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Manual for urGENOVA
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Disclaimer of Warranty

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Abstract

urGENOVA is an ANSI C computer program for estimating random effects variance components for both balanced and unbalanced designs that are complete in the sense that all interactions are included. urGENOVA was created primarily for use in generalizability theory, which typically involves designs with single observations per cell. However, urGENOVA has broader applicability. Specifically, urGENOVA estimates random effects variance components for balanced designs, as well as for designs that are unbalanced with respect to (a) nesting, (b) numbers of observations per cell, and/or (c) empty cells.

In urGENOVA, random effects variance components are estimated using the so-called analogous-ANOVA procedure, which is sometimes called Henderson's (1953) Method 1. No distributional-form assumptions are made. If the design is balanced, the estimates are the usual ANOVA estimates.

The algorithms used in urGENOVA are quite general and do not require operations with large matrices. Consequently, urGENOVA can process very large data sets in a relatively short amount of time.

Introduction

urGENOVA is an ANSI C computer program for estimating random effects variance components for designs that are complete in the sense that all interactions are included. urGENOVA was created primarily for applications in generalizability theory (Cronbach, Gleser, Nanda, & Rajaratnam, 1972; Brennan, 2001a) that involve unbalanced designs with single observartions per cell, although urGENOVA has broader applicability, as discussed later. The notational conventions, equations, algorithms, and procedures used in urGENOVA are described in detail in Brennan (2001a, see especially, Chapter 7). Indeed, urGENOVA is intentionally coordinated with Brennan (2001a).

urGENOVA estimates random effects variance components for balanced designs, as well as for designs that are unbalanced with respect to (a) nesting, (b) numbers of observations per cell, and/or (c) empty cells. These three categories of unbalanced designs are useful primarily for descriptive purposes; theoretically, all unbalanced designs are simply "non-orthogonal."

For generalizability analyses, these capabilities imply, for example, that urGENOVA can handle designs in which different numbers of items are nested within reading passages, as well as crossed or nested designs that have some missing data. urGENOVA was written primarily for designs that are unbalanced with respect to nesting. The control card setups are easier for such designs than for those that involve missing data.

urGENOVA extends GENOVA (Crick & Brennan, 1983; see also Brennan, 1992, Appendix E, and Brennan, 2001a, Appendix F) in the sense that urGENOVA can handle unbalanced designs.[1] However, urGENOVA estimates G study random effects variance components, only; it has no D study capabilities. (See Brennan, 2001a, Section 7.2 for a discussion of D study issues with unbalanced designs.) In these senses, urGENOVA is more restrictive than GENOVA.

mGENOVA (Brennan, 2001b), a program for multivariate generalizability analyses, can also estimate random effects variance components, but only for a selected set of designs. In addition, for these designs, mGENOVA can estimate certain D study results.

The primary theoretical problem presented by most unbalanced designs is that there are many possible estimators of random effects variance components and no unambiguously clear basis for determining which estimators are best. Among the practical problems with unbalanced designs are that some estimation methods require distributional form assumptions, which are often difficult to justify in generalizability analyses.

In urGENOVA, random effects variance components are estimated using the so-called analogous-ANOVA procedure (see, Brennan, 2001a, Section 7.1; Searle, 1971, p. 425ff.; Searle, Casella, & McCulloch, 1992, p. 181ff.). No

[1] For balanced designs, urGENOVA and GENOVA give identical results if there are no negative estimates. If there are negative estimates, results are identical if ALGORITHM is specified in the GENOVA control cards.

distributional-form assumptions are made. If the design is balanced, the estimates are the usual ANOVA estimates. Sometimes these estimates are also referred to as Henderson's (1953) Method 1 estimates. For at least some designs that are unbalanced with respect to nesting, Henderson's Methods 1 and 3 give the same results (see Jarjoura & Brennan, 1981). It is not claimed that ANOVA estimates are "best" in some compelling theoretical sense, but their lack of distributional-form assumptions is a distinct advantage for use in generalizability theory.

Another practical limitation of many estimation methods for unbalanced designs of the type encountered in generalizability theory (single observations per cell) is that they are often implemented using matrices on the order of size $N \times N$, where N is the number observations in the data. With many thousands of observations, which is quite common in large-scale testing programs, working with such large matrices is always difficult, and sometimes prohibitive, in terms of required memory and/or processing time. Some elegant algorithms permit using matrices smaller that $N \times N$, but any matrix-based estimation procedure is likely to encounter very real practical problems in many, if not most, generalizability theory applications.

The algorithms, equations, and procedures used in urGENOVA are quite general. Among them are equations for computing T terms (i.e., so-called "uncorrected sums of squares"), an algorithm for computing sums of squares, and procedures for determining the expected values of T terms and expected mean squares. None of these algorithms or procedures require working with $N \times N$ matrices, and urGENOVA can process an almost unlimited number of observations very rapidly. Even analyses with several thousand observations require only a few seconds of processing time with typical personal computers.

Output includes a banner (optional), echoing the control cards, a list of some or all of the input data records (optional), means for main effects, an ANOVA table containing estimated random effects variance components, expected values of T terms (optional), expected mean squares (optional), estimated standard errors and confidence intervals for estimated variance components (optional and appropriate only for balanced designs under normality assumptions), and processing time (optional).

urGENOVA is available for both Macintosh Power PCs and Windows-based PCs. To execute the urGENOVA Macintosh application, the user double-clicks the urGENOVA icon. urGENOVA then prompts the user for the name of the file containing the control cards. *This file must be a text file.* The input file should be in the same folder as the urGENOVA application, or the full pathname must be specified. After the user types a return, urGENOVA completes execution. The same process applies to executing urGENOVA.exe on a Windows-based PC but, again, the input file must be a text file. For both the Mac and PC versions, it is strongly suggested that white space between parameters in the input file be generated by typing spaces rather than tabs. Sometimes, for example, one tab appears as if it is equivalent to two or more spaces, but one tab is read as one unit of white space.

Table 1: Illustrative Control Cards

```
COLUMNS  1111111111222222222233333333334444444444555555555
123456789012345678901234567890123456789012345678901234567890123456789

GSTUDY   Example: (p:c) x (i:r) Design
OPTIONS  NREC 4  "*.out" EMS TIME
EFFECT     c     2
EFFECT   * p:c   4 6
EFFECT     r     3
EFFECT     i:r   3 6 3
FORMAT   3 2
PROCESS  "Dummy data pstar"
```

Control Cards

All runs of urGENOVA require a file containing a set of control cards, and a data set that immediately follows the control cards or is external to them (i.e., in a different file). An illustrative set of control cards is in Table 1.

Each control card begins with a card identifier, with parameters starting in column 10. All parameters are separated from each other by any number of spaces and/or tabs. Unless otherwise specified, the order in which parameters are provided is fixed, and they must occur somewhere in columns 10–99, inclusive. All control cards should end with a newline character, which is generated by typing a return.[2]

The urGENOVA control cards have an appearance similar to those for GENOVA, but the user is warned that the control card conventions for the two programs are not identical. The urGENOVA control cards must occur in the order that they are discussed below. All of them are required except the COMMENT card(s) and the OPTIONS card.

Unless otherwise indicated, card identifiers and parameters should be input as indicated in uppercase letters.

GSTUDY Card

GSTUDY in columns 1–6.

title an alphanumeric title that may be placed anywhere in columns 10–99, inclusive. For purposes of centering the title in the output, the title is taken to extend from column 10 to the newline

[2]Strictly speaking, each control card is terminated by any uninterrupted sequence of newline and/or return characters, but it is simplest to use only a single newline character.

character that is generated by typing a return. Even if no title is provided, the GSTUDY card must be present. urGENOVA does not interpret any information provided in the title. So, for example, in the illustrative control cards in Table 1, the text in *title* that specifies the design $[(p\!:\!c) \times (i\!:\!r)]$ is nothing more than uninterpreted text to urGENOVA.

COMMENT Card(s)

COMMENT in columns 1–6 followed by any text.

Any number of COMMENT cards (including none) are permitted, but all COMMENT cards must occur immediately after the GSTUDY card.

OPTIONS Card

OPTIONS in columns 1–7.

NREC d — the characters "NREC" followed by an integer (d) designate that the first d input records should be printed in the output. It is suggested that users print out at least one record to make sure that urGENOVA is reading the data as intended.

outname — a filename enclosed in double quotes that is used to identify the file for printed output. If *outname* is not provided, the system's standard output file is used. If *outname* begins with *, the output filename is the concatenation of the control cards filename and the characters immediately following the * in *outname*. For example, if the illustrative control cards in Table 1 were in a file named "cc.manual" (without the quotes), then the output filename would be "cc.manual.out" (without the quotes). If *outname* is provided without using a full pathname, the output file is in the same folder as the urGENOVA application. This output file can be opened with any text editor or word processor.

ET — means that the expected T term equations should be printed.

EMS — means that expected mean square equations should be printed.

SECI γ — indicates that standard errors and Ting et al. (1990) $100\gamma\%$ confidence intervals should be reported for the estimated variance components. Note that γ should be a floating point number between 0.0 and 1.0, exclusive. This output is appropriate only for balanced designs under normality assumptions.

