Discrete Mathematics 1, Test 1 Group 1

> Each task must be solved on paper, then you need to take a photo of their solution or scan them. When you are ready upload the solution to Canvas. Please write your name and Neptun code on each paper. Please use **PDF file format** for uploading, only this will be accepted. Many free applications allow you to scan and take photos in pdf format (e.g. Cam Scanner, Office Lens, MS Word...). You have 90+15 minutes for writing the test and uploading solutions. **Late submissions will not be accepted** so be careful. Grade boundaries: 42, 34, 25 and 17 points for grades 5, 4, 3 and 2, respectively.

1. (a) (4 marks) Give three example sets A, B and C, for which $(B\backslash(A\cup C))\cup((A\cap C)\backslash B) = (A\cap C)\cup(B\backslash C)$ is **NOT true**. Then give three, for which it is **true**.

   (b) (4 marks) Prove **by definition**, that for arbitrary sets A, B and C the following statement holds: $(B\cap C)\backslash A = (B\backslash A)\cap(C\backslash A)$.

2. (a) (4 marks) Let R be a homogeneous binary relation on set X. Decide, whether R is reflexive, symmetric, transitive and anti-symmetric, if the relation is the following:

$$R = \{(1,2),(1,3),(2,1),(2,4),(3,1),(4,2)\} \quad X = \{1,2,3,4\}.$$

   (b) (4 marks) A homogeneous binary relation $R \subseteq \mathbb{R}\times\mathbb{R}$, $R = \{(x,y)\in\mathbb{R}\times\mathbb{R} \mid \quad y^2 = 2x^2 - 4\}$ is given. Find $dmn(R)$, $rng(R)$ and $R(\{2,3\})$.

3. (a) (4 marks) Let $R = \{(1,c),(2,a),(2,c),(4,b),(4,d)\}$ and $S = \{(a,2),(a,3),(b,2),(c,4),(d,3),(d,1)\}$ be binary relations.

   i. Find the inverse image $R^{-1}(\{b,d,e\})$

   ii. Find the relation $S\circ R$ and write it down as a set of ordered pairs.

   (b) (4 marks) For each of the following examples, decide if the relation is an equivalence relation, justifying your answer.

   i. $R_1 \subseteq \mathbb{R}\times\mathbb{R}$, $R_1 = \{(x,y)\in\mathbb{R}\times\mathbb{R} \mid \quad |x| = y\}$

   ii. $R_2 \subseteq \mathbb{Z}\times\mathbb{Z}$, $R_2 = \{(x,y)\in\mathbb{Z}\times\mathbb{Z} \mid \quad x^2 - y^2 \text{ is divisible by } 3\}$

4. (8 marks) Decide about each of the relations below if it is a function, justifying your answer. For each function, decide if it is injective, surjective and/or bijective.

   (a) $f_1 \subset X\times X$, $f_1 = \{(a,c),(b,d),(c,a),(d,b),(e,e)\}$, where $X = \{a,b,c,d,e\}$.

   (b) $f_2 \subset \mathbb{R}\times\mathbb{R}$, $f_2 = \{(x,y)\in\mathbb{R}\times\mathbb{R} \mid \quad x^3 - 3 = \sqrt{y+2}\}$.

5. (10 marks) Give the polar form of the following complex number:

$$z = \frac{(\sqrt{2}+\sqrt{2}i)^7}{(3-3\sqrt{3}i)^9}.$$

   Find all complex numbers w such that $w^4 = z$.

6. (8 marks) Represent the following sets in the Gaussian plane:

   (a) $\{z\in\mathbb{C} \mid \quad Re z > Im z \wedge |z-1-i| \le 1\}$.

   (b) $\{z\in\mathbb{C} \mid \quad |z+3-2i| \le 4 \wedge |z+3-i| \ge 2\}$.