

## Homework 2

Let  $A = [1..6]$  be a statespace,  $S \subseteq A \times (\bar{A} \cup \{fail\})^{**}$  a program over the statespace  $A$ .

$$S = \left\{ \begin{array}{lll} 1 \rightarrow \langle 1, 2, 5, 1 \rangle & 2 \rightarrow \langle 2, 4, 3, 5, 6 \rangle & 2 \rightarrow \langle 2, 2, 2, \dots \rangle \\ 3 \rightarrow \langle 3, 1 \rangle & 3 \rightarrow \langle 3, 2, 4 \rangle & 3 \rightarrow \langle 3, 5, 2, 4, 1 \rangle \\ 4 \rightarrow \langle 4, 1, fail \rangle & 5 \rightarrow \langle 5, 3, 2, 4 \rangle & 5 \rightarrow \langle 5, 3, 6, 1 \rangle \\ 6 \rightarrow \langle 6, 1, 4 \rangle & 6 \rightarrow \langle 6, 1, fail \rangle & \end{array} \right\}$$

Let  $F \subseteq A \times A$  denote the following problem:  $F = \{ (1, 1), (3, 4), (3, 1), (5, 1), (5, 2), (5, 4) \}$

### Question

Let  $S_1$  and  $S_2$  be programs, let  $F_1$  and  $F_2$  be problems over the same statespace.

Statement1: if  $S_1$  solves  $F_1$  and  $F_1 \subseteq F_2$  then  $S_1$  solves  $F_2$ .

Statement2: if  $S_1$  solves  $F_1$  and  $F_2 \subseteq F_1$  then  $S_1$  solves  $F_2$ .

Statement3: if  $S_1$  solves  $F_1$  and  $S_1 \subseteq S_2$  then  $S_2$  solves  $F_1$ .

Statement4: if  $S_1$  solves  $F_1$  and  $S_2 \subseteq S_1$  then  $S_2$  solves  $F_1$ .

Informally, for example, Statement4 says that if a program  $S_1$  solves a given problem  $F_1$  then a smaller program  $S_2$  also solves the same problem  $F_1$ . Interestingly, only this fourth statement is true out of the four statements.

Your task is to find a counter-example for Statement3.

More precisely: consider the program  $S$  and problem  $F$  given above in this exercise. Find a program  $S_2$  such that  $S \subseteq S_2$  and  $S_2$  does not solve  $F$ . Explain why your  $S_2$  program does not solve  $F$ .

### Answer

Here is the  $S_2$  that has subset  $S$  and it does not Solve the  $F$ .

$$S_2 = \left\{ \begin{array}{lll} 1 \rightarrow \langle 1, 2, 5, 1 \rangle & 2 \rightarrow \langle 2, 4, 3, 5, 6 \rangle & 2 \rightarrow \langle 2, 2, 2, \dots \rangle \\ 3 \rightarrow \langle 3, 1 \rangle & 3 \rightarrow \langle 3, 2, 4 \rangle & 3 \rightarrow \langle 3, 5, 2, 4, fail \rangle \\ 4 \rightarrow \langle 4, 1, fail \rangle & 5 \rightarrow \langle 5, 3, 2, 4 \rangle & 5 \rightarrow \langle 5, 3, 6, 1 \rangle \\ 6 \rightarrow \langle 6, 1, 4 \rangle & 6 \rightarrow \langle 6, 1, fail \rangle & \end{array} \right\}$$

So it does not Satisfy the Condition So it  $S_2$  does not Solve the  $F_1$

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### Recall

$S$  program solves problem  $F$  if

1.  $\mathcal{D}_F \subseteq \mathcal{D}_{P[S]}$
2.  $\forall a \in \mathcal{D}_F: P[S](a) \subseteq F(a)$