DICTIONARIES: + HASH TABLES

Data collections + insertion, search, deletion } ops.

+ BINARY SEARCH TREES (BST)

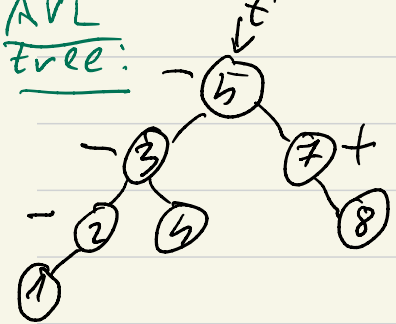
− AVL TREES } (RED-BLACK TREES...)

− B+ TREES } THIS SEMESTER

AVL TREES: 1966

(Height) balanced BST

AVL tree:



$|t| = 7$

$h(t) = 3$

The balance of $(*p)$: HEIGHT

$p \to b \stackrel{def}{=} h(p \to right) - h(p \to left)$

$+ \sim +1$	$++ \sim +2$	$= \sim 0$
$- \sim -1$	$-- \sim -2$	

DEF: AVL tree: balanced BST

$(*p)$ is balanced $\stackrel{def}{\iff} p \to b \in \{-1, 0, 1\}$

D. A binary tree is balanced iff each node of the tree is balanced.

Theorem: If t is a nonempty balanced binary tree, $n = |t|$ (number of nodes), $h = h(t)$

$\Rightarrow \lfloor \log n \rfloor \leq h \leq 1.55 \cdot \log n$ ($h \in \Theta(\log n)$)

$\lfloor \log n \rfloor \leq h$: for each bin. tree

$h \leq n-1$

- ins(t, k)
+ search(t, k): Node*
+ min(t): Node*
+ max(t): Node*
- remMin(t, minp)
- remMax(t, maxp)
- del(t, k)

running time: $O(h)$

AVL tree $\Rightarrow h \in O(\log n)$

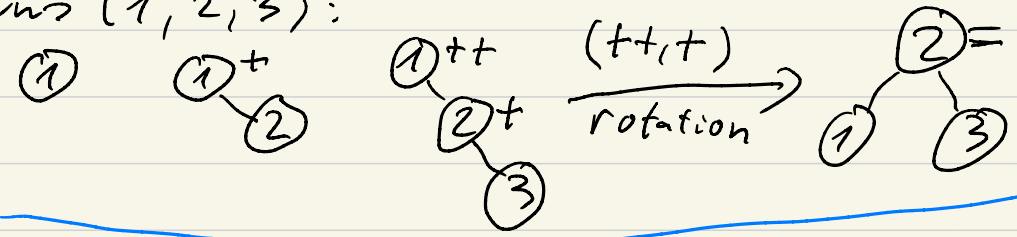
all these ops are $O(\log n)$

THESE SHOULD KEEP the balanced property (HOW?)

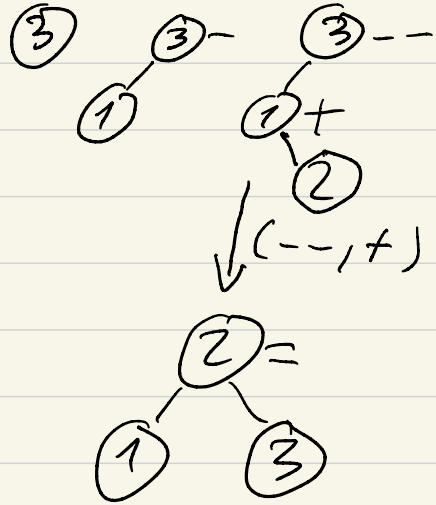
Problem: ins, remMin, remMax, del modify the tree: it may become imbalanced: efficiency may be lost

Examples: Q. insert 1, 2, 3 in some order

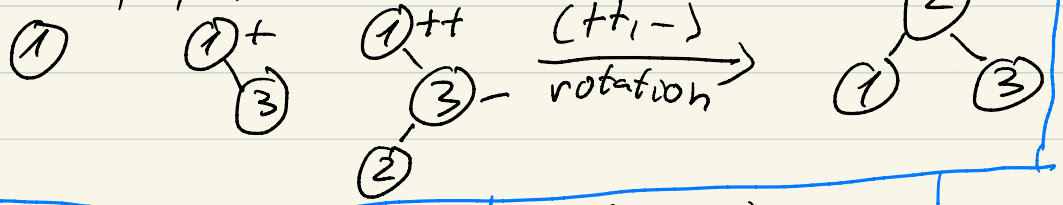
ins(1, 2, 3):



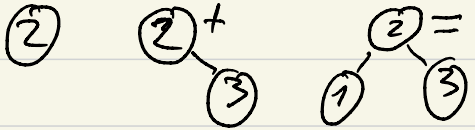
ins(3, 1, 2)



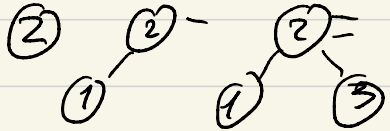
ins(1, 3, 2)



