Natural Language Processing

Anoop Sarkar
anoopsarkar.github.io/nlp-class

Simon Fraser University

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Part 1: Generative Models for Word Alignment

Statistical Machine Translation

Generative Model of Word Alignment

- Word Alignments: IBM Model 3
- Word Alignments: IBM Model 1
- Finding the best alignment: IBM Model 1
- Learning Parameters: IBM Model 1

Statistical Machine Translation

Noisy Channel Model

$$\mathbf{e}^* = \arg\max_{\mathbf{e}} \underbrace{\Pr(\mathbf{e})}_{\textbf{Language Model}} \cdot \underbrace{\Pr(\mathbf{f} \mid \mathbf{e})}_{\textbf{Alignment Model}}$$

Alignment Task

$$\begin{matrix} \mathbf{e} \searrow \\ \mathbf{f} \nearrow \end{matrix} \text{Program} \longrightarrow \Pr(\mathbf{e} \mid \mathbf{f})$$

Training Data

- **Alignment Model**: learn a mapping between $\mathbf{f}$and $\mathbf{e}$.
  Training data: lots of translation pairs between $\mathbf{f}$and $\mathbf{e}$.

Statistical Machine Translation

The IBM Models

- The first statistical machine translation models were developed at IBM Research (Yorktown Heights, NY) in the 1980s
- The models were published in 1993:
  Brown et. al. The Mathematics of Statistical Machine Translation.
  Computational Linguistics. 1993.
  `http://aclweb.org/anthology/J/J93/J93-2003.pdf`
- These models are the basic SMT models, called:
  IBM Model 1, IBM Model 2, IBM Model 3, IBM Model 4,
  IBM Model 5
  as they were called in the 1993 paper.
- We use **e**and **f** in the equations in honor of their system which translated from French to English.
  Trained on the Canadian Hansards (Parliament Proceedings)

Statistical Machine Translation

Generative Model of Word Alignment

- Word Alignments: IBM Model 3
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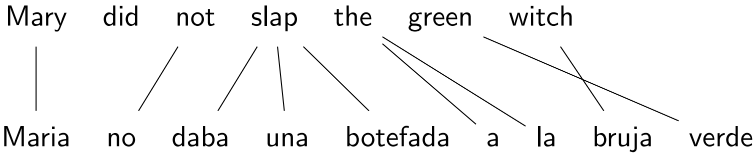
Generative Model of Word Alignment

- English **e**: Mary did not slap the green witch
- "French" **f**: Maria no daba una botefada a la bruja verde
- Alignment **a**: $\{1, 3, 4, 4, 4, 5, 5, 7, 6\}$
  e.g. $(f_8, e_{a_8}) = (f_8, e_7) = (\text{bruja, witch})$

Visualizing alignment **a**



Generative Model of Word Alignment

Data Set

- Data set $\mathcal{D}$ of N sentences:
$$\mathcal{D} = \{(\mathbf{f}^{(1)}, \mathbf{e}^{(1)}), \ldots, (\mathbf{f}^{(N)}, \mathbf{e}^{(N)})\}$$
- French **f**: $(f_1, f_2, \ldots, f_I)$
- English **e**: $(e_1, e_2, \ldots, e_J)$
- Alignment **a**: $(a_1, a_2, \ldots, a_I)$

