Retake Exam – 2021.07.01.

Due Jul 1 at 1:53pm **Points** 15 **Questions** 15
Available Jul 1 at 1pm - Jul 1 at 1:53pm about 1 hour **Time Limit** 45 Minutes

Instructions

Only one question is visible at once, and after you have submitted your answer to a question, you cannot go back to change your choice. There is exactly 1 correct answer for each question. Every correct answer is worth 1 point.

The maximum is 15 points. Under 8 points, the exam is finished with a fail (1) grade. If one achives at least 8 points, then the grade satisfactory (2), from 12 points, the grade average (3) is offered and one may attend the second, oral part of the exam for better grades.

Attempt History

	Attempt	Time	Score
LATEST	Attempt 1	25 minutes	12 out of 15

ⓘ Correct answers are hidden.

Score for this quiz: **12** out of 15
Submitted Jul 1 at 1:32pm
This attempt took 25 minutes.

Question 1

1 / 1 pts

Which one of the below numbers is not in the set of binary machine numbers $M(3,-3,4)$?

(A) 0.2
(B) 0
(C) 1
(D) 8

- A
- B
- C
- D

Incorrect

Question 2

0 / 1 pts

We are to solve the system of linear equations $Ax = b$ using Gaussian elimination. Which one of the below statements is true?

(A) If $\det(A) = 0$, then the Gaussian elimination can not be completed without switching rows or columns.

(B) If $\det(A) = 0$, then the system might not have a solution.

(C) If $\det(A) \neq 0$, then the Gaussian elimination can be completed without switching rows or columns.

(D) If $\det(A) \neq 0$, then the system might have two distinct solutions.

- A
- B
- C
- D

Question 3

1 / 1 pts

Let A denote an invertible matrix and $\mathrm{cond}(A)$ its condition number in a natural norm! Which below statement is false?

(A) $\mathrm{cond}(A^{-1}) = \big(\mathrm{cond}(A)\big)^{-1}$
(B) $\mathrm{cond}(A) \geq 1$
(C) Given a number $c \neq 0$ we have $\mathrm{cond}(cA) = \mathrm{cond}(A)$.
(D) If A is symmetric, then $\mathrm{cond}_2(A)$ is the fraction of the biggest and smallest eigenvalues (in absolute value) of A.

- (●) A
- ( ) B
- ( ) C
- ( ) D

Question 4

1 / 1 pts

We encounter $\omega_3(x) = (x+1)(x-1)(x-2)(x-3)$ in a polynomial interpolation problem. Which one is the Lagrange base polynomial $\ell_2(x)$?

(A) $\dfrac{(x+1)(x-1)(x-3)}{-2}$
(B) $\dfrac{(x+1)(x-1)(x-3)}{2}$
(C) $\dfrac{(x+1)(x-1)(x-3)}{-3}$
(D) $\dfrac{(x+1)(x-1)(x-3)}{3}$

- A
- B
- C
- D

Question 5

1 / 1 pts

What is the condition number of the matrix

$$A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with respect to the Euclidean norm?

(A) $\text{cond}_2(A) = 1$
(B) $\text{cond}_2(A) = 2$
(C) $\text{cond}_2(A) = 4$
(D) The condition number is not defined for this matrix.

- A
- B
- C
- D

Question 6

1 / 1 pts

Which one is the most accurate estimate we can state about the location of the roots of the polynomial $2x^3 + 3x^2 - 4x + 4$ based on the studied methods?

(A) $1.33 \leq |x_k| \leq 4$
(B) $0.50 \leq |x_k| \leq 3$
(C) $0.20 \leq |x_k| \leq 3$
(D) $0.33 \leq |x_k| \leq 5$

- ○ A
- ◉ B
- ○ C
- ○ D

Incorrect

Question 7

0 / 1 pts

Which formula is incorrect to calculate the Frobenius norm of the matrix $A \in \mathbb{R}^{n \times n}$? Or all three of them are correct?

(A) $\|A\|_F = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} |a_{ij}|^2}$
(B) $\|A\|_F = \mathrm{tr}(A^T A)$
(C) $\|A\|_F = \sqrt{\sum_{i=1}^{n} \lambda_i (A^T A)}$
(D) all above three are correct

- A
- B
- C
- D

Incorrect

Question 8

0 / 1 pts

Let $a, b \in \mathbb{R}$ arbitrary and

$$c := a - b, \qquad d := a + b.$$

What can we say about the relative error bounds for c and d?

