Exam Sample 2

Due No due date **Points** 15 **Questions** 15 **Time Limit** 45 Minutes
Allowed Attempts Unlimited

Instructions

This is a sample exam, so you can have an impression of the actual exam quiz part. (There is also an optional second oral part of exam.)

Only one question is visible at once, and after you have submitted your answer to a question, you cannot go back to change your choice. You can start the sample exam as many times you wish. But of course on an actual exam, the quiz may be filled only once. The questions of the sample exam are fixed, but the questions and answers of an actual exam will be randomized.

Please also read the description of the course exam requirements announced separately!

Take the Quiz Again

Attempt History

	Attempt	Time	Score
KEPT	Attempt 5	2 minutes	15 out of 15
LATEST	Attempt 5	2 minutes	15 out of 15
	Attempt 4	3 minutes	13 out of 15
	Attempt 3	25 minutes	12 out of 15
	Attempt 2	35 minutes	9 out of 15
	Attempt 1	3 minutes	7 out of 15

Correct answers are hidden.

Submitted May 31 at 5:31pm

Question 1 1 / 1 pts

Which one of the below numbers is contained by the set of machine numbers $M(5, -5, 5)$?

(A) [10101|-6]
(B) [111011|2]
(C) [10101|-3]
(D) [01101|-1]

- A
- B
- C
- D

Question 2

1 / 1 pts

We are to solve the system of linear equations $Ax = b$ using Gaussian elimination. Which one of the below statements is true?

(A) If $\det(A) = 0$, then the Gaussian elimination can not be completed without switching rows or columns.
(B) If $\det(A) = 0$, then the system might not have a solution.
(C) If $\det(A) \neq 0$, then the Gaussian elimination can be completed without switching rows or columns.
(D) If $\det(A) \neq 0$, then the system might have two distinct solutions.

A

B

C

D

Question 3

1 / 1 pts

Let $x_0 = 0, x_1 = 1, x_2 = 2$ the support points of the interpolation and $y_0 = 1, y_1 = 1, y_2 = 0$ the prescribed function values. Which one is the appropriate interpolating polynomial?

(A) $L_2(x) = -x^2 + x + 1$
(B) $L_2(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1$
(C) $L_2(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 1$
(D) $L_2(x) = \frac{1}{2}x^2 + \frac{1}{2}x + 1$

A

B

C

D

Question 4

1 / 1 pts

Consider the contraction $\varphi : [a, b] \to [a, b]$ on the interval $[a, b]$ with contraction coefficient q. In this case

(A) $\exists x^* \in [a, b] : x^* = \varphi(x^*)$.
(B) $\forall x^* \in [a, b] : x^* = \varphi(x^*)$.
(C) $\exists! x_0 \in [a, b]$ such that the sequence $x_{k+1} := \varphi(x_k)$ $(k \in \mathbb{N})$ is convergent and $\lim_{k\to\infty} x_k = x^*$.
(D) if $q > 1$, then $\forall x_0 \in [a, b]$ the sequence $x^{k+1} := \varphi(x^k)$ $(k \in \mathbb{N})$ is convergent and $\lim_{k\to\infty} x_k = x^*$.

- (•) A
- ( ) B
- ( ) C
- ( ) D

Question 5

1 / 1 pts

What is the condition number of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

with respect to the Euclidean norm?

(A) $\text{cond}_2(A) = 1$
(B) $\text{cond}_2(A) = 2$
(C) $\text{cond}_2(A) = 4$
(D) The condition number is not defined for this matrix.

A

B

C

D

Question 6

1 / 1 pts

Choose the correct equality about the interpolatory quadrature formula $\sum_{k=0}^{n} A_k f(x_k)$ to estimate the integral $\int_a^b f(x)\,dx$.

(A) $\sum_{k=0}^{n} A_k = \frac{b^2 - a^2}{2}$

(B) $\sum_{k=0}^{n} x_k = \frac{b^2 - a^2}{2}$

(C) $\sum_{k=0}^{n} A_k x_k = \frac{b^2 - a^2}{2}$

(D) $\sum_{k=0}^{n} A_k x_k^2 = \frac{b^2 - a^2}{2}$

A

B

C

D

Question 7

1 / 1 pts

Assume that we completed the Gaussian elimination on the matrix $A \in \mathbb{R}^{n \times n}$ and we arrived at $a_{n,n}^{(n-1)} = 0$. What does this mean?

(A) We had to switch rows or columns at least one time during the elimination.

(B) A linear system of equations with matrix A has no solutions.

(C) The matrix A does not have a unique LU decomposition.

(D) The last component of the solution of a linear system of equations with matrix A is arbitrary.

