



# Rectified Diffusion: Straightness Is Not Your Need in Rectified Flow

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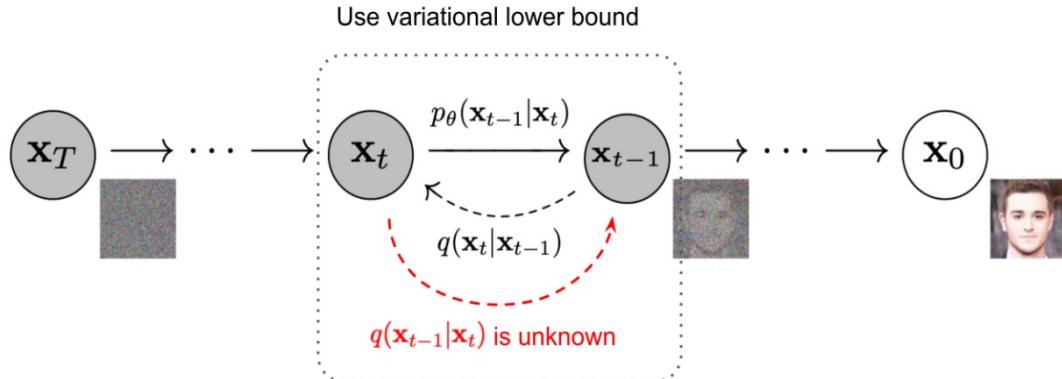
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<https://github.com/G-U-N/Rectified-Diffusion>



# Diffusion Models: Markovian Perspective



## ■ Assumption:

- $p(\mathbf{x}_{0:T}) = p(\mathbf{x}_T) \prod_{t=1}^T p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)$

## ■ Forward Process:

- $q(\mathbf{x}_t|\mathbf{x}_{t-1}) = \mathcal{N}(\mathbf{x}_t; \sqrt{\alpha_t}\mathbf{x}_{t-1}, (1 - \alpha_t)\mathbf{I})$
- $p(\mathbf{x}_T) = \mathcal{N}(\mathbf{x}_T; \mathbf{0}, \mathbf{I})$

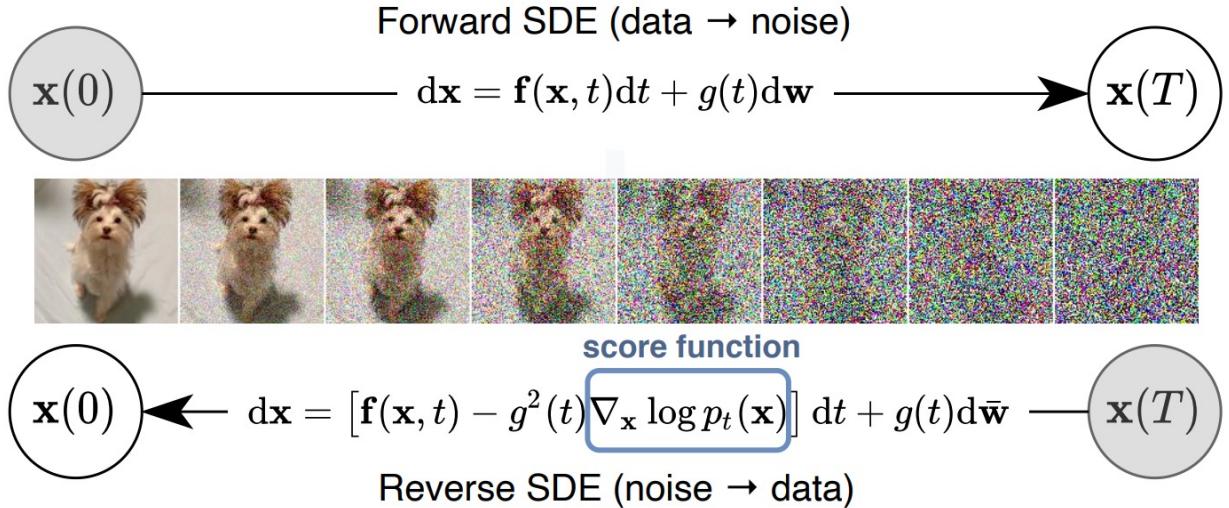
## ■ Reverse Process:

- $q(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{x}_0) = \frac{q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0)q(\mathbf{x}_t|\mathbf{x}_0)}{q(\mathbf{x}_{t-1}|\mathbf{x}_0)}$

## ■ Maximum Likelihood Estimation (MLE) is Equivalent to

$$\arg \min_{\theta} \mathbb{E}_{t \sim U\{2, T\}} [\mathbb{E}_{q(\mathbf{x}_t|\mathbf{x}_0)} [D_{KL}(q(\mathbf{x}_{t-1}|\mathbf{x}_t, \mathbf{x}_0) \parallel p_\theta(\mathbf{x}_{t-1}|\mathbf{x}_t)]]]$$

# Diffusion Models: Stochastic Differential Equation Perspective



The only unknown term is the **score function**.

Train a neural network through score matching!

Probability Flow ODE:

A deterministic reverse process

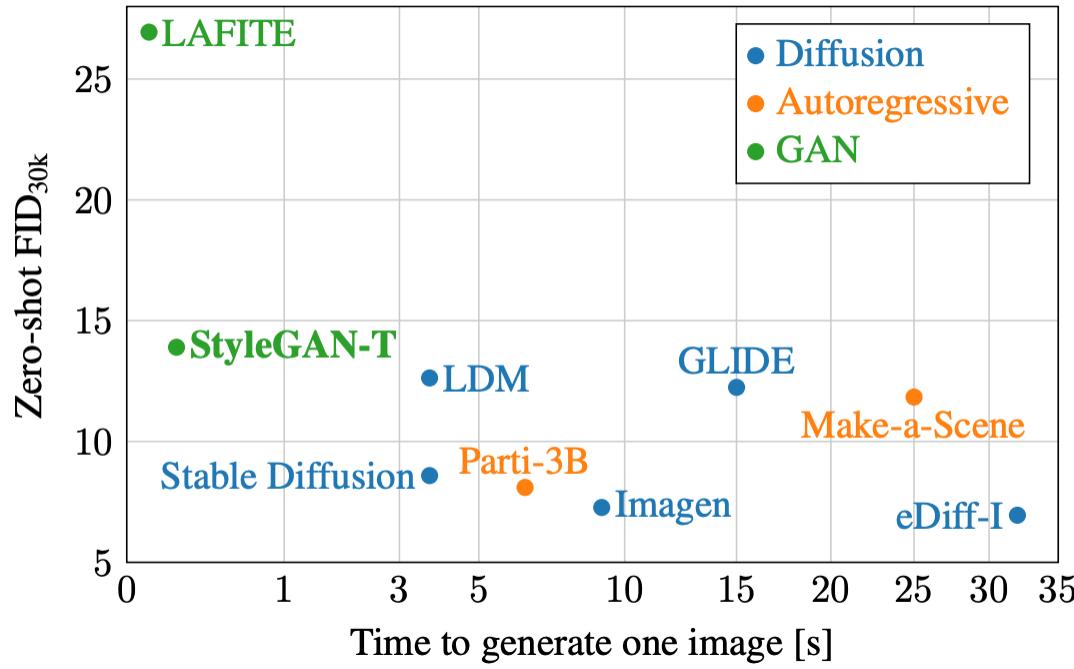
$$d\mathbf{x} = \left[ \mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g(t)^2\nabla_{\mathbf{x}} \log p_t(\mathbf{x}) \right] dt$$

Exact Solution form of PF-ODE

$$\mathbf{x}_t = \frac{\alpha_t}{\alpha_s} \mathbf{x}_s - \alpha_t \int_{\lambda_s}^{\lambda_t} e^{-\lambda} \hat{\epsilon}_{\theta}(\hat{\mathbf{x}}_{\lambda}, \lambda) d\lambda.$$



# Diffusion Models: Slow Inference Speed



How to speed up the diffusion generation?

