Notes on Dirac gamma matrices

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1st ver : 2023/08/19 − −09/15

2nd ver : 2024/05/08 − −05/12

I have been confused by the representations of Dirac's gamma matrices in textbooks of Quantum field theory since I was a graduate student. I never understand how to work one representation out since those authors take these representations for granted and there is no reference.

A few months ago, I learned one method that Fermi used to work out Pauli matrices in his lecture notes on quantum mechanics, and I realized it could be used to derive the representation of Dirac's gamma matrices as well. This note is an elementary exercise to derive these presentations for myself.

2023 / 08 / 19

Useful formulae

Conventions :

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \mathbb{I}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbb{I}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{1}$$

Pauli matrices :

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2}$$

which satisfies

$$\sigma^i \sigma^i = \mathbb{I}_2 \,, \qquad \sigma^i \sigma^j + \sigma^j \sigma^i = 0 \,, \qquad \sigma^i \sigma^j - \sigma^j \sigma^i = i\, 2\, \epsilon^{ijk} \sigma^k \,. \tag{3}$$

or written in one equation $\quad \sigma^i \sigma^j = \delta^{ij} + i\epsilon^{ijk}\sigma^k\,.$

Gamma matrices in popular textbooks

1. Peskin & Schroeder, *An Introduction to Quantum Field Theory*, Westview.
2. Schwartz, *Quantum Field Theory and the Standard Model*, Cambridge.
3. Srednicki, *Quantum Field Theory*, Cambridge. used convention $g^{\mu\nu} = (-1, 1, 1, 1)$.

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}. \tag{4}$$

4. Itzykson & Zuber, *Quantum Field Theory*, Dover.
5. Zee, *Quantum Field Theory in a Nutshell*, Princeton.
6. Ryder, *Quantum Field Theory*, Cambridge.

$$\gamma^0 = \begin{pmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}. \tag{5}$$

7. Weinberg, *The Quantum Theory of Fields*, Cambridge.

P.216, Eq.(5.4.17)
$$\gamma^0 = -i\begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix}, \qquad \gamma^i = -i\begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}. \tag{6}$$

Convention $g^{\mu\nu} = (-1, 1, 1, 1)$, that is why there are $-i$ in the gamma matrices.

Properties of Gamma matrices

Dirac gamma matrices satisfy Clifford algebra:

$$\{\gamma^\mu\,,\,\gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\mathbb{I}\,,\quad \mu,\nu \in \{0,1,2,3\}\,. \tag{7}$$

$\det(\gamma^\mu\gamma^\nu) = \det(-\gamma^\nu\gamma^\mu) = (-1)^d\det(\gamma^\mu\gamma^\nu)$, so dimensionality of gamma matrices must be even, $d = 2n$, $n \in \mathbb{Z}^*$. But $n = 1$ fails since there are only three independent matrices satisfy anticommutative relation. Thus the minimal dimensionality of gamma matrices is 4 as $n = 2$.

- $\left(\gamma^0\right)^2 = \mathbb{I}\,,\quad \left(\gamma^i\right)^2 = -\mathbb{I}\,.$
- $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = i\epsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\,,\quad \epsilon_{0123} = 1\,.$

$$\Rightarrow\qquad \left(\gamma_5\right)^2 = \mathbb{I}\,,\quad \{\gamma_5\,,\,\gamma^\mu\} = 0\,. \tag{8}$$

- $\mathrm{Tr}\left(\gamma^\mu\right) = 0\,.$

$$\begin{aligned}
\mu \neq \nu \quad &\Rightarrow \quad \gamma^\mu\gamma^\nu = -\gamma^\nu\gamma^\mu \\
&\Rightarrow \quad \gamma^\mu\gamma^\nu\gamma^\nu = -\gamma^\nu\gamma^\mu\gamma^\nu \\
&\Rightarrow \quad \gamma^\mu g^{\nu\nu} = -\gamma^\nu\gamma^\mu\gamma^\nu \\
&\Rightarrow \quad \mathrm{Tr}\left(\gamma^\mu\right)g^{\nu\nu} = -\mathrm{Tr}\left(\gamma^\nu\gamma^\mu\gamma^\nu\right) = -\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\gamma^\nu\right) = -\mathrm{Tr}\left(\gamma^\mu\right)g^{\nu\nu} \\
g^{\nu\nu} \neq 0 \quad &\Rightarrow \quad \mathrm{Tr}\left(\gamma^\mu\right) = 0\,.
\end{aligned}$$

