Natural Language Processing

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September 16, 2014

Trigram Models

- The trigram model:
  $P(w_1, w_2, \ldots, w_n) =$
  $P(w_1) \times P(w_2 \mid w_1) \times P(w_3 \mid w_1, w_2) \times P(w_4 \mid w_2, w_3) \times$
  $\ldots P(w_i \mid w_{i-2}, w_{i-1}) \ldots \times P(w_n \mid w_{n-2}, \ldots, w_{n-1})$

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  $\ldots P(w_i \mid w_{i-2}, w_{i-1}) \ldots \times P(w_n \mid w_{n-2}, \ldots, w_{n-1})$
- Notice that the length of the sentence n is variable
- What is the event space?

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\begin{array}{rl}
a \text{ stop} & 0.5 \\
b \text{ stop} & 0.5 \\
aa \text{ stop} & 0.5^2 \\
bb \text{ stop} & 0.5^2 \\
& \vdots
\end{array}
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- But $P(a) + P(b) + P(aa) + P(bb) = 1.5$!!

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- $P(\text{stop}) = 0.5$, $P(a\ \text{stop}) = P(b\ \text{stop}) = 0.25 \times 0.5 = 0.125$, $P(aa\ \text{stop}) = 0.25^2 \times 0.5 = 0.03125$ (now the sum is no longer greater than one)

The stop symbol

- With this new stop symbol we can show that $\sum_w P(w) = 1$
  Notice that the probability of any sequence of length n is $0.25^n \times 0.5$
  Also there are 2^n sequences of length n

$$\sum_w P(w) =$$

$$\sum_{n=0}^{\infty} 2^n \times 0.25^n \times 0.5$$

$$\sum_{n=0}^{\infty} 0.5^n \times 0.5 = \sum_{n=0}^{\infty} 0.5^{n+1}$$

$$\sum_{n=1}^{\infty} 0.5^n = 1$$

Bigram Models

- In practice:

$$\begin{aligned} P(\text{Mork read a book}) = \\ & P(\text{Mork} \mid <\text{start}>) \times P(\text{read} \mid \text{Mork}) \times \\ & P(\text{a} \mid \text{read}) \times P(\text{book} \mid \text{a}) \times \\ & P(<\text{stop}> \mid \text{book}) \end{aligned}$$

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\end{aligned}$$

- $P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$
  On unseen data, $c(w_{i-1}, w_i)$ or worse $c(w_{i-1})$ could be zero

$$\sum_{w_i} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})} =?$$

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- Not just unobserved events: what about events observed once?

Add-one Smoothing

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- Let V be the number of words in our vocabulary
  Assign count of 1 to unseen bigrams

Add-one Smoothing

$$
\begin{aligned}
&P(\text{Mindy read a book}) = \\
&\quad P(\text{Mindy} \mid <\text{start}>) \times P(\text{read} \mid \text{Mindy}) \times \\
&\quad P(\text{a} \mid \text{read}) \times P(\text{book} \mid \text{a}) \times \\
&\quad P(<\text{stop}> \mid \text{book})
\end{aligned}
$$

- Without smoothing:

$$P(\text{read} \mid \text{Mindy}) = \frac{c(\text{Mindy, read})}{c(\text{Mindy})} = 0$$

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- Without smoothing:

$$P(\text{read} \mid \text{Mindy}) = \frac{c(\text{Mindy, read})}{c(\text{Mindy})} = 0$$

- With add-one smoothing (assuming c(Mindy) = 1 but c(Mindy, read) = 0):

$$P(\text{read} \mid \text{Mindy}) = \frac{1}{V+1}$$

Additive Smoothing: (Lidstone 1920, Jeffreys 1948)

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-one smoothing works horribly in practice. Seems like 1 is too large a count for unobserved events.

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- $0 < \delta \leq 1$
  Still works horribly in practice, but better than add-one smoothing.

Good-Turing Smoothing: (Good, 1953)

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- Chance you will observe a new kind of seafood: $\frac{3}{18}$
- How likely are you to see another plate of salmon: should be $< \frac{2}{18}$

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- How many types of seafood (words) were seen once? Use this to predict probabilities for unseen events
  Let n_1 be the number of events that occurred once: $p_0 = \frac{n_1}{N}$

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- n_r: number of different objects seen r times

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- How likely is new data? Let n_1 be the number of items occurring once, which is 3 in this case. N is the total, which is 18.

$$p_0 = \frac{n_1}{N} = \frac{3}{18} = 0.166$$

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- To compute 1^*, we need $n_1 = 3$ and $n_2 = 1$

$$1^* = 2 \times \frac{1}{3} = \frac{2}{3}$$

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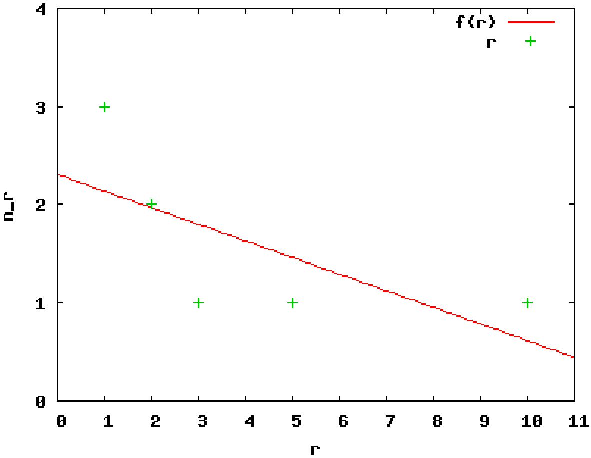
- What happens when $n_{r+1} = 0$? (smoothing before smoothing)

