NUMERICAL METHODS

2020/2021 Spring Semester

HOMEWORK #1

Deadline of submission: midnight before the first midterm

1. Let the set of machine numbers be $M(5,-4,4)$. Identify the special machine numbers ϵ_0, M_∞, ϵ_M! Map the following numbers into this set:

$$\frac{1}{7}, \quad 0,015, \quad 31, \quad 1,68.$$

2. What is the machine epsilon ϵ_M , if chopping is applied instead of rounding? Explain!

3. What are the results of the operations

$$\mathrm{fl}(\sqrt{3}) \oplus \mathrm{fl}(\sqrt{5}), \qquad \mathrm{fl}(\sqrt{5}) \ominus \mathrm{fl}(\sqrt{3})$$

in the set $M(4,-5,5)$?

4. Prove, that for the set of lower triangular matrices

$$\mathcal{L} = \Big\{ L \in \mathbb{R}^{n\times n}: \ l_{ij} = 0, \text{ if } i < j \ \ (i,j = 1,\dots,n) \Big\}$$

if $L_1, L_2 \in \mathcal{L}$, then $L_1 \cdot L_2 \in \mathcal{L}$, and if L_1 is invertable, then $L_1^{-1} \in \mathcal{L}$.

5. Prove, that for the set of lower triangular matrices with entries 1 in the diagonal

$$\mathcal{L}^{(1)} = \Big\{ L \in \mathbb{R}^{n\times n}: \ l_{ij} = 0, \text{ if } i < j, l_{ii} = 1 \ \ (i,j = 1,\dots,n) \Big\}$$

if $L_1, L_2 \in \mathcal{L}^{(1)}$, then $L_1 \cdot L_2 \in \mathcal{L}^{(1)}$, and if L_1 is invertible, then $L_1^{-1} \in \mathcal{L}^{(1)}$.

6. Using Gaussian elimination find the inverse of the following $n \times n$ matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 1 & 1 & 1 & 1 & 1 & \dots & 1 \end{bmatrix}$$

7. Consider the following $n \times n$ matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 2 & -1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 2 & -1 & \dots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \\ 0 & 0 & 0 & \dots & -1 & 2 & -1 \\ 0 & 0 & 0 & \dots & \dots & -1 & 2 \end{bmatrix}$$

   (a) Using Gaussian elimination find the LU decomposition of the matrix A. Find an iterative algorithm for the elements of L and U.

   (b) Using the LU decomposition of the matrix A find its determinant.

8. Prove that if A is a 2×2 matrix, then

$$\mathrm{cond}_1(A) = \mathrm{cond}_\infty(A)$$

9. Prove that for all $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$

$$\|\mathbf{a}\mathbf{b}^T\|_2 = \|\mathbf{a}\|_2 \cdot \|\mathbf{b}\|_2$$

10. Prove that for all $\mathbf{a}, \mathbf{b} \in \mathbb{R}^n$

$$\|\mathbf{a} \cdot \mathbf{b}^T\|_1 = \|\mathbf{a}\|_1 \cdot \|\mathbf{b}\|_\infty, \qquad \|\mathbf{a} \cdot \mathbf{b}^T\|_\infty = \|\mathbf{a}\|_\infty \cdot \|\mathbf{b}\|_1.$$