Generative Model of Word Alignment

Find the best alignment for each translation pair

$$\mathbf{a}^* = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$$

Alignment probability

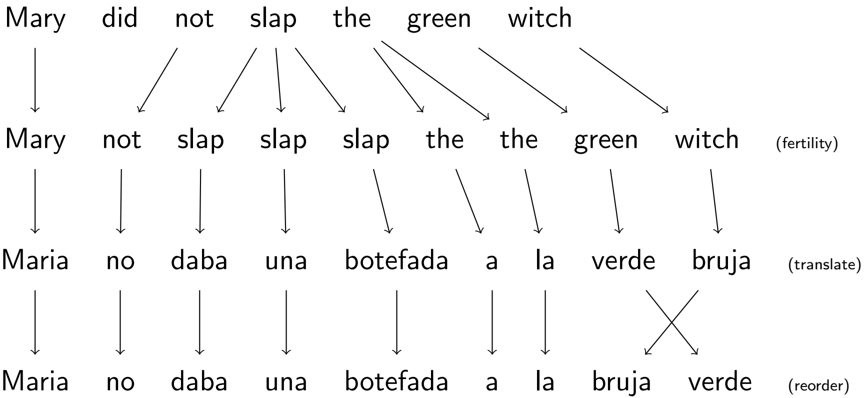
$$\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a}, \mathbf{e})}{\Pr(\mathbf{f}, \mathbf{e})} \\
&= \frac{\Pr(\mathbf{e}) \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{e}) \Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}
\end{aligned}$$

Statistical Machine Translation

Generative Model of Word Alignment
Word Alignments: IBM Model 3
Word Alignments: IBM Model 1
Finding the best alignment: IBM Model 1
Learning Parameters: IBM Model 1

Word Alignments: IBM Model 3

Generative "story" for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$



Word Alignments: IBM Model 3

Fertility parameter

$$n(\phi_j \mid e_j) : n(3 \mid slap); n(0 \mid did)$$

Translation parameter

$$t(f_i \mid e_{a_i}) : t(bruja \mid witch)$$

Distortion parameter

$$d(f_{pos} = i \mid e_{pos} = j, I, J) : d(8 \mid 7, 8, 6)$$

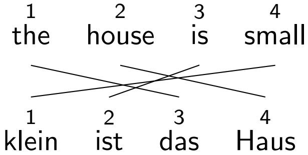
Word Alignments: IBM Model 3

Generative model for $P(\mathbf{f}, \mathbf{a} \mid \mathbf{e})$

$$\begin{aligned} P(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) &= \prod_{j=1}^{J} n(\phi_i \mid e_i) \\ &\times \prod_{i=1}^{I} t(f_i \mid e_{a_j}) \\ &\times \prod_{i=1}^{I} d(i \mid j, I, J) \end{aligned}$$

Word Alignments: IBM Model 3

Sentence pair with alignment $\mathbf{a} = (4, 3, 1, 2)$

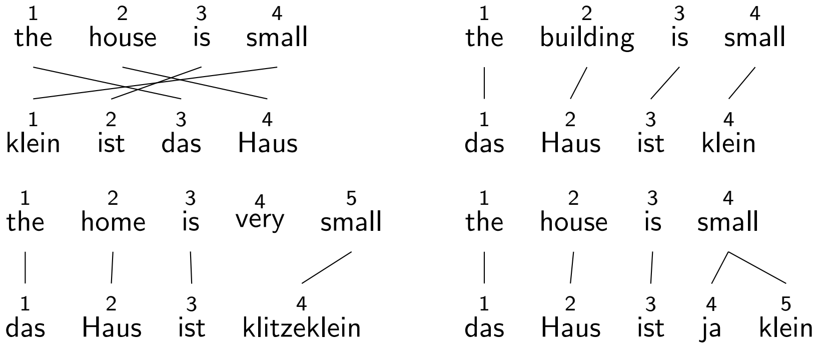


If we know the parameter values we can easily compute the probability of this aligned sentence pair.

$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) =$

$$\begin{array}{lllll}
n(1 \mid \text{the}) & \times & t(\text{das} \mid \text{the}) & \times & d(3 \mid 1, 4, 4) \times \\
n(1 \mid \text{house}) & \times & t(\text{Haus} \mid \text{house}) & \times & d(4 \mid 2, 4, 4) \times \\
n(1 \mid \text{is}) & \times & t(\text{ist} \mid \text{is}) & \times & d(2 \mid 3, 4, 4) \times \\
n(1 \mid \text{small}) & \times & t(\text{klein} \mid \text{small}) & \times & d(1 \mid 4, 4, 4)
\end{array}$$