- Reducing the number of function evaluation (NFE).

- Better Solvers.

- Adversarial post-training.

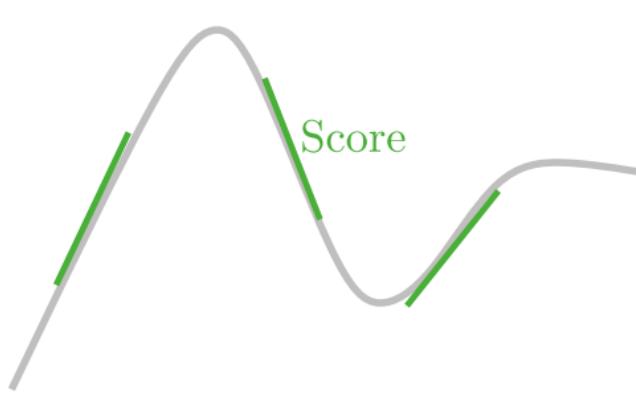
- Distillation.
  - Naïve distillation.
  - Guided distillation.
  - Score distillation.
  - Consistency distillation.
  - Rectification.



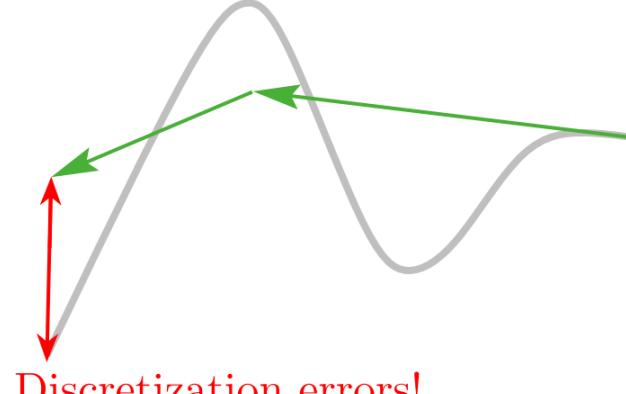
# Understanding Diffusion Models from the PF-ODE path

We know the derivative w.r.t. time  $t$ .

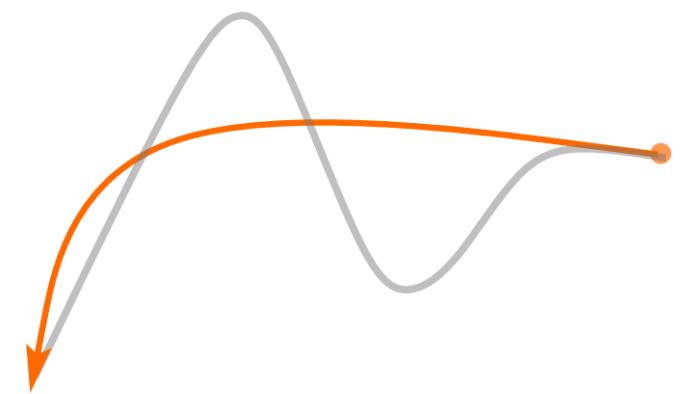
$$d\mathbf{x} = \left[ \mathbf{f}(\mathbf{x}, t) - \frac{1}{2}g(t)^2 \nabla_{\mathbf{x}} \log p_t(\mathbf{x}) \right] dt$$



PF-ODE



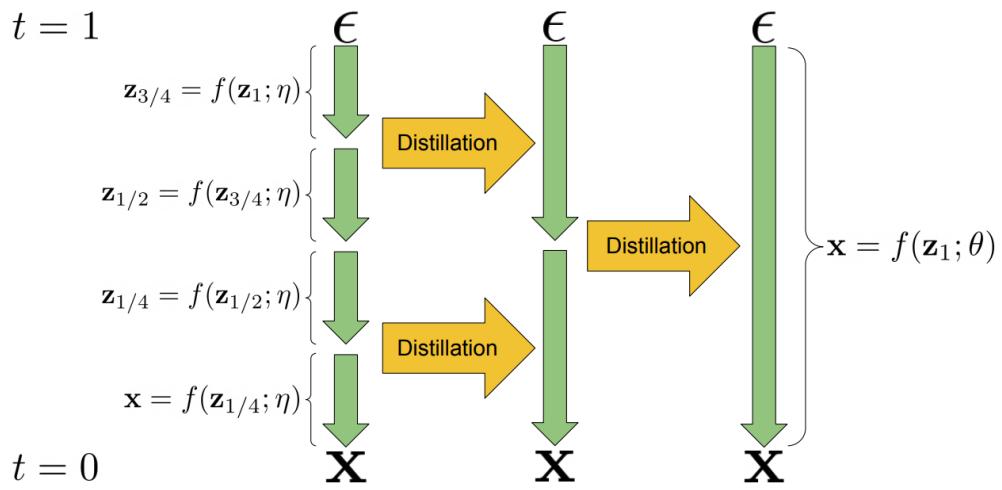
Discretized numerical solving.



Naïve distillation.



# Distillation Techniques: Progressive Distillation




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## Algorithm 2 Progressive distillation

