



# Natural Language Processing

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[anoopsarkar.github.io/nlp-class](https://anoopsarkar.github.io/nlp-class)

Simon Fraser University

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Part 1: Generative Models for Word Alignment

## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

IBM Model 2

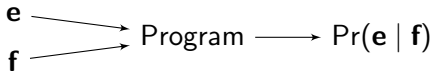
Back to IBM Model 3

# Statistical Machine Translation

## Noisy Channel Model

$$\mathbf{e}^* = \arg \max_{\mathbf{e}} \underbrace{\Pr(\mathbf{e})}_{\text{Language Model}} \cdot \underbrace{\Pr(\mathbf{f} | \mathbf{e})}_{\text{Alignment Model}}$$

## Alignment Task



## Training Data

- **Alignment Model:** learn a mapping between **f** and **e**.  
Training data: lots of translation pairs between **f** and **e**.

# Statistical Machine Translation

## The IBM Models

- ▶ The first statistical machine translation models were developed at IBM Research (Yorktown Heights, NY) in the 1980s
- ▶ The models were published in 1993:  
Brown et. al. The Mathematics of Statistical Machine Translation. *Computational Linguistics*. 1993.  
<http://aclweb.org/anthology/J/J93/J93-2003.pdf>
- ▶ These models are the basic SMT models, called:  
IBM Model 1, IBM Model 2, IBM Model 3, IBM Model 4, IBM Model 5  
as they were called in the 1993 paper.
- ▶ We use **e** and **f** in the equations in honor of their system which translated from French to English.  
Trained on the Canadian Hansards (Parliament Proceedings)

## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

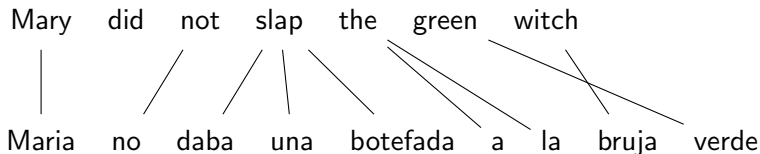
IBM Model 2

Back to IBM Model 3

# Generative Model of Word Alignment

- ▶ English **e**: Mary did not slap the green witch
- ▶ “French” **f**: Maria no daba una botefada a la bruja verde
- ▶ Alignment **a**:  $\{1, 3, 4, 4, 4, 5, 5, 7, 6\}$   
e.g.  $(f_8, e_{a_8}) = (f_8, e_7) = (\text{bruja}, \text{witch})$

## Visualizing alignment **a**





# Generative Model of Word Alignment

## Data Set

- ▶ Data set  $\mathcal{D}$  of  $N$  sentences:

$$\mathcal{D} = \{(\mathbf{f}^{(1)}, \mathbf{e}^{(1)}), \dots, (\mathbf{f}^{(N)}, \mathbf{e}^{(N)})\}$$

- ▶ French  $\mathbf{f}$ :  $(f_1, f_2, \dots, f_I)$
- ▶ English  $\mathbf{e}$ :  $(e_1, e_2, \dots, e_J)$
- ▶ Alignment  $\mathbf{a}$ :  $(a_1, a_2, \dots, a_I)$
- ▶  $\text{length}(\mathbf{f}) = \text{length}(\mathbf{a}) = I$

# Generative Model of Word Alignment

Find the best alignment for each translation pair

$$\mathbf{a}^* = \arg \max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$$

Alignment probability

$$\begin{aligned} \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a}, \mathbf{e})}{\Pr(\mathbf{f}, \mathbf{e})} \\ &= \frac{\Pr(\mathbf{e}) \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{e}) \Pr(\mathbf{f} \mid \mathbf{e})} \\ &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\ &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})} \end{aligned}$$

## Statistical Machine Translation

### Generative Model of Word Alignment

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Back to IBM Model 3

# Word Alignments: IBM Model 3

Generative “story” for  $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$



# Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid \textit{slap}); n(0 \mid \textit{did})$$

Translation parameter

$$t(f_i \mid e_{a_i}) : t(\textit{bruja} \mid \textit{witch})$$

Distortion parameter

$$d(f_{pos} = i \mid e_{pos} = j, I, J) : d(8 \mid 7, 9, 7)$$

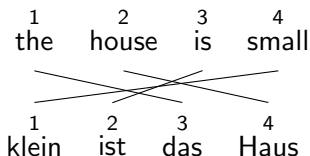
# Word Alignments: IBM Model 3

Generative model for  $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

$$\begin{aligned} P(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) &= \prod_{i=1}^I n(\phi_{a_i} \mid e_{a_i}) \\ &\times t(f_i \mid e_{a_i}) \\ &\times d(i \mid a_i, I, J) \end{aligned}$$

# Word Alignments: IBM Model 3

Sentence pair with alignment  $\mathbf{a} = (4, 3, 1, 2)$





If we know the parameter values we can easily compute the probability of this aligned sentence pair.


