Analysis II., Test 2, 13th of May 2021.

1. *(5+4+4 points)* Evaluate the following integrals:

(a) $\int \frac{(x-1)^2}{x^2+4}\,dx \;\; (x \in \mathbb{R})$;

(b) $\int \frac{\sin x}{\cos^4 x}\,dx \;\; (x \in (-\pi/2; +\pi/2))$;

(c) $\int_1^2 \frac{1}{\sqrt{2-x}}\,dx$.

2. *(6 points)* Determine the area of the bounded region enclosed by the following curves:

$$y = x, \;\; y = \frac{x}{9}, \;\; y = \frac{1}{x}, \;\; \text{and} \;\; y = \frac{9}{x}.$$

3. *(7 points)* Find the volume of the rotation solid obtained by rotating the graph of the following function around axes x:

$$f(x) = \sqrt{\frac{3x-5}{x^2-3x+2}} \;\; (x \in [3;4]).$$

4. *(6 points)* Evalute the integral of $f(x,y) := x \;\; ((x,y) \in \mathbb{R}^2)$ over the given bounded domain D enclosed by the following curves:

$$y = 0, \;\; x = 2, \;\; y = \ln x,$$

in the first quadrant of the plane.

5. *(8 points)* Find the local extrema (place and value) for the following function:

$$f(x,y) := \frac{x^3}{3} + \frac{y^3}{3} - x^2 + y^2 \;\; ((x,y) \in \mathbb{R}^2).$$