Problem 1:

$S_1 = \{4, 5, 6\}$
$S_2 = \{i : i \text{ is odd}\}$
$S_3 = \{i : i \text{ is div. by } 3\}$

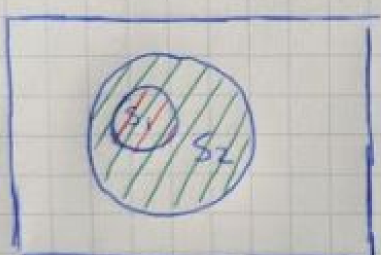
$S_4 = 2^{S_1} = \{\emptyset, \{4\}, \{5\}, \{6\}, \{4,5\}, \{4,6\}, \{5,6\}, \{4,5,6\}\}$
(power set) set is finite

$S_5 = S_1 \cap S_2 = \{5\}$ set is finite

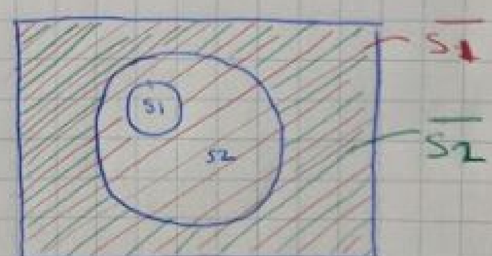
$S_6 = S_2 \cap S_3 = \{i : i \text{ is odd and div by } 3\}$ infinite

Problem 2:

Prove that if $S_1 \subseteq S_2$ then $\overline{S_2} \subseteq \overline{S_1}$



for S_1 to be a subset of S2 all elem. of S1 must be present in S2, as shown above.



The complement of S1 are all elements outside S1 & complement of S2 are all elements outside S2. Therefore every element present in $\overline{S2}$ is also present in $\overline{S1}$

Problem 3

(a) prove $1^3+2^3+3^3+\dots+n^3=\left(\frac{n(n+1)}{2}\right)^2$ for $\forall$ int $n \geq 1$

Induction:

Basis $n(1)$ $\quad 1^3=\left(\frac{1(1+1)}{2}\right)^2 \equiv 1=\left(\frac{2}{2}\right)^2 \equiv \boxed{1=1}$ ✓

Suppose $n(k)$ $\quad k^3=\boxed{\left(\frac{k(k+1)}{2}\right)^2}$ ①

Consider $n(k+1)$

$$k^3+(k+1)^3=\left(\frac{(k+1)((k+1)+1)}{2}\right)^2$$

per ①

$$\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3=\frac{(k+1)^2(k+2)^2}{4}$$

$$\frac{k^2(k^2+2k+1)}{4}+k^3+3k^2+3k+1=\frac{k^4+6k^3+13k^2+12k+4}{4}$$

(convert fraction)

$$\frac{k^4+2k^3+k^2}{4}+\frac{4k^3+12k^2+12k+4}{4}=\frac{k^4+6k^3+13k^2+12k+4}{4}$$

add

$$\boxed{\frac{k^4+6k^3+13k^2+12k+4}{4}=\frac{k^4+6k^3+13k^2+12k+4}{4}}$$

LHS = RHS ✓

hence proved

Problem 3 part (b)

prove $\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}$ for $\forall$ int $n \geq 2$

induction:

Basis $n(2)$ $\sum_{i=1}^{2-1} 1(1+1) = \frac{2(2-1)(2+1)}{3}$

$$2 = \frac{2(1)(3)}{3} \equiv 2 = \frac{6}{3}$$

$$\boxed{2 = 2}$$

Suppose $n(k)$ $\sum_{i=1}^{k-1} (k-1)((k-1)+1) = \frac{k(k-1)(k+1)}{3}$

$$\equiv \boxed{\frac{k^3-k}{3}} \text{ ①}$$

Consider $n(k+1)$ $\sum_{i=1}^{(k+1)-1} k(k+1) = \frac{(k+1)((k+1)-1)((k+1)+1)}{3}$

$$(k-1) + k(k+1) = \frac{(k+1)(k)(k+2)}{3}$$

per ①

$$\frac{k^3-k}{3} + k(k+1) = \frac{k^3+3k^2+2k}{3}$$

conv. to fraction

$$\frac{k^3-k}{3} + \frac{3k^2+3k}{3} = \frac{k^3+3k^2+2k}{3}$$

add

$$\boxed{\frac{k^3+3k^2+2k}{3} = \frac{k^3+3k^2+2k}{3}}$$

LHS = RHS thus proving equality

Problem 4

Prove inequality $\frac{n}{n+1} < \frac{n+1}{n+2}$, $n \geq 0$

$$\Rightarrow \frac{n}{n+1} - \frac{n+1}{n+2} < 0$$

$$\Rightarrow \frac{n(n+2)}{(n+1)(n+2)} - \frac{(n+1)(n+1)}{(n+1)(n+2)} < 0$$

$$\Rightarrow \frac{n^2+2n-n^2-2n-1}{(n+1)(n+2)} < 0$$

$$\Rightarrow \frac{-1}{n^2+3n+2} < 0$$

$$\Rightarrow -\frac{1}{n^2+3n+2} < 0$$

$\therefore$ this is true for any val of $n \geq 0$

$n(0) = -\frac{1}{2} < 0$ true

$n(1) = -\frac{1}{4} < 0$ true

$n(2) \; -\frac{1}{12} < 0$ true

$\vdots$

as n gets larger the LHS gets smaller, therefore inequality will hold

Problem 5: find DFA for the following language on $\Sigma = \{a, b\}$ $L = \{w : |w| \bmod 4 = 0\}$

for alphabet $\{a, b\}$ we want language accepted by the determinate finite accepters to be the set of strings where string w % 4 has no remainder.