SAT — indicates that Satterthwaite $100\gamma\%$ confidence intervals should be reported as well as Ting et al. (1990) intervals. This parameter is functional only if "SECI γ" is specified.

TIME means that the time and date when processing began should be printed along with the number of seconds of processing time (truncated value) required for execution.

NOBANNER indicates that the urGENOVA banner should not be printed.

Note that the OPTIONS card may be absent, and the order of the fields in the OPTIONS card is irrelevant.

EFFECT Cards

urGENOVA requires as many EFFECT cards as there are main effects in the design. In the terminology of generalizability theory, this is one more than the number of facets. As many as six EFFECT cards may be present in any run of urGENOVA. Exactly one of the EFFECT cards must begin with an asterisk (*) that designates the effect (or facet) associated with records in the input data set. The order of the EFFECT cards is crucial and must correspond to the order in which the data are provided to urGENOVA. The effect associated with the slowest moving subscript should be first, the effect associated with the next slowest moving subscript should be second, etc. This aspect of the control cards conventions in urGENOVA is the same as that in GENOVA.

EFFECT in columns 1–7.

[*] One and only one EFFECT card must begin with a single asterisk. urGENOVA goes to the next physical record in the input data set each time the main effect index associated with this effect changes its value (see discussion later in this manual).

facet This is a sequence of letters and colons designating a main effect in the manner described in Brennan (2001a, Section 3.2.1), which employs the same conventions as in GENOVA. The first character (or index) must be a single alphanumeric character. This first character is sometimes referred to as the main effect index. The alphanumeric characters are case-sensitive which means, for example, that "a" and "A" are different. (Alphanumeric characters are *not* case sensitive in GENOVA.)

$n_1[n_2 \ldots]$ These are integer(s) designating sample size(s). If *facet* is a single non-nested main effect, only one sample size is required, which is the number of levels for the effect. If *facet* is a nested main effect, the number of sample sizes required equals the product of the sample sizes for all nesting indices in *facet*. Usually, all sample sizes are provided in the same record immediately after *facet*, and sample sizes may occur beyond column 99. However, sample sizes may be provided on a subsequent record or records. So, the EFFECT cards in Table 1 could be specified as:

```
EFFECT     c     2
EFFECT   * p:c   4
6
EFFECT     r     3
EFFECT     i:r
3
6
3
```

It is crucial that the user specify the correct number of sample sizes (and, of course, the correct sample sizes, too).

FORMAT Card

FORMAT	in columns 1–6.
skip	number of characters to skip before reading data.
width	field width for reading data in fixed format. If *width* = 0, urGENOVA assumes all data values are separated by white space (blanks, horizontal and vertical tabs, newlines, and form-feeds).

See subsequent discussion of "Input Data Records" for more information about these parameters.

PROCESS Card

PROCESS	in columns 1–7.
records	a filename enclosed in double quotes containing the input data records. The *records* filename can occur anywhere in columns 10-99, inclusive. This file must be in the same folder as the urGENOVA application, if a full pathname is not provided as the filename. If no filename is provided, urGENOVA assumes the input data records immediately follow the PROCESS card.

Input Data Records

urGENOVA reads data from the input data set (records file) based on (a) the order of the EFFECT cards; (b) the position of the starred effect; and (c) the *skip* and *width* specifications in the FORMAT card. The EFFECT card associated with the slowest moving main effect index should be specified first, the EFFECT card associated with the next slowest moving main effect index should be second, etc. This aspect of the control cards conventions in urGENOVA is the same as that in GENOVA.

The main effect index preceded by an asterisk (i.e., the starred effect) is associated with a record in the input data set. All the observations for a single

Table 2: "Dummy data pstar"

```
COLUMNS  1111111111222222222233333333
1234567890123456789012345678901234567 8

    5 6 5 5 5 3 4 5 6 7 3 3  Record  1
    7 1 5 5 5 3 3 3 5 5 3 0  Record  2
    3 4 3 3 5 3 3 5 6 5 1 6  Record  3
    7 5 5 3 3 1 4 3 5 3 3 5  Record  4
    9 2 9 7 7 7 3 7 2 7 5 3  Record  5
    3 4 3 5 3 3 6 3 4 5 1 2  Record  6
    7 3 7 7 7 5 5 7 5 5 5 4  Record  7
    5 8 5 7 7 5 5 4 3 2 1 1  Record  8
    9 9 8 8 6 6 6 5 5 8 1 1  Record  9
    4 4 4 3 3 5 6 5 5 7 1 1  Record 10
```

level of the starred effect should be contained in a single logical record. When this number of observations has been read, the index for the starred effect is incremented, and urGENOVA goes to the next record. It follows that the number of records read is associated with the sample sizes in the EFFECT cards prior to, and including, the starred effect. Each record should be terminated by a newline character, which is generated by typing a return.[3]

Suppose, for example, that the dummy data in Table 2 are used with the illustrative set of control cards in Table 1. Recall that the EFFECT cards are:

```
EFFECT     c     2
EFFECT   * p:c   4 6
EFFECT     r     3
EFFECT     i:r   3 6 3
```

The second main effect index, p, is associated with the starred effect. (The main effect per se is $p{:}c$, but the second main effect index is simply p.) The first two EFFECT cards jointly indicate that there should be 10 records, corresponding to the $4 + 6 = 10$ levels of the main effect index p (4 in the first level of c and 6 in the second level of c). Since $skip = 3$, three characters are skipped before reading data. Since $width = 2$, the data are read in fields with a fixed width of two characters.

The last two EFFECT cards (those that immediately follow the starred effect) indicate that $3 + 6 + 3 = 12$ observations should be read from each record. The first 3 observations are associated with the first level of r, the

[3]Strictly speaking, each control card is terminated by any uninterrupted sequence of newline and/or return characters, but it is simplest to use only a single newline character.

Table 3: Dummy Data Respecified

```
COLUMNS  1111111111222
1234567890123456789012

Record  1 565553456733
Record  2 715553335530
Record  3 343353356516
Record  4 755331435335
Record  5 929777372753
Record  6 343533634512
Record  7 737775575554
Record  8 585775543211
Record  9 998866655811
Record 10 444335655711
```

next 6 with the second level of r, and the last 3 with the third level of r. The characters following column 27 (3 skipped characters plus 12 fields of width 2) in each record are ignored.

The ordering of the EFFECT cards indicates that c is the slowest moving index, p is the next slowest, r is the next slowest, and i is the fastest moving index. This ordering of slowest-to-fastest moving index means that:

$$\begin{array}{cccccc}
c_1p_1r_1i_1 = 5 & c_1p_1r_1i_2 = 6 & c_1p_1r_1i_3 = 5 & c_1p_1r_2i_1 = 5 & \cdots & c_1p_1r_3i_3 = 3 \\
c_1p_2r_1i_1 = 7 & c_1p_2r_1i_2 = 1 & c_1p_2r_1i_3 = 5 & c_1p_2r_2i_1 = 5 & \cdots & c_1p_2r_3i_3 = 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
c_2p_6r_1i_1 = 4 & c_2p_6r_1i_2 = 4 & c_2p_6r_1i_3 = 4 & c_2p_6r_2i_1 = 3 & \cdots & c_2p_6r_3i_3 = 1
\end{array}$$

Since the data values in Table 2 are separated by a space, the FORMAT card could have been specified with $width = 0$. In that case, urGENOVA would simply skip a space before reading a value.

Suppose the dummy data set were specified as in Table 3. In this case, the parameters on the FORMAT card should be specified as $skip = 10$ and $width = 1$.

The illustrative data in Tables 2 and 3 are single-digit integers. This is not a restriction for urGENOVA. The data may consist of integers or floating-point numbers. (Each observation is read using a format that provides approximately six decimal digits of accuracy for the internal representation of the data.)

Strictly speaking, if $width = 0$, the observations for a logical record (i.e., a record implied by the position of the starred effect) need not be in a single physical record (i.e., a record terminated by a newline character). If the obser-

Table 4: Dummy Data with two Physical Records for each Logical Record

```
COLUMNS  11111111112222
12345678901234567890123

Record  1   5 6 5 5 5 3
 4 5 6 7 3 3
Record  2   7 1 5 5 5 3
 3 3 5 5 3 0
Record  3   3 4 3 3 5 3
 3 5 6 5 1 6
Record  4   7 5 5 3 3 1
 4 3 5 3 3 5
Record  5   9 2 9 7 7 7
 3 7 2 7 5 3
Record  6   3 4 3 5 3 3
 6 3 4 5 1 2
Record  7   7 3 7 7 7 5
 5 7 5 5 5 4
Record  8   5 8 5 7 7 5
 5 4 3 2 1 1
Record  9   9 9 8 8 6 6
 6 5 5 8 1 1
Record 10   4 4 4 3 3 5
 6 5 5 7 1 1
```

vations for a logical record have not all been read when the newline character is encountered, urGENOVA looks for the next value at the beginning of the next record. So, for example, with *skip* = 9 and *width* = 0, the dummy data could be provided to urGENOVA as in Table 4.

