Pavel Ghazaryan
ID: 10756505

Mathematics OB219812 03/06/21

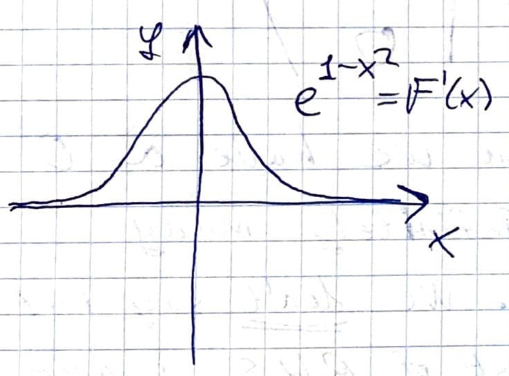
Take-Home Exam Paper

1. $F(x) = \int_0^x e^{-t^2+1}\,dt$

a) $F'(x) = e^{-x^2+1}$ because $F(x)$ is accumulation function.

b) Stationary points occur when $F'(x)=0$ but e^{1-x^2} cannot be 0 for any value of x, so there are no stationary points. $e^{1-x^2} > 0$ for all values of x

c) $\lim_{x\to\infty} F'(x) = \lim_{x\to\infty} e^{1-x^2} = e^{1-\infty^2} = e^{-\infty} = 0$ this means that the gradient goes to zero (0) when $x\to\infty$ of $F(x)$.
This can be seen in graph of $F'(x)$:



On the graph we can see that when $x \to \infty$ $F'(x) \to 0$ and so gradient of $F(x) \to 0$.

Answer: 0

2.
a) $\left(\begin{array}{cccc|c} 5 & 3 & 15 & -4 & 10 \\ 0 & 5 & -8 & 0 & -9 \\ 0 & -1 & 0 & 2 & 1 \\ 5 & 0 & 19 & -3 & 15 \end{array}\right)$

This matrix we get when writing down coefficients on LHS and constants on RHS.

Lets take the matrix to row echelon form.

1. $R_4 \to R_4 - R_1$

$\left(\begin{array}{cccc|c} 5 & 3 & 15 & -4 & 10 \\ 0 & 5 & -8 & 0 & -9 \\ 0 & -1 & 0 & 2 & 1 \\ 0 & -3 & 4 & 1 & 5 \end{array}\right)$

2. $R_3 \to R_3 + \frac{R_2}{5}$

$$\left(\begin{array}{cccc|c} 5 & 3 & 15 & -4 & 10 \\ 0 & 5 & -8 & 0 & -9 \\ 0 & 0 & -\frac{8}{5} & 2 & -\frac{4}{5} \\ 0 & -3 & 4 & 1 & 5 \end{array}\right)$$

3. $R_4 \to R_4 + \frac{3}{5} R_2$

$$\left(\begin{array}{cccc|c} 5 & 3 & 15 & -4 & 10 \\ 0 & 5 & -8 & 0 & -9 \\ 0 & 0 & -\frac{8}{5} & 2 & -\frac{4}{5} \\ 0 & 0 & -\frac{4}{5} & 1 & -\frac{2}{5} \end{array}\right)$$

4. $R_4 \to R_4 - \frac{1}{2} R_3$

$$\left(\begin{array}{cccc|c} 5 & 3 & 15 & -4 & 10 \\ 0 & 5 & -8 & 0 & -9 \\ 0 & 0 & -\frac{8}{5} & 2 & -\frac{4}{5} \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

As we may see in Row echelon form we have a 0 row which means that there are infinitely many solutions for the system of equations. We <u>don't</u> say that it is inconsistent be caues the constanst of RHS is also ~~0~~ zero. <u>Answer</u>: Infinitely many solutions because we get zero row in ref.

b) The system will be <u>inconsistent</u> and will have no solution. This is because in the R_4 in ref for this matrix all entries will be zero besides the constant RHS. As we have changed 15 to 20 this means that for sure RHS of last row will not be zero meaning <u>inconsistent</u>

