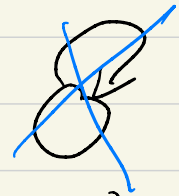
Graph representations

① Mathematical representation

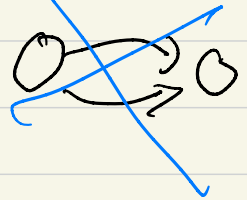
$|V| < \infty$

$G = (V, E)$ (V: vertices, E: edges)

$E \subseteq V \times V \setminus \{(u,u) \mid u \in V\}$ (no looping edges)



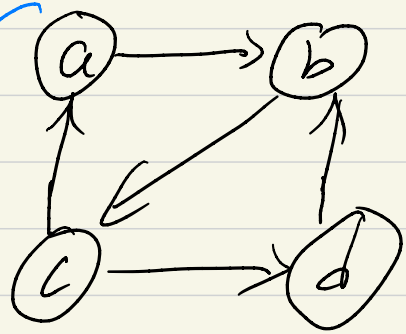
Directed: $(u,v) \neq (v,u)$ $(\{(u,v)\} \in E)$



Undirected: $(u,v) = (v,u)$

② Graphical representation

($a=1$, $b=2$
$c=3$, $d=4$, ...)

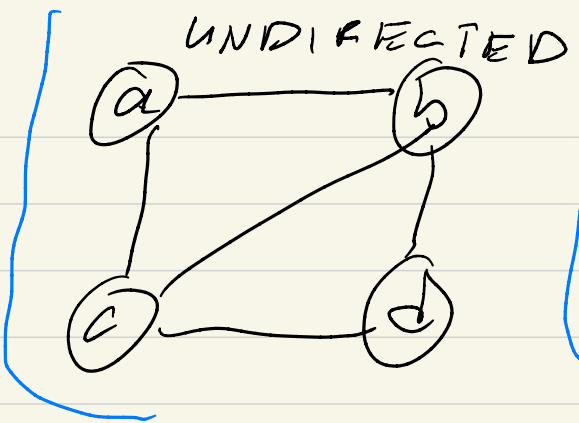


DIRECTED

③ Textual repr.

a → b.
b → c.
c → a; d.
d → b.

undirected equivalent of the directed graph above



a — b ; c.
b — c ; d.
c — d.
d.

⑤ Adjecency matrix repres. $V = \{v_1, \dots, v_n\}$

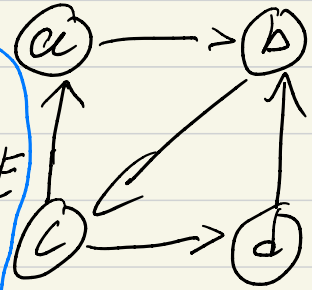
$A[1:|B[1,1]$

General case
$A[1:|B[n,n]$
$n = |V|$

	a	b	c	d
a	0	1	0	0
b	0	0	1	0
c	1	0	0	1
d	0	1	0	0

General case

$A[i,j] = \begin{cases} 1 \text{ if } (v_i, v_j) \in E \\ 0 \text{ if } (v_i, v_j) \notin E \end{cases}$



$n = 4$ here

n^2 bits are needed

Adj. mtx.

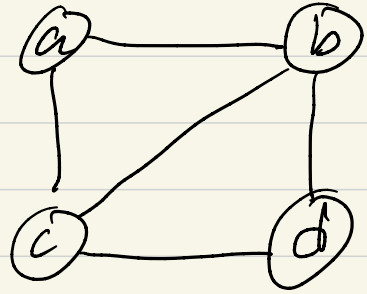
Graphical repn.

n^2 bits ~

Undirected case

	a	b	c	d
a	0	1	1	0
b	1	0	1	1
c	1	1	0	1
d	0	1	1	0

~ here $n = 4$



A/1 : IB[n, n]

↓ represended by

B : IB[n *(n-1)/2]

$\left(\frac{n^2}{2} - \frac{n}{2}\right)$ bits are needed

0	a_{12}	a_{13}	a_{14}
a_{21}	0	a_{23}	a_{24}
a_{31}	a_{32}	0	a_{34}
a_{41}	a_{42}	a_{43}	0