- $\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) = 4g^{\mu\nu}$ in 4-dimensional space.

$$\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) = \mathrm{Tr}\left(2g^{\mu\nu}\mathbb{I}\right) - \mathrm{Tr}\left(\gamma^\nu\gamma^\mu\right) = \mathrm{Tr}\left(2g^{\mu\nu}\mathbb{I}\right) - \mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right)$$

$$\Rightarrow\qquad 2\mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) = \mathrm{Tr}\left(2g^{\mu\nu}\mathbb{I}\right)\qquad\Rightarrow\qquad \mathrm{Tr}\left(\gamma^\mu\gamma^\nu\right) = g^{\mu\nu}\mathrm{Tr}\left(\mathbb{I}\right) = 4g^{\mu\nu}\,.$$

- $\mathrm{Tr}\Big(\underbrace{\gamma^\alpha\,\cdots\,\gamma^\omega}_{2n+1}\Big) = 0\,.$

$$\mathrm{Tr}\Big(\underbrace{\gamma^\alpha\,\cdots\,\gamma^\omega}_{2n+1}\Big) = \mathrm{Tr}\Big(\left(\gamma_5\right)^2\gamma^\alpha\,\cdots\,\gamma^\omega\Big) = (-1)^{2n+1}\mathrm{Tr}\left(\gamma_5\gamma^\alpha\,\cdots\,\gamma^\omega\gamma_5\right) = -\mathrm{Tr}\Big(\gamma^\alpha\,\cdots\,\gamma^\omega\Big)$$

$$\Rightarrow\qquad \mathrm{Tr}\Big(\underbrace{\gamma^\alpha\,\cdots\,\gamma^\omega}_{2n+1}\Big) = 0\,.$$

- $\mathrm{Tr}\left(\gamma_5\right) = 0\,.$
- $\mathrm{Tr}\left(\gamma_5\gamma^\mu\right) = 0\,.$

- $\mathrm{Tr}\big(\gamma_5\gamma^\mu\gamma^\nu\big) = 0\,.$

$$\begin{cases} \mu=\nu & \mathrm{Tr}\big(\gamma_5\gamma^\mu\gamma^\mu\big) = \mathrm{Tr}\big(\gamma_5\big)g^{\mu\mu} = 0 \\ \mu\neq\nu & \mathrm{Tr}\big(\gamma_5\gamma^\mu\gamma^\nu\big) = i\epsilon_{\alpha\beta\nu\mu}\mathrm{Tr}\big(\gamma^\alpha\gamma^\beta\gamma^\nu\gamma^\mu\gamma^\mu\gamma^\nu\big) = i4\epsilon_{\alpha\beta\nu\mu}g^{\alpha\beta}g^{\mu\mu}g^{\nu\nu} = 0\,. \end{cases}$$

- $\mathrm{Tr}\big(\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\big) = 0\,.$

- $\mathrm{Tr}\big(\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\big) = -i\,4\,\epsilon_{\mu\nu\alpha\beta}\,.$

$$\begin{aligned} \mathrm{Tr}\big(\gamma_5\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\big) &= i\epsilon_{\beta\alpha\nu\mu}\mathrm{Tr}\big(\gamma^\beta\gamma^\alpha\gamma^\nu\gamma^\mu\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta\big) \\ &= i(-1)^3\,\epsilon_{\beta\alpha\nu\mu}\mathrm{Tr}\big(\mathbb{I}_4\big) = -i\,4\,\epsilon_{\mu\nu\alpha\beta}\,. \end{aligned}$$

Representations of Gamma matrices

$$\{\gamma^\mu\,,\,\gamma^\nu\} = 2g^{\mu\nu}\mathbb{I} \quad \Rightarrow \quad \det\left(\gamma^0\gamma^0\right) = \left(\det\gamma^0\right)^2 = 1 \quad \Rightarrow \quad \det\gamma^0 = \pm 1\,. \tag{9}$$

Gamma matrix γ^0 is nonsingular, so it has eigenvalues and eigenvectors $\gamma^0\mathbf{v} = \lambda\mathbf{v}$. Then using $\gamma^0\gamma^0 = \mathbb{I}$, we get $\mathbf{v} = \gamma^0\gamma^0\mathbf{v} = \lambda^2\mathbf{v}$, thus the eigenvalues can only be $\lambda = \pm 1$.

