Natural Language Processing

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anoopsarkar.github.io/nlp-class

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Part 1: Generative Models for Word Alignment

Statistical Machine Translation

Generative Model of Word Alignment
- Word Alignments: IBM Model 3
- Word Alignments: IBM Model 1
- Finding the best alignment: IBM Model 1
- Learning Parameters: IBM Model 1
- IBM Model 2
- Back to IBM Model 3

Statistical Machine Translation

Noisy Channel Model

$$\mathbf{e}^* = \arg\max_{\mathbf{e}} \underbrace{\Pr(\mathbf{e})}_{\textbf{Language Model}}$$

Statistical Machine Translation

Noisy Channel Model

$$\mathbf{e}^* \quad = \quad \arg\max_{\mathbf{e}} \quad \underbrace{\Pr(\mathbf{e})}_{\textbf{Language Model}} \quad \cdot \quad \underbrace{\Pr(\mathbf{f} \mid \mathbf{e})}_{\textbf{Alignment Model}}$$

Alignment Task

$$\begin{tikzcd} \mathbf{e} \arrow[dr] & & \\ & \text{Program} \arrow[r] & \Pr(\mathbf{e} \mid \mathbf{f}) \\ \mathbf{f} \arrow[ur] & & \end{tikzcd}$$

Alignment Task

$$\begin{array}{l} \mathbf{e} \searrow \\ \qquad \text{Program} \longrightarrow \Pr(\mathbf{e} \mid \mathbf{f}) \\ \mathbf{f} \nearrow \end{array}$$

Training Data

- **Alignment Model**: learn a mapping between $\mathbf{f}$and $\mathbf{e}$.
  Training data: lots of translation pairs between $\mathbf{f}$and $\mathbf{e}$.

Statistical Machine Translation

The IBM Models

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  as they were called in the 1993 paper.

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- These models are the basic SMT models, called:
  IBM Model 1, IBM Model 2, IBM Model 3, IBM Model 4, IBM Model 5
  as they were called in the 1993 paper.
- We use **e**and **f** in the equations in honor of their system which translated from French to English.
  Trained on the Canadian Hansards (Parliament Proceedings)

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Generative Model of Word Alignment

- English **e**: Mary did not slap the green witch
- "French" **f**: Maria no daba una botefada a la bruja verde
- Alignment **a**: $\{1, 3, 4, 4, 4, 5, 5, 7, 6\}$
  e.g. $(f_8, e_{a_8}) = (f_8, e_7) =$ (bruja, witch)

Generative Model of Word Alignment

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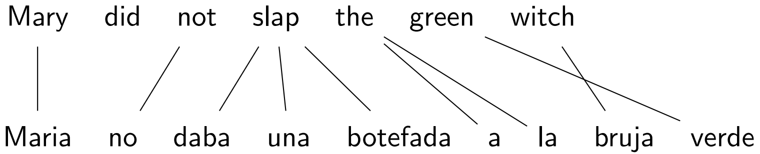
Visualizing alignment **a**



Generative Model of Word Alignment

Data Set

- Data set $\mathcal{D}$ of N sentences:
$$\mathcal{D} = \{(\mathbf{f}^{(1)}, \mathbf{e}^{(1)}), \ldots, (\mathbf{f}^{(N)}, \mathbf{e}^{(N)})\}$$

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- $\text{length}(\mathbf{f}) = \text{length}(\mathbf{a}) = I$

Generative Model of Word Alignment

Find the best alignment for each translation pair

$$\mathbf{a}^* = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$$

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Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a}, \mathbf{e})}{\Pr(\mathbf{f}, \mathbf{e})}$$

Generative Model of Word Alignment

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&= \frac{\Pr(\mathbf{e}) \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{e}) \Pr(\mathbf{f} \mid \mathbf{e})}
\end{aligned}$$

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&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}
\end{aligned}$$

Statistical Machine Translation

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Word Alignments: IBM Model 3

Generative “story” for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

Mary did not slap the green witch

Mary not slap slap slap the the green witch (fertility)

Word Alignments: IBM Model 3

Generative "story" for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

Mary did not slap the green witch

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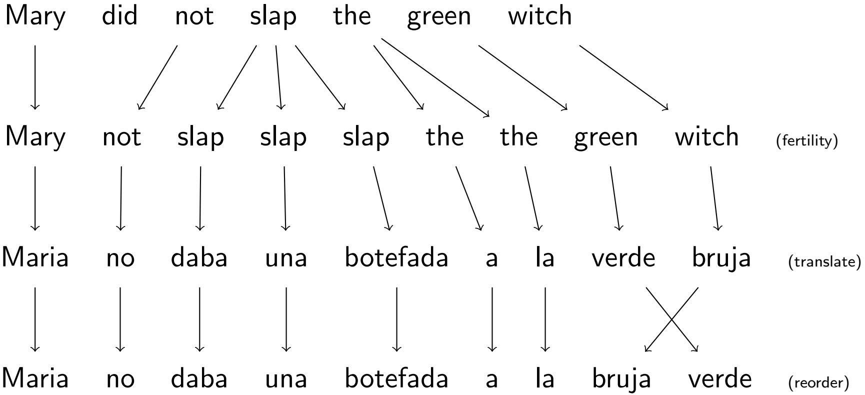
Maria no daba una botefada a la verde bruja (translate)

Word Alignments: IBM Model 3

Generative "story" for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$



Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid \textit{slap}); n(0 \mid \textit{did})$$

Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid \mathit{slap}); n(0 \mid \mathit{did})$$

Translation parameter

$$t(f_i \mid e_{a_i}) : t(\mathit{bruja} \mid \mathit{witch})$$

Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid slap); n(0 \mid did)$$

Translation parameter

$$t(f_i \mid e_{a_i}) : t(bruja \mid witch)$$

Distortion parameter

$$d(f_{pos} = i \mid e_{pos} = j, I, J) : d(8 \mid 7, 9, 7)$$

Word Alignments: IBM Model 3

Generative model for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

$$P(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) \quad = \quad \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i})$$

Word Alignments: IBM Model 3

Generative model for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

$$
\begin{aligned}
P(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) &= \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \\
&\times \quad t(f_i \mid e_{a_i})
\end{aligned}
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Word Alignments: IBM Model 3

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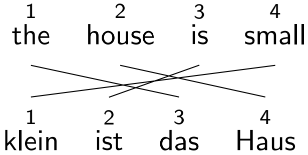
Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$



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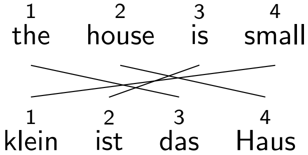


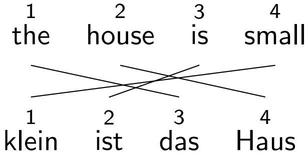
If we know the parameter values we can easily compute the probability of this aligned sentence pair.

