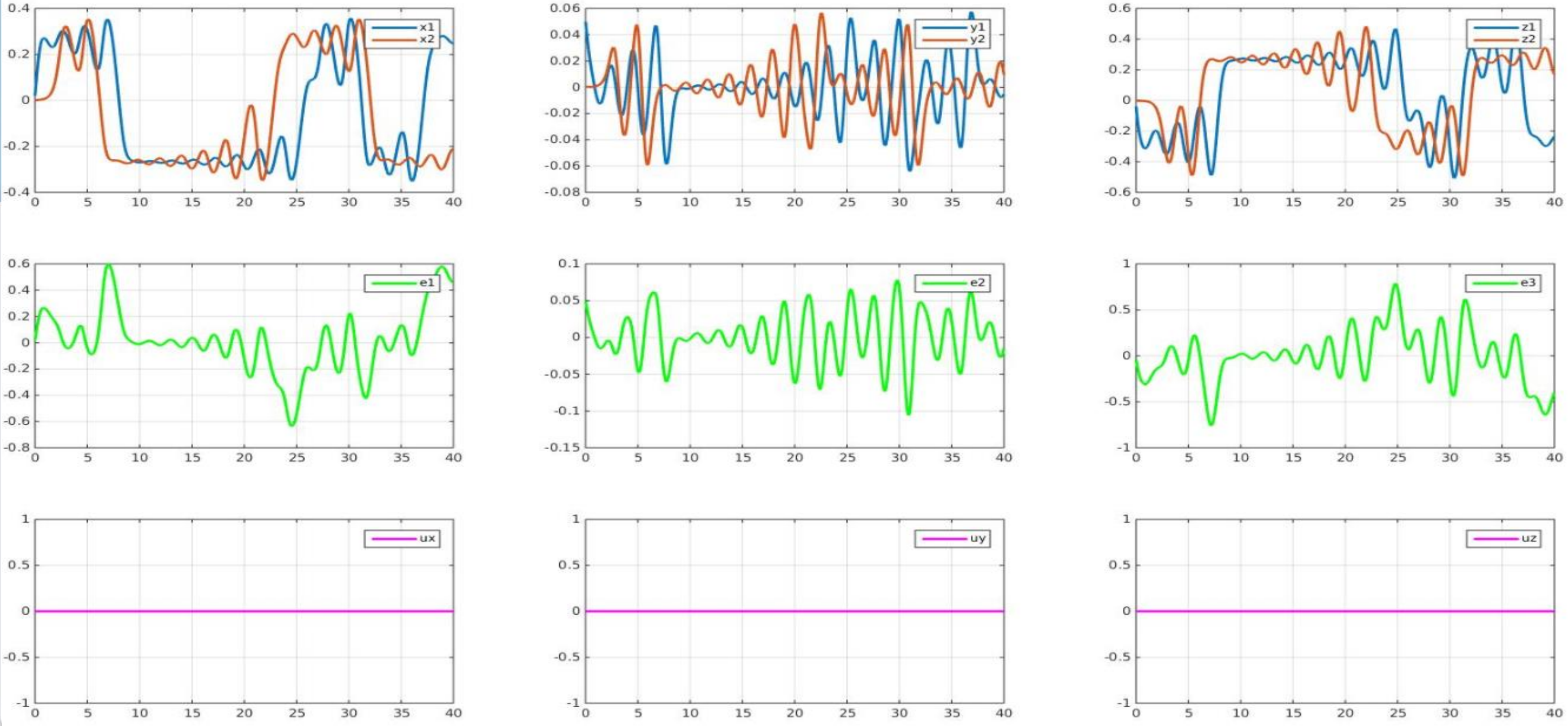
Ampyx Power



[0;0;0]
Constant
t <= 3
> 0
Switch
1
u(t)
u
To Workspace
u[t]
x_m
1
2
x_s
Interpreted MATLAB Fcn
Interpreted MATLAB Function
Delay
z-1
x
ode
dx
MATLAB Function
1/s
Integrator
1
x_master
Chaotic system - Master
x_m
x_m
x_s
ToWorkspace
x_m
u(t)
x_s
Control
Chaotic system - Slave
x_s
1
In1
x
u
ode
dx
MATLAB Function2
1/s
Integrator2
1
x_ slave

