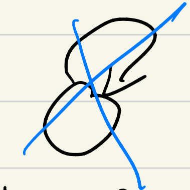
Graph representations

① Mathematical representation

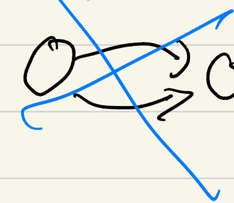
$|V| < \infty$

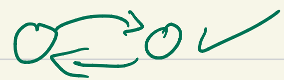


$G = (V, E)$ (V: vertices, E: edges)

$E \subseteq V \times V \setminus \{(u,u) \mid u \in V\}$ (no looping edges)

Directed: $(u,v) \neq (v,u)$ ($\left\{\begin{matrix}(u,v)\\(v,u)\end{matrix}\right\} \in E$)

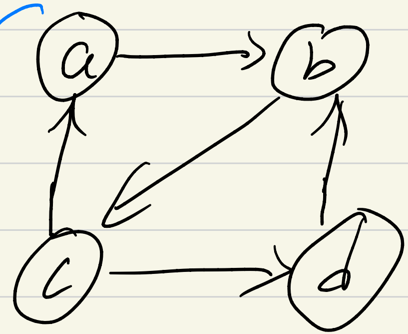




Undirected: $(u,v) = (v,u)$

② Graphical representation

($a = 1$, $b = 2$,
$c = 3$, $d = 4$, ...)



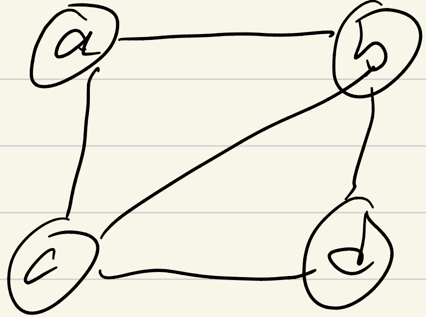
DIRECTED

③ Textual repr.

a -> b.
b -> c.
c -> a; d.
d -> b.

undirected equivalent of the directed graph above

UNDIRECTED



a — b; c.
b — c; d.
c — d.
d.

5 Adjacency matrix repr. $V=\{v_1,\dots,v_n\}$

$A/1 : \mathbb{B}[4,4]$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 0 & 1 & 0 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 1 & 0 & 0 \end{array}$$

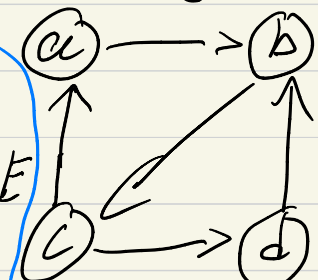
General case

$A/1 : \mathbb{B}[n,n]$

$n = |V|$

General case

$$A[i,j] = \begin{cases} 1 & \text{if } (v_i,v_j)\in E \\ 0 & \text{if } (v_i,v_j)\notin E \end{cases}$$



$n=4$ here

n^2 bits are needed

n^2 bits ~

Undirected case

$A/1 : \mathbb{B}[n,n]$

↓ represended by

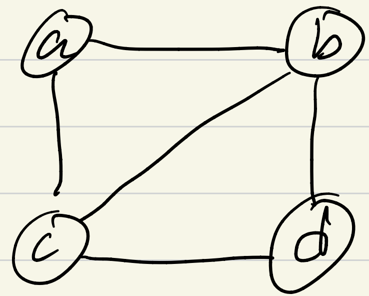
$B : \mathbb{B}[n*(n-1)/2]$ — $\Theta(n^2)$ — $\underbrace{\left(\frac{n^2}{2} - \frac{n}{2}\right)}$ bits are needed

Adj. mtx.

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 1 & 0 \\ b & 1 & 0 & 1 & 1 \\ c & 1 & 1 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

~ here $n = 4$

Graphical repn.



$$\begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{21} & 0 & a_{23} & a_{24} \\ a_{31} & a_{32} & 0 & a_{34} \\ a_{41} & a_{42} & a_{43} & 0 \end{bmatrix}$$

$a_{12} \doteq a_{21}$

$\vdots$

$a_{34} \doteq a_{43}$

$\boxed{a_{ij} = a_{ji}}$

$B = \langle a_{21}, a_{31}, a_{32}, a_{41}, a_{42}, a_{43}, \ldots a_{n1}, a_{n2}, \ldots, a_{n(n-1)} \rangle$

