Exam – 2021.06.09.

Due Jun 9, 2021 at 9:55am **Points** 15 **Questions** 15
Available until Jun 9, 2021 at 9:55am **Time Limit** 45 Minutes

Instructions

Only one question is visible at once, and after you have submitted your answer to a question, you cannot go back to change your choice. There is exactly 1 correct answer for each question. Every correct answer is worth 1 point.

The maximum is 15 points. Under 8 points, the exam is finished with a fail (1) grade. If one achives at least 8 points, then the grade satisfactory (2), from 12 points, the grade average (3) is offered and one may attend the second, oral part of the exam for better grades.

This quiz is no longer available as the course has been concluded.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	14 minutes	7 out of 15

Correct answers are hidden.

Score for this quiz: **7** out of 15
Submitted Jun 9, 2021 at 9:55am
This attempt took 14 minutes.

Question 1

1 / 1 pts

Which one of the below numbers is contained by the set of machine numbers $M(5,-5,5)$?

(A) [10101|-6]
(B) [111011|2]
(C) [10101|-3]
(D) [01101|-1]

- A
- B
- C
- D

Question 2

1 / 1 pts

We are to solve the system of linear equations $Ax = b$ using Gaussian elimination. Which one of the below statements is true?

(A) If $\det(A) = 0$, then the Gaussian elimination can not be completed without switching rows or columns.

(B) If $\det(A) = 0$, then the system might not have a solution.

(C) If $\det(A) \neq 0$, then the Gaussian elimination can be completed without switching rows or columns.

(D) If $\det(A) \neq 0$, then the system might have two distinct solutions.

- A
- B
- C
- D

Question 3

1 / 1 pts

Let $x_0 = 0, x_1 = 1, x_2 = 2$ the support points of the interpolation and $y_0 = 1, y_1 = 1, y_2 = 0$ the prescribed function values. Which one is the appropriate interpolating polynomial?

(A) $L_2(x) = -x^2 + x + 1$
(B) $L_2(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1$
(C) $L_2(x) = -\frac{1}{2}x^2 + \frac{1}{2}x + 1$
(D) $L_2(x) = \frac{1}{2}x^2 + \frac{1}{2}x + 1$

- ( ) A
- ( ) B
- (•) C
- ( ) D

Question 4

1 / 1 pts

Consider the contraction $\varphi : [a, b] \to [a, b]$ on the interval $[a, b]$ with contraction coefficient q. In this case

(A) $\exists x^* \in [a, b] : x^* = \varphi(x^*)$.

(B) $\forall x^* \in [a, b] : x^* = \varphi(x^*)$.

(C) $\exists! x_0 \in [a, b]$ such that the sequence $x_{k+1} := \varphi(x_k)$ $(k \in \mathbb{N})$ is convergent and $\lim_{k\to\infty} x_k = x^*$.

(D) if $q > 1$, then $\forall x_0 \in [a, b]$ the sequence $x^{k+1} := \varphi(x^k)$ $(k \in \mathbb{N})$ is convergent and $\lim_{k\to\infty} x_k = x^*$.

- (selected) A
- B
- C
- D

Question 5

1 / 1 pts

What is the condition number of the matrix

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

with respect to the Euclidean norm?

(A) $\mathrm{cond}_2(A) = 1$
(B) $\mathrm{cond}_2(A) = 2$
(C) $\mathrm{cond}_2(A) = 4$
(D) The condition number is not defined for this matrix.

- A
- B
- C
- D

Incorrect

Question 6

0 / 1 pts

Choose the correct equality about the interpolatory quadrature formula $\sum_{k=0}^{n} A_k f(x_k)$ to estimate the integral $\int_a^b f(x)\,dx$.

(A) $\sum_{k=0}^{n} A_k = \dfrac{b^2 - a^2}{2}$

(B) $\sum_{k=0}^{n} x_k = \dfrac{b^2 - a^2}{2}$

(C) $\sum_{k=0}^{n} A_k x_k = \dfrac{b^2 - a^2}{2}$

(D) $\sum_{k=0}^{n} A_k x_k^2 = \dfrac{b^2 - a^2}{2}$

- A
- B
- C
- D

Incorrect

Question 7

0 / 1 pts

Assume that we completed the Gaussian elimination on the matrix $A \in \mathbb{R}^{n \times n}$ and we arrived at $a_{n,n}^{(n-1)} = 0$. What does this mean?

(A) We had to switch rows or columns at least one time during the elimination.

(B) A linear system of equations with matrix A has no solutions.

(C) The matrix A does not have a unique LU decomposition.

(D) The last component of the solution of a linear system of equations with matrix A is arbitrary.

- ○ A
- ○ B
- ○ C
- ◉ D

Incorrect

Question 8

0 / 1 pts

Let $a, b \in \mathbb{R}$ arbitrary and

$$c := a - b, \qquad d := a + b.$$

What can we say about the relative error bounds for c and d?

