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Mathematics 0D2
Take-home Exam
MATH 19872

1. a) $r = 1 - \sin(\theta)$

$x + y = 0 \rightarrow$ ~~illegible~~

$y = -x \Rightarrow m = -1$ and $c = 0 \Rightarrow \tan\theta = -1$ (Polar coord)

$r = 1 - \sin\theta$

two answers because period π

$\tan\theta = -1 \Rightarrow \theta = \arctan -1 \Rightarrow \theta = -0{,}78539$ or $2{,}35619449$

1st $r = 1 - \sin(-0{,}78539) = 1{,}707106$ so coord $(1{,}707106; \theta_1)$

2nd $r = 1 - \sin(2{,}35619449) = 0{,}292893$ so coord $(0{,}292893; \theta_2)$

We found two intersection points in Polar form, now we need to make them Cartesian:

$(1{,}707106; -0{,}785398)$

$x = r\cos\theta =$ ~~illegible~~ $1{,}207106229 \approx 1{,}207106$

$y = r\sin\theta = -1{,}207106$ first point $(1{,}207106; -1{,}207106)$

$(0{,}292893; 2{,}356194)$

$x = r\cos\theta = -0{,}207107$
$y = r\sin\theta = 0{,}207107$ $\Rightarrow$ Second point $(-0{,}207107; 0{,}207107)$

Answers: $(1{,}207106; -1{,}207106)$

&

$(-0{,}207107; 0{,}207107)$

b) $r = \cos\theta - \sin\theta$ ~~$\text{Area} = \int (\cos\theta - \sin\theta)^2 d\theta$~~

~~$= \int (\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta) d\theta$~~

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$$= \int_{\pi}^{\pi} (\cos\theta)^2 - \int_{\pi}^{\pi} 2\cos\theta\sin\theta + \int_{\pi}^{\pi} (\sin\theta)^2 d\theta$$

The limits for integral are when $r>0$ and $r \geq 0$ when $\cos\theta \geq \sin\theta \Rightarrow \theta$ is interval $\frac{\pi}{2} \leq \theta < \pi$ and $0 < \theta < \frac{\pi}{4}$.

Will get back to this question in the end.

c) $r = 2 + 2\cos\theta \qquad \theta_0 = -\pi \quad \theta_1 = \pi$

$$L = \int_{\theta_0}^{\theta_1} \sqrt{r(\theta)^2 + r'(\theta)^2} d\theta =$$

$$= \int_{-\pi}^{\pi} \sqrt{(2+2\cos\theta)^2 + (-2\sin\theta)^2} d\theta = \int_{-\pi}^{\pi} \sqrt{4 + 8\cos\theta + \cos^2\theta + 4\sin^2\theta} = \int_{-\pi}^{\pi} \sqrt{5 + 8\cos\theta + 3\sin^2\theta}\, d\theta =$$

$$= \int_{-\pi}^{\pi} 2\sqrt{2} \cdot \sqrt{1 + \cos\theta}\, d\theta = 2\sqrt{2} \int_{-\pi}^{\pi} \sqrt{1 + \cos\theta}\, d\theta =$$

$$= \underline{16}$$

$\boxed{\text{Answer: } 16}$

b)

$$A = \frac{1}{2} \int_{-\pi}^{\pi} f(\theta)^2 d\theta = \frac{1}{2} \int_{-\pi}^{\pi} (\cos\theta - \sin\theta)^2 = \frac{1}{2} \cdot \left(\int_{-\pi}^{\pi} (\cos\theta)^2 - \right.$$

$$- \int_{-\pi}^{\pi} 2\cos\theta\sin\theta + \int_{-\pi}^{\pi} (\sin\theta)^2 = \frac{1}{2} \cdot \left[\frac{1}{2}\left(x + \frac{1}{2}\sin(2\theta)\right) - \sin^2\theta + \right.$$

$$+ \frac{1}{2}\left(\theta - \frac{1}{2}\sin(2\theta)\right) = \frac{1}{2} \cdot \left[\theta - \sin^2\theta\right]_{-\pi}^{\pi} =$$

$$= \frac{1}{2} \cdot (\pi - 0 + \pi + 0) = \frac{1}{2} \cdot 2\pi = \underline{\pi}$$