(A) A smaller relative error bound can be given for c, than for d.
(B) A smaller relative error bound can be given for d, than for c.
(C) It depends on the signs of a and b.
(D) Independent of a and b, the relative error bounds for c and d are approximately the same.

- A
- B
- C
- D

Question 9

1 / 1 pts

Which formula is incorrect about the Chebyshev polynomials?

(A) $T_3(x) = 4x^3 - 3x$
(B) $2 \cdot t_2(x) = 2x^2 - 1$
(C) $8 \cdot t_4(x) = 8x^4 - 8x^2 + 1$
(D) $4 \cdot T_2(x) = t_2(x)$

- A
- B
- C
- D

Question 10

1 / 1 pts

Let us consider the interpolatory quadrature formula $\sum_{k=0}^{n} A_k f(x_k)$ to estimate the integral $\int_a^b f(x)dx$? Which below statement holds?

(A) $\sum_{k=0}^{n} A_k = 1$
(B) $\sum_{k=0}^{n} A_k = b - a$
(C) $\sum_{k=0}^{n} A_k x_k = b - a$
(D) $\sum_{k=0}^{n} A_k x_k = b^2 - a^2$

- A
- B
- C
- D

Question 11

1 / 1 pts

In the proof of the monotone convergence theorem of Newton's method, we approximate the function f with its first degree Taylor polynomial and evaluate it at some point. But what is the center of it and where do we evaluate?

(A) It is centered at x_k and evaluated at x^*.
(B) It is centered at x_k and evaluated at x_{k+1}.
(C) It is centered at x^* and evaluated at x^k.
(D) It is centered at x^* and evaluated at x_{k+1}.

- ○ A
- ◉ B
- ○ C
- ○ D

Question 12

1 / 1 pts

Decide whether these functions are contractions on the interval $[1, 2]$ or not!

$$\varphi_1(x) = \sqrt{x+1}, \qquad \varphi_2(x) = \frac{x+1}{3}.$$

(A) only φ_1 is
(B) only φ_2 is
(C) both of them are
(D) none of them is

- A
- B
- C
- D

Question 13

1 / 1 pts

Does such a vector x always (in case of every square matrix A) exist that maximizes the fraction $\frac{\|Ax\|_1}{\|x\|_1}$?

(A) Yes, an eigenvector of the eigenvalue $\lambda_{\max}(A)$ always does.
(B) Yes, a canonical unit vector always does.
(C) Yes, there is always one among the vectors with $x_i = \pm 1$ $(i = 1, \ldots, n)$.
(D) No, the maximum does not always exist, only the existence of the supremum is guaranteed.

- A
- B
- C
- D

Question 14

1 / 1 pts

What form is appropriate for the polynomial $L_k(x) - L_{k-1}(x)$ with the Lagrange interpolation polynomials of the degree indicated in their indices, and the set of their support points x_0, x_1 etc. differing in only one point?

(A) $L_k(x) - L_{k-1}(x) = c_k(x - x_0)(x - x_1)\cdots(x - x_k)$
(B) $L_k(x) - L_{k-1}(x) = (x - x_0)(x - x_1)\cdots(x - x_{k-1})$
(C) $L_k(x) - L_{k-1}(x) = c_k(x - x_0)(x - x_1)\cdots(x - x_{k-1})$
(D) $L_k(x) - L_{k-1}(x) = (x - x_0)(x - x_1)\cdots(x - x_k)$

- ○ A
- ○ B
- ◉ C
- ○ D

Question 15

1 / 1 pts

Assume that we are using the Newton–Cotes quadrature formulas $M(f), T(f), S(f)$ to estimate the integral of the polynomial $f(x) := \alpha x^3 + \beta x$ $(\alpha, \beta \in \mathbb{R})$ on the interval $[0, 2]$.
Which below statement is incorrect?

(A) $M(f) = \alpha + \beta$
(B) $\int_0^2 f(x)\, dx - S(f) = 0$
(C) $\forall x \in [0, 2] \ : \ |f''(x)| \leq 12\,|\alpha|$
(D) $\left|\int_0^2 f(x)\, dx - T(f)\right| \leq 8\,|\alpha|$

- (•) A
- ( ) B
- ( ) C
- ( ) D

Quiz Score: **12** out of 15