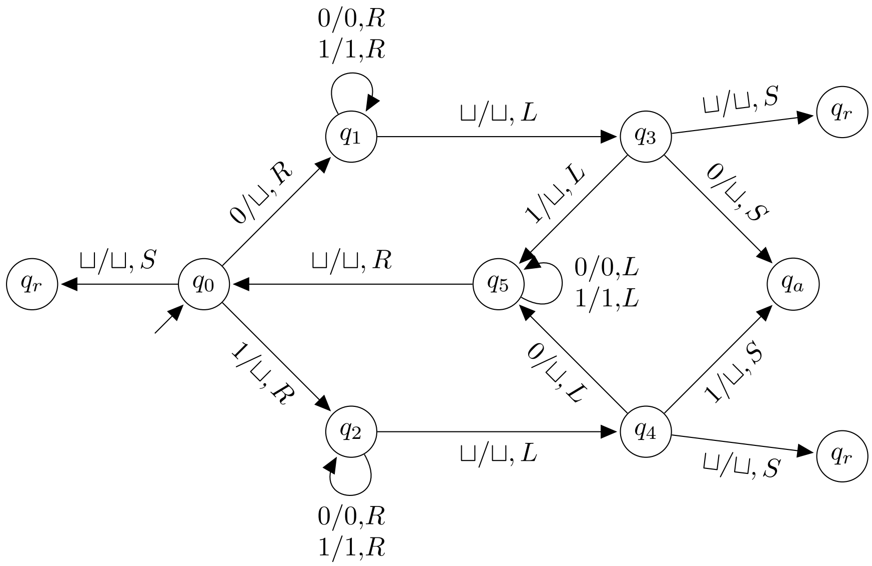
Fundamentals of theory of computation 2 – 2nd test

1. Transitions of a deterministic Turing machine $M = \langle \{q_0, q_1, q_2, q_3, q_4, q_5, q_a, q_r\}, \{0,1\}, \{0,1,\sqcup\}, \delta, q_0, q_i, q_n \rangle$ is given by the following transition diagram. (Of course, there is a single q_r, figure contains 3 of them due to readability reasons.)



   (a) Give the sequence of configurations for input 011 till a halting configuration.
   (b) Determine $L(M)$, the language recognized by M. Support your answer by an argument.
   (c) Give an asymptotically sharp asymptotic upper bound for the time complexity of M. (8 points)

2. (a) Give a Turing machine computing the function $f(b^n) = (ab)^{\lfloor n/2 \rfloor}$ $(n \in \mathbb{N})$
   (Examples: $f(bbb) = ab$, $f(bbbb) = abab$, $f(bbbbb) = abab$.)
   (b) Give an asymptotically sharp asymptotic upper bound for the time complexity of your machine. (7 points)

3. (a) Give a Turing machine recognizing $L = \left\{ x_1 a x_2 y_1 a y_2 \;\middle|\; x_1, x_2, y_1, y_2 \in \{a,b\}^*, |x_1 a x_2| = |y_1 a y_2| \right\}$. ($L$ consists of words of even length, having at least one a in both halves of the word.)
   (b) Give an asymptotically sharp asymptotic upper bound for the time complexity of your machine. (8 points)

4. Is there an instance D of Post Correspondence Problem satisfying all of the following properies?
   (i) $|D| \geq 3$,
   (i) the alphabet of the dominos is $\{0,1\}$,
   (ii) D has a solution,
   (iv) there are dominos d and d' of D, such that if we exchange the top words on d and d' and leave other dominos unchanged, then the new set of dominos D' has no solution.
   Support your answer by an argument. (6 points)

Choose one from the following 2 exercises. You do not have to solve the other one. Points will be given only for the chosen one.

5A. Prove that $L_{\text{EQComp}} = \{\langle M_1, M_2 \rangle \mid L(M_1) = \overline{L(M_2)}\} \notin R$, where M_1, M_2 are Turing machines of input alphabet $\{0,1\}$ and $\langle M_1, M_2 \rangle$ is a code of a 2-tuple (M_1, M_2) of TM's.