Note that the data in Table 4 would not be read correctly with *skip* = 11 and *width* = 2, because the newline character terminating each physical record would cause the data for the seventh value in each logical record to be read incorrectly. That is, with a fixed length field (*width* > 0), the newline character is not skipped in reading data and all observations for a logical record must be in a single physical record.

Output

Appendix A provides the output for the illustrative control cards and dummy data in Tables 1 and 2, respectively. Each section of the output contains a descriptive heading and the title from the GSTUDY card. The first part of the output echoes the control cards exactly as provided to urGENOVA. Note that the second line in the heading preceding the list of control cards is:

```
Control Cards File Name:  cc.manual
```

This means that the user (in this case, the author of urGENOVA) specified that the file containing the control cards was "cc.manual" (without the quotes). Since the OPTIONS card specifies "*.out" (without the quotes) the output is directed to a file named "cc.manual.out" (without the quotes) in the same folder as "cc.manual" (without the quotes).

Since "NREC 4" is specified in the OPTIONS card, the next section of the output provides the first four records of the input data set. Ten observations are printed per line.

Then, the output provides means for the levels of each of the main effects. Ten means are printed per line. The means are followed immediately by an ANOVA table. The ANOVA table lists the effects (using the notation in Brennan, 2001a), degrees of freedom (df), uncorrected sums of squares (T), sums of squares (SS), mean squares (MS), and G study estimated random effects variance components (VC). Immediately below the ANOVA table is the grand mean over all observations in the input data set.

Since "EMS" is specified in the OPTIONS card, the expected mean square equations are provided. Because "TIME" is specified in the OPTIONS card, the output ends with the date and time that processing began, and the number of seconds (truncated) required for execution. In the sample output, 0 seconds means that less than one second was required for execution.

Multiple Runs

Multiple runs of urGENOVA can be performed in a single job by providing multiple sets of control cards in the control cards file. After processing the data for any given run, urGENOVA immediately looks for a new set of control cards beginning with a GSTUDY card. Execution terminates when no new GSTUDY card is found. The multiple runs capability of urGENOVA is the reason why the output headings in Appendix A specify "RUN 1".

Appendix B provides the control card setups for four runs of urGENOVA, each of which produces the same estimated random effects variance components as in Appendix A. The first set of control cards (Run 1) is identical to those in Table 1, except for a slight change in the heading and the order of the parameters in the OPTIONS card.

In the second set of control cards (Run 2) the first main effect, c, is the starred effect. The data set immediately follows the PROCESS card, and it is identical to the data set in Table 2. Since c is the starred effect in the second run, and there are two levels of c, urGENOVA will read two logical records—one with $4 \times 12 = 48$ observations and one with $6 \times 12 = 72$ observations. Since each physical record in the second run contains 12 observations, urGENOVA will read the first four physical records to get the observations for the first level of c, and the next six physical records to get the observations for the second level of c. For example, after reading the 12th observation from the first physical record, since $width = 0$ in the FORMAT card, urGENOVA reads past the newline character to the second physical record in order to read the 13th observation for the first level of c.

The principal difference between the setups for Runs 1 and 2 is that, for Run 1, a logical record coincides with a physical record, whereas for Run 2, a logical record involves multiple physical records. Of course, the data set for Run 2 could have been specified with the first 48 observations on one physical record and the last 72 observations on another physical record. Then a logical record would coincide with a physical record.

In the third set of control cards (Run 3) the third main effect, r, is the starred effect. Note that the input data set has three observations in the first physical record, six in the second, and three in the third—and this pattern is repeated in the rest of the data set. The first physical record provides the observations for the three levels of i nested within the first level of r. The second physical record provides the observations for the six levels of i nested within the second level of r. The third physical record provides the observations for the three levels of i nested within the third level of r. In other words, a new record is required each time the level of r changes.

In the fourth set of control cards (Run 4) the fourth main effect index, i, is associated with the starred effect. (The main effect per se is $i{:}r$, but the fourth main effect index is simply i.) A new record is required each time the level of i changes. Since i is the fastest moving subscript, there is a new level of i for each observation, and there must be one observation per record.

Standard Errors and Confidence Intervals for Balanced Designs

If the OPTIONS card contains "SECI" followed by a confidence coefficient (a floating point number between 0.0 and 1.0, exclusive), and the design is balanced, urGENOVA provides a table of standard errors and confidence intervals for each of the estimated variance components. These estimated standard errors and confidence intervals assume that score effects have a multivariate normal distribution. Note that this is the only segment of output that requires normality assumptions.

Consider, for example, the control cards in Table 5 on the following page, based on Synthetic Data Set Number 4 in Brennan (2001a, p. 73). The output for this run of urGENOVA is given in Appendix C. The squares of the standard errors (SEVC) are unbiased estimates of the variances of the estimated variance components, assuming score effects have a multivariate normal distribution. These standard errors are based on using $df + 2$ in the sense discussed by Searle (1971, p. 417) and Brennan (2001a, Section 6.1.1). The ratio reported in the fourth column is VC/SEVC. All things considered, large values for ratio signify more trustworthy estimated variance components than small values of ratio.

Table 5: An Illustrative Balanced Design

```
COLUMNS  111111111122222222223333333333444444444455555555556666666666
123456789012345678901234567890123456789012345678901234567890123456789

GSTUDY   Example for p x (i:r) Design for Synthetic Data Set Number 4
OPTIONS  NREC 5  "*.out" EMS  SECI .8  NOBANNER
EFFECT   * p    10
EFFECT     r     3
EFFECT     i:r   4 4 4
FORMAT   0 2
PROCESS
 5 6 5 5 5 3 4 5 6 7 3 3
 9 3 7 7 7 5 5 5 7 7 5 2
 3 4 3 3 5 3 3 5 6 5 1 6
 7 5 5 3 3 1 4 3 5 3 3 5
 9 2 9 7 7 7 3 7 2 7 5 3
 3 4 3 5 3 3 6 3 4 5 1 2
 7 3 7 7 7 5 5 7 5 5 5 4
 5 8 5 7 7 5 5 4 3 2 1 1
 9 9 8 8 6 6 6 5 5 8 1 1
 4 4 4 3 3 5 6 5 5 7 1 1
```

The default procedure for confidence intervals in urGENOVA is the procedure developed by Ting et al. (1990). It is succinctly discussed by Burdick and Graybill (1990, pp. 36–42) and summarized by Brennan (2001a, Section 6.2.3). Confidence intervals for the Ting et al. (1990) procedure are provided in the last two columns. These are approximate confidence intervals under the assumption of multivariate normality of the score effects.

If "SAT" is specified in the OPTIONS card, then confidence intervals are also provided based on Satterthwaite's (1941, 1946) procedure, as discussed by Brennan (2001a, Section 6.2.2). The Satterthwaite intervals are provided immediately below the Ting et al. intervals.

If "SECI" is specified in the OPTIONS card and the design is unbalanced, these standard errors and confidence intervals are not provided, because their appropriateness is highly questionable even under normality assumptions. The principal problem is that analogous sums of squares for unbalanced designs are generally not independent (see Searle, 1971, p. 433ff), but the methods used in urGENOVA to estimate standard errors and confidence intervals assume independence.

Different Numbers of Observations Per Cell and Empty Cells

Although urGENOVA was developed primarily for use with designs that are unbalanced with respect to nesting, it can handle designs that are unbalanced in the sense of different numbers of observations per cell and/or missing data. The programming logic that enables doing so is based on the nested effects notation used in urGENOVA. This is explained most easily in terms of examples.

Table 6 on the next page provides the control card setup for a run based on a synthetic data example in Searle et al. (1992, pp. 189–190). This is a 2×3 factorial design with different numbers of observations per cell (2, 1, and 1 for the cells in the first row; 1, 2, and 1 for the cells in the second row). The cell sample sizes are provided to urGENOVA in:

```
EFFECT r:ph 2 1 1 1 2 1
```

This feature of urGENOVA is nothing more than a use of the nested effect notational conventions.

The output for this run is in Appendix D. Note that the $r\!:\!ph$ effect is placed in the last row of the output after the ph interaction. This is done for consistency with the usual ordering of effects when there are multiple observations per cell. In conventional statistical texts the $r\!:\!ph$ effect is usually identified as e, or the residual.

Table 7 on the following page provides the control card setup for the same design as in Table 6, except that one of the cells is declared to be empty through specifying 0 observations in the $r\!:\!ph$ EFFECT card:

```
EFFECT r:ph 2 0 1 1 2 1
```

The output is in Appendix E. Note that, since h is the starred effect, there must be a record for the second cell in the first row, even though that cell contains no observations, and no data are read for that cell. Recall that urGENOVA goes to a new record each time there is a change in the level of the starred effect. After reading the two observations for the first cell in the first row, urGENOVA goes to the next record and discovers that no observations are to be read. It then increments the value of h and moves to the following record to read the single observation for the third cell in the first row.

