Homework 2

Let $A = [1..6]$ be a statespace, $S \subseteq A \times (\bar{A} \cup \{fail\})^{**}$ a program over the statespace A.

$$S = \begin{Bmatrix} 1 \to < 1, 2, 5, 1 > & 2 \to < 2, 4, 3, 5, 6 > & 2 \to < 2, 2, 2, \ldots > \\ 3 \to < 3, 1 > & 3 \to < 3, 2, 4 > & 3 \to < 3, 5, 2, 4, 1 > \\ 4 \to < 4, 1, fail > & 5 \to < 5, 3, 2, 4 > & 5 \to < 5, 3, 6, 1 > \\ 6 \to < 6, 1, 4 > & 6 \to < 6, 1, fail > & \end{Bmatrix}$$

Let $F \subseteq A \times A$ denote the following problem: $F = \{ (1, 1), (3, 4), (3, 1), (5, 1), (5, 2), (5, 4) \}$

Question
Let S_1 and S_2 be programs, let F_1 and F_2 be problems over the same statespace.
Statement1: if S_1 solves F_1 and $F_1 \subseteq F_2$ then S_1 solves F_2.
Statement2: if S_1 solves F_1 and $F_2 \subseteq F_1$ then S_1 solves F_2.
Statement3: if S_1 solves F_1 and $S_1 \subseteq S_2$ then S_2 solves F_1.
Statement4: if S_1 solves F_1 and $S_2 \subseteq S_1$ then S_2 solves F_1.

Informally, for example, Statement4 says that if a program S_1 solves a given problem F_1 then a smaller program S_2 also solves the same problem F_1. Interestingly, only this fourth statement is true out of the four statements.

Your task is to find a counter-example for Statement3.
More precisely: consider the program S and problem F given above in this exercise. Find a program S_2 such that $S \subseteq S_2$ and S_2 does not solve F. Explain why your S_2 program does not solve F.

Recall
S program solves problem F if

1. $\mathcal{D}_F \subseteq \mathcal{D}_{P[S]}$
2. $\forall a \in \mathcal{D}_F \colon P[S](a) \subseteq F(a)$