Natural Language Processing

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anoopsarkar.github.io/nlp-class

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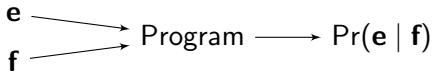
Part 1: Generative Models for Word Alignment

Statistical Machine Translation

Statistical Machine Translation

Noisy Channel Model

$$\mathbf{e}^* = \arg\max_{\mathbf{e}} \underbrace{\Pr(\mathbf{e})}_{\textbf{Language Model}} \cdot \underbrace{\Pr(\mathbf{f} \mid \mathbf{e})}_{\textbf{Alignment Model}}$$

Alignment Task

$$\mathbf{e} \searrow$$
$$\text{Program} \longrightarrow \Pr(\mathbf{e} \mid \mathbf{f})$$
$$\mathbf{f} \nearrow$$

Training Data

- **Alignment Model**: learn a mapping between $\mathbf{f}$ and $\mathbf{e}$.
  Training data: lots of translation pairs between $\mathbf{f}$ and $\mathbf{e}$.

Statistical Machine Translation

The IBM Models

- The first statistical machine translation models were developed at IBM Research (Yorktown Heights, NY) in the 1980s

- The models were published in 1993:
  Brown et. al. The Mathematics of Statistical Machine Translation. *Computational Linguistics*. 1993.
  http://aclweb.org/anthology/J/J93/J93-2003.pdf

- These models are the basic SMT models, called:
  IBM Model 1, IBM Model 2, IBM Model 3, IBM Model 4, IBM Model 5
  as they were called in the 1993 paper.

- We use **e** and **f** in the equations in honor of their system which translated from French to English.
  Trained on the Canadian Hansards (Parliament Proceedings)

Generative Model of Word Alignment

Generative Model of Word Alignment

- English **e**: Mary did not slap the green witch
- "French" **f**: Maria no daba una botefada a la bruja verde
- Alignment **a**: $\{1, 3, 4, 4, 4, 5, 5, 7, 6\}$
  e.g. $(f_8, e_{a_8}) = (f_8, e_7) = $ (bruja, witch)

Visualizing alignment **a**

Mary   did   not   slap   the   green   witch

Maria   no   daba   una   botefada   a   la   bruja   verde
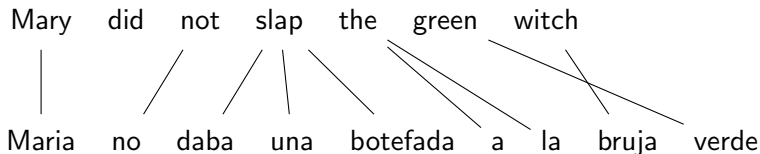
Generative Model of Word Alignment

Data Set

- Data set $\mathcal{D}$ of N sentences:
  $$\mathcal{D} = \{(\mathbf{f}^{(1)}, \mathbf{e}^{(1)}), \ldots, (\mathbf{f}^{(N)}, \mathbf{e}^{(N)})\}$$

- French $\mathbf{f}$: $(f_1, f_2, \ldots, f_I)$
- English $\mathbf{e}$: $(e_1, e_2, \ldots, e_J)$
- Alignment $\mathbf{a}$: $(a_1, a_2, \ldots, a_I)$

Generative Model of Word Alignment

Find the best alignment for each translation pair

$$\mathbf{a}^* = \arg \max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$$

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a}, \mathbf{e})}{\Pr(\mathbf{f}, \mathbf{e})} \\
&= \frac{\Pr(\mathbf{e}) \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{e}) \Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}
\end{aligned}
$$

Word Alignments: IBM Model 3

Generative "story" for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

Mary	did	not	slap	the	green	witch	
Mary	not	slap	slap	slap	the	the	green	witch	(fertility)
Maria	no	daba	una	botefada	a	la	verde	bruja	(translate)
Maria	no	daba	una	botefada	a	la	bruja	verde	(reorder)

Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid slap); n(0 \mid did)$$

Translation parameter

$$t(f_i \mid e_{a_i}) : t(bruja \mid witch)$$

Distortion parameter

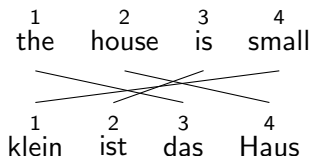$$d(f_{pos} = i \mid e_{pos} = j, I, J) : d(8 \mid 7, 8, 6)$$

Word Alignments: IBM Model 3

Generative model for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

$$
\begin{aligned}
P(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) &= \prod_{j=1}^{J} n(\phi_i \mid e_i) \\
&\times \prod_{i=1}^{I} t(f_i \mid e_{a_j}) \\
&\times \prod_{i=1}^{I} d(i \mid j, I, J)
\end{aligned}
$$

Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$

If we know the parameter values we can easily compute the probability of this aligned sentence pair.

