



# Natural Language Processing

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Part 1: Probability models of Language

# The Language Modeling problem

## Setup

- ▶ Assume a (finite) vocabulary of words:

$$\mathcal{V} = \{killer, crazy, clown\}$$

- ▶ Use  $\mathcal{V}$  to construct an infinite set of *sentences*

$$\mathcal{V}^+ = \left\{ \begin{array}{l} clown, killer clown, crazy clown, \\ crazy killer clown, killer crazy clown, \\ \dots \end{array} \right\}$$

- ▶ A *sentence* is **defined** as each  $s \in \mathcal{V}^+$

# The Language Modeling problem

## Data

Given a training data set of example sentences  $s \in \mathcal{V}^+$

## Language Modeling problem

Estimate a probability model:

$$\sum_{s \in \mathcal{V}^+} p(s) = 1.0$$

- ▶  $p(\text{clown}) = 1\text{e-}5$
- ▶  $p(\text{killer}) = 1\text{e-}6$
- ▶  $p(\text{killer clown}) = 1\text{e-}12$
- ▶  $p(\text{crazy killer clown}) = 1\text{e-}21$
- ▶  $p(\text{crazy killer clown killer}) = 1\text{e-}110$
- ▶  $p(\text{crazy clown killer killer}) = 1\text{e-}127$

Why do we want to do this?

# Scoring Hypotheses in Speech Recognition

## From acoustic signal to candidate transcriptions

Hypothesis	Score
the station signs are in deep in english	-14732
the stations signs are in deep in english	-14735
the station signs are in deep into english	-14739
the station 's signs are in deep in english	-14740
the station signs are in deep in the english	-14741
the station signs are indeed in english	-14757
the station 's signs are indeed in english	-14760
the station signs are indians in english	-14790
the station signs are indian in english	-14799
the stations signs are indians in english	-14807
the stations signs are indians and english	-14815

# Scoring Hypotheses in Machine Translation

From source language to target language candidates

Hypothesis	Score
we must also discuss a vision .	-29.63
we must also discuss on a vision .	-31.58
it is also discuss a vision .	-31.96
we must discuss on greater vision .	-36.09
⋮	⋮

# Scoring Hypotheses in Decryption

## Character substitutions on ciphertext to plaintext candidates

Hypothesis	Score
Heopaj, zk ukq swjp pk gjks w oaynap?	-93
Urbcnw, mx hxd fjwc cx twxf j bnanc?	-92
Wtdepy, oz jzf hlye ez vyzh l dpncpe?	-91
Mjtufo, ep zpv xbou up lopx b tfdsfu?	-89
Nkuvgp, fq aqw ycpv vq mpqy c ugetgv?	-87
Gdnozi, yj tjp rvio oj fijr v nzxmzo?	-86
Czjkve, uf pfl nrek kf befn r jvtivk?	-85
Yvfgra, qb lbh jnag gb xabj n frperg?	-84
Zwghsb, rc mci kobh hc ybck o gsqfsh?	-83
Byijud, te oek mqdj je adem q iushuj?	-77
Jgqrcl, bm wms uylr rm ilmu y qcapcr?	-76
Listen, do you want to know a secret?	-25

# Scoring Hypotheses in Spelling Correction

Substitute spelling variants to generate hypotheses

Hypothesis	Score
... stellar and versatile <b>acress</b> whose combination of sass and glamour has defined her ...	-18920
... stellar and versatile <b>acres</b> whose combination of sass and glamour has defined her ...	-10209
... stellar and versatile <b>actress</b> whose combination of sass and glamour has defined her ...	-9801



# Probability models of language

## Question

- ▶ Given a finite vocabulary set  $\mathcal{V}$
- ▶ We want to build a probability model  $P(s)$  for all  $s \in \mathcal{V}^+$
- ▶ **But** we want to consider sentences  $s$  of each length  $\ell$  separately.
- ▶ Write down a new model over  $\mathcal{V}^+$  such that  $P(s \mid \ell)$  is in the model
- ▶ **And** the model should be equal to  $\sum_{s \in \mathcal{V}^+} P(s)$ .
- ▶ Write down the model

$$\sum_{s \in \mathcal{V}^+} P(s) = \dots$$

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Part 2:  $n$ -grams for Language Modeling