For simplicity, at first we write $\gamma^0 = diag(a_{11}, a_{22}, a_{33}, a_{44})$, since we can always transform a non-singular matrix into a diagonal matrix, and its characteristic equation is

$$\det(\gamma^0 - \lambda\,\mathbb{I}_4) = (\lambda - a_{11})(\lambda - a_{22})(\lambda - a_{33})(\lambda - a_{44}) = (\lambda - 1)^k(\lambda + 1)^{4-k} = 0 \tag{10}$$

Only $k = 2$ satisfies the requirement $\mathrm{Tr}\left(\gamma^0\right) = 0$, so there are two $+1$ and two -1 in the diagonal of γ^0.

Same arguments on γ^i show their eigenvalues are $\pm\, i$.

There are different arrangement of numbers in the diagonal matrix γ^0

(I)

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} = \begin{pmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix}\,, \quad \gamma^i = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}\,. \tag{11}$$

$$\gamma^0\gamma^i + \gamma^i\gamma^0 = \begin{pmatrix} 2a_{11} & 2a_{12} & 0 & 0 \\ 2a_{21} & 2a_{22} & 0 & 0 \\ 0 & 0 & -2a_{33} & -2a_{34} \\ 0 & 0 & -2a_{43} & -2a_{44} \end{pmatrix} = 0 \quad \Rightarrow \quad \gamma^i = \begin{pmatrix} 0 & A^i \\ B^i & 0 \end{pmatrix}\,. \tag{12}$$

$$\gamma^i\gamma^i = \begin{pmatrix} A^iB^i & 0 \\ 0 & B^iA^i \end{pmatrix} = \begin{pmatrix} -\mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix} \quad \Rightarrow \quad A^iB^i = B^iA^i = -\mathbb{I}_2\,. \tag{13}$$

$$\gamma^i\gamma^j + \gamma^j\gamma^i = \begin{pmatrix} A^iB^j + A^jB^i & 0 \\ 0 & A^iB^j + A^jB^i \end{pmatrix} = 0 \quad \Rightarrow \quad A^iB^j + A^jB^i = 0\,. \tag{14}$$

A simple choice conforming last two requirements is $A^i = \sigma^i, B^i = -\sigma^i$, thus we have

$$\gamma^0 = \begin{pmatrix} \mathbb{I}_2 & 0 \\ 0 & -\mathbb{I}_2 \end{pmatrix}\,, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}\,. \tag{15}$$

This is the Dirac representation in Itzykson & Zuber

(II) We can arrange ± 1 in a different way, then we have

$$\gamma^0 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}. \tag{16}$$

$$\gamma^0\gamma^i + \gamma^i\gamma^0 = \begin{pmatrix} 2a_{11} & 0 & 2a_{13} & 0 \\ 0 & -2a_{22} & 0 & -2a_{24} \\ 2a_{31} & 0 & 2a_{33} & 0 \\ 0 & -2a_{42} & 0 & -2a_{44} \end{pmatrix} = 0$$

$$\Rightarrow \quad \gamma^i = \begin{pmatrix} A^i & B^i \\ C^i & D^i \end{pmatrix}, \quad \text{where} \;\; X^i = \begin{pmatrix} 0 & X^i_{12} \\ X^i_{21} & 0 \end{pmatrix}, \quad X \in \{A, B, C, D\} \tag{17}$$