$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$

$$
\begin{array}{lll}
n(1 \mid \text{the}) & \times \quad t(\text{das} \mid \text{the}) & \times \quad d(3 \mid 1, 4, 4) \times \\
n(1 \mid \text{house}) & \times \quad t(\text{Haus} \mid \text{house}) & \times \quad d(4 \mid 2, 4, 4) \times \\
n(1 \mid \text{is}) & \times \quad t(\text{ist} \mid \text{is}) & \times \quad d(2 \mid 3, 4, 4) \times \\
n(1 \mid \text{small}) & \times \quad t(\text{klein} \mid \text{small}) & \times \quad d(1 \mid 4, 4, 4)
\end{array}
$$

Word Alignments: IBM Model 3

Parameter Estimation

- What is $n(1 \mid \text{very}) = ?$ and $n(0 \mid \text{very}) = ?$
- What is $t(\text{Haus} \mid \text{house}) = ?$ and $t(\text{klein} \mid \text{small}) = ?$
- What is $d(1 \mid 4, 4, 4) = ?$ and $d(1 \mid 1, 4, 4) = ?$

Word Alignments: IBM Model 3

1 the	2 house	3 is	4 small	
1 klein	2 ist	3 das	4 Haus	

1 the	2 building	3 is	4 small	
1 das	2 Haus	3 ist	4 klein	

1 the	2 home	3 is	4 very	5 small
1 das	2 Haus	3 ist	4 klitzeklein	

1 the	2 house	3 is	4 small	
1 das	2 Haus	3 ist	4 ja	5 klein

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{l} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, \mathbf{f}_{\text{len}}, \mathbf{e}_{\text{len}})$$

Word Alignments: IBM Model 3

Summary

- If we know the parameter values we can easily compute the probability $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$ given an aligned sentence pair
- If we are given a corpus of sentence pairs with alignments we can easily learn the parameter values by using relative frequencies.
- If we do not know the alignments then perhaps we can produce all possible alignments each with a certain probability?

IBM Model 3 is too hard: Let us try learning only $t(f_i \mid e_{a_i})$

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, \mathbf{f}_{\text{len}}, \mathbf{e}_{\text{len}})$$

Generative Model of Word Alignment

Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

Example alignment

1	2	3	4
the	house	is	small
$\mid$	$\mid$	$\mid$	$\mid$
1	2	3	4
das	Haus	ist	klein

$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{l} t(f_i \mid e_{a_i})$

$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$
$\quad t(\text{das} \mid \text{the}) \times$
$\quad t(\text{Haus} \mid \text{house}) \times$
$\quad t(\text{ist} \mid \text{is}) \times$
$\quad t(\text{klein} \mid \text{small})$

Word Alignments: IBM Model 1

Generative "story" for Model 1

the	house	is	small

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

das   Haus   ist   klein   (translate)

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{l} t(f_i \mid e_{a_i})$$

Finding the best word alignment: IBM Model 1

Compute the arg max word alignment

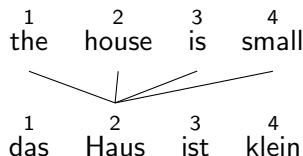$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{e}, \mathbf{f})$$

▶ For each f_i in $(f_1, \ldots, f_I)$ build $\mathbf{a} = (\hat{a_1}, \ldots, \hat{a_I})$

$$\hat{a_i} = \arg\max_{a_i} t(f_i \mid e_{a_i})$$

Many to one alignment ✓

```
 1    2     3    4
the  house  is  small


 1    2     3    4
das  Haus  ist  klein
```

One to many alignment ✗

```
 1    2     3    4
the  house  is  small


 1    2     3    4
das  Haus  ist  klein
```

Learning parameters [from P.Koehn SMT book slides]