$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$

$$n(1 \mid \text{the}) \quad \times \quad t(\text{das} \mid \text{the}) \quad \times \quad d(3 \mid 1, 4, 4) \times$$

Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$



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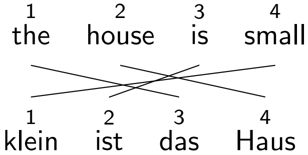
$$
\begin{array}{lllll}
n(1 \mid \text{the}) & \times & t(\text{das} \mid \text{the}) & \times & d(3 \mid 1, 4, 4) \times \\
n(1 \mid \text{house}) & \times & t(\text{Haus} \mid \text{house}) & \times & d(4 \mid 2, 4, 4) \times
\end{array}
$$

Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$



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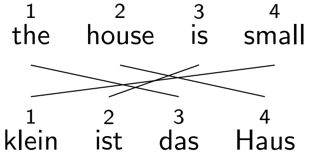
n(1 \mid \text{is}) & \times & t(\text{ist} \mid \text{is}) & \times & d(2 \mid 3, 4, 4) \times
\end{array}
$$

Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$



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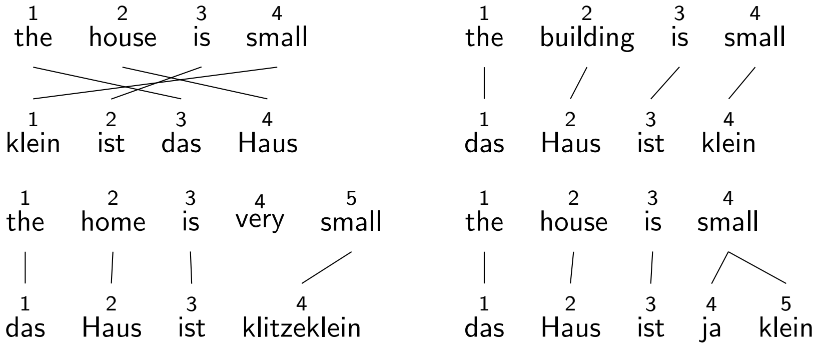
n(1 \mid \text{is}) & \times & t(\text{ist} \mid \text{is}) & \times & d(2 \mid 3, 4, 4) \times \\
n(1 \mid \text{small}) & \times & t(\text{klein} \mid \text{small}) & \times & d(1 \mid 4, 4, 4)
\end{array}$$

Word Alignments: IBM Model 3



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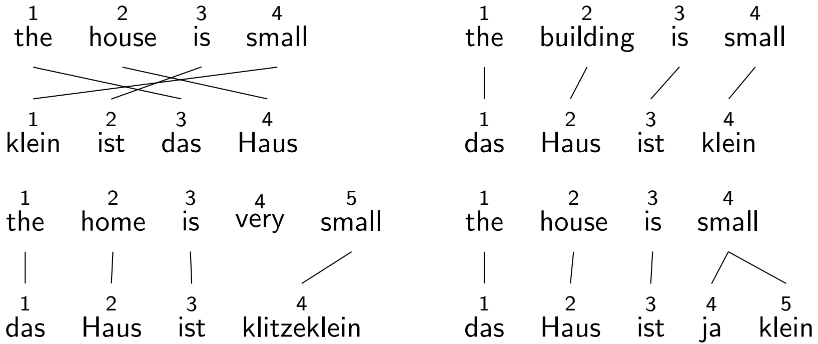
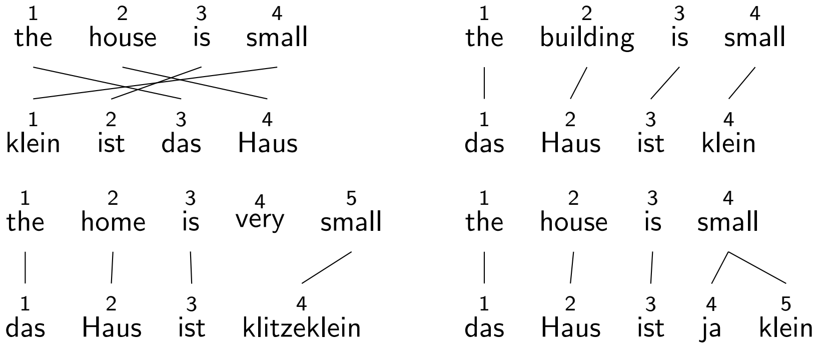


Parameter Estimation

- What is $n(1 \mid \mathrm{very}) = ?$ and $n(0 \mid \mathrm{very}) = ?$

Word Alignments: IBM Model 3



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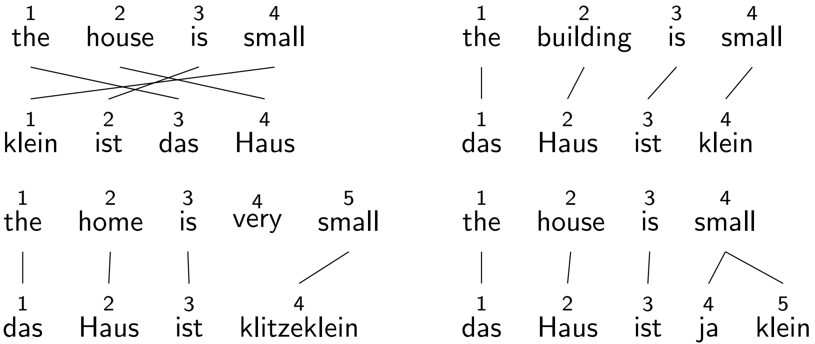
Word Alignments: IBM Model 3