No Synchronization

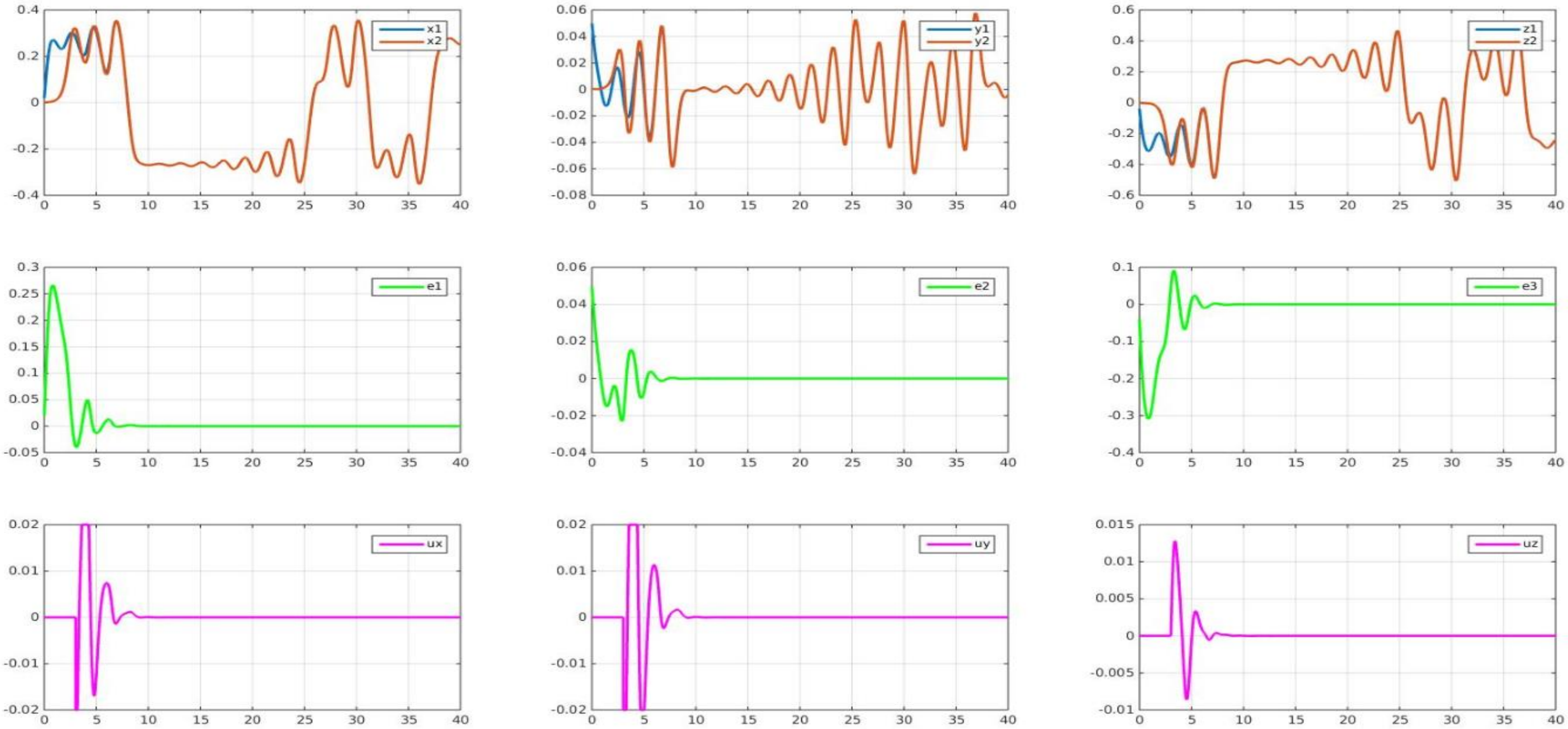
i.c. ($x_1 = 0.02\ x_2 = 0.0002,\ y_1 = 0.05\ y_2 = 0.0005,\ z_1 = 0.04\ z_2 = 0.0004$)



