Matrices

Definition:

A **matrix** is a rectangular array (or table) of numbers, symbols, or expressions arranged in rows and columns. Think of it as a grid where data is organized in an orderly fashion.

Notation and Dimensions:

A matrix is usually written inside brackets or parentheses. For example, a matrix AAA with 3 rows and 2 columns (written as a 3×2 matrix) looks like:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Matrix Dimensions (Size)

The size (or dimensions) of a matrix is determined by how many rows and columns it has. The dimensions are written as $m \times n$, where:

- m is the number of rows.
- n is the number of columns.

Matrix Elements

Each individual number in the matrix is called an **element**. The element in the i-th row and j-th column is denoted as A_{ij}.

For example, in the matrix A:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

- The element in the first row and first column is $A_{11} = 1$.
- The element in the second row and third column is $A_{23} = 6$.

Identity Matrix/Unit Matrix:

A square matrix where all the **diagonal elements are 1**, and all other elements are 0.

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

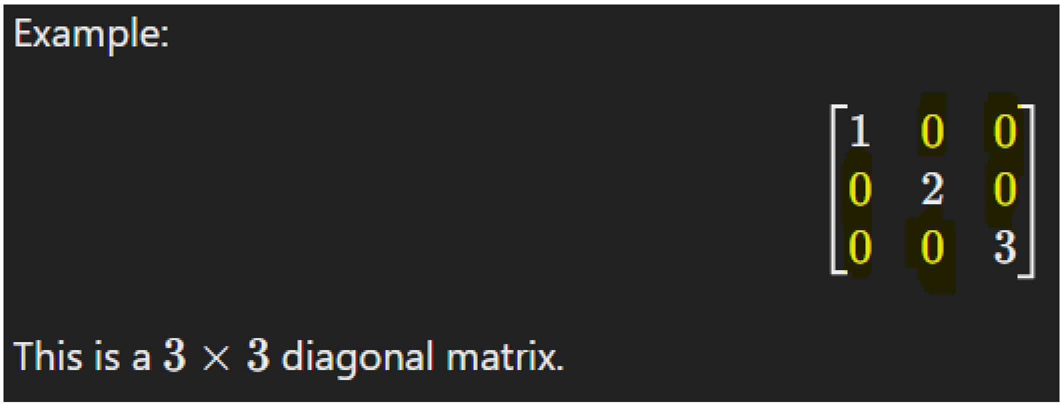
This is a 2×2 identity matrix, often denoted as I.

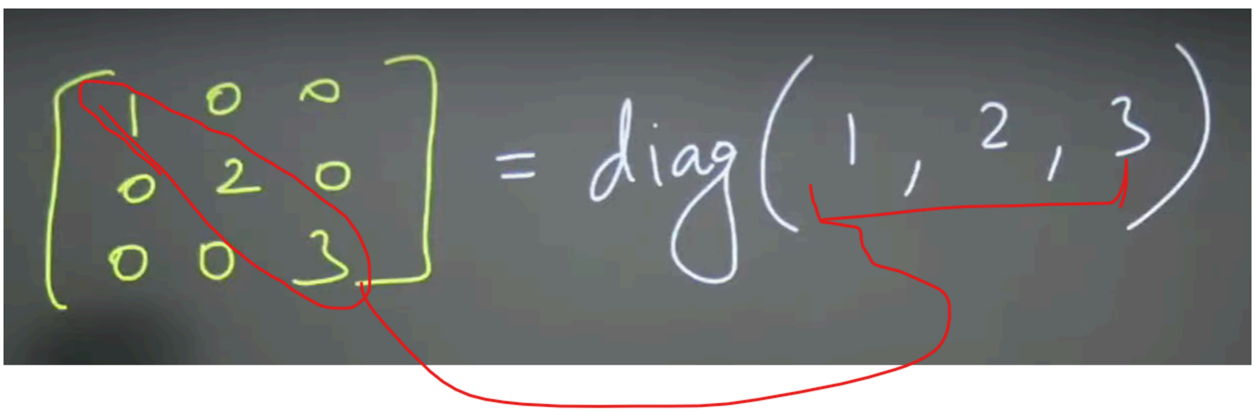
💡 **It is called Identity Matrix because → If you multiply it with any matrix, you will gate the same matrix.**

- It's like Number 1.
    - 1 X Anything = That Number

Diagonal Matrix:

A square matrix where all **non-diagonal elements are zero.**





Scalar Matrix

- All diagonal elements are equal

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Symmetric Matrix

A matrix that is equal to its transpose, i.e., $A = A^T$.