The designs in Tables 6 and 7 are relatively common in traditional statistical contexts, but they seldom occur in generalizability theory. They might be called designs with replications (r) within cells. By contrast, the designs in generalizability theory typically have one observation per cell. When data are missing in a generalizability analysis, in the sense of one or more empty cells, the levels of the facet associated with the empty cell are often discarded to make the design balanced. Doing so, greatly simplifies estimation, and probably is not too problematic if only a small amount of data is discarded and sample sizes are

Table 6: Searle et al. (1992, pp. 189–190) (Output in Appendix D)

```
COLUMNS  11111111112222222222333333333344444444445555555555666666
123456789012345678901234567890123456789012345678901234567890123456

GSTUDY   r:(p x h) Design
COMMENT    From Searle, Casella, and McCulloch (1992, pp. 189-190)
COMMENT    Note that Searle et al. have an error for the first
COMMENT    estimated variance component.  They report
COMMENT    -454.6/121 = -1.6909 which is obviously incorrect.
OPTIONS  NREC 6  "*.out"  ET  EMS  TIME  NOBANNER
EFFECT     p      2
EFFECT   * h      3
EFFECT     r:ph   2 1 1 1 2 1
FORMAT   0 0
PROCESS
    7   9   p1 h1
    6       p1 h2
    2       p1 h3
    8       p2 h1
    4   8   p2 h2
   12       p2 h3
```

Table 7: Searle et al. (1992, pp. 189–190) with Empty Cell (Output in Appendix E)

```
COLUMNS  11111111112222222222333333333344444444445555555555666666
123456789012345678901234567890123456789012345678901234567890123456

GSTUDY   r:(p x h) Design with one empty cell
COMMENT    From Searle, Casella, and McCulloch (1992, pp. 189-190)
COMMENT    excluding the data for cell (p1,h2)
OPTIONS  NREC 6  "*.out"  ET  EMS  TIME  NOBANNER
EFFECT     p      2
EFFECT   * h      3
EFFECT     r:ph   2 0 1 1 2 1
FORMAT   0 0
PROCESS
    7   9   p1 h1
    6       p1 h2  NOT READ BUT RECORD MUST BE PRESENT
    2       p1 h3
    8       p2 h1
    4   8   p2 h2
   12       p2 h3
```

large. However, if sample sizes are relatively small, investigators are justifiably uncomfortable discarding data.

Using the type of control card setup in Table 7, urGENOVA can estimate variance components when some of the cells are empty in a generalizability analysis. For example, Table 8 on the next page provides the control card setup for a $p \times i$ design in which the data are dichotomous and some of the data are missing. The example comes from a paper by Huynh (1977). The output is in Appendix F. For the example in Table 8, the replication effect r is not a facet, as that term is used in generalizability theory. Rather, the effect r is used simply to index which cells are empty. For this reason, there is no $r\!:\!pi$ effect reported in the output in Appendix F, and the degrees of freedom for pi is reduced by the number of empty cells.

The meaning of the 0's and 1's in the $r\!:\!pi$ EFFECT card should not be confused with the meaning of the 0's and 1's in the data. The latter indicate correct (1) and incorrect (0) responses; the former indicate that data are present (1) or missing (0). So, for example, the response of the third person to the first item is missing (0 in the $r\!:\!pi$ EFFECT card). Therefore, urGENOVA will not attempt to read data for the response of the third person to the first item. Given the ordering of the data and the position of the starred effect (p), this means that there should be no data provided for the response of the third person to the first item. The appearance of the data set in Table 8 makes it clear which data are missing through the use of spaces where data otherwise would be. This is permissible because the FORMAT card specifies $width = 0$. The data could have been specified as provided below, but doing so masks the location of the missing data.

```
1 1 1 1 0 1
1 0 1 1 1 1
1 1 1 0 0
0 1 0 1
1 0 1 0
1 1 1 1 0 1
0 1 0 0
0 1 1 1 1
1 1 0 1 0
0 0 0 0
1 1 1 1 1 1
0 0 0 1
```

The crucial points are: (a) the location of missing data is designated solely through the location of 0's in the $r\!:\!pi$ EFFECT card; and (b) urGENOVA does not attempt to read any missing data. Table 9 on page 17 provides a missing data example for the more complicated $p \times (i\!:\!h)$ design. The output is in Appendix G.

The missing-data capability of urGENOVA should not be overused. It is not intended for situations in which there are large amounts of missing data.

Table 8: Huynh (1977, p. 14) (Output in Appendix F)

```
COLUMNS  111111111122222222223333333333444444444455555555556666666666
123456789012345678901234567890123456789012345678901234567890123456789

GSTUDY   p x i design with missing data -- from Huynh (1977, p. 14)
OPTIONS  NREC 12 "*.out" ET EMS TIME NOBANNER
EFFECT   * p    12
EFFECT     i     6
EFFECT     r:pi  1 1 1 1 1 1
                 1 1 1 1 1 1
                 0 1 1 1 1 1
                 1 0 0 1 1 1
                 1 0 1 1 1 0
                 1 1 1 1 1 1
                 1 0 1 0 1 1
                 1 0 1 1 1 1
                 1 1 1 1 0 1
                 0 1 1 1 0 1
                 1 1 1 1 1 1
                 0 1 0 1 1 1
FORMAT   0 0
PROCESS
 1 1 1 1 0 1
 1 0 1 1 1 1
   1 1 1 0 0
 0     1 0 1
 1   0 1 0
 1 1 1 1 0 1
 0   1   0 0
 0   1 1 1 1
 1 1 0 1   0
   0 0 0   0
 1 1 1 1 1 1
   0   0 0 1
```

Other Issues, Designs, and Capabilities

For most, if not all, practical purposes urGENOVA can process an unlimited amount of data. Strictly speaking, however, the number of levels for a facet cannot exceed the maximum value of C's "int" type. This can vary depending on the machine, but a common value is 32,767.

urGENOVA makes use of several functions in Press, Teukolsky, Vetterling, and Flannery (1992).

Table 9: $p \times (i\!:\!h)$ Design with Missing Data (Output in Appendix G)

```
COLUMNS  111111111122222222223333333333444444444455555555556666
123456789012345678901234567890123456789012345678901234567890123

GSTUDY   p x (i:h) Design as r:[p x (i:h)] with six empty cells
OPTIONS  NREC 12  "*.out" EMS ET  NOBANNER
EFFECT   * p      10
EFFECT     h       3
EFFECT     i:h     3 6 3
EFFECT     r:pi:h  1 1 1 1 1 1 1 0 1 1 1 1
                   1 1 1 0 1 1 1 1 1 1 1 1
                   1 1 1 1 1 1 1 1 1 0 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
                   1 1 1 1 1 0 1 1 1 1 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
                   1 1 0 1 1 1 1 1 1 1 1 1
                   1 1 1 1 1 1 1 0 1 1 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
FORMAT   0 0
PROCESS
  5 6 5 5 5 3 4   6 7 3 3
  7 1 5   5 3 3 3 5 5 3 0
  3 4 3 3 5 3 3 5 6   1 6
  7 5 5 3 3 1 4 3 5 3 3 5
  9 2 9 7 7   3 7 2 7 5 3
  3 4 3 5 3 3 6 3 4 5 1 2
  7 3   7 7 5 5 7 5 5 5 4
  5 8 5 7 7 5 5   3 2 1 1
  9 9 8 8 6 6 6 5 5 8 1 1
  4 4 4 3 3 5 6 5 5 7 1 1
```

Simple Matrix Sampling Designs

The nested effects notation employed in urGENOVA permits processing simple matrix sampling designs, such as $(p \times i):r$. An illustrative control card setup is provided in Table 10, and the output is in Appendix H. Note that urGENOVA cannot process matrix sampling designs with structures that are more complicated than $(p \times i):r$.

Table 10: Simple Matrix Sampling Design (Output in Appendix H)

```
COLUMNS  11111111112222222222333333333344444444445555
1234567890123456789012345678901234567890123456789012345

GSTUDY   Matrix Sampling Design (p x i):r
COMMENT ********************************************
COMMENT Same first nesting index for two main effects
COMMENT ********************************************
OPTIONS  NREC 6  "*.out"  ET EMS NOBANNER
EFFECT      r  3
EFFECT    * p:r    6  8 10
EFFECT      i:r    5 10 15
FORMAT   0 2
PROCESS
 5 6 5 5 5
 3 4 3 3 5
 9 2 9 7 7
 7 3 7 7 7
 9 9 8 8 6
 5 6 5 5 5
 3 4 3 3 5 3 3 5 6 5
 9 2 9 7 7 7 3 7 2 7
 7 3 7 7 7 5 5 7 5 5
 9 9 8 8 6 6 6 5 5 8
 5 6 5 5 5 3 4 5 6 7
 3 4 3 3 5 3 3 5 6 5
 9 2 9 7 7 7 3 7 2 7
 7 3 7 7 7 5 5 7 5 5
 9 9 8 8 6 6 6 5 5 8 1 1 4 4 4
 5 6 5 5 5 3 4 5 6 7 3 3 7 1 5
 3 4 3 3 5 3 3 5 6 5 1 6 7 5 5
 9 2 9 7 7 7 3 7 2 7 5 3 3 4 3
 7 3 7 7 7 5 5 7 5 5 5 4 5 8 5
 9 9 8 8 6 6 6 5 5 8 1 1 4 4 4
 5 6 5 5 5 3 4 5 6 7 3 3 7 1 5
 3 4 3 3 5 3 3 5 6 5 1 6 7 5 5
 9 2 9 7 7 7 3 7 2 7 5 3 3 4 3
 7 3 7 7 7 5 5 7 5 5 5 4 5 8 5
```