Parameter Estimation

- What is $n(1 \mid \mathrm{very}) = ?$ and $n(0 \mid \mathrm{very}) = ?$
- What is $t(\mathrm{Haus} \mid \mathrm{house}) = ?$ and $t(\mathrm{klein} \mid \mathrm{small}) = ?$
- What is $d(1 \mid 4, 4, 4) = ?$ and $d(1 \mid 1, 4, 4) = ?$

Word Alignments: IBM Model 3

1	2	3	4
the	house	is	small
1	2	3	4
klein	ist	das	Haus

1	2	3	4
the	building	is	small
1	2	3	4
das	Haus	ist	klein

1	2	3	4	5
the	home	is	very	small
1	2	3	4	
das	Haus	ist	klitzeklein	

1	2	3	4	
the	house	is	small	
1	2	3	4	5
das	Haus	ist	ja	klein

Word Alignments: IBM Model 3

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1	2	3	4	5
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1	2	3	4	5
das	Haus	ist	ja	klein

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, I, J)$$

Word Alignments: IBM Model 3

Summary

- If we know the parameter values we can easily compute the probability $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$ given an aligned sentence pair

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- If we do not know the alignments then perhaps we can produce all possible alignments each with a certain probability?

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IBM Model 3 is too hard: Let us try learning only $t(f_i \mid e_{a_i})$

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Statistical Machine Translation

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Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

Word Alignments: IBM Model 1

Alignment probability

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Example alignment

1	2	3	4
the	house	is	small
|	|	|	|
1	2	3	4
das	Haus	ist	klein

Word Alignments: IBM Model 1

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$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = t(\text{das} \mid \text{the}) \times t(\text{Haus} \mid \text{house}) \times t(\text{ist} \mid \text{is}) \times t(\text{klein} \mid \text{small})$$

Word Alignments: IBM Model 1

Generative "story" for Model 1

the	house	is	small	
↓	↓	↓	↓	
das	Haus	ist	klein	(translate)

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

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Finding the best word alignment: IBM Model 1

Compute the arg max word alignment

$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{e}, \mathbf{f})$$

- For each f_i in $(f_1, \ldots, f_I)$ build $\mathbf{a} = (\hat{a}_1, \ldots, \hat{a}_I)$

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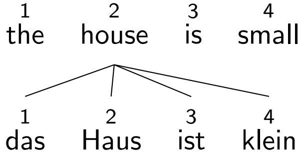
Many to one alignment ✓



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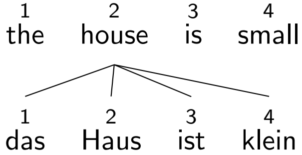
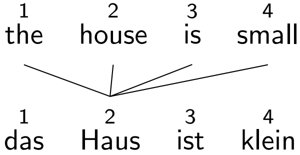
$$\hat{a}_i = \arg\max_{a_i} t(f_i \mid e_{a_i})$$

Many to one alignment ✓



One to many alignment ✗



Statistical Machine Translation

Generative Model of Word Alignment
  Word Alignments: IBM Model 3
  Word Alignments: IBM Model 1
  Finding the best alignment: IBM Model 1
  Learning Parameters: IBM Model 1
  IBM Model 2
  Back to IBM Model 3

Learning parameters[from P.Koehn SMT book slides]

- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus

Learning parameters[from P.Koehn SMT book slides]

- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus
- ... but we do not have the alignments

Learning parameters[from P.Koehn SMT book slides]

- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus
- ... but we do not have the alignments
- Chicken and egg problem

Learning parameters[from P.Koehn SMT book slides]

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  - if we had the *alignments*,
    $\rightarrow$ we could estimate the *parameters* of our generative model

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- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus
- ... but we do not have the alignments
- Chicken and egg problem
  - if we had the *alignments*,
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  - if we had the *parameters*,
    $\rightarrow$ we could estimate the *alignments*

EM Algorithm[from P.Koehn SMT book slides]

- Incomplete data

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  - if we had *complete data*, we could estimate *model*

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- Expectation Maximization (EM) in a nutshell

EM Algorithm [from P.Koehn SMT book slides]

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  1. initialize model parameters (e.g. uniform)

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  2. assign probabilities to the missing data

EM Algorithm [from P.Koehn SMT book slides]

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  3. estimate model parameters from completed data

EM Algorithm[from P.Koehn SMT book slides]

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  - if we had *complete data*, we could estimate *model*
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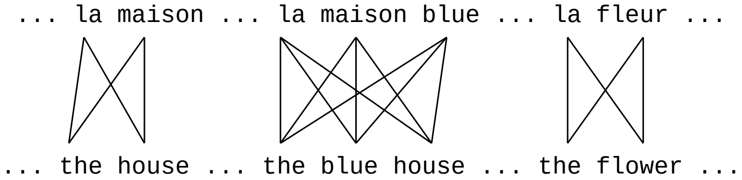
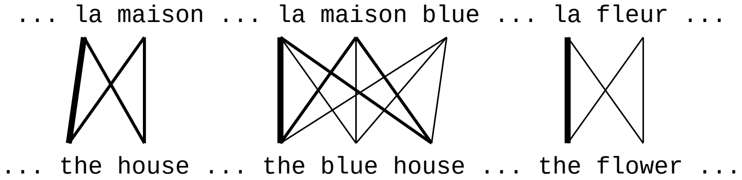
- Expectation Maximization (EM) in a nutshell
  1. initialize model parameters (e.g. uniform)
  2. assign probabilities to the missing data
  3. estimate model parameters from completed data
  4. iterate steps 2–3 until convergence