- ○ A
- ○ B
- ◉ C
- ○ D

Question 8

1 / 1 pts

Let $a, b \in \mathbb{R}$ arbitrary and

$$c := a - b, \qquad d := a + b.$$

What can we say about the relative error bounds for c and d?

(A) A smaller relative error bound can be given for c, than for d.

(B) A smaller relative error bound can be given for d, than for c.

(C) It depends on the signs of a and b.

(D) Independent of a and b, the relative error bounds for c and d are approximately the same.

A

B

C

D

Question 9

1 / 1 pts

Let $x_0 = 0, x_1 = 1, x_2 = 2$ the support points of the interpolation. What is the value of the sum $\ell_0(x) + \ell_1(x)$ with the Lagrange base polynomials?

(A) $\ell_0(x) + \ell_1(x) = \dfrac{x(x-1)}{2}$

(B) $\ell_0(x) + \ell_1(x) = -\dfrac{x(x-1)}{2}$

(C) $\ell_0(x) + \ell_1(x) = 1 - \dfrac{x(x-1)}{2}$

(D) $\ell_0(x) + \ell_1(x) = -1 + \dfrac{x(x-1)}{2}$

A

B

C

D

Question 10

1 / 1 pts

Which statement is correct about the line $y = ax + b$ fitted according to the least squares method, given the points $(x_i, y_i) \quad (i = 1, \ldots, N)$.

(A) $\sum_{i=1}^{N} |y_i - ax_i - b|$ is minimal

(B) $N \cdot b + \left(\sum_{i=1}^{N} x_i \right) a = \sum_{i=1}^{N} y_i$

(C) $\sum_{i=1}^{N} (y_i + ax_i + b)^2$ is minimal

(D) $\sum_{i=1}^{N} (y_i - ax_i - b)^2 = 0$

- A
- B
- C
- D

Question 11

1 / 1 pts

In the proof of the local convergence theorem of Newton's method, we approximate the function f with its Taylor polynomial.
But which one?

(A) The Taylor polynomial of degree two around x^*.

(B) The Taylor polynomial of degree one around x_{k+1}.

(C) The Taylor polynomial of degree two around x_k.

(D) The Taylor polynomial of degree one around x_k.

A

B

C

D

Question 12

1 / 1 pts

Consider the formulas $M(f)$ and $T(f)$ for a function $f \in C^2[a, b]$ estimating $\int_a^b f(x)\,dx$ with $M_2 = \max_{[a,b]} |f''|$. Based on their error formulas, which below estimate is the sharpest correct one?

(A) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{4}$

(B) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{8}$

(C) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{16}$

(D) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{24}$

A

B

C

D

Question 13

1 / 1 pts

Let $t \in \mathbb{N}^+$ and consider the set of machine numbers $M(t, t, t)$! What is the difference of the cardinality and the biggest number of this set?

$$|M| - M_\infty = ?$$

(A) 0
(B) 2
(C) t
(D) 2^{t+1}

- A
- B
- C
- D

Question 14

1 / 1 pts

What form is appropriate for the polynomial $L_k(x) - L_{k-1}(x)$ with the Lagrange interpolation polynomials of the degree indicated in their indices, and the set of their support points x_0, x_1 etc. differing in only one point?

(A) $L_k(x) - L_{k-1}(x) = c_k(x - x_0)(x - x_1)\cdots(x - x_k)$
(B) $L_k(x) - L_{k-1}(x) = (x - x_0)(x - x_1)\cdots(x - x_{k-1})$
(C) $L_k(x) - L_{k-1}(x) = c_k(x - x_0)(x - x_1)\cdots(x - x_{k-1})$
(D) $L_k(x) - L_{k-1}(x) = (x - x_0)(x - x_1)\cdots(x - x_k)$

A

B

C

D

Question 15

1 / 1 pts

Which below formula is incorrect about Simpson's rule $S_m(f)$ (with m being even)?

$$S_m(f) = \dots$$

(A) $\frac{b-a}{3m}\left(f(x_0) + 4\sum_{k=1}^{m-1} f(x_k) + f(x_m) - 2\sum_{k=1}^{m/2} f(x_{2k})\right)$

(B) $\frac{b-a}{3m}\left(f(x_0) + 2\sum_{k=1}^{m-1} f(x_k) + f(x_m) + 2\sum_{k=1}^{m/2} f(x_{2k-1})\right)$

(C) $\frac{2}{3}\left(T_m(f) + \frac{b-a}{m} \cdot \sum_{k=1}^{m/2} f(x_{2k-1})\right)$

(D) $\frac{T_m(f) + T_{m/2}(f)}{3}$

A

B

C

D