Exam – 2021.06.02.

Due Jun 2 at 9:55am **Points** 15 **Questions** 15
Available Jun 2 at 9am - Jun 2 at 9:55am about 1 hour **Time Limit** 45 Minutes

Instructions

Only one question is visible at once, and after you have submitted your answer to a question, you cannot go back to change your choice. There is exactly 1 correct answer for each question. Every correct answer is worth 1 point.

The maximum is 15 points. Under 8 points, the exam is finished with a fail (1) grade. If one achives at least 8 points, then the grade satisfactory (2), from 12 points, the grade average (3) is offered and one may attend the second, oral part of the exam for better grades.

This quiz was locked Jun 2 at 9:55am.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	45 minutes	10 out of 15

Correct answers are hidden.

Score for this quiz: **10** out of 15
Submitted Jun 2 at 9:54am
This attempt took 45 minutes.

Question 1

1 / 1 pts

Which one of the below numbers is not in the set of binary machine numbers $M(4, -3, 3)$?

(A) -0.75
(B) 0
(C) 1
(D) 8

- A
- B
- C
- D

Question 2

1 / 1 pts

According to the theorems about the error bounds of the basic arithmetic operations, which below statement is true?

(A) Forming the sum of two nearby numbers will significantly increase the relative error bound of the result.
(B) Forming the sum of two nearby numbers will significantly increase the absolute error bound of the result.
(C) Forming the difference of two nearby numbers will significantly increase the relative error bound of the result.
(D) Forming the difference of two nearby numbers will significantly increase the absolute error bound of the result.

A

B

C

D

Incorrect

Question 3

0 / 1 pts

We are to determine the solution of the below system of linear equations using Gaussian elimination with full pivoting. What does the algorithm do before the first step of the elimination?

$$\begin{pmatrix} 1 & -2 & 5 \\ 3 & 6 & -9 \\ 8 & 2 & 7 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(A) Nothing, just carries on with the elimination.
(B) Switches columns 1 and 3.
(C) Switches columns 1 and 3, and rows 1 and 2.
(D) Switches rows 1 and 3.

A

B

C

D

Question 4

1 / 1 pts

We encounter $\omega_3(x) = (x+1)(x-1)(x-2)(x-3)$ in a polynomial interpolation problem. Which one is the Lagrange base polynomial $\ell_2(x)$?

(A) $\dfrac{(x+1)(x-1)(x-3)}{3}$

(B) $\dfrac{(x+1)(x-1)(x-3)}{-3}$

(C) $\dfrac{(x+1)(x-1)(x-3)}{2}$

(D) $\dfrac{(x+1)(x-1)(x-3)}{-2}$

- A
- B
- C
- D

Question 5

1 / 1 pts

Upon considering an overdetermined system with matrix A, right-hand-side b and unknowns x, the Gaussian normal equations have the form...

(A) $AA^\top x = Ab$

(B) $AA^\top x = A^\top b$

(C) $A^\top Ax = Ab$

(D) $A^\top Ax = A^\top b$

A

B

C

D

Question 6

1 / 1 pts

Fix the support points $x_0 = 0$, $x_1 = 1$, $x_2 = 2$ and denote the resulting interpolating polynomial $L_2(x)$. Now add a new support point $x_3 = 3$. What is the third degree interpolating polynomial based on support points x_0, x_1, x_2, x_3, provided we know the value $f[0,1,2,3] = \frac{1}{2}$?

(A) $L_2(x) + \frac{1}{2} \cdot x(x-1)(x-2)(x-3)$
(B) $\frac{1}{2} \cdot L_2(x) + x(x-1)(x-2)$
(C) $L_2(x) + \frac{1}{2} \cdot x(x-1)(x-2)$
(D) $\frac{1}{2} \cdot L_2(x) + x(x-1)(x-2)(x-3)$

A

B

C

D

Incorrect

Question 7

0 / 1 pts

Which formula is incorrect to calculate the Frobenius norm of the matrix $A \in \mathbb{R}^{n\times n}$?

(A) $\|A\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} |a_{ij}|}$ ✓

(B) $\|A\|_F = \sqrt{\mathrm{tr}(A^T A)}$

(C) $\|A\|_F = \sqrt{\sum_{i=1}^{n} \lambda_i(A^T A)}$

(D) all above three are correct

A

B

C

D

Question 8

1 / 1 pts

Which condition is not necessary for the local convergence theorem of Newton's method to be applicable?