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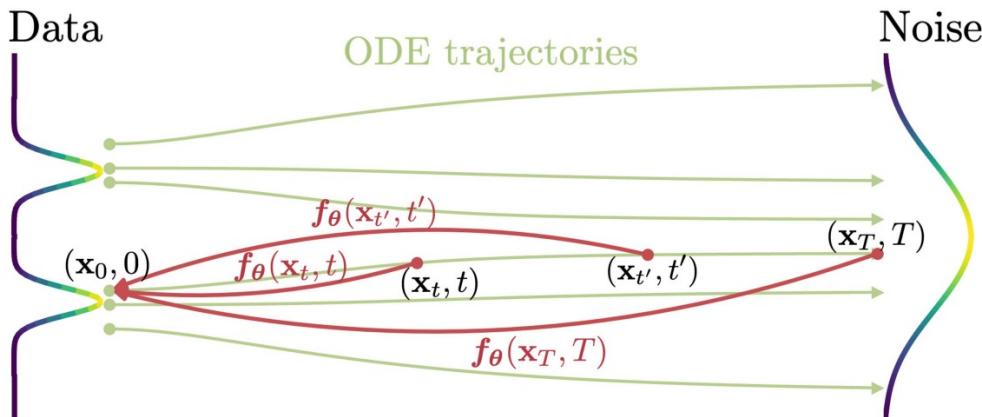
Require: Trained teacher model  $\hat{\mathbf{x}}_\eta(\mathbf{z}_t)$ 
Require: Data set  $\mathcal{D}$ 
Require: Loss weight function  $w()$ 
Require: Student sampling steps  $N$ 
for  $K$  iterations do
     $\theta \leftarrow \eta$                                  $\triangleright$  Init student from teacher
    while not converged do
         $\mathbf{x} \sim \mathcal{D}$ 
         $t = i/N, i \sim \text{Cat}[1, 2, \dots, N]$ 
         $\epsilon \sim N(0, I)$ 
         $\mathbf{z}_t = \alpha_t \mathbf{x} + \sigma_t \epsilon$ 
        # 2 steps of DDIM with teacher
         $t' = t - 0.5/N, t'' = t - 1/N$ 
         $\mathbf{z}_{t'} = \alpha_{t'} \hat{\mathbf{x}}_\eta(\mathbf{z}_t) + \frac{\sigma_{t'}}{\sigma_t} (\mathbf{z}_t - \alpha_t \hat{\mathbf{x}}_\eta(\mathbf{z}_t))$ 
         $\mathbf{z}_{t''} = \alpha_{t''} \hat{\mathbf{x}}_\eta(\mathbf{z}_{t'}) + \frac{\sigma_{t''}}{\sigma_{t'}} (\mathbf{z}_{t'} - \alpha_{t'} \hat{\mathbf{x}}_\eta(\mathbf{z}_{t'}))$ 
         $\tilde{\mathbf{x}} = \frac{\mathbf{z}_{t''} - (\sigma_{t''}/\sigma_t) \mathbf{z}_t}{\alpha_{t''} - (\sigma_{t''}/\sigma_t) \alpha_t}$   $\triangleright$  Teacher  $\hat{\mathbf{x}}$  target
         $\lambda_t = \log[\alpha_t^2/\sigma_t^2]$ 
         $L_\theta = w(\lambda_t) \|\tilde{\mathbf{x}} - \hat{\mathbf{x}}_\theta(\mathbf{z}_t)\|_2^2$ 
         $\theta \leftarrow \theta - \gamma \nabla_\theta L_\theta$ 
    end while
     $\eta \leftarrow \theta$   $\triangleright$  Student becomes next teacher
     $N \leftarrow N/2$   $\triangleright$  Halve number of sampling steps
end for

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# Distillation Techniques: Consistency Distillation



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## Algorithm 2 Consistency Distillation (CD)

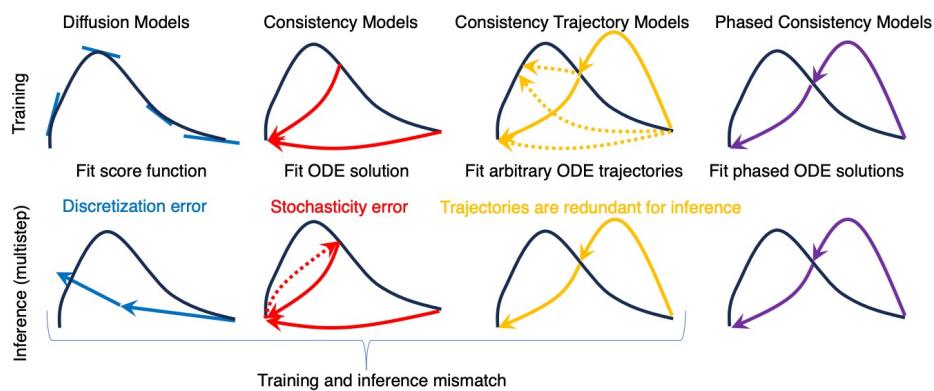
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**Input:** dataset  $\mathcal{D}$ , initial model parameter  $\theta$ , learning rate  $\eta$ , ODE solver  $\Phi(\cdot, \cdot; \phi)$ ,  $d(\cdot, \cdot)$ ,  $\lambda(\cdot)$ , and  $\mu$   
 $\theta^- \leftarrow \theta$   
**repeat**  
    Sample  $\mathbf{x} \sim \mathcal{D}$  and  $n \sim \mathcal{U}[1, N - 1]$   
    Sample  $\mathbf{x}_{t_{n+1}} \sim \mathcal{N}(\mathbf{x}; t_{n+1}^2 \mathbf{I})$   
     $\hat{\mathbf{x}}_{t_n}^\phi \leftarrow \mathbf{x}_{t_{n+1}} + (t_n - t_{n+1})\Phi(\mathbf{x}_{t_{n+1}}, t_{n+1}; \phi)$   
     $\mathcal{L}(\theta, \theta^-; \phi) \leftarrow$   
         $\lambda(t_n)d(f_\theta(\mathbf{x}_{t_{n+1}}, t_{n+1}), f_{\theta^-}(\hat{\mathbf{x}}_{t_n}^\phi, t_n))$   
     $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}(\theta, \theta^-; \phi)$   
     $\theta^- \leftarrow \text{stopgrad}(\mu \theta^- + (1 - \mu) \theta)$   
**until** convergence

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# Distillation Techniques: Phased Consistency Distillation



**Algorithm 1** Phased Consistency Distillation with CFG-augmented ODE solver (PCD)

**Input:** dataset  $\mathcal{D}$ , initial model parameter  $\theta$ , learning rate  $\eta$ , ODE solver  $\Psi(\cdot, \cdot, \cdot, \cdot)$ , distance metric  $d(\cdot, \cdot)$ , EMA rate  $\mu$ , noise schedule  $\alpha_t, \sigma_t$ , guidance scale  $[w_{\min}, w_{\max}]$ , number of ODE step  $k$ , discretized timesteps  $t_0 = \epsilon < t_1 < t_2 < \dots < t_N = T$ , edge timesteps  $s_0 = t_0 < s_1 < s_2 < \dots < s_M = t_N \in \{t_i\}_{i=0}^N$  to split the ODE trajectory into  $M$  sub-trajectories.

Training data :  $\mathcal{D}_x = \{(\mathbf{x}, \mathbf{c})\}$

$\theta^- \leftarrow \theta$

**repeat**

    Sample  $(\mathbf{z}, \mathbf{c}) \sim \mathcal{D}_z$ ,  $n \sim \mathcal{U}[0, N - k]$  and  $\omega \sim [\omega_{\min}, \omega_{\max}]$

    Sample  $\mathbf{x}_{t_{n+k}} \sim \mathcal{N}(\alpha_{t_{n+k}} \mathbf{z}; \sigma_{t_{n+k}}^2 \mathbf{I})$

    Determine  $[s_m, s_{m+1}]$  given  $n$

$\mathbf{x}_{t_n}^\phi \leftarrow (1 + \omega)\Psi(\mathbf{x}_{t_{n+k}}, t_{n+k}, t_n, \mathbf{c}) - \omega\Psi(\mathbf{x}_{t_{n+k}}, t_{n+k}, t_n, \emptyset)$

$\tilde{\mathbf{x}}_{s_m} = \mathbf{f}_\theta^m(\mathbf{x}_{t_{n+k}}, t_{n+k}, \mathbf{c})$  and  $\hat{\mathbf{x}}_{s_m} = \mathbf{f}_{\theta^-}(\mathbf{x}_{t_n}^\phi, t_n, \mathbf{c})$

