Pavel Ghazaryan | Exercise Sheet 5 | ID: 10756505

CExercise 1: $(P \to Q) \vee (Q \to R) \equiv T$

a) $(P \to Q) \vee (Q \to R) \equiv (\sim P \vee Q) \vee (\sim Q \vee R) \equiv \sim P \vee Q \vee \sim Q \vee R \equiv$
$\equiv \sim P \vee R \vee T \equiv (\sim P \vee R) \vee T \equiv T$

b) $(A \to B) \vee (B \to C) \equiv (\sim A \vee Q) \vee (\sim B \vee C) \equiv \sim A \vee B \vee \sim B \vee C \equiv$
$\equiv \sim A \vee C \vee T \equiv T$

CExercise 2: a) $((P \to Q) \wedge (Q \to R)) \to (\sim P \wedge R)$

(i) $((P \to Q) \wedge (Q \to R)) \to (\sim P \wedge R) \equiv ((\sim P \vee Q) \wedge (\sim Q \vee R)) \to$
$(\sim P \wedge R) \equiv \sim((\sim P \vee Q) \wedge (\sim Q \vee R) \wedge \sim(\sim P \wedge R)) \equiv$
$\equiv \sim((\sim P \vee Q) \wedge (\sim Q \vee R) \wedge (P \vee \sim R))$ CNF

(ii) $\equiv ((\sim P \vee Q) \wedge (\sim Q \vee R)) \to (\sim P \wedge R) \equiv$
$\equiv \sim((\sim P \vee Q) \wedge (\sim Q \vee R)) \vee (\sim P \wedge R) \equiv ((P \wedge Q) \vee (Q \wedge \sim R)) \vee$
$\vee (\sim P \wedge R) \equiv (P \wedge Q) \vee (Q \wedge \sim R) \vee (\sim P \wedge R)$ DNF

b)

(i) $((P \wedge Q) \to R) \wedge (\sim(P \wedge Q) \to R) \equiv ((\sim P \vee \sim Q) \vee R) \wedge$
$\wedge ((P \wedge Q) \vee R) \equiv ((\sim P \vee \sim Q) \vee R) \wedge (P \vee R) \wedge (Q \vee R) \equiv$
$\equiv ((\sim P \vee \sim Q \vee R)) \wedge (P \vee R) \wedge (Q \vee R)$ CNF

(ii) $((\sim P \vee \sim Q) \vee R) \wedge ((P \wedge Q) \vee R) \equiv R \vee ((\sim P \vee \sim Q) \wedge P \wedge Q) \equiv$
$\equiv R \vee ((\sim P \wedge P) \vee (\sim Q \wedge P)) \wedge Q) \equiv R \vee ((\sim P \wedge P \wedge Q) \vee (Q \wedge \sim Q \wedge P)$
$\equiv R \vee (\sim P \wedge P \wedge Q) \vee (Q \wedge \sim Q \wedge P) \equiv R \vee (\perp \wedge Q) \vee (\perp \wedge P)$
$\equiv R \vee \perp \vee \perp$ DNF

CExercise 3: $(Q \to P) \to (\sim P \wedge \sim Q) \equiv$

$\equiv (\sim Q \vee P) \to (\sim P \wedge \sim Q) \equiv$

$\equiv (\sim(\sim Q \vee P)) \vee (\sim P \wedge \sim Q) \equiv (Q \wedge \sim P) \vee (\sim P \wedge \sim Q) \equiv$

$\equiv \sim P \wedge (Q \vee \sim Q) \equiv \sim P \wedge T \equiv \sim P$

So this is not a tautology.

EExercise 4: $(A \to B) \vee (B \to C) \equiv T \Rightarrow$

$\Rightarrow ((X \setminus S_A) \cup S_B) \cup ((X \setminus S_B) \cup S_C) \equiv$

$\equiv (X \setminus S_A) \cup (X \setminus S_B) \cup S_B \cup S_C \equiv$

$\equiv X \cup S_B \cup S_C \equiv X \equiv T$ /Proved/

EExercise 5: $B^3 \to B$

$(\sim P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R)$ ⇒ This is proposition for the truth table

$\equiv \sim P \wedge ((Q \wedge \sim R) \vee (Q \wedge R)) \equiv$

$\equiv \sim P \wedge (Q \wedge (\sim R \vee R)) \equiv \sim P \wedge Q \wedge (\sim R \vee R) \equiv$ CNF

$\equiv \sim P \wedge Q \wedge T \equiv \sim P \wedge Q$ (more simplified)

Pavel Ghazaryan

ID: 10756505