Exam – 2021.06.16.

Due Jun 16, 2021 at 9:53am **Points** 15 **Questions** 15
Available until Jun 16, 2021 at 9:53am **Time Limit** 45 Minutes

Instructions

Only one question is visible at once, and after you have submitted your answer to a question, you cannot go back to change your choice. There is exactly 1 correct answer for each question. Every correct answer is worth 1 point.

The maximum is 15 points. Under 8 points, the exam is finished with a fail (1) grade. If one achives at least 8 points, then the grade satisfactory (2), from 12 points, the grade average (3) is offered and one may attend the second, oral part of the exam for better grades.

This quiz is no longer available as the course has been concluded.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	35 minutes	6 out of 15

Correct answers are hidden.

Score for this quiz: **6** out of 15
Submitted Jun 16, 2021 at 9:43am
This attempt took 35 minutes.

Incorrect

Question 1 0 / 1 pts

We are to determine the solution of the below system of linear equations using Gaussian elimination with partial pivoting. What does the algorithm do before the first step of the elimination?

$$\begin{pmatrix} 1 & -2 & 5 \\ 3 & 6 & -9 \\ 8 & 2 & 7 \end{pmatrix} \cdot x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

(A) Nothing, just carries on with the elimination.
(B) Switches columns 1 and 3.
(C) Switches columns 1 and 3, and rows 1 and 2.
(D) Switches rows 1 and 3.

- A
- B
- C
- D

Question 2

1 / 1 pts

According to the theorems about the error bounds of the basic arithmetic operations, which below statement is true about the two nearby numbers of the same sign a and b?

(A) δ_{a+b} can be orders of magnitude bigger than δ_a and δ_b
(B) δ_{a-b} can be orders of magnitude bigger than δ_a and δ_b
(C) Δ_{a+b} can be orders of magnitude bigger than Δ_a and Δ_b
(D) Δ_{a-b} can be orders of magnitude bigger than Δ_a and Δ_b

- ○ A
- ◉ B
- ○ C
- ○ D

Incorrect

Question 3

0 / 1 pts

Let A denote an invertible matrix and $\mathrm{cond}(A)$ its condition number in a natural norm! Which below statement is false?

(A) $\mathrm{cond}(A^{-1}) = \mathrm{cond}(A)$

(B) $\mathrm{cond}(A) \geq 1$

(C) Given a number $c \neq 0$ we have $\mathrm{cond}(cA) = \mathrm{cond}(A)$.

(D) If A is symmetric, then $\mathrm{cond}_\infty(A)$ is the fraction of the biggest and smallest eigenvalues (in absolute value) of A.

- A
- B
- C
- D

Question 4

1 / 1 pts

What is not true about the problem of least squares?

(A) If the base points are different, then the solution is unique.

(B) The solution minimizes the sum of the Euclidean distances of the given points from the graph of the found polynomial.

(C) The problem can be solved using the Gaussian normal equations.

(D) The degree of the least squares fitted polynomial is less than the number of data points.

- ○ A
- ◉ B
- ○ C
- ○ D

Incorrect

Question 5

0 / 1 pts

Which below statement is true concerning the LU decomposition of the matrix A?

(A) If $\det(A) \neq 0$ and the LU decomposition of A exists, then it is unique.

(B) If $\det(A) = 0$, then the LU decomposition of A does not exist.

(C) If $\det(A) \neq 0$, then the LU decomposition of A exists.

(D) If $\det(A) \neq 0$, then the LU decomposition of A exists and it is unique.

- A
- B
- C
- D

Question 6

1 / 1 pts

Which one is the most accurate estimate we can state about the location of the roots of the polynomial $2x^3 + 3x^2 - 4x + 1$ based on the studied methods?

(A) $1.33 \leq |x_k| \leq 4$
(B) $0.50 \leq |x_k| \leq 3$
(C) $0.20 \leq |x_k| \leq 3$
(D) $0.33 \leq |x_k| \leq 5$

- A
- B
- C
- D

Question 7

1 / 1 pts

Assume that a number $x \in \mathbb{R}_M$ converted to the set of machine numbers $M(8, -16, 16)$ has the exponent 10. Which one is the sharpest correct error bound for $fl(x)$?

(A) 0.5
(B) 1
(C) 2
(D) 4

- A
- B
- C
- D

Incorrect

Question 8

0 / 1 pts

We are using Newton's method to solve the equation $f(x) := e^x - \pi x = 0$. For which starting point(s) do we have monotone convergence?