- ▶ We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus
- ▶ ... but we do not have the alignments
- ▶ Chicken and egg problem
  - ▶ if we had the *alignments*,
    $\rightarrow$ we could estimate the *parameters* of our generative model
  - ▶ if we had the *parameters*,
    $\rightarrow$ we could estimate the *alignments*
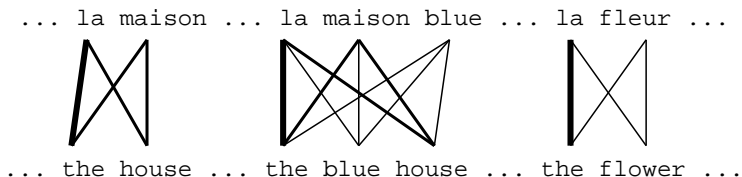
EM Algorithm[from P.Koehn SMT book slides]

- Incomplete data
    - if we had *complete data*, would could estimate *model*
    - if we had *model*, we could fill in the *gaps in the data*
- Expectation Maximization (EM) in a nutshell
    1. initialize model parameters (e.g. uniform)
    2. assign probabilities to the missing data
    3. estimate model parameters from completed data
    4. iterate steps 2–3 until convergence

```
... la maison ... la maison blue ... la fleur ...
... the house ... the blue house ... the flower ...
```
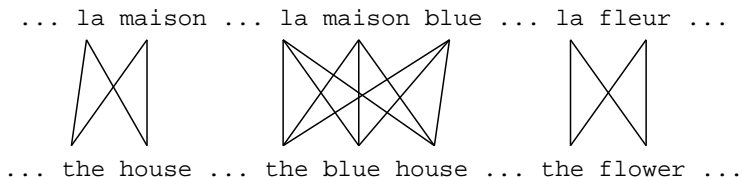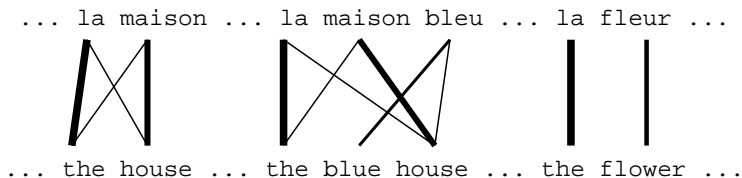
- Initial step: all alignments equally likely
- Model learns that, e.g., *la* is often aligned with *the*

EM Algorithm

```
... la maison ... la maison blue ... la fleur ...
```

```
... the house ... the blue house ... the flower ...
```

- After one iteration
- Alignments, e.g., between *la* and *the* are more likely

... la maison ... la maison bleu ... la fleur ...

... the house ... the blue house ... the flower ...

- ▶ After another iteration
- ▶ It becomes apparent that alignments, e.g., between *fleur* and *flower* are more likely (pigeon hole principle)

... la maison ... la maison bleu ... la fleur ...

... the house ... the blue house ... the flower ...

- Convergence
- Inherent hidden structure revealed by EM

EM Algorithm [from P.Koehn SMT book slides]


```
... la maison ... la maison bleu ... la fleur ...

... the house ... the blue house ... the flower ...
```

$$p(\text{la}|\text{the}) = 0.453$$
$$p(\text{le}|\text{the}) = 0.334$$
$$p(\text{maison}|\text{house}) = 0.876$$
$$p(\text{bleu}|\text{blue}) = 0.563$$
$$\dots$$

► Parameter estimation from the aligned corpus

IBM Model 1 and the EM Algorithm[from P.Koehn SMT book slides]

- ▶ EM Algorithm consists of two steps
- ▶ Expectation-Step: Apply model to the data
  - ▶ parts of the model are hidden (here: alignments)
  - ▶ using the model, assign probabilities to possible values
- ▶ Maximization-Step: Estimate model from data
  - ▶ take assign values as fact
  - ▶ collect counts (weighted by probabilities)
  - ▶ estimate model from counts
- ▶ Iterate these steps until convergence

IBM Model 1 and the EM Algorithm [from P.Koehn SMT book slides]

- ► We need to be able to compute:
  - ► Expectation-Step: probability of alignments
  - ► Maximization-Step: count collection

Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})}$$

$$= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

$$= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})}$$

Computing the denominator

▶ The denominator above is summing over J^I alignments

▶ An interlude on how compute the denominator faster ...

