

1. (2 pts) **Memory leak:**

- What is memory leak?
- Write a C/C++ function that has memory leak problem.

2. (4 pts) **Shallow copy:** A program is used to record each student's quiz scores.

Answer the following sub-questions:

a. Class definition:

```
class student {  
    public:  
        student(int n=5){count=n; score=new int[count];};  
        ~student(){delete [] score;};  
        int *score, count;  
        string name;  
};
```

b. Main program:

```
int main(){  
    student a(3); a.name="John"; a.score[0]=70; a.score[1]=60; a.score[2]=30;  
    student b=a; b.name="Mary"; b.score[2]=90;  
}
```

- What are the contents of variables a and b?
- What are the two potential problems of this program?

3. (4 pts) **STL vectors**: Give 3 advantages and 1 disadvantage of STL vectors when compared with the standard C/C++ arrays.

4. (3 pts) **STL vectors**: Given an STL vector `x`, answer the following sub-questions.

- a. What does "`x.reserve(25)`" mean?
- b. What is the difference between `x[i]` and `x.at(i)`?
- c. What is the difference between `x.size()` and `x.capacity()`?

1. (2 pts) **Row-based vs. column-based operations:** There are two ways to compute the total of elements in a traditional two-dimensional arrays in C/C++, as shown next.
 - a. Compute the column sum first, then compute the overall total.
 - b. Compute the row sum first, then compute the overall total.

In general, which one is faster? Why?

2. (3 pts) **Insertion sort:** Show each major step of insertion sort on the sequence {7, 2, 8, 5, 6, 9, 4, 1, 3}.

3. (2 pts) **Binary search:** There are two ways to compute the middle position during a binary search, as shown next.
 - a. $\text{mid} = (\text{left} + \text{right}) / 2$
 - b. $\text{mid} = \text{left} + (\text{right} - \text{left}) / 2$

What is the advantage of each method?

4. (3 pts) **Search complexity:** What is the time complexity of linear search, binary search, and hash search, and when are they used?

5. (4 pts) **Multiple-key binary search:** If you want to do binary search over multiple keys, what do you need to do in the following two stages:

- Preprocessing stage
- Search stage

6. What else can we use to compute mid?

1. (3 pts) **Definition of big-Oh:** Given functions $f(n)$ and $g(n)$, under what condition do we say that $f(n)$ is $O(g(n))$ (Note that $g(n)$ is one of the basic functions, such as n^3 .)

2. (3 pts) **Example of big-Oh:** Use the definition of big Oh to explain why $5n^3 + 2(n+10)^2 + (n+50)\log(n+20)$ is $O(n^3)$.

3. (4 pts) **Tries:** Given a key sequence of {stop, steer, skip, step, top, took}, draw the trie data structure. (You need to indicate the end-of-key nodes clearly.)

4. (3 pts) **Binary search vs. tries:** What are the complexities of tries with n keys, each with m characters?
- a. Time complexity for construction
 - b. Time complexity for Search
 - c. Space complexity
5. (4 pts) **Associative arrays in STL:** There are two implementations of associative arrays (dictionaries) in STL, including `<map>` and `<unordered_map>`. Explain at least two differences between these two implementations.
6. Prove that $F(n) < 2^n$ where $F(n)$ is the Fibonacci series

1. (2 pts) **Advantages of postfix notation:** What are the two major advantages of using postfix notation?
2. (4 pts) **Infix to postfix:** Use a tabular form to show how to convert the following infix notation into postfix.

$$f-((a+b)*c+d*e)$$

(Note that you need to show the contents of the stack at each step.)

3. (2 pts) **BFS vs. DFS:** In a constraint satisfaction problem for search in a game (such as 2048 or nonograms), why do we prefer to use DFS (depth-first search) instead of BFS (breadth-first search)?