    Obtain  $\tilde{\mathbf{x}}_s$  and  $\hat{\mathbf{x}}_s$  through adding noise to  $\tilde{\mathbf{x}}_{s_m}$  and  $\hat{\mathbf{x}}_{s_m}$

$\mathcal{L}(\theta, \theta^-) = d(\tilde{\mathbf{x}}_{s_m}, \hat{\mathbf{x}}_{s_m}) + \lambda(\text{ReLU}(1 + \tilde{\mathbf{x}}_s) + \text{ReLU}(1 - \hat{\mathbf{x}}_s))$

$\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}(\theta, \theta^-)$

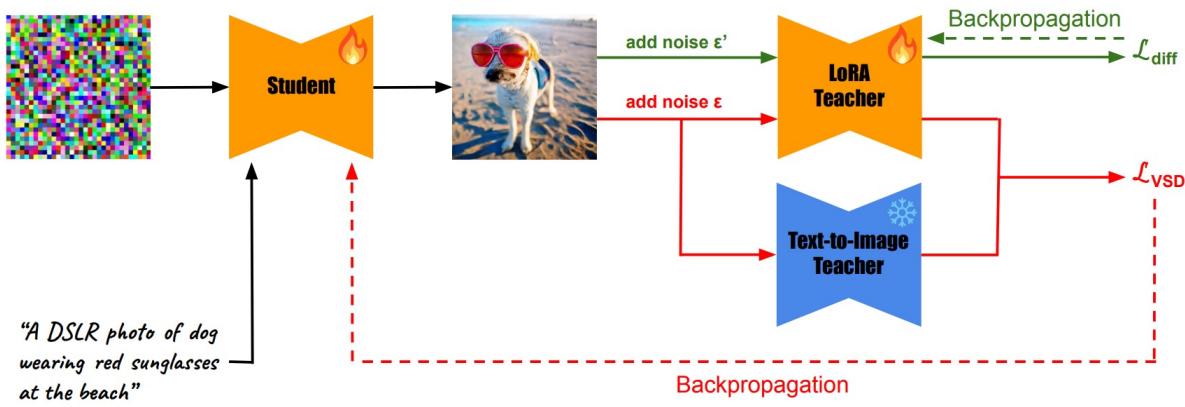
$\theta^- \leftarrow \text{stopgrad}(\mu \theta^- + (1 - \mu) \theta)$

**until** convergence



# Distillation Techniques: Score Distillation

$$\nabla_{\theta} \mathcal{L}_{\text{VSD}}(\theta) \triangleq \mathbb{E}_{t, \epsilon, c} \left[ \omega(t) (\epsilon_{\text{pretrain}}(\mathbf{x}_t, t, y^c) - \epsilon_{\phi}(\mathbf{x}_t, t, c, y)) \frac{\partial g(\theta, c)}{\partial \theta} \right]$$




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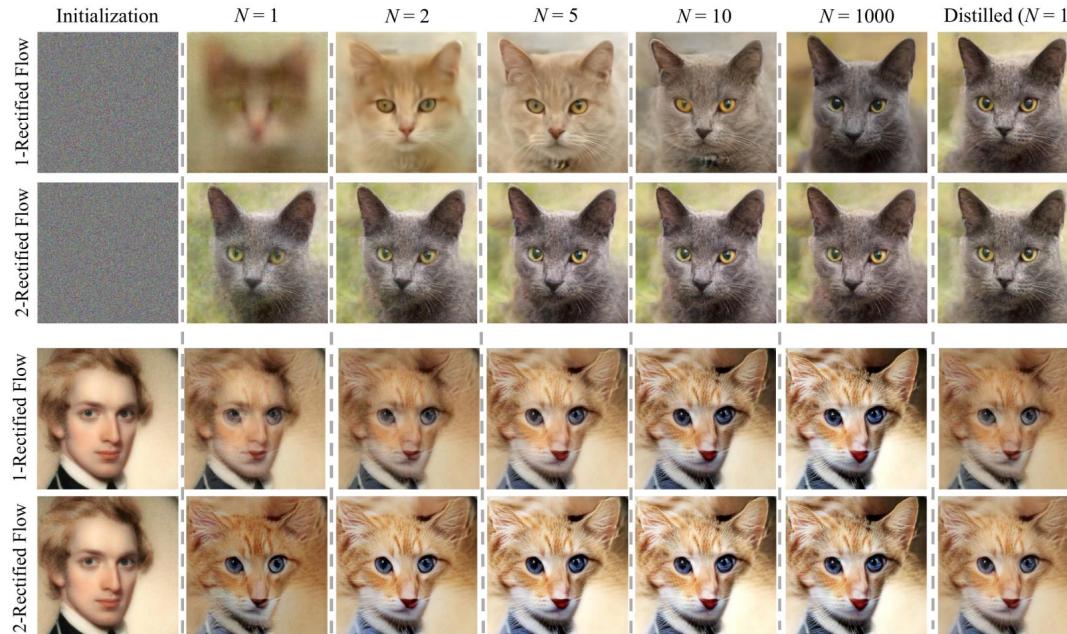
**Algorithm 1** SwiftBrush Distillation

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- 1: **Require:** a pretrained text-to-image teacher  $\epsilon_{\psi}$ , a LoRA teacher  $\epsilon_{\phi}$ , a student model  $f_{\theta}$ , two learning rates  $\eta_1$  and  $\eta_2$ , a weighting function  $\omega$ , a prompts dataset  $Y$ , the maximum number of time steps  $T$  and the noise schedule  $\{(\alpha_t, \sigma_t)\}_{t=1}^T$  of the teacher model
  - 2: **Initialize:**  $\phi \leftarrow \psi, \theta \leftarrow \psi$
  - 3: **while** not converged **do**
  - 4:     Sample input noise  $z \sim \mathcal{N}(0, I)$
  - 5:     Sample text caption input  $y \sim Y$
  - 6:     Compute student output  $\hat{x}_0 = f_{\theta}(z, y)$
  - 7:     Sample timestep  $t \sim \mathcal{U}(0.02T, 0.98T)$
  - 8:     Sample added noise  $\epsilon \sim \mathcal{N}(0, I)$
  - 9:     Compute noisy sample  $\hat{x}_t = \alpha_t \hat{x}_0 + \sigma_t \epsilon$
  - 10:      $\theta \leftarrow \theta - \eta_1 [\omega(t) (\epsilon_{\psi}(\hat{x}_t, t, y) - \epsilon_{\phi}(\hat{x}_t, t, y)) \frac{\partial \hat{x}_0}{\partial \theta}]$
  - 11:     Sample timestep  $t' \sim \mathcal{U}(0, T)$
  - 12:     Sample added noise  $\epsilon' \sim \mathcal{N}(0, I)$
  - 13:     Compute noisy sample  $\hat{x}_{t'} = \alpha_{t'} \hat{x}_0 + \sigma_{t'} \epsilon'$
  - 14:      $\phi \leftarrow \phi - \eta_2 \nabla_{\phi} \|\epsilon_{\phi}(\hat{x}_{t'}, t', y) - \epsilon'\|^2$
  - 15: **end while**
  - 16: **return** trained student model  $f_{\theta}$
- 



# Distillation Techniques: Rectified Flow



Advantages:

- High-quality few-step generation.
- Flexibility on inference steps.
- Simple forms.