Table 11: Deleting Records Example

```
COLUMNS  1111111111222222222233333333334444444444555555555
12345678901234567890123456789012345678901234567890123456 7
```

```
GSTUDY   p x (i:r) Design for Synthetic Data Set Number 4
OPTIONS  NREC 5  "*.out" EMS ET  SECI .8  DSUM 0
EFFECT   * p    13
EFFECT     r     3
EFFECT     i:r   4 4 4
FORMAT   0 2
PROCESS
 5 6 5 5 5 3 4 5 6 7 3 3
 9 3 7 7 7 5 5 5 7 7 5 2
 3 4 3 3 5 3 3 5 6 5 1 6
 0 0 0 0 0 0 0 0 0 0 0 0
 7 5 5 3 3 1 4 3 5 3 3 5
 9 2 9 7 7 7 3 7 2 7 5 3
 3 4 3 5 3 3 6 3 4 5 1 2
 0 0 0 0 0 0 0 0 0 0 0 0
 7 3 7 7 7 5 5 7 5 5 5 4
 5 8 5 7 7 5 5 4 3 2 1 1
 9 9 8 8 6 6 6 5 5 8 1 1
 4 4 4 3 3 5 6 5 5 7 1 1
 0 0 0 0 0 0 0 0 0 0 0 0
```

Deleting Records

urGENOVA has a limited capability for deleting records that have a user-specified sum, provided the starred effect is non-nested. This feature is enabled by specifying DSUM followed by an integer or floating point number ("sum") in the OPTIONS card. Records are eliminated that have total scores (over all facets following the starred effect) equaling "sum." An illustrative control card setup is provided in Table 11. Note that the sample size for effect p is 13 because there are indeed 13 records, even though only 10 of them are valid (in the sense that their sums are not 0). The numerical output for this run is the same as that in Appendix C.

The DSUM options should be used sparingly. It is not a terribly efficient way to preprocess data, and it involves use of temporary storage that is almost as large as the size of the data set.

D Studies

It is important to remember that random effects variance components per se characterize the population and universe; their interpretation has nothing to do with unbalanced designs. Rather, the fact that a data collection design is unbalanced creates complexities in estimating these variance components.

The estimated variance components from urGENOVA can be used as input to GENOVA using GENOVA's VCOMPONENT control cards, and then GENOVA's D study output can be obtained. This is a sensible way to proceed when unbalanced data do not occur by design. In such cases, an investigator's principal interest is in a measurement procedure characterized by a balanced design.

Sometimes, however, data are indeed unbalanced by design. This is often the case for tests in which different number of items are associated with different reading passages. Presumably, a new form (i.e., replication) of such tests would involve the same (or at least a similar) unbalanced pattern. In such cases, the procedure for estimating D study results mentioned in the previous paragraph is not sensible. Brennan (2001a, Section 7.2) treats such matters in some detail.

Appendix A
Output for Control Cards and Dummy Data in Tables 1 and 2

```
****************************************************************************************
****************************************************************************************


                 GGGGGGG     EEEEEEEE    N       N     OOOOO      V       V       AAA
                G       G    E           N N     N   O       O    V       V      A   A
U   U  RRRR     G            E           N  N    N   O       O    V       V     A     A
U   U  R   R    G            EEEEE       N   N   N   O       O    V       V    AAAAAAAAA
U   U  RRRR     G     GGG    E           N    N  N   O       O     V     V     A       A
U   U  R  R     G       G    E           N     N N   O       O      V   V      A       A
 UUU   R   R     GGGGGGG     EEEEEEEE    N       N     OOOOO         V        A       A


                Version 2.1                           Robert L. Brennan
                 June 2001                          Iowa Testing Programs
            All Rights Reserved                       University of Iowa

****************************************************************************************
****************************************************************************************


                             CONTROL CARDS FOR RUN 1
                      Control Cards File Name:  cc.manual
                         Example: (p:c) x (i:r) Design

GSTUDY   Example: (p:c) x (i:r) Design
OPTIONS  NREC 4  "*.out" EMS TIME
EFFECT     c     2
EFFECT   * p:c   4 6
EFFECT     r     3
EFFECT    i:r    3 6 3
FORMAT   3 2
PROCESS  "Dummy data pstar"



                             INPUT RECORDS FOR RUN 1
                         Example: (p:c) x (i:r) Design

RECORD NUMBER 1:
  5.000   6.000   5.000   5.000   5.000   3.000   4.000   5.000   6.000   7.000
  3.000   3.000
RECORD NUMBER 2:
  7.000   1.000   5.000   5.000   5.000   3.000   3.000   3.000   5.000   5.000
  3.000   0.000
RECORD NUMBER 3:
  3.000   4.000   3.000   3.000   5.000   3.000   3.000   5.000   6.000   5.000
  1.000   6.000
RECORD NUMBER 4:
  7.000   5.000   5.000   3.000   3.000   1.000   4.000   3.000   5.000   3.000
  3.000   5.000

                        MEANS FOR MAIN EFFECTS FOR RUN 1
                         Example: (p:c) x (i:r) Design

Means for c

  4.083   4.861
```

```
Means for p:c

  4.750   3.750   3.917   3.917   5.667   3.500   5.583   4.417   6.000   4.000

Means for r

  5.300   4.717   3.467

Means for i:r

  5.900   4.600   5.400   5.300   5.100   4.100   4.500   4.700   4.600   5.400
  2.400   2.600


                           ANOVA TABLE FOR RUN 1
                      Example: (p:c) x (i:r) Design
-----------------------------------------------------------------------------
Effect             df            T            SS            MS            VC
-----------------------------------------------------------------------------
c                   1    2501.72222      17.42222      17.42222       0.09835
p:c                 8    2572.16667      70.44444       8.80556       0.44198
r                   2    2538.05000      53.75000      26.87500       0.42936
i:r                 9    2612.20000      74.15000       8.23889       0.45817
cr                  2    2567.41667      11.94444       5.97222       0.08635
ci:r                9    2673.66667      32.10000       3.56667       0.22632
pr:c               16    2691.16667      53.30556       3.33160       0.22701
pi:cr              72    2976.00000     178.58333       2.48032       2.48032
-----------------------------------------------------------------------------
Mean                     2484.30000
-----------------------------------------------------------------------------
Total             119                   491.70000
-----------------------------------------------------------------------------
        Grand Mean:         4.55000


                        EXPECTED MEAN SQUARE EQUATIONS FOR RUN 1
                             Example: (p:c) x (i:r) Design


EMS(c)       =   1.000*VC(pi:cr) +   4.500*VC(pr:c)  +   4.800*VC(ci:r)
                                 +  21.600*VC(cr)    +  12.000*VC(p:c)
                                 +  57.600*VC(c)

EMS(p:c)     =   1.000*VC(pi:cr) +   4.500*VC(pr:c)  +  12.000*VC(p:c)

EMS(r)       =   1.000*VC(pi:cr) +   3.750*VC(pr:c)  +   5.200*VC(ci:r)
                                 +  19.500*VC(cr)    +  10.000*VC(i:r)
                                 +  37.500*VC(r)

EMS(i:r)     =   1.000*VC(pi:cr) +   5.200*VC(ci:r)  +  10.000*VC(i:r)

EMS(cr)      =   1.000*VC(pi:cr) +   3.750*VC(pr:c)  +   4.800*VC(ci:r)
                                 +  18.000*VC(cr)

EMS(ci:r)    =   1.000*VC(pi:cr) +   4.800*VC(ci:r)

EMS(pr:c)    =   1.000*VC(pi:cr) +   3.750*VC(pr:c)

EMS(pi:cr)   =   1.000*VC(pi:cr)

*** EMS matrix is upper diagonal***


Date and time at beginning of Run 1:  Thu Jun  7 14:46:44 2001
Processor time for run: 0 seconds
```