## Language models

### $n$ -grams for Language Modeling

#### Smoothing $n$ -gram Models

##### Smoothing Counts

- Add-one Smoothing

- Additive Smoothing

- Interpolation: Jelinek-Mercer Smoothing

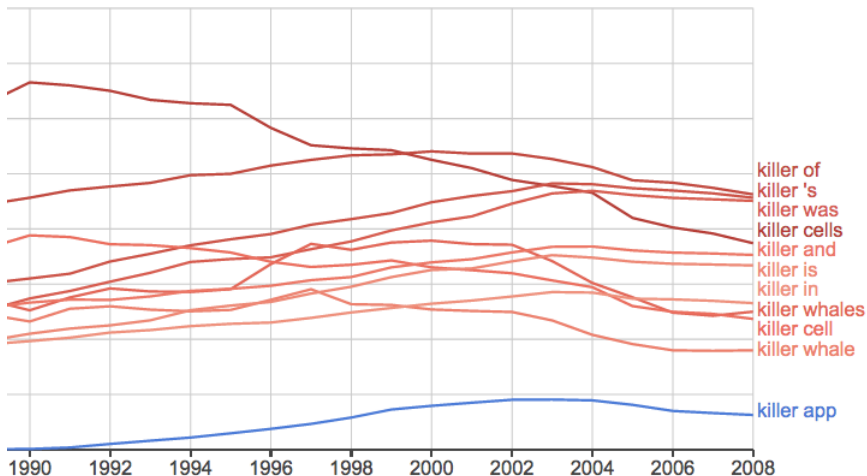
##### Backoff Smoothing with Discounting

- Backoff Smoothing with Discounting

#### Evaluating Language Models

# $n$ -gram Models

Google  $n$ -gram viewer



# Learning Language Models

- ▶ Directly count using a training data set of sentences:  
 $w_1, \dots, w_n$ :

$$p(w_1, \dots, w_n) = \frac{n(w_1, \dots, w_n)}{N}$$

- ▶  $n$  is a function that counts how many times each sentence occurs
- ▶  $N$  is the sum over all possible  $n(\cdot)$  values
- ▶ Problem: does not generalize to new sentences unseen in the training data.
- ▶ What are the chances you will see a sentence: crazy killer clown crazy killer?
- ▶ In NLP applications we often need to assign non-zero probability to previously unseen sentences.

# Learning Language Models

Apply the Chain Rule: the unigram model

$$\begin{aligned} p(w_1, \dots, w_n) &\approx p(w_1)p(w_2) \dots p(w_n) \\ &= \prod_i p(w_i) \end{aligned}$$

Big problem with a unigram language model

$p(\text{the the the the the the the}) > p(\text{we must also discuss a vision .})$

# Learning Language Models

Apply the Chain Rule: the bigram model

$$\begin{aligned} p(w_1, \dots, w_n) &\approx p(w_1)p(w_2 \mid w_1) \dots p(w_n \mid w_{n-1}) \\ &= p(w_1) \prod_{i=2}^n p(w_i \mid w_{i-1}) \end{aligned}$$

Better than unigram

$p(\text{the the the the the the the}) < p(\text{we must also discuss a vision .})$

# Learning Language Models

Apply the Chain Rule: the trigram model

$$\begin{aligned} p(w_1, \dots, w_n) &\approx \\ &p(w_1)p(w_2 \mid w_1)p(w_3 \mid w_1, w_2) \dots p(w_n \mid w_{n-2}, w_{n-1}) \\ &p(w_1)p(w_2 \mid w_1) \prod_{i=3}^n p(w_i \mid w_{i-2}, w_{i-1}) \end{aligned}$$

Better than bigram, but ...

$p(\text{we must also discuss a vision .})$  might be zero because we have not seen  $p(\text{discuss} \mid \text{must also})$



# Maximum Likelihood Estimate

## Using training data to learn a trigram model

- ▶ Let  $c(u, v, w)$  be the count of the trigram  $u, v, w$ , e.g.  $c(\text{crazy}, \text{killer}, \text{clown})$
- ▶ Let  $c(u, v)$  be the count of the bigram  $u, v$ , e.g.  $c(\text{crazy}, \text{killer})$
- ▶ For any  $u, v, w$  we can compute the conditional probability of generating  $w$  given  $u, v$ :

$$p(w \mid u, v) = \frac{c(u, v, w)}{c(u, v)}$$

- ▶ For example:

$$p(\text{clown} \mid \text{crazy}, \text{killer}) = \frac{c(\text{crazy}, \text{killer}, \text{clown})}{c(\text{crazy}, \text{killer})}$$

# Number of Parameters

How many probabilities in each  $n$ -gram model

- ▶ Assume  $\mathcal{V} = \{killer, crazy, clown, UNK\}$

Question

How many unigram probabilities:  $P(x)$  for  $x \in \mathcal{V}$ ?