<u>Answer</u>: No solutions because inconsistent

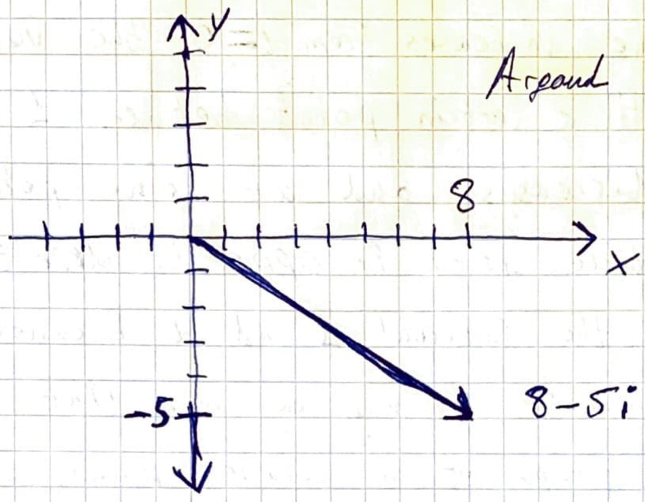
(Because we do same operations to bring to REF; 20 will not be zero like 15 in the end)

3/

a) In complex numbers i represents $\sqrt{-1}$; $i=\sqrt{-1}$.

b) Complex number is a number which can be expressed as $a+bi$ where $a,b \in \mathbb{R}$ (are Real numbers) and $i=\sqrt{-1}$. a is real part of complex number and b is imaginary part of a complex number.

c). $z = 8-5i$



4/

a) $\frac{dv}{dt} = v - t^2$

Note: Pencil curve is answer for b)

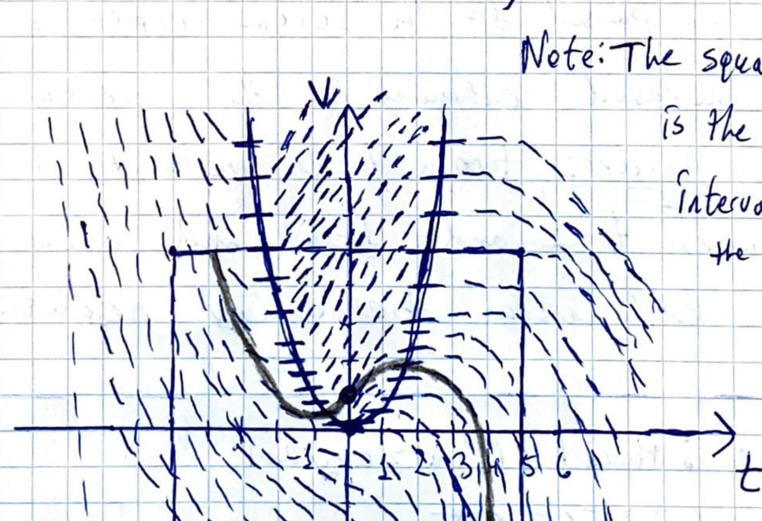
Lest find the curve for stationary points

It is when $\frac{dv}{dt}=0$

or when $v=t^2$

This means that on curve t^2 all points are stationary

$v=t^2$ is a parabola.



Note: The square is the provided interval for the question

Now let's find where is the function $v(t)$ increasing. It is increasing when $\frac{dv}{dt} \geq 0 \Rightarrow v > t^2$ this is the region above $v=t^2$ parabola. see the increasing gradient lines on graph.

Now lets find the decreasing interval i.e. where the ~~interval~~ $v(t)$ is decreasing. That is when $\frac{dv}{dt} < 0$ or $v < t^2$. This is the region below $v = t^2$ parabola. see gradient lines on previous page graph.

Now we obtained the direction field graph for the differential equation.

b) Please see on the graph previous page.

c) ~~use~~ As time increases from $t=0$ the velocity of a particle increases until a certain point between 1 and 2 (t). After that point it decreases and at some point becomes negative meaning particle going in opposite direction.

We know the interval 1 and 2 because on the graph when correctly sketched it may be seen that $V = t^2$ when $t = 2$ is $v = 4$ and on the graph of the solution passing through $(0;1)$ it's slightly less. Only way it ~~is~~ can be more than the 4 it is if the solution is a multiple of parabola but in that case it would n't have passed to the second region of decreasing but would have increased continuing in the increasing region. (+)

So **Answer:** from $t=0$ velocity increases to some point between $t=1$ and $t=2$ and afterwards decreases, in the continuation becoming negative as seen on graph.

Name: Pavel Ghazaryan

ID: 10756505

Date: June 3^{rd}, 2021

In total 4 pages.