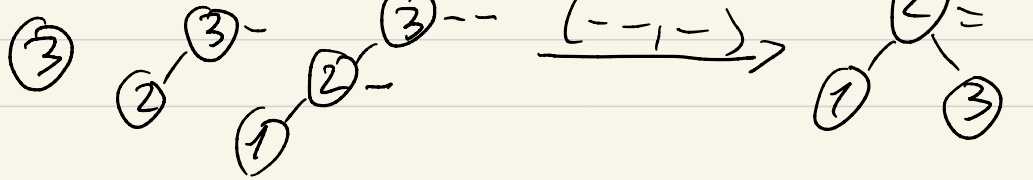
ins(2, 3, 1)

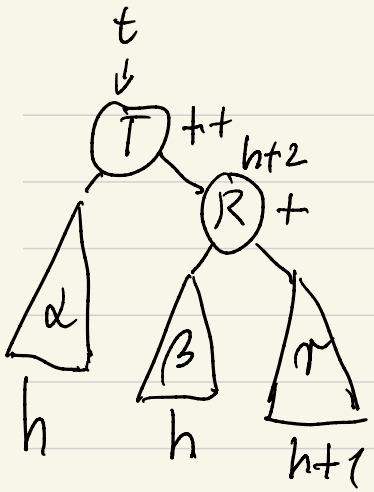


ins(2, 1, 3)



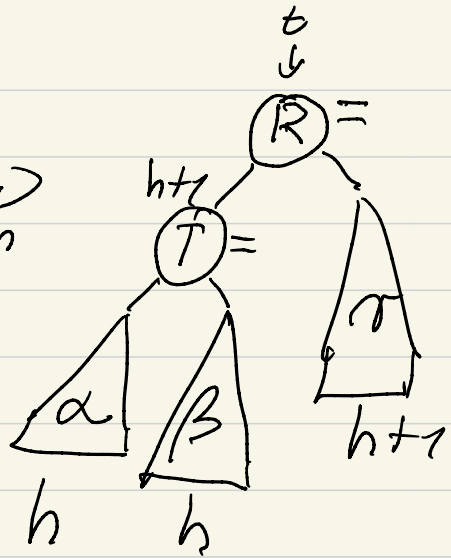
ins(3, 2, 1):





