Programming theory
endterm test - sample

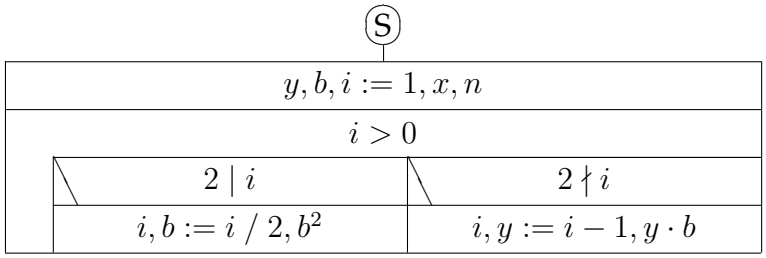
1. $A = (x{:}\mathbb{N}, n{:}\mathbb{N}, y{:}\mathbb{N})$

   $B = (x'{:}\mathbb{N}, n'{:}\mathbb{N})$

   $Q = (x = x' \wedge n = n' \wedge x > 0)$

   $R = (Q \wedge y = x^n)$

   $b{:}\mathbb{N}$ and $i{:}\mathbb{N}$ are auxiliary variables of the program S.



   $Q' = (Q \wedge y = 1 \wedge b = x \wedge i = n)$ is the intermediate condition of the program, $Inv = (Q \wedge y \cdot b^i = x^n)$ is the loop invariant, $t : i$ is the variant function of the loop. Prove that $Q \implies lf(S, R)$ holds. Detailed explanation is required.
   (25 points)

2. $A = (a{:}\mathbb{Z}^n, b{:}\mathbb{Z}^n)$

   $i{:}\mathbb{N}$ and $j{:}\mathbb{N}$ are auxiliary variables.
   Prove that the following program is free from deadlock.

   parbegin $S_1 \| S_2$ **parend**

S_1 :	S_2 :
$\{Inv\}$ while $i \leq n$ do $\{Inv \wedge i \leq n\}$ await $i = j$ then $x, i := a[i], i + 1$ ta $\{Inv\}$ do $\{Inv \wedge i = n + 1\}$	$\{Inv\}$ while $j \leq n$ do $\{Inv \wedge j \leq n\}$ await $i > j$ then $b[j], j := x, j + 1$ ta $\{Inv\}$ od $\{Inv \wedge j = n + 1\}$

   $Inv = (a = a' \wedge 0 \leq i - 1 \leq j \leq i \leq n + 1 \wedge \forall k \in [1..j-1]\colon b[k] = a[k]) \wedge (i > j \rightarrow x = a[i-1]))$
   (10 points)

3. $A = (x{:}\mathbb{Z})$

S_1 :	S_2 :
$\{x = 0 \vee x = 3\}$ $x := x + 2$ $\{x = 2 \vee x = 5\}$	$\{x = 0 \vee x = 2\}$ $x := x + 3$ $\{x = 3 \vee x = 5\}$

   Prove that $x = 0 \implies wp(\textbf{parbegin } S_1 \parallel S_2 \textbf{ parend}, x = 5)$ holds. (10 points)