Answer: π

2/ $f(x) = x^3 - x^2 + 2x - 1$ $\quad x \in [0, 1]$

$f(0) = 0 - 0 + 0 - 1 = -1 < 0 \qquad x_1 = 0$

$f(1) = 1 - 1 + 2 - 1 = 1 > 0 \qquad x_2 = 1$

1st step $\quad x_3 = \dfrac{x_1 + x_2}{2} = \dfrac{0+1}{2} = \dfrac{1}{2}$

$f(x_3) = \dfrac{1}{8} - \dfrac{1}{4} + 1 - 1 = \dfrac{1}{8} - \dfrac{1}{4} < 0$

we keep x_3 and x_2

2nd step $\quad x_4 = \dfrac{x_3 + x_2}{2} = \dfrac{0,5+1}{2} = 0,75$

$f(x_4) = 0,75^3 - 0,75^2 + 1,5 - 1 = 0,359\ldots > 0 \Rightarrow$

$\Rightarrow$ we keep x_3 and x_4

3rd step $\quad x_5 = \dfrac{x_3 + x_4}{2} = \dfrac{0,75 + 0,5}{2} = 0,625$

The value found after three steps of Bisection metod is $\underline{\underline{x = 0,625}}$ <u>Answer</u>: $\boxed{0,625}$

3/ $I_n = \int_0^1 x^n \sin(\pi x)\,dx$

a) $I_n = \int_0^1 x^n \cdot \sin(\pi x)\,dx = \dfrac{x^{n+1}}{n+1} \cdot \sin(\pi x)\Big|_0^1 -$

$- \int_0^1 \dfrac{x^{n+1}}{n+1} \cdot \cos(\pi x) \cdot \pi$

$\dfrac{x^{n+1}}{n+1} \cdot \sin\pi - 0 = \dfrac{x^{n+1}}{n+1} \cdot 0 = 0 \Rightarrow$

Here I integrate $\sin \pi x$ and derive x^n

$$I_n = \int_0^1 x^n \sin(\pi x)\,dx = x^n \cdot \frac{1}{\pi} \cdot (-\cos \pi x)\Big|_0^1 + \int_0^1 x^{\overline{n-1}} \cdot \frac{\cos \pi x}{\pi} =$$

$$x^n \cdot \left(\frac{-\cos \pi x}{\pi}\right)\Big|_0^1 = x^n \cdot \frac{+1}{\pi} - x^n \frac{-1}{\pi} = \cancel{x^n \cdot \left(\frac{1}{\pi} + \frac{1}{\pi}\right)} =$$

$$= \frac{1^n}{\pi} - 0 = \frac{1}{\pi}$$

I integrate $\cos \pi x$ derive x^{n-1}.

$$+\int_0^1 x^{\overline{n-1}} \cdot \frac{\cos \pi x}{\pi} = \underbrace{x^{\overline{n-1}} \cdot \frac{\sin \pi x}{\pi^2}\Big|_0^1}_{0} - \int_0^1 x^{\overline{n-2}} \cdot \frac{\sin \pi x}{\pi^2}$$

$$\cancel{I_n = \frac{1}{\pi} - \frac{I_{n+2}}{(n+2)\cdot(n+1)\cdot\pi^2}}$$

$$\boxed{\cancel{I_n = \frac{1}{\pi} - \frac{I_{n+2}}{(n+2)\cdot(n+1)\cdot\pi^2}}}$$ or $I_n \geq \frac{1}{\pi}$

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$\cancel{x^{n-1}\cdot\frac{\sin \pi x}{\pi^2}\Big|_0^1 = 1\cdot 0 - 0 = 0}$

$\cancel{\int_0^1 x^{n}}$

$$\int_0^1 x^{n-1}\frac{\cos \pi x}{\pi} = \frac{x^{n-1}\sin \pi x}{\pi^2}\Big|_0^1 - \int_0^1 x^{n-2}\frac{\sin \pi x}{\pi^2} =$$

$$= 0 - \frac{1}{\pi^2} I_{n-2} = -\frac{1}{\pi^2} I_{n-2}$$

$$\boxed{I_n = \frac{1}{\pi} + \left(\left(-\frac{1}{\pi^2}\right)\cdot I_{n-2}\right)}$$

answer for 3 a)

3/b) $I_0 = \frac{2}{\pi}$ $\quad I_2 = \frac{1}{\pi} - \frac{1}{\pi^2}\cdot\frac{2}{\pi} =$

$I_1 = \frac{1}{\pi}$ $\quad = \frac{1}{\pi} - \frac{2}{\pi^3} = \frac{\pi^2-2}{\pi^3}$

$$I_3 = \frac{1}{\pi} - \frac{1}{\pi^2}\cdot\frac{1}{\pi} = \frac{1}{\pi} - \frac{1}{\pi^3} = \frac{\pi^2-1}{\pi^3}$$

$$I_4 = \frac{1}{\cancel{4}\pi} - \frac{1}{\pi^2}\cdot\left(\frac{\pi^2-2}{\pi^3}\right)$$

$$\boxed{I_6 = \frac{1}{\pi} - \frac{1}{\pi}\cdot\left(\frac{1}{\pi} - \frac{1}{\pi^2}\cdot\frac{(\pi^2-2)}{\pi^3}\right)}$$

Answer fo 3b

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