$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$


$$\begin{aligned} n(1 \mid \text{the}) &\times t(\text{das} \mid \text{the}) &\times d(3 \mid 1, 4, 4) &\times \\ n(1 \mid \text{house}) &\times t(\text{Haus} \mid \text{house}) &\times d(4 \mid 2, 4, 4) &\times \\ n(1 \mid \text{is}) &\times t(\text{ist} \mid \text{is}) &\times d(2 \mid 3, 4, 4) &\times \\ n(1 \mid \text{small}) &\times t(\text{klein} \mid \text{small}) &\times d(1 \mid 4, 4, 4) \end{aligned}$$

# Word Alignments: IBM Model 3

1	2	3	4
the	house	is	small
			
1	2	3	4
klein	ist	das	Haus

1	2	3	4
the	building	is	small
			
1	2	3	4
das	Haus	ist	klein

1	2	3	4	5
the	home	is	very	small
				
1	2	3	4	
das	Haus	ist	klitzeklein	

1	2	3	4	
the	house	is	small	
				
1	2	3	4	5
das	Haus	ist	ja	klein

## Parameter Estimation

- ▶ What is  $n(1 \mid \text{very}) = ?$  and  $n(0 \mid \text{very}) = ?$
- ▶ What is  $t(\text{Haus} \mid \text{house}) = ?$  and  $t(\text{klein} \mid \text{small}) = ?$
- ▶ What is  $d(1 \mid 4, 4, 4) = ?$  and  $d(1 \mid 1, 4, 4) = ?$



## Word Alignments: IBM Model 3

1	2	3	4
the	house	is	small
1	2	3	4
klein	ist	das	Haus

1	2	3	4
the	building	is	small
1	2	3	4
das	Haus	ist	klein

1	2	3	4	5
the	home	is	very	small
1	2	3	4	
das	Haus	ist	klitzeklein	

1	2	3	4	
the	house	is	small	
1	2	3	4	5
das	Haus	ist	ja	klein

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^I n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, I, J)$$

# Word Alignments: IBM Model 3

## Summary

- ▶ If we know the parameter values we can easily compute the probability  $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$  given an aligned sentence pair
- ▶ If we are given a corpus of sentence pairs with alignments we can easily learn the parameter values by using relative frequencies.
- ▶ If we do not know the alignments then perhaps we can produce all possible alignments each with a certain probability?

IBM Model 3 is too hard: Let us try learning only  $t(f_i \mid e_{a_i})$

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^I n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, I, J)$$

## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

IBM Model 2

Back to IBM Model 3

# Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

Example alignment

1	2	3	4
the	house	is	small
1	2	3	4
das	Haus	ist	klein

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^I t(f_i \mid e_{a_i})$$

$$\begin{aligned} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = & t(\text{das} \mid \text{the}) \times \\ & t(\text{Haus} \mid \text{house}) \times \\ & t(\text{ist} \mid \text{is}) \times \\ & t(\text{klein} \mid \text{small}) \end{aligned}$$

# Word Alignments: IBM Model 1

Generative “story” for Model 1

the	house	is	small	
↓	↓	↓	↓	
das	Haus	ist	klein	(translate)

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^I t(f_i \mid e_{a_i})$$

## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

IBM Model 2

Back to IBM Model 3

# Finding the best word alignment: IBM Model 1

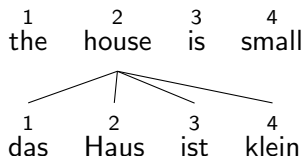
Compute the arg max word alignment

$$\hat{\mathbf{a}} = \arg \max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{e}, \mathbf{f})$$

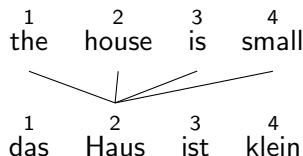
- For each  $f_i$  in  $(f_1, \dots, f_I)$  build  $\mathbf{a} = (\hat{a}_1, \dots, \hat{a}_I)$

$$\hat{a}_i = \arg \max_{a_i} t(f_i \mid e_{a_i})$$

Many to one alignment ✓



One to many alignment ✗



## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

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Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

IBM Model 2

Back to IBM Model 3



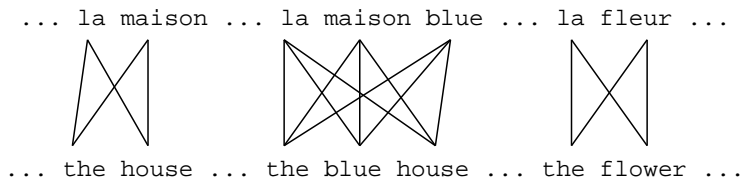
# Learning parameters<sub>[from P.Koehn SMT book slides]</sub>