Word Alignments: IBM Model 1

Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) \;=\; \sum_{a_1=1}^{J} \sum_{a_2=1}^{J} \cdots \sum_{a_J=1}^{J} \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Assume (f_1, f_2, f_3) and (e_1, e_2)

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

$J^I = 2^3$ terms to be added:

$$
\begin{array}{ccccc}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_2) &
\end{array}
$$

Word Alignments: IBM Model 1

Factor the terms:

$$(t(f_1 \mid e_1) \times t(f_2 \mid e_1)) \quad \times \quad (t(f_3 \mid e_1) + t(f_3 \mid e_2)) \quad +$$
$$(t(f_1 \mid e_1) \times t(f_2 \mid e_2)) \quad \times \quad (t(f_3 \mid e_1) + t(f_3 \mid e_2)) \quad +$$
$$(t(f_1 \mid e_2) \times t(f_2 \mid e_1)) \quad \times \quad (t(f_3 \mid e_1) + t(f_3 \mid e_2)) \quad +$$
$$(t(f_1 \mid e_2) \times t(f_2 \mid e_2)) \quad \times \quad (t(f_3 \mid e_1) + t(f_3 \mid e_2))$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \begin{pmatrix} t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & + \\ t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & + \\ t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & + \\ t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) \end{pmatrix}$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \begin{pmatrix} t(f_1 \mid e_1) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) & + \\ t(f_1 \mid e_2) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) \end{pmatrix}$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\prod_{i=1}^{3} \sum_{a_i=1}^{2} t(f_i \mid e_{a_i})$$

$I \times J = 2 \times 3$ terms to be added:

$$\begin{aligned}
(t(f_1 \mid e_1) \;+\; t(f_1 \mid e_2)) \;\times\; \\
(t(f_2 \mid e_1) \;+\; t(f_2 \mid e_2)) \;\times\; \\
(t(f_3 \mid e_1) \;+\; t(f_3 \mid e_2))
\end{aligned}$$

Word Alignments: IBM Model 1

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j)}
\end{aligned}
$$

Learning Parameters: IBM Model 1

$$
\begin{array}{cc}
\overset{1}{\text{the}} & \overset{2}{\text{house}} \\
| & / \\
\underset{1}{\text{das}} & \underset{2}{\text{Haus}}
\end{array}
\qquad
\begin{array}{cc}
\overset{1}{\text{the}} & \overset{2}{\text{book}} \\
| & \backslash \\
\underset{1}{\text{das}} & \underset{2}{\text{Buch}}
\end{array}
\qquad
\begin{array}{cc}
\overset{1}{\text{a}} & \overset{2}{\text{book}} \\
| & \backslash \\
\underset{1}{\text{ein}} & \underset{2}{\text{Buch}}
\end{array}
$$

Learning parameters $t(f|e)$ when alignments are known

$$
t(das \mid the) = \frac{c(das,the)}{\sum_f c(f,the)} \qquad t(house \mid Haus) = \frac{c(Haus,house)}{\sum_f c(f,house)}
$$
$$
t(ein \mid a) = \frac{c(ein,the)}{\sum_f c(f,the)} \qquad t(Buch \mid book) = \frac{c(Buch,book)}{\sum_f c(f,book)}
$$

$$
t(f|e) = \sum_{s=1}^{N} \sum_{f \to e \in \mathbf{f}^{(s)}, \mathbf{e}^{(s)}} \frac{c(f,e)}{\sum_f c(f,e)}
$$

Learning Parameters: IBM Model 1

Learning parameters $t(f|e)$ when alignments are *unknown*

Also list alignments for *(the book, das Buch)* and *(a book, ein Buch)*

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

$t(Haus \mid the)$	=	0.25		$t(das \mid house)$	=	0.5
$t(das \mid the)$	=	0.5		$t(Haus \mid house)$	=	0.5
$t(Buch \mid the)$	=	0.25		$t(Buch \mid house)$	=	0.0

Compute posterior for each alignment
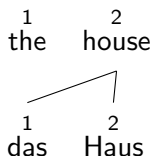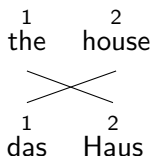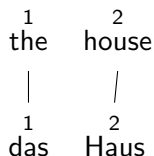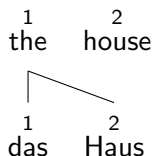
$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} = \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j)}$$