4. (2 pts) **BFS vs. DFS:** Is DFS more related to stacks or queues? How about BFS?
5. (4 pts) **Evaluation of postfix notation:** Use a tabular form to evaluate the following postfix notation:

$abc-d+/ea-*c^*$

(Note that you need to show the contents of the stack at each step.)

1 (5 pts) **About BT:** Given a binary tree and a proper binary tree with the root located at depth 0, the height of the tree is defined as the maximum depth.

- a What is the maximum number of nodes at depth d of a binary tree?
- b What is the maximum number of nodes in a binary tree of height h ?
- c If a binary tree has 25 nodes, what is its maximum height?
- d If a binary tree has 25 nodes, what is its minimum height?
- e If a proper binary tree has 35 nodes, what are the numbers of external nodes?

2 (2 pts) **Vector representation for BT:** In order to access the tree nodes quickly, the following binary tree is stored level-by-level in a one-dimensional array A .

What are the indices for nodes L and M , respectively?

3 (3 pts) **Traversal of BT:** Give the preorder, inorder, and postorder traversals of the following expression tree.

4 (3 pts) **BT from inorder and preorder sequences:** Draw a binary tree with inorder sequence $[a\ b\ c\ d\ e\ f\ g\ h\ i]$ and preorder sequence $[f\ b\ a\ e\ d\ c\ h\ g\ i]$.

5 (3 pts) **BT from inorder and postorder sequences:**

Draw a binary tree with inorder sequence $[a\ b\ c\ d\ e\ f\ g\ h\ i]$ and postorder sequence $[b\ a\ e\ d\ g\ i\ h\ f\ c]$.

6 (3 pts) **Conversion to BT:** Use the left child-right sibling representation to convert the following tree into a binary tree.

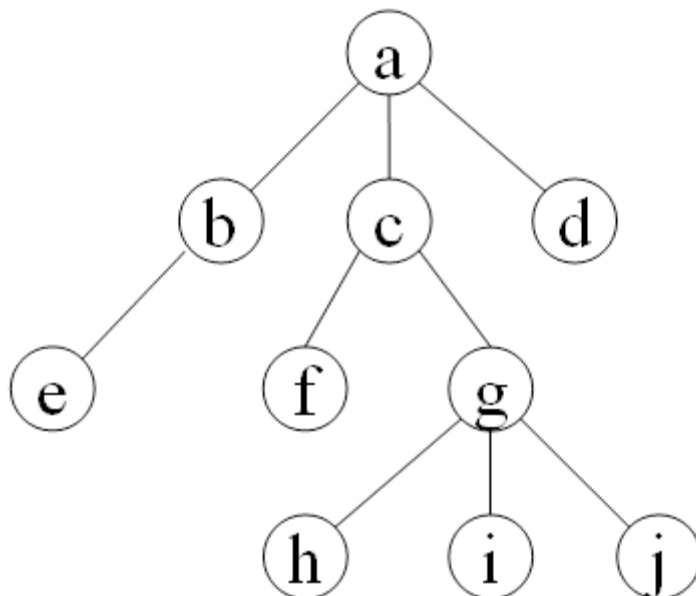
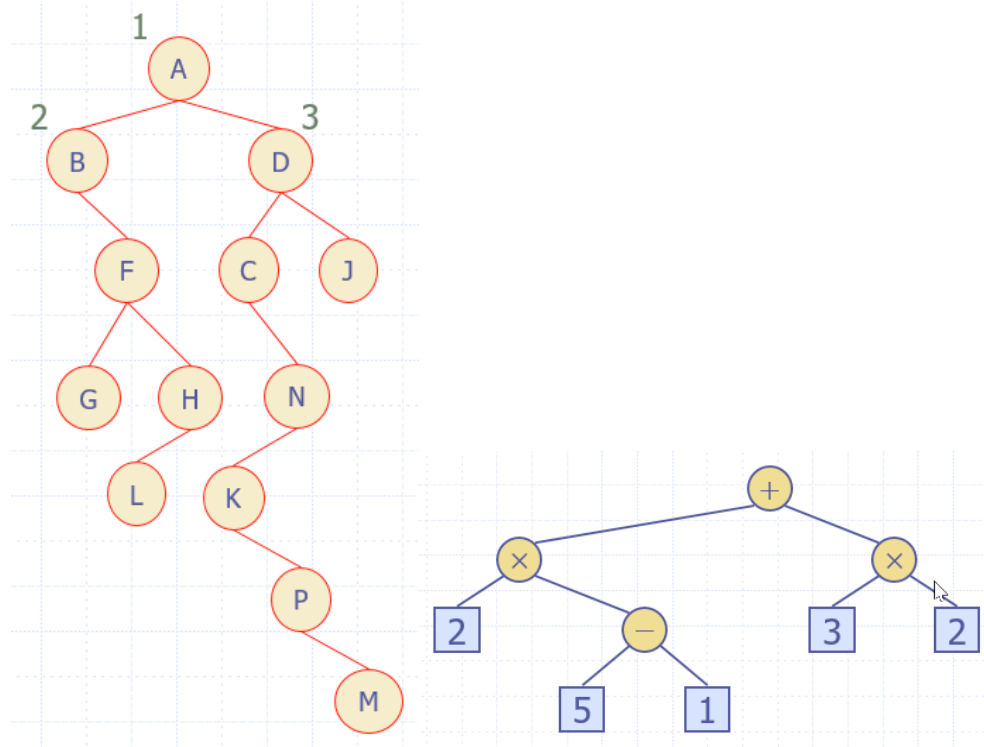
7. prove that $n_e = n_i + 1$