- ▶ We would like to estimate the lexical translation probabilities  $t(e|f)$  from a parallel corpus
- ▶ ... but we do not have the alignments
- ▶ Chicken and egg problem
  - ▶ if we had the *alignments*,
    - we could estimate the *parameters* of our generative model
  - ▶ if we had the *parameters*,
    - we could estimate the *alignments*

# EM Algorithm<sub>[from P.Koehn SMT book slides]</sub>

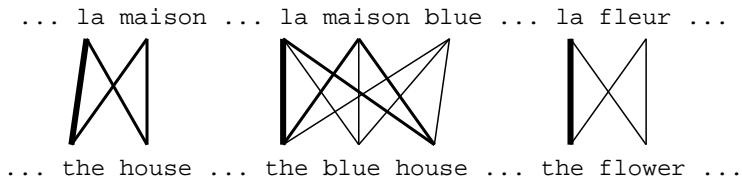
- ▶ Incomplete data
  - ▶ if we had *complete data*, we could estimate *model*
  - ▶ if we had *model*, we could fill in the *gaps in the data*
- ▶ Expectation Maximization (EM) in a nutshell
  1. initialize model parameters (e.g. uniform)
  2. assign probabilities to the missing data
  3. estimate model parameters from completed data
  4. iterate steps 2–3 until convergence

# EM Algorithm [from P.Koehn SMT book slides]



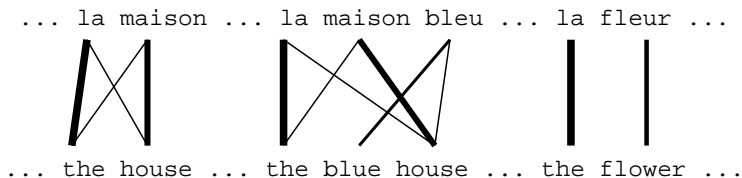
- ▶ Initial step: all alignments equally likely
- ▶ Model learns that, e.g., *la* is often aligned with *the*

# EM Algorithm [from P.Koehn SMT book slides]



- ▶ After one iteration
- ▶ Alignments, e.g., between *la* and *the* are more likely

# EM Algorithm [from P.Koehn SMT book slides]



- ▶ After another iteration
- ▶ It becomes apparent that alignments, e.g., between *fleur* and *flower* are more likely (pigeon hole principle)

# EM Algorithm [from P.Koehn SMT book slides]

... la maison ... la maison bleu ... la fleur ...  
/ | | X | |  
... the house ... the blue house ... the flower ...

- ▶ Convergence
- ▶ Inherent hidden structure revealed by EM

# EM Algorithm<sub>[from P.Koehn SMT book slides]</sub>

... la maison ... la maison bleu ... la fleur ...  
/ | | X | |  
... the house ... the blue house ... the flower ...



$p(\text{la}|\text{the}) = 0.453$   
 $p(\text{le}|\text{the}) = 0.334$   
 $p(\text{maison}|\text{house}) = 0.876$   
 $p(\text{bleu}|\text{blue}) = 0.563$   
...

- Parameter estimation from the aligned corpus

# IBM Model 1 and the EM Algorithm<sub>[from P.Koehn SMT book slides]</sub>

- ▶ EM Algorithm consists of two steps
- ▶ Expectation-Step: Apply model to the data
  - ▶ parts of the model are hidden (here: alignments)
  - ▶ using the model, assign probabilities to possible values
- ▶ Maximization-Step: Estimate model from data
  - ▶ take assign values as fact
  - ▶ collect counts (weighted by probabilities)
  - ▶ estimate model from counts
- ▶ Iterate these steps until convergence



# IBM Model 1 and the EM Algorithm<sub>[from P.Koehn SMT book slides]</sub>

- ▶ We need to be able to compute:
  - ▶ Expectation-Step: probability of alignments
  - ▶ Maximization-Step: count collection

# Word Alignments: IBM Model 1

## Alignment probability

$$\begin{aligned}\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\ &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})} \\ &= \frac{\prod_{i=1}^I t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^I t(f_i \mid e_{a_i})}\end{aligned}$$

## Computing the denominator

- ▶ The denominator above is summing over  $J^I$  alignments
- ▶ An interlude on how compute the denominator faster ...

