NLP - Fall 2016 - Midterm Exam

(1) **Probability Models and Smoothing**:

a. (4pts) You are given the following training data for the prepositional phrase (PP) attachment task.

v	n1	p	n2	Attachment
join	board	as	director	V
is	chairman	of	N.V.	N
using	crocidolite	in	filters	V
⋮	⋮	⋮	⋮	⋮

In order to resolve PP attachment ambiguity we can train a probability model: $P(A = N \mid v, n1, p, n2)$ which predicts the attachment A as N if $P > 0.5$ and V otherwise. Since we are unlikely to see the same four words $v, n1, p, n2$ in novel unseen data, in order for this probability model to be useful we need to take care of zero counts.

Provide a Jelinek-Mercer *interpolation* smoothing model $\hat{P}(A = N \mid v, n1, p, n2)$ for this PP attachment probability model. Assume that our training data is large enough to contain all the prepositions we might observe in unseen data. You cannot assume that all the verbs and nouns in the unseen data were seen in training.

In the interpolation model if you use any new variables then provide the constraints that the variables must obey such that $\hat{P}$ continues to be a valid probability.

Answer:

$$\begin{aligned}\hat{P}(A = N \mid v, n1, p, n2) &= \lambda_1 P(A = N \mid v, n1, p, n2) \\ &\quad +\lambda_2 P(A = N \mid v, n1, p) \\ &\quad +\lambda_3 P(A = N \mid n1, p) \\ &\quad +\lambda_4 P(A = N \mid v, p) \\ &\quad +\lambda_5 P(A = N \mid p)\end{aligned}$$

To be a well-formed interpolation model, $\sum_i \lambda_i = 1$. There are many other solutions such as for instance, the 4-gram model can be recursively interpolated with the 3-gram model and so on just like th JM model for n-grams.

b. (2pts) You are given a text of words: $w_1, \ldots, w_N$ and you proceed to estimate bigram probabilities with the maximum likelihood estimate using the bigram frequencies $c(w_i, w_{i-1})$ and unigram frequencies $c(w_i)$:

$$\hat{P}(w_i \mid w_{i-1}) = \frac{c(w_i, w_{i-1})}{c(w_{i-1})}$$

The frequency for all individual words w_i are non-zero, i.e. $c(w_i) > 0$ for all w_i but many bigrams w_i, w_{i-1} have zero counts. To avoid zeroes in the numerator of the above equation, you decide to add a small factor δ to each count in the following manner:

$$\hat{P}(w_i \mid w_{i-1}) = \frac{\delta + c(w_i, w_{i-1})}{c(w_{i-1})}$$

Where $0 < \delta < 1$. Is this equation correct? If not, what is the basic condition on $P(w_i \mid w_{i-1})$ violated in this formula? If incorrect, provide a correction for this formula.

Answer: The following condition is violated:

$$1 = \sum_{w_i} \hat{P}(w_i \mid w_{i-1})$$

Assuming a vocabulary size of V the number of δ terms added across all the numerators will be δV so to make it sum to 1:

$$\hat{P}(w_i \mid w_{i-1}) = \frac{\delta + c(w_i, w_{i-1})}{\delta V + c(w_i)}$$

c. (4pts) For bigram probabilities, Katz backoff smoothing is defined as follows:

$$P_{katz}(w_i \mid w_{i-1}) = \begin{cases} \frac{c^*(w_{i-1}, w_i)}{c(w_{i-1})} & \text{if } c(w_{i-1}, w_i) > 0 \\ \alpha(w_{i-1}) P_{katz}(w_i) & \text{otherwise} \end{cases}$$

where $\alpha(w_{i-1})$ is chosen to make sure that $P_{katz}(w_i \mid w_{i-1})$ is a proper probability

$$\alpha(w_{i-1}) = 1 - \sum_{w_i} \frac{c^*(w_{i-1}, w_i)}{c(w_{i-1})}$$

Provide two definitions for the new count $c^*(w_{i-1}, w_i)$, (1) using the Good-Turing method, and (2) the absolute discounting method. Assume that $1 = \sum_{w_i} P_{katz}(w_i)$.