8. Given a proper binary tree explain why is the number of nodes always odd?

If n is 51 what is the number of internal and external nodes?

9. n_k is the number of nodes with k children.

Proof that $n_0 = n_2 + 1$



1. (2 pts) **About heaps:** Given a heap of n nodes, what are its minimum and maximum values of height? Why?
2. (4 pts) **Complexity of operations on heaps:** Give the time complexity of the following operations on a min-heap of n nodes: (a) size (b) min (c) insert (d) removeMin.
3. (4 pts) **Heap construction in $O(n)$:** Given a vector [16 15 4 12 6 9 23 20 25 5 11 27 7 8 10], construct a min-heap from these elements based on a $O(n)$ algorithm. Please plot the result at each step to demonstrate the algorithm clearly.
4. (4 pts) **Insertion and deletion in heaps:** Draw a max-heap tree after each of the following operations is performed, beginning with an empty

heap: insert 43, insert 31, insert 68, insert 24, delete-max, insert 51,
insert 44, insert 53, insert 69, insert 71, delete-max.

5. (6 pts) **In-place heap sort:** Given a vector of [8 5 1 3 7], how do you perform in-place heap sort? Please plot the 5 heaps (both vector and tree representations) during heap construction, and the other 5 during output, as described in class.



1. (2 pts) **Two components of a hash function:** What are the two functions that are generally used to compose a hash function?
2. (2 pts) **Horner's rule:** Explain the Horner's rule for efficient polynomial evaluation of $p(z)=a_0+a_1z+a_2z^2+\dots+a_{n-1}z^{n-1}$.
3. (2 pts) **Basic compression function:** What is the most commonly used compression function for a hash function?
4. (2 pts) **Definition of collision:** What is the definition of collision in terms of the operation of a hash table?
5. (4 pts) **Collision handling:**
 - a. What are the basic two methods for handling collision?
 - b. What is the major difference between the above two methods?

6. (4 pts) **Linear probing:** If we are using linear probing for handling collisions in a hash table, with a hash function $h(x) = x \bmod 13$ Show the hash table contents after inserting keys of 18, 41, 22, 44, 59, 32, 31, 73.

7. (4 pts) **Design criteria for a hash function:** What are the most important two considerations in designing a hash function?