$$\gamma^i\gamma^i = \begin{pmatrix} A^i_{12}A^i_{21} + B^i_{12}C^i_{21} & 0 & A^i_{12}B^i_{21} + B^i_{12}D^i_{21} & 0 \\ 0 & A^i_{12}A^i_{21} + B^i_{21}C^i_{12} & 0 & A^i_{21}B^i_{12} + B^i_{21}D^i_{12} \\ A^i_{21}C^i_{12} + C^i_{21}D^i_{12} & 0 & B^i_{21}C^i_{12} + D^i_{12}D^i_{21} & 0 \\ 0 & A^i_{12}C^i_{21} + C^i_{12}D^i_{21} & 0 & B^i_{12}C^i_{21} + D^i_{12}D^i_{21} \end{pmatrix} = -\mathbb{I}_4$$

$$\Rightarrow \quad \begin{cases} A^i_{12}A^i_{21} + B^i_{12}C^i_{21} = -1, \\ A^i_{12}A^i_{21} + B^i_{21}C^i_{12} = -1, \\ D^i_{12}D^i_{21} + B^i_{12}C^i_{21} = -1, \\ D^i_{12}D^i_{21} + B^i_{21}C^i_{12} = -1, \\ C^i_{12}A^i_{21} + D^i_{12}C^i_{21} = 0, \\ A^i_{12}C^i_{21} + D^i_{21}C^i_{12} = 0, \\ A^i_{12}B^i_{21} + B^i_{12}D^i_{21} = 0, \\ B^i_{12}A^i_{21} + B^i_{21}D^i_{12} = 0. \end{cases} \tag{18}$$

For γ^1, we can set $A^1_{12} = A^1_{21} = 0$, which leads to $D^1_{21} = D^1_{12} = 0$ and $B^1_{12}C^1_{21} = B^1_{21}C^1_{12} = -1$, next choosing $B^1_{12} = B^1_{21} = C^1_{12} = C^1_{21} = i$, we have

$$\gamma^1 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\sigma^1 \\ i\sigma^1 & 0 \end{pmatrix}. \tag{19}$$

$$\gamma^1\gamma^2+\gamma^2\gamma^1=\begin{pmatrix} i(B_{12}^2+C_{21}^2) & 0 & i(A_{12}^2+D_{21}^2) & 0 \\ 0 & i(B_{12}^2+C_{21}^2) & 0 & i(A_{21}^2+D_{12}^2) \\ i(A_{21}^2+D_{12}^2) & 0 & i(B_{21}^2+C_{12}^2) & 0 \\ 0 & i(A_{12}^2+D_{21}^2) & 0 & i(B_{12}^2+C_{21}^2) \end{pmatrix}=0$$

$$\Rightarrow \quad B_{12}^2=-C_{21}^2\,,\quad B_{21}^2=-C_{12}^2\,,\quad A_{12}^2=-D_{21}^2\,,\quad A_{21}^2=-D_{12}^2\,. \tag{20}$$

Let $B_{12}^2=B_{21}^2=0$, then we have only $A_{12}^2A_{21}^2=-1$, choosing $A_{12}^2=1$ gives

$$\gamma^2=\begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}=\begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}\,. \tag{21}$$

$$\gamma^2\gamma^3+\gamma^3\gamma^2=\begin{pmatrix} -A_{12}^3+A_{21}^3 & 0 & -B_{12}^3+B_{21}^3 & 0 \\ 0 & -A_{12}^3+A_{21}^3 & 0 & -B_{21}^3+B_{12}^3) \\ -C_{12}^3+C_{21}^3 & 0 & -D_{12}^3+D_{21}^3 & 0 \\ 0 & -C_{12}^3+C_{21}^3 & 0 & -D_{12}^2+D_{21}^3 \end{pmatrix}=0$$

$$\Rightarrow \quad A_{12}^3=A_{21}^3=-D_{12}^3=-D_{21}^3\,,\quad B_{12}^3=B_{21}^3=-C_{12}^3=-C_{21}^3\,. \tag{22}$$

Let $B_{12}^3=0$, which leads to $A_{12}^3=i$ then we can find γ^3

$$\gamma^3=\begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix} \tag{23}$$

$$\gamma^0=\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix}, \gamma^1=\begin{pmatrix} 0 & i\sigma^1 \\ i\sigma^1 & 0 \end{pmatrix}, \gamma^2=\begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix}, \gamma^3=\begin{pmatrix} i\sigma^1 & 0 \\ 0 & -i\sigma^1 \end{pmatrix} \tag{24}$$

$$\gamma^5\equiv i\gamma^0\gamma^1\gamma^2\gamma^3=\begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}\,. \tag{25}$$

I did not find this form of gamma matrices in any references, and have no idea if it is convenient for some special cases.