$h(t) = h+3$

(++,+)
rotation



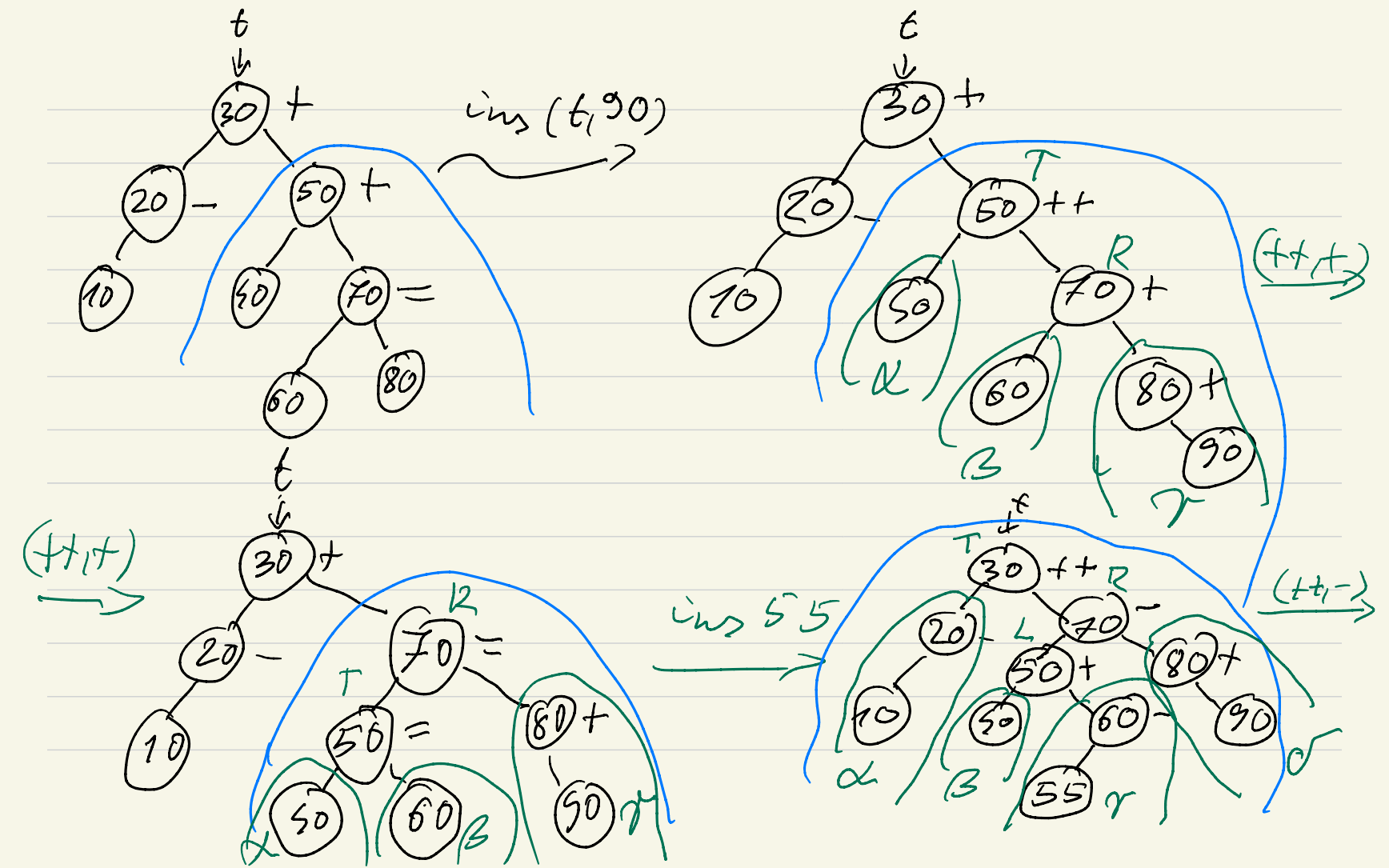
$h(t) = h+2$

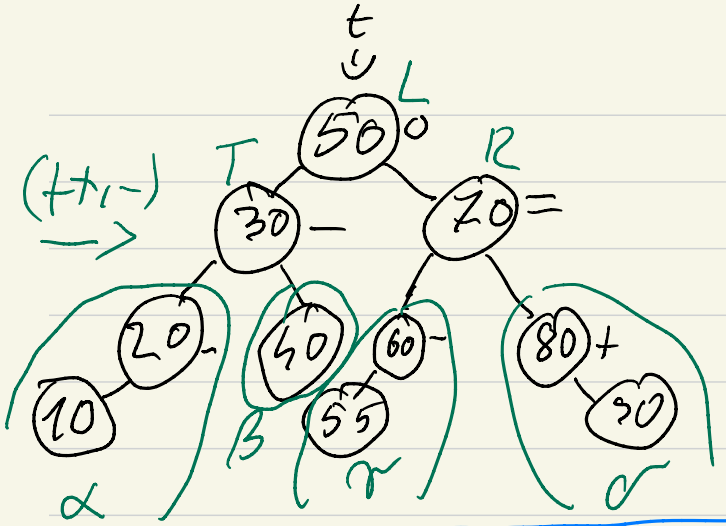
AFTER INSERTION, MAXIMUM ONE ROTATION IS NEEDED

$\alpha < T < \beta < R < \gamma$

$h(t)=h+2 \xrightarrow{ins} h(t)=h+3 \xrightarrow{(++,+)} h(t)=h+2$

THE ORIGINAL HEIGHT OF t IS RESTORED

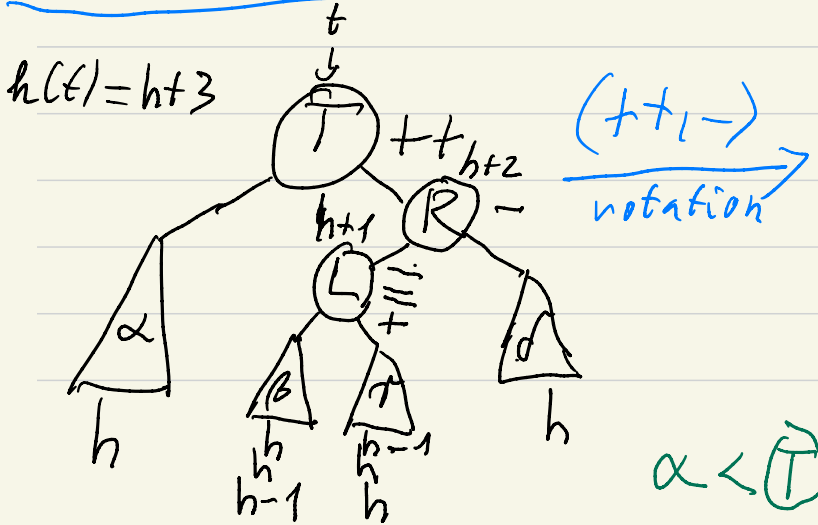

t
30 +
20 −
50 +
10
40
70 =
60
80
ins (t, 90)
t
30 +
20 −
50 ++
10
40
70 +
60
80 +
90
(++,+)
t
30 +
20 −
70 =
10
50 =
80 +
50
60
90
(++,+)
ins 5 5
t
30 ++
20 −
70 −
10
50 +
80 +
50
60
90
55
(+,+,−)