OR

5B. UHP $= \{\langle G, s, t \rangle \mid$ undirected graph G has a Hamiltonian path with endpoints s and $t\}$
UHP1END $= \{\langle G, t \rangle \mid$ undirected graph G has a Hamiltonian path with one of the endpoints $t\}$
Prove, that UHP $\leq_p$ UHP1END. (7 points)

Turing machines in exercies 2 and 3 can have multiple tapes, but they should be deterministic.

1)

a) 011

q_0 011 ; q_1 $\sqcup$11 ; q_1 11 ; q_1 11$\sqcup$

q_3 11 ; q_5 1 ; q_5 $\sqcup$1 ; q_0 1 ; q_2 $\sqcup$;

q_4 $\sqcup$; q_r

b) Languages of the form $V_1 1 w 1 V_2$ or $V_1 0 w 0 V_2$

where V_1 and V_2 have same length. $V_1, V_2, w \in \{0,1\}^*$

Machine is checking for same endpoints on each iteration.

if endpoints are same we accept, otherwise delete them

and start with the new endpoints.

c) 000000111111

c) assume we have length K on some iteration.

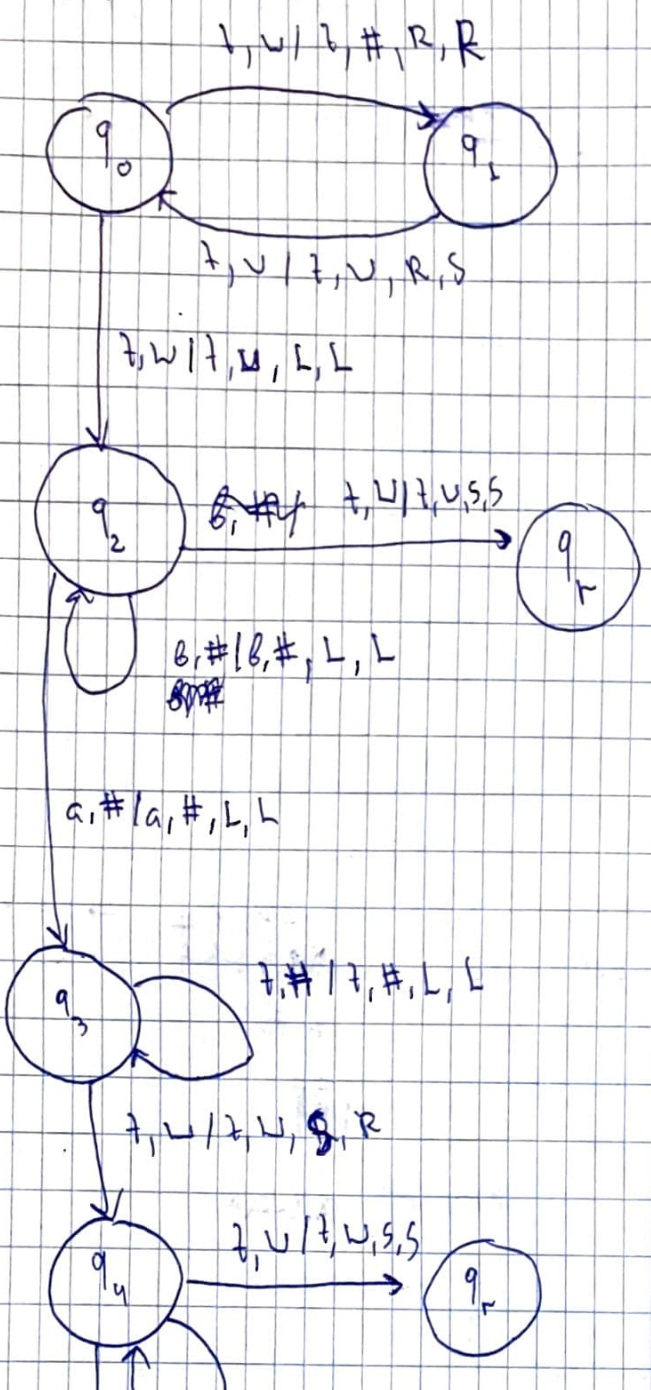
we need $\Theta(K)$ steps to reach the end and come back. (worst case)

