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Urban Traffic Flow Prediction Based on Road Network Model

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Abstract—This paper addresses an issue of short-term traffic flow prediction in urban traffic networks with traffic signals in intersections. An effective spatial prediction approach is proposed based on a macroscopic urban traffic network model. In contrast with other time series based or spatio-temporal correlation methods, this research focuses on the substantial mechanism of vehicles transmission on road segments and the spatial model of the entire urban network. Furthermore, this approach employs a simple speed-density model based on the macroscopic fundamental diagram (MFD) to obtain a more accurate vehicle travel time on the link. Finally, the microscopic traffic simulation software, CORSIM, is adopted to simulate the real urban traffic, and the proposed method is used to predict the traffic flows generated by CORSIM. The simulation results illustrate that our approach performs effective prediction timely in the rush hours, as well as the suddenly changed traffic states.

Index Terms—Traffic Flow Prediction; Spatial Model; Urban Road Network; MFD

I. INTRODUCTION

With the rapid development of the metropolises, intelligent transportation systems (ITS) are widely adopted all over the world to solve or relieve the traffic related problems. In ITS, the urban traffic management system collects mass traffic condition data in the urban surface traffic network, including traffic flow rate, volume, occupancy and speed, with various facilities such as GPS, loop detectors, RFID, and float probe vehicles. As one of the major elements of ITS and key technology for the advanced traffic management and traveler information services, traffic flow prediction takes advantage of these data to produce accurate and timely short-term traffic states like traffic flux, travel time, and vehicle density.

Since the early 1980s, researchers have taken use of the historical data to predict short-term traffic flow by different methods, mostly in freeways. For instance, prediction through the Kalman filtering [1], nonparametric regression method [2], autoregressive integrated moving average (ARIMA) [3], artificial neural network (ANN) approach [4], fuzzy logic-based method [5], etc. On the basis of considering the traffic flow

as time series, these approaches mostly could promise well in freeway or a location on an artery in urban, instead of the whole complicated and variable urban road network.

Recent years, approaches incorporating temporal and spatial characteristics have been presented to predict the urban traffic flow. Sun *et al.* [6] proposed an approach based on Bayesian network taking account of historical data from both current and neighbor junctions in order to achieve better effectiveness. Vlahogianni *et al.* [7] proposed multilayer perceptions (MLP) that were fed with volume data from sequential locations to improve the accuracy of short-term forecasts. Min and Wynter [8] predicted road traffic by taking into account the spatial characteristics of a road network in the way that reflects not only the distance but also the average speed on the links.

These convenient methods are proposed within the emphasis of fitting the variational tendency of traffic flow with some function tools based on historical data or upstream links, without considering of the movement mechanism of the network traffic flow. Consequently, these approaches could generate good effect in normal traffic statuses, *i.e.*, the traffic flow changes smoothly or gradually, whereas, inaccurate predictions usually occur when traffic flow suddenly changes. For this reason, Lin *et al.* [9] tried to forecast the traffic flow in short term using the macroscopic urban traffic network model, and obtained promising results. However, some approximations of the model parameters made the prediction lack of robustness. In this paper, we first amend some inadequacies of Lin's model, and then apply a speed-density model based on the macroscopic fundamental diagram (MFD) of traffic flow to the urban traffic network.

This paper is structured as follows. Section II specifies the urban traffic network, urban topology, and signalized link model. Section III describes the speed-density model method to obtain link average speed on the link. A simulation case study is expounded in section IV. Some concluding remarks and directions for the future work are given in Section V.

II. URBAN TRAFFIC NETWORK MODEL

In this section, the UTN model described by Lin *et al.* [10] and the link model by Berg *et al.* [11] are briefly introduced, meanwhile, some improvements are employed to make the UTN model more accurate.

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Considering the models described in the following sections, the notations are listed in Table I.