4

# Number of Parameters

How many probabilities in each  $n$ -gram model

- ▶ Assume  $\mathcal{V} = \{killer, crazy, clown, UNK\}$

## Question

How many bigram probabilities:  $P(y|x)$  for  $x, y \in \mathcal{V}$ ?

$$4^2 = 16$$

# Number of Parameters

How many probabilities in each  $n$ -gram model

- ▶ Assume  $\mathcal{V} = \{killer, crazy, clown, UNK\}$

## Question

How many trigram probabilities:  $P(z|x, y)$  for  $x, y, z \in \mathcal{V}$ ?

$$4^3 = 64$$

# Number of Parameters

## Question

- ▶ Assume  $|\mathcal{V}| = 50,000$  (a realistic vocabulary size for English)
- ▶ What is the minimum size of training data in tokens?
  - ▶ If you wanted to observe all unigrams at least once.
  - ▶ If you wanted to observe all trigrams at least once.

125,000,000,000,000 (125 Ttokens)

Some trigrams should be zero since they do not occur in the language,  $P(\textit{the} \mid \textit{the}, \textit{the})$ .

But others are simply unobserved in the training data,  $P(\textit{idea} \mid \textit{colourless}, \textit{green})$ .

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Part 3: Smoothing Probability Models

## Language models

### $n$ -grams for Language Modeling

#### Smoothing $n$ -gram Models

##### Smoothing Counts

Add-one Smoothing

Additive Smoothing

Interpolation: Jelinek-Mercer Smoothing

##### Backoff Smoothing with Discounting

Backoff Smoothing with Discounting

### Evaluating Language Models

# Bigram Models

- In practice:

$$\begin{aligned} P(\text{crazy killer clown}) = \\ P(\text{crazy} \mid \langle s \rangle) \times P(\text{killer} \mid \text{crazy}) \times \\ P(\text{clown} \mid \text{killer}) \times P(\langle /s \rangle \mid \text{clown}) \end{aligned}$$

- $P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$   
On unseen data,  $c(w_{i-1}, w_i)$  or worse  $c(w_{i-1})$  could be zero

$$\sum_{w_i} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})} = ?$$



# Smoothing

- ▶ **Smoothing** deals with events that have been observed zero times
- ▶ Smoothing algorithms also tend to improve the accuracy of the model

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- ▶ Not just unobserved events: what about events observed once?

## Add-one Smoothing

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-one Smoothing:

$$P(w_i \mid w_{i-1}) = \frac{1 + c(w_{i-1}, w_i)}{V + c(w_{i-1})}$$

- Let  $V$  be the number of words in our vocabulary  
Assign count of 1 to unseen bigrams

## Add-one Smoothing

$$\begin{aligned} P(\text{insane killer clown}) = \\ P(\text{insane} \mid \langle s \rangle) \times P(\text{killer} \mid \text{insane}) \times \\ P(\text{clown} \mid \text{killer}) \times P(\langle /s \rangle \mid \text{clown}) \end{aligned}$$

- ▶ Without smoothing:

$$P(\text{killer} \mid \text{insane}) = \frac{c(\text{insane}, \text{killer})}{c(\text{insane})} = 0$$

- ▶ With add-one smoothing (assuming initially that  $c(\text{insane}) = 1$  and  $c(\text{insane}, \text{killer}) = 0$ ):

$$P(\text{killer} \mid \text{insane}) = \frac{1}{V + 1}$$

## Additive Smoothing: (Lidstone 1920, Jeffreys 1948)

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- ▶ Why add 1? 1 is an overestimate for unobserved events.
- ▶ Additive Smoothing:

$$P(w_i \mid w_{i-1}) = \frac{\delta + c(w_{i-1}, w_i)}{(\delta \times V) + c(w_{i-1})}$$

- ▶  $0 < \delta \leq 1$

## Interpolation: Jelinek-Mercer Smoothing

$$P_{ML}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- ▶  $P_{JM}(w_i \mid w_{i-1}) = \lambda P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda)P_{ML}(w_i)$   
where,  $0 \leq \lambda \leq 1$
- ▶ Jelinek and Mercer (1980) describe an elegant form of this **interpolation**:

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1 - \lambda)P_{JM}(n - 1\text{gram})$$