# Word Alignments: IBM Model 1

Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{a_1=1}^J \sum_{a_2=1}^J \dots \sum_{a_I=1}^J \prod_{i=1}^I t(f_i \mid e_{a_i})$$

Assume  $(f_1, f_2, f_3)$  and  $(e_1, e_2)$

$$\sum_{a_1=1}^2 \sum_{a_2=1}^2 \sum_{a_3=1}^2 t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

# Word Alignments: IBM Model 1

Assume  $(f_1, f_2, f_3)$  and  $(e_1, e_2)$ :  $I = 3$  and  $J = 2$

$$\sum_{a_1=1}^2 \sum_{a_2=1}^2 \sum_{a_3=1}^2 t(f_1 | e_{a_1}) \times t(f_2 | e_{a_2}) \times t(f_3 | e_{a_3})$$

$J' = 2^3$  terms to be added:

$$\begin{array}{llllll} t(f_1 | e_1) & \times & t(f_2 | e_1) & \times & t(f_3 | e_1) & + \\ t(f_1 | e_1) & \times & t(f_2 | e_1) & \times & t(f_3 | e_2) & + \\ t(f_1 | e_1) & \times & t(f_2 | e_2) & \times & t(f_3 | e_1) & + \\ t(f_1 | e_1) & \times & t(f_2 | e_2) & \times & t(f_3 | e_2) & + \\ t(f_1 | e_2) & \times & t(f_2 | e_1) & \times & t(f_3 | e_1) & + \\ t(f_1 | e_2) & \times & t(f_2 | e_1) & \times & t(f_3 | e_2) & + \\ t(f_1 | e_2) & \times & t(f_2 | e_2) & \times & t(f_3 | e_1) & + \\ t(f_1 | e_2) & \times & t(f_2 | e_2) & \times & t(f_3 | e_2) & \end{array}$$

# Word Alignments: IBM Model 1

Factor the terms:

$$\begin{aligned} & (t(f_1 | e_1) \times t(f_2 | e_1)) \times (t(f_3 | e_1) + t(f_3 | e_2)) + \\ & (t(f_1 | e_1) \times t(f_2 | e_2)) \times (t(f_3 | e_1) + t(f_3 | e_2)) + \\ & (t(f_1 | e_2) \times t(f_2 | e_1)) \times (t(f_3 | e_1) + t(f_3 | e_2)) + \\ & (t(f_1 | e_2) \times t(f_2 | e_2)) \times (t(f_3 | e_1) + t(f_3 | e_2)) \end{aligned}$$

$$(t(f_3 | e_1) + t(f_3 | e_2)) \left( \begin{array}{l} t(f_1 | e_1) \times t(f_2 | e_1) + \\ t(f_1 | e_1) \times t(f_2 | e_2) + \\ t(f_1 | e_2) \times t(f_2 | e_1) + \\ t(f_1 | e_2) \times t(f_2 | e_2) \end{array} \right)$$

$$(t(f_3 | e_1) + t(f_3 | e_2)) \left( \begin{array}{l} t(f_1 | e_1) \times (t(f_2 | e_1) + t(f_2 | e_2)) \\ t(f_1 | e_2) \times (t(f_2 | e_1) + t(f_2 | e_2)) \end{array} + \right)$$

# Word Alignments: IBM Model 1

Assume  $(f_1, f_2, f_3)$  and  $(e_1, e_2)$ :  $I = 3$  and  $J = 2$

$$\prod_{i=1}^3 \sum_{a_i=1}^2 t(f_i | e_{a_i})$$

$I \times J = 2 \times 3$  terms to be added:

$$\begin{aligned} & (t(f_1 | e_1) + t(f_1 | e_2)) \times \\ & (t(f_2 | e_1) + t(f_2 | e_2)) \times \\ & (t(f_3 | e_1) + t(f_3 | e_2)) \end{aligned}$$

# Word Alignments: IBM Model 1

Alignment probability

$$\begin{aligned}\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\ &= \frac{\prod_{i=1}^I t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^I t(f_i \mid e_{a_i})} \\ &= \frac{\prod_{i=1}^I t(f_i \mid e_{a_i})}{\prod_{i=1}^I \sum_{j=1}^J t(f_i \mid e_j)}\end{aligned}$$

# Learning Parameters: IBM Model 1

<sup>1</sup> the	<sup>2</sup> house
	/
<sup>1</sup> das	<sup>2</sup> Haus

<sup>1</sup> the	<sup>2</sup> book
<sup>1</sup> das	<sup>2</sup> Buch

<sup>1</sup> a	<sup>2</sup> book
	\
<sup>1</sup> ein	<sup>2</sup> Buch

Learning parameters  $t(f|e)$  when alignments are known

$$\begin{aligned} t(das | the) &= \frac{c(das, the)}{\sum_f c(f, the)} & t(house | Haus) &= \frac{c(Haus, house)}{\sum_f c(f, house)} \\ t(ein | a) &= \frac{c(ein, a)}{\sum_f c(f, a)} & t(Buch | book) &= \frac{c(Buch, book)}{\sum_f c(f, book)} \end{aligned}$$

$$t(f|e) = \sum_{s=1}^N \sum_{f \rightarrow e \in \mathbf{f}^{(s)}, \mathbf{e}^{(s)}} \frac{c(f, e)}{\sum_f c(f, e)}$$