TABLE I
NOTATIONS DEFINED IN UTN

variable	description
T	sampling time interval (<i>sec</i>)
$D \in \{W, N, E, S\}$	orientation of the link in the network element (i.e. <i>west, north, east, south</i>)
$t \in \{s, l, r\}$	direction of the traffic turn movement in the link (i.e. <i>straight, left, right</i>)
$v_D^0(i, j)$	free flow speed in the link (<i>m/s</i>)
$v_D(i, j, k)^*$	average flow speed in the link from link entrance to tail of the queue (<i>km/h</i>)
$C_D(i, j)$	capacity of the link (<i>veh</i>)
$W_D(i, j)$	number of lanes in the link
L_{veh}	average length of vehicles (<i>m</i>)
t_h^*	mean discharge headway (<i>sec</i>)
s_{Dt}	saturated flow rate turning t (<i>veh/s</i>)
$\beta_{Dt}(i, j, k)$	turning rate at the stop line in the link at time k
$d_{Dt}(i, j, k)$	number of vehicles discharge from the link to t at time k (<i>veh</i>)
$d_{in,D}(i, j, k)$	number of vehicles enter the link at time k (<i>veh</i>)
$d_{out,D}(i, j, k)$	number of vehicles depart from the link at time k (<i>veh</i>)
$x_D(i, j, k)$	number of vehicles waiting in the link at time k (<i>veh</i>)
$a_D(i, j, k)$	number of vehicles arriving at the tail of the waiting queue in the link at time k (<i>veh</i>)
$f_D(i, j, k)$	available free space in the link at time k (<i>veh</i>)
$f_{Dt,ds}(i, j, k)$	free space in the downstream link of the departure vehicles turning t at the time k (<i>veh</i>)
$r_D(i, j, k)^*$	density of vehicles in the link at time t (<i>veh/m</i>)
$r_{jam,D}(i, j)^*$	jam density in the link (<i>veh/m</i>)
$q_D(i, j, k)^*$	flow rate in the link at time t (<i>veh/s</i>)
$q_{m,D}(i, j)^*$	maximum flow rate in the link (<i>veh/s</i>)
$g_{Dt}(i, j, k)$	signal symbol for vehicles turning t , 1 when signal is green, 0 when signal is red

* Extra parameters in our updated UTN model.

A. Network Model

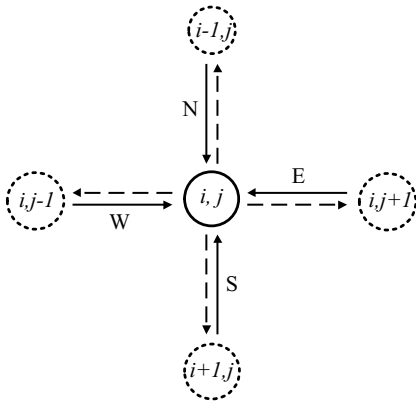


Fig. 1. Urban traffic network element

According to the topology of UTN, it is decomposed into

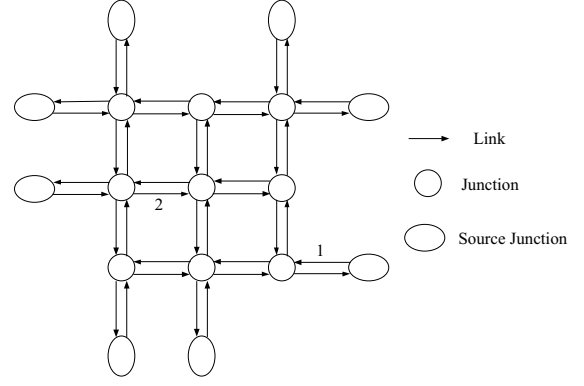


Fig. 2. A basic urban traffic network

three types of network elements (including a node and all links pointing to it, as Fig. 1 shows). Then, the whole UTN can be constructed by assembling each element together. Based on the decomposition method, the basic UTN showed in Fig. 2 is a network with 5 lines and 5 columns, and is made up by three kinds of network elements, i.e., “Cross”, “T-shape”, and “Source”. Among them, “T-shape” and “Source” are categorized by their directions again.

Cross as the solid lines shown in Fig. 1 which contains a node and joint links running to it, explains the typical calculating element in our model. The signalized junction of the UTN element is defined by the coordinates in the form of (i, j) (i and j represent line and column number of the node). The links assembling to the junction are labeled by their orientations, i.e., west link (W), south link (S), east link (E), and north link (N). The whole UTN element is marked as $E(i, j)$.