# Distillation Techniques: Rectified Flow

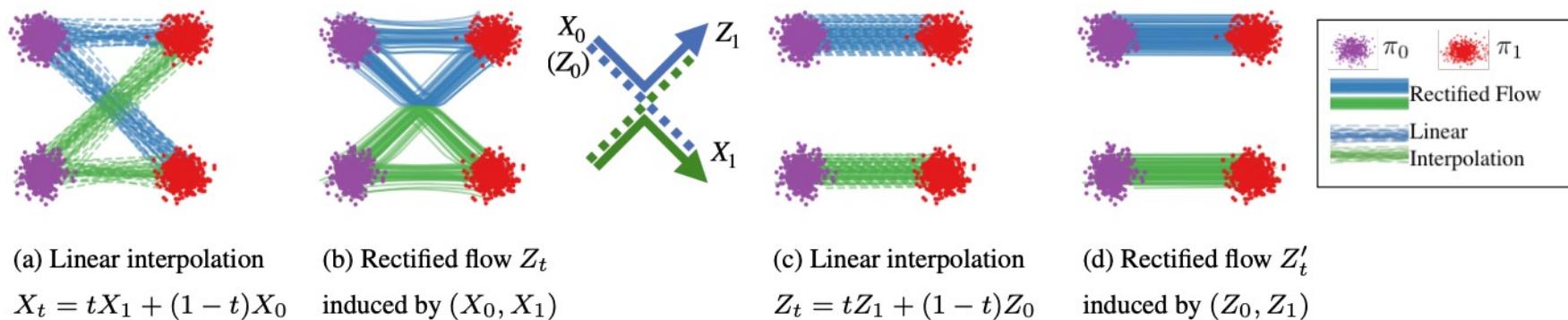
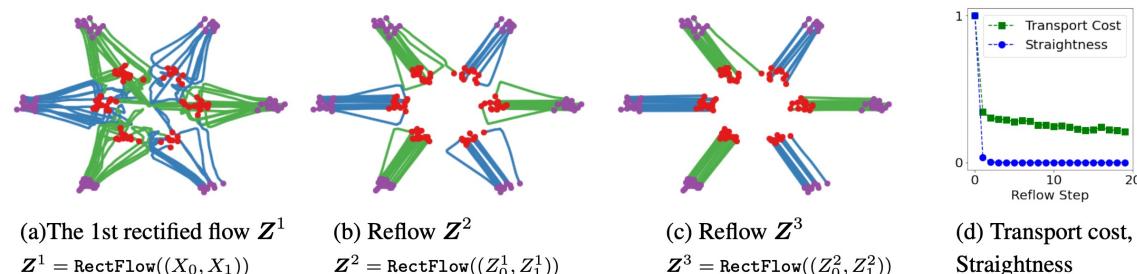
- Linear interpolation.

$$X_t = tX_1 + (1 - t)X_0$$

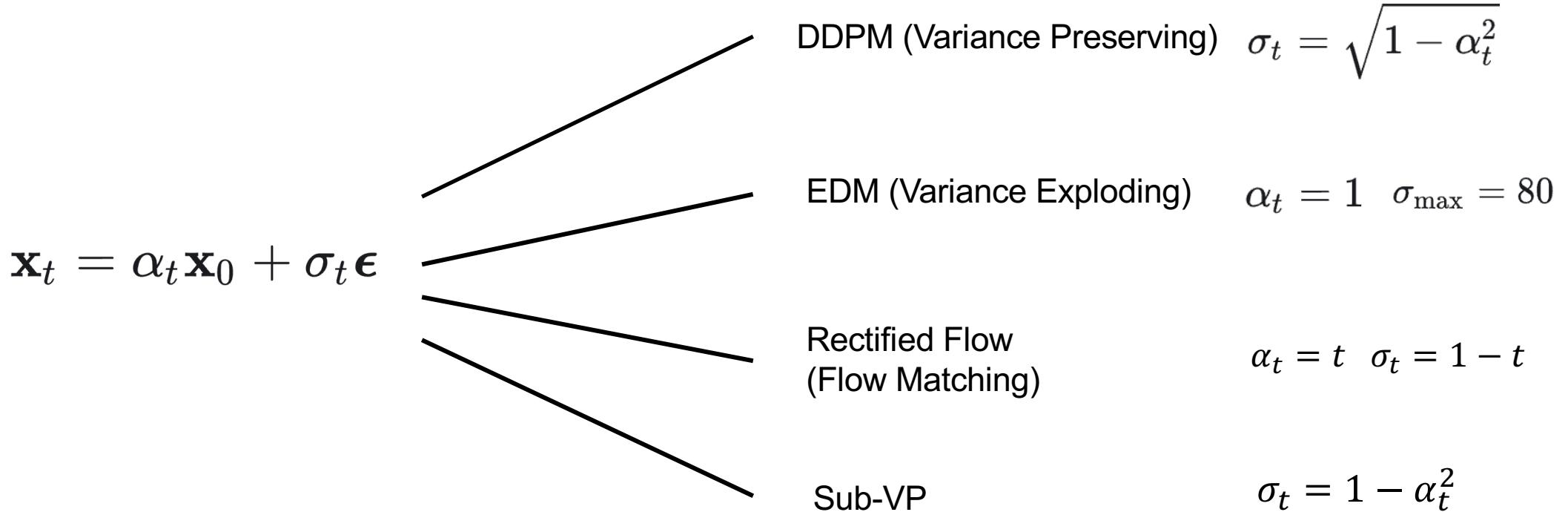
- $v$ -prediction.

$$dX_t = (X_1 - X_0)dt$$

- Rectification (Reflow).



# Diffusion Models: A (relative) Unified Perspective



# The Magic of Rectified Flow: Retraining with Matched Noise-Sample Pairs

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## Algorithm 1 Flow Matching $v$ -Prediction

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**Input:**

Sample  $\mathbf{x}_0$  from the data distribution  
Sample time  $t$  from a predefined schedule or uniformly from  $[0, 1]$   
Sample noise  $\epsilon$  from normal distribution  
Compute  $\mathbf{x}_t$ :  $\mathbf{x}_t = (1 - t) \cdot \mathbf{x}_0 + t \cdot \epsilon$   
Predict velocity  $\hat{\mathbf{v}}$  using the model:  $\hat{\mathbf{v}} = \text{Model}(\mathbf{x}_t, t)$   
Compute loss:  $\mathcal{L} = \|\hat{\mathbf{v}} - (\mathbf{x}_0 - \epsilon)\|_2^2$   
Backpropagate and update parameters

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## Algorithm 3 Rectified Flow $v$ -Prediction

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**Input: noise-data pair  $(\epsilon, \hat{\mathbf{x}}_0)$** 