Example:

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$$

Singular Matrix:

A matrix that **does not have an inverse**. This occurs when the **determinant** of the matrix is 0.

Null Matrix

- All elements are zero

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Matrix Operations

Matrix Addition and Subtraction

- You can add or subtract two matrices if they have the **same dimensions/order**. The operation is performed **element by element**.

- **Example:**
  If

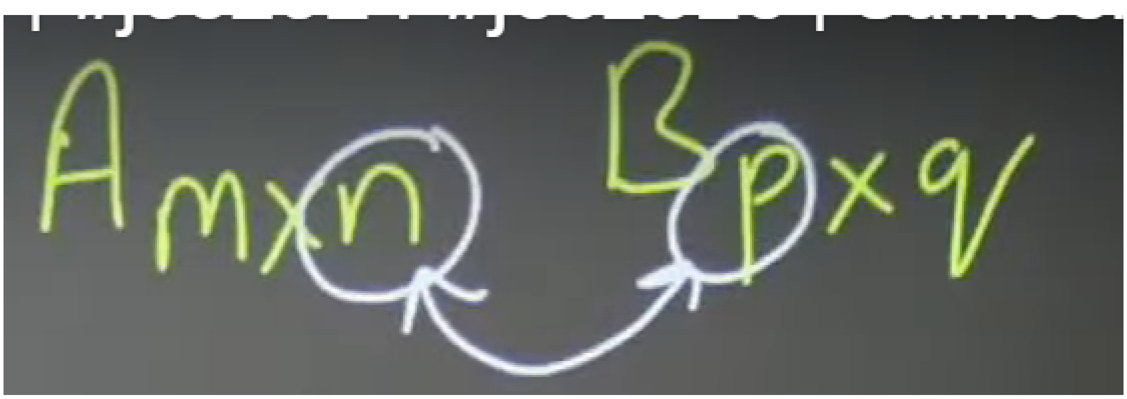
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix},$$

  then:

$$A + B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}.$$

Matrix Multiplication

- To multiply two matrices, **the number of columns in the first matrix must equal the number of rows in the second matrix.**



> 💡 **No. of columns in A should be equal to number of row in B**

- **Row x Column**
- The resultant matrix will have order $m * q$
- You multiply the rows of the first matrix by the columns of the second matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

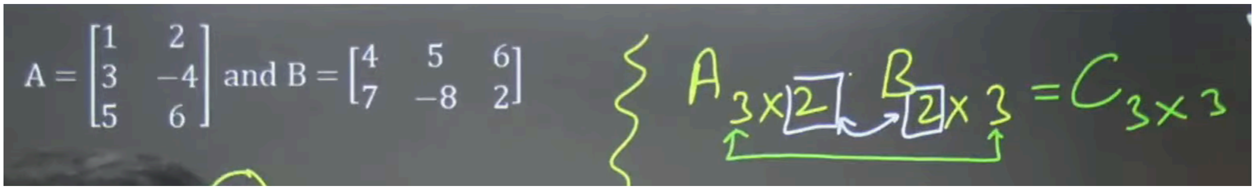
$$C = A \times B = \begin{bmatrix} 1 \times 5 + 2 \times 7 & 1 \times 6 + 2 \times 8 \\ 3 \times 5 + 4 \times 7 & 3 \times 6 + 4 \times 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

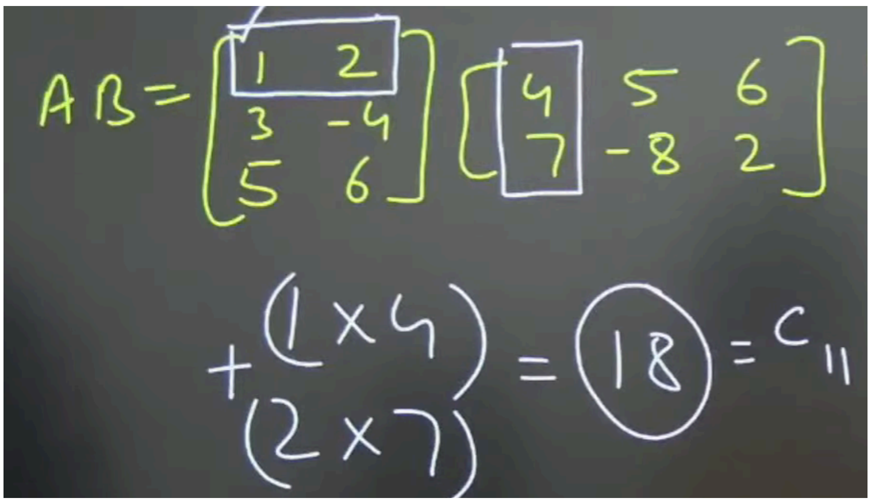
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