EM Algorithm [from P.Koehn SMT book slides]



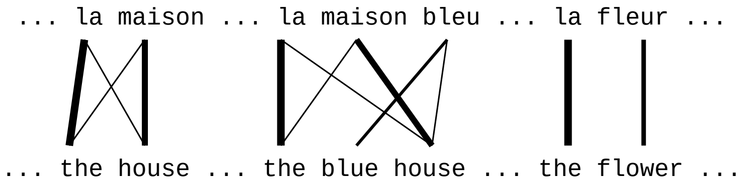
- Initial step: all alignments equally likely
- Model learns that, e.g., *la* is often aligned with *the*

EM Algorithm[from P.Koehn SMT book slides]



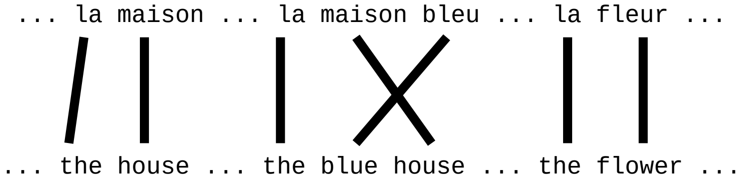
- After one iteration
- Alignments, e.g., between *la* and *the* are more likely

EM Algorithm[from P.Koehn SMT book slides]



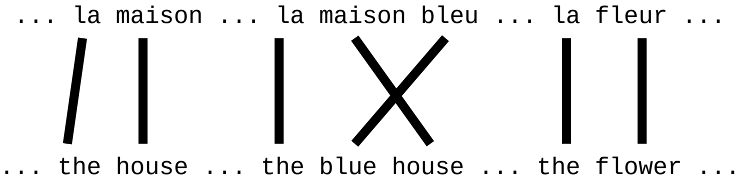
- After another iteration
- It becomes apparent that alignments, e.g., between *fleur* and *flower* are more likely (pigeon hole principle)

EM Algorithm[from P.Koehn SMT book slides]



- Convergence
- Inherent hidden structure revealed by EM

EM Algorithm [from P.Koehn SMT book slides]




```
p(la|the) = 0.453
p(le|the) = 0.334
p(maison|house) = 0.876
p(bleu|blue) = 0.563
...
```

- Parameter estimation from the aligned corpus

IBM Model 1 and the EM Algorithm[from P.Koehn SMT book slides]

- EM Algorithm consists of two steps

IBM Model 1 and the EM Algorithm[from P.Koehn SMT book slides]

- EM Algorithm consists of two steps
- Expectation-Step: Apply model to the data

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IBM Model 1 and the EM Algorithm [from P.Koehn SMT book slides]

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IBM Model 1 and the EM Algorithm [from P.Koehn SMT book slides]

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IBM Model 1 and the EM Algorithm [from P.Koehn SMT book slides]

- EM Algorithm consists of two steps
- Expectation-Step: Apply model to the data
  - parts of the model are hidden (here: alignments)
  - using the model, assign probabilities to possible values
- Maximization-Step: Estimate model from data
  - take assign values as fact
  - collect counts (weighted by probabilities)
  - estimate model from counts
- Iterate these steps until convergence

IBM Model 1 and the EM Algorithm[from P.Koehn SMT book slides]

- We need to be able to compute:

IBM Model 1 and the EM Algorithm[from P.Koehn SMT book slides]

- We need to be able to compute:
  - Expectation-Step: probability of alignments

IBM Model 1 and the EM Algorithm [from P.Koehn SMT book slides]

- We need to be able to compute:
  - Expectation-Step: probability of alignments
  - Maximization-Step: count collection

Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})}$$

Word Alignments: IBM Model 1

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}
\end{aligned}
$$

Word Alignments: IBM Model 1

Alignment probability

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\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})}
\end{aligned}$$

Word Alignments: IBM Model 1

Alignment probability

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&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})}
\end{aligned}$$

Computing the denominator

- The denominator above is summing over J^I alignments
- An interlude on how compute the denominator faster ...

Word Alignments: IBM Model 1

Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) \;=\; \sum_{a_1=1}^{J} \sum_{a_2=1}^{J} \dots \sum_{a_I=1}^{J} \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Word Alignments: IBM Model 1

Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) \;=\; \sum_{a_1=1}^{J} \sum_{a_2=1}^{J} \dots \sum_{a_I=1}^{J} \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Assume (f_1, f_2, f_3) and (e_1, e_2)

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

$J^I = 2^3$ terms to be added:

$$\begin{array}{ccccccc}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_2) &
\end{array}$$

Word Alignments: IBM Model 1

Factor the terms:

$$
\begin{array}{lcll}
(t(f_1 \mid e_1) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_1) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) &
\end{array}
$$

Word Alignments: IBM Model 1

Factor the terms:

$$
\begin{array}{lclc}
(t(f_1 \mid e_1) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_1) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) &
\end{array}
$$

$$
(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lclc}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) &
\end{array} \right)
$$

Word Alignments: IBM Model 1

Factor the terms:

$$\begin{array}{lcll}
(t(f_1 \mid e_1) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_1) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) &
\end{array}$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lcll}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) &
\end{array} \right)$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lcll}
t(f_1 \mid e_1) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) & + \\
t(f_1 \mid e_2) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) &
\end{array} \right)$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\prod_{i=1}^{3} \sum_{a_i=1}^{2} t(f_i \mid e_{a_i})$$

$I \times J = 2 \times 3$ terms to be added:

$$\begin{array}{lcll} (t(f_1 \mid e_1) & + & t(f_1 \mid e_2)) & \times \\ (t(f_2 \mid e_1) & + & t(f_2 \mid e_2)) & \times \\ (t(f_3 \mid e_1) & + & t(f_3 \mid e_2)) & \end{array}$$

Word Alignments: IBM Model 1

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
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\end{aligned}
$$