~~Sample  $\mathbf{x}_0$  from the data distribution~~  
~~Sample time  $t$  from a predefined schedule or uniformly from  $[0, 1]$~~   
~~Sample noise  $\epsilon$  from normal distribution~~  
~~Compute  $\mathbf{x}_t$ :  $\mathbf{x}_t = (1 - t) \cdot \hat{\mathbf{x}}_0 + t \cdot \epsilon$~~   
~~Predict velocity  $\hat{\mathbf{v}}$  using the model:  $\hat{\mathbf{v}} = \text{Model}(\mathbf{x}_t, t)$~~   
~~Compute loss:  $\mathcal{L} = \|\hat{\mathbf{v}} - (\hat{\mathbf{x}}_0 - \epsilon)\|_2^2$~~   
Backpropagate and update parameters

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# Flow Matching Training Is a Subset of Diffusion Training

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## Algorithm 1 Flow Matching $v$ -Prediction

---

**Input:**

Sample  $\mathbf{x}_0$  from the data distribution  
Sample time  $t$  from a predefined schedule or uniformly from  $[0, 1]$   
Sample noise  $\epsilon$  from normal distribution  
Compute  $\mathbf{x}_t : \mathbf{x}_t = (1 - t) \cdot \mathbf{x}_0 + t \cdot \epsilon$   
Predict velocity  $\hat{\mathbf{v}}$  using the model:  $\hat{\mathbf{v}} = \text{Model}(\mathbf{x}_t, t)$   
Compute loss:  $\mathcal{L} = \|\hat{\mathbf{v}} - (\mathbf{x}_0 - \epsilon)\|_2^2$   
Backpropagate and update parameters

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## Algorithm 2 Diffusion Training $\epsilon$ -Prediction

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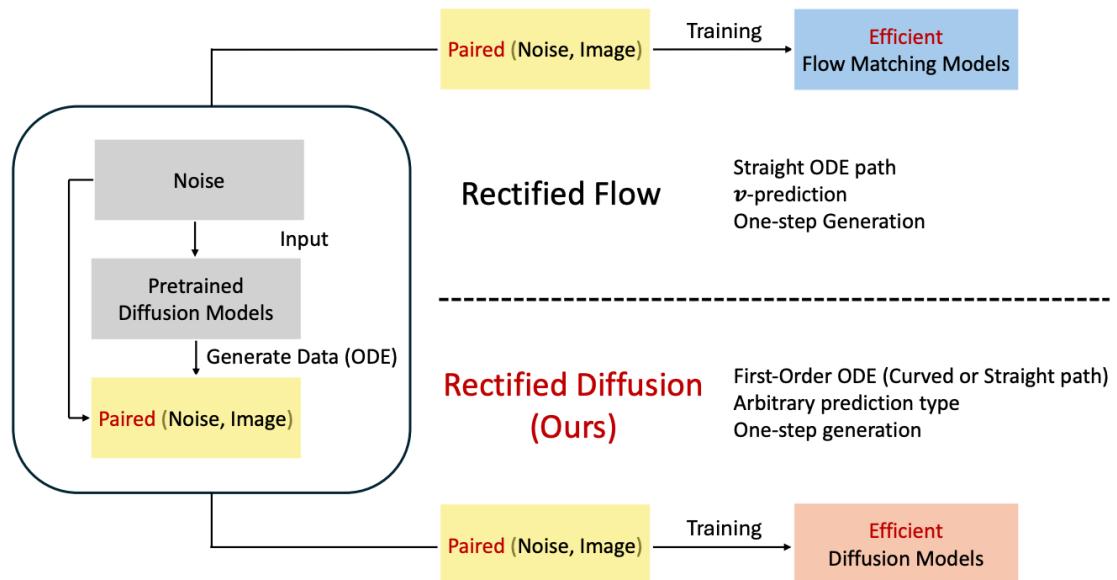
**Input:**  $\alpha_t, \sigma_t$

Sample  $\mathbf{x}_0$  from the data distribution  
Sample time  $t$  from a predefined schedule or uniformly from  $[0, 1]$   
Sample noise  $\epsilon$  from normal distribution  
Compute  $\mathbf{x}_t : \mathbf{x}_t = \alpha_t \cdot \mathbf{x}_0 + \sigma_t \cdot \epsilon$   
Predict noise  $\hat{\epsilon}$  using the model:  $\hat{\epsilon} = \text{Model}(\mathbf{x}_t, t)$   
Compute loss:  $\mathcal{L} = \|\hat{\epsilon} - \epsilon\|_2^2$   
Backpropagate and update parameters

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# Rectified Diffusion: Extending Rectified Flow to General Diffusion Models



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## Algorithm 4 Rectified Diffusion $\epsilon$ -Prediction

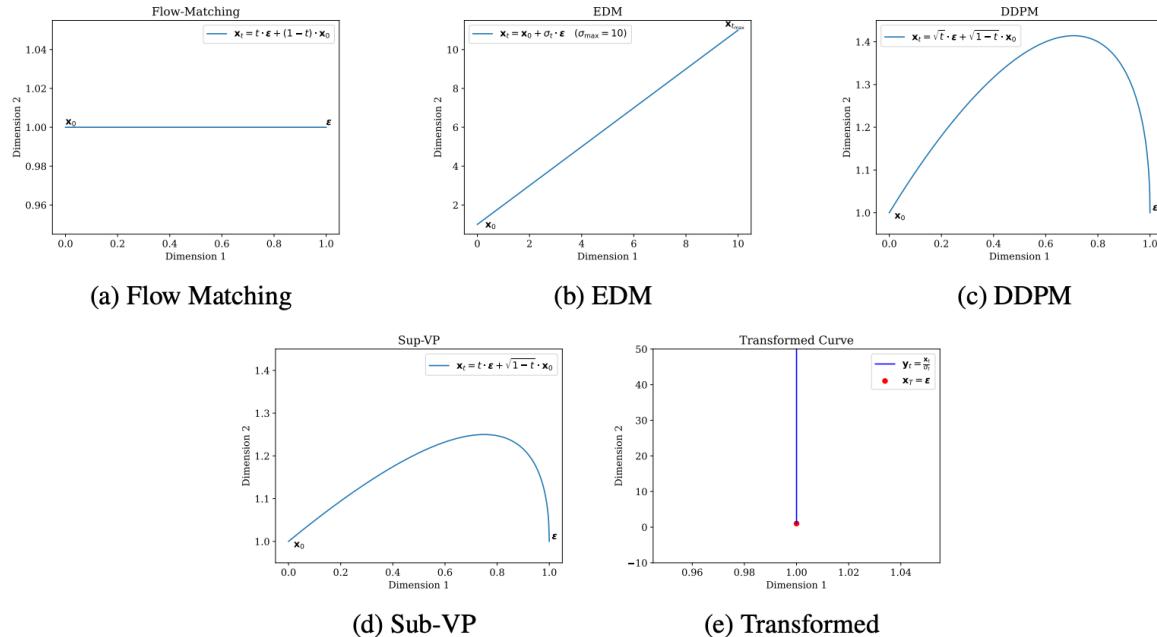
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**Input:** noise-data pair  $(\epsilon, \hat{x}_0)$ ,  $\alpha_t, \sigma_t$   
Sample  $x_0$  from the data distribution  
Sample time  $t$  from a predefined schedule or uniformly from  $[0, 1]$   
Sample noise  $\epsilon$  from normal distribution  
Compute  $x_t : x_t = \alpha_t \cdot \hat{x}_0 + \sigma_t \cdot \epsilon$   
Predict noise  $\hat{\epsilon}$  using the model:  $\hat{\epsilon} = \text{Model}(x_t, t)$   
Compute loss:  $\mathcal{L} = \|\hat{\epsilon} - \epsilon\|_2^2$   
Backpropagate and update parameters