Simple Good-Turing: linear interpolation for missing n_{r+1}



$$
\begin{aligned}
f(r) &= a + b * r \\
a &= 2.3 \\
b &= -0.17
\end{aligned}
$$

r	$n_r = f(r)$
1	2.14
2	1.97
3	1.80
4	1.63
5	1.46
6	1.29
7	1.12
8	0.95
9	0.78
10	0.61
11	0.44

Comparison between Add-one and Good-Turing

freq r	num with freq r n_r	NS p_r	Add1 p_r	SGT p_r
0	0	0	0.0294	0.12
1	3	0.04	0.0588	0.03079
2	2	0.08	0.0882	0.06719
3	1	0.12	0.1176	0.1045
5	1	0.2	0.1764	0.1797
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- Important: we added a new word type for unseen words. Let's call it UNK, the unknown word.
- Check that: $1.0 == \sum_r n_r \times p_r$
  $0.12 + (3 * 0.03079) + (2 * 0.06719) + 0.1045 + 0.1797 + 0.3691 = 1.0$

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- NS = No smoothing: $p_r = \frac{r}{N}$
- Add1 = Add-one smoothing: $p_r = \frac{1+r}{V+N}$
- SGT = Simple Good-Turing: $p_0 = \frac{n_1}{N}$, $p_r = \frac{(r+1)\frac{n_{r+1}}{n_r}}{N}$
  with linear interpolation for missing values where $n_{r+1} = 0$
  (Gale and Sampson, 1995) *http://www.grsampson.net/AGtf1.html*

Using unigrams to smooth bigrams: incorrect version

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- Works for trigrams too: back off to bigrams and then unigrams
- Problem: probabilities get mixed up (unseen bigrams, for example will get higher probabilities than seen bigrams)

Interpolation: Jelinek-Mercer Smoothing

$$P_{ML}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- $P_{JM}(w_i \mid w_{i-1}) = \lambda P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda) P_{ML}(w_i)$
  where, $0 \leq \lambda \leq 1$

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- Jelinek-Mercer (1980) describe an elegant form of this **interpolation**:

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1 - \lambda) P_{JM}(n - 1\text{gram})$$

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- What about $P_{JM}(w_i)$?
For missing unigrams: $P_{JM}(w_i) = \lambda P_{ML}(w_i) + (1-\lambda)\frac{\delta}{V}$

Interpolation: Finding λ

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- Deleted Interpolation (Jelinek, Mercer)
  compute λ values to minimize cross-entropy on **held-out** data which is deleted from the initial set of training data
- Improved JM smoothing, a separate λ for each w_{i-1}:

$$P_{JM}(w_i \mid w_{i-1}) = \lambda(w_{i-1}) P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda(w_{i-1})) P_{ML}(w_i)$$

$$\text{where } \sum_i \lambda(w_i) = 1 \text{ because } \sum_{w_i} P(w_i \mid w_{i-1}) = 1$$

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- Katz Backoff (include Good-Turing with Backoff Smoothing)

$$P_{katz}(y \mid x) = \begin{cases} \frac{c^*(xy)}{c(x)} & \text{if } c(xy) > 0 \\ \alpha(x) P_{katz}(y) & \text{otherwise} \end{cases}$$

Backoff Smoothing: Katz Backoff

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- where $\alpha(x)$ is chosen to make sure that $P_{katz}(y \mid x)$ is a proper probability

$$\alpha(x) = 1 - \sum_y \frac{c^*(xy)}{c(x)}$$

Backoff Smoothing: Katz Backoff

x	$c(x)$	$c^*(x)$	$\frac{c^*(x)}{c(the)}$
the	48		
the,dog	15	14.5	14.5/48
the,woman	11	10.5	10.4/48
the,man	10	9.5	9.5/48
the,park	5	4.5	4.5/48
the,job	2	1.5	4.5/48
the,telescope	1	0.5	0.5/48
the,manual	1	0.5	0.5/48
the,afternoon	1	0.5	0.5/48
the,country	1	0.5	0.5/48
the,street	1	0.5	0.5/48
TOTAL			0.9479
the,UNK	0		0.052

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- Absolute discounting (Ney, Essen, Kneser)

$$P_{abs}(y \mid x) = \begin{cases} \frac{c(xy)-D}{c(x)} & \text{if } c(xy) > 0 \\ \alpha(x) P_{abs}(y) & \text{otherwise} \end{cases}$$

compute $\alpha(x)$ as was done in Katz smoothing

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  - *stew* is common, **and** occurs in many contexts
  - Hence weight the interpolation based on number of contexts for the word using discounting

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- Let $T = s_0, \ldots, s_m$ be a text corpus with sentences s_0 through s_m
- What is $P(T)$?
  Let us assume that we trained $P(\cdot)$ on some *training data*, and T is the *test data*

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- To do this we use the *per word* perplexity of the model:

$$PP_P(T) = P(T)^{-\frac{1}{W_T}} = \sqrt[W_T]{\frac{1}{P(T)}}$$

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- Therefore, $H_P(T) = \log_2 PP_P(T) = -\frac{1}{W_T}\log_2 P(T)$
- Above we use a unigram model $P(w)$, but the same derivation holds for bigram, trigram, ...

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  Correlation with performance of the language model in various applications
- Performance of a language model is its cross-entropy or perplexity on *test data* (unseen data)
  corresponds to the number bits required to encode that data
- On various real life datasets, typical perplexity values yielded by n-gram models on English text range from about 50 to almost 1000 (corresponding to cross entropies from about 6 to 10 bits/word)