After building the UTN element model, the macroscopic traffic behavior at the signalized junction can be represented by mathematical formulas. Therefore, for any link D in $E(i, j)$, the number of the departure vehicles turning t is given by

$$d_{Dt}(i, j, k) = \begin{cases} \min\{x_{Dt}(i, j, k) + a_{Dt}(i, j, k), \\ f_{Dt,ds}(i, j, k), \\ s_{Dt}(i, j)T\} & \text{if } g_{Dt}(i, j, k) = 1 \\ 0 & \text{if } g_{Dt}(i, j, k) = 0 \end{cases} \quad (1)$$

where $x_{Dt}(i, j, k)$ is the number of the vehicles waiting in the link D and turning t at time k , $a_{Dt}(i, j, k)$ is the number of the vehicles arriving at end of the tail of the link, Both of them are expressed by the predefined turning rate, shown as follows:

$$\begin{cases} x_{Dt}(i, j, k) = x_D(i, j, k)\beta_{Dt}(i, j, k) \\ a_{Dt}(i, j, k) = a_D(i, j, k)\beta_{Dt}(i, j, k) \end{cases} \quad (2)$$

Additionally, in Lin's model, the saturated flow rate was set to be an uniform value for every link. This setting

would decrease the accuracy of the model. Therefore, in this work, $s_{Dt}(i, j)$, the maximal number of vehicles can pass the junction turning t in the link, is calculated by the mean discharge headway t_h when the vehicle leaves the stop line at the end of the link (see Equation 3). In order to obtain a good approximation to the real traffic, vehicles that turn left or right will queue in the leftmost or rightmost lane, the vehicles go straight can wait in the remanent lanes.

$$s_{Dt}(i, j) = \begin{cases} 1/t_h & \text{if } t = \{l, r\} \\ 1/t_h \cdot (W_D(i, j) - 1) & \text{if } t = s, T\text{-shape} \\ 1/t_h \cdot (W_D(i, j) - 2) & \text{if } t = s, Cross \end{cases} \quad (3)$$

B. Link Model

The link mode we used is based on the model proposed by Berg *et al.* [11]. To simulate the real road link, a basic diagram of road segment between two signalized junction is represented as follows:

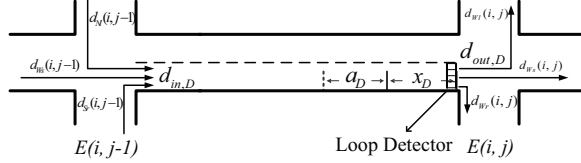


Fig. 3. UTN link model

In the process of vehicles transmission in the link, we assume that there is no vehicle access or disappear in the segment. In another words, all vehicles that enter a link will depart the link after a period of time. Based on this assumption, the free space of link D in $E(i, j)$ is updated by

$$f_D(i, j, k+1) = f_D(i, j, k) - d_{in,D}(i, j, k) + d_{out,d}(i, j, k) \quad (4)$$

where

$$d_{out,D}(i, j, k) = d_{Ds}(i, j, k) + d_{Dl}(i, j, k) + d_{Dr}(i, j, k) \quad (5)$$

The set of upstream links of link D is marked as D_{upl} ; $d_{Du,D}(i, j, k)$ means the number of vehicles that depart from upstream link D_u to link D at time k . Thus, $d_{in,D}(i, j, k)$ can be obtained by

$$d_{in,D}(i, j, k) = \sum_{D_u \in D_{upl}} d_{Du,D}(i, j, k) \quad (6)$$

We can represent the number of vehicles arriving at the tail of the waiting queues in link D in $E(i, j)$ as follows:

$$a_D(i, j, k) = \left(\frac{T - \gamma_D(i, j, k)}{T} \right) d_{in,D}(i, j, k - \delta_D(i, j, k) - \sigma) + \left(\frac{\gamma_D(i, j, k)}{T} \right) d_{in,D}(i, j, k - \delta_D(i, j, k) - 1 - \sigma) \quad (7)$$

where

$$\begin{cases} \delta_D(i, j, k) = fix \left(\frac{(C_D(i, j) - x_D(i, j, k)) L_{veh}}{W_D(i, j) v_D(i, j, k) T} \right) \\ \gamma_D(i, j, k) = rem \left(\frac{(C_D(i, j) - x_D(i, j, k)) L_{veh}}{W_D(i, j) v_D(i, j, k) T} \right) \end{cases} \quad (8)$$

and σ is the parameter corresponding to the time during which the vehicles passing through the junction freely.