(A) A smaller relative error bound can be given for c, than for d.
(B) A smaller relative error bound can be given for d, than for c.
(C) It depends on the signs of a and b.
(D) Independent of a and b, the relative error bounds for c and d are approximately the same.

- ○ A
- ◉ B
- ○ C
- ○ D

Question 9

1 / 1 pts

Let $x_0 = 0, x_1 = 1, x_2 = 2$ the support points of the interpolation. What is the value of the sum $\ell_0(x) + \ell_1(x)$ with the Lagrange base polynomials?

(A) $\ell_0(x) + \ell_1(x) = \dfrac{x(x-1)}{2}$

(B) $\ell_0(x) + \ell_1(x) = -\dfrac{x(x-1)}{2}$

(C) $\ell_0(x) + \ell_1(x) = 1 - \dfrac{x(x-1)}{2}$

(D) $\ell_0(x) + \ell_1(x) = -1 + \dfrac{x(x-1)}{2}$

- A
- B
- C (selected)
- D

Incorrect

Question 10

0 / 1 pts

Which statement is correct about the line $y = ax + b$ fitted according to the least squares method, given the points $(x_i, y_i) \quad (i = 1, \ldots, N)$.

(A) $\sum\limits_{i=1}^{N} |y_i - ax_i - b|$ is minimal

(B) $N \cdot b + \left(\sum\limits_{i=1}^{N} x_i\right) a = \sum_{i=1}^{N} y_i$

(C) $\sum\limits_{i=1}^{N} (y_i + ax_i + b)^2$ is minimal

(D) $\sum\limits_{i=1}^{N} (y_i - ax_i - b)^2 = 0$

- A
- B
- C
- D

Incorrect

Question 11

0 / 1 pts

In the proof of the local convergence theorem of Newton's method, we approximate the function f with its Taylor polynomial.
But which one?

(A) The Taylor polynomial of degree two around x^*.
(B) The Taylor polynomial of degree one around x_{k+1}.
(C) The Taylor polynomial of degree two around x_k.
(D) The Taylor polynomial of degree one around x_k.

- A
- B
- C
- D

Incorrect

Question 12

0 / 1 pts

Consider the formulas $M(f)$ and $T(f)$ for a function $f \in C^2[a,b]$ estimating $\int_a^b f(x)\,dx$ with $M_2 = \max_{[a,b]} |f''|$. Based on their error formulas, which below estimate is the sharpest correct one?

(A) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{4}$

(B) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{8}$

(C) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{16}$

(D) $|T(f) - M(f)| \leq \dfrac{M_2(b-a)^3}{24}$

- A
- B
- C
- D

Incorrect

Question 13

0 / 1 pts

Let $t \in \mathbb{N}^+$ and consider the set of machine numbers $M(t,t,t)$!
What is the difference of the cardinality and the biggest number of this set?

$$|M| - M_\infty = ?$$

(A) 0
(B) 2
(C) t
(D) 2^{t+1}

- A
- B
- C
- D

Question 14

1 / 1 pts

What form is appropriate for the polynomial $L_k(x) - L_{k-1}(x)$ with the Lagrange interpolation polynomials of the degree indicated in their indices, and the set of their support points x_0, x_1 etc. differing in only one point?

(A) $L_k(x) - L_{k-1}(x) = c_k(x - x_0)(x - x_1)\cdots(x - x_k)$
(B) $L_k(x) - L_{k-1}(x) = (x - x_0)(x - x_1)\cdots(x - x_{k-1})$
(C) $L_k(x) - L_{k-1}(x) = c_k(x - x_0)(x - x_1)\cdots(x - x_{k-1})$
(D) $L_k(x) - L_{k-1}(x) = (x - x_0)(x - x_1)\cdots(x - x_k)$

- A
- B
- C (selected)
- D

Incorrect

Question 15

0 / 1 pts

Which below formula is incorrect about Simpson's rule $S_m(f)$ (with m being even)?

$$S_m(f) = \dots$$

(A) $\frac{b-a}{3m}\left(f(x_0) + 4\sum_{k=1}^{m-1} f(x_k) + f(x_m) - 2\sum_{k=1}^{m/2} f(x_{2k})\right)$

(B) $\frac{b-a}{3m}\left(f(x_0) + 2\sum_{k=1}^{m-1} f(x_k) + f(x_m) + 2\sum_{k=1}^{m/2} f(x_{2k-1})\right)$

(C) $\frac{2}{3}\left(T_m(f) + \frac{b-a}{m} \cdot \sum_{k=1}^{m/2} f(x_{2k-1})\right)$

(D) $\frac{T_m(f) + T_{m/2}(f)}{3}$

- ○ A
- ◉ B
- ○ C
- ○ D

Quiz Score: **7** out of 15