6.1

1. the data point lie on the margin in the correct side
2. the data point lie between the hyperpane and the margin but in the correct side
3. the data point lie on the wrong side of the hyperpane
4. left: C=0.1, right: C=10

6.2

1. support vector are the vectors of the data points which has the closest distance to the hyperplane for classification of data.
2. since x_0 and x_1 lies on the hyperpane H, they satisfy the definition of H, i.e.

$$w^T x_0 = b$$
$$w^T x_1 = b$$

consider the dot product of vectors $(x_1 - x_0)$ and w,

$$w \cdot (x_1 - x_0) = w \cdot x_1 - w \cdot x_0 = w^T x_0 - w^T x_1 = b - b = 0$$

which by definition dot product for 2 vectors $A \cdot B = |A|cos\theta|B|$

Assume the Hyperplane H is properly defined, hence $w \neq (0, 0, 0)$,

and the two points x_0 and x_1 are two seperate points,

The two vector in length, $|(x_1 - x_0)|$ and $|w|$ must be greater than 0,

thus $w \cdot (x_1 - x_0) = |w|cos\theta|x_1 - x_0|$ must imply the angle between $(x_1 - x_0)$ and w are perpendicular such that $cos\theta = 0$

3. since distance D between a point $z(x_0, x_1, x_2)$ and a plane H is measure on a line passing through z and perpendicular to plane H,

we first find the vector normal to the plane H,

which from Q2., vector w is normal to any two points given on the plane.

Reduce the length of the noraml vector to 1 such that it forms the unit vector in the direction normal to the polane H

$$n = \frac{w}{|w|}$$

Next consider a vector A starting from any arbitary point $p(x_1, y_1, z_1)$ on the plane H pointing towards z

$$A = p - z = (x_1 - x_0, y_1 - y_0, z_1 - z_0)$$

we project the vector A to the normal vector n by taking dot product of A and the normal vector n, which will give out the distance D between the point z to on a line perpendicular to plane H as we are finding

$$\begin{aligned} D &= |A \cdot n| \\ &= \frac{|(p - z) \cdot w|}{|w|} \\ &= \frac{|w^T p - w^T z|}{|w|} \end{aligned}$$

Since p lies on plane H, by the plane definition $w^T p = b$

$$\begin{aligned} D &= \frac{|-b - w^T z|}{|w|} \\ &= \frac{|b + w^T z|}{|w|} \end{aligned}$$

Assume w is defined as (p, q, r), we can expand the distance formula as follow:

$$\begin{aligned} D &= \frac{|b + [p, q, r]^T [x_0, y_0, z_0]|}{\sqrt{p^2 + q^2 + r^2}} \\ &= \frac{p x_0 + q y_0 + r z_0 + b}{\sqrt{p^2 + q^2 + r^2}} \end{aligned}$$