Finally, the number of vehicles waiting in the queue turning t in link D in element $E(i, j)$ is updated by

$$x_{Dt}(i, j, k+1) = x_{Dt}(i, j, k) + a_{Dt}(i, j, k) - d_{Dt}(i, j, k) \quad (9)$$

In the model proposed by Lin *et al.* [10], the delay time of the vehicles running from the beginning of link D to the tail of the queues was given by Equation 8. According to this model, the average speed of the vehicles in link D from beginning to the tail of queue was regarded as the free flow speed. But actually, vehicles can not always keep free flow speed $v_D^0(i, j)$ on the link, especially located in a high density segment. In order to obtain a more precise prediction, we amend the average speed $v_D(i, j, k)$ based on the speed-density model, which is described in section III.

III. SPEED-DENSITY MODEL

In order to predict short-term traffic flow in UTN more accurately, normal traffic phenomena should be better reproduced in the UTN model. According to the description in the previous section, the UTN model proposed by Lin *et al.* [10] neglected the mechanism that vehicles move on road segments, and simply treated the average speed of vehicles as the free flow speed. This approximation always generates high deviation if the traffic on the road is heavy even blocked. For this reason, we apply a speed-density model based on the macroscopic traffic flow model to the link model to make the travel time more close to the real traffic situation.

The fundamental diagram represents all possible stationary traffic states as an approximation of reality. Recent experimental research on relations of traffic variables has shown that the average traffic flow rate and average traffic density within certain urban networks are related by a unique, reproducible curve known as the Macroscopic Fundamental Diagram (MFD) [12]. Classical forms of the MFD are the triangular shape and the parabolic shape. For the sake of simple and effective calculation, we adopt the triangular shape MFD as Fig. 4 shows. In our case, the MFD is defined by three values: the maximum flow rate (maximum number of vehicles through the link per hour) q_m , the traffic jam density r_{jam} of the link, the velocity v^0 in the free flow state.

The triangular shaped curve shown in Fig. 4 consists of two vectors. The first vector is the free flow side of the curve, in which the velocity identically equals to the free flow speed v^0 . The second vector is the congested branch, which starts with the maximum flow rate q_m to the state with zero flow and jam density r_{jam} . The congested branch has a negative slope, which implies that the higher the density on the congested branch, the lower the flow rate, and dovetail nicely with

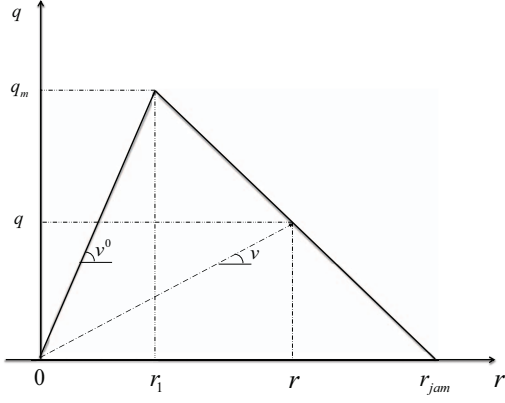


Fig. 4. Triangular shaped fundamental diagram

real traffic condition. The vertex of triangular shaped MFD (r_1, q_m) can be obtained by Equation 10.

$$r_1 = q_m(i, j) / v_D^0(i, j) \quad (10)$$

Based on the previous assumption that neither vehicles be created nor destroyed in the road segment and the conservation of number of vehicles, relationship between vehicle density, flow rate and average velocity can be described by:

$$q(i, j) = \begin{cases} v_D^0(i, j)r & 0 \leq r < r_1 \\ -\frac{q_m}{r_{jam} - r_1}r + \frac{q_m r_{jam}}{r_{jam} - r_1} & r_1 \leq r \leq r_{jam} \end{cases} \quad (11)$$

where $r = [1 - f_D(i, j, k) / C_D(i, j)] / L_{veh}$, $r_{jam} = W_D(i, j) / L_{veh}$, and $q_m = W_D(i, j) / t_h$. Hence, the average speed in link D of $E(i, j)$ at time k can be calculated as follows:

$$v_D(i, j, k) = \frac{q}{r} = \begin{cases} v_D^0(i, j) & 0 \leq r < r_1 \\ \frac{q_m v_D^0(i, j)}{r_{jam} v_D^0(i, j) - q_m} \cdot \frac{r_{jam} - r}{r} & r_1 \leq r \leq r_{jam} \end{cases} \quad (12)$$

Finally, we update $v_D(i, j, k)$ in Equation 8 to implement the more accurate estimation to the vehicle delay time in the link.

