Exercise Sheet 12: Pavel Ghazaryan : Mathematical Techniques

CExercise 120 | a) Base case: len ((map f) [ ]) = len [ ] = 0

len [ ] = 0

Ind hyp: len((map f) ℓ) = len(ℓ)

Step case: len ((map f) [ℓ : ℓ_s]) = len [ℓ : ℓ_s]

Left side: len ((map f) [ℓ : ℓ_s]) = len (f ℓ : (map f) ℓ) = 1 + len ((map f) ℓ) =

= 1 + len(ℓ) (Ind. Hyp.)

Right side: len [ℓ : ℓ_s] = 1 + len ℓ_s = 1 + len(ℓ)

Left side and Right side are the same => Proved. ✓

b) Base case: (map f) ([ ] ++ [ ]) = (map f) [ ] = [ ]

(map f) [ ] ++ (map f) [ ] = [ ] ++ [ ] = [ ]

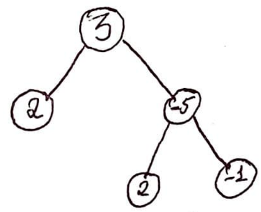
Ind. Hyp: (map f) (ℓ ++ ℓ') = (map f) ℓ ++ (map f) ℓ'

Step Case: (map f)((ℓ : ℓ_s) ++ (ℓ' : ℓ'_s)) = (map f)(ℓ : ℓ_s) ++ (map f)(ℓ' : ℓ'_s)

(map f)((ℓ : ℓ_s) ++ (ℓ' : ℓ'_s)) = (map f) (ℓ : (ℓ_s ++ ℓ_s)) = f ℓ : (map f) (ℓ_s ++ ℓ_s) ?

(map f) (ℓ : ℓ_s) ++ (map f) (ℓ' : ℓ'_s) = f ℓ : (map f) ℓ_s ++ f ℓ' : (map f) ℓ_s'

CExercise 121 | a)



b) Base: sum (tree s_N) = N

Step case: sum (trees$_N$ (t, t')) =

= ~~sum~~ N + t + t'

c) sum(tree$_3$(~~2~~ tree 2, tree$_{-5}$ (tree 2, tree -1)) =

= ~~sum~~ 3 + sum (tree 2, tree$_{-5}$(tree 2, tree -1)) = 3 + 2 + sum(tree$_{-5}$(tree 2, tree -1)) =

= 3 + 2 + (-5) + sum (tree 2, tree -1) = 5 + (-5) + 2 + sum (tree -1) = 5 + (-5) + 2 + (-1) =

= 1

d) -

(e) ?

CExercise 123 | a) map f : ~~tree~~ ( ) = ( ) Base case

Step case map f : (tree$_N$ ~~..~~ (tree$_M$)) = f(tree$_N$) ~~..~~ (map f (tree$_M$)).

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b) $tree_{17}(tree_5(tree3, tree19), tree25(tree19, tree27)) \to FBTrees$

map f: ~~FBT~~Trees = ~~34~~.$tree_{34}$ (map f(everything, 1.)) = $tree_{39}(tree_{10}(map()), tree50(map)) =$

$= tree_{34}(tree_{10}(tree6, tree38), tree50(tree38, tree54)).$

c) hght((map f)t) = hght t

Base Case.

hght((map f) trees) = hght(()) = 0.

~~54~~ Ind. hypo.

hght((map f) $trees_5$(t, t')) = hght(trees[redacted](n, m)) = max {hight n, hght n} +

+1.