Word Alignments: IBM Model 3



Parameter Estimation

- What is $n(1 \mid \text{very}) = ?$ and $n(0 \mid \text{very}) = ?$
- What is $t(\text{Haus} \mid \text{house}) = ?$ and $t(\text{klein} \mid \text{small}) = ?$
- What is $d(1 \mid 4, 4, 4) = ?$ and $d(1 \mid 1, 4, 4) = ?$

Word Alignments: IBM Model 3

1	2	3	4
the	house	is	small
1	2	3	4
klein	ist	das	Haus

1	2	3	4
the	building	is	small
1	2	3	4
das	Haus	ist	klein

1	2	3	4	5
the	home	is	very	small
1	2	3	4	
das	Haus	ist	klitzeklein	

1	2	3	4	5
the	house	is	small	
1	2	3	4	5
das	Haus	ist	ja	klein

Parameter Estimation: Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, \mathbf{f}_{\text{len}}, \mathbf{e}_{\text{len}})$$

Word Alignments: IBM Model 3

Summary

- If we know the parameter values we can easily compute the probability $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e})$ given an aligned sentence pair
- If we are given a corpus of sentence pairs with alignments we can easily learn the parameter values by using relative frequencies.
- If we do not know the alignments then perhaps we can produce all possible alignments each with a certain probability?

IBM Model 3 is too hard: Let us try learning only $t(f_i \mid e_{a_i})$

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{\mathbf{a}} \prod_{i=1}^{I} n(\phi_{a_i} \mid e_{a_i}) \times t(f_i \mid e_{a_i}) \times d(i \mid a_i, \mathbf{f}_{\mathrm{len}}, \mathbf{e}_{\mathrm{len}})$$

Statistical Machine Translation

Generative Model of Word Alignment

Word Alignments: IBM Model 3

Word Alignments: IBM Model 1

Finding the best alignment: IBM Model 1

Learning Parameters: IBM Model 1

Word Alignments: IBM Model 1

Alignment probability

$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}$$

Example alignment

1	2	3	4
the	house	is	small
1	2	3	4
das	Haus	ist	klein

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

$$\begin{aligned}\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \\ & t(\text{das} \mid \text{the}) \times \\ & t(\text{Haus} \mid \text{house}) \times \\ & t(\text{ist} \mid \text{is}) \times \\ & t(\text{klein} \mid \text{small})\end{aligned}$$

Word Alignments: IBM Model 1

Generative "story" for Model 1

$$
\begin{array}{cccc}
\text{the} & \text{house} & \text{is} & \text{small} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\text{das} & \text{Haus} & \text{ist} & \text{klein}
\end{array}
\quad \text{(translate)}
$$

$$\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Statistical Machine Translation

Generative Model of Word Alignment

- Word Alignments: IBM Model 3
- Word Alignments: IBM Model 1
- Finding the best alignment: IBM Model 1
- Learning Parameters: IBM Model 1

Finding the best word alignment: IBM Model 1

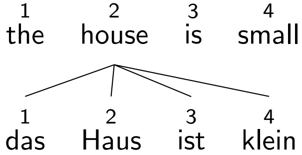
Compute the arg max word alignment

$$\hat{\mathbf{a}} = \arg\max_{\mathbf{a}} \Pr(\mathbf{a} \mid \mathbf{e}, \mathbf{f})$$

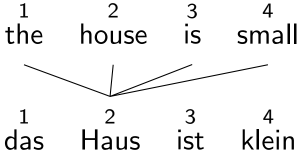
- For each f_i in $(f_1, \ldots, f_I)$ build $\mathbf{a} = (\hat{a}_1, \ldots, \hat{a}_I)$

$$\hat{a}_i = \arg\max_{a_i} t(f_i \mid e_{a_i})$$

Many to one alignment ✓



One to many alignment ✗



Statistical Machine Translation

Generative Model of Word Alignment

- Word Alignments: IBM Model 3
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- Learning Parameters: IBM Model 1