Appendix B
Control Card Setups Illustrating Multiple Runs and Different Positions for Starred Effect

```
GSTUDY   Run 1 : (p:c) x (i:h) Design
OPTIONS  "*.out"   ET   NREC 4  TIME  NOBANNER
EFFECT     c     2
EFFECT   * p:c   4 6
EFFECT     h     3
EFFECT     i:h   3 6 3
FORMAT   3 2
PROCESS  "Dummy data pstar"
GSTUDY   Run 2: (p:c) x (i:r) Design
OPTIONS  NREC 2  "*.out"  EMS  ET  TIME    NOBANNER
EFFECT   * c     2
EFFECT     p:c   4 6
EFFECT     r     3
EFFECT     i:r   3 6 3
FORMAT   3 0
PROCESS
    5 6 5 5 5 3 4 5 6 7 3 3
    7 1 5 5 5 3 3 3 5 5 3 0
    3 4 3 3 5 3 3 5 6 5 1 6
    7 5 5 3 3 1 4 3 5 3 3 5
    9 2 9 7 7 7 3 7 2 7 5 3
    3 4 3 5 3 3 6 3 4 5 1 2
    7 3 7 7 7 5 5 7 5 5 5 4
    5 8 5 7 7 5 5 4 3 2 1 1
    9 9 8 8 6 6 6 5 5 8 1 1
    4 4 4 3 3 5 6 5 5 7 1 1
GSTUDY   Run 3: (p:c) x (i:r) Design
OPTIONS  NREC 4  "*.out" EMS  TIME    NOBANNER
EFFECT     c     2
EFFECT     p:c   4 6
EFFECT   * r     3
EFFECT     i:r   3 6 3
FORMAT   3 2
PROCESS
    5 6 5
    5 5 3 4 5 6
    7 3 3
    7 1 5
    5 5 3 3 3 5
    5 3 0
    3 4 3
    3 5 3 3 5 6
    5 1 6
    7 5 5
    3 3 1 4 3 5
    3 3 5
    9 2 9
    7 7 7 3 7 2
    7 5 3
    3 4 3
    5 3 3 6 3 4
    5 1 2
```

```
    7 3 7
    7 7 5 5 7 5
    5 5 4
    5 8 5
    7 7 5 5 4 3
    2 1 1
    9 9 8
    8 6 6 6 5 5
    8 1 1
    4 4 4
    3 3 5 6 5 5
    7 1 1
GSTUDY   Run 4: (p:c) x (i:h) Design
OPTIONS  NREC 5  "*.out" TIME    NOBANNER
EFFECT     c     2
EFFECT     p:c   4 6
EFFECT     h     3
EFFECT   * i:h   3 6 3
FORMAT   3 2
PROCESS
    5
    6
    5
    5
    5
(next 100 observations -- one per record)
    5
    5
    7
    1
    1
```

Appendix C
Output for Balanced Design Illustrating Estimated Standard Errors and Confidence Intervals

```
                          CONTROL CARDS FOR RUN 2
                   Control Cards File Name:  cc.manual
          Example for p x (i:r) Design for Synthetic Data Set Number 4

GSTUDY   Example for p x (i:r) Design for Synthetic Data Set Number 4
OPTIONS  NREC 5  "*.out" EMS  SECI .8  NOBANNER
EFFECT   * p    10
EFFECT     r     3
EFFECT     i:r   4 4 4
FORMAT   0 2
PROCESS
```

```
                          INPUT RECORDS FOR RUN 2
          Example for p x (i:r) Design for Synthetic Data Set Number 4

RECORD NUMBER 1:
  5.000   6.000   5.000   5.000   5.000   3.000   4.000   5.000   6.000   7.000
  3.000   3.000
RECORD NUMBER 2:
  9.000   3.000   7.000   7.000   7.000   5.000   5.000   5.000   7.000   7.000
  5.000   2.000
RECORD NUMBER 3:
  3.000   4.000   3.000   3.000   5.000   3.000   3.000   5.000   6.000   5.000
  1.000   6.000
RECORD NUMBER 4:
  7.000   5.000   5.000   3.000   3.000   1.000   4.000   3.000   5.000   3.000
  3.000   5.000
RECORD NUMBER 5:
  9.000   2.000   9.000   7.000   7.000   7.000   3.000   7.000   2.000   7.000
  5.000   3.000
```

```
                     MEANS FOR MAIN EFFECTS FOR RUN 2
          Example for p x (i:r) Design for Synthetic Data Set Number 4

Means for p

  4.750   5.750   3.917   3.917   5.667   3.500   5.583   4.417   6.000   4.000

Means for r

  5.500   4.800   3.950

Means for i:r

  6.100   4.800   5.600   5.500   5.300   4.300   4.700   4.900   4.800   5.600
  2.600   2.800
```

```
                           ANOVA TABLE FOR RUN 2
        Example for p x (i:r) Design for Synthetic Data Set Number 4
-----------------------------------------------------------------------------
Effect            df           T            SS            MS            VC
-----------------------------------------------------------------------------
p                  9    2800.16667      92.66667      10.29630       0.47315
r                  2    2755.70000      48.20000      24.10000       0.32515
i:r                9    2835.40000      79.70000       8.85556       0.64753
pr                18    2931.50000      83.13333       4.61852       0.55957
pi:r              81    3204.00000     192.80000       2.38025       2.38025
-----------------------------------------------------------------------------
Mean                    2707.50000
-----------------------------------------------------------------------------
Total            119                   496.50000
-----------------------------------------------------------------------------
        Grand Mean:        4.75000


    STANDARD ERRORS AND CONFIDENCE INTERVALS FOR VARIANCE COMPONENTS FOR RUN 2
                            TING ET AL. INTERVALS
           Example for p x (i:r) Design for Synthetic Data Set Number 4
----------------------------------------------------------------------------------
Effect                VC         SEVC        ratio     80.00%  Confidence Interval
----------------------------------------------------------------------------------
p                0.47315      0.38558      1.11503     (      0.0606,       1.4657)
r                0.32515      0.43799      0.53064     (      0.0000,       5.4144)
i:r              0.64753      0.37941      1.54495     (      0.2987,       1.6724)
pr               0.55957      0.37663      1.41279     (      0.1746,       1.3202)
pi:r             2.38025      0.36949      6.36396     (      1.9738,       2.9581)
----------------------------------------------------------------------------------
NOTE: SEVC*SEVC from the third column is an unbiased estimate that uses df[]+2.
NOTE: ratio = VC/SEVC where SEVC*SEVC is a biased estimate based on
      using df[] rather than df[]+2.
NOTE: Standard errors and confidence intervals are theoretically justified
      under normality assumptions for balanced designs only (see manual).



                      EXPECTED MEAN SQUARE EQUATIONS FOR RUN 2
               Example for p x (i:r) Design for Synthetic Data Set Number 4


EMS(p)      =   1.000*VC(pi:r) +   4.000*VC(pr)   +  12.000*VC(p)

EMS(r)      =   1.000*VC(pi:r) +   4.000*VC(pr)   +  10.000*VC(i:r)  +  40.000*VC(r)

EMS(i:r)    =   1.000*VC(pi:r) +  10.000*VC(i:r)

EMS(pr)     =   1.000*VC(pi:r) +   4.000*VC(pr)

EMS(pi:r)   =   1.000*VC(pi:r)

*** EMS matrix is upper diagonal***
```

Appendix D
Output for Searle et al. (1992) Example with Different Numbers of Observations per Cell

```
                          CONTROL CARDS FOR RUN 3
                   Control Cards File Name:  cc.manual
                            r:(p x h) Design

GSTUDY   r:(p x h) Design
COMMENT    From Searle, Casella, and McCulloch (1992, pp. 189-190)
COMMENT    Note that Searle et al. have an error for the first
COMMENT    estimated variance component.  They report
COMMENT    -454.6/121 = -1.6909 which is obviously incorrect.
OPTIONS  NREC 6  "*.out"  ET  EMS  TIME  NOBANNER
EFFECT     p      2
EFFECT   * h      3
EFFECT     r:ph   2 1 1 1 2 1
FORMAT   0 0
PROCESS
```

```
                          INPUT RECORDS FOR RUN 3
                            r:(p x h) Design

RECORD NUMBER 1:
  7.000   9.000
RECORD NUMBER 2:
  6.000
RECORD NUMBER 3:
  2.000
RECORD NUMBER 4:
  8.000
RECORD NUMBER 5:
  4.000   8.000
RECORD NUMBER 6:
 12.000
```

```
                     MEANS FOR MAIN EFFECTS FOR RUN 3
                            r:(p x h) Design

Means for p

  6.000   8.000

Means for h

  8.000   6.000   7.000
```

ANOVA TABLE FOR RUN 3
r:(p x h) Design

Effect	df	T	SS	MS	VC
p	1	400.00000	8.00000	8.00000	-3.75702
h	2	398.00000	6.00000	3.00000	-7.70248
ph	2	448.00000	42.00000	21.00000	13.30248
r:ph	2	458.00000	10.00000	5.00000	5.00000
Mean		392.00000			
Total	7		66.00000		

Grand Mean: 7.00000

```
              EXPECTED VALUES OF T TERMS FOR RUN 3
ET() terms do not include (total number of observations) X (grand mean squared)
                        r:(p x h) Design


 ET(p)      =    2.00*VC(r:ph) +    3.00*VC(ph)   +    3.00*VC(h)    +    8.00*VC(p)

 ET(h)      =    3.00*VC(r:ph) +    4.33*VC(ph)   +    8.00*VC(h)    +    4.33*VC(p)

 ET(ph)     =    6.00*VC(r:ph) +    8.00*VC(ph)   +    8.00*VC(h)    +    8.00*VC(p)

 ET(r:ph)   =    8.00*VC(r:ph) +    8.00*VC(ph)   +    8.00*VC(h)    +    8.00*VC(p)

 ET(Mean)   =    1.00*VC(r:ph) +    1.50*VC(ph)   +    2.75*VC(h)    +    4.00*VC(p)


              EXPECTED MEAN SQUARE EQUATIONS FOR RUN 3
                        r:(p x h) Design


EMS(p)      =   1.000*VC(r:ph) +   1.500*VC(ph)   +   0.250*VC(h)    +   4.000*VC(p)

EMS(h)      =   1.000*VC(r:ph) +   1.417*VC(ph)   +   2.625*VC(h)    +   0.167*VC(p)

EMS(ph)     =   1.000*VC(r:ph) +   1.083*VC(ph)   +  -0.125*VC(h)    +  -0.167*VC(p)

EMS(r:ph)   =   1.000*VC(r:ph)

Date and time at beginning of Run 3:  Thu Jun  7 14:46:44 2001
Processor time for run: 0 seconds
```