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

$t(Haus \mid the)$	=	0.25		$t(das \mid house)$	=	0.5
$t(das \mid the)$	=	0.5		$t(Haus \mid house)$	=	0.5
$t(Buch \mid the)$	=	0.25		$t(Buch \mid house)$	=	0.0

Compute $\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})$ for each alignment
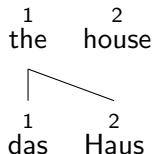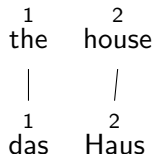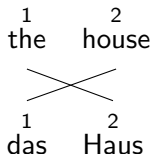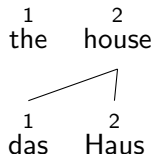
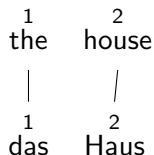|			
0.5×0.25	0.5×0.5	0.25×0.5	0.5×0.5
0.125	0.25	0.125	0.25

Learning Parameters: IBM Model 1

Compute $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})}$

$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$
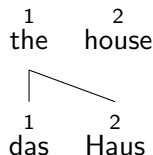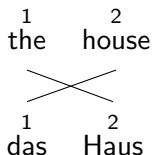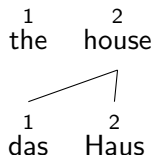

$$\frac{0.125}{0.75}$$
0.167

$$\frac{0.25}{0.75}$$
0.334

$$\frac{0.125}{0.75}$$
0.167

$$\frac{0.25}{0.75}$$
0.334

Compute fractional counts $c(f, e)$

$c(Haus, the)$	=	$0.125 + 0.125$	$c(das, house)$	=	$0.125 + 0.25$
$c(das, the)$	=	$0.125 + 0.25$	$c(Haus, house)$	=	$0.25 + 0.25$
$c(Buch, the)$	=	0.0	$c(Buch, house)$	=	0.0

Learning Parameters: IBM Model 1
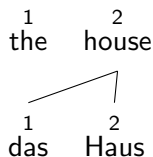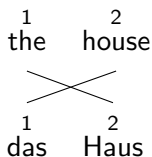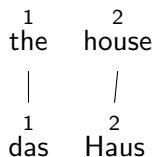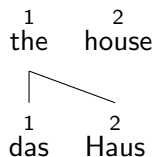
$$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$$

Expectation step: expected counts $g(f, e)$

$$g(Haus, the) = \frac{0.125+0.125}{0.75} \qquad g(das, house) = \frac{0.125+0.25}{0.75}$$
$$g(das, the) = \frac{0.125+0.25}{0.75} \qquad g(Haus, house) = \frac{0.25+0.25}{0.75}$$
$$g(Buch, the) = 0.0 \qquad g(Buch, house) = 0.0$$

Maximization step: get new $t^{(1)}(f \mid e) = \frac{g(f,e)}{\sum_f g(f,e)}$

Learning Parameters: IBM Model 1

Expectation step: expected counts $g(f, e)$

$g(Haus, the)$	=	0.334	$g(das, house)$	=	0.5
$g(das, the)$	=	0.5	$g(Haus, house)$	=	0.667
$g(Buch, the)$	=	0.0	$g(Buch, house)$	=	0.0
total	=	0.834	**total**	=	1.167

Maximization step: get new $t^{(1)}(f \mid e) = \frac{g(f,e)}{\sum_f g(f,e)}$

$t(Haus \mid the)$	=	0.4	$t(das \mid house)$	=	0.43
$t(das, \mid the)$	=	0.6	$t(Haus \mid house)$	=	0.57
$t(Buch \mid the)$	=	0.0	$t(Buch \mid house)$	=	0.0

Keep iterating: Compute $t^{(0)}, t^{(1)}, t^{(2)}, \ldots$ until convergence

Parameter Estimation: IBM Model 1

EM learns the parameters $t(\cdot \mid \cdot)$ that maximizes the log-likelihood of the training data:

$$\arg\max_t L(t) = \arg\max_t \sum_s \log \Pr(\mathbf{f}^{(s)} \mid \mathbf{e}^{(s)}, t)$$

- Start with an initial estimate t_0
- Modify it iteratively to get $t_1, t_2, \ldots$
- Re-estimate t from parameters at previous time step t_{-1}
- The convergence proof of EM guarantees that $L(t) \geq L(t_{-1})$
- EM converges when $L(t) - L(t_{-1})$ is zero (or almost zero).

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All mistakes are my own.