DOT Prdct

$$\begin{bmatrix} (1 \times 5) + (7 \times 2) & (6 \times 1) + (8 \times 2) \\ (3 \times 5) + (7 \times 7) & (6 \times 3) + (8 \times 4) \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

Ex.





- **We made the row vertical → Multified it with column and add the two.**
  - this will generate 1 element
- Likewise, pick all the columns with first row and write them in a row
- Then pick the second row and columns-1,2,3 and so on...

Transpose of a Matrix

- The **transpose** of a matrix A, written A^T, is formed by turning rows into columns and vice versa.

If

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad (3 \times 2),$$

then:

$$A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} \quad (2 \times 3).$$

Inverse of a Matrix

The inverse of a matrix A is another matrix A^{-1} such that:

$$A \times A^{-1} = A^{-1} \times A = I$$

Where,

I is the **identity matrix (all diagonal elements are 1, all others are 0)**

Not all matrices have an inverse—only those with a non-zero determinant.

For a 2x2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The inverse is:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

identity matrix.

Determinant

The **determinant** of a square matrix provides important information about the matrix. It's a scalar value that can be calculated for any square matrix. The determinant can tell us:

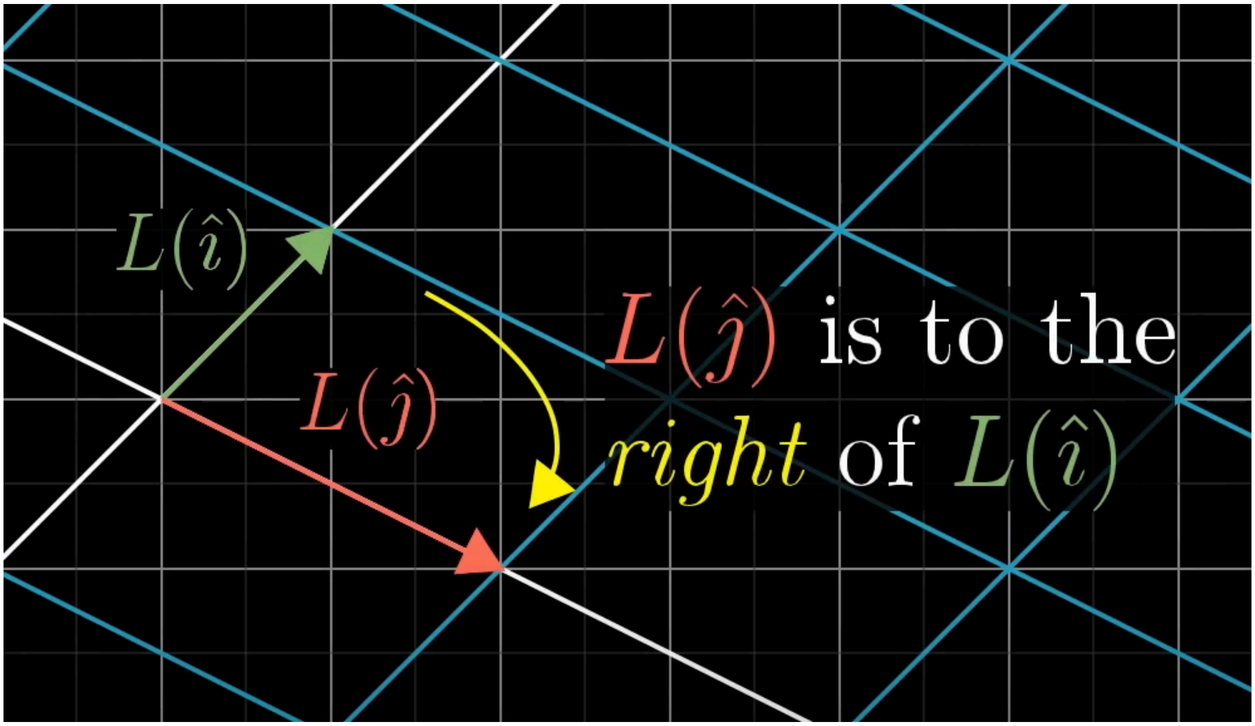
- Whether the matrix is **invertible** (non-zero determinant) or **singular** (zero determinant).
- The **area** or **volume** of the space defined by the matrix.