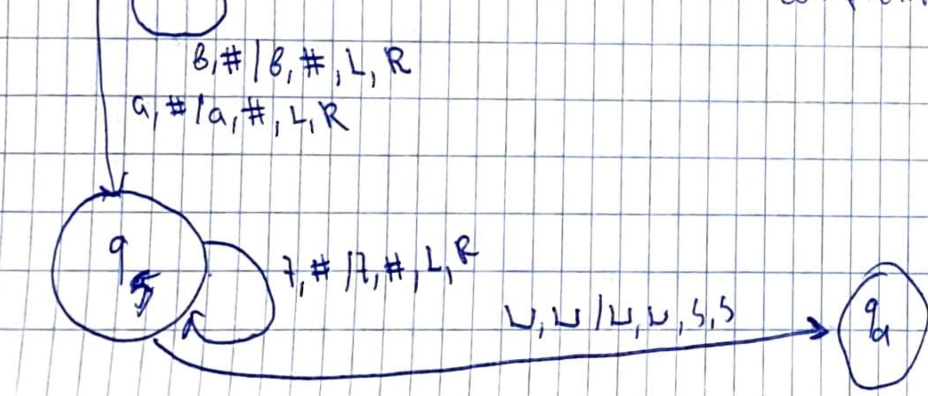
$$\sum_{K=0}^{n} \Theta(K) = \Theta\left(\frac{n(n+1)}{2}\right) = \Theta(n^2)$$

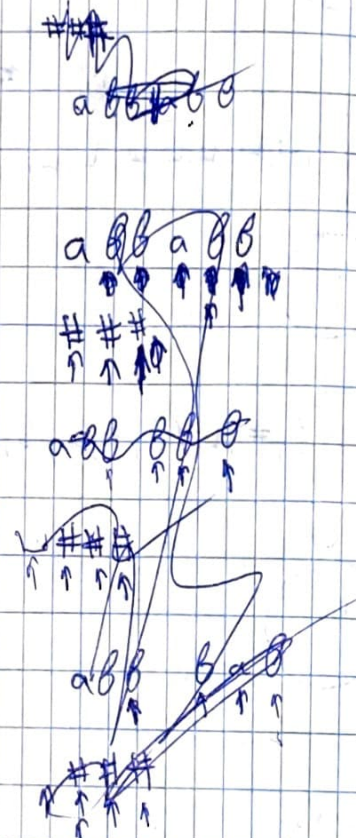
3) $L = \{ x_1 a x_2 y_1 a y_2 \mid$

even length, having at least one a in both halves.

- $z \in \{a, b\}$







Time Complexity: $\Theta(n)$

we make #-s in second tape in $\Theta(n)$
after that we just go through the input once and thrugh #-s twice.
so complexity is $\Theta(n)$

3) 4) $|D| \geq 3$

$\{0,1\}$.

d d' exchang tops no solution.

example:

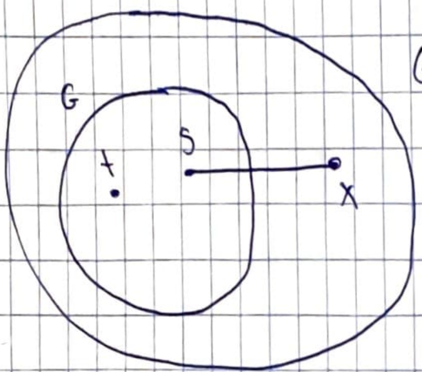
$$D = \left\{ \frac{0}{0}, \frac{1}{1}, \frac{0}{11} \right\} \rightarrow \frac{1}{0} \quad \frac{0}{1} \quad \frac{0}{11}$$

d_1 d_2 d_3

- D has solution (d_1 or d_2)
- exchange top words of d_1 and d_2 we get
  $\frac{1}{0}$ $\frac{0}{1}$ $\frac{0}{11}$
- and this has no solution, since none of dominos can be start of a solution.