# Learning Parameters: IBM Model 1

<sup>1</sup> the    <sup>2</sup> house  
┌───┐  
└───┘  
<sup>1</sup> das    <sup>2</sup> Haus

<sup>1</sup> the    <sup>2</sup> book  
┌───┐  
└───┘  
<sup>1</sup> das    <sup>2</sup> Buch

<sup>1</sup> a    <sup>2</sup> book  
┌───┐  
└───┘  
<sup>1</sup> ein    <sup>2</sup> Buch

Learning parameters  $t(f|e)$  when alignments are *unknown*

<sup>1</sup> the    <sup>2</sup> house  
└───┐  
┌───┘  
<sup>1</sup> das    <sup>2</sup> Haus

<sup>1</sup> the    <sup>2</sup> house  
|       |  
<sup>1</sup> das    <sup>2</sup> Haus

<sup>1</sup> the    <sup>2</sup> house  
└───┐  
┌───┘  
<sup>1</sup> das    <sup>2</sup> Haus

<sup>1</sup> the    <sup>2</sup> house  
└───┐  
┌───┘  
<sup>1</sup> das    <sup>2</sup> Haus

Also list alignments for (*the book, das Buch*) and (*a book, ein Buch*)

# Learning Parameters: IBM Model 1

Initialize  $t^0(f|e)$

$$t(\text{Haus} | \text{the}) = 0.25$$

$$t(\text{das} | \text{the}) = 0.5$$

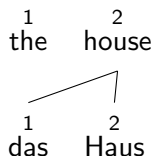
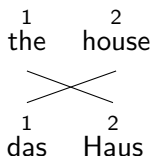
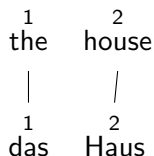
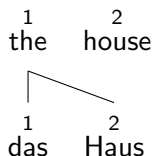
$$t(\text{Buch} | \text{the}) = 0.25$$

$$t(\text{das} | \text{house}) = 0.5$$

$$t(\text{Haus} | \text{house}) = 0.5$$

$$t(\text{Buch} | \text{house}) = 0.0$$

Compute posterior for each alignment



$$\Pr(\mathbf{a} | \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} | \mathbf{e})}{\Pr(\mathbf{f} | \mathbf{e})} = \frac{\prod_{i=1}^I t(f_i | e_{a_i})}{\prod_{i=1}^I \sum_{j=1}^J t(f_i | e_j)}$$

# Learning Parameters: IBM Model 1

Initialize  $t^0(f|e)$

$$t(\text{Haus} \mid \text{the}) = 0.25$$

$$t(\text{das} \mid \text{the}) = 0.5$$

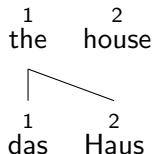
$$t(\text{Buch} \mid \text{the}) = 0.25$$

$$t(\text{das} \mid \text{house}) = 0.5$$

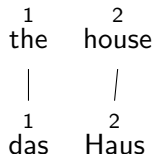
$$t(\text{Haus} \mid \text{house}) = 0.5$$

$$t(\text{Buch} \mid \text{house}) = 0.0$$

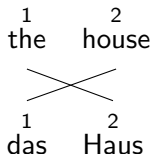
Compute  $\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})$  for each alignment



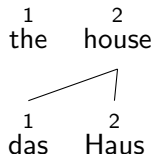
$$0.5 \times 0.25$$
$$0.125$$



$$0.5 \times 0.5$$
$$0.25$$



$$0.25 \times 0.5$$
$$0.125$$

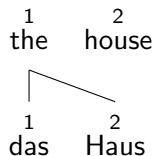


$$0.5 \times 0.5$$
$$0.25$$

# Learning Parameters: IBM Model 1

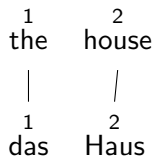
$$\text{Compute } \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})}$$

