

DDI Analysis

Equations

$$U_t = U_{xx} + \gamma \left(\frac{U^2}{(1+kU^2)V} - cU \right)$$

$$V_t = V_{xx} + \gamma(U^2 - eV + S)$$

Jacobian

$$J_{(U,V)} = \gamma \cdot \begin{bmatrix} \frac{2U}{V(1+kU^2)^2} - c & -\frac{U^2}{(1+kU^2)V^2} \\ 2U & -e \end{bmatrix}$$

Numerically computed fixed point

$$\bar{U} = 1.11031, \bar{V} = 61.63890 \text{ (for } \gamma = 15,000, k = 0.65, c = 0.01, e = 0.02 \text{ and } S = 0)$$

DDI 1 & 2

$$M = J_{(1.11, 61.64)} = \begin{bmatrix} 16.5454 & -2.70917 \\ 33,309.3 & -3,000 \end{bmatrix}$$

$$\text{Tr}(M) < 0$$

$$\det(M) > 0$$

DDI 3 & 4

$$D = \begin{bmatrix} \frac{1}{200} & 0 \\ 0 & 1 \end{bmatrix}$$

$$M - k^2 D = \begin{bmatrix} 16.5454 - \frac{k^2}{200} & -2.70917 \\ 33,309.3 & -(3,000 + k^2) \end{bmatrix}$$

$$h(k^2) = \det(M - k^2 D) = \frac{(k^2)^2}{200} + (15 - 16.5454)k^2 + 40,604.4$$