SFU NatLangLab

Natural Language Processing

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Part 1: Probability models of Language

The Language Modeling problem

Setup

- Assume a (finite) vocabulary of words:
$$\mathcal{V} = \{\mathit{killer}, \mathit{crazy}, \mathit{clown}\}$$

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- Use $\mathcal{V}$ to construct an infinite set of *sentences*
$$\begin{aligned}\mathcal{V}^+ = \; \{ & \\ & \text{clown, killer clown, crazy clown,} \\ & \text{crazy killer clown, killer crazy clown,} \\ & \ldots \\ \} & \end{aligned}$$

The Language Modeling problem

Setup

- Assume a (finite) vocabulary of words:
$$\mathcal{V} = \{\textit{killer}, \textit{crazy}, \textit{clown}\}$$
- Use $\mathcal{V}$ to construct an infinite set of *sentences*
$$\mathcal{V}^+ = \{ \text{clown, killer clown, crazy clown, crazy killer clown, killer crazy clown, } \ldots \}$$
- A *sentence* is **defined** as each $s \in \mathcal{V}^+$

The Language Modeling problem

Data

Given a training data set of example sentences $s \in \mathcal{V}^+$

The Language Modeling problem

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Language Modeling problem

Estimate a probability model:

$$\sum_{s \in \mathcal{V}^+} p(s) = 1.0$$

- p(clown) = 1e-5
- p(killer) = 1e-6
- p(killer clown) = 1e-12
- p(crazy killer clown) = 1e-21
- p(crazy killer clown killer) = 1e-110
- p(crazy clown killer killer) = 1e-127

Why do we want to do this?

Scoring Hypotheses in Speech Recognition

From acoustic signal to candidate transcriptions

Hypothesis	Score
the station signs are in deep in english	-14732
the stations signs are in deep in english	-14735
the station signs are in deep into english	-14739
the station 's signs are in deep in english	-14740
the station signs are in deep in the english	-14741
the station signs are indeed in english	-14757
the station 's signs are indeed in english	-14760
the station signs are indians in english	-14790
the station signs are indian in english	-14799
the stations signs are indians in english	-14807
the stations signs are indians and english	-14815

Scoring Hypotheses in Machine Translation

From source language to target language candidates

Hypothesis	Score
we must also discuss a vision .	-29.63
we must also discuss on a vision .	-31.58
it is also discuss a vision .	-31.96
we must discuss on greater vision .	-36.09
⋮	⋮

Scoring Hypotheses in Decryption

Character substitutions on ciphertext to plaintext candidates

Hypothesis	Score
Heopaj, zk ukq swjp pk gjks w oaynap?	-93
Urbcnw, mx hxd fjwc cx twxf j bnlanc?	-92
Wtdepy, oz jzf hlye ez vyzh l dpncpe?	-91
Mjtufo, ep zpv xbou up lopx b tfdsfu?	-89
Nkuvgp, fq aqw ycpv vq mpqy c ugetgv?	-87
Gdnozi, yj tjp rvio oj fijr v nzxmzo?	-86
Czjkve, uf pfl nrek kf befn r jvtivk?	-85
Yvfgra, qb lbh jnag gb xabj n frperg?	-84
Zwghsb, rc mci kobh hc ybck o gsqfsh?	-83
Byijud, te oek mqdj je adem q iushuj?	-77
Jgqrcl, bm wms uylr rm ilmu y qcapcr?	-76
Listen, do you want to know a secret?	-25

Scoring Hypotheses in Spelling Correction

Substitute spelling variants to generate hypotheses

Hypothesis	Score
… stellar and versatile **acress** whose combination of sass and glamour has defined her …	-18920
… stellar and versatile **acres** whose combination of sass and glamour has defined her …	-10209
… stellar and versatile **actress** whose combination of sass and glamour has defined her …	-9801

Probability models of language

Question

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Probability models of language

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- **And** the model should be equal to $\sum_{s \in \mathcal{V}^+} P(s)$.