$$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$$



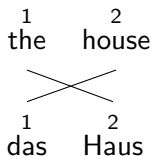
$$\frac{0.125}{0.75}$$

0.167



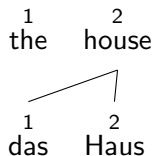
$$\frac{0.25}{0.75}$$

0.334



$$\frac{0.125}{0.75}$$

0.167



$$\frac{0.25}{0.75}$$

0.334

Compute fractional counts  $c(f, e)$

$$c(\text{Haus}, \text{the}) = 0.125 + 0.125$$

$$c(\text{das}, \text{the}) = 0.125 + 0.25$$

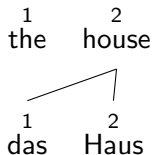
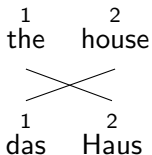
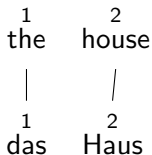
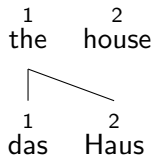
$$c(\text{Buch}, \text{the}) = 0.0$$

$$c(\text{das}, \text{house}) = 0.125 + 0.25$$

$$c(\text{Haus}, \text{house}) = 0.25 + 0.25$$

$$c(\text{Buch}, \text{house}) = 0.0$$

# Learning Parameters: IBM Model 1



$$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$$

Expectation step: expected counts  $g(f, e)$

$$g(das, the) = \frac{0.125+0.25}{0.75}$$

$$g(Haus, the) = \frac{0.125+0.125}{0.75}$$

$$g(Buch, the) = 0.0$$

$$g(das, house) = \frac{0.125+0.25}{0.75}$$

$$g(Haus, house) = \frac{0.25+0.25}{0.75}$$

$$g(Buch, house) = 0.0$$

Maximization step: get new  $t^{(1)}(f \mid e) = \frac{g(f, e)}{\sum_f g(f, e)}$

# Learning Parameters: IBM Model 1

Expectation step: expected counts  $g(f, e)$

$g(das, the)$	$= 0.5$	$g(das, house)$	$= 0.5$
$g(Haus, the)$	$= 0.334$	$g(Haus, house)$	$= 0.667$
$g(Buch, the)$	$= 0.0$	$g(Buch, house)$	$= 0.0$
<b>total</b>	$= 0.834$	<b>total</b>	$= 1.167$

Maximization step: get new  $t^{(1)}(f | e) = \frac{g(f, e)}{\sum_f g(f, e)}$

$t(Haus   the)$	$= 0.4$	$t(das   house)$	$= 0.43$
$t(das,   the)$	$= 0.6$	$t(Haus   house)$	$= 0.57$
$t(Buch   the)$	$= 0.0$	$t(Buch   house)$	$= 0.0$

Keep iterating: Compute  $t^{(0)}, t^{(1)}, t^{(2)}, \dots$  until convergence

# Parameter Estimation: IBM Model 1

EM learns the parameters  $t(\cdot \mid \cdot)$  that maximizes the log-likelihood of the training data:

$$\arg \max_t L(t) = \arg \max_t \sum_s \log \Pr(\mathbf{f}^{(s)} \mid \mathbf{e}^{(s)}, t)$$

- ▶ Start with an initial estimate  $t_0$
- ▶ Modify it iteratively to get  $t_1, t_2, \dots$
- ▶ Re-estimate  $t$  from parameters at previous time step  $t_{-1}$
- ▶ The convergence proof of EM guarantees that  $L(t) \geq L(t_{-1})$
- ▶ EM converges when  $L(t) - L(t_{-1})$  is zero (or almost zero).

## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

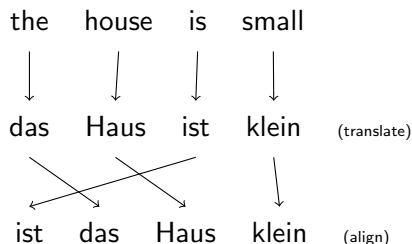
IBM Model 2

Back to IBM Model 3



# Word Alignments: IBM Model 2

## Generative “story” for Model 2



$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^I t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)$$

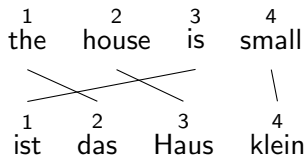
# Word Alignments: IBM Model 2

## Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^I t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)$$

## Example alignment



$$\begin{aligned} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = & t(\text{das} \mid \text{the}) \times a(1 \mid 2, 4, 4) \times \\ & t(\text{Haus} \mid \text{house}) \times a(2 \mid 3, 4, 4) \times \\ & t(\text{ist} \mid \text{is}) \times a(3 \mid 1, 4, 4) \times \\ & t(\text{klein} \mid \text{small}) \times a(4 \mid 4, 4, 4) \end{aligned}$$

## Word Alignments: IBM Model 2

Alignment probability

$$\begin{aligned}\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\&= \frac{\prod_{i=1}^I t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)}{\sum_{\mathbf{a}} \prod_{i=1}^I t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)} \\&= \frac{\prod_{i=1}^I t(f_i \mid e_{a_i}) \times a(a_i \mid i, I, J)}{\prod_{i=1}^I \sum_{j=1}^J t(f_i \mid e_j) \times a(j \mid i, I, J)}\end{aligned}$$

# Word Alignments: IBM Model 2

## Learning the parameters

- ▶ EM training for IBM Model 2 works the same way as IBM Model 1
- ▶ We can do the same factorization trick to efficiently learn the parameters
- ▶ The EM algorithm:
  - ▶ Initialize parameters  $t$  and  $a$  (prefer the diagonal for alignments)
  - ▶ Expectation step: We collect expected counts for  $t$  and  $a$  parameter values
  - ▶ Maximization step: add up expected counts and normalize to get new parameter values
  - ▶ Repeat EM steps until convergence.