Word Alignments: IBM Model 1

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
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&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j)}
\end{aligned}
$$

Learning Parameters: IBM Model 1

1	2
the	house
das	Haus
1	2

1	2
the	book
das	Buch
1	2

1	2
a	book
ein	Buch
1	2

Learning Parameters: IBM Model 1

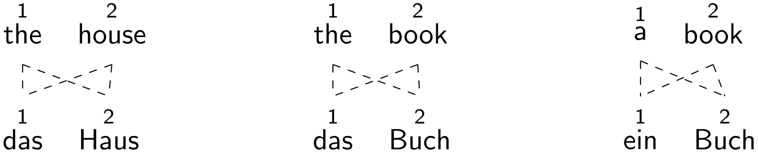
1	2		1	2		1	2
the	house		the	book		a	book
1	2		1	2		1	2
das	Haus		das	Buch		ein	Buch

Learning parameters $t(f|e)$ when alignments are known

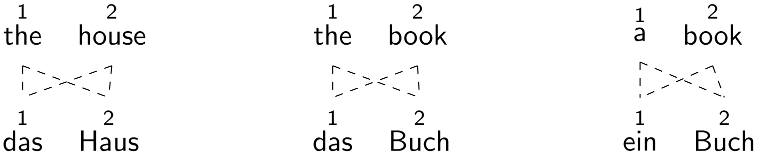
$$t(das \mid the) = \frac{c(das,the)}{\sum_f c(f,the)} \quad t(house \mid Haus) = \frac{c(Haus,house)}{\sum_f c(f,house)}$$
$$t(ein \mid a) = \frac{c(ein,a)}{\sum_f c(f,a)} \quad t(Buch \mid book) = \frac{c(Buch,book)}{\sum_f c(f,book)}$$

$$t(f|e) = \sum_{s=1}^{N} \sum_{f \rightarrow e \in \mathbf{f}^{(s)},\mathbf{e}^{(s)}} \frac{c(f,e)}{\sum_f c(f,e)}$$

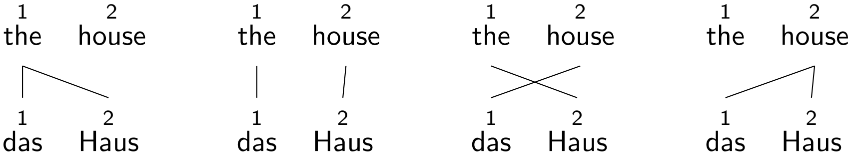
Learning Parameters: IBM Model 1



Learning Parameters: IBM Model 1



Learning parameters $t(f|e)$ when alignments are *unknown*



Also list alignments for *(the book, das Buch)* and *(a book, ein Buch)*

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

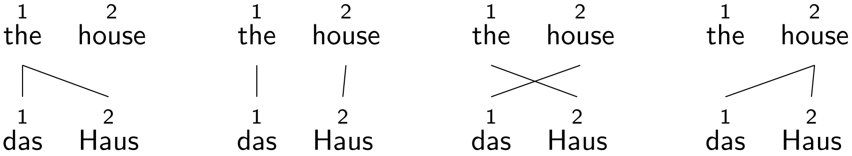
$$\begin{aligned} t(Haus \mid the) &= 0.25 \\ t(das \mid the) &= 0.5 \\ t(Buch \mid the) &= 0.25 \end{aligned} \qquad \begin{aligned} t(das \mid house) &= 0.5 \\ t(Haus \mid house) &= 0.5 \\ t(Buch \mid house) &= 0.0 \end{aligned}$$

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

$t(Haus \mid the) = 0.25$		$t(das \mid house) = 0.5$	
$t(das \mid the) = 0.5$		$t(Haus \mid house) = 0.5$	
$t(Buch \mid the) = 0.25$		$t(Buch \mid house) = 0.0$	

Compute posterior for each alignment



$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} = \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j)}$$

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

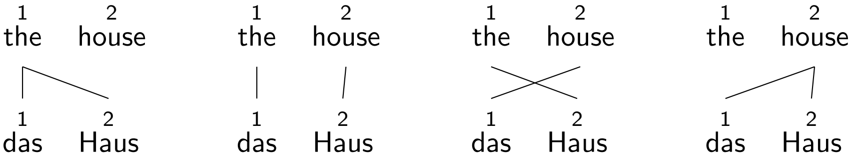
$$
\begin{aligned}
t(Haus \mid the) &= 0.25 \\
t(das \mid the) &= 0.5 \\
t(Buch \mid the) &= 0.25
\end{aligned}
\qquad
\begin{aligned}
t(das \mid house) &= 0.5 \\
t(Haus \mid house) &= 0.5 \\
t(Buch \mid house) &= 0.0
\end{aligned}
$$

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

$$
\begin{aligned}
t(\mathit{Haus} \mid \mathit{the}) &= 0.25 \qquad & t(\mathit{das} \mid \mathit{house}) &= 0.5 \\
t(\mathit{das} \mid \mathit{the}) &= 0.5 & t(\mathit{Haus} \mid \mathit{house}) &= 0.5 \\
t(\mathit{Buch} \mid \mathit{the}) &= 0.25 & t(\mathit{Buch} \mid \mathit{house}) &= 0.0
\end{aligned}
$$

Compute $\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})$ for each alignment

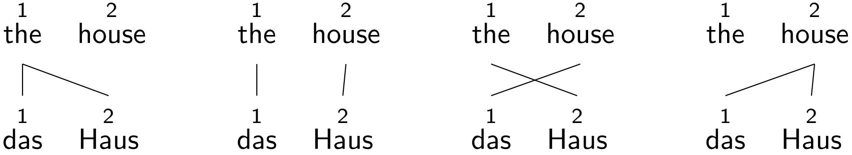


0.5×0.25	0.5×0.5	0.25×0.5	0.5×0.5
0.125	0.25	0.125	0.25

Learning Parameters: IBM Model 1

Compute $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{a}, \mathbf{f} | \mathbf{e})}{\Pr(\mathbf{f} | \mathbf{e})}$