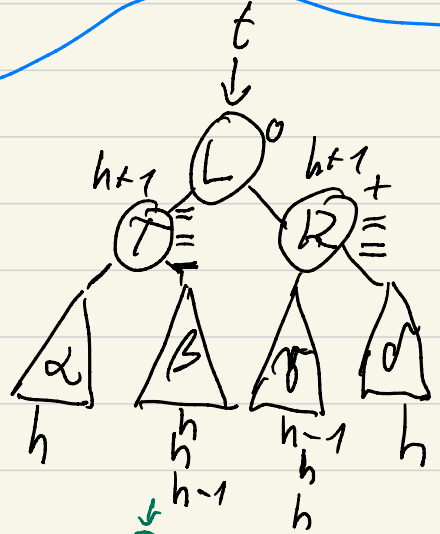
(+t, −) →

$\{(--,-), (--,+)\}$ and similarly for $\{(++,+), (++,-)\}$

HW (check in printed Lecture notes)

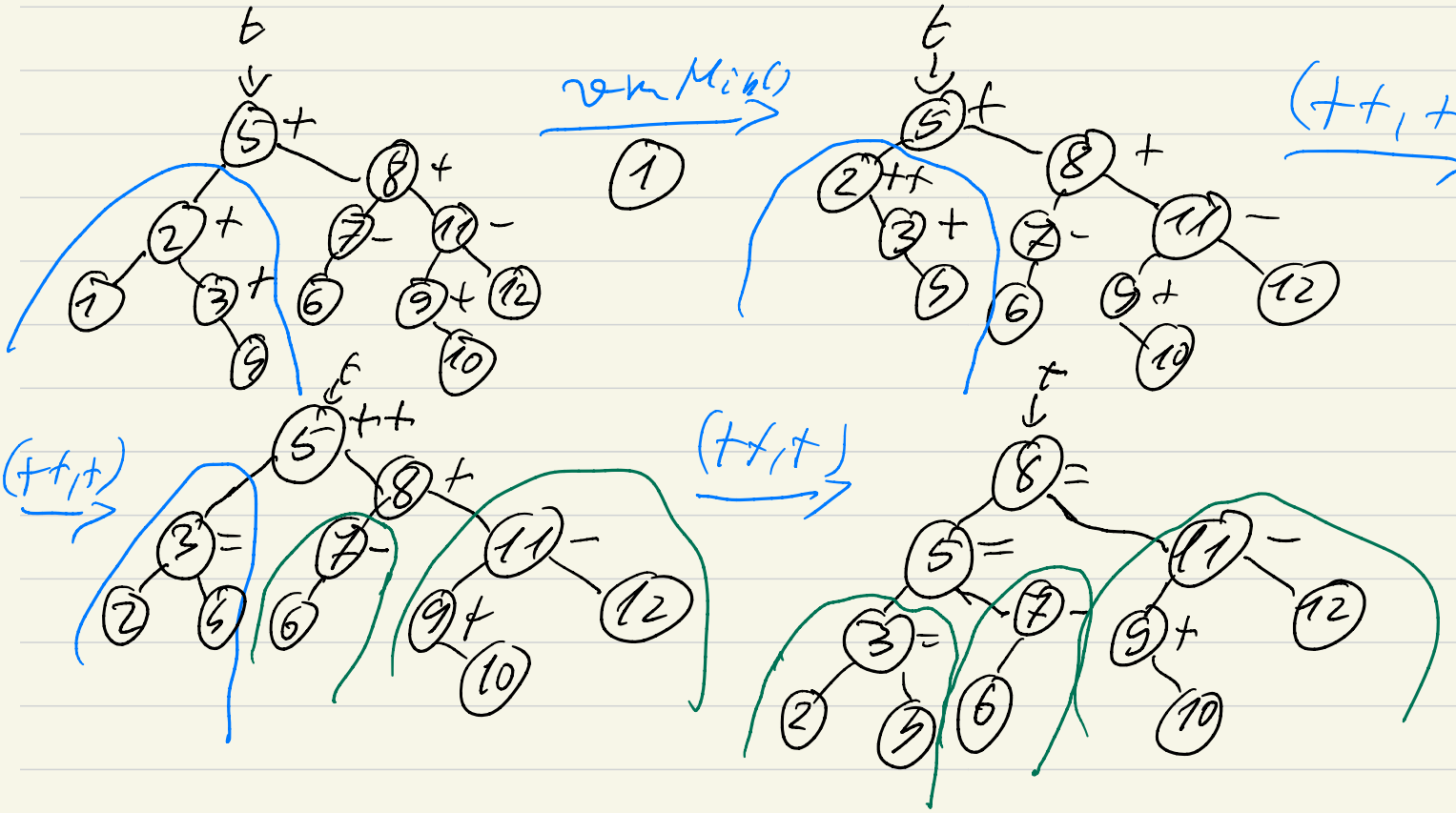


(+t, −) notation

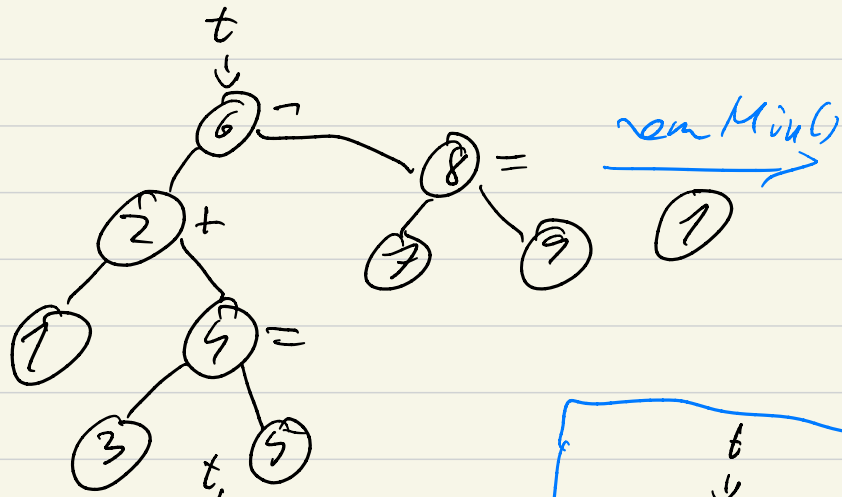


$\alpha < (T) < \beta < (L) < \gamma < (R) < \delta$