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- Given a finite vocabulary set $\mathcal{V}$
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- Write down a new model over $\mathcal{V}^+$ such that $P(s \mid \ell)$ is in the model
- **And** the model should be equal to $\sum_{s \in \mathcal{V}^+} P(s)$.
- Write down the model

$$\sum_{s \in \mathcal{V}^+} P(s) = \ldots$$

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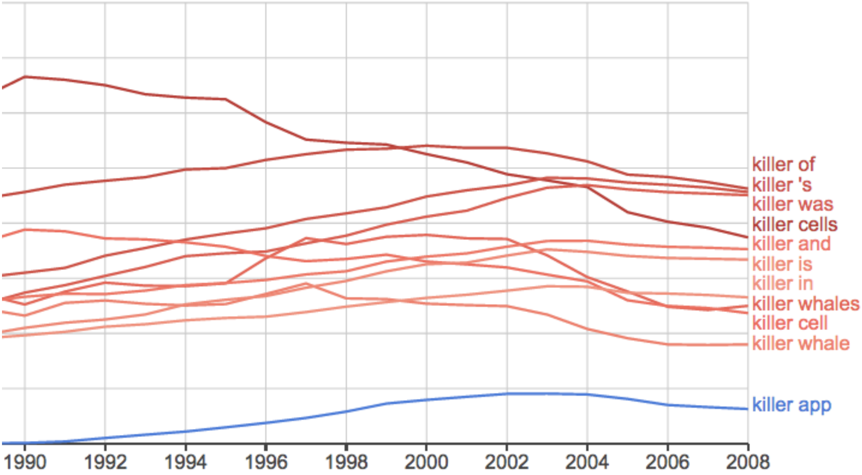
Part 2: *n*-grams for Language Modeling

n-gram Models

Google *n*-gram viewer



Learning Language Models

- Directly count using a training data set of sentences: $w_1, \ldots, w_n$:

$$p(w_1, \ldots, w_n) = \frac{n(w_1, \ldots, w_n)}{N}$$

Learning Language Models

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- Problem: does not generalize to new sentences unseen in the training data.

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- What are the chances you will see a sentence: `crazy killer clown crazy killer`?

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- N is the sum over all possible $n(\cdot)$ values
- Problem: does not generalize to new sentences unseen in the training data.
- What are the chances you will see a sentence: `crazy killer clown crazy killer`?
- In NLP applications we often need to assign non-zero probability to previously unseen sentences.

Learning Language Models

Apply the Chain Rule: the unigram model

$$\begin{aligned} p(w_1, \ldots, w_n) &\approx p(w_1)p(w_2)\ldots p(w_n) \\ &= \prod_i p(w_i) \end{aligned}$$

Learning Language Models

Apply the Chain Rule: the unigram model

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Big problem with a unigram language model

p(the the the the the the the) $>$ p(we must also discuss a vision .)

Learning Language Models

Apply the Chain Rule: the bigram model

$$
\begin{aligned}
p(w_1, \ldots, w_n) &\approx p(w_1)p(w_2 \mid w_1) \ldots p(w_n \mid w_{n-1}) \\
&= p(w_1) \prod_{i=2}^{n} p(w_i \mid w_{i-1})
\end{aligned}
$$

Learning Language Models

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p(w_1, \ldots, w_n) &\approx p(w_1)p(w_2 \mid w_1)\ldots p(w_n \mid w_{n-1}) \\
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\end{aligned}
$$

Better than unigram

$p(\text{the the the the the the the}) < p(\text{we must also discuss a vision .})$

Learning Language Models

Apply the Chain Rule: the trigram model

$$
\begin{aligned}
& p(w_1, \ldots, w_n) \approx \\
& \qquad p(w_1)p(w_2 \mid w_1)p(w_3 \mid w_1, w_2) \ldots p(w_n \mid w_{n-2}, w_{n-1}) \\
& \qquad p(w_1)p(w_2 \mid w_1) \prod_{i=3}^{n} p(w_i \mid w_{i-2}, w_{i-1})
\end{aligned}
$$

Learning Language Models

Apply the Chain Rule: the trigram model

$$
\begin{aligned}
p(w_1, \ldots, w_n) \approx& \\
& p(w_1)p(w_2 \mid w_1)p(w_3 \mid w_1, w_2) \ldots p(w_n \mid w_{n-2}, w_{n-1}) \\
& p(w_1)p(w_2 \mid w_1)\prod_{i=3}^{n} p(w_i \mid w_{i-2}, w_{i-1})
\end{aligned}
$$