$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$



$\frac{0.125}{0.75}$ 0.167

$\frac{0.25}{0.75}$ 0.334

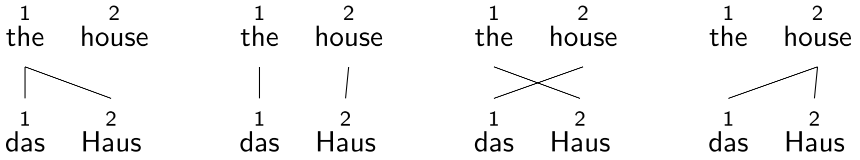
$\frac{0.125}{0.75}$ 0.167

$\frac{0.25}{0.75}$ 0.334

Learning Parameters: IBM Model 1

Compute $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{a},\mathbf{f}|\mathbf{e})}{\Pr(\mathbf{f}|\mathbf{e})}$

$$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$$

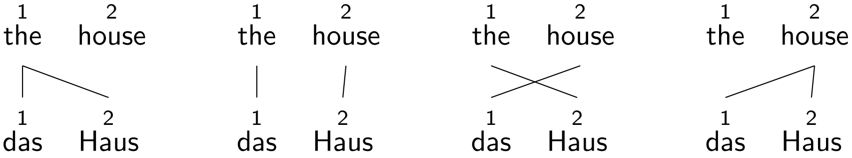


$\frac{0.125}{0.75}$	$\frac{0.25}{0.75}$	$\frac{0.125}{0.75}$	$\frac{0.25}{0.75}$
0.167	0.334	0.167	0.334

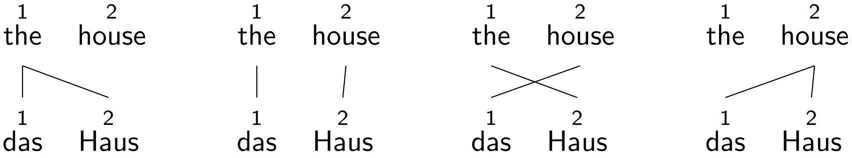
Compute fractional counts $c(f, e)$

$$\begin{aligned} c(Haus, the) &= 0.125 + 0.125 & c(das, house) &= 0.125 + 0.25 \\ c(das, the) &= 0.125 + 0.25 & c(Haus, house) &= 0.25 + 0.25 \\ c(Buch, the) &= 0.0 & c(Buch, house) &= 0.0 \end{aligned}$$

Learning Parameters: IBM Model 1



Learning Parameters: IBM Model 1

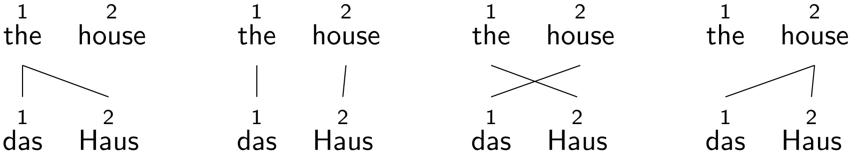


$$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$$

Expectation step: expected counts $g(f, e)$

$$\begin{aligned} g(das, the) &= \frac{0.125+0.25}{0.75} & g(das, house) &= \frac{0.125+0.25}{0.75} \\ g(Haus, the) &= \frac{0.125+0.125}{0.75} & g(Haus, house) &= \frac{0.25+0.25}{0.75} \\ g(Buch, the) &= 0.0 & g(Buch, house) &= 0.0 \end{aligned}$$

Learning Parameters: IBM Model 1



$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$

Expectation step: expected counts $g(f, e)$

$$
\begin{aligned}
g(das, the) &= \tfrac{0.125+0.25}{0.75} & \quad g(das, house) &= \tfrac{0.125+0.25}{0.75} \\
g(Haus, the) &= \tfrac{0.125+0.125}{0.75} & \quad g(Haus, house) &= \tfrac{0.25+0.25}{0.75} \\
g(Buch, the) &= 0.0 & \quad g(Buch, house) &= 0.0
\end{aligned}
$$

Maximization step: get new $t^{(1)}(f \mid e) = \frac{g(f,e)}{\sum_f g(f,e)}$

Learning Parameters: IBM Model 1

Expectation step: expected counts $g(f, e)$

$$\begin{aligned} g(das, the) &= 0.5 \\ g(Haus, the) &= 0.334 \\ g(Buch, the) &= 0.0 \\ \textbf{total} &= 0.834 \end{aligned}$$

$$\begin{aligned} g(das, house) &= 0.5 \\ g(Haus, house) &= 0.667 \\ g(Buch, house) &= 0.0 \\ \textbf{total} &= 1.167 \end{aligned}$$

Learning Parameters: IBM Model 1

Expectation step: expected counts $g(f, e)$

$$
\begin{array}{lcl@{\qquad\qquad}lcl}
g(das, the) & = & 0.5 & g(das, house) & = & 0.5 \\
g(Haus, the) & = & 0.334 & g(Haus, house) & = & 0.667 \\
g(Buch, the) & = & 0.0 & g(Buch, house) & = & 0.0 \\
\textbf{total} & = & 0.834 & \textbf{total} & = & 1.167
\end{array}
$$

Maximization step: get new $t^{(1)}(f \mid e) = \frac{g(f,e)}{\sum_f g(f,e)}$

$$
\begin{array}{lcl@{\qquad\qquad}lcl}
t(Haus \mid the) & = & 0.4 & t(das \mid house) & = & 0.43 \\
t(das, \mid the) & = & 0.6 & t(Haus \mid house) & = & 0.57 \\
t(Buch \mid the) & = & 0.0 & t(Buch \mid house) & = & 0.0
\end{array}
$$

Keep iterating: Compute $t^{(0)}, t^{(1)}, t^{(2)}, \ldots$ until convergence

Parameter Estimation: IBM Model 1

EM learns the parameters $t(\cdot \mid \cdot)$ that maximizes the log-likelihood of the training data:

$$\arg\max_t L(t) = \arg\max_t \sum_s \log \Pr(\mathbf{f}^{(s)} \mid \mathbf{e}^{(s)}, t)$$

