

Analysis 2, Test 1, 24th of March 2021

1. (8 points) Find $a, b \in \mathbb{R}$ so that f to be differentiable at every real point :

$$f(x) = \begin{cases} 2 \cdot \ln(1-x) - \frac{b}{x-1} & \text{if } x \in (-\infty; 0) \\ \sqrt{2x+1} - a & \text{if } x \in [0; +\infty) \end{cases}$$

2.

- a) (4 points) Determine the tangent line to the graph of the following function f at point $a = 1$

$$f(x) := \sqrt{x^2 + 1} \quad (x \in \mathbb{R})$$

- b) (5 points) Evaluate the T_2 Taylor polynomial for f centered at $a = 1$.

3. (6 points) Evaluate the following limit :

$$\lim_{x \rightarrow 0} \frac{xe^x - \sin x}{1 - \cos^3 x}$$

4. (7 points) Find the absolute minimum and maximum places and values of f on the set $[0; 4]$, if

$$f(x) = x^3 - \frac{7}{2}x^2 + 2x + 3 \quad (x \in [0; +\infty))$$

5. (10 points) After the complete discussion sketch the graph of the function

$$f(x) = \frac{x-1}{e^x} \quad (x \in \mathbb{R})$$