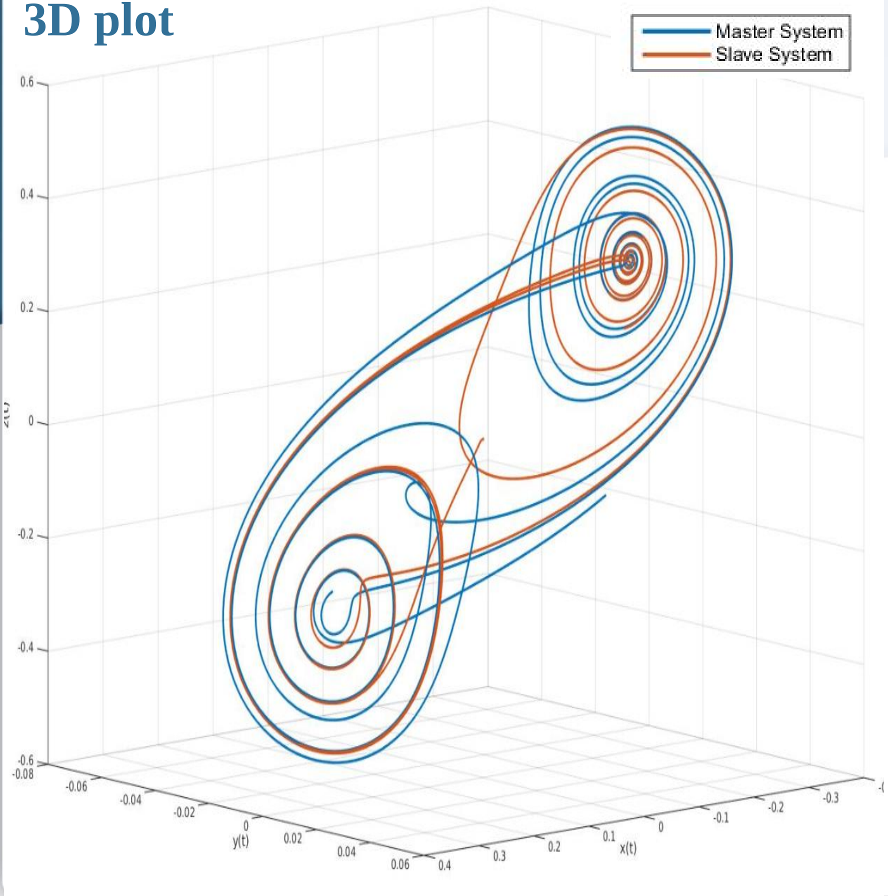
Synchronization via MPC (MPT toolbox)