(A) $f \in C^2[a,b]$

(B) $\exists m > 0 : \forall x \in (a,b) : |f'(x)| < m$

(C) $\exists M > 0 : \forall x \in (a,b) : |f''(x)| < M$

(D) all above three are necessary

- A
- B
- C
- D

Question 9

1 / 1 pts

Which order of magnitude is appropriate about the quadrature rules $T_m(f)$ and $S_m(f)$ under the conditions of their error formulae?

(A) $\int_a^b f(x)dx - T_m(f) = \mathcal{O}(1/m^3)$

(B) $\int_a^b f(x)dx - T_m(f) = \mathcal{O}(1/m^4)$

(C) $\int_a^b f(x)dx - S_m(f) = \mathcal{O}(1/m^5)$

(D) $\int_a^b f(x)dx - S_m(f) = \mathcal{O}(1/m^4)$

- A
- B
- C
- D

Question 10

1 / 1 pts

Let us consider the interpolatory quadrature formula $\sum_{k=0}^{n} A_k f(x_k)$ to estimate the integral $\int_a^b f(x)dx$? Which below statement holds?

(A) $\sum_{k=0}^{n} A_k = 1$
(B) $\sum_{k=0}^{n} A_k = b - a$
(C) $\sum_{k=0}^{n} A_k x_k = b - a$
(D) $\sum_{k=0}^{n} A_k x_k = b^2 - a^2$

- A
- B
- C
- D

Question 11

1 / 1 pts

Assume that in case of the set of machine numbers $M(t, k^-, 1)$ we have $\varepsilon_1 < \varepsilon_0$. Which below statement is true about the precision of the floating point conversion for all convertible $x \in \mathbb{R}_M$ numbers?

(A) $|x - fl(x)| \leq \frac{1}{2} \cdot \varepsilon_0 \cdot |x|$
(B) $|x - fl(x)| \leq \varepsilon_0$
(C) $|x - fl(x)| \leq \frac{1}{2} \cdot \varepsilon_1 \cdot |x|$
(D) $|x - fl(x)| \leq \frac{1}{2} \cdot \varepsilon_1$

A

B

C

D

Incorrect

Question 12

0 / 1 pts

What estimate can we state about the location of the roots of the polynomial $P(x) = -4x^4 + 3x^3 - 7x^2 + x$ based on the studied methods?

(A) $\frac{1}{8} < |x_k| < \frac{11}{4}$
(B) $\frac{4}{11} < |x_k| < 8$
(C) $|x_k| < \frac{11}{4}$
(D) none

A

B

C

D

Incorrect

Question 13

0 / 1 pts

Does such a vector x always (in case of every square matrix A) exist that maximizes the fraction $\frac{\|Ax\|_\infty}{\|x\|_\infty}$?

(A) Yes, an eigenvector of the eigenvalue $\lambda_{max}(A)$ always does.
(B) Yes, a canonical unit vector always does.
(C) Yes, there is always one among the vectors with $x_i = \pm 1$ $(i = 1, \ldots, n)$.
(D) No, the maximum does not always exist, only the existence of the supremum is guaranteed.

- A
- B
- C
- D

Question 14

1 / 1 pts

Which one of the below listed theorems ensures superlinear convergence?

(A) the convergence theorem of the false position method
(B) Banach's fixed point theorem
(C) the monotone convergence theorem of Newton's method
(D) all above three state only first order convergence

A

B

C

D

Incorrect

Question 15

0 / 1 pts

Choose the correct formula related to the coefficients of the interpolatory quadrature formula $\sum_{k=0}^{n} A_k f(x_k)$.

(A) $A_k = c \cdot \int\limits_a^b \frac{\omega_n(x)}{x - x_k}\, dx$ with a constant $c \neq 0$ ✓

(B) $A_k = \int\limits_a^b \frac{\omega_n(x)}{x - x_k}\, dx$

(C) $A_k = c \cdot \int\limits_a^b \omega_n(x) dx$ with a constant $c \neq 0$

(D) $A_k = \int\limits_a^b \omega_n(x)\, dx$

A

B

C

D

Quiz Score: **10** out of 15