Appendix E
Output for Searle et al. (1992) Example with Different Numbers of Observations per Cell and One Empty Cell

```
                          CONTROL CARDS FOR RUN 4
                   Control Cards File Name:  cc.manual
                   r:(p x h) Design with one empty cell

GSTUDY   r:(p x h) Design with one empty cell
COMMENT    From Searle, Casella, and McCulloch (1992, pp. 189-190)
COMMENT    excluding the data for cell (p1,h2)
OPTIONS  NREC 6  "*.out"  ET  EMS  TIME  NOBANNER
EFFECT     p      2
EFFECT   * h      3
EFFECT     r:ph   2 0 1 1 2 1
FORMAT   0 0
PROCESS



                          INPUT RECORDS FOR RUN 4
                   r:(p x h) Design with one empty cell

RECORD NUMBER 1:
  7.000   9.000
RECORD NUMBER 2:

RECORD NUMBER 3:
  2.000
RECORD NUMBER 4:
  8.000
RECORD NUMBER 5:
  4.000   8.000
RECORD NUMBER 6:
 12.000


                     MEANS FOR MAIN EFFECTS FOR RUN 4
                   r:(p x h) Design with one empty cell

Means for p

  6.000   8.000

Means for h

  8.000   6.000   7.000
```

```
                             ANOVA TABLE FOR RUN 4
                     r:(p x h) Design with one empty cell
------------------------------------------------------------------------------
Effect             df             T             SS             MS           VC
------------------------------------------------------------------------------
p                   1     364.00000        6.85714        6.85714     -4.67840
h                   2     362.00000        4.85714        2.42857    -11.16287
ph                  2     412.00000       43.14286       21.57143     13.24983
r:ph                1     422.00000       10.00000       10.00000     10.00000
------------------------------------------------------------------------------
Mean                      357.14286
------------------------------------------------------------------------------
Total               6                     64.85714
------------------------------------------------------------------------------
        Grand Mean:         7.14286


                            EXPECTED VALUES OF T TERMS FOR RUN 4
     ET() terms do not include (total number of observations) X (grand mean squared)
                            r:(p x h) Design with one empty cell


 ET(p)      =    2.00*VC(r:ph) +    3.17*VC(ph)    +    3.17*VC(h)    +    7.00*VC(p)

 ET(h)      =    3.00*VC(r:ph) +    4.67*VC(ph)    +    7.00*VC(h)    +    4.67*VC(p)

 ET(ph)     =    6.00*VC(r:ph) +    7.00*VC(ph)    +    7.00*VC(h)    +    7.00*VC(p)

 ET(r:ph)   =    7.00*VC(r:ph) +    7.00*VC(ph)    +    7.00*VC(h)    +    7.00*VC(p)

 ET(Mean)   =    1.00*VC(r:ph) +    1.57*VC(ph)    +    2.43*VC(h)    +    3.57*VC(p)


                          EXPECTED MEAN SQUARE EQUATIONS FOR RUN 4
                            r:(p x h) Design with one empty cell


EMS(p)      =   1.000*VC(r:ph) +   1.595*VC(ph)    +   0.738*VC(h)    +   3.429*VC(p)

EMS(h)      =   1.000*VC(r:ph) +   1.548*VC(ph)    +   2.286*VC(h)    +   0.548*VC(p)

EMS(ph)     =   1.000*VC(r:ph) +   0.369*VC(ph)    +  -0.369*VC(h)    +  -0.548*VC(p)

EMS(r:ph)   =   1.000*VC(r:ph)

Date and time at beginning of Run 4:  Thu Jun  7 14:46:44 2001
Processor time for run: 0 seconds
```

Appendix F
Output for Huynh (1997) Example of a $p \times i$ Design with Missing Observations

```
                         CONTROL CARDS FOR RUN 5
                   Control Cards File Name:  cc.manual
     p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)

GSTUDY   p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)
OPTIONS  NREC 12 "*.out" ET EMS TIME NOBANNER
EFFECT   * p    12
EFFECT     i     6
EFFECT     r:pi  1 1 1 1 1 1
                 1 1 1 1 1 1
                 0 1 1 1 1 1
                 1 0 0 1 1 1
                 1 0 1 1 1 0
                 1 1 1 1 1 1
                 1 0 1 0 1 1
                 1 0 1 1 1 1
                 1 1 1 1 0 1
                 0 1 1 1 0 1
                 1 1 1 1 1 1
                 0 1 0 1 1 1
FORMAT   0 0
PROCESS
```

```
                         INPUT RECORDS FOR RUN 5
     p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)

RECORD NUMBER 1:
  1.000   1.000   1.000   1.000   0.000   1.000
RECORD NUMBER 2:
  1.000   0.000   1.000   1.000   1.000   1.000
RECORD NUMBER 3:
  1.000   1.000   1.000   0.000   0.000
RECORD NUMBER 4:
  0.000   1.000   0.000   1.000
RECORD NUMBER 5:
  1.000   0.000   1.000   0.000
RECORD NUMBER 6:
  1.000   1.000   1.000   1.000   0.000   1.000
RECORD NUMBER 7:
  0.000   1.000   0.000   0.000
RECORD NUMBER 8:
  0.000   1.000   1.000   1.000   1.000
RECORD NUMBER 9:
  1.000   1.000   0.000   1.000   0.000
RECORD NUMBER 10:
  0.000   0.000   0.000   0.000
RECORD NUMBER 11:
  1.000   1.000   1.000   1.000   1.000   1.000
RECORD NUMBER 12:
  0.000   0.000   0.000   1.000
```

```
                        MEANS FOR MAIN EFFECTS FOR RUN 5
      p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)

Means for p

  0.833   0.833   0.600   0.500   0.500   0.833   0.250   0.800   0.600   0.000
  1.000   0.250

Means for i

  0.667   0.625   0.700   0.818   0.300   0.636


                            ANOVA TABLE FOR RUN 5
     p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)
-----------------------------------------------------------------------------
Effect            df             T            SS            MS            VC
-----------------------------------------------------------------------------
p                 11      27.80000       4.59661       0.41787       0.04725
i                  5      24.74318       1.53979       0.30796       0.01174
pi                42      37.00000       7.66021       0.18239       0.18401
-----------------------------------------------------------------------------
Mean                      23.20339
-----------------------------------------------------------------------------
Total             58                    13.79661
-----------------------------------------------------------------------------
        Grand Mean:        0.62712


                           EXPECTED VALUES OF T TERMS FOR RUN 5
     ET() terms do not include (total number of observations) X (grand mean squared)
           p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)


 ET(p)      =   12.00*VC(pi)   +   12.00*VC(i)    +   59.00*VC(p)

 ET(i)      =    6.00*VC(pi)   +   59.00*VC(i)    +    6.00*VC(p)

 ET(pi)     =   59.00*VC(pi)   +   59.00*VC(i)    +   59.00*VC(p)

 ET(Mean)   =    1.00*VC(pi)   +    9.95*VC(i)    +    5.07*VC(p)


                         EXPECTED MEAN SQUARE EQUATIONS FOR RUN 5
           p x i design with some missing 0/1 data -- from Huynh (1977, p. 14)


EMS(p)      =   1.000*VC(pi)   +   0.186*VC(i)    +   4.903*VC(p)

EMS(i)      =   1.000*VC(pi)   +   9.810*VC(i)    +   0.186*VC(p)

EMS(pi)     =   1.000*VC(pi)   +  -0.049*VC(i)    +  -0.022*VC(p)


Date and time at beginning of Run 5:  Thu Jun  7 14:46:44 2001
Processor time for run: 0 seconds
```