Control stabilizes starting from t = 3 s

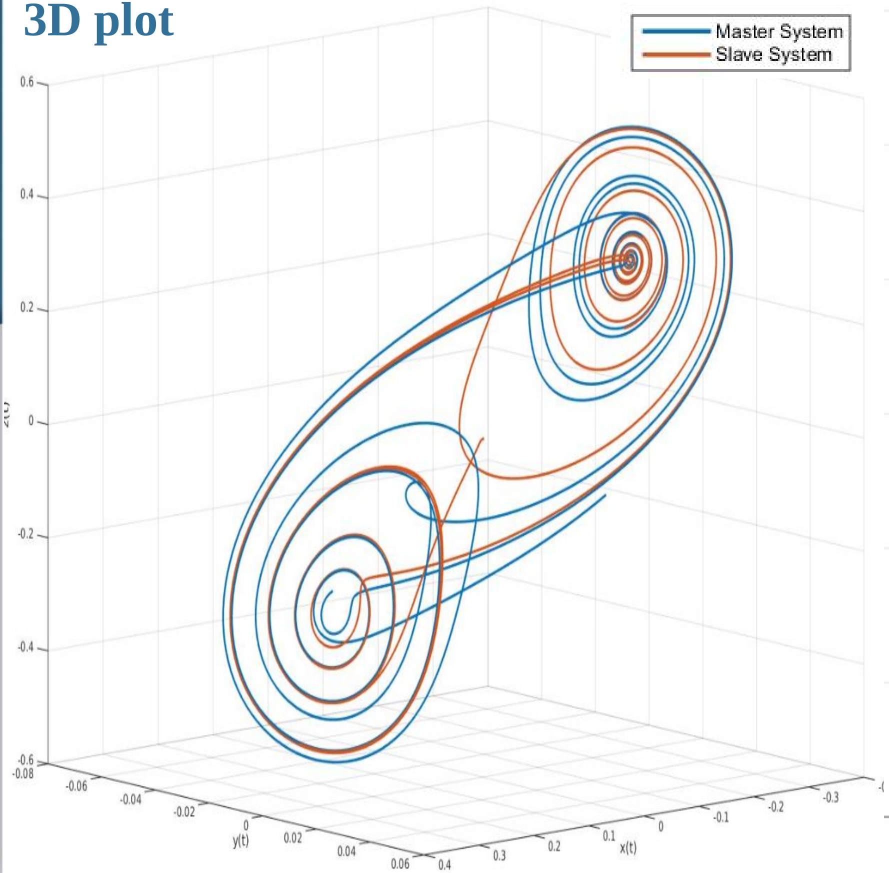
Ampyx Power



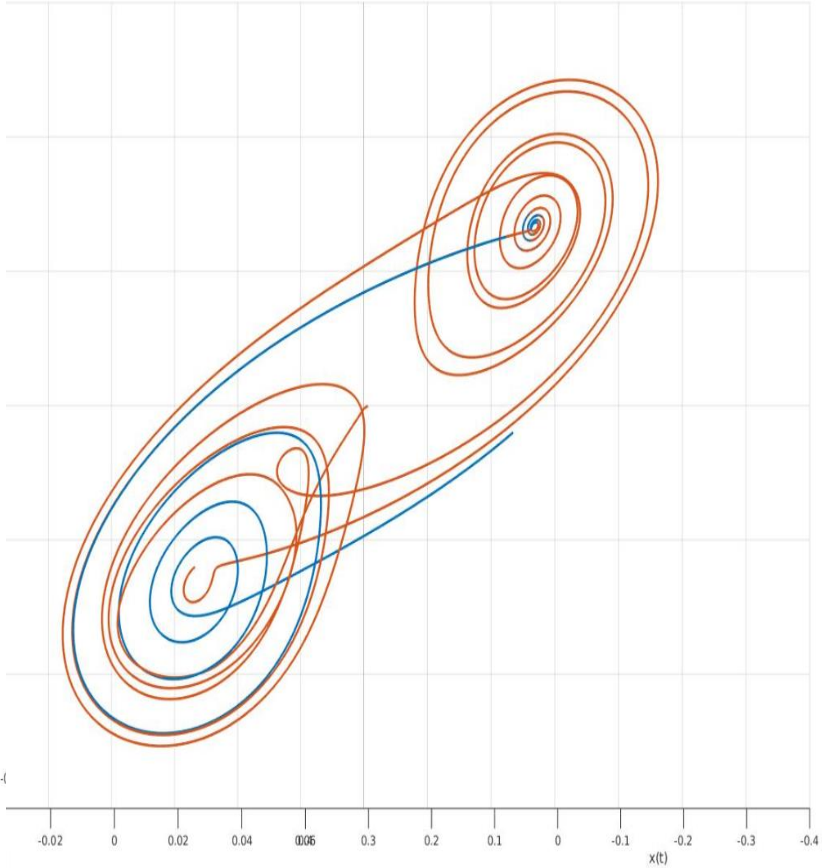
3D plot



3D plot

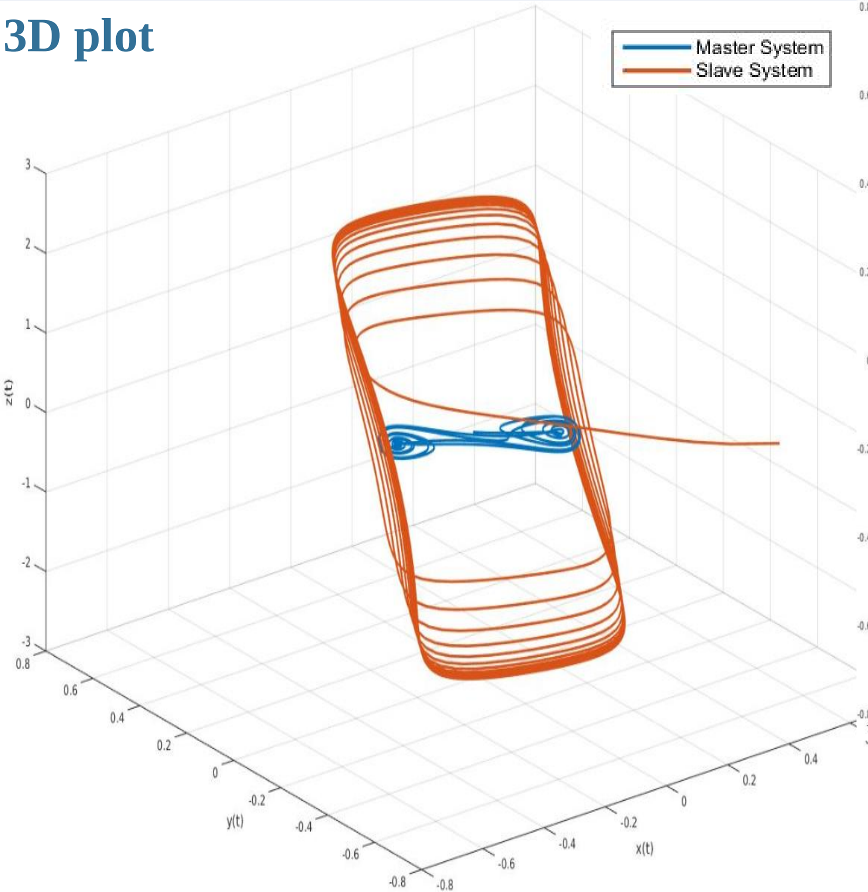


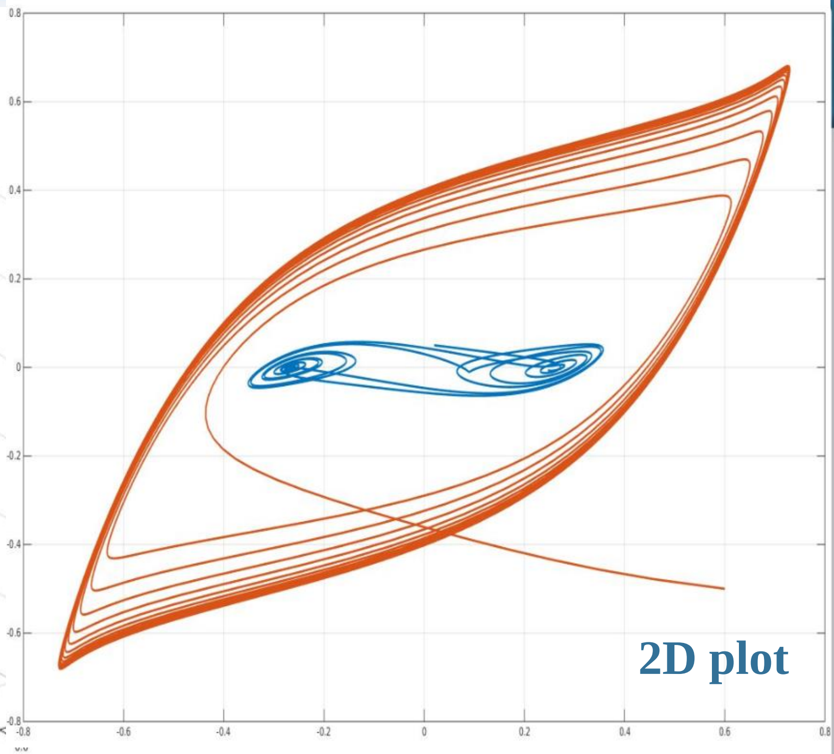
Master System
Slave System
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.08
-0.06
-0.04
-0.02
0
y(t)
0.02
0.04
0.06
0.4
0.3
0.2
0.1
x(t)
0
-0.1
-0.2
-0.3



-0.02
0
0.02
0.04
0.06
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
x(t)

