5. $f(x,y) := \frac{x^3}{3} + \frac{y^3}{3} - x^2 + y^2 \quad ((x,y) \in \mathbb{R}^2)$.

① $\partial_1 f(x,y) = \frac{\cancel{3}x^2}{\cancel{3}} - 2x = x^2 - 2x \quad (0,2)$

② $\partial_2 f(x,y) = \frac{\cancel{3}y^2}{\cancel{3}} + 2y = y^2 + 2y \quad (0,-2)$

$\partial_1 f'(x,y) = (x^2-2x)' = 2x-2$

$\partial_2 f'(x,y) = 2y+2$

① $x^2 - 2x = 0$

$x(x-2) = 0 \Rightarrow x=0$ or $x=2$. $\quad (0,0) \quad (0,-2)$

② $y^2 + 2y = 0 \quad (2,-2) \quad (2,0)$

$y(y+2) = 0 \Rightarrow y=0$ or $y=-2$.

$\Rightarrow$ possible local extremai in $(0,0)$ and $(2,-2)$

$$f''(a) = \begin{pmatrix} \partial_{11} f(a) & \partial_{12} f(a) \\ \partial_{21} f(a) & \partial_{22} f(a) \end{pmatrix}$$

$$f''(x,y) = \begin{pmatrix} 2x-2 & 0 \\ 0 & 2y+2 \end{pmatrix}$$

$f''(0,0) = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$ $\det = -2\times2 - 0 = -4 < 0$, indefinite.

$f''(0,-2) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ $\det = 4 > 0$ $\partial_{11} f(a) < 0 \Rightarrow$ max in a.

$f''(2,-2) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$ $\det = -4 < 0$, indefinite

$f''(2,0) = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ $\det = 4 > 0$ $\cdot \partial_{11} f(a) > 0 \Rightarrow$ min in a.