Appendix G
Output for a $p \times (i\!:\!h)$ Design with Missing Observations

```
                          CONTROL CARDS FOR RUN 6
                      Control Cards File Name:  cc.manual
             p x (i:h) Design as r:[p x (i:h)] with six empty cells

GSTUDY   p x (i:h) Design as r:[p x (i:h)] with six empty cells
OPTIONS  NREC 12  "*.out" EMS ET  NOBANNER
EFFECT   * p      10
EFFECT     h       3
EFFECT     i:h     3 6 3
EFFECT     r:pi:h  1 1 1 1 1 1 1 0 1 1 1 1
                   1 1 1 0 1 1 1 1 1 1 1 1
                   1 1 1 1 1 1 1 1 1 0 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
                   1 1 1 1 1 0 1 1 1 1 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
                   1 1 0 1 1 1 1 1 1 1 1 1
                   1 1 1 1 1 1 1 0 1 1 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
                   1 1 1 1 1 1 1 1 1 1 1 1
FORMAT   0 0
PROCESS
```

```
                          INPUT RECORDS FOR RUN 6
             p x (i:h) Design as r:[p x (i:h)] with six empty cells

RECORD NUMBER 1:
  5.000   6.000   5.000   5.000   5.000   3.000   4.000   6.000   7.000   3.000
  3.000
RECORD NUMBER 2:
  7.000   1.000   5.000   5.000   3.000   3.000   3.000   5.000   5.000   3.000
  0.000
RECORD NUMBER 3:
  3.000   4.000   3.000   3.000   5.000   3.000   3.000   5.000   6.000   1.000
  6.000
RECORD NUMBER 4:
  7.000   5.000   5.000   3.000   3.000   1.000   4.000   3.000   5.000   3.000
  3.000   5.000
RECORD NUMBER 5:
  9.000   2.000   9.000   7.000   7.000   3.000   7.000   2.000   7.000   5.000
  3.000
RECORD NUMBER 6:
  3.000   4.000   3.000   5.000   3.000   3.000   6.000   3.000   4.000   5.000
  1.000   2.000
RECORD NUMBER 7:
  7.000   3.000   7.000   7.000   5.000   5.000   7.000   5.000   5.000   5.000
  4.000
RECORD NUMBER 8:
  5.000   8.000   5.000   7.000   7.000   5.000   5.000   3.000   2.000   1.000
  1.000
RECORD NUMBER 9:
  9.000   9.000   8.000   8.000   6.000   6.000   6.000   5.000   5.000   8.000
  1.000   1.000
RECORD NUMBER 10:
  4.000   4.000   4.000   3.000   3.000   5.000   6.000   5.000   5.000   7.000
  1.000   1.000
```

```
                     MEANS FOR MAIN EFFECTS FOR RUN 6
           p x (i:h) Design as r:[p x (i:h)] with six empty cells

Means for p

  4.727   3.636   3.818   3.917   5.545   3.500   5.455   4.455   6.000   4.000

Means for h

  5.241   4.679   3.414

Means for i:h

  5.900   4.600   5.222   5.333   5.100   3.778   4.500   4.750   4.600   5.444
  2.400   2.600
```

```
                          ANOVA TABLE FOR RUN 6
         p x (i:h) Design as r:[p x (i:h)] with six empty cells
------------------------------------------------------------------------------
Effect            df             T            SS            MS            VC
------------------------------------------------------------------------------
p                  9    2390.53788      82.03788       9.11532       0.43859
h                  2    2360.44089      51.94089      25.97044       0.45415
i:h                9    2436.26667      75.82578       8.42509       0.60321
ph                18    2512.00000      69.52123       3.86229       0.32937
pi:h              75    2787.00000     199.17422       2.65566       2.67060
------------------------------------------------------------------------------
Mean                    2308.50000
------------------------------------------------------------------------------
Total            113                   478.50000
------------------------------------------------------------------------------
        Grand Mean:         4.50000
```

```
                        EXPECTED VALUES OF T TERMS FOR RUN 6
     ET() terms do not include (total number of observations) X (grand mean squared)
                 p x (i:h) Design as r:[p x (i:h)] with six empty cells


 ET(p)      =   10.00*VC(pi:h) +   42.55*VC(ph)   +   10.00*VC(i:h)  +   42.55*VC(h)
                               +  114.00*VC(p)

 ET(h)      =    3.00*VC(pi:h) +   11.50*VC(ph)   +   28.77*VC(i:h)  +  114.00*VC(h)
                               +   11.50*VC(p)

 ET(i:h)    =   12.00*VC(pi:h) +   12.00*VC(ph)   +  114.00*VC(i:h)  +  114.00*VC(h)
                               +   12.00*VC(p)

 ET(ph)     =   30.00*VC(pi:h) +  114.00*VC(ph)   +   30.00*VC(i:h)  +  114.00*VC(h)
                               +  114.00*VC(p)

 ET(pi:h)   =  114.00*VC(pi:h) +  114.00*VC(ph)   +  114.00*VC(i:h)  +  114.00*VC(h)
                               +  114.00*VC(p)

 ET(Mean)   =    1.00*VC(pi:h) +    4.26*VC(ph)   +    9.54*VC(i:h)  +   42.26*VC(h)
                               +   11.42*VC(p)
```

```
                    EXPECTED MEAN SQUARE EQUATIONS FOR RUN 6
            p x (i:h) Design as r:[p x (i:h)] with six empty cells


EMS(p)      =   1.000*VC(pi:h) +   4.254*VC(ph)   +   0.051*VC(i:h)  +   0.031*VC(h)
                               +  11.398*VC(p)

EMS(h)      =   1.000*VC(pi:h) +   3.621*VC(ph)   +   9.614*VC(i:h)  +  35.868*VC(h)
                               +   0.042*VC(p)

EMS(i:h)    =   1.000*VC(pi:h) +   0.055*VC(ph)   +   9.470*VC(i:h)  +   0.055*VC(p)

EMS(ph)     =   1.000*VC(pi:h) +   3.567*VC(ph)   +   0.043*VC(i:h)  +  -0.016*VC(h)
                               +  -0.005*VC(p)

EMS(pi:h)   =   1.000*VC(pi:h) +  -0.007*VC(ph)   +  -0.016*VC(i:h)  +  -0.007*VC(p)
```

Appendix H
Output for Multiple Matrix Sampling Design

```
                              CONTROL CARDS FOR RUN 7
                       Control Cards File Name:  cc.manual
                         Matrix Sampling Design (p x i):r

GSTUDY   Matrix Sampling Design (p x i):r
COMMENT *********************************************
COMMENT Same first nesting index for two main effects
COMMENT *********************************************
OPTIONS  NREC 6  "*.out"  ET EMS NOBANNER
EFFECT      r  3
EFFECT    * p:r    6  8 10
EFFECT      i:r    5 10 15
FORMAT   0 2
PROCESS
```

```
                              INPUT RECORDS FOR RUN 7
                         Matrix Sampling Design (p x i):r

RECORD NUMBER 1:
  5.000   6.000   5.000   5.000   5.000
RECORD NUMBER 2:
  3.000   4.000   3.000   3.000   5.000
RECORD NUMBER 3:
  9.000   2.000   9.000   7.000   7.000
RECORD NUMBER 4:
  7.000   3.000   7.000   7.000   7.000
RECORD NUMBER 5:
  9.000   9.000   8.000   8.000   6.000
RECORD NUMBER 6:
  5.000   6.000   5.000   5.000   5.000
```

```
                         MEANS FOR MAIN EFFECTS FOR RUN 7
                         Matrix Sampling Design (p x i):r

Means for r

  5.833   5.463   5.080

Means for p:r

  5.200   3.600   6.800   6.200   8.000   5.200   4.000   6.000   5.800   7.000
  5.100   4.000   6.000   5.800   5.600   4.667   4.267   5.200   5.667   5.600
  4.667   4.267   5.200   5.667

Means for i:r

  6.333   5.000   6.167   5.833   5.833   6.500   4.125   6.375   5.875   6.125
  4.875   4.000   6.000   4.625   6.125   6.600   4.800   6.400   6.000   6.000
  4.800   4.200   5.800   4.800   6.400   3.000   3.400   5.200   4.400   4.400
```

```
                           ANOVA TABLE FOR RUN 7
                     Matrix Sampling Design (p x i):r
------------------------------------------------------------------------------
Effect            df           T            SS            MS            VC
------------------------------------------------------------------------------
r                  2   7278.90583     17.84429       8.92215      -0.05200
p:r               21   7456.30000    177.39417       8.44734       0.51416
i:r               27   7522.04167    243.13583       9.00503       0.70806
pi:r             209   8288.00000    588.56417       2.81610       2.81610
------------------------------------------------------------------------------
Mean                   7261.06154
------------------------------------------------------------------------------
Total            259                1026.93846
------------------------------------------------------------------------------
        Grand Mean:        5.28462


                       EXPECTED VALUES OF T TERMS FOR RUN 7
     ET() terms do not include (total number of observations) X (grand mean squared)
                         Matrix Sampling Design (p x i):r


 ET(r)      =    3.00*VC(pi:r) +   24.00*VC(i:r)  +   30.00*VC(p:r)  +  260.00*VC(r)

 ET(p:r)    =   24.00*VC(pi:r) +   24.00*VC(i:r)  +  260.00*VC(p:r)  +  260.00*VC(r)

 ET(i:r)    =   30.00*VC(pi:r) +  260.00*VC(i:r)  +   30.00*VC(p:r)  +  260.00*VC(r)

 ET(pi:r)   =  260.00*VC(pi:r) +  260.00*VC(i:r)  +  260.00*VC(p:r)  +  260.00*VC(r)

 ET(Mean)   =    1.00*VC(pi:r) +    8.92*VC(i:r)  +   12.31*VC(p:r)  +  114.62*VC(r)


                       EXPECTED MEAN SQUARE EQUATIONS FOR RUN 7
                         Matrix Sampling Design (p x i):r


EMS(r)      =   1.000*VC(pi:r) +   7.538*VC(i:r)  +   8.846*VC(p:r)  +  72.692*VC(r)

EMS(p:r)    =   1.000*VC(pi:r) +  10.952*VC(p:r)

EMS(i:r)    =   1.000*VC(pi:r) +   8.741*VC(i:r)

EMS(pi:r)   =   1.000*VC(pi:r)

*** EMS matrix is upper diagonal***
```

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