Q1

By Typed Calculation

Assume the simple linear model objective is Traffic Density (veh/km) and it takes in variable of only Traffic Volume (veh/hr). Let y be the objective and x be the variables.

$$X = [10, 35, 18, 45], Y = [500, 1200, 1000, 1800]$$

from equation 10 and 11 from lecture 2, w_0 and w_1 are estimated from the dataset X and Y:

$$w_1 = \frac{\sum_{n=1}^{N} x_n y_n - N\bar{x}\bar{y}}{\sum_{n=1}^{N} (x_n)^2 - N(\bar{x})^2}$$
$$w_0 = \bar{y} - w_1\bar{x}$$

Substituting the dataset of X and Y

$$w_1 = \frac{(10 \times 500 + 35 \times 1200 + 18 \times 1000 + 45 \times 1800) - 4 \times (\frac{10+35+18+45}{4}) \times (\frac{500+1200+1000+1800}{4})}{(10^2 + 35^2 + 18^2 + 45^2) - 4 \times (\frac{10+35+18+45}{4})^2}$$
$$= 32.322$$
$$w_0 = \frac{500 + 1200 + 1000 + 1800}{4} - 32.322 \times \frac{500 + 1200 + 1000 + 1800}{4}$$
$$= 252.309$$

By Jupyter Notebook

Assume the simple linear model objective is Traffic Density (veh/km) and it takes in variable of only Traffic Volume (veh/hr). Let y be the objective and x be the variables. Declare the x and y dataset with the values taken from Table 1:

In [1]:
```
X = [10,35,18,45]
Y = [500,1200,1000,1800]
```

Conduct some basic statistic analysis

In [2]:
```
x_mean = sum(X)/len(X)
y_mean = sum(Y)/len(Y)
N = len(X)
sum_xy = sum([i*j for i,j in zip(X,Y)])
sum_x_sq = sum([i**2 for i in X])
```

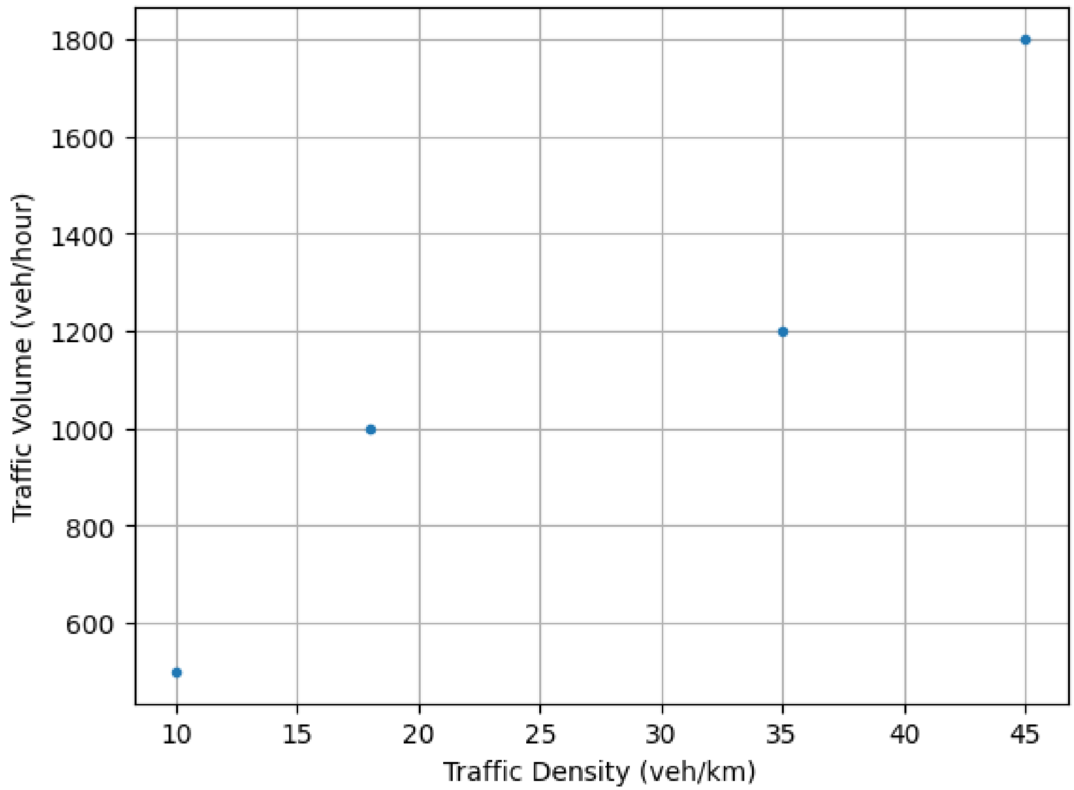
Using the closed form solution formula (Eq. 9 from Lec. 2 slide 9) to estimate the linear model parameter w0 and w1