(III) Another form of γ^0 is

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} , \qquad \gamma^i = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} . \tag{26}$$

$$\gamma^0\gamma^i + \gamma^i\gamma^0 = \begin{pmatrix} a_{13}+a_{31} & a_{14}+a_{32} & a_{11}+a_{33} & a_{12}+a_{34} \\ a_{23}+a_{41} & a_{24}+a_{42} & a_{21}+a_{43} & a_{22}+a_{44} \\ a_{11}+a_{33} & a_{12}+a_{34} & a_{13}+a_{31} & a_{14}+a_{32} \\ a_{21}+a_{43} & a_{22}+a_{44} & a_{23}+a_{41} & a_{24}+a_{42} \end{pmatrix} = 0 .$$

$$\Rightarrow \qquad \gamma^i = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ -a_{13} & -a_{14} & -a_{11} & -a_{12} \\ -a_{23} & -a_{24} & -a_{21} & -a_{22} \end{pmatrix} = \begin{pmatrix} A^i & B^i \\ -B^i & -A^i \end{pmatrix} . \tag{27}$$

$$\gamma^i\gamma^i = \begin{pmatrix} A^iA^i - B^iB^i & A^iB^i - B^iA^i0 \\ A^iB^i - B^iA^i & A^iA^i - B^iB^i \end{pmatrix} = -\mathbb{I}_4 \quad \Rightarrow \quad \begin{cases} A^iA^i - B^iB^i = -\mathbb{I}_2 , \\ A^iB^i - B^iA^i = 0 . \end{cases} \tag{28}$$

$$\gamma^i\gamma^j + \gamma^j\gamma^i = \begin{pmatrix} A^iA^j + A^jA^i - B^iB^j - B^jB^i & A^iB^j + A^jB^i - B^iA^j - B^jA^i \\ A^iB^j + A^jB^i - B^iA^j - B^jA^i & A^iA^j + A^jA^i - B^iB^j - B^jB^i \end{pmatrix} = 0$$

$$\Rightarrow \quad \begin{cases} A^iA^j + A^jA^i - B^iB^j - B^jB^i = 0 , \\ A^iB^j + A^jB^i - B^iA^j - B^jA^i = 0 . \end{cases} \tag{29}$$

(III.a) Assuming $A^i = 0$, then there are conditions for B^i

$$B^iB^i = \mathbb{I}_2 , \qquad B^iB^j + B^jB^i = 0 . \qquad i,j \in \{1,2,3\} . \tag{30}$$

we see this familiar algebra again, so we can choose $B^i = \sigma^i$.

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix} , \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} . \tag{31}$$

This is the Weyl or chiral representation in Peskin & Schroeder . While the chiral representation in Itzykson & Zuber is

$$\gamma^0 = \begin{pmatrix} 0 & -\mathbb{I}_2 \\ -\mathbb{I}_2 & 0 \end{pmatrix} , \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} . \tag{32}$$

(III.b) Assuming $B^i = 0$, then there are conditions for A^i

$$A^i A^i = -\mathbb{I}_2 \,, \qquad A^i A^j + A^j A^i = 0 \,. \qquad i, j \in \{1, 2, 3\} \,. \tag{33}$$

we see this familiar algebra again, and we can choose $A^i = i\sigma^i$.

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{I}_2 \\ \mathbb{I}_2 & 0 \end{pmatrix} , \qquad \gamma^i = \begin{pmatrix} i\sigma^i & 0 \\ 0 & -i\sigma^i \end{pmatrix} . \tag{34}$$

Other representations could be worked out using the same procedure, but these representations might not be as convenient as Chiral representation and Dirac representation.