## Statistical Machine Translation

### Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

IBM Model 2

Back to IBM Model 3

# Learning Parameters: IBM Model 3

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^I n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, I, J)$$

# Sampling the Alignment Space<sub>[from P.Koehn SMT book slides]</sub>

- ▶ Training IBM Model 3 with the EM algorithm
  - ▶ The trick that reduces exponential complexity does not work anymore
  - Not possible to exhaustively consider all alignments
- ▶ Finding the most probable alignment by hillclimbing
  - ▶ start with initial alignment
  - ▶ change alignments for individual words
  - ▶ keep change if it has higher probability
  - ▶ continue until convergence
- ▶ Sampling: collecting variations to collect statistics
  - ▶ all alignments found during hillclimbing
  - ▶ neighboring alignments that differ by a move or a swap

# Higher IBM Models<sub>[from P.Koehn SMT book slides]</sub>

IBM Model 1	lexical translation
IBM Model 2	adds absolute reordering model
IBM Model 3	adds fertility model
IBM Model 4	relative reordering model
IBM Model 5	fixes deficiency

- ▶ Only IBM Model 1 has global maximum
  - ▶ training of a higher IBM model builds on previous model
- ▶ Computationally biggest change in Model 3
  - ▶ trick to simplify estimation does not work anymore
  - exhaustive count collection becomes computationally too expensive
  - ▶ sampling over high probability alignments is used instead



# Summary [from P.Koehn SMT book slides]

- ▶ IBM Models were the pioneering models in statistical machine translation
- ▶ Introduced important concepts
  - ▶ generative model
  - ▶ EM training
  - ▶ reordering models
- ▶ Only used for niche applications as translation model
- ▶ ... but still in common use for word alignment (e.g., GIZA++, mgiza toolkit)

# Natural Language Processing

Anoop Sarkar

[anoopsarkar.github.io/nlp-class](https://anoopsarkar.github.io/nlp-class)

Simon Fraser University

Part 2: Word Alignment

# Word Alignment

[from P.Koehn SMT book slides]

Given a sentence pair, which words correspond to each other?

	michael	geht	davon	aus	,	dass	er	im	haus	bleibt
michael										
assumes										
that										
he										
will										
stay										
in										
the										
house										

# Word Alignment?

[from P.Koehn SMT book slides]

	john	wohnt	hier	nicht
john				
does		?		?
not				
live				
here				

Is the English word *does* aligned to the German *wohnt* (verb) or *nicht* (negation) or neither?

# Word Alignment? [from P.Koehn SMT book slides]

	john	biss	ins	grass
john				
kicked				
the				
bucket				

How do the idioms *kicked the bucket* and *biss ins grass* match up?  
Outside this exceptional context, *bucket* is never a good translation for *grass*

# Measuring Word Alignment Quality<sub>[from P.Koehn SMT book slides]</sub>

- ▶ Manually align corpus with *sure* ( $S$ ) and *possible* ( $P$ ) alignment points ( $S \subseteq P$ )
- ▶ Common metric for evaluation word alignments: Alignment Error Rate (AER)

$$\text{AER}(S, P; A) = \frac{|A \cap S| + |A \cap P|}{|A| + |S|}$$

- ▶  $\text{AER} = 0$ : alignment  $A$  matches all sure, any possible alignment points
- ▶ However: different applications require different precision/recall trade-offs

# Word Alignment with IBM Models<sub>[from P.Koehn SMT book slides]</sub>

- ▶ IBM Models create a **many-to-one** mapping
  - ▶ words are aligned using an alignment function
  - ▶ a function may return the same value for different input  
(one-to-many mapping)
  - ▶ a function can not return multiple values for one input  
(no many-to-one mapping)
- ▶ Real word alignments have **many-to-many** mappings

# Symmetrizing Word Alignments [from F. Koehn SMT book slides]

	michael	geht	davon	aus	.	dass	er	im	haus	bleibt
michael	█									
assumes		█	█	█						
that						█				
he							█			
will										
stay										█
in								█		
the										
house									█	

