Exercise Sheet 4 : Pavel Ghazaryan | 25/10/2021

Ex. 3 (ii) $P \wedge Q$ and $\neg(P \to \neg Q)$

P	Q	$P \wedge Q$	$\neg(P \to \neg Q)$
T	T	T	T (T implies F $\Rightarrow$ F but $\neg F = T$)
T	F	F	F (T implies T = T but $\neg T = F$)
F	T	F	F (F implies F = T $\neg T = F$)
F	F	F	F (F implies T = T $\neg T = F$)

(iv) $\neg(P \leftrightarrow Q)$ and $(P \vee Q) \wedge (\neg P \vee \neg Q)$

P	Q	$\neg(P \leftrightarrow Q)$	$(P \vee Q) \wedge (\neg P \vee \neg Q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

(vi) $P \wedge (Q \vee R)$ and $(P \wedge Q) \vee (P \wedge R)$

P	Q	R	$P \wedge (Q \vee R)$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
F	T	T	F	F
F	F	T	F	F
F	T	F	F	F
T	F	F	F	F
F	F	F	F	F

Ex. 6 (i) $(P \vee Q) \to \neg(P \wedge Q) = (0 \vee 1) \to \neg(0 \wedge 1) =$

$= 1 \to \neg(0) = 1 \to 1 = 1$

(ii) [illegible]

$P \vee Q = vP \cup vQ = \{a, b, c\} \cup \{c\} = \{a, b, c\}$

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$P \wedge Q = vP \cap vQ = \{a,b,c\} \cap \{c\} = \{c\}$

$\neg(P \wedge Q) = X \setminus \{c\} = \{a,b\}$

$(P \vee Q) \to \neg(P \wedge Q) = (X \setminus \{a,b,c\}) \cup \{a,b\} = \emptyset \cup \{a,b\} =$

$= \{a,b\}$ //

Ex. 7 (i)

P	Q	$(Q \to P)$	$P \to (Q \to P)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Tautology

(ii)

P	Q	$(\neg P \vee Q)$	$P \to (\neg P \vee Q)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Satisfiable

(iii)

P	Q	$(P \to Q)$	$((P \to Q) \to Q) \to Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Satisfiable

(iv)

P	Q	$(P \to Q)$	$((P \to Q) \to P) \to P$
T	T	T	T
T	F	F	T
F	T	T	T
F	F	T	T

Tautology

(v)

P	Q	R	$(P \to Q)$	$(P \to Q) \vee (Q \to R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T
F	T	F	T	T
T	F	F	F	T
F	F	F	T	T

Tautology

(vi)

P	Q	R	$(((P \vee Q) \wedge (P \to R)) \wedge (Q \to R)) \to R$
T	T	T	T
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	T
F	T	F	T
T	F	F	T
F	F	F	T

Tautology

Ex. 4 (i) 00001000 means only once there can be True out of all 8 combinations with three variables: Formula: $(\neg(P \to Q) \wedge \neg R)$ which is True only when $P=T; Q=F; R=F$.

(ii) 10001010 means three True values out of 8.

Formula: $(P \to Q) \wedge \neg R)$ which is True 3 times: 1. $P=T; Q=T; R=F$
2. $P=F; Q=T; R=F$
3. $P=F; Q=F; R=F$

(iii) 00000000 means contradiction.

Formula: $(\neg P \wedge Q) \wedge (P \wedge R))$ which is never True $\Rightarrow$ is contradiction.

Ex. 9 (i) A is alway True; $A \to B$ is always True

$A \to B$ is True when B is True or when both A and B are False. But because A is always True i.e. is Tautology then only way $A \to B$ to be tautology is when B is always True $\Rightarrow$ B is Tautology

(ii) $A \to B$ is tautology; B is tautology:

$A \to B$ can be true even when A is false i.e. $F \to T \equiv T \Rightarrow$ $\Rightarrow$ A doesn't have to be alway true for $A \to B$ to be tautology even if B is tautology.

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