IV. A SIMULATION CASE STUDY

In this study, for the purpose of evaluation of the prediction model based on road network, the microscopic traffic simulation software package TSIS-CORSIM exploited by FHWA [13] is employed to simulate the real traffic. The basic urban road network model shown in Fig. 2 is first built in CORSIM. The whole experiment continues 4 hours totally, predicting the future average traffic flux at the interval of 5 minutes. In addition, we assign fix-timed signal time controller at all intersections. Towards every step of the prediction, the prediction model is fed with the initial traffic states that are detected on all over the traffic network, including number of the waiting vehicles, current number of vehicles on the link, the number of the departure vehicles in the previous sampling

time interval, the future signal information (i.e., cycle length, offset time, phase, split, etc.), and the future network input traffic flows at the source junctions.

During the simulation process, the free flow speed is set to be 30 km/h , the average vehicle length L_{veh} is 5 m , mean discharge headway t_h is equal to 1.8 s . The time for which the vehicles pass through the junction freely: $\sigma = 3 \text{ s}$. Furthermore, all the capacities of links $C_D(i, j)$, number of lanes $W_D(i, j)$, and the turning movement percentages $\beta_{Dt}(i, j, k)$ at the stop line are considered to be definite and known. The sampling time interval T is equal to 1 s , both for CORSIM and the proposed UTN model. In addition, to attain our object that test the proposed prediction model comprehensively using the simulation case, the flow rates with significant changes, which are generated from each source junction are defined as Fig. 5 shows. As shown in the figure, two rush hours are designed in the input flow rates. The first rush hour achieves 2000 veh/h in the middle of the simulation, and continues for 20 minutes followed by a sharp flow drop. Moreover, another small rush hour appears at the last quarter of the simulation. Furthermore, two steady flow rates are also inputted into the test road network at the beginning and end of the simulation. Thence our input flow rates contain common traffic states in real urban traffic such as rush hours, sharp rise, sharp drop, minor rise, minor drop, steady flow and so on.

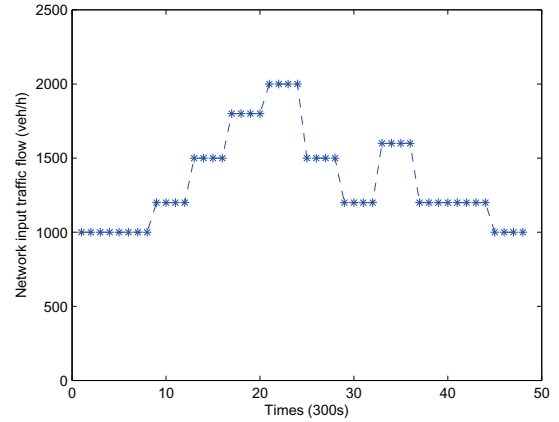


Fig. 5. Network input flow rate

The output of our prediction model is the average traffic flux in 5 minutes, which denotes the mean traffic flow rate discharged from a link and converted into veh/h . In the basic urban network, two typical links are selected to investigate the effect of the proposed prediction model, i.e., link 1 and 2, marked in Fig. 2. Furthermore, three different measures of effectivity are used in this research for evaluating the performance of Lin's and the proposed model: maximum absolute percent error (MaxAPE), minimum absolute percent error (MinAPE), and mean absolute percent error (MAPE).

$$MaxAPE = \max_{1 \leq k \leq K} \left\{ \frac{|Q_k - \hat{Q}_k|}{Q_k} \cdot 100 \right\} \quad (13)$$

$$MinAPE = \min_{1 \leq k \leq K} \left\{ \frac{|Q_k - \hat{Q}_k|}{Q_k} \cdot 100 \right\} \quad (14)$$

$$MAPE = \frac{1}{K} \sum_{k=1}^K \frac{|Q_k - \hat{Q}_k|}{Q_k} \cdot 100 \quad (15)$$

where K is the total number of intervals during the experiment, Q_k denotes the average traffic flux generated by CORSIM, \hat{Q}_k is the prediction value produced by Lin's model or the proposed model.