English to German

	michael	geht	davon	aus	.	dass	er	im	haus	bleibt
michael	█									
assumes		█								
that						█				
he							█			
will										█
stay										
in								█		
the										
house									█	

German to English

	michael	geht	davon	aus	.	dass	er	im	haus	bleibt
michael	█									
assumes		█	█	█						
that						█				
he							█			
will										█
stay										█
in								█		
the										
house									█	

Intersection / Union

- Intersection plus grow additional alignment points [Och and Ney, CompLing2003]



## Growing heuristic<sub>[from P.Koehn SMT book slides]</sub>

**grow-diag-final**(e2f,f2e)

- 1: neighboring =  $\{(-1,0),(0,-1),(1,0),(0,1),(-1,-1),(-1,1),(1,-1),(1,1)\}$
- 2: alignment  $A = \text{intersect}(e2f,f2e)$ ; **grow-diag**(); **final**(e2f); **final**(f2e);

**grow-diag**()

- 1: **while** new points added **do**
- 2:     **for all** English word  $e \in [1...e_n]$ , foreign word  $f \in [1...f_n]$ ,  $(e, f) \in A$  **do**
- 3:         **for all** neighboring alignment points  $(e_{\text{new}}, f_{\text{new}})$  **do**
- 4:             **if**  $(e_{\text{new}}$  unaligned OR  $f_{\text{new}}$  unaligned) AND  $(e_{\text{new}}, f_{\text{new}}) \in \text{union}(e2f,f2e)$  **then**
- 5:                 add  $(e_{\text{new}}, f_{\text{new}})$  to  $A$
- 6:             **end if**
- 7:         **end for**
- 8:     **end for**
- 9: **end while**

**final**()

- 1: **for all** English word  $e_{\text{new}} \in [1...e_n]$ , foreign word  $f_{\text{new}} \in [1...f_n]$  **do**
- 2:     **if**  $(e_{\text{new}}$  unaligned OR  $f_{\text{new}}$  unaligned) AND  $(e_{\text{new}}, f_{\text{new}}) \in \text{union}(e2f,f2e)$  **then**
- 3:         add  $(e_{\text{new}}, f_{\text{new}})$  to  $A$
- 4:     **end if**
- 5: **end for**

# More Recent Work on Symmetrization<sub>[from P.Koehn SMT book slides]</sub>

- ▶ Symmetrize after each iteration of IBM Models [Matusov et al., 2004]
  - ▶ run one iteration of E-step for each direction
  - ▶ symmetrize the two directions
  - ▶ count collection (M-step)
- ▶ Use of posterior probabilities in symmetrization
  - ▶ generate n-best alignments for each direction
  - ▶ calculate how often an alignment point occurs in these alignments
  - ▶ use this posterior probability during symmetrization

# Link Deletion / Addition Models<sub>[from P.Koehn SMT book slides]</sub>

- ▶ Link deletion [Fossum et al., 2008]
  - ▶ start with union of IBM Model alignment points
  - ▶ delete one alignment point at a time
  - ▶ uses a neural network classifiers that also considers aspects such as how useful the alignment is for learning translation rules
- ▶ Link addition [Ren et al., 2007] [Ma et al., 2008]
  - ▶ possibly start with a skeleton of highly likely alignment points
  - ▶ add one alignment point at a time

# Discriminative Training Methods<sub>[from P.Koehn SMT book slides]</sub>

- ▶ Given some annotated training data, supervised learning methods are possible
- ▶ Structured prediction
  - ▶ not just a classification problem
  - ▶ solution structure has to be constructed in steps
- ▶ Many approaches: maximum entropy, neural networks, support vector machines, conditional random fields, MIRA, ...
- ▶ Small labeled corpus may be used for parameter tuning of unsupervised aligner [Fraser and Marcu, 2007]

# Better Generative Models<sub>[from P.Koehn SMT book slides]</sub>

- ▶ Aligning phrases
  - ▶ joint model [Marcu and Wong, 2002]
  - ▶ problem: EM algorithm likes really long phrases
- ▶ Fraser and Marcu: LEAF
  - ▶ decomposes word alignment into many steps
  - ▶ similar in spirit to IBM Models
  - ▶ includes step for grouping into phrase

# Summary

[from P.Koehn SMT book slides]

- ▶ Lexical translation
- ▶ Alignment
- ▶ Expectation Maximization (EM) Algorithm
- ▶ Noisy Channel Model
- ▶ IBM Models 1–5
  - ▶ IBM Model 1: lexical translation
  - ▶ IBM Model 2: alignment model
  - ▶ IBM Model 3: fertility
  - ▶ IBM Model 4: relative alignment model
  - ▶ IBM Model 5: deficiency
- ▶ Word Alignment

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