3D plot



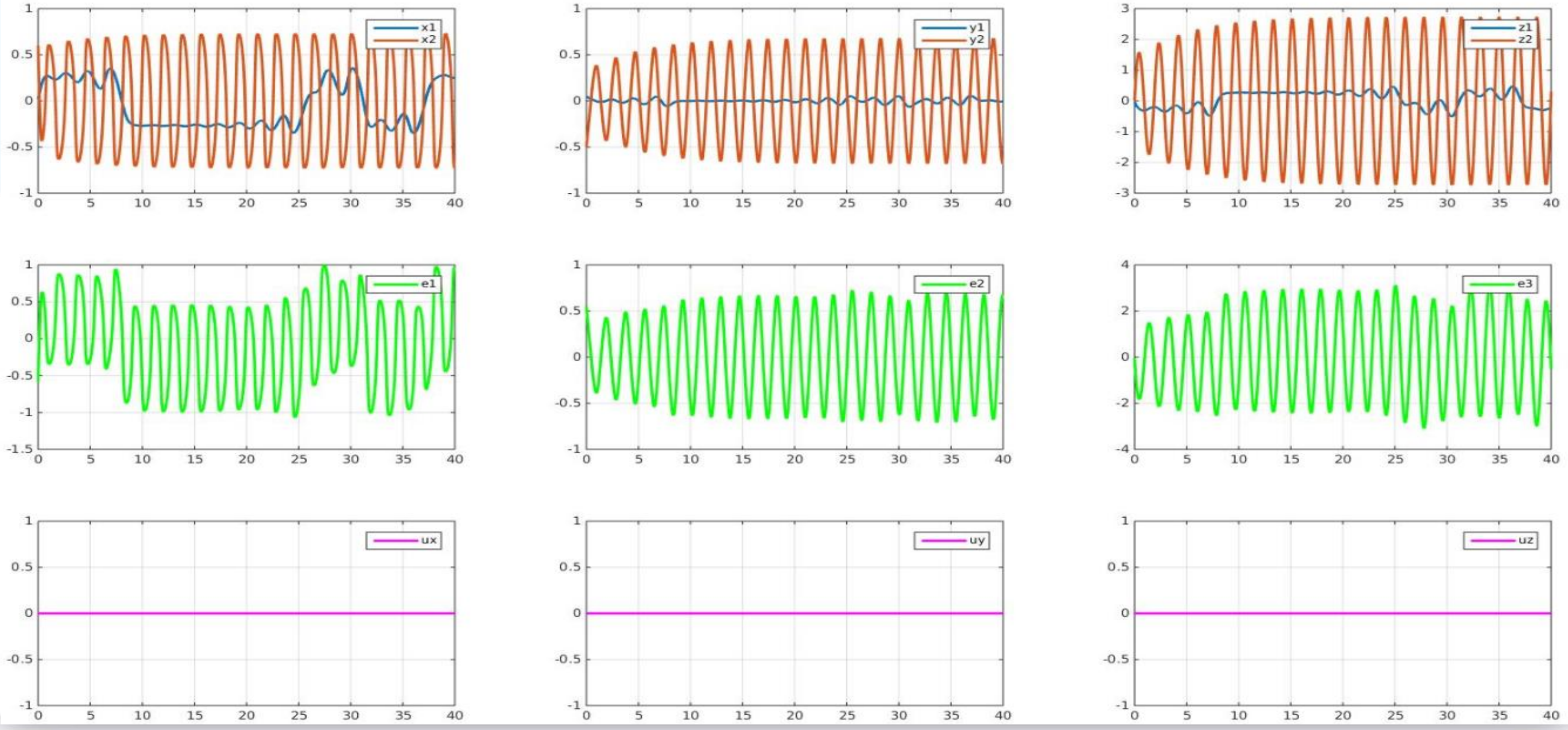


2D plot

No Synchronization

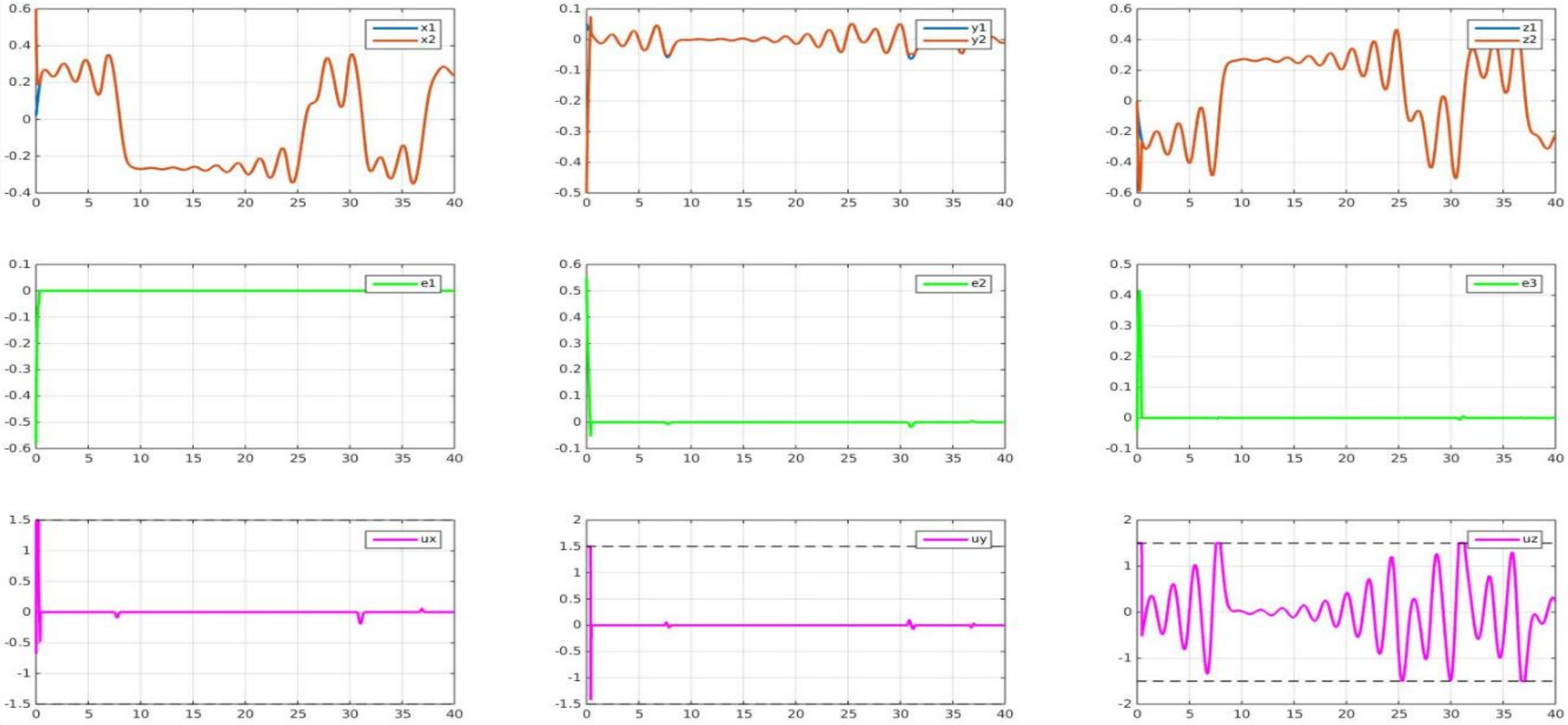
$i.c.(x_1 = 0.02,\ x_2 = 0.6,\ y_1 = \ 0.05,\ y_2 = -0.5,\ z_1 = -0.04,\ z_2 = 0.0004)$

