Mathematics ON1 19861 | Take Home Exam | Pavel Ghazaryan

Section B

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B1: $[-5;-3]$ closed interval $\subseteq D = \{x \in \mathbb{R}: x^2 > 8\}$. Prove.

Solution: Let's assume $x \in [-5;-3]$ which can also be written as $-5 \leq x \leq -3$. Now let's multiply by minus all parts of this inequality. We will get $5 \geq x \geq 3$ or $3 \leq x \leq 5$. Now lets square all parts. $\Rightarrow$

$\Rightarrow$ $9 \leq x^2 \leq 25$ this means that in the given interval $[-5;-3]$ x^2 is more or equal to nine and less or equal to 25. This also means $\Rightarrow$ $8 < 9 \leq x^2 \leq 25$ showing that indeed in this interval $x^2 > 8$ showing that All elements of interval $[-5;-3]$ are in Set D meaning that $[-5;-3] \subseteq D$. $\Rightarrow$ Proved

B2: show $p \vee (p \to q)$ is Tautology. Solution:

$p \vee (p \to q) \equiv p \vee (\sim p \vee q) \equiv (p \vee \sim p) \vee q \equiv T \vee q \equiv$

$\equiv T \Rightarrow$ is a tautology because is always true.

Proved

B3: a) $(\forall x)\ (x > 1) \to (x^3 > 1)$

b) Converse: $(\forall x)\ (x^3 > 1) \to (x > 1)$

Contrapositive: $(\forall x) \sim (x^3 > 1) \to \sim (x > 1)$ or

$(\forall x)\ (x^3 \leq 1) \to (x \leq 1)$

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B3: c) Contrapositive and the statement are true.

Converse is also true. Indeed, it can only be false when $x^3>1$ and $x<1$. However we know that all negative numbers have negative cubes i.e. if $x<0$ then $x^3<0$. This means that for all x; statement, contrapositive and converse are true. For example, $-5 = x$

statement: $-5>1 \longrightarrow -125>1$ $F \to F$ is True
False False

contrapositive: $-125 \leq 1 \longrightarrow -5 \leq 1$ $T \to T$ is True
True True

converse: $-125>1 \longrightarrow -5>1$ $F \to F$ is True
False False

B4: Prove $8n \leq 4^n$ for $n \geq 2$.

Basic Step: $n=2$

$$8 \cdot 2 \leq 4^2$$

$$16 \leq 16 \to \text{is True for } n=2$$

Inductive Step:

Assume is true for $n=k$

$$8k \leq 4^k$$

Lets check for $n=k+1$:

$$8(k+1) \leq 4^{k+1}$$

To prove this it will be enough to show that right side of inequality grows faster than left side of inequality. To show this, subtract $n=k$ sides from $n=k+1$ sides.

We will get $\Rightarrow$

$\Rightarrow \quad 8(k+1) - 8k = 8k + 8 - 8k = 8$

$$4^{k+1} - 4^k = 4 \cdot 4^k - 4^k = 4^k \cdot (4-1) = 3 \cdot 4^k$$

Indeed, it is obvious that $3 \cdot 4^k \geq 8$ for $k \geq 2$ meaning that right side grows faster than left side which shows that $8(k+1)$ is indeed small or equal to 4^{k+1}.

All this shows that we proved $8n \leq 4^n$ for $n \geq 2$ through induction: Proved

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