Answer:

Good Turing

$$c^*(w_{i-1}, w_i) = (c(w_{i-1}, w_i) + 1) \times \frac{n_{c(w_{i-1}, w_i)+1}}{n_{c(w_{i-1}, w_i)}}$$

$$c^*(w_i) = (c(w_i) + 1) \times \frac{n_{c(w_i)+1}}{n_{c(w_i)}}$$

where $n_{c(w_{i-1}, w_i)}$ and $n_{c(w_i)}$ stands for the number of different w_{i-1}, w_i or w_i types observed for count $c(w_{i-1}, w_i)$ and $c(w_i)$ respectively.

Absolute Discounting

$$c^*(w_{i-1}, w_i) = c(w_{i-1}, w_i) - D$$

where D is set to some value less than one using held out set.

(2) **Word Alignment and Machine Translation**:

IBM Model 1 provides the probability of an alignment **a** and source sentence **f** given a target sentence **e** using the parameters t follows:

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

a. (5pts) In order to compute the most likely alignment **a** we need to compute $P(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$. Derive the following formula and provide the value of Z using parameters t:

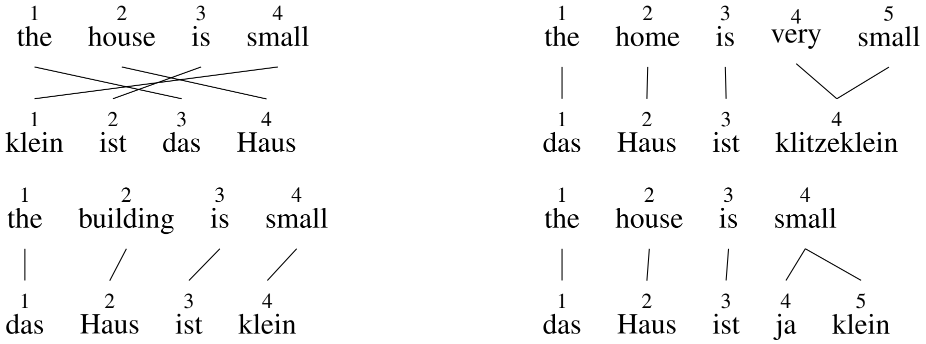
$$\log \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \sum_{i=1}^{I} \log t(f_i \mid e_{a_i}) - \log Z$$

Answer:

$$\log \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \log \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) - \log Z$$

$$\begin{aligned}\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\ &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})} \\ &= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\underbrace{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})}_{Z}}\end{aligned}$$

b. (5pts) Consider the following sentence aligned and word aligned parallel corpus.


1 the 2 house 3 is 4 small
1 klein 2 ist 3 das 4 Haus
1 the 2 home 3 is 4 very 5 small
1 das 2 Haus 3 ist 4 klitzeklein
1 the 2 building 3 is 4 small
1 das 2 Haus 3 ist 4 klein
1 the 2 house 3 is 4 small
1 das 2 Haus 3 ist 4 ja 5 klein

Provide the values of the following parameters:

i $t(\text{klein} \mid \text{small})$

Answer: $\frac{3}{5}$

ii $t(\text{klitzeklein} \mid \text{small})$

Answer: $\frac{1}{5}$

iii $t(\text{ja} \mid \text{small})$

Answer: $\frac{1}{5}$

iv $t(\text{ja} \mid \text{very})$

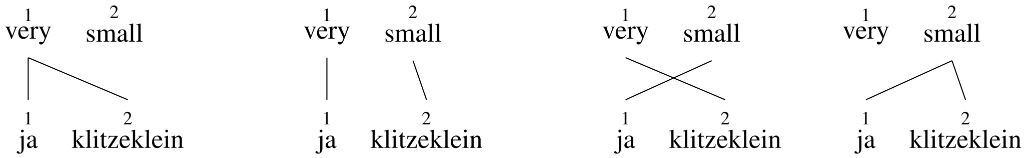
Answer: 0

v $t(\text{klitzeklein} \mid \text{very})$

Answer: 1

c. (5pts) Compute Pr(**ja klitzeklein | very small**) using IBM Model 1 without NULLs. Show all the steps in the computation and write your answer as a fraction $\frac{x}{y}$.

Answer: First compute Pr(**f**, **a** | **e**) for each alignment **a**:



$$0 \times 1 = 0 \qquad 0 \times \frac{1}{5} = 0 \qquad \frac{1}{5} \times 1 = \frac{1}{5} \qquad \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$$

Then sum over all alignments to get Pr(**f** | **e**):

$$0 + 0 + \frac{1}{5} + \frac{1}{25} = \frac{30}{125} = 0.24$$