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# Rectified Diffusion: the Essential Training Target Is First-Order ODE



## Important Points of First-Order ODE:

- It has the same form of predefined diffusion forms.

$$\mathbf{x}_t = \alpha_t \mathbf{x}_0 + \sigma_t \epsilon$$

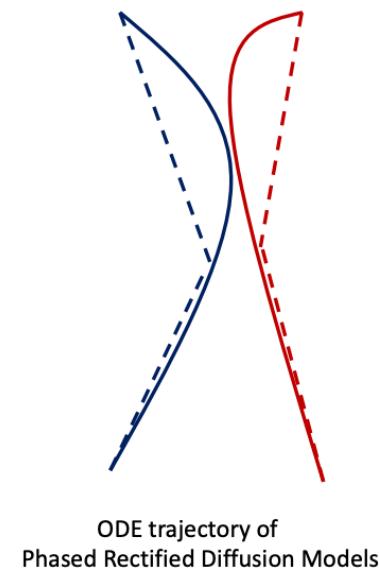
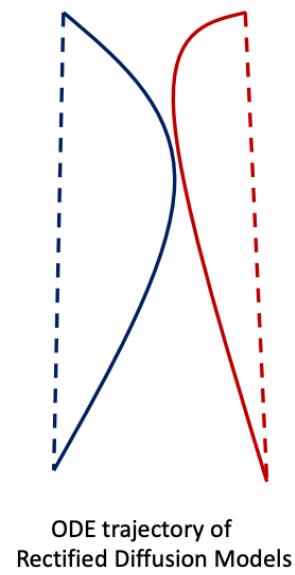
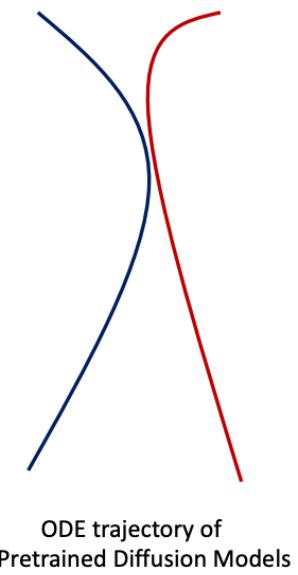
- It can be inherently curved.

- It can be transformed into straight lines with timestep dependent scaling.

$$\mathbf{y}_t = \frac{\alpha_t}{\sigma_t} \mathbf{x}_0 + \epsilon$$



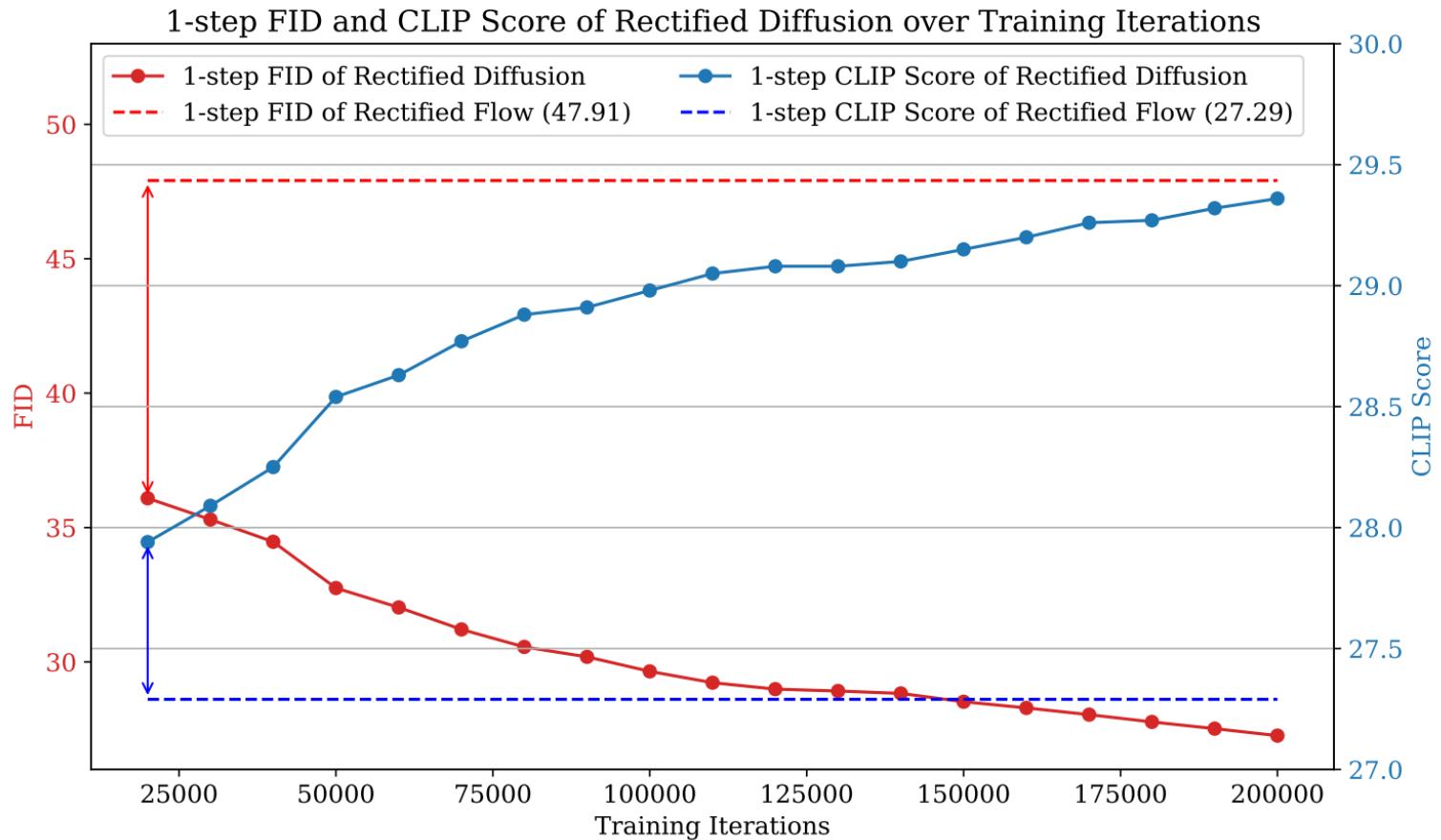
## Rectified Diffusion (Phased)



$$\epsilon = \frac{\frac{\mathbf{x}_{s_{m-1}}}{\alpha_{s_{m-1}}} - \frac{\mathbf{x}_{s_m}}{\alpha_{s_m}}}{\frac{\sigma_{s_{m-1}}}{\alpha_{s_{m-1}}} - \frac{\sigma_{s_m}}{\alpha_{s_m}}} = \frac{\Delta \mathbf{z}}{\Delta \text{NSR}}$$



