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Mathematics OJ2 19852

Take-Home Exam

04/06/2021

1/

$m_1 = 1400kg$
$m_2 = 600kg$
$a = 1,3 ms^{-2}$

(i) $V = 13 ms^{-1}$
$S - ?$

$V = u + at \Rightarrow u = 0 \Rightarrow V = at$

$13 = a \cdot t \Rightarrow t = \frac{13}{1,3} = 10s.$

$S = ut + \frac{at^2}{2} = 0 \cdot t + \frac{1,3 \cdot 10^2}{2} =$

$= 65m$

Answer: for (i) $S = 65m$.

(ii) In order for the car and trailer to have $a = 1,3 ms^{-2} \Rightarrow$ means there should a total $F_T = (m_1 + m_2) \cdot a = 2600N$

As given in the problem $F_{car} = 3900N$ $R_c = 800N$; P & $T - ?$

Now let's find individual forces on trailer and car to have a.

$F_{car} = m_1 \cdot a = 1820 N.$

$F_{\text{after Resistance}} = 3900 - 800 = 3100N.$
AR

$F_{TRAILER} = T$

So we need to have 2600N in total but we have 3100N, this means that resistance

$P = F_{AR} - F_T = 500N.$ Answer: 500N.

(iii) from previous part the sum of all forces should be

$3900N \Rightarrow T + 500 + 1820 + 800 = 3900 \Rightarrow \underline{T = 780N}$

Answer: 780N.

Question 2 on next page: $\Longrightarrow$

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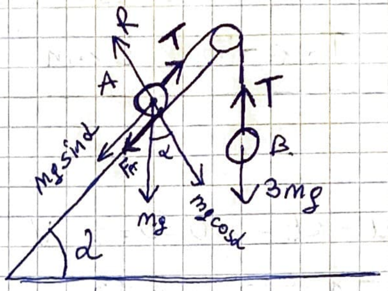
$\tan\alpha = \frac{4}{3}$

$m_1 = m$

$m_2 = 3m$

$a = \frac{1}{2}g$

(i)



(ii) because $a = \frac{1}{2}g \Rightarrow T = F = m_2 a = 3m\frac{1}{2}g = \frac{3}{2}mg$

Answer: $\frac{3}{2}mg = T$

(iii) $\tan\alpha = \frac{4}{3} \Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{4}{3} \Rightarrow \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{4}{3} \Rightarrow$

$\Rightarrow \frac{3}{4}\sin\alpha = \sqrt{1-\sin^2\alpha} \Rightarrow \sin^2\alpha\cdot\left(\frac{9}{16}+1\right) = 1 \Rightarrow \sin^2\alpha = \frac{16}{25} \Rightarrow$

$\Rightarrow \sin\alpha = \frac{4}{5}$ and $\cos\alpha = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Answer:

$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

(iv) A also has acceleration $a = \frac{1}{2}g$ because A and B are on the same string; forces parallel the plane should show this:

$$\frac{T - mg\sin\alpha - \mu mg\cos\alpha}{m} = \frac{1}{2}g.$$

$$\frac{\frac{3}{2}mg - mg\sin\alpha - \mu mg\cos\alpha}{m} = \frac{1}{2}g.$$

$$\frac{3}{2} - \sin\alpha - \mu\cos\alpha = \frac{1}{2}$$

$$\frac{3}{2} - \frac{4}{5} - \frac{3}{5}\mu = \frac{1}{2} \Rightarrow \mu = \frac{1}{3} \text{ or } 0,333..$$

Answer: $\mu = \frac{1}{3}$ or $\approx 0,333.$

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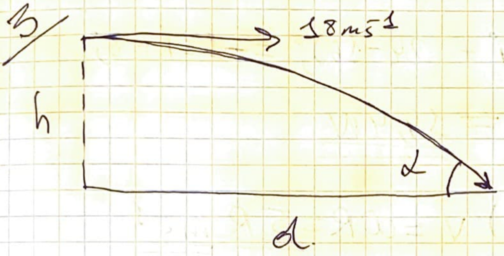