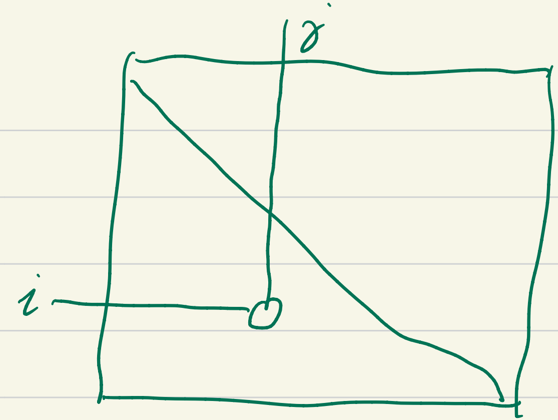
$A[i,j] = B[(i-1)*(i-2)/2 + (j-1)]$ if $i > j$

$A[i,j] = A[j,i]$ if $i < j$

$A[i,i] = 0$

$i > j$

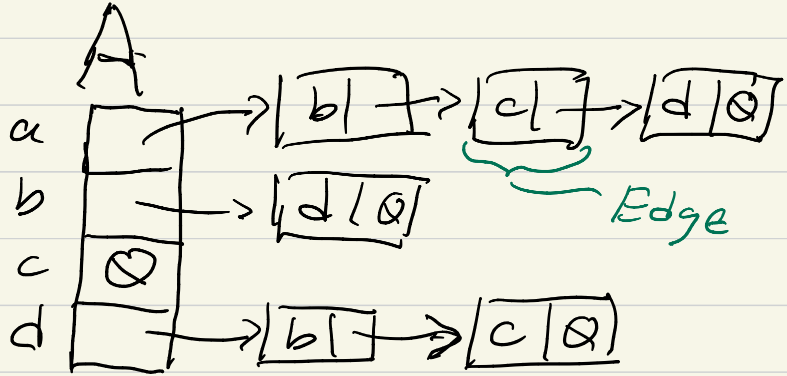
$$A[i,j] = B\left[\overbrace{1+2+\dots+(i-2)}^{\frac{(i-1)(i-2)}{2}} + (j-1)\right]$$

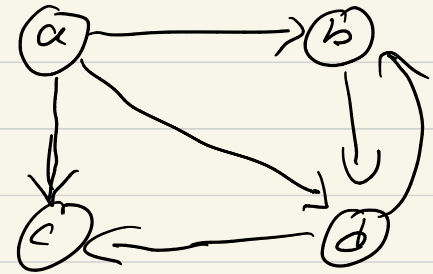


$$+\begin{cases} 1: & a_{21} \\ 2: & a_{31} \quad a_{32} \\ \vdots & \vdots \\ i-2: & a_{(i-1)1} \quad a_{(i-1)2} \cdots a_{(i-1)(i-2)} \\ j: & a_{i1} \quad a_{i2} \dots a_{i(j-1)} \quad a_{ij} \end{cases}$$

%

⑤ Adjacency list representation (most often used)





$A/1 : Edge*[n]$ ($n=4$ in the example above)

Edge
$+v : \mathbb{N}$
$+next : Edge*$

$G=(V,E)$

$n=|V|$

$m=|E|$

Sparse graph:

$m \in O(n)$ Mem: $\Theta(n+m) \prec \Theta(n^2)$ in sparse graphs

($m \leq 4*n$ in most practical applications)

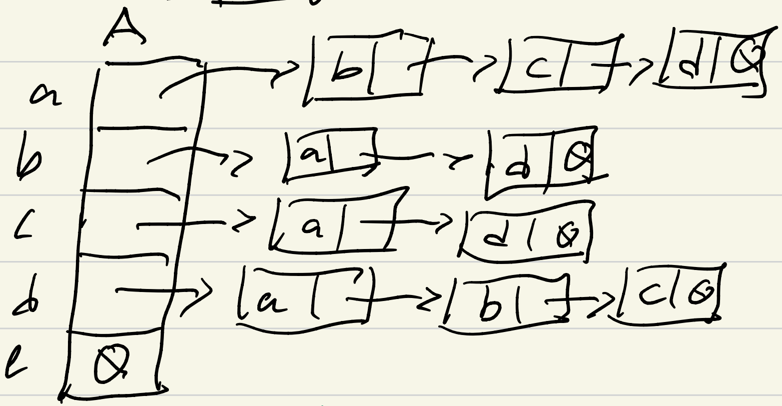
Adj. list is preferred

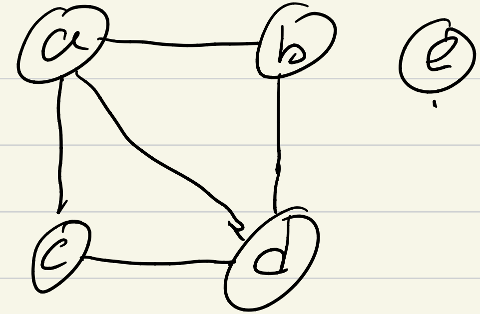
Dense graphs:

$m \in \Theta(n^2)$

Adj. mtx is preferred

Undirected graphs

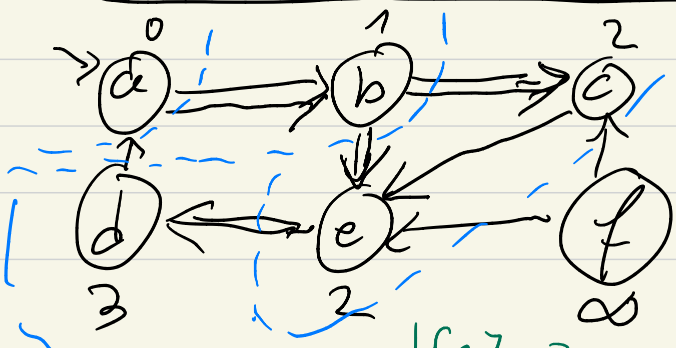




Mem: $\Theta(n+m)$

Each edge is represented twice

Breadth-first search: to find the shortest paths from a source vertex.



d[e]=2

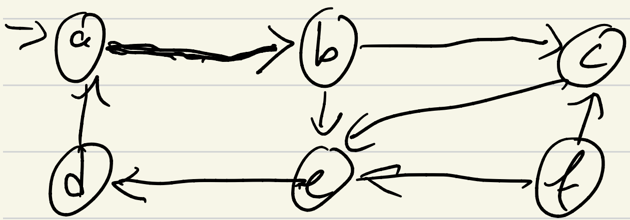
We find the shortest paths tree.

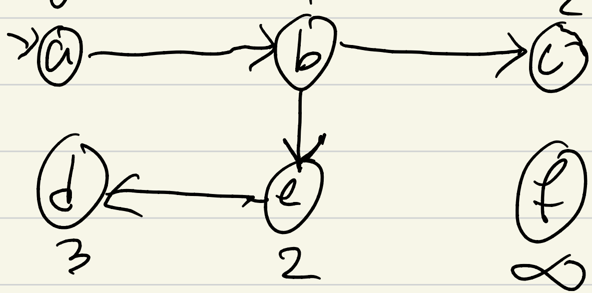
It contains all the vertices available from the source vertex.

$\pi[1:N[n]]$ $\pi[i]$ is the parent of vertex i in the shortest paths tree

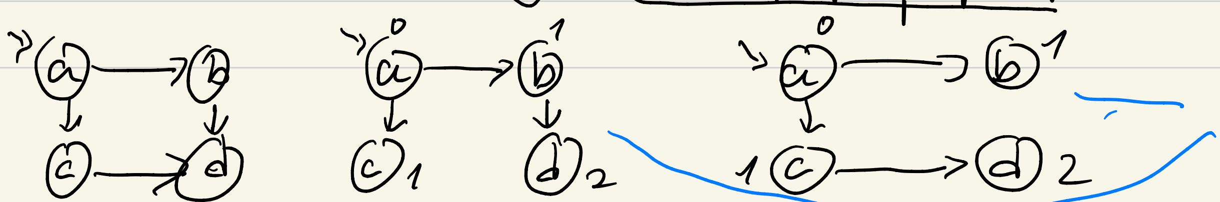
$\pi[s]=0$ for source s.

$\pi[v]=0$ for unavailable vertex v, too.





d: a	b	c	d	e	f	expanded vertex	Q: Queue	π: a	b	c	d	e	f
0	∞	∞	∞	∞	∞	—	⟨a⟩	0	0	0	0	0	0
	1					a:0	⟨b⟩		a				
		2		2		b:1	⟨c, e⟩			b		b	
						c:2	⟨e⟩						
			3			e:2	⟨d⟩				e		
						d:3	⟨ ⟩						
0	1	2	3	2	∞			0	a	b	e	b	0



shortest paths trees.

$BFS(G : \mathcal{G} ; s : \mathcal{V})$

$G = (V, E)$ $\quad \mathcal{G} = \text{graph}$

$\mathcal{V} = \text{vertex}$

s is the source vertex

$\forall v \in G.V$

- $d(v) := \infty$; $\pi(v) := \emptyset$ $\quad \} \; \Theta(n)$

$d(s) := 0$; $Q : Queue$

$Q.add(s)$ $\quad \} \; \Theta(1)$

$\neg Q.isEmpty()$ $\quad \} \; O(n)$

- $u := Q.rem()$ $\quad \} \; O(n)$
- $\forall v : (u, v) \in G.E$ $\quad \} \; O(m)$
  - $d(v) = \infty$
    - $d(v) := d(u) + 1$
    - $\pi(v) := u$
    - $Q.add(v)$
  - X

runtime: $O(n+m)$

$MT(n,m) \in \Theta(n+m)$

(when all the vertices are available from s)