Natural Language Processing

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Part 1: Generative Models for Word Alignment

Statistical Machine Translation

Noisy Channel Model

$$\mathbf{e}^* = \arg\max_{\mathbf{e}} \underbrace{\Pr(\mathbf{e})}_{\textbf{Language Model}} \cdot \underbrace{\Pr(\mathbf{f} \mid \mathbf{e})}_{\textbf{Alignment Model}}$$

Alignment Task

$$\begin{array}{l} \mathbf{e} \searrow \\ \quad \text{Program} \longrightarrow \Pr(\mathbf{e} \mid \mathbf{f}) \\ \mathbf{f} \nearrow \end{array}$$

Training Data

- **Alignment Model**: learn a mapping between $\mathbf{f}$and $\mathbf{e}$.
  Training data: lots of translation pairs between $\mathbf{f}$and $\mathbf{e}$.

Statistical Machine Translation

The IBM Models

- The first statistical machine translation models were developed at IBM Research (Yorktown Heights, NY) in the 1980s
- The models were published in 1993:
  Brown et. al. The Mathematics of Statistical Machine Translation.
  Computational Linguistics. 1993.
  `http://aclweb.org/anthology/J/J93/J93-2003.pdf`
- These models are the basic SMT models, called:
  IBM Model 1, IBM Model 2, IBM Model 3, IBM Model 4, IBM Model 5
  as they were called in the 1993 paper.
- We use $\mathbf{e}$and $\mathbf{f}$in the equations in honor of their system which translated from French to English.
  Trained on the Canadian Hansards (Parliament Proceedings)

Statistical Machine Translation

Generative Model of Word Alignment

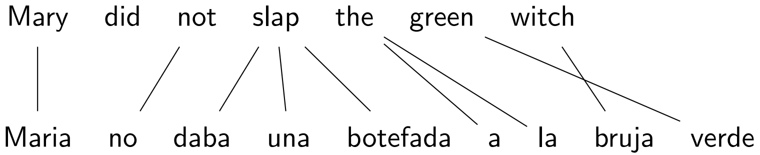
Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Generative Model of Word Alignment

- English **e**: Mary did not slap the green witch
- "French" **f**: Maria no daba una botefada a la bruja verde
- Alignment **a**: $\{1, 3, 4, 4, 4, 5, 5, 7, 6\}$
  e.g. $(f_8, e_{a_8}) = (f_8, e_7) = $ (bruja, witch)

Visualizing alignment **a**



Generative Model of Word Alignment

Data Set

- Data set $\mathcal{D}$ of N sentences:
$$\mathcal{D} = \{(\mathbf{f}^{(1)}, \mathbf{e}^{(1)}), \ldots, (\mathbf{f}^{(N)}, \mathbf{e}^{(N)})\}$$
- French $\mathbf{f}$: $(f_1, f_2, \ldots, f_I)$
- English $\mathbf{e}$: $(e_1, e_2, \ldots, e_J)$
- Alignment $\mathbf{a}$: $(a_1, a_2, \ldots, a_I)$

Generative Model of Word Alignment

Find the best alignment for each translation pair

$$\mathbf{a}^* = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$$

Alignment probability

$$\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{a}, \mathbf{f}, \mathbf{e})}{\Pr(\mathbf{f}, \mathbf{e})} \\
&= \frac{\Pr(\mathbf{e}) \Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})}{\Pr(\mathbf{e}) \Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})}
\end{aligned}$$

Statistical Machine Translation

Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Word Alignments: IBM Model 3

Generative "story" for $P(\mathbf{a}, \mathbf{f} \mid \mathbf{e})$

Mary did not slap the green witch (fertility)

Mary not slap slap slap the the green witch (translate)

Maria no daba una botefada a la verde bruja (reorder)