remMin()

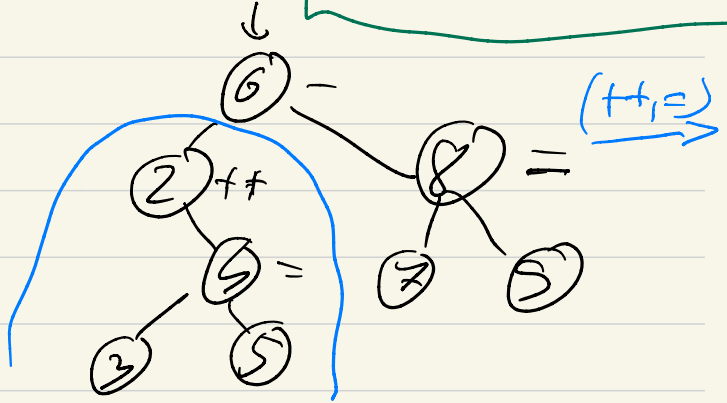


rem Min() : (++1=) variation



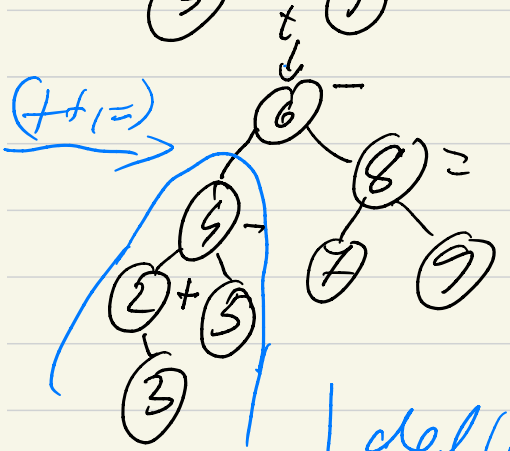
rem Min()

HW rem Max()

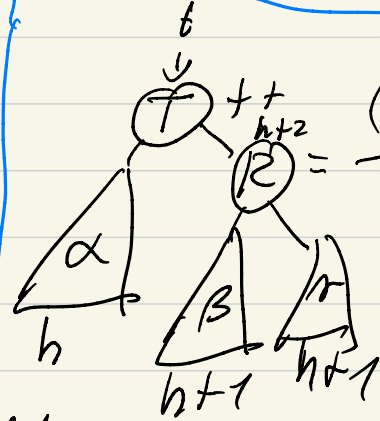


(++1=)

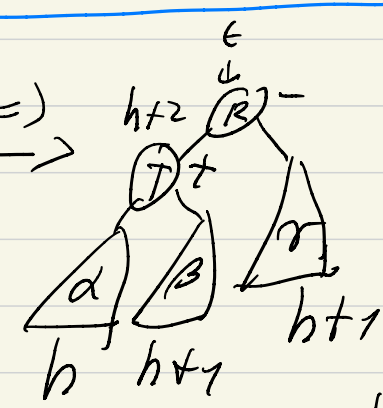
(++1=)



del(5)