Ampyx Power

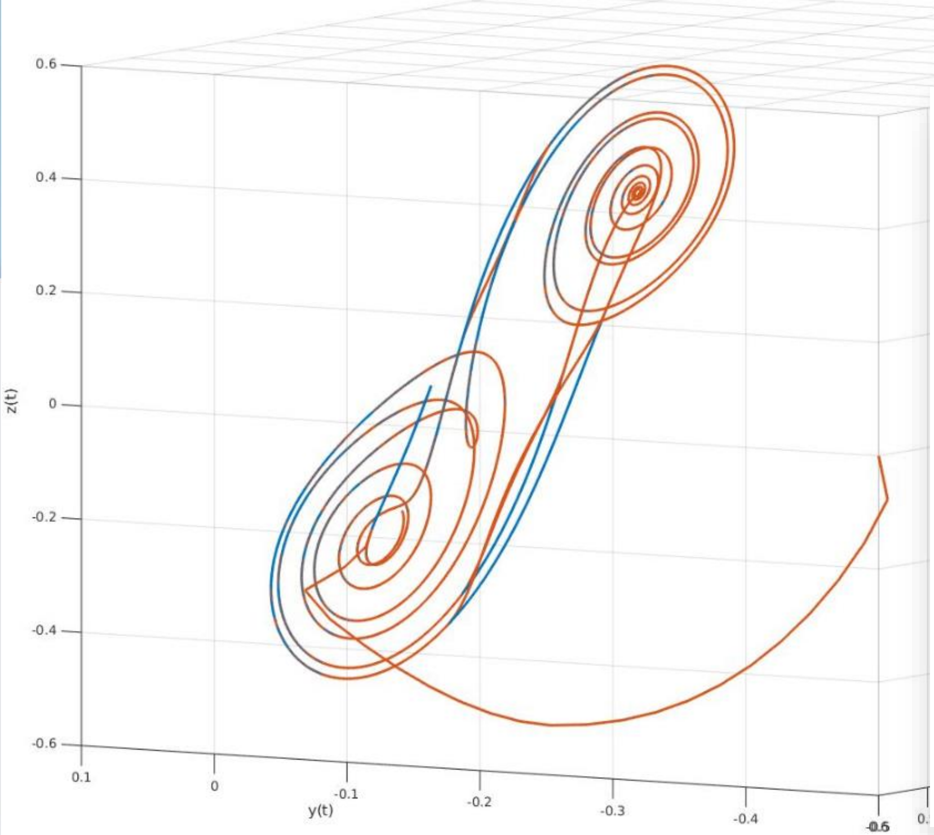


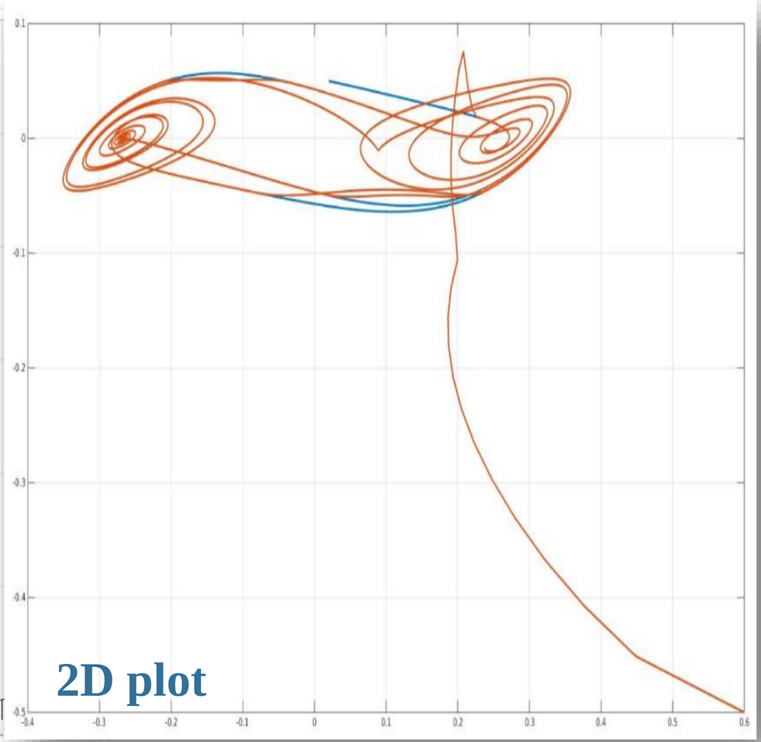
Synchronization via NMPC (ACADO Toolkit)