Maria no daba una botefada a la bruja verde

Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid slap); n(0 \mid did)$$

Translation parameter

$$t(f_i \mid e_{a_i}) : t(bruja \mid witch)$$

Distortion parameter

$$d(f_{pos} = i \mid e_{pos} = j, I, J) : d(8 \mid 7, 8, 6)$$

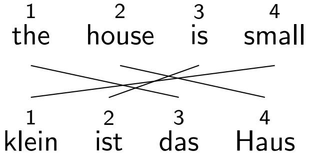
Word Alignments: IBM Model 3

Generative model for $P(\mathbf{a}, \mathbf{f} \mid \mathbf{e})$

$$\begin{aligned} P(\mathbf{a}, \mathbf{f} \mid \mathbf{e}) &= \prod_{j=1}^{J} n(\phi_i \mid e_i) \\ &\times \prod_{i=1}^{I} t(f_i \mid e_{a_j}) \\ &\times \prod_{i=1}^{I} d(i \mid j, I, J) \end{aligned}$$

Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$

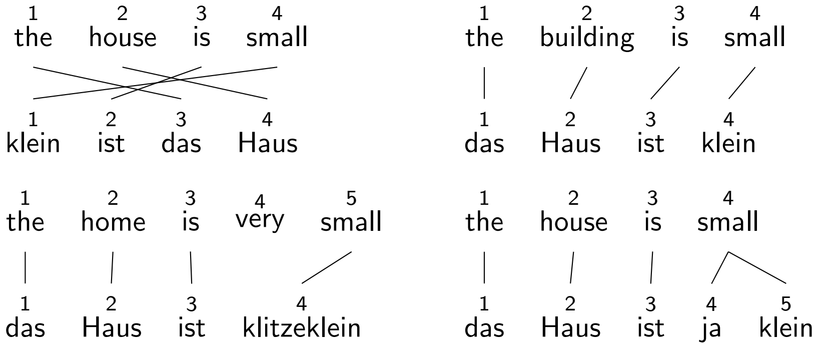


If we know the parameter values we can easily compute the probability of this aligned sentence pair.

$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$

$n(1 \mid \text{the})$	$\times$	$t(\text{das} \mid \text{the})$	$\times$	$d(3 \mid 1, 4, 4) \times$
$n(1 \mid \text{house})$	$\times$	$t(\text{Haus} \mid \text{house})$	$\times$	$d(4 \mid 2, 4, 4) \times$
$n(1 \mid \text{is})$	$\times$	$t(\text{ist} \mid \text{is})$	$\times$	$d(2 \mid 3, 4, 4) \times$
$n(1 \mid \text{small})$	$\times$	$t(\text{klein} \mid \text{small})$	$\times$	$d(1 \mid 4, 4, 4)$

Word Alignments: IBM Model 3



Parameter Estimation

- What is $n(1 \mid \text{very}) = ?$ and $n(0 \mid \text{very}) = ?$
- What is $t(\text{Haus} \mid \text{house}) = ?$ and $t(\text{klein} \mid \text{small}) = ?$
- What is $d(1 \mid 4, 4, 4) = ?$ and $d(1 \mid 1, 4, 4) = ?$

Word Alignments: IBM Model 3

1	2	3	4
the	house	is	small
klein	ist	das	Haus
1	2	3	4

1	2	3	4
the	building	is	small
das	Haus	ist	klein
1	2	3	4

1	2	3	4	5
the	home	is	very	small
das	Haus	ist	klitzeklein	
1	2	3	4	

1	2	3	4	
the	house	is	small	
das	Haus	ist	ja	klein
1	2	3	4	5

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, \mathbf{f}_{\mathrm{len}}, \mathbf{e}_{\mathrm{len}})$$

Word Alignments: IBM Model 3

Summary

- If we know the parameter values we can easily compute the probability $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$ given an aligned sentence pair
- If we are given a corpus of sentence pairs with alignments we can easily learn the parameter values by using relative frequencies.
- If we do not know the alignments then perhaps we can produce all possible alignments each with a certain probability?