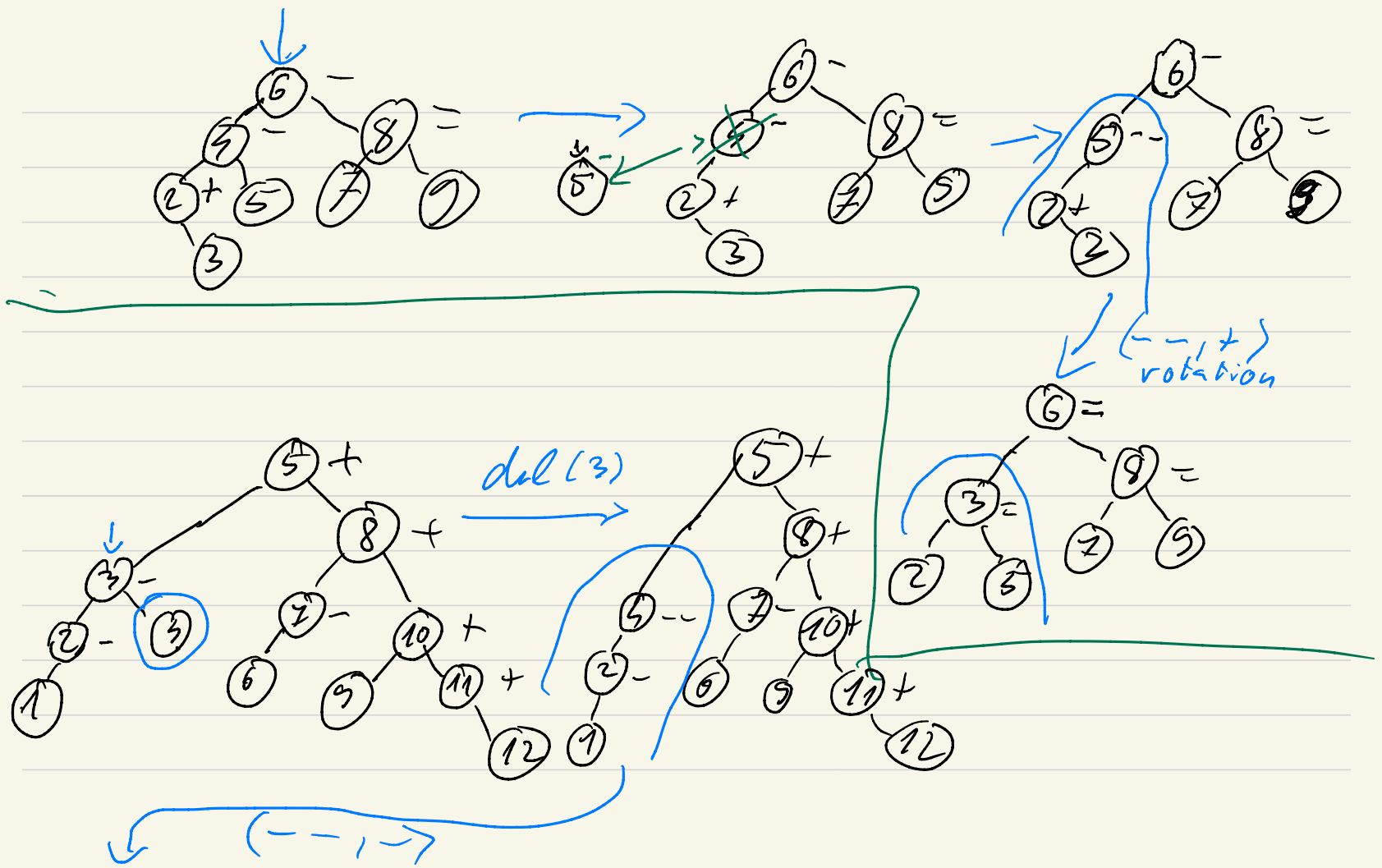
(++1=)