Better than bigram, but …

p(we must also discuss a vision .) might be zero because we have not seen p(discuss | must also)

Maximum Likelihood Estimate

Using training data to learn a trigram model

- Let $c(u, v, w)$ be the count of the trigram u, v, w, e.g. $c(\mathit{crazy}, \mathit{killer}, \mathit{clown})$

Maximum Likelihood Estimate

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- Let $c(u, v)$ be the count of the bigram u, v, e.g. $c(crazy, killer)$
- For any u, v, w we can compute the conditional probability of generating w given u, v:

$$p(w \mid u, v) = \frac{c(u, v, w)}{c(u, v)}$$

Maximum Likelihood Estimate

Using training data to learn a trigram model

- Let $c(u, v, w)$ be the count of the trigram u, v, w, e.g. $c(crazy, killer, clown)$
- Let $c(u, v)$ be the count of the bigram u, v, e.g. $c(crazy, killer)$
- For any u, v, w we can compute the conditional probability of generating w given u, v:

$$p(w \mid u, v) = \frac{c(u, v, w)}{c(u, v)}$$

- For example:

$$p(clown \mid crazy, killer) = \frac{c(crazy, killer, clown)}{c(crazy, killer)}$$

Number of Parameters

How many probabilities in each *n*-gram model

- Assume $\mathcal{V} = \{\textit{killer}, \textit{crazy}, \textit{clown}, \textit{UNK}\}$

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Question

How many unigram probabilities: $P(x)$ for $x \in \mathcal{V}$?

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How many bigram probabilities: $P(y|x)$ for $x, y \in \mathcal{V}$?

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How many bigram probabilities: $P(y|x)$ for $x, y \in \mathcal{V}$?

$4^2 = 16$

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How many probabilities in each *n*-gram model

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Number of Parameters

How many probabilities in each *n*-gram model

- Assume $\mathcal{V} = \{killer, crazy, clown, UNK\}$

Question

How many trigram probabilities: $P(z|x, y)$ for $x, y, z \in \mathcal{V}$?

Number of Parameters

How many probabilities in each *n*-gram model

- Assume $\mathcal{V} = \{killer, crazy, clown, UNK\}$

Question

How many trigram probabilities: $P(z|x, y)$ for $x, y, z \in \mathcal{V}$?

$4^3 = 64$

Number of Parameters

Question

- Assume $|\mathcal{V}| = 50{,}000$ (a realistic vocabulary size for English)
- What is the minimum size of training data in tokens?
  - If you wanted to observe all unigrams at least once.
  - If you wanted to observe all trigrams at least once.

Some trigrams should be zero since they do not occur in the language, $P(\mathit{the} \mid \mathit{the}, \mathit{the})$.
But others are simply unobserved in the training data, $P(\mathit{idea} \mid \mathit{colourless}, \mathit{green})$.

Number of Parameters

Question

- Assume $\mid \mathcal{V} \mid = 50{,}000$ (a realistic vocabulary size for English)
- What is the minimum size of training data in tokens?
  - If you wanted to observe all unigrams at least once.
  - If you wanted to observe all trigrams at least once.

125,000,000,000,000 (125 Ttokens)

Some trigrams should be zero since they do not occur in the language, $P(\mathit{the} \mid \mathit{the}, \mathit{the})$.
But others are simply unobserved in the training data, $P(\mathit{idea} \mid \mathit{colourless}, \mathit{green})$.