(A) $x_0 = 0$ ✓
(B) $x_0 = 1$
(C) both above points
(D) none of the above points

- [x] A
- [ ] B
- [ ] C
- [ ] D

Question 9

1 / 1 pts

Which formula is incorrect about the Chebyshev polynomials?

(A) $T_3(x) = 4x^3 - 3x$
(B) $4 \cdot t_2(x) = 4x^2 - 2$
(C) $4 \cdot t_4(x) = 4x^4 - 4x^2 + 1$
(D) $2 \cdot t_2(x) = T_2(x)$

- A
- B
- C
- D

Incorrect

Question 10

0 / 1 pts

The following Horner's scheme is given for the polynomial $P(x)$

1	−4	−7	11
−2	−2	12	−10
1	−6	5	1
−2	−2	16	
1	−8	21	
−2	−2		
1	−10		

Which of the following statements is true for $P(x)$?

(A) $P(x) = (x-2)(x^2-6x+5)+1$
(B) $P(x) = (x+2)(x^2-6x+5)$
(C) $P'(-2)\cdot P''(-2) = -210$
(D) $P'(-2)\cdot P''(-2) = -420$ ✓

- A
- B
- C
- D

Question 11

1 / 1 pts

Consider the system of linear equations $Ax = b$ with the orthogonal matrix A, and consider also the perturbed problem $A(x + \Delta x) = b + \Delta b$. Which below statement is true based on the relevant theorems?

(A) $\frac{\|\Delta x\|_2}{\|x\|_2} = \frac{\|\Delta b\|_2}{\|b\|_2}$

(B) $\frac{\|\Delta x\|_2}{\|x\|_2} > \frac{\|\Delta b\|_2}{\|b\|_2}$

(C) $\frac{\|\Delta x\|_2}{\|x\|_2} < \frac{\|\Delta b\|_2}{\|b\|_2}$

(D) $\|\Delta x\|_2 \leq \mathrm{cond}_2(A) \cdot \|\Delta b\|_2$

- (●) A
- ( ) B
- ( ) C
- ( ) D

Incorrect

Question 12

0 / 1 pts

Decide whether these functions are contractions on the interval $[1,2]$ or not!

$$\varphi_1(x) = \sqrt{x+1}, \qquad \varphi_2(x) = \frac{x+1}{3}.$$

(A) only φ_1 is
(B) only φ_2 is
(C) both of them are ✓
(D) none of them is

- A
- B
- C
- D

Incorrect

Question 13

0 / 1 pts

Let $L_n(x)$ be the interpolating polynomial of the function $f(x) = x^{n+1}$ with respect to the distinct support points $x_0, x_1, \ldots, x_n$. Choose the correct formula about the error of the interpolation!

(A) $f(x) - L_n(x) = \dfrac{1}{(n+1)!} \cdot \omega_n(x)$ ✓

(B) $f(x) - L_n(x) = \omega_n(x)$

(C) $f(x) - L_n(x) = \dfrac{x}{(n+1)!} \cdot \omega_n(x)$

(D) $f(x) - L_n(x) = \dfrac{1}{n!} \cdot \omega_n(x)$

- (•) A
- ( ) B
- (•) C
- ( ) D

Incorrect

Question 14

0 / 1 pts

$t=3$ $k^+=4$

Assume that in case of a set of machine numbers $M(t, k^-, k^+)$ we have $|M| = 57$, $M_\infty = 14$ and $\varepsilon_1 = \frac{1}{4}$.
Which below statement is true?

(A) $k^- = -2$ ✓
(B) $k^- = -3$
(C) the value of k^- can not be determined by these conditions
(D) no such set of machine numbers exists

- [x] A
- [ ] B
- [ ] C
- [ ] D

Incorrect

Question 15

0 / 1 pts

Assume that we are using the Newton–Cotes quadrature formulas $M(f), T(f), S(f)$ to estimate the integral of the polynomial $f(x) := \alpha x^3 + \beta x$ $(\alpha, \beta \in \mathbb{R})$ on the interval $[0, 2]$.
Which below statement is incorrect?

(A) $M(f) = 2(\alpha + \beta)$
(B) $\int_0^2 f(x)\, dx - S(f) = 0$
(C) $\forall x \in [0, 2] \ : \ |f''(x)| \leq 12\,|\alpha|$
(D) $\left|\int_0^2 f(x)\, dx - T(f)\right| \leq 6\,|\alpha|$ ✓

- A
- B
- C
- D

Quiz Score: **6** out of 15