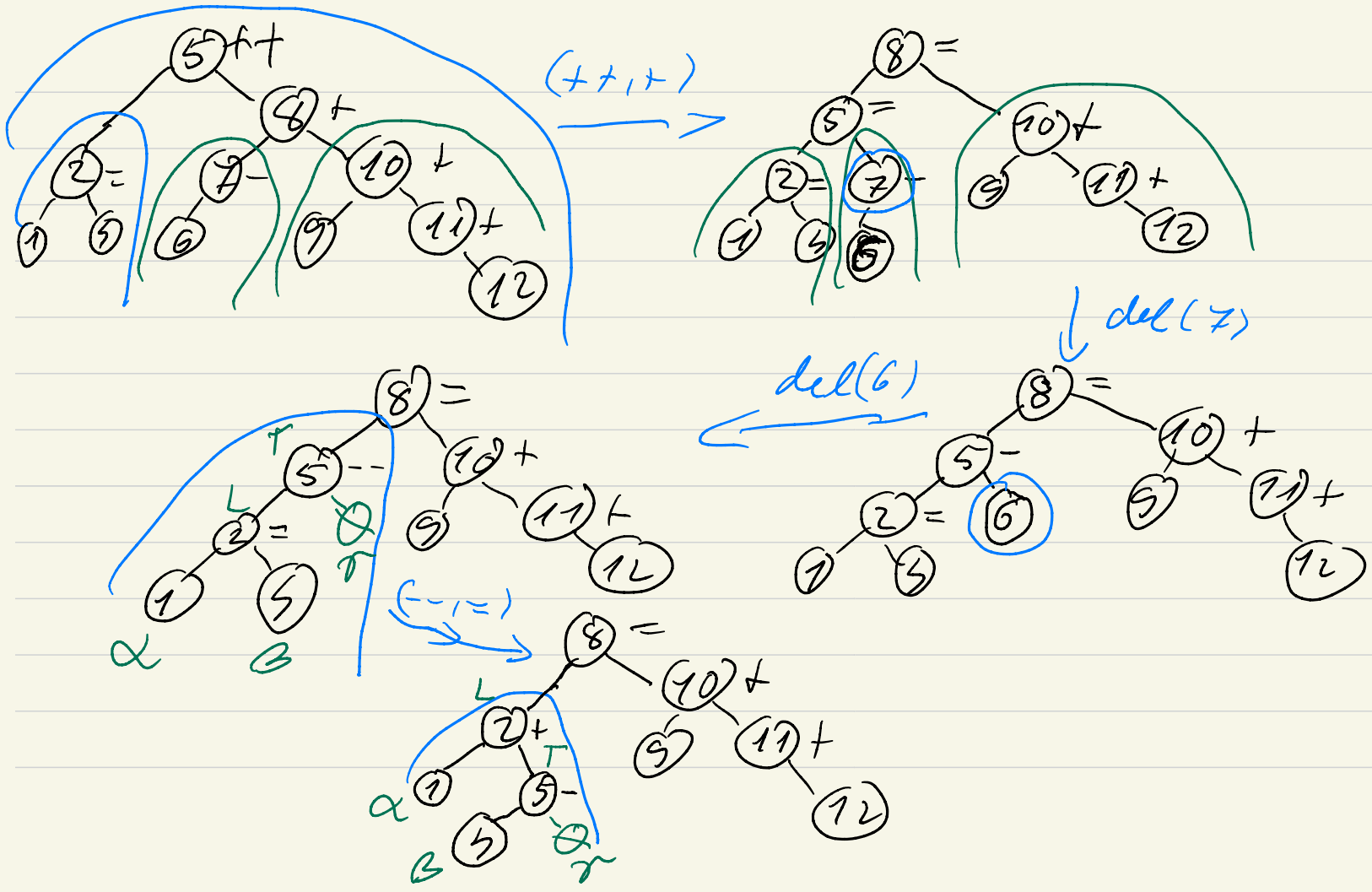


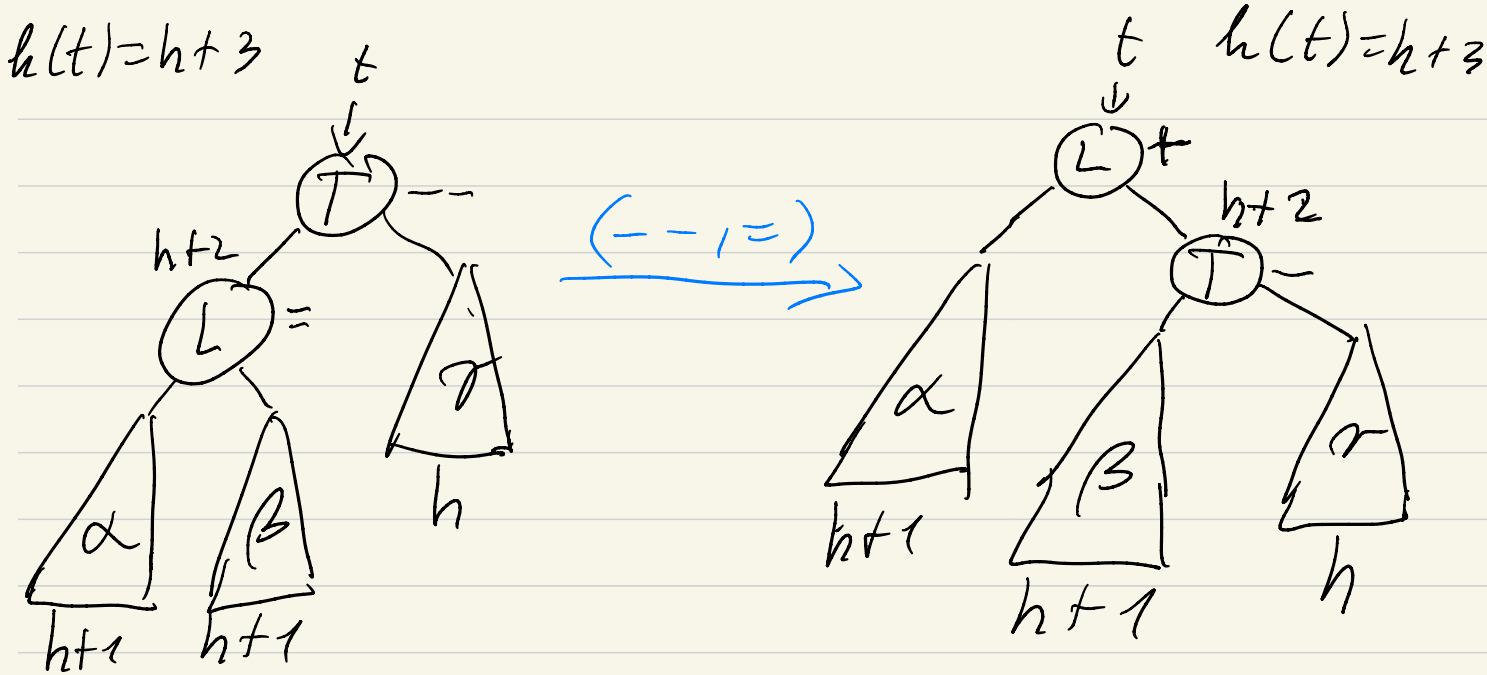
h(t)=h+3

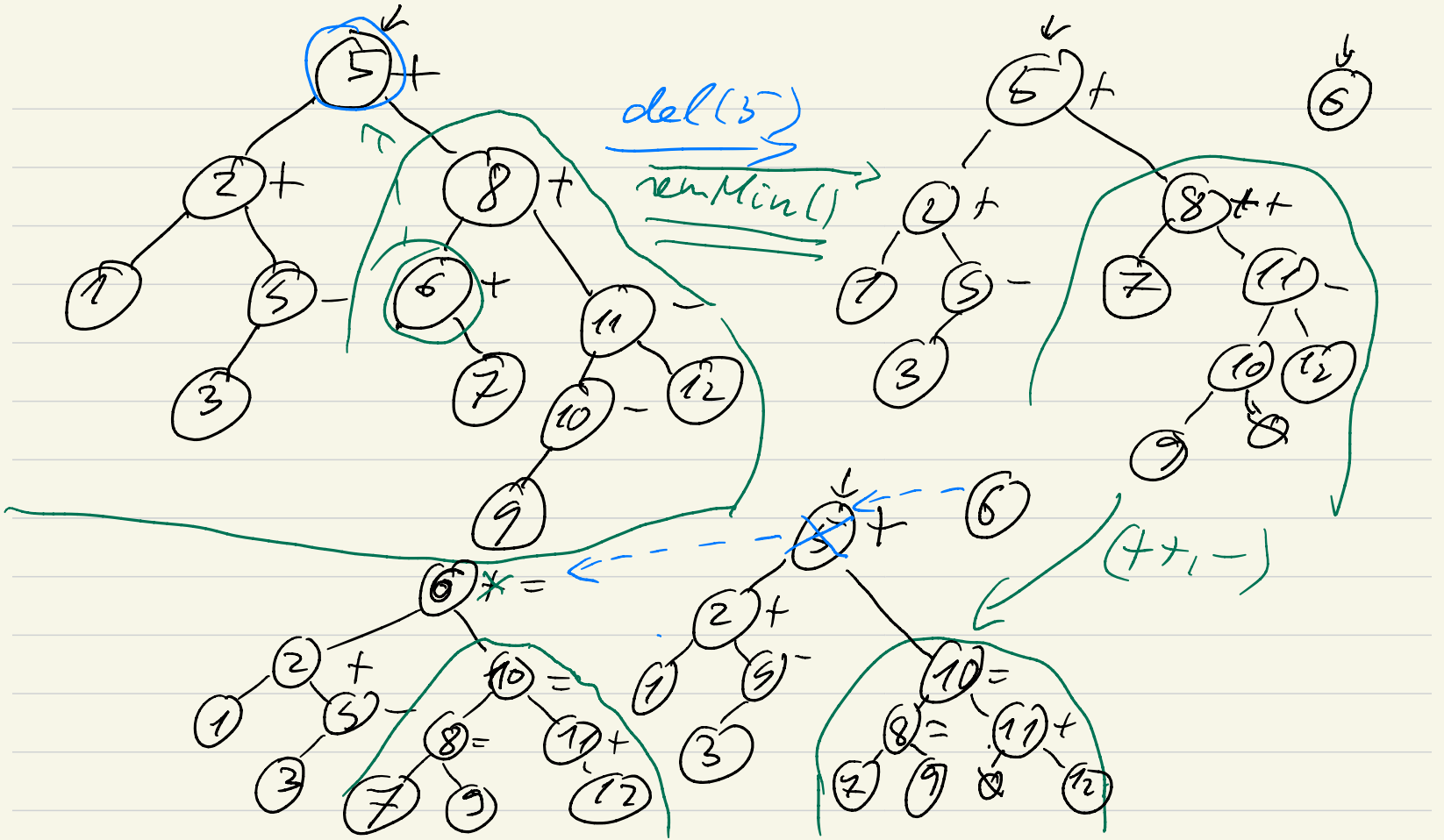
HW (--1=)

h(t)=h+3


del(3)
rotation


(++,+)
del(7)
del(6)
(--,=)


h(t)=h+3
t
T
--
h+2
L
=
α
β
γ
h+1
h+1
h
(--,=)
t
h(t)=h+3
L
+
h+2
T
-
α
β
γ
h+1
h+1
h


del(5)
remMin()
(++, --)