$t = 0,6s$
$V_x = 18ms^{-1}$

(i) in order find h let's look at the ~~sp~~ vertical velocity V_y which is initially 0. Using ~~~~ distance formula we can find h having $g = a$ and $t = 0,6s$:

$$S = h = \frac{at^2}{2} = \frac{g \cdot 0,36}{2} = 1,7658 \approx \underline{\underline{1,77\,m}} \text{ (3. sig. fig)}$$

(ii) $d = V\cos\alpha T = V_x t = 18 \cdot 0,6 = \underline{\underline{10,8\,m}}$

By using horizontal velocity

(iii) Speed in the end is $V = V_x i + V_y j \Rightarrow |V| = \sqrt{V_x^2 + V_y^2}$

$V_x = 18ms^{-2}$ $V_y = u + at = 0 + gt = 0,6g$

$|V| = \sqrt{324 + 0,36g^2} = 18,9379 \approx \underline{\underline{18,9\,ms^{-1}}}$

$|V|\cos\alpha = V_x = 18 \Rightarrow \cos\alpha = \frac{18}{|V|} \Rightarrow \alpha = 18,1077 \approx \underline{\underline{18,1^0}}$

<u>Answers</u>: <u>$18,9ms^{-1}$</u>; <u>$18,1^0$</u>

4/ a) $v = (t^3 - 15t - 5)i + (6t - t^2)j$

(i) $a = \frac{dv}{dt} = \underline{\underline{(3t^2 - 15)i + (6 - 2t)j}}$

(ii) $m = 4kg$

$F = ma = 4 \cdot (3t^2 - 15)i + 4 \cdot (6 - 2t)j =$

$= \underline{\underline{(12t^2 - 60)i + (24 - 8t)j}}$

(iii) $t=2 \Rightarrow F = (12\cdot 4 - 60)i + (24-16)j = -12i + 8j$

$|F| = \sqrt{144+64} = 14{,}4222 = \underline{14{,}4\,N}$

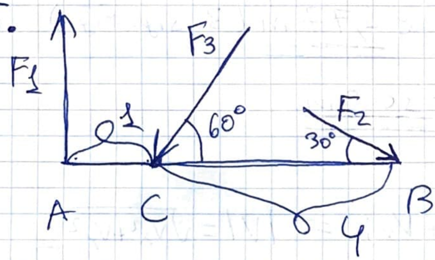
b) $m = 2kg$. $\omega = 5 \text{ rad } s^{-1}$; (i) $v = \omega R = \underline{5R}\ \underline{ms^{-1}}$

$R = R$

$l = R$ (ii) $a = \omega^2 R = \frac{v^2}{R} = \underline{25R}\ \underline{ms^{-2}}$

(iii) $T = F_{\text{Centripetal}} = m\omega^2 R = \underline{50R}\ \underline{N}$

5.



$AB = 5m$ and A is $(0,0)$
$|F_1| = P$ $|F_2| = Q$ $|F_3| = 2\sqrt{3}$

(i) $F_{1x} = 0$ $F_{1y} = P$ $F_{2x} = Q\cos 30^\circ = \frac{Q\sqrt{3}}{2}$ $F_{2y} = \frac{Q}{2}$

$F_{3x} = F_3 \cos 60^\circ = \sqrt{3}$ $F_{3y} = F_3 \sin 60^\circ = 3$

(ii) Resolving forces Horizontally we get $F_{3x} = F_{2x} \Rightarrow$

$\Rightarrow \sqrt{3} = \frac{Q\sqrt{3}}{2} \Rightarrow \underline{Q = 2N}$ and Resolving verticall because

it is couple $F_{3y} + F_{2y} = F_{1y} \Rightarrow 3 + \frac{Q}{2} = P \Rightarrow \underline{P = 4N}$

(iii) Magnitude of couple = Moment of couple, lets moment around

$A. \Rightarrow -F_{3y}\cdot 1 - F_{2y}\cdot 5 = -3 - 5 = -8\ N\cdot m$

This is also the magnitude of the couple $= \underline{8N}$

(iv) Because moment in (iii) $< 0 \Rightarrow$ means direction

is clockwise.

