# Exercise Sheet 14: Pavel Ghazaryan

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## CExercise 142

a) $\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r-1}$

B.C.: $n=0 \Rightarrow r^0 = 1 = \frac{r^1 - 1}{r-1} = \frac{r-1}{r-1} = 1$

I.H: $n=n \Rightarrow r^i + \dots r^n = \frac{r^{n+1} - 1}{r-1}$

S.C: $n=n+1$ $r^i + \dots r^n + r^{n+1} = \frac{r^{n+2} - 1}{r-1}$

$$\frac{r^{n+1} - 1}{r-1} + r^{n+1} = \frac{r^{n+2} - 1}{r-1} \Rightarrow$$

$$\Rightarrow \frac{r^{n+1} - 1 + r^{n+2} - r^{n+1}}{r-1} = \frac{r^{n+2} - 1}{r-1}$$ ✓ Proved

f) $1 + \sum_{i=0}^{n} 2^i = 2^{n+1}$

B.C $n=0 \Rightarrow 1 + 2^0 = 2^1 \Rightarrow 2=2$ ✓

I.H. $n=k$ $1 + \sum_{i=0}^{k} 2^i = 2^{k+1}$

S.C. $n=k+1$

$$1+\sum_{i=0}^{k+1} 2^i = 2^{k+2}$$

$$2^{k+1} + 2^{k+1} = 2^{k+2}$$

$$1+2^i \dots 2^k = 2^{k+1}$$

$$1+2^i \dots + 2^k + 2^{k+1} = 2^{k+2}$$

$$2^{k+1}+2^{k+1} = 2\cdot(2^k+2^k) = 2\cdot(2\cdot 2^k) = 4\cdot 2^k =$$

$$= 2^2\cdot 2^k = 2^{k+2}$$ ✓ Proven.

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**Exercise 144** a) $f(n) = 2n$

$f(0) = 0$

$f(n) = 2n$

$f(n+1) = 2n+2$

$f(n+1) = fn + 2$

$fn + 2 = 2n+2 \Rightarrow fn = 2n \Rightarrow 2n+2 = 2n+2$ ✓

d). $f0 = 0$; $f1 = 1$; $f(n+2) = 2f(n+1) - f_n$

$f(2) = 2 - 0 = 2$

$f(3) = 2\cdot 2 - 1 = 3$

$f(4) = 2\cdot 3 - 2 = 4$

$f(5) = 8 - 3 = 5$

$f(6) = 6 \Rightarrow$

$\Rightarrow f(n) = n$

$f(0) = 0$

$f(n) = n$

$f(n+1) = n+1$

$f(n+2) = n+2$

$2 \cdot f(n+1) - n = n+2$

$2n+2-n = n+2$ ✓ Proved.

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Exercise 149 ? a) Domain will be all Trees except the ones without any nodes: Basically with at least one node.

b). Domain: ~~At least~~ Any tree with at least 2 nodes.

c) ?

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