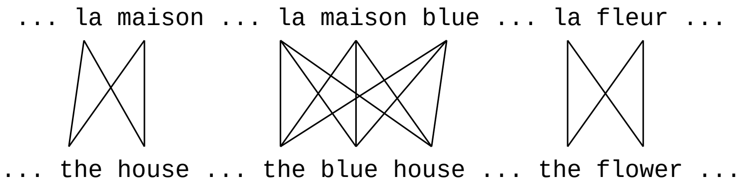
Learning parameters[from P.Koehn SMT book slides]

- We would like to estimate the lexical translation probabilities $t(e|f)$ from a parallel corpus
- ... but we do not have the alignments
- Chicken and egg problem
  - if we had the *alignments*,
    $\rightarrow$ we could estimate the *parameters* of our generative model
  - if we had the *parameters*,
    $\rightarrow$ we could estimate the *alignments*

EM Algorithm [from P.Koehn SMT book slides]

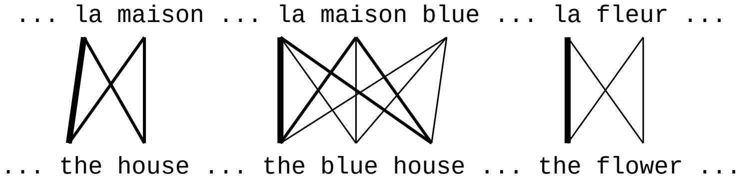
- Incomplete data
  - if we had *complete data*, we could estimate *model*
  - if we had *model*, we could fill in the *gaps in the data*
- Expectation Maximization (EM) in a nutshell
  1. initialize model parameters (e.g. uniform)
  2. assign probabilities to the missing data
  3. estimate model parameters from completed data
  4. iterate steps 2–3 until convergence

EM Algorithm[from P.Koehn SMT book slides]



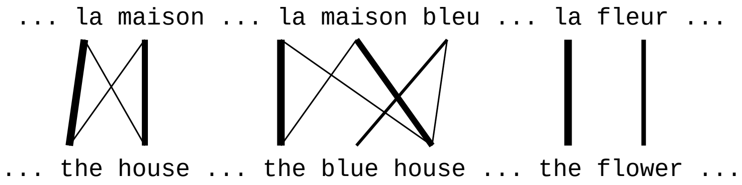
- Initial step: all alignments equally likely
- Model learns that, e.g., *la* is often aligned with *the*

EM Algorithm[from P.Koehn SMT book slides]



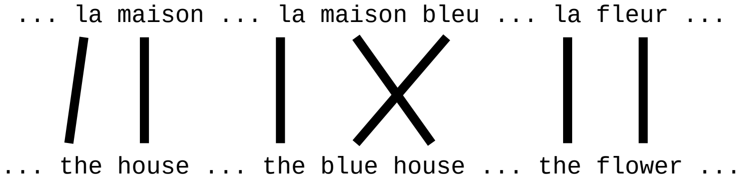
- After one iteration
- Alignments, e.g., between *la* and *the* are more likely

EM Algorithm[from P.Koehn SMT book slides]



- After another iteration
- It becomes apparent that alignments, e.g., between *fleur* and *flower* are more likely (pigeon hole principle)

EM Algorithm[from P.Koehn SMT book slides]



- Convergence
- Inherent hidden structure revealed by EM

EM Algorithm[from P.Koehn SMT book slides]

... la maison ... la maison bleu ... la fleur ...

... the house ... the blue house ... the flower ...

p(la|the) = 0.453
p(le|the) = 0.334
p(maison|house) = 0.876
p(bleu|blue) = 0.563
...

- Parameter estimation from the aligned corpus

IBM Model 1 and the EM Algorithm[from P.Koehn SMT book slides]

- EM Algorithm consists of two steps
- Expectation-Step: Apply model to the data
  - parts of the model are hidden (here: alignments)
  - using the model, assign probabilities to possible values
- Maximization-Step: Estimate model from data
  - take assign values as fact
  - collect counts (weighted by probabilities)
  - estimate model from counts
- Iterate these steps until convergence

IBM Model 1 and the EM Algorithm [from P.Koehn SMT book slides]

- We need to be able to compute:
  - Expectation-Step: probability of alignments
  - Maximization-Step: count collection

Word Alignments: IBM Model 1

Alignment probability

$$\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})}
\end{aligned}$$

Computing the denominator

- The denominator above is summing over J^I alignments
- An interlude on how compute the denominator faster ...

