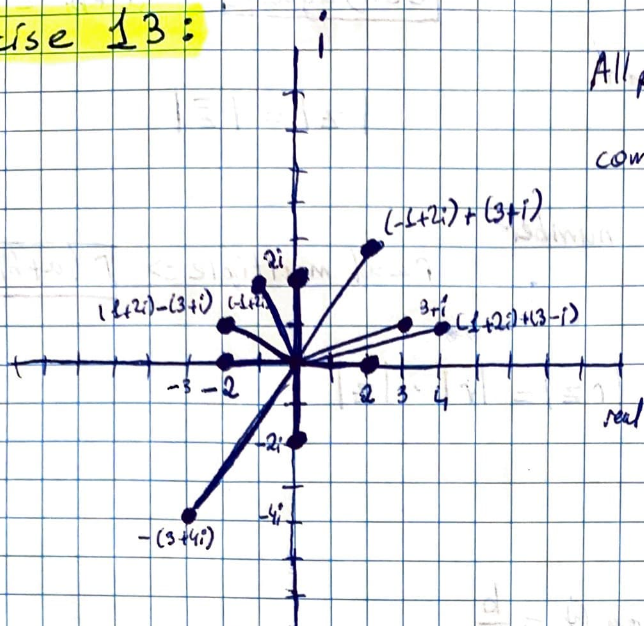
Exercise Sheet 1: Week 1 | Pavel Ghazaryan

CExercise 13:



All points may be seen on the complex plane.

a) $-z$, for drawing $-z$ of z that my friend draw I would suggest him to take the z and rotate it by $180°$ around Origin. Or by continuing the line passing through the origin and point it self go to the other side of origin the same amount of distance away.

b) $2z$, same direction, continue drawing one more z, starting from z. Basically draw another z totally the same way but instead Of starting from origin; start from the ending of z. Or just ~~draw~~ multiply z components by two and draw them. (Stretch 2 the original length)

c) $3z$, assumming he already knows how to draw $2z$, draw $2z$ starting at the end of z; same direction. ~~Or~~ Or, again multiply components by 3 and draw the result. (Stretch 3 times original length)

d) rz, take the ruler measure the length of z starting from origin. ~~Aran~~ take the ruler put the beginning of the ruler at the end of the z and draw z remembering the size. then ~~t~~do the same Steps r times.

going back to the graph I should find absolute values of a complex from each quadrant.

I quadrant: $(3+i) \Rightarrow |3+i| = \sqrt{9+1} = \sqrt{10}$

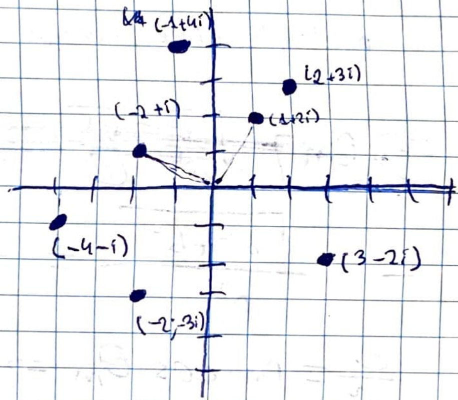
II quadrant: $(1+2i)-(3+i) = -2+i \Rightarrow \sqrt{4+1} = \sqrt{5}$

III quadrant: $-(3+4i) = (-3-4i) \Rightarrow |-3-4i| = \sqrt{9+16} = \sqrt{25} = 5$

IV quadrant: $-2i \Rightarrow |-2i| = \sqrt{0+4} = \sqrt{4} = 2$

CExercise 17:

The chosen numbers:



$(1+2i)$ | Lets multiply each by i

$(-1+4i)$ | $i\cdot(1+2i) = i+2i^2 = i-2 = -2+i$

$(-2-3i)$ | $i(-1+4i) = -i-4 = -4-i$

$(3-2i)$ | $i\cdot(-2-3i) = 3-2i$

$i(3-2i) = 2+3i$

Instructing my friend: rotate z around the origin by 90° to draw iz.

CExercise 22: Show that $z\bar{z} = |z|^2$

Assume: $z = a+bi \Rightarrow \bar{z} = a-bi \Rightarrow z\cdot\bar{z} = (a+bi)\cdot(a-bi) =$

$= a^2 - abi + abi - b^2i^2$ | Now $-abi$ and $+abi$ equal 0 and $-b^2i^2 =$

$= +b^2$ because $i^2 = -1$ | $\Rightarrow a^2 - abi + abi - b^2i^2 = a^2+b^2 \Rightarrow z\cdot\bar{z} = a^2+b^2$

$|z| = \sqrt{a^2+b^2} \Rightarrow |z|^2 = a^2+b^2$ where we se we got the same thing meaning $z\cdot\bar{z} = |z|^2$: Proved.

EExercise 19: prove lemma 1.3

$z^{-1} = \frac{1}{a+bi}$ => we should rationaliz it by multiplying with $\bar{z}$ =>

=> $\frac{1}{a+bi} = \frac{a-bi}{(a+bi)\cdot((a-bi))} = \frac{a-bi}{a^2+b^2}$ => $\boxed{\frac{a}{(a^2+b^2)} - \frac{b}{(a^2+b^2)} i}$ which was needed to ~~approve~~. prove.

EExercis 20: polar form of multiplicative inverse.

$r = \sqrt{a^2+b^2}$

$\varphi = \tan^{-1}\left(\frac{b}{a}\right)$ or $a = r\cos\theta$

multiplicative inverse $= \frac{a}{(a^2+b^2)} - \frac{b}{(a^2+b^2)} i$ => ~~Dlatter~~ $\frac{a}{r^2} - \frac{b}{r^2} i$ =>

=> $\cancel{r_m = \sqrt{\frac{a^2}{r^2} + \frac{b^2}{r^2}} = \sqrt{\frac{a^2+b^2}{r^2}} = \sqrt{r}}$ => $r_m = \sqrt{\frac{a^2}{r^4} + \frac{b^2}{r^4}} = \sqrt{\frac{a^2+b^2}{r^4}} =$

$= \frac{\sqrt{a^2+b^2}}{r^2} = \frac{r}{r^2} = \frac{1}{r}$ => $r_m = \frac{1}{r}$

$\frac{a}{r^2} = r_m \cdot \cos\theta_m$ => $\frac{a}{r^2} = \frac{1}{r} \cdot \cos\theta_m$ => $\cos\theta_m = \frac{a}{r} = \frac{r\cos\theta}{r}$ =>

=> $\cos\theta_m = \cos\theta$ => $\theta_m = \theta_m$ => polar form of multiplicative inverse is $\boxed{\left(\frac{1}{r}; \theta\right)}$