Fig. 6 shows the average traffic flux forecasting curve for link 1. As one step (the link is directly connected to the source junction, which variation is known) prediction, the forecasting curve matches quite close to the tendency of the average flux generated by CORSIM, especially during the early 3 hours. As the comparison shows, the traffic peak in CORSIM can be predicted more accurately than Lin's model, see Table II. Furthermore, when traffic flux suddenly drops after the traffic peak, the prediction model can also reflect exactly and timely.

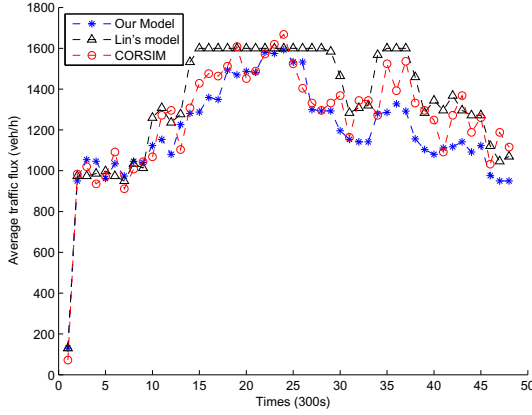


Fig. 6. Prediction of average traffic flux for link 1

TABLE II
ERROR ANALYSIS OF LINK 1

	MaxAPE(%)	MinAPE(%)	MAPE(%)
Lin's Model	23.46	0.50	8.22
Proposed Model	20.03	0.21	7.51

The comparison diagrams of another link 2 located at the center of the UTN is shown in Fig. 7 and Table III. Although the prediction curve does not perform as good as result in link 1, it can represent the tendency of CORSIM to a certain degree.

As the previous experimental results shows, the forecasting curve produced by the proposed macroscopic UTN model can basically match with traffic data generated by microscopic model CORSIM. Furthermore, the computing time for each prediction step is observed to vary between 0.7 and 0.9 CPU seconds by using a personal computer with a 2.8GHz

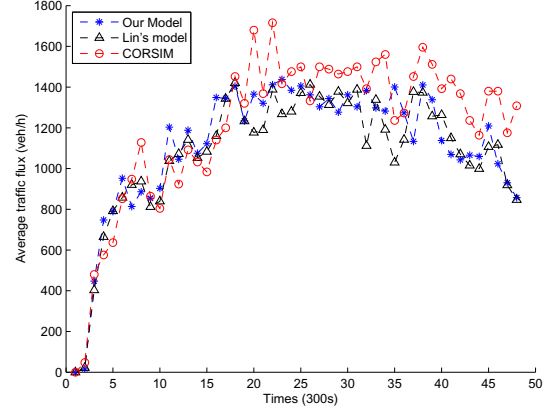


Fig. 7. Prediction of average traffic flux for link 2

TABLE III
ERROR ANALYSIS OF LINK 2

	MaxAPE(%)	MinAPE(%)	MAPE(%)
Lin's Model	35.26	0.64	12.74
Proposed Model	34.47	0.13	11.16

processor and 4GB memory, even though the time is yielded against the backdrop that the sampling time interval equals to 1 second. Obviously, the larger sampling interval can reduce the computing cost. Therefore, it can satisfy the requirement of real-time prediction.

V. CONCLUSIONS

This paper presents a short-term traffic flow prediction approach based on the macroscopic UTN model for urban arteries with fix-timed signalized junctions. Our method is totally on the basis of the macroscopic spatial structure of UTN and the mechanism of traffic flow transmission. In the study of calculation of vehicle delay time from beginning of the link to tail of queue, we employ a simple speed-density model based on triangular shape MFD to obtain more accurate approximation. Finally, experiments compared with CORSIM indicate that the model can predict the short-term traffic flow timely and accurately, and is robust to the suddenly change as well as traffic peaks. Moreover, the proposed macroscopic UTN based approach has lower computation complexity, which is propitious to real time calculation.

Nonetheless, this prediction model requires several known parameters as input variables. Sometimes the exact values of these parameters (e.g. number of the waiting vehicles at the stop line currently, traffic flow feed in the network in the future) are not easy to be obtained, as the result, the model always leads to an inaccurate prediction.

Therefore, the further steps of this study are to acquire more precise queue on the link. and then, to test this model in real urban road network and compare with other prediction methods.

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