- ▶ What about  $P_{JM}(w_i)$ ?  
For missing unigrams:  $P_{JM}(w_i) = \lambda P_{ML}(w_i) + (1 - \lambda)\frac{\delta}{V}$

## Interpolation: Finding $\lambda$

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1 - \lambda)P_{JM}(n - 1\text{gram})$$

- ▶ Deleted Interpolation (Jelinek, Mercer)  
compute  $\lambda$  values to minimize cross-entropy on **held-out** data  
which is **deleted** from the initial set of training data
- ▶ Improved JM smoothing, a separate  $\lambda$  for each  $w_{i-1}$ :

$$P_{JM}(w_i \mid w_{i-1}) = \lambda(w_{i-1})P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda(w_{i-1}))P_{ML}(w_i)$$

# Backoff Smoothing with Discounting

- ▶ Discounting (Ney, Essen, Kneser)

$$P_{abs}(y \mid x) = \begin{cases} \frac{c(xy)-D}{c(x)} & \text{if } c(xy) > 0 \\ \alpha(x)P_{abs}(y) & \text{otherwise} \end{cases}$$

- ▶ where  $\alpha(x)$  is chosen to make sure that  $P_{katz}(y \mid x)$  is a proper probability

$$\alpha(x) = 1 - \sum_y \frac{c(xy) - D}{c(x)}$$

# Backoff Smoothing with Discounting

$x$	$c(x)$	$c(x) - D$	$\frac{c(x)-D}{c(the)}$
the	48		
the,dog	15	14.5	14.5/48
the,woman	11	10.5	10.5/48
the,man	10	9.5	9.5/48
the,park	5	4.5	4.5/48
the,job	2	1.5	4.5/48
the,telescope	1	0.5	0.5/48
the>manual	1	0.5	0.5/48
the,afternoon	1	0.5	0.5/48
the,country	1	0.5	0.5/48
the,street	1	0.5	0.5/48
TOTAL			0.9479
the,UNK	0		0.052



## Language models

### $n$ -grams for Language Modeling

### Smoothing $n$ -gram Models

#### Smoothing Counts

- Add-one Smoothing

- Additive Smoothing

- Interpolation: Jelinek-Mercer Smoothing

#### Backoff Smoothing with Discounting

- Backoff Smoothing with Discounting

## Evaluating Language Models

# Evaluating Language Models

- ▶ So far we've seen the probability of a sentence:  $P(w_0, \dots, w_n)$
- ▶ What is the probability of a collection of sentences, that is what is the probability of an unseen test corpus  $T$
- ▶ Let  $T = s_0, \dots, s_m$  be a test corpus with sentences  $s_i$
- ▶  $T$  is assumed to be separate from the training data used to train our language model  $P(s)$
- ▶ What is  $P(T)$ ?

# Evaluating Language Models: Independence assumption

- ▶  $T = s_0, \dots, s_m$  is the text corpus with sentences  $s_0$  through  $s_m$
- ▶  $P(T) = P(s_0, s_1, s_2, \dots, s_m)$  – but each sentence is independent from the other sentences
- ▶  $P(T) = P(s_0) \cdot P(s_1) \cdot P(s_2) \cdot \dots \cdot P(s_m) = \prod_{i=0}^m P(s_i)$
- ▶  $P(s_i) = P(w_0^i, \dots, w_n^i)$  – which can be any  $n$ -gram language model
- ▶ A language model is better if the value of  $P(T)$  is higher for unseen sentences  $T$ , we want to maximize:

$$P(T) = \prod_{i=0}^m P(s_i)$$

# Evaluating Language Models: Computing the Average

- ▶ However,  $T$  can be any arbitrary size
- ▶  $P(T)$  will be lower if  $T$  is larger.
- ▶ Instead of the probability for a given  $T$  we can compute the *average* probability.
- ▶  $M$  is the total number of tokens in the test corpus  $T$ :

$$M = \sum_{i=1}^m \text{length}(s_i)$$

- ▶ The average *log* probability of the test corpus  $T$  is:

$$\frac{1}{M} \log_2 \prod_{i=1}^m P(s_i) = \frac{1}{M} \sum_{i=1}^m \log_2 P(s_i)$$

# Evaluating Language Models: Perplexity

- ▶ The average *log* probability of the test corpus  $T$  is:

$$\ell = \frac{1}{M} \sum_{i=1}^m \log_2 P(s_i)$$

- ▶ Note that  $\ell$  is a negative number
- ▶ We evaluate a language model using *Perplexity* which is  $2^{-\ell}$

# Evaluating Language Models

## Question

Show that:

$$2^{-\frac{1}{M} \log_2 \prod_{i=1}^m P(s_i)} = \frac{1}{\sqrt[M]{\prod_{i=1}^m P(s_i)}}$$

# Evaluating Language Models

## Question

What happens to  $2^{-\ell}$  if any  $n$ -gram probability for computing  $P(T)$  is zero?