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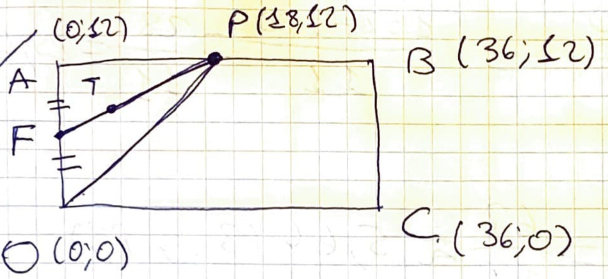

$m_1 = 8kg$

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(i) Because P is midpoint between AB $\Rightarrow P(18;12)$ coords. The Centre of mass of a triangle is on median ~~and~~ ~~on point where~~. The point is $\frac{1}{3}$ away of that of a Vertex of the triangle. Now consider the median PF in OAP. F has coords $(0;6)$ because midpoint of OA. then $\Rightarrow$ ~~ie~~ $\Rightarrow \vec{FP} = (18;6)$ $\vec{T}$ = position vector of centre of mass = $= \vec{OF} + \frac{1}{3}\vec{FP} = (0;6) + (6;2) = \underline{(6;8)}$ and $T = \rightarrow (6;8)$ coords.

Answer: T $\underline{(6;8)}$.

(ii) First of let's find centre of mass after OAP is removed. $A_{OAP} = \frac{1}{4} \cdot A_{OABC} \Rightarrow M_{OAP} = \frac{m_1}{4} = 2kg$. Use formula for $\bar{x}$ and $\bar{y}$ for two bodies but $m_{OAP} = -2$ because it is removed $\Rightarrow$ (Also C of M of rectangle is $G(18;6)$)

$$\bar{x} = \frac{8 \cdot 18 - 2 \cdot 6}{8 + (-2)} = 22$$

$$\bar{y} = \frac{8 \cdot 6 - 2 \cdot 8}{8 + (-2)} = \frac{16}{3}$$

So After Removing triangle Centre of mass is $\left(22; \frac{16}{3}\right)$

Now let's do the same formula with add Q where $m_2 = M_Q = 5kg$ and $Q(30,6)$:

Note mass of new object is $8 - 2 = 6 = m_{OPBC} = 6kg$

Next page $\Longrightarrow$

$$\bar{X} = \frac{m_{opsc}\cdot 22 + m_a \cdot 30}{m_{opsc} + m_1} = \frac{132+150}{11} = \frac{282}{11} = 25,6$$

$$\bar{y} = \frac{6\cdot\frac{16}{3} + 5\cdot 6}{11} = \frac{62}{11} \approx 5,64 \text{ (3 sig. fig.)}$$

Answer: $\left(\frac{282}{11} ; \frac{62}{11}\right)$ or $(25,6; 5,64)$

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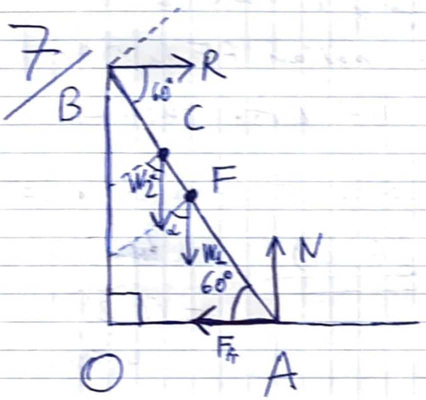

$AB = 6m$ $\mu = 0,35$

$\alpha = 60°$

$W_1 = 22g$ N

$W_2 = 90g$ N

F is Centre of Mass $\Rightarrow FN = FB = 3m$

(i) Please see above.

(ii) Lets check moment around point A it should be zero in order the ladder to not slip. First checking forces verticall we see that $N = W_1 + W_2$ and horizontally $R = F_{fr}$. We also know that $F_{fr} = N\mu = (W_1 + W_2)\cdot\mu \Rightarrow R = \mu\cdot(W_1 + W_2)$

Take moments around A.

$$-R\sin 60°\cdot 6 + W_2\cdot\cos 60°\cdot AC_{max} + W_1\cdot\cos 60°\cdot 3 = 0$$

$$90g\cdot\frac{1}{2}\cdot AC_{max} + 22g\cdot\frac{1}{2}\cdot 3 = 0,35\cdot 112g\cdot\frac{\sqrt{3}}{2}\cdot 6$$

$$45 AC_{max} + 33 = 117,6\cdot\sqrt{3}$$

$$AC_{max} = 3,79309 \approx 3,79m$$

Answer: 3,79m