Word Alignments: IBM Model 1

Sum over all alignments

$$\sum_{\mathbf{a}} \Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e}) = \sum_{a_1=1}^{J} \sum_{a_2=1}^{J} \dots \sum_{a_I=1}^{J} \prod_{i=1}^{I} t(f_i \mid e_{a_i})$$

Assume (f_1, f_2, f_3) and (e_1, e_2)

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\sum_{a_1=1}^{2} \sum_{a_2=1}^{2} \sum_{a_3=1}^{2} t(f_1 \mid e_{a_1}) \times t(f_2 \mid e_{a_2}) \times t(f_3 \mid e_{a_3})$$

$J^I = 2^3$ terms to be added:

$$\begin{array}{ccccccc}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & \times & t(f_3 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) & \times & t(f_3 \mid e_2) &
\end{array}$$

Word Alignments: IBM Model 1

Factor the terms:

$$\begin{array}{lcll}
(t(f_1 \mid e_1) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_1) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_1)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) & + \\
(t(f_1 \mid e_2) \times t(f_2 \mid e_2)) & \times & (t(f_3 \mid e_1) + t(f_3 \mid e_2)) &
\end{array}$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lcll}
t(f_1 \mid e_1) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_1) & \times & t(f_2 \mid e_2) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_1) & + \\
t(f_1 \mid e_2) & \times & t(f_2 \mid e_2) &
\end{array} \right)$$

$$(t(f_3 \mid e_1) + t(f_3 \mid e_2)) \left(\begin{array}{lcll}
t(f_1 \mid e_1) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) & + \\
t(f_1 \mid e_2) & \times & (t(f_2 \mid e_1) + t(f_2 \mid e_2)) &
\end{array} \right)$$

Word Alignments: IBM Model 1

Assume (f_1, f_2, f_3) and (e_1, e_2): $I = 3$ and $J = 2$

$$\prod_{i=1}^{3} \sum_{a_i=1}^{2} t(f_i \mid e_{a_i})$$

$I \times J = 2 \times 3$ terms to be added:

$$\begin{array}{ccccc}
(t(f_1 \mid e_1) & + & t(f_1 \mid e_2)) & \times \\
(t(f_2 \mid e_1) & + & t(f_2 \mid e_2)) & \times \\
(t(f_3 \mid e_1) & + & t(f_3 \mid e_2)) &
\end{array}$$

Word Alignments: IBM Model 1

Alignment probability

$$
\begin{aligned}
\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) &= \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\sum_{\mathbf{a}} \prod_{i=1}^{I} t(f_i \mid e_{a_i})} \\
&= \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j)}
\end{aligned}
$$

Learning Parameters: IBM Model 1

1	2	1	2	1	2
the	house	the	book	a	book
1	2	1	2	1	2
das	Haus	das	Buch	ein	Buch

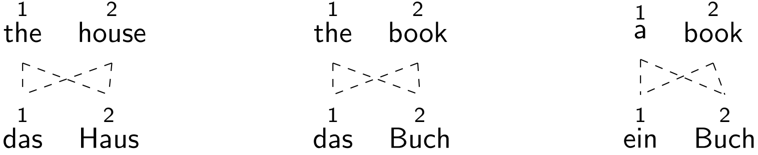
Learning parameters $t(f|e)$ when alignments are known

$$t(das \mid the) = \frac{c(das,the)}{\sum_f c(f,the)} \quad t(house \mid Haus) = \frac{c(Haus,house)}{\sum_f c(f,house)}$$