# Evaluating Language Models: Typical Perplexity Values

From 'A Bit of Progress in Language Modeling' by Chen and Goodman

Model	Perplexity
unigram	955
bigram	137
trigram	74



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Part 4: Event space in Language Models

# Trigram Models

- ▶ The trigram model:

$$P(w_1, w_2, \dots, w_n) = \\ P(w_1) \times P(w_2 \mid w_1) \times P(w_3 \mid w_1, w_2) \times P(w_4 \mid w_2, w_3) \times \\ \dots P(w_i \mid w_{i-2}, w_{i-1}) \dots \times P(w_n \mid w_{n-2}, \dots, w_{n-1})$$

- ▶ Notice that the length of the sentence  $n$  is variable
- ▶ What is the event space?

# The stop symbol

- ▶ Let  $\mathcal{V} = \{a, b\}$  and the language  $L$  be  $\mathcal{V}^*$
- ▶ Consider a unigram model:  $P(a) = P(b) = 0.5$
- ▶ So strings in this language  $L$  are:

$a$ stop	$0.5$
$b$ stop	$0.5$
$aa$ stop	$0.5^2$
$bb$ stop	$0.5^2$
$\vdots$	

- ▶ The sum over all strings in  $L$  should be equal to 1:

$$\sum_{w \in L} P(w) = 1$$

- ▶ But  $P(a) + P(b) + P(aa) + P(bb) = 1.5$  !!

# The stop symbol

- ▶ What went wrong?  
We need to model variable length sequences
- ▶ Add an explicit probability for the stopsymbol:

$$P(a) = P(b) = 0.25$$

$$P(\text{stop}) = 0.5$$

- ▶  $P(\text{stop}) = 0.5$ ,  $P(a \text{ stop}) = P(b \text{ stop}) = 0.25 \times 0.5 = 0.125$ ,  
 $P(aa \text{ stop}) = 0.25^2 \times 0.5 = 0.03125$  (now the sum is no longer greater than one)

## The stop symbol

- ▶ With this new stop symbol we can show that  $\sum_w P(w) = 1$   
Notice that the probability of any sequence of length  $n$  is  $0.25^n \times 0.5$   
Also there are  $2^n$  sequences of length  $n$

$$\begin{aligned}\sum_w P(w) &= \\& \sum_{n=0}^{\infty} 2^n \times 0.25^n \times 0.5 \\& \sum_{n=0}^{\infty} 0.5^n \times 0.5 = \sum_{n=0}^{\infty} 0.5^{n+1} \\& \sum_{n=1}^{\infty} 0.5^n = 1\end{aligned}$$

## The stop symbol

- ▶ With this new stop symbol we can show that  $\sum_w P(w) = 1$   
Using  $p_s = P(\text{stop})$  the probability of any sequence of length  $n$  is  $p(n) = p(w_1, \dots, w_{n-1}) \times p_s(w_n)$

$$\begin{aligned}\sum_w P(w) &= \sum_{n=0}^{\infty} p(n) \sum_{w_1, \dots, w_n} p(w_1, \dots, w_n) \\ &= \sum_{n=0}^{\infty} p(n) \sum_{w_1, \dots, w_n} \prod_{i=1}^n p(w_i)\end{aligned}$$

$$\begin{aligned}\sum_{w_1, \dots, w_n} \prod_i p(w_i) &= \\ \sum_{w_1} \sum_{w_2} \dots \sum_{w_n} p(w_1)p(w_2) \dots p(w_n) &= 1\end{aligned}$$

## The stop symbol

$$\sum_{w_1} \sum_{w_2} \dots \sum_{w_n} p(w_1)p(w_2) \dots p(w_n) = 1$$

$$\begin{aligned} \sum_{n=0}^{\infty} p(n) &= \sum_{n=0}^{\infty} p_s(1 - p_s)^n \\ &= p_s \sum_{n=0}^{\infty} (1 - p_s)^n \\ &= p_s \frac{1}{1 - (1 - p_s)} = p_s \frac{1}{p_s} = 1 \end{aligned}$$

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