Exercise Sheet 3 | Pavel Ghazaryan

Exercise 37:

a) $f: \mathbb{R} \to \mathbb{R}$

$x \to \sin x \Rightarrow$ $x \Rightarrow \sin x$

$x' \to \sin x'$

assume $\sin x = \sin x'$ but this doesn't mean $x = x'$

counter proof let $x = 45°$

$x' = 135°$

$\sin x = \frac{\sqrt{2}}{2}$

$\sin x' = \frac{\sqrt{2}}{2}$ $\Big| \Rightarrow \sin x = \sin x'$ but $x' \neq x$

However if $x \in [0; \frac{\pi}{2}]$ or $[-\frac{\pi}{2}; \frac{\pi}{2}]$ then $\sin x$ is injective

f) $x \to x(-i)$

$x' \to x'(-i)$

assume $x'(-i) = x(-i)$

$i = \sqrt{-1} \Rightarrow -i = -\sqrt{-1} \Rightarrow x'(-i) = x(-i) \Rightarrow$

$\Rightarrow x' = x \Rightarrow$ injective

h) $x \longmapsto \{x\}$

$f: S \longmapsto PS$

$x \to \{x\}$

$x' \to \{x'\}$

if $\{x\}$ is the same as $\{x'\}$ because there are no repeating powersets, i.e. all are unique

It means this function is injective

Exercise 41:

b) $f: \mathbb{R} \to \mathbb{R}$

$x \to x^4 - 100$

$x^4 - 100 = f x = n$

$n = x^4 - 100$

$\sqrt[4]{n + 100} = x \Rightarrow$ surjective

Check: $x \to (\sqrt[4]{n+100})^4 - 100 = n + 100 - 100 = n.$

e) It is surjective

g) $N \times N \mapsto N$

$(x,y) \mapsto x$

$x = f(x,y) = n \Leftrightarrow x = n$ ↩?

Exercise 43: a) $\{0,1,2,3,4\}$

$x \mapsto x \bmod 3$

$x' \mapsto x' \bmod 3$ $\quad x \bmod 3 = x' \bmod 3 \Rightarrow x = x'$ so injective for given se

$x \mapsto x \bmod 3$

$x \bmod 3 = f x = n$

$x = 3k + n$ $\quad f(3k+n) = (3k+n) \bmod 3 = n \Rightarrow$ surjective

d) Is not injective because two people's first names can have same first letters.

Surjective -?