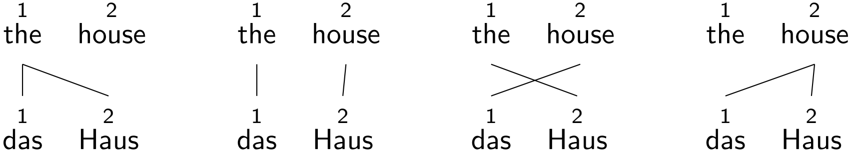
$$t(ein \mid a) = \frac{c(ein,a)}{\sum_f c(f,a)} \quad t(Buch \mid book) = \frac{c(Buch,book)}{\sum_f c(f,book)}$$

$$t(f|e) = \sum_{s=1}^{N} \sum_{f \to e \in \mathbf{f}^{(s)},\mathbf{e}^{(s)}} \frac{c(f,e)}{\sum_f c(f,e)}$$

Learning Parameters: IBM Model 1



Learning parameters $t(f|e)$ when alignments are *unknown*



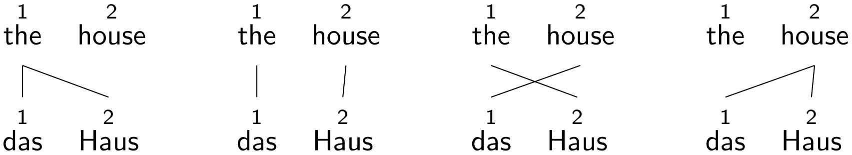
Also list alignments for *(the book, das Buch)* and *(a book, ein Buch)*

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

$$\begin{aligned} t(Haus \mid the) &= 0.25 & \quad t(das \mid house) &= 0.5 \\ t(das \mid the) &= 0.5 & \quad t(Haus \mid house) &= 0.5 \\ t(Buch \mid the) &= 0.25 & \quad t(Buch \mid house) &= 0.0 \end{aligned}$$

Compute posterior for each alignment



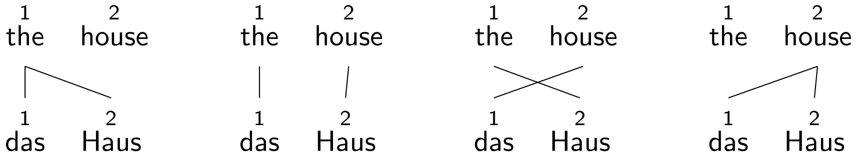
$$\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{f}, \mathbf{a} \mid \mathbf{e})}{\Pr(\mathbf{f} \mid \mathbf{e})} = \frac{\prod_{i=1}^{I} t(f_i \mid e_{a_i})}{\prod_{i=1}^{I} \sum_{j=1}^{J} t(f_i \mid e_j)}$$

Learning Parameters: IBM Model 1

Initialize $t^0(f|e)$

$$\begin{aligned} t(Haus \mid the) &= 0.25 \\ t(das \mid the) &= 0.5 \\ t(Buch \mid the) &= 0.25 \end{aligned} \qquad \begin{aligned} t(das \mid house) &= 0.5 \\ t(Haus \mid house) &= 0.5 \\ t(Buch \mid house) &= 0.0 \end{aligned}$$

Compute $\Pr(\mathbf{a}, \mathbf{f} \mid \mathbf{e})$ for each alignment

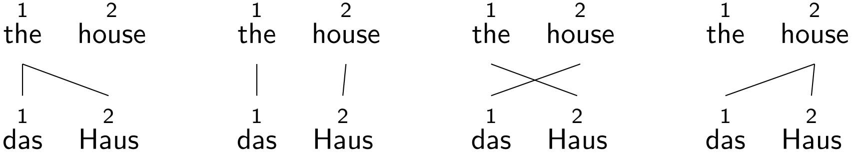


0.5×0.25	0.5×0.5	0.25×0.5	0.5×0.5
0.125	0.25	0.125	0.25

Learning Parameters: IBM Model 1

Compute $\Pr(\mathbf{a} \mid \mathbf{f}, \mathbf{e}) = \frac{\Pr(\mathbf{a},\mathbf{f}|\mathbf{e})}{\Pr(\mathbf{f}|\mathbf{e})}$