5) G has an directed H path between s, t iff

G' has an directed H-path with endpoint t.



add x vertex and connect it to just s.

- It is polynomial time construction.
- "$\Rightarrow$" if we have path $t\, v_1 v_2 \dots v_k\, s$ in G

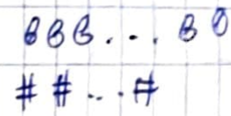
then $t\, v_1 v_2 \dots v_k\, s\, x$ is in G'.

- "$\Leftarrow$" if we have H-path with one endpoint t in G'

then second endpoint must be x. since it has degree 1, it can't be in the middle of a path.

so we have $t\, v_1 v_2 \dots v_k\, s\, x$ in G'. so we have $t\, v_1 v_2 \dots v_k\, s$ in G.

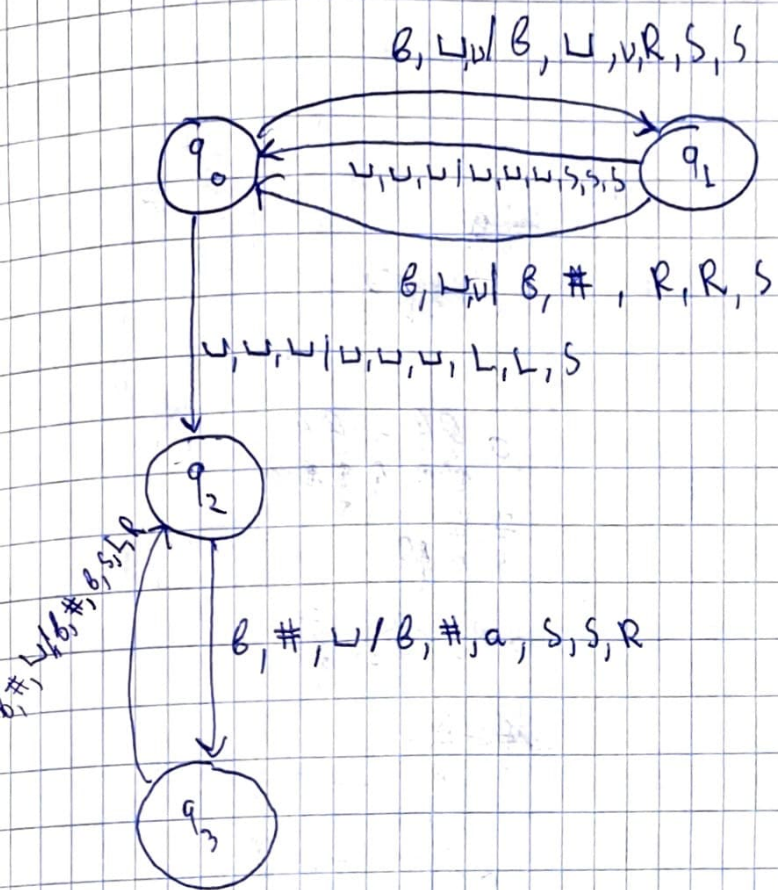
2) $f(b^n) = (ab)^{\lfloor n/2 \rfloor}$



tape 1: input

tape 2: length/2

tape 3: output.



Complexity:

we need $\Theta(n)$ to calculate length/2 and write it on second tape.

and $\Theta(n)$ to write (ab) - pairs.

so total is $\Theta(n)$