Exercise Sheet 2 | Week 2 | Pavel Ghazaryan

Exercise 25 p. 82: b) $x \in N$ and x is even and x is prime

Def. of prime number is that no number can divide $x \Rightarrow$

$\Rightarrow x \bmod R \neq 0$

x is even means $\Rightarrow$ $x \bmod 2 = 0$. Here we see two opposing statements $x \bmod R \neq 0$

$x \bmod 2 = 0$ Now 2 is part of R so

If one of the statements is incorrect other one should be correct meaning of the same both can't be true.

No such x exists. // Not true.

d) $x \bmod 4 = 0$

$4 = 2 \cdot 2$ meaning 4 is a multiple of 2 $\Rightarrow$

$\Rightarrow$ $x \bmod 4 = 0 \Rightarrow$ $x = 4k \Rightarrow x = 2 \cdot 2 \cdot k = 2^2 k \Rightarrow$

$\Rightarrow x$ is multiple of $2 \Rightarrow$ 2 divides x if 4 divides x, $\Rightarrow$ Valid / True

g) $x \in \mathbb{Z} \setminus \{-1, 0, 1, 3\}$ $x \operatorname{div} y = 2$

~~$x \operatorname{div} y = 2$ means x is even: for all even numbers~~

$x \operatorname{div} y = 2$ means y divides x in two halves.

In the or x is even number. But as both y and x are in set of integers only in case of x being even $x \operatorname{div} y = 2$ will be true, i.e. $x = 5$ is not true $\Rightarrow$

$\Rightarrow$ For all x and for some y; $\forall x \exists y$ is not true

Exercise 27 p.85:

a) $A \cap (B \cap C) = (A \cap B) \cap C$

this associative (Associative property on sets).

$N \cap (Z \cap R) = (N \cap Z) \cap R = N \Rightarrow$ Associative

e) $ave(ave(r, r'), r'') = \dfrac{\frac{r+r'}{2} + r''}{2}$ Associative

g) Associative because concatenating strings gives you the attached version so $"s" + ("d" + "m") = ("s" + "d") + "m"$

Exercise 28 p.88

b) $\mathbb{N}$ $\quad 5 - 4 = 1 \quad 1 \neq -1 \Rightarrow$ not commutative

$4 - 5 = -1$

g) And is commutative because. The order in the and statement does not matter; matters the truth values of the exparessions.

True and False = False

False and True = False