Parameter Estimation: IBM Model 1

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- Start with an initial estimate t_0

Parameter Estimation: IBM Model 1

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- Start with an initial estimate t_0
- Modify it iteratively to get $t_1, t_2, \ldots$

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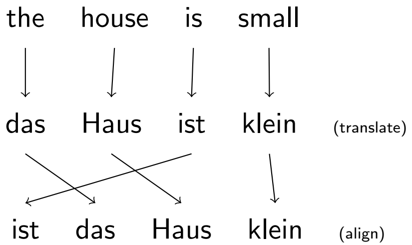
- Start with an initial estimate t_0
- Modify it iteratively to get $t_1, t_2, \ldots$
- Re-estimate t from parameters at previous time step t_{-1}
- The convergence proof of EM guarantees that $L(t) \geq L(t_{-1})$
- EM converges when $L(t) - L(t_{-1})$ is zero (or almost zero).

Statistical Machine Translation

Generative Model of Word Alignment
Word Alignments: IBM Model 3
Word Alignments: IBM Model 1
Finding the best alignment: IBM Model 1
Learning Parameters: IBM Model 1
IBM Model 2
Back to IBM Model 3

Word Alignments: IBM Model 2

Generative "story" for Model 2



$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)$$

Word Alignments: IBM Model 2

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

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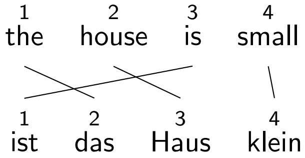
Example alignment



Word Alignments: IBM Model 2

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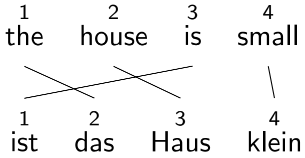

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$$
$$t(\text{das} \mid \text{the}) \times a(1 \mid 2, 4, 4) \times$$
$$t(\text{Haus} \mid \text{house}) \times a(2 \mid 3, 4, 4) \times$$
$$t(\text{ist} \mid \text{is}) \times a(3 \mid 1, 4, 4) \times$$
$$t(\text{klein} \mid \text{small}) \times a(4 \mid 4, 4, 4)$$

Word Alignments: IBM Model 2

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)}
\end{aligned}
$$

Word Alignments: IBM Model 2

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&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j) \times a(j \mid i, I, J)}
\end{aligned}
$$

Word Alignments: IBM Model 2

Learning the parameters

- EM training for IBM Model 2 works the same way as IBM Model 1

Word Alignments: IBM Model 2

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- We can do the same factorization trick to efficiently learn the parameters

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Word Alignments: IBM Model 2

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Word Alignments: IBM Model 2

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Word Alignments: IBM Model 2

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  - Maximization step: add up expected counts and normalize to get new parameter values

Word Alignments: IBM Model 2

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  - Maximization step: add up expected counts and normalize to get new parameter values
  - Repeat EM steps until convergence.

Statistical Machine Translation

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IBM Model 2
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Learning Parameters: IBM Model 3

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, I, J)$$

Sampling the Alignment Space [from P.Koehn SMT book slides]

- Training IBM Model 3 with the EM algorithm
  - The trick that reduces exponential complexity does not work anymore

  $\rightarrow$ Not possible to exhaustively consider all alignments
- Finding the most probable alignment by hillclimbing
  - start with initial alignment
  - change alignments for individual words
  - keep change if it has higher probability
  - continue until convergence
- Sampling: collecting variations to collect statistics
  - all alignments found during hillclimbing
  - neighboring alignments that differ by a move or a swap

Higher IBM Models[from P.Koehn SMT book slides]

IBM Model 1	lexical translation
IBM Model 2	adds absolute reordering model
IBM Model 3	adds fertility model
IBM Model 4	relative reordering model
IBM Model 5	fixes deficiency

- Only IBM Model 1 has global maximum
  - training of a higher IBM model builds on previous model
- Computationally biggest change in Model 3
  - trick to simplify estimation does not work anymore
  - $\rightarrow$ exhaustive count collection becomes computationally too expensive
  - sampling over high probability alignments is used instead

Summary[from P.Koehn SMT book slides]

- IBM Models were the pioneering models in statistical machine translation
- Introduced important concepts
  - generative model
  - EM training
  - reordering models

- Only used for niche applications as translation model
- ... but still in common use for word alignment (e.g., GIZA++, mgiza toolkit)

Natural Language Processing

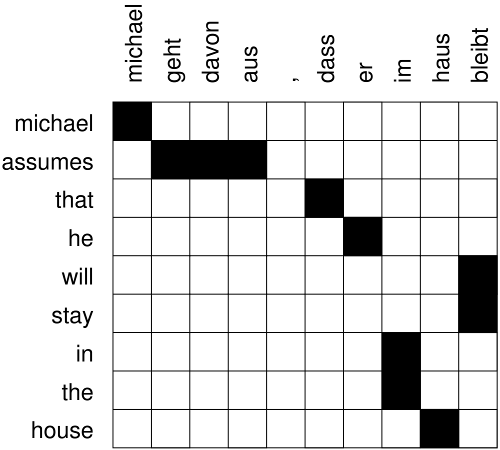
Anoop Sarkar
anoopsarkar.github.io/nlp-class

Simon Fraser University

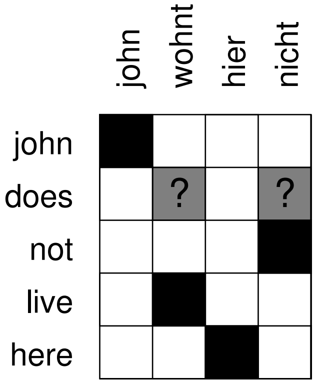
Part 2: Word Alignment

Word Alignment [from P.Koehn SMT book slides]

Given a sentence pair, which words correspond to each other?