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Part 3: Smoothing Probability Models

Bigram Models

- In practice:

$$
\begin{aligned}
&P(\texttt{crazy killer clown}) = \\
&\quad P(\texttt{crazy} \mid \texttt{<s>}) \times P(\texttt{killer} \mid \texttt{crazy}) \times \\
&\quad P(\texttt{clown} \mid \texttt{killer}) \times P(\texttt{</s>} \mid \texttt{clown})
\end{aligned}
$$

Bigram Models

- In practice:

$$P(\texttt{crazy killer clown}) = \\ P(\texttt{crazy} \mid \texttt{<s>}) \times P(\texttt{killer} \mid \texttt{crazy}) \times \\ P(\texttt{clown} \mid \texttt{killer}) \times P(\texttt{</s>} \mid \texttt{clown})$$

- $P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$
  On unseen data, $c(w_{i-1}, w_i)$ or worse $c(w_{i-1})$ could be zero

$$\sum_{w_i} \frac{c(w_{i-1}, w_i)}{c(w_{i-1})} = ?$$

Smoothing

- **Smoothing** deals with events that have been observed zero times

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- Smoothing algorithms also tend to improve the accuracy of the model

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Smoothing

- **Smoothing** deals with events that have been observed zero times
- Smoothing algorithms also tend to improve the accuracy of the model

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Not just unobserved events: what about events observed once?

Add-one Smoothing

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-one Smoothing:

$$P(w_i \mid w_{i-1}) = \frac{1 + c(w_{i-1}, w_i)}{V + c(w_{i-1})}$$

Add-one Smoothing

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Add-one Smoothing:

$$P(w_i \mid w_{i-1}) = \frac{1 + c(w_{i-1}, w_i)}{V + c(w_{i-1})}$$

- Let V be the number of words in our vocabulary
  Assign count of 1 to unseen bigrams

Add-one Smoothing

$$
\begin{aligned}
&P(\texttt{insane killer clown}) = \\
&\quad P(\texttt{insane} \mid \texttt{<s>}) \times P(\texttt{killer} \mid \texttt{insane}) \times \\
&\quad P(\texttt{clown} \mid \texttt{killer}) \times P(\texttt{</s>} \mid \texttt{clown})
\end{aligned}
$$

- Without smoothing:

$$P(\texttt{killer} \mid \texttt{insane}) = \frac{c(\texttt{insane, killer})}{c(\texttt{insane})} = 0$$

Add-one Smoothing

$$
\begin{aligned}
&P(\texttt{insane killer clown}) = \\
&\quad P(\texttt{insane} \mid \texttt{<s>}) \times P(\texttt{killer} \mid \texttt{insane}) \times \\
&\quad P(\texttt{clown} \mid \texttt{killer}) \times P(\texttt{</s>} \mid \texttt{clown})
\end{aligned}
$$

- Without smoothing:

$$P(\texttt{killer} \mid \texttt{insane}) = \frac{c(\texttt{insane, killer})}{c(\texttt{insane})} = 0$$

- With add-one smoothing (assuming initially that $\texttt{c(insane)} = 1$ and $\texttt{c(insane, killer)} = 0$):

$$P(\texttt{killer} \mid \texttt{insane}) = \frac{1}{V+1}$$

Additive Smoothing: (Lidstone 1920, Jeffreys 1948)

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Why add 1? 1 is an overestimate for unobserved events.

Additive Smoothing: (Lidstone 1920, Jeffreys 1948)

$$P(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- Why add 1? 1 is an overestimate for unobserved events.
- Additive Smoothing:

$$P(w_i \mid w_{i-1}) = \frac{\delta + c(w_{i-1}, w_i)}{(\delta \times V) + c(w_{i-1})}$$

Additive Smoothing: (Lidstone 1920, Jeffreys 1948)

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- Why add 1? 1 is an overestimate for unobserved events.
- Additive Smoothing:

$$P(w_i \mid w_{i-1}) = \frac{\delta + c(w_{i-1}, w_i)}{(\delta \times V) + c(w_{i-1})}$$

- $0 < \delta \leq 1$

Interpolation: Jelinek-Mercer Smoothing

$$P_{ML}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

- $P_{JM}(w_i \mid w_{i-1}) = \lambda P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda) P_{ML}(w_i)$ where, $0 \leq \lambda \leq 1$

Interpolation: Jelinek-Mercer Smoothing

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- $P_{JM}(w_i \mid w_{i-1}) = \lambda P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda) P_{ML}(w_i)$ where, $0 \leq \lambda \leq 1$
- Jelinek and Mercer (1980) describe an elegant form of this **interpolation**:

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1 - \lambda) P_{JM}(n - 1\text{gram})$$

Interpolation: Jelinek-Mercer Smoothing

$$P_{ML}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

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- Jelinek and Mercer (1980) describe an elegant form of this **interpolation**:

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1 - \lambda) P_{JM}(n - 1\text{gram})$$

- What about $P_{JM}(w_i)$?
  For missing unigrams: $P_{JM}(w_i) = \lambda P_{ML}(w_i) + (1 - \lambda)\frac{\delta}{V}$

Interpolation: Finding λ

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1-\lambda) P_{JM}(n-1\text{gram})$$

- Deleted Interpolation (Jelinek, Mercer)
  compute λ values to minimize cross-entropy on **held-out** data which is deleted from the initial set of training data

Interpolation: Finding λ

$$P_{JM}(n\text{gram}) = \lambda P_{ML}(n\text{gram}) + (1-\lambda) P_{JM}(n-1\text{gram})$$

- Deleted Interpolation (Jelinek, Mercer)
  compute λ values to minimize cross-entropy on **held-out** data which is deleted from the initial set of training data
- Improved JM smoothing, a separate λ for each w_{i-1}:

$$P_{JM}(w_i \mid w_{i-1}) = \lambda(w_{i-1}) P_{ML}(w_i \mid w_{i-1}) + (1 - \lambda(w_{i-1})) P_{ML}(w_i)$$

Backoff Smoothing with Discounting

- Discounting (Ney, Essen, Kneser)

$$P_{abs}(y \mid x) = \begin{cases} \frac{c(xy) - D}{c(x)} & \text{if } c(xy) > 0 \\ \alpha(x) P_{abs}(y) & \text{otherwise} \end{cases}$$

Backoff Smoothing with Discounting

- Discounting (Ney, Essen, Kneser)

$$P_{abs}(y \mid x) = \begin{cases} \frac{c(xy)-D}{c(x)} & \text{if } c(xy) > 0 \\ \alpha(x) P_{abs}(y) & \text{otherwise} \end{cases}$$

- where $\alpha(x)$ is chosen to make sure that $P_{katz}(y \mid x)$ is a proper probability

$$\alpha(x) = 1 - \sum_{y} \frac{c(xy) - D}{c(x)}$$

Backoff Smoothing with Discounting

x	$c(x)$	$c(x) - D$	$\frac{c(x)-D}{c(the)}$
the	48		
the,dog	15	14.5	14.5/48
the,woman	11	10.5	10.4/48
the,man	10	9.5	9.5/48
the,park	5	4.5	4.5/48
the,job	2	1.5	4.5/48
the,telescope	1	0.5	0.5/48
the,manual	1	0.5	0.5/48
the,afternoon	1	0.5	0.5/48
the,country	1	0.5	0.5/48
the,street	1	0.5	0.5/48
TOTAL			0.9479
the,UNK	0		0.052

Evaluating Language Models

- So far we've seen the probability of a sentence: $P(w_0, \ldots, w_n)$

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- What is the probability of a collection of sentences, that is what is the probability of an unseen test corpus T

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- What is the probability of a collection of sentences, that is what is the probability of an unseen test corpus T
- Let $T = s_0, \ldots, s_m$ be a test corpus with sentences s_i

Evaluating Language Models

- So far we've seen the probability of a sentence: $P(w_0, \ldots, w_n)$
- What is the probability of a collection of sentences, that is what is the probability of an unseen test corpus T
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- T is assumed to be separate from the training data used to train our language model $P(s)$

Evaluating Language Models

- So far we've seen the probability of a sentence: $P(w_0, \ldots, w_n)$
- What is the probability of a collection of sentences, that is what is the probability of an unseen test corpus T
- Let $T = s_0, \ldots, s_m$ be a test corpus with sentences s_i
- T is assumed to be separate from the training data used to train our language model $P(s)$
- What is $P(T)$?