Ampyx Power



3D plot



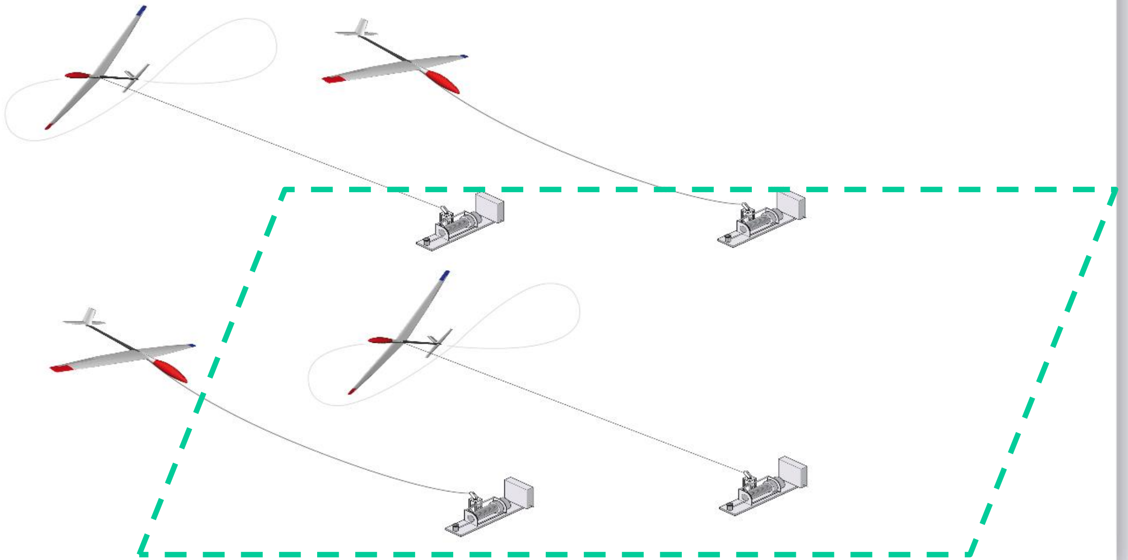


2D plot

If it is possible to synchronize chaotic systems, it should be simpler to synchronize deterministic ones...

What's Ampyx Power?

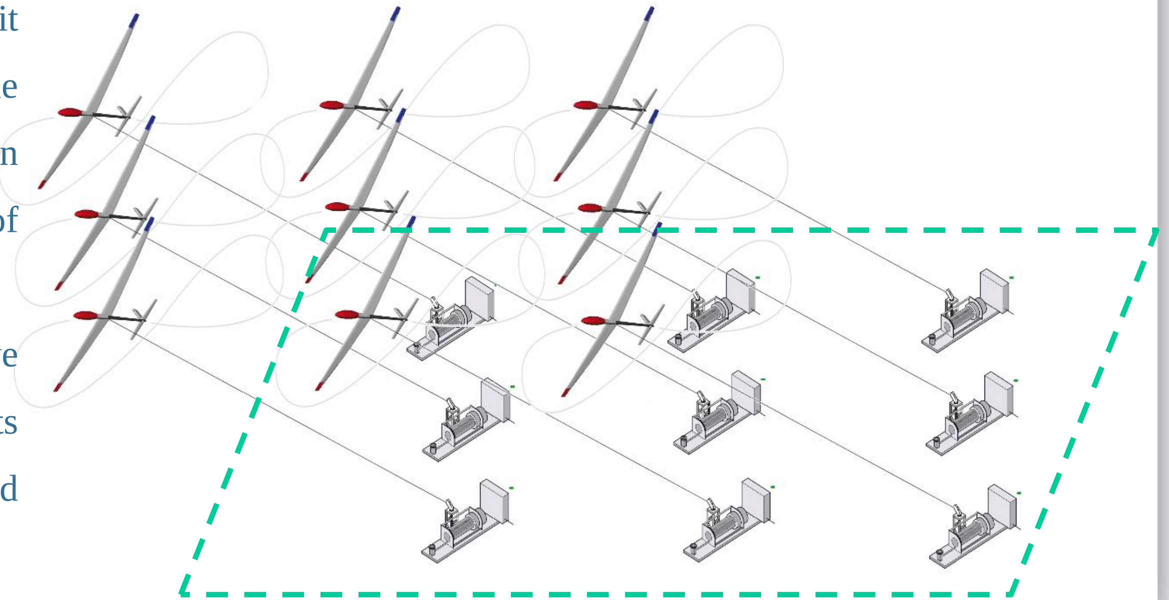
Ampyx Power is a start-up company developing a novel wind energy technology. Its product, the PowerPlane®, a tethered high-strength autonomously controlled glider. The basic principle of power generation uses a “pumping cycle” that uses strong tether tension during roll-out to drive a generator at the ground



Why this concept could be useful for Ampyx Power?

Given a fixed amount of land, it would be possible minimize the distance between these system in order to increase the number of them.

Moreover applying predictive schemes, geometric constraints could be enforced in order to avoid any crash.




Thanks for your
attention