In [8]:
```
w1 = (sum_xy - N*x_mean*y_mean)/(sum_x_sq-N*x_mean*x_mean)
w0 = y_mean - w1*x_mean
print(f'coefficient w0 and w1 are {w0:.3f} and {w1:.3f} respectively')
```

```
coefficient w0 and w1 are 252.309 and 32.322 respectively
```

Plot out data and the linear fitted model

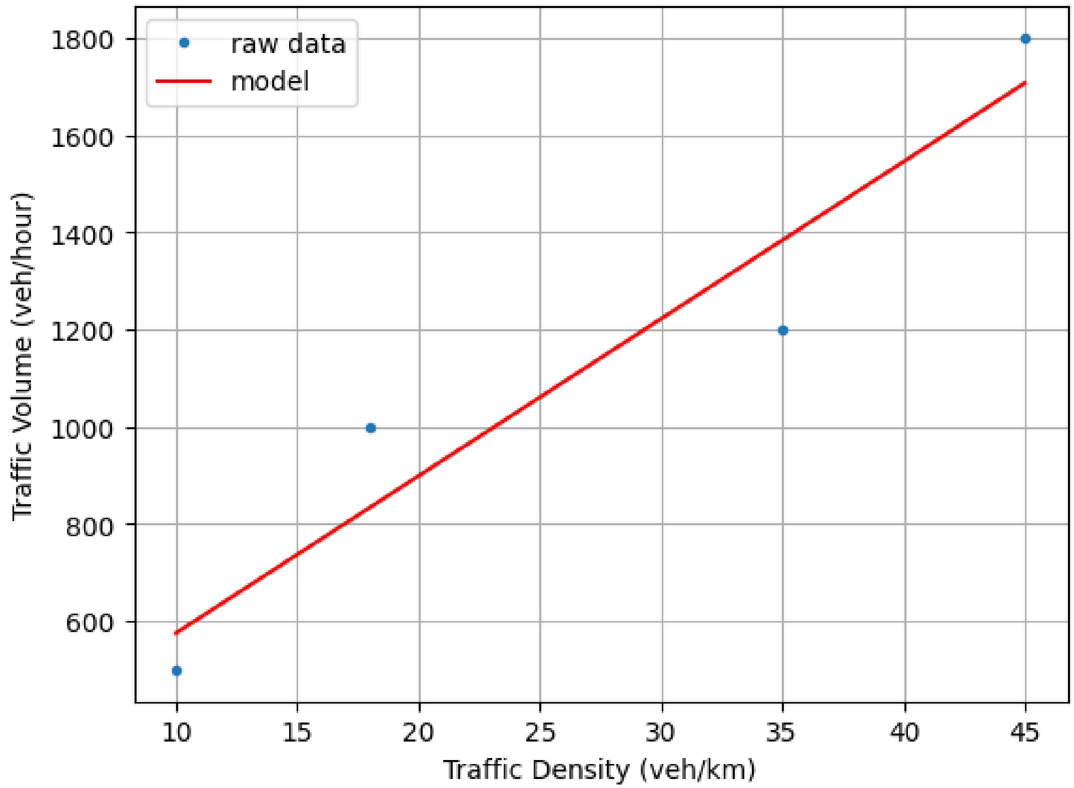
In [4]:
```
import matplotlib.pyplot as plt

plt.plot(X, Y, '.', label='raw data')
plt.xlabel('Traffic Density (veh/km)')
plt.ylabel('Traffic Volume (veh/hour)')
plt.grid()
```



Adding model in red for comparison

In [5]:
```
plt.plot(X, Y, '.', label='raw data')
plt.plot([min(X),max(X)], [min(X)*w1+w0,max(X)*w1+w0], '-r', label='model')
plt.xlabel('Traffic Density (veh/km)')
plt.ylabel('Traffic Volume (veh/hour)')
plt.grid()
plt.legend(loc=0)
```

Out[5]:
```
<matplotlib.legend.Legend at 0x2534a33a560>
```



If the predicted traffic using this arterial is 1000 veh/hour, what is the estimated traffic density?

In [10]:
```
print(f"estimated traffic density for traffic volume of 1000 veh/hour is {1000*w1+w0:.3f} veh/km")
```

```
estimated traffic density for traffic volume of 1000 veh/hour is 32574.208 veh/km
```