Evaluating Language Models: Independence assumption

- $T = s_0, \ldots, s_m$ is the text corpus with sentences s_0 through s_m

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- $P(s_i) = P(w_0^i, \ldots, w_n^i)$ – which can be any n-gram language model
- A language model is better if the value of $P(T)$ is higher for unseen sentences T, we want to maximize:

$$P(T) = \prod_{i=0}^{m} P(s_i)$$

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- The average *log* probability of the test corpus T is:

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- Note that ℓ is a negative number
- We evaluate a language model using *Perplexity* which is $2^{-\ell}$

Evaluating Language Models

Question

Show that:

$$2^{-\frac{1}{M}\log_2 \prod_{i=1}^{m} P(s_i)} = \frac{1}{\sqrt[M]{\prod_{i=1}^{m} P(s_i)}}$$

Evaluating Language Models

Question

What happens to $2^{-\ell}$ if any n-gram probability for computing $P(T)$ is zero?

Evaluating Language Models: Typical Perplexity Values

From 'A Bit of Progress in Language Modeling' by Chen and Goodman

Model	Perplexity
unigram	955
bigram	137
trigram	74

Natural Language Processing

Anoop Sarkar
anoopsarkar.github.io/nlp-class

Simon Fraser University

Part 4: Event space in Language Models

Trigram Models

- The trigram model:
  $P(w_1, w_2, \ldots, w_n) =$
  $P(w_1) \times P(w_2 \mid w_1) \times P(w_3 \mid w_1, w_2) \times P(w_4 \mid w_2, w_3) \times$
  $\ldots P(w_i \mid w_{i-2}, w_{i-1}) \ldots \times P(w_n \mid w_{n-2}, \ldots, w_{n-1})$

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  $\ldots P(w_i \mid w_{i-2}, w_{i-1}) \ldots \times P(w_n \mid w_{n-2}, \ldots, w_{n-1})$
- Notice that the length of the sentence n is variable
- What is the event space?

The stop symbol

- Let $\mathcal{V} = \{a, b\}$ and the language L be $\mathcal{V}^*$

The `stop` symbol

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- But $P(a) + P(b) + P(aa) + P(bb) = 1.5$!!

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- Add an explicit probability for the stopsymbol:

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$$P(\text{stop}) = 0.5$$

- $P(\text{stop}) = 0.5$, $P(a\ \text{stop}) = P(b\ \text{stop}) = 0.25 \times 0.5 = 0.125$, $P(aa\ \text{stop}) = 0.25^2 \times 0.5 = 0.03125$ (now the sum is no longer greater than one)

The `stop` symbol

- With this new `stop` symbol we can show that $\sum_w P(w) = 1$
  Notice that the probability of any sequence of length n is
  $0.25^n \times 0.5$
  Also there are 2^n sequences of length n

$$\sum_w P(w) =$$

$$\sum_{n=0}^{\infty} 2^n \times 0.25^n \times 0.5$$

$$\sum_{n=0}^{\infty} 0.5^n \times 0.5 = \sum_{n=0}^{\infty} 0.5^{n+1}$$

$$\sum_{n=1}^{\infty} 0.5^n = 1$$

The stop symbol

- With this new stop symbol we can show that $\sum_w P(w) = 1$
  Using $p_s = P(\mathtt{stop})$ the probability of any sequence of length n is $p(n) = p(w_1, \ldots, w_{n-1}) \times p_s(w_n)$

$$\begin{aligned}\sum_w P(w) &= \sum_{n=0}^{\infty} p(n) \sum_{w_1,\ldots,w_n} p(w_1, \ldots, w_n) \\ &= \sum_{n=0}^{\infty} p(n) \sum_{w_1,\ldots,w_n} \prod_{i=1}^{n} p(w_i)\end{aligned}$$

$$\sum_{w_1,\ldots,w_n} \prod_i p(w_i) =$$

$$\sum_{w_1} \sum_{w_2} \ldots \sum_{w_n} p(w_1)p(w_2)\ldots p(w_n) = 1$$

The stop symbol

$$\sum_{w_1}\sum_{w_2}\ldots\sum_{w_n} p(w_1)p(w_2)\ldots p(w_n) \quad = \quad 1$$

$$\begin{aligned}
\sum_{n=0}^{\infty} p(n) \quad &= \quad \sum_{n=0}^{\infty} p_s(1-p_s)^n \\
&= \quad p_s\sum_{n=0}^{\infty}(1-p_s)^n \\
&= \quad p_s\frac{1}{1-(1-p_s)} = p_s\frac{1}{p_s} = 1
\end{aligned}$$

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All mistakes are my own.