IBM Model 3 is too hard: Let us try learning only $t(f_i \mid e_{a_i})$

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, \mathbf{f}_{\mathrm{len}}, \mathbf{e}_{\mathrm{len}})$$

Statistical Machine Translation

Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

Example alignment

1	2	3	4
the	house	is	small
|	|	|	|
1	2	3	4
das	Haus	ist	klein

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

$$\begin{aligned}\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \\ & t(\text{das} \mid \text{the}) \times \\ & t(\text{Haus} \mid \text{house}) \times \\ & t(\text{ist} \mid \text{is}) \times \\ & t(\text{klein} \mid \text{small})\end{aligned}$$

Word Alignments: IBM Model 1

Generative "story" for Model 1

the	house	is	small	(translate)
↓	↓	↓	↓	
das	Haus	ist	klein	

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Finding the best word alignment: IBM Model 1

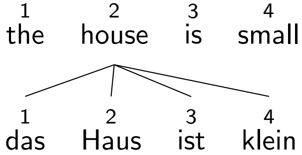
Compute the arg max word alignment

$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{e}, \mathbf{f})$$

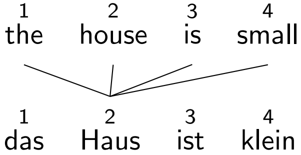
- For each f_i in $(f_1, \ldots, f_I)$ build $\mathbf{a} = (\hat{a}_1, \ldots, \hat{a}_I)$

$$\hat{a}_i = \arg\max_{a_i} t(f_i \mid e_{a_i})$$

Many to one alignment ✓



One to many alignment ✗



Word Alignments: IBM Model 1

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})}
\end{aligned}
$$

Computing the denominator

- The denominator above is summing over J^I alignments
- An interlude on how compute the denominator faster ...

Word Alignments: IBM Model 1

Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{a_1=1}^{J} \sum_{a_2=1}^{J} \cdots \sum_{a_I=1}^{J} \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Assume (f_1, f_2, f_3) and (e_1, e_2)

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

$J^I = 2^3$ terms to be added:

$t(f_1 \mid e_1)$	$\times$	$t(f_2 \mid e_1)$	$\times$	$t(f_3 \mid e_1)$	$+$
$t(f_1 \mid e_1)$	$\times$	$t(f_2 \mid e_1)$	$\times$	$t(f_3 \mid e_2)$	$+$
$t(f_1 \mid e_1)$	$\times$	$t(f_2 \mid e_2)$	$\times$	$t(f_3 \mid e_1)$	$+$
$t(f_1 \mid e_1)$	$\times$	$t(f_2 \mid e_2)$	$\times$	$t(f_3 \mid e_2)$	$+$
$t(f_1 \mid e_2)$	$\times$	$t(f_2 \mid e_1)$	$\times$	$t(f_3 \mid e_1)$	$+$
$t(f_1 \mid e_2)$	$\times$	$t(f_2 \mid e_1)$	$\times$	$t(f_3 \mid e_2)$	$+$
$t(f_1 \mid e_2)$	$\times$	$t(f_2 \mid e_2)$	$\times$	$t(f_3 \mid e_1)$	$+$
$t(f_1 \mid e_2)$	$\times$	$t(f_2 \mid e_2)$	$\times$	$t(f_3 \mid e_2)$	

Word Alignments: IBM Model 1

Factor the terms:

$$\begin{array}{lcll}
(t(f_1 \mid e_1) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_1) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) &
\end{array}$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lcll}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) &
\end{array} \right)$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lcll}
t(f_1 \mid e_1) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) & + \\
t(f_1 \mid e_2) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) &
\end{array} \right)$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\prod_{i=1}^{3} \sum_{a_i=1}^{2} t(f_i \mid e_{a_i})$$

$I \times J = 2 \times 3$ terms to be added:

$$\begin{array}{lcll}
(t(f_1 \mid e_1) & + & t(f_1 \mid e_2)) & \times \\
(t(f_2 \mid e_1) & + & t(f_2 \mid e_2)) & \times \\
(t(f_3 \mid e_1) & + & t(f_3 \mid e_2)) &
\end{array}$$

Parameter Estimation: IBM Model 1

We wish to learn the parameters $t(\cdot \mid \cdot)$ that maximize the log-likelihood of the training data:

$$\arg\max_t L(t) = \arg\max_t \sum_s \log \Pr(\mathbf{f}^{(s)} \mid \mathbf{e}^{(s)}, t)$$

- We start with an initial estimate t_0
- Modify it iteratively to get $t_1, t_2, \ldots$
- Create t from previous time step t_{-1}

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