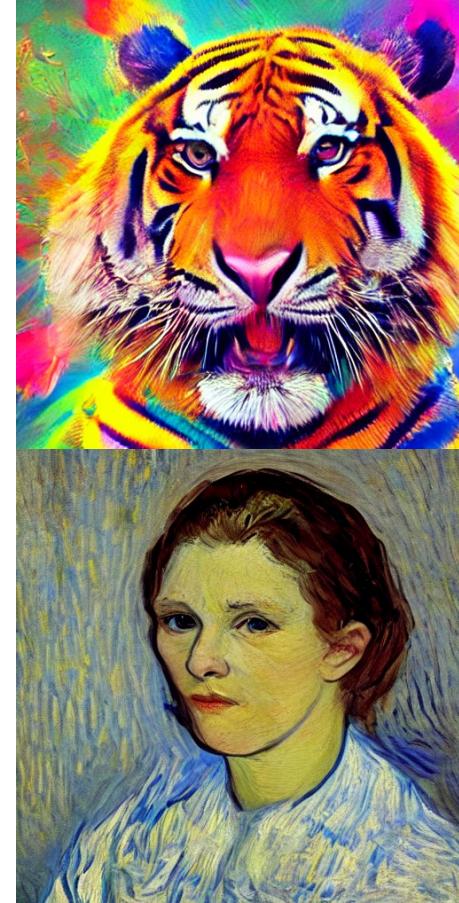
# Rectified Diffusion Vs Rectified Flow



# Rectified Diffusion Vs Rectified Flow



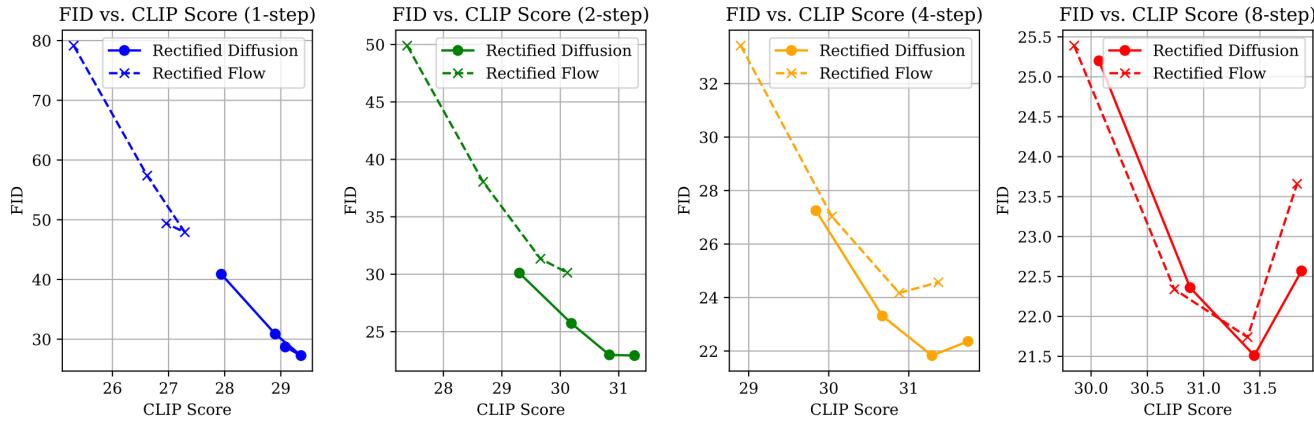
Rectified-Flow



Rectified-Diffusion



# Rectified Diffusion Vs Rectified Flow



Method	Res.	Time (↓)	# Steps	# Param.	FID (↓)	CLIP (↑)
SDv1-5+DPMSSolver (Upper-Bound) (Lu et al., 2022)	512	0.88s	25	0.9B	20.1	0.318
Rectified Flow (Liu et al., 2023)	512	0.88s	25	0.9B	21.65	0.315
Rectified Flow (Liu et al., 2023)	512	0.09s	1	0.9B	47.91	0.272
Rectified Flow (Liu et al., 2023)	512	0.13s	2	0.9B	31.35	0.296
Rectified Diffusion (Ours)	512	0.09s	25	0.9B	21.28	0.316
Rectified Diffusion (Ours)	512	0.09s	1	0.9B	27.26	0.295
Rectified Diffusion (Ours)	512	0.13s	2	0.9B	22.98	0.309
Rectified Flow (Distill) (Liu et al., 2023)	512	0.09s	1	0.9B	23.72	0.302
Rectified Flow (Distill) (Liu et al., 2023)	512	0.13s	2	0.9B	73.49	0.261
Rectified Flow (Distill) (Liu et al., 2023)	512	0.21s	4	0.9B	103.48	0.245
Rectified Diffusion (CD) (Ours)	512	0.09s	1	0.9B	22.83	0.305
Rectified Diffusion (CD) (Ours)	512	0.13s	2	0.9B	21.66	0.312
Rectified Diffusion (CD) (Ours)	512	0.21s	4	0.9B	21.43	0.314
PeRFFlow (Yan et al., 2024)	512	0.21s	4	0.9B	22.97	0.294
Rectified Diffusion (Phased) (Ours)	512	0.21s	4	0.9B	20.64	0.311
PeRFFlow-SDXL (Yan et al., 2024)	1024	0.71s	4	3B	27.06	0.335
Rectified Diffusion-SDXL (Phased) (Ours)	1024	0.71s	4	3B	25.81	0.341



## References

- [1] Denoising Diffusion Probabilistic Models.
- [2] Denoising Diffusion Implicit Models.
- [3] Score-Based Generative Modeling through Stochastic Differential Equations.
- [4] Flow Matching for Generative Modeling.
- [5] Elucidating the Design Space of Diffusion-Based Generative Models.
- [6] DPM-Solver: A Fast ODE Solver for Diffusion Probabilistic Model Sampling in Around 10 Steps.
- [7] Discrete Flow Matching.
- [8] Consistency Models.
- [9] Consistency Models Made Easy.
- [10] Latent Consistency Models: Synthesizing High-Resolution Images with Few-step Inference.
- [11] Phased Consistency Model.
- [12] Multistep Consistency Models.
- [13] PeRFlow: Piecewise Rectified Flow as Universal Plug-and-Play Accelerator.
- [14] Flow Straight and Fast: Learning to Generate and Transfer Data with Rectified Flow.
- [15] InstaFlow: One Step is Enough for High-Quality Diffusion-Based Text-to-Image Generation.
- [16] StyleGAN-T: Unlocking the Power of GANs for Fast Large-Scale Text-to-Image Synthesis.
- [17] Stable Consistency Tuning: Understanding and Improving Consistency Models.
- [18] SwiftBrush : One-Step Text-to-Image Diffusion Model with Variational Score Distillation.
- [19] One-step Diffusion with Distribution Matching Distillation.
- [20] Progressive Distillation for Fast Sampling of Diffusion Models



## Our Works

- [1] Stable Consistency Tuning: Understanding and Improving Consistency Models.
- [2] Rectified Diffusion: Straightness Is Not Your Need in Rectified Flow.
- [3] Phased Consistency Model.
- [4] AnimateLCM: Computation-Efficient Personalized Style Video Generation without Personalized Video Data.



# Thank you!

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