

Analysis I., Sample Test 2

1. Determine the sums of the following series:

$$\text{a) } \sum_{n=1} \frac{2^{2n+1} - 3 \cdot 2^{n+3}}{6 \cdot 5^n} \quad \text{b) } \sum_{n=1} \frac{1}{n(n+3)}$$

2. Determine whether the following series are convergent or not:

$$\begin{array}{lll} \text{a) } \sum_{n=1} \left(1 + \frac{1}{n}\right)^n & \text{b) } \sum_{n=1} \frac{n+3}{\sqrt{n^5 + n^3 + n + 2}} & \text{c) } \sum_{n=1} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}} \\ \text{d) } \sum_{n=1} \left(\frac{n+2}{2n}\right)^n & \text{e) } \sum_{n=1} \frac{2^n \cdot n!}{n^n} & \text{f) } \sum_{n=1} (-1)^n \cdot \frac{1}{\sqrt{n+1}} \end{array}$$

3. Determine the radius of convergence and the convergence set of the following power series:

$$\sum_{n=1} \frac{1}{n^2 \cdot 3^n} \cdot (x-2)^n$$

4. Determine the following limits (if they exist):

$$\begin{array}{lll} \text{a) } \lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1} & \text{b) } \lim_{x \rightarrow 2} \frac{x^2 + x + 1}{x^2 - 3x + 2} & \text{c) } \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 1} - 1}{\sqrt{x^2 + 16} - 4} \\ \text{d) } \lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x}{1 - \sin x - \cos x} & \text{e) } \lim_{x \rightarrow 0} \frac{2 - e^x - e^{-x}}{\cos x - 1} \end{array}$$