$a_{12} = a_{21}$

$a_{34} = a_{43}$

$a_{ij} = a_{ji}$

$B = \{ a_{21}, a_{31}, a_{32}, a_{41}, a_{42}, a_{43}$

$\ldots a_{n1}, a_{n2}, \ldots, a_{n(n-1)} \}$

$A[i,j] = B[(i-1)*(i-2)/2 + (j-1)]$ if $i > j$

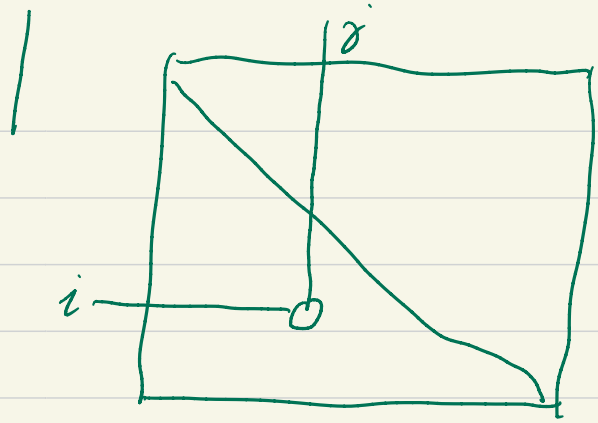
$A[i,j] = A[j,i]$ if $i < j$

$A[0,i] = 0$

$i > j$ ⇒

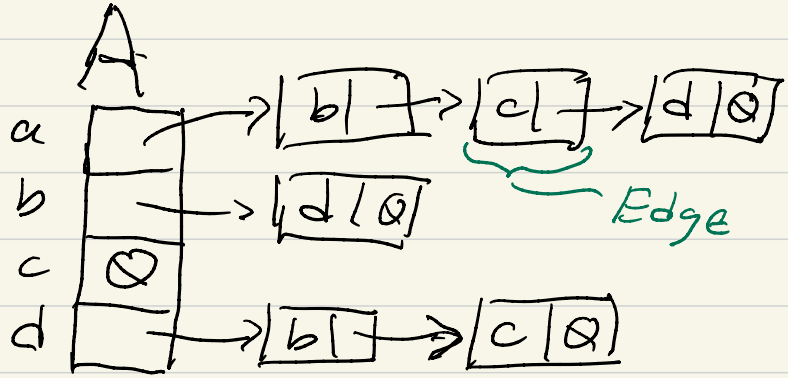
$$A[i,j] = B\left[\overbrace{1+2+\dots+(i-2)}^{\frac{(i-1)(i-2)}{2}} + (j-1)\right]$$

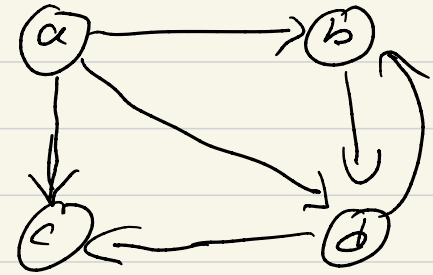
$$+ \begin{cases} 1: & a_{21} \\ 2: & a_{31}\ a_{32} \\ \vdots & \vdots \\ i-2: & a_{(i-1)1}\ a_{(i-1)2} \dots a_{(i-1)(i-2)} \\ j: & a_{i1}\ a_{i2} \dots a_{i(j-1)}\ a_{ij} \end{cases}$$



0/0

⑤ Adjacency list representation (most often used)





$Adj[1 : Edge*[n]$

Edge
+ v : ℕ
+ next : Edge*

$G = (V, E)$

$n = |V|$

$m = |E|$

Sparse graph:

$m \in O(n)$ Mem.: $\Theta(n+m) < \Theta(n^2)$ in sparse graphs

($m \leq 4 * n$ in most practical applications)

Adj. list is preferred

Dense graphs:

$m \in \Theta(n^2)$

Adj. mtx is preferred