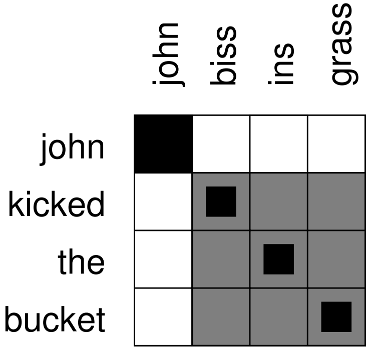


Word Alignment? [from P.Koehn SMT book slides]



Is the English word *does* aligned to
the German *wohnt* (verb) or *nicht* (negation) or neither?

Word Alignment? [from P.Koehn SMT book slides]



How do the idioms *kicked the bucket* and *biss ins grass* match up?
Outside this exceptional context, *bucket* is never a good translation for *grass*

Measuring Word Alignment Quality[from P.Koehn SMT book slides]

- Manually align corpus with *sure* (S) and *possible* (P) alignment points ($S \subseteq P$)
- Common metric for evaluation word alignments: Alignment Error Rate (AER)

$$\mathrm{AER}(S, P; A) = \frac{|A \cap S| + |A \cap P|}{|A| + |S|}$$

- AER $= 0$: alignment A matches all sure, any possible alignment points
- However: different applications require different precision/recall trade-offs

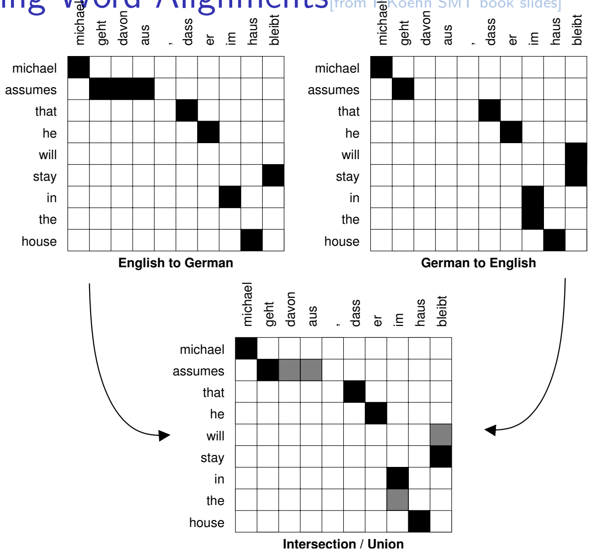
Word Alignment with IBM Models[from P.Koehn SMT book slides]

- IBM Models create a **many-to-one** mapping
  - words are aligned using an alignment function
  - a function may return the same value for different input

    (one-to-many mapping)
  - a function can not return multiple values for one input

    (no many-to-one mapping)
- Real word alignments have **many-to-many** mappings

Symmetrizing Word Alignments [from P.Koehn SMT book slides]



- Intersection plus grow additional alignment points [Och and Ney, CompLing2003]

Growing heuristic [from P.Koehn SMT book slides]

grow-diag-final(e2f,f2e)

```
neighboring = {(-1,0),(0,-1),(1,0),(0,1),(-1,-1),(-1,1),(1,-1),(1,1)}
alignment A = intersect(e2f,f2e); grow-diag(); final(e2f); final(f2e);
```

grow-diag()

```
while new points added do
    for all English word e ∈ [1...e_n], foreign word f ∈ [1...f_n], (e, f) ∈ A do
        for all neighboring alignment points (e_new, f_new) do
            if (e_new unaligned OR f_new unaligned) AND
            (e_new, f_new) ∈ union(e2f,f2e) then
                add (e_new, f_new) to A
            end if
        end for
    end for
end while
```

final()

```
for all English word e_new ∈ [1...e_n], foreign word f_new ∈ [1...f_n] do
    if (e_new unaligned OR f_new unaligned) AND (e_new, f_new) ∈ union(e2f,f2e)
    then
        add (e_new, f_new) to A
    end if
end for
```

More Recent Work on Symmetrization [from P.Koehn SMT book slides]

- Symmetrize after each iteration of IBM Models [Matusov et al., 2004]
  - run one iteration of E-step for each direction
  - symmetrize the two directions
  - count collection (M-step)

- Use of posterior probabilities in symmetrization
  - generate n-best alignments for each direction
  - calculate how often an alignment point occurs in these alignments
  - use this posterior probability during symmetrization

Link Deletion / Addition Models[from P.Koehn SMT book slides]

- Link deletion [Fossum et al., 2008]
  - start with union of IBM Model alignment points
  - delete one alignment point at a time
  - uses a neural network classifiers that also considers aspects such as how useful the alignment is for learning translation rules
- Link addition [Ren et al., 2007] [Ma et al., 2008]
  - possibly start with a skeleton of highly likely alignment points
  - add one alignment point at a time

Discriminative Training Methods[from P.Koehn SMT book slides]

- Given some annotated training data, supervised learning methods are possible
- Structured prediction
  - not just a classification problem
  - solution structure has to be constructed in steps
- Many approaches: maximum entropy, neural networks, support vector machines, conditional random fields, MIRA, ...
- Small labeled corpus may be used for parameter tuning of unsupervised aligner [Fraser and Marcu, 2007]

Better Generative Models[from P.Koehn SMT book slides]

- Aligning phrases
  - joint model [Marcu and Wong, 2002]
  - problem: EM algorithm likes really long phrases

- Fraser and Marcu: LEAF
  - decomposes word alignment into many steps
  - similar in spirit to IBM Models
  - includes step for grouping into phrase

Summary[from P.Koehn SMT book slides]

- Lexical translation
- Alignment
- Expectation Maximization (EM) Algorithm
- Noisy Channel Model
- IBM Models 1–5
  - IBM Model 1: lexical translation
  - IBM Model 2: alignment model
  - IBM Model 3: fertility
  - IBM Model 4: relative alignment model
  - IBM Model 5: deficiency
- Word Alignment

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All mistakes are my own.