- **When you apply transformation, the area changes. Determinant tells us by what factor has the area changed.**
  - **The determinant of a transformation will be 3 if it increases the area by the factor of 3.**
  - **The determinant will be 0.5 if it squishes down the area by half.**
  - **The determinant will be 0 if it squishes all of space onto a line or a point.**

Determinants can have negative values

- It's about orientation
- It inverts the orientation of space

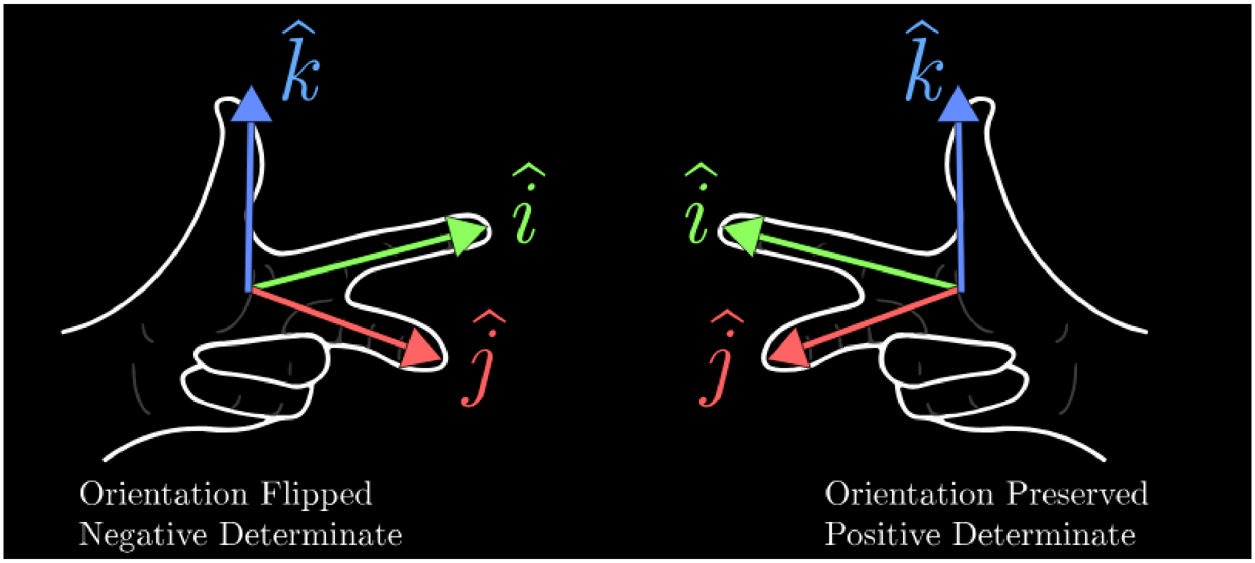
- It flips the plane



For a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the determinant is:

$$\det(A) = ad - bc$$

- In 3D, determinant tells information about the **Volume.**
- Right hand rule



$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$\det(A) = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)$$

$$\det(M_1 M_2) = \det(M_1)\det(M_2)$$

Eigenvalues and Eigenvectors

- An **eigenvector** is a vector that, when multiplied by a matrix, only gets **stretched or shrunk** (no change in direction). The amount of stretching or shrinking is called the **eigenvalue**.

For a matrix A, an eigenvector $\mathbf{v}$ and its corresponding eigenvalue λ satisfy this equation:

$$A\mathbf{v} = \lambda\mathbf{v}$$

- A is a square matrix (it could be, for example, a 2×2 or 3×3 matrix).
- $\mathbf{v}$ is the **eigenvector**, and it's a vector that **does not change direction** after being multiplied by A (it just gets stretched or shrunk).
- λ is the **eigenvalue**, which tells you how much the eigenvector is stretched or shrunk.