$$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$$

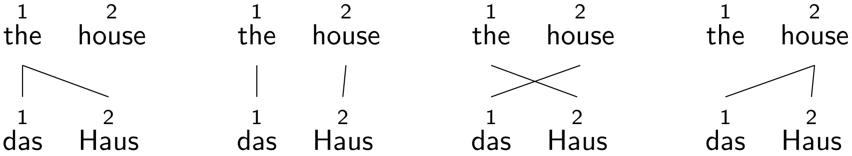


$\frac{0.125}{0.75}$	$\frac{0.25}{0.75}$	$\frac{0.125}{0.75}$	$\frac{0.25}{0.75}$
0.167	0.334	0.167	0.334

Compute fractional counts $c(f, e)$

$$\begin{aligned} c(Haus, the) &= 0.125 + 0.125 & c(das, house) &= 0.125 + 0.25 \\ c(das, the) &= 0.125 + 0.25 & c(Haus, house) &= 0.25 + 0.25 \\ c(Buch, the) &= 0.0 & c(Buch, house) &= 0.0 \end{aligned}$$

Learning Parameters: IBM Model 1



$\Pr(\mathbf{f} \mid \mathbf{e}) = 0.125 + 0.25 + 0.125 + 0.25 = 0.75$

Expectation step: expected counts $g(f, e)$

$$
\begin{aligned}
g(\textit{das}, \textit{the}) &= \tfrac{0.125+0.25}{0.75} & \quad g(\textit{das}, \textit{house}) &= \tfrac{0.125+0.25}{0.75} \\
g(\textit{Haus}, \textit{the}) &= \tfrac{0.125+0.125}{0.75} & \quad g(\textit{Haus}, \textit{house}) &= \tfrac{0.25+0.25}{0.75} \\
g(\textit{Buch}, \textit{the}) &= 0.0 & \quad g(\textit{Buch}, \textit{house}) &= 0.0
\end{aligned}
$$

Maximization step: get new $t^{(1)}(f \mid e) = \frac{g(f,e)}{\sum_f g(f,e)}$

Learning Parameters: IBM Model 1

Expectation step: expected counts $g(f, e)$

$$\begin{array}{lcl} g(das, the) & = & 0.5 \\ g(Haus, the) & = & 0.334 \\ g(Buch, the) & = & 0.0 \\ \textbf{total} & = & 0.834 \end{array} \qquad \begin{array}{lcl} g(das, house) & = & 0.5 \\ g(Haus, house) & = & 0.667 \\ g(Buch, house) & = & 0.0 \\ \textbf{total} & = & 1.167 \end{array}$$

Maximization step: get new $t^{(1)}(f \mid e) = \frac{g(f,e)}{\sum_f g(f,e)}$

$$\begin{array}{lcl} t(Haus \mid the) & = & 0.4 \\ t(das, \mid the) & = & 0.6 \\ t(Buch \mid the) & = & 0.0 \end{array} \qquad \begin{array}{lcl} t(das \mid house) & = & 0.43 \\ t(Haus \mid house) & = & 0.57 \\ t(Buch \mid house) & = & 0.0 \end{array}$$

Keep iterating: Compute $t^{(0)}, t^{(1)}, t^{(2)}, \ldots$ until convergence

Parameter Estimation: IBM Model 1

EM learns the parameters $t(\cdot \mid \cdot)$ that maximizes the log-likelihood of the training data:

$$\arg\max_t L(t) = \arg\max_t \sum_s \log \Pr(\mathbf{f}^{(s)} \mid \mathbf{e}^{(s)}, t)$$

- Start with an initial estimate t_0
- Modify it iteratively to get $t_1, t_2, \ldots$
- Re-estimate t from parameters at previous time step t_{-1}
- The convergence proof of EM guarantees that $L(t) \geq L(t_{-1})$
